

Ray or Geometrical Optics by Prof. Subhashish Chattopadhyay SKMClasses Bangalore Useful for IIT-JEE, CBSE, I.Sc. PU-II, Boards, CET, CEE, PET, IGCSE IB AP-Physics and other exams



Spoon Feeding Ray or Geometrical Optics



Simplified Knowledge Management Classes Bangalore

My name is [Subhashish Chattopadhyay](#). I have been teaching for IIT-JEE, Various International Exams (such as IMO [International Mathematics Olympiad], IPhO [International Physics Olympiad], IChO [International Chemistry Olympiad]), IGCSE (IB), CBSE, I.Sc, Indian State Board exams such as WB-Board, Karnataka PU-II etc since 1989. As I write this book in 2016, it is my 27 th year of teaching. I was a Visiting Professor to BARC Mankhurd, Chembur, Mumbai, Homi Bhabha Centre for Science Education (HBCSE) Physics Olympics camp BARC Campus.

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I am Life Member of ...

- [IAPT \(Indian Association of Physics Teachers \)](#)
- [IPA \(Indian Physics Association \)](#)
- [AMTI \(Association of Mathematics Teachers of India \)](#)
- [National Human Rights Association](#)
- [Men's Rights Movement \(India and International \)](#)
- [MGTOV Movement \(India and International \)](#)

And also of

[IACT \(Indian Association of Chemistry Teachers \)](#)



The selection for National Camp (for Official Science Olympiads - Physics, Chemistry, Biology, Astronomy) happens in the following steps

1) NSEP (National Standard Exam in Physics) and NSEC (National Standard Exam in Chemistry) held around 24 rth November. Approx 35,000 students appear for these exams every year. The exam fees is Rs 100 each. Since 1998 the IIT JEE toppers have been topping these exams and they get to know their rank / performance ahead of others.

2) INPhO (Indian National Physics Olympiad) and INChO (Indian National Chemistry Olympiad). Around 300 students in each subject are allowed to take these exams. Students coming from outside cities are paid fair from the Govt of India.

3) The Top 35 students of each subject are invited at HBCSE (Homi Bhabha Center for Science Education) Mankhurd, near Chembur, BARC, Mumbai. After a 2-3 weeks camp the top 5 are selected to represent India. The flight tickets and many other expenses are taken care by Govt of India.

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Since last 50 years there has been no dearth of “Good Books“. Those who are interested in studies have been always doing well. This e-Book does not intend to replace any standard text book. These topics are very old and already standardized.

There are 3 kinds of Text Books

- The thin Books - Good students who want more details are not happy with these. Average students who need more examples are not happy with these. Most students who want to “Cram” quickly and pass somehow find the thin books “good” as they have to read less !!

- The Thick Books - Most students do not like these, as they want to read as less as possible. Average students are “busy” with many other things and have no time to read all these.

- The Average sized Books - Good students do not get all details in any one book. Most bad students do not want to read books of “this much thickness“ also !!

We know there can be no shoe that’s fits in all.

Printed books are not e-Books! Can’t be downloaded and kept in hard-disc for reading “later”

So if you read this book later, you will get all kinds of examples in a single place. This becomes a very good “Reference Material”. I sincerely wish that all find this “very useful”.

Students who do not practice lots of problems, do not do well. The rules of “doing well” had never changed Will never change !

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After 2016 CBSE Mathematics exam, lots of students complained that the paper was tough!

The screenshot shows a news article on the IBNLive website. At the top, it says 'Updated 8:47 am Mar 22, 2016'. The website logo 'IBNLive' and 'CNN' are visible. There are language options for 'ENGLISH', 'HINDI', and 'MARATHI'. A navigation bar includes 'READ', 'WATCH', 'CRICKET', and 'TECH'. Below this, a secondary navigation bar lists 'LATEST', 'BUDGET 2016', 'POLITICS', 'INDIA', 'SPORTS', 'FOOTBALL', 'MOVIES', 'LIVE TV', 'BUZZ', and 'WC'. The article is under the 'INDIA' category. The main headline is 'CBSE assures remedial measures for tricky and tough Class XII Math paper'. It is dated 'Posted on: 12:17 PM IST Mar 17, 2016' and 'Updated on: 12:20 pm, Mar 17, 2016 IST'. The article text states: 'After several students claimed that the Central Board of Secondary Education (CBSE) Class XII board Mathematics examination paper was 'tricky' and tough, the board has issued a clarification on remedial measures which are likely to be taken before evaluation. The CBSE says that feedback received from various stakeholders like students, subject teachers and examiners will be put before the committee of subject experts.'

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On 21 st May 2016 the CBSE standard 12 result was declared. I loved the headline

INDIATODAY.IN NEW DELHI, MAY 21, 2016 | UPDATED 16:40 IST

CBSE Class 12 Results out: No leniency in Maths paper, high paper standard to be maintained in future

The CBSE Class 12 Mathematics board exam on March 14 reduced many students to tears as they found the paper quite lengthy and tough and many couldn't finish it on time. The results show an overall lowering of marks received in the Maths paper.



RELATED STORIES

- CBSE Board result 2016 declared! Thiruvananthapuram obtains the highest part percentage, check how your region scored
- Meet CBSE topper Sukriti Gupta: Check her percentage here!
- CBSE Class 12 Boards 2016: Results announced ahead of time!
- CBSE results declared at www.cbse.nic.in: Steps to check online
- Exclusive! CBSE declares Class 12 Results at www.cbseresults.nic.in and cbse.nic.in

The CBSE (Central Board of Secondary Education) Class 12 Board exam results have been announced today, i.e on May 21, around 10:30 am ahead of time. Students may check their scores at the official website, www.cbseresults.nic.in. (Read: **CBSE Class 12 Boards 2016: Results announced ahead of time! Check your score at cbseresults.nic.in**)

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In 2015 also the same complain was there by many students



The screenshot shows a news article on the Zee News website. The header includes the Zee News logo, language options (Hindi, Marathi, Bangla), and mobile app icons (Apple, Android, Facebook). The navigation bar lists categories like INDIA, STATES, WORLD, S ASIA, BIZ, SPORTS, CRICKET, SCI-TECH, SHOWBIZ, HEALTH, BLOG, and EXCLUSIVE. The article title is "CBSE Class 12 exam: Issue of tough maths paper raised in Parliament". The sub-headline reads: "A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue 'seriously'". The article is dated "Last Updated: Thursday, March 19, 2015 - 14:41". It has 2547 shares (Facebook, Twitter, G+), 16 comments, and 33 comments. A "Follow @ZeeNews" button is also visible. The main text of the article is partially visible, starting with "New Delhi: A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue 'seriously'".

So we see that by raising frivolous requests, even upto parliament, actually does not help. Many times requests from several quarters have been put to CBSE, or Parliament etc for easy Math Paper. These kinds of requests actually can-not be entertained, never will be.

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In March 2016, students of Karnataka PU-II also complained the same, regarding standard 12 (PU-II Mathematics Exam). Even though the Math Paper was identical to previous year, most students had not even solved the 2015 Question Paper.

Friday, March 25, 2016 - 13:28

The **NEWS** Minute



HOME NEWS ANDHRA KARNATAKA KERALA TAMIL NADU TELANGANA CULTURE MEDIA BLOG

Exams

Online petition for lenient evaluation of K'taka II PU math paper gets over 8000 supporters

The campaign, which was launched on Monday, has garnered over 8000 supporters

TNM Staff | Wednesday, March 16, 2016 - 09:32

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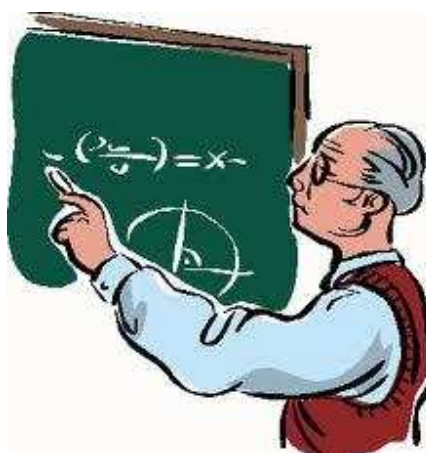
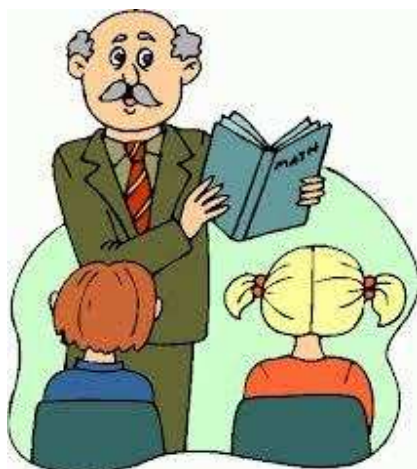
Reddit

Following a "very tough" math paper that left many II PU students in tears, Saket Ravindran a student launched an online campaign demanding lenient evaluation.

These complains are not new. In fact since last 40 years, (since my childhood), I always see this; every year the same setback, same complain!

In this e-Book I am trying to solve this problem. Those students who practice can learn.

No one can help those who are not studying, or practicing.



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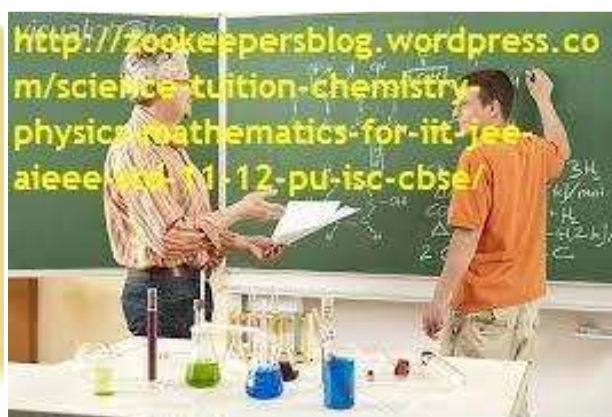
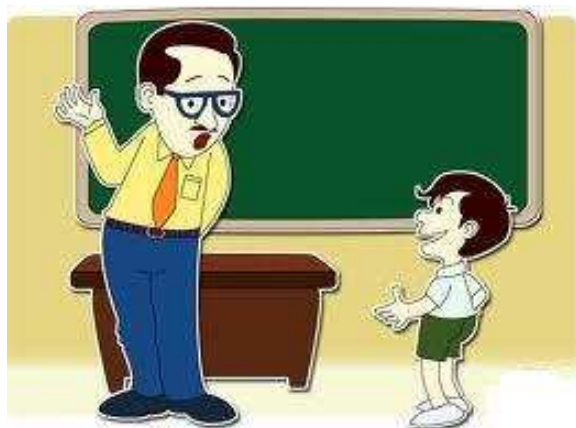
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Twitter - <https://twitter.com/ZookeeperPhy>

Facebook - <https://www.facebook.com/IIT.JEE.by.Prof.Subhashish/>

Blog - <http://skmclasses.kinja.com>



A very polite request :

I wish these e-Books are read only by Boys and Men. Girls and Women, better read something else; learn from somewhere else.

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Preface

We all know that in the species “Homo Sapiens “, males are bigger than females. The reasons are explained in standard 10, or 11 (high school) Biology texts. **This shapes or size, influences all of our culture.** Before we recall / understand the reasons once again, let us see some random examples of the influence

Random - 1

If there is a Road rage, then who all fight ? (generally ?). Imagine two cars driven by adult drivers. Each car has a woman of similar age as that of the Man. The cars “ touch “ or “ some issue happens”. Who all comes out and fights ? Who all are most probable to drive the cars ?



(Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win)

Random - 2

Heavy metal music artists are all Men. Metallica, Black Sabbath, Motley Crue, Megadeth, Motorhead, AC/DC, Deep Purple, Slayer, Guns & Roses, Led Zeppelin, Aerosmith [the list can be in thousands.](#) All these are grown-up Boys, known as Men.



(**Men strive for perfection. Men are eager to excel. Men work hard. Men want to win.**)



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Random - 3

Apart from Marie Curie, only one more woman got Nobel Prize in Physics. (Maria Goeppert Mayer - 1963). So, ... almost all are men.



(**Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.**)

Random - 4

The best Tabla Players are all Men.



(**Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.**)

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Random - 5

History is all about, [which all Kings ruled](#). Kings, their men, and Soldiers went for wars. [History is all about wars, fights, and killings by men](#). Who won, and who controlled !



Boys start fighting from school days. Girls do not fight like this



([Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.](#))

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Random - 6

The highest award in Mathematics, the “ Fields Medal “ is around since decades. Till date only one woman could get that. (Maryam Mirzakhani - 2014). So, ... almost all are men.



(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 7

Actor is a gender neutral word. Could the movie like “ Top Gun “ be made with Female actors ? The best pilots, astronauts, Fighters are all Men.



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Random - 8

In my childhood had seen a movie named “ The Tower in Inferno “. In the movie when the tall tower is in fire, women were being saved first, as only one lift was working....



Many decades later another movie is made. A box office hit. “ The Titanic “. In this also As the ship is sinking women are being saved. Men are disposable. Men may get their turn later... (never) !!



Movies are not training programs. Movies do not teach people what to do, or not to do. Movies only reflect the prevalent culture. Men are disposable; is the culture in the society. Knowingly, unknowingly, the culture is depicted in Movies, Theaters, Stories, Poems, Rituals, etc. I or you can't write a story, or make a movie in which after a minor car accident the Male passengers keep seating in the back seat, while the both the women drivers come out of the car and start fighting very bitterly on the road. There has been no story in this world, or no movie made, where after an accident or calamity, Men are being helped for safety first, and women are told to wait.

Random - 9

Artists generally follow the prevalent culture of the Society. In paintings, sculptures, stories, poems, movies, cartoon, Caricatures, knowingly / unknowingly, “ **the prevalent Reality** “ is depicted. **The opposite will not go well with people.** If deliberately “ the opposite “ is shown then it may only become a special art, considered as a special mockery.

पत्नी (सल्टू से): मुझे नई साड़ी ला दो प्लीज।
 सल्टू : पर तुम्हारी दो-दो अलमारियां साईडों से ही तो भरी है।
 पत्नी - वह सारी तो पूरे मोहल्ले वालों ने देख रखी है।
 सल्टू - तो साड़ी लेने के बजाए मोहल्ला बदल लेते हैं।



Random - 10

Men go to “girl / woman’s house” to marry / win, and bring her to his home. That is a sort of winning her. When a boy gets a “ Girl-Friend “, generally he and his friends consider that as an achievement. The boy who “ got / won “ a girl-friend feels proud. His male friends feel, jealous, competitive and envious. Millions of stories have been written on these themes. Lakhs of movies show this. Boys / Men go for “ bike race “, or say “ Car Race “, where the winner “ gets “ the most beautiful girl of the college.



(Men want to excel. Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.)

Prithviraj Chauhan ‘ went ` to “ pickup “ or “ abduct “ or “ win “ or “ bring “ his love. There was a Hindi movie (hit) song ... “ Pasand ho jaye, to ghar se utha laye “. It is not other way round. Girls do not go to Boy’s house or man’s house to marry. Nor the girls go in a gang to “ pick-up “ the boy / man and bring him to their home / place / den.

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Random - 11

We have the word "ice cold". While, when it snows heavily, the cleaning of the roads is done by Men. Ice avalanche is cleared by Guns, by Men.



Can women do this please ?



Random - 12



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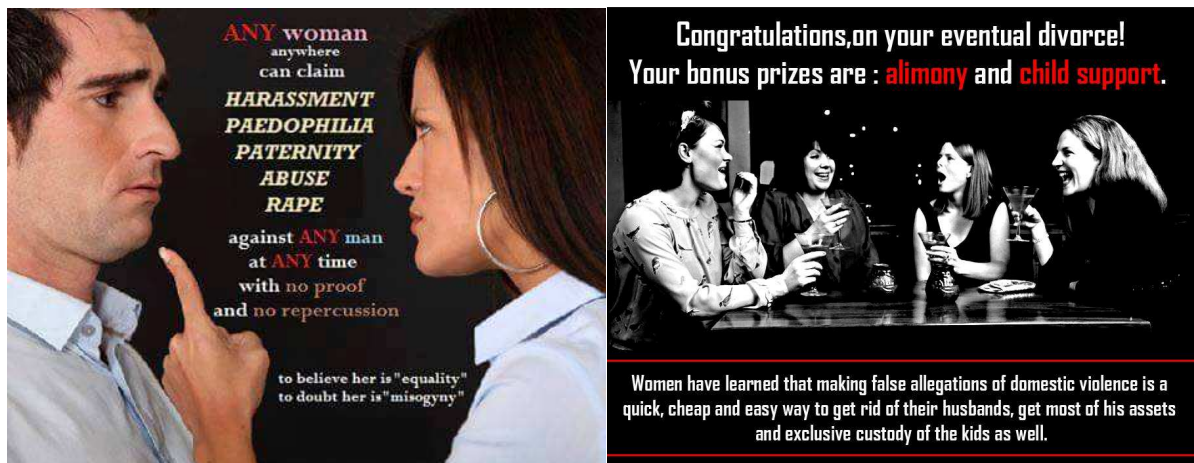
There are many remote mines in this world which are connected by rails through Hilly regions. These railroads move through steep ups and downs. Optimum speed of the train has to be maintained so that the brakes do not burn out, but the next climbing can be done. Sudden braking is not possible as the load of the wagons will derail the train, and will mean huge loss and deaths. The **Drivers are Men** who risk their lives in every journey.



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Random - 13

Almost all of us are very biased. Instead of I asking some questions, see the following images

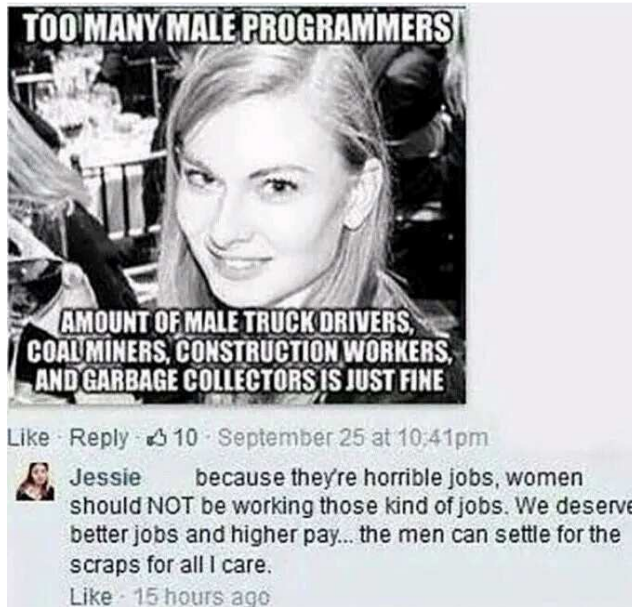




**All women are born
evil. Some just realize
their potential later in
life than others.**

QUOTEHD.COM

Chad A. Gamble



Proof that girls are evil

First we state that girls require time and money.

$$\text{GIRLS} = \text{TIME} \times \text{MONEY}$$

And as we all know "time is money"

$$\text{TIME} = \text{MONEY}$$

Therefore:

$$\text{GIRLS} = \text{MONEY} \times \text{MONEY} = (\text{MONEY})^2$$

And because "money is the root of all evil":

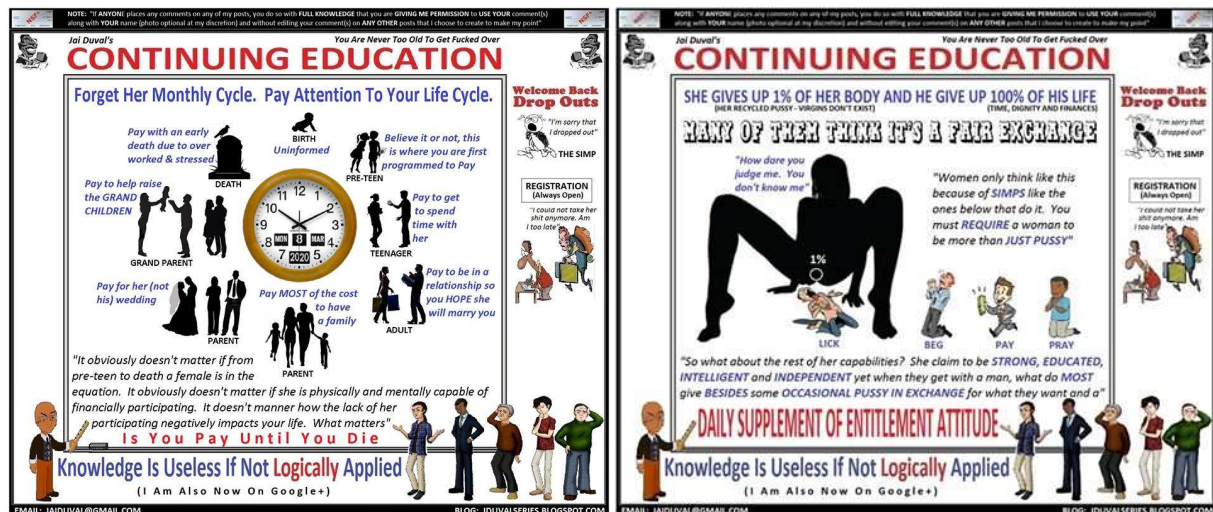
$$\text{MONEY} = \sqrt{\text{EVIL}}$$

Therefore:

$$\text{GIRLS} = (\sqrt{\text{EVIL}})^2$$

We are forced to conclude that:

$$\text{GIRLS} = \text{EVIL}$$



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Random - 14

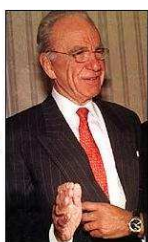
Rich people; often are very hard working. Successful business men, establish their business (empire), amass lot of wealth, with lot of difficulty. Lots of sacrifice, lots of hard work, gets into this. **Rich people's wives had no contribution in this wealth creation.** Women are smart, and successful upto the extent to choose the right/rich man to marry. So generally what happens in case of Divorces ? Search the net on " most costly divorces " and you will know. The women;(who had no contribution at all, in setting up the business / empire), often gets in Billions, or several Millions in divorce settlements.

Number 1

Rupert & Anna Murdoch -- \$1.7 billion

One of the richest men in the world, **Rupert Murdoch** developed his worldwide media empire when he inherited his father's Australian newspaper in 1952. He married Anna Murdoch in the '60s and they remained together for 32 years, springing off three children.

They split amicably in 1998 but soon Rupert forced Anna off the board of News Corp and the gloves came off. The divorce was finalized in June 1999 when Rupert agreed to let his ex-wife leave with \$1.7 billion worth of his assets, \$110 million of it in cash. Seventeen days later, Rupert married Wendi Deng, one of his employees.



Ted Danson & Casey Coates -- \$30 million

Ted Danson's claim to fame is undoubtedly his decade-long stint as Sam Malone on NBC's celebrated sitcom Cheers. While he did other TV shows and movies, he will always be known as the bartender of that place where everybody knows your name. He met his future first bride Casey, a designer, in 1976 while doing Erhard Seminars Training.

Ten years his senior, she suffered a paralyzing stroke while giving birth to their first child in 1979. In order to nurse her back to health, Danson took a break from acting for six months. But after two children and 15 years of marriage, the infatuation fell to pieces. Danson had started seeing Whoopi Goldberg while filming the comedy, Made in America and this precipitated the 1992 divorce. Casey got \$30 million for her trouble.

See <https://zookeepersblog.wordpress.com/misandry-and-men-issues-a-short-summary-at-single-place/>

See <http://skmclasses.kinja.com/save-the-male-1761788732>

It was Boys and Men, who brought the girls / women home. The Laws are biased, completely favoring women. The men are paying for their own mistakes.

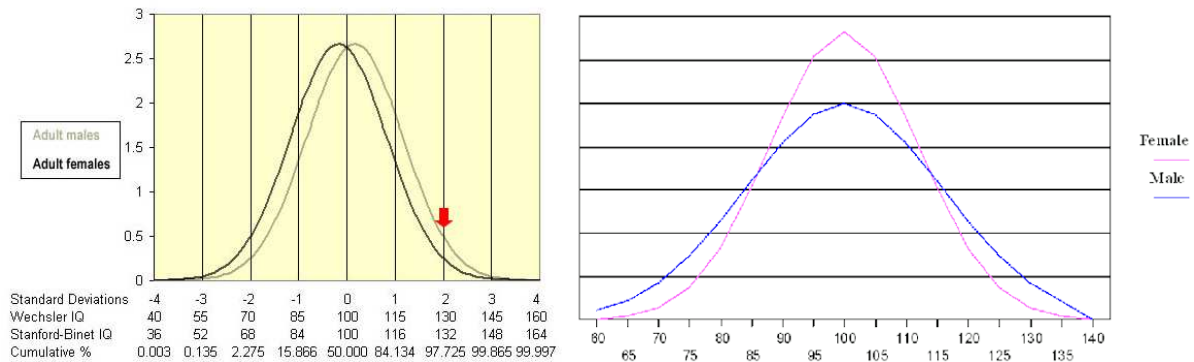
See <https://zookeepersblog.wordpress.com/biased-laws/>

(Man brings the Woman home. When she leaves, takes away her share of big fortune!)

Random - 15

A standardized test of Intelligence will never be possible. It never happened before, nor ever will happen in future. No IQ test results will be acceptable by all. **In the net there are thousands of charts which show that the intelligence scores of girls / women are lesser.** Debates of Trillion words, does not improve performance of Girls.

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I am not wasting a single second debating or discussing with anyone, on this. **I am simply accepting ALL the results.** IQ is only one of the variables which is required for success in life. Thousands of books have been written on “ Networking Skills “, EQ (Emotional Quotient), Drive, Dedication, Focus, “ Tenacity towards the end goal “ ... etc. **In each criteria, and in all together, women (in general) do far worse than men.** Bangalore is known as “ capital of India “. [Fill in the blanks]. The blanks are generally filled as “ Software Capital “, “ IT Capital “, “ Startup Capital “, etc. I am member in several startup eco-systems / groups.

I have attended hundreds of meetings, regarding “ technology startups “, or “ idea startups “. **These meetings have very few women.** (Generally in most meetings there are no women at all !). Starting up new companies are all “ **Men’s Game** “ / “ **Men’s business** “. Only in Divorce settlements women will take their goodies, due to Biased laws. **There is no dedication, towards wealth creation, by women.** Women want easy money.

Random - 16

Many men, as fathers, very unfortunately treat their daughters as “ Princess “. Every “ non-performing “ woman / wife was “ princess daughter “ of some loving father. Pampering the girls, in name of “ equal opportunity “, or “ women empowerment “, have led to nothing.

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See <http://skmclasses.kinja.com/progressively-daughters-become-monsters-1764484338>

See <http://skmclasses.kinja.com/vivacious-vixens-1764483974>

There can be thousands of more such random examples, where “ Bigger Shape / size “ of males have influenced our culture, our Society. Let us recall the reasons, that we already learned in standard 10 - 11, Biology text Books. In humans, women have a long gestation period, and also spends many years (almost a decade) to grow, nourish, and stabilize the child. (Million years of habit) Due to survival instinct Males want to inseminate. Boys and Men fight for the “ facility (of womb + care) “ the girl / woman may provide. Bigger size for males, has a winning advantage. Whoever wins, gets the “ woman / womb / facility “. The male who is of “ Bigger Size “, has an advantage to win.... Leading to Natural selection over millions of years. In general “ Bigger Males “; the “ fighting instinct “ in men; have led to wars, and solving tough problems (Mathematics, Physics, Technology, startups of new businesses, Wealth creation, Unreasonable attempts to make things [such as planes], Hard work)

So let us see the IIT-JEE results of girls. Statistics of several years show that there are around 17, (or less than 20) girls in top 1000 ranks, at all India level. Some people will yet not understand the performance, till it is said that ... year after year we have around 980 boys in top 1000 ranks. Generally we see only 4 to 5 girls in top 500. **In last 50 years not once any girl topped in IIT-JEE advanced.** Forget about Single digit ranks, double digit ranks by girls have been extremely rare. It is all about “ good boys “, “ hard working “, “ focused “, “Bel-esprit “ boys.

In 2015, Only 2.6% of total candidates who qualified are girls (upto around 12,000 rank). while 20% of the Boys, amongst all candidates qualified. The Total number of students who appeared for the exam were around 1.4 million for IIT-JEE main. Subsequently 1.2 lakh (around 120 thousands) appeared for IIT-JEE advanced.

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IIT-JEE results and analysis, of many years is given at

<https://zookeepersblog.wordpress.com/iit-jee-iseet-main-and-advanced-results/>

In Bangalore it is rare to see a girl with rank better than 1000 in IIT-JEE advanced. We hardly see 6-7 boys with rank better than 1000. Hardly 2-3 boys get a rank better than 500.

See <http://skmclasses.weebly.com/everybody-knows-so-you-should-also-know.html>

So what " some women " are doing ?

Thousands of people are exposing the heinous crimes that Motherly Women are doing, or Female Teachers are committing. See <https://www.facebook.com/WomenCriminals/>

Some Random Examples must be known by all

It is extremely unfortunate that the " woman empowerment " has created. This is the kind of society and women we have now. I and many other sensible Men hate such women. Be away from such women, be aware of reality.



Mother Admits On Facebook to Sleeping with 15 Yr Old Son, They Have a Baby Together - Alwaysturntup

Sometimes it hard to believe w From Alwaysturntup

ALWAYZTURNUP.ME



'Sex with my son is incredible - we're in love and we want a baby'

Ben Ford, who ditched his wife when he met his mother Kim West after 30 years, claims what the couple are doing 'isn't incest'

MIRROR.CO.UK

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Woman sent to jail for the rest of her life after raping her four grandchildren is described as the 'most evil person' the judge has ever seen

Edwina Louis rape...

[See More](#)



Former Shelbyville ISD teacher who had sex with underage student gets 3 years in prison

After a two day break over the weekend, A Shelby County jury was back in the courtroom looking to conclude the trial of a former Shelbyville ISD teacher who had...

KLTV.COM | BY CALEB BEAMES

<http://www.thenativecanadian.com/.../eastern-ontario-teacher-...>



Woman sent to jail for raping her four grandchildren

A Ohio grandmother has been sentenced to four consecutive life terms after being found guilty of the rape of her own grandchildren. Edwina Louis, 53, will spend the rest of her life behind bars.

DAILYMAIL.CO.UK



The N.C. Chronicles.: Eastern Ontario teacher charged with 36 sexual offences

anti feminism, Child abuse, children's rights, Feminist hypocrisy,

THENATIVECANADIAN.COM | BY BLACKWOLF



Hyd woman kills newborn boy as she wanted daughter - Times of India

Having failed to bear a daughter for the third time, a shopkeeper's wife slit the throat of her 24day-old son with a shaving blade and left him to die in a street on Tuesday night.Pumima's first child was a stillborn boy, followed by another boy born five years ago.

TIMESOFINDIA.INDIATIMES.COM

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Montgomery's son, Alan Vonn Webb, took the stand and was a key witness in her conviction.

"I want to see her placed somewhere she can never do that to children

....

See More




Woman sentenced to 40 years in prison for raping her children

A Murfreesboro mother found guilty of raping her own children learned her fate on Wednesday.

WAFF.COM | BY DENNIS FERRIER

gentler sex? Violence against men.'s photo.



Up to 64,000 women in UK are child-sex offenders

~ The Guardian

Women, the gentler sex? Violence against men.

April 8 at 1:38am

Like Page

In fact, the past decade has seen a dramatic increase in the number of incidents of women raping and sexually assaulting boys and men. On May 2014, Jezebel repo...

In Facebook, and internet + whatsapp etc we have unending number of posts describing frustration of men / husbands on naughty unreasonable women. **Most women are very illogical, Punic, perfidious, treacherous, naughty, gamey bitches.**

We also see zillions of Jokes which basically describe how unreasonable women / girls are. How stupid they are, making life of Boys / Men / Husband a hell.

While each of these girls was someones daughter. Millions of foolish Dads are into Fathers rights movement, who want their daughter back for pampering.

Most girls are being cockered, coddled, cosseted, mollycoddled, featherbedded, spoilt into brats.

Foolish fathers are breeding Monsters who are filing false rape cases. Enacting Biased Laws. Filing False domestic violence cases. Filing false sexual assault cases. Asking for alimony, and taking custody of the Daughter, not allowing the " monster " to meet dad. The cycle goes on and on and on.

Foolish men keep pampering future demons who make other Men's life a hell. (Now read this again from beginning). Every day we see the same posts of frustration.

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**When I grow up
I will beat my
husband
No one will care
No one will stop me**



October is Domestic Violence Awareness Month

**53% of Domestic Violence
Victims are Men
Stop the Silence
Stop the Violence**



FUNNY.

NOT FUNNY.

**DOUBLE STANDARDS HURT
EVERYONE.**

<https://nicewomen.wordpress.com/>

Each women as described below was someone's Pampered Princess ...

End violence against women ...



North Carolina Grandma Eats Her Daughter's New Born Baby After Smoking Bath Salts

Henderson, North Carolina—A North Carolina grandmother of 4 and recovering drug addict, is now in custody after she allegedly ate her daughter's newborn baby....

AZ-365.TOP



28-Year-Old Texas Teacher Accused of Sending Nude Picture to 14-Year-Old Former Student

BREITBART.COM

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<http://latest.com/.../attractive-girl-gang-lured-men-alleywa.../>



Attractive Girl Gang Lured Men Into Alleyways Where Female Body Builder Would Attack Them

A Mexican street gang made up entirely of women has been accused of using their feminine wiles to lure men into alleyways and then beating them up and...

LATEST.COM

<http://www.wfmj.com/.../youngstown-woman-convicted-of-raping-...>



Youngstown woman convicted of raping a 1 year old is back in jail

A Youngstown woman who went to prison for raping a 1-year-old boy fifteen years ago is in trouble with the law again.

WFMJ.COM

End violence against women



Women are raping boys and young men

Rape advocacy has been maligned and twisted into a political agenda controlled by radicalized activists. Tim Patten takes a razor keen and well supported look into the manufactured rape culture and...

AVOICEFORMEN.COM | BY TIM PATTEN

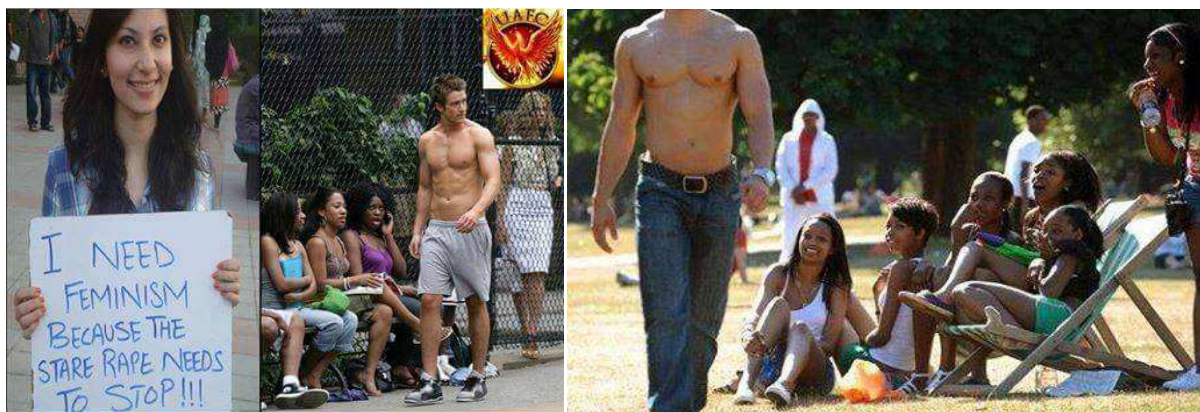


Bronx Woman Convicted of Poisoning and Drowning Her Children

Lisette Barnenga researched methods on the Internet before she killed her son and daughter in 2012.

NYTIMES.COM | BY MARC SANTORA

Monster women have very easy and cozy life. Easy to demand anything and get law in favor !



If the lawmakers submit to these strange demands of say ... " Stare Rape ! "; then we can easily see what kind of havoc that will create.

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55%
of Biological Parents
Who Kill Their Children are
Mothers

Homicidal Encounters: A Study of Homicide in Australia 1989-1999.
Australian Institute of Criminology (Monash, 2000, p. 142, Fig. 74).

ManKind
Equality Network



Woman charged with killing baby also had previous infant die

Woman charged with killing baby also had previous infant die

ABC7.COM | BY ROB MCMILLAN



Female Sex Predators: A Crime Epidemic shared a link.

Yesterday at 12:40am · 🌐



WTVB.com | Woman pleads guilty to having sex with a dog

PITTSBORO, Miss. (WTVB) -- A Calhoun County woman has pleaded guilty to charges she had unnatural intercourse with a dog. Sheriff Greg Poll

WTVB.COM

👍 Like 💬 Comment ➦ Share

👍 🗨️ 🗨️ Mhra Leander Pallat, Eric Antonio Alvarado and 31 others Top Comments ▾



Oklahoma Teacher Receives 15-Year Prison Sentence For Sex With 15-Year-Old Boy

A former Oklahoma middle school teacher has pleaded guilty to 6 counts of rape, child enticement...

THREEPERCENTERNATION.COM

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A Russian-born newlywed slowly butchered her German husband — feeding strips of his flesh to their dog until he took his last breath. Svetlana Batukova, 46, was...

See More



She killed her husband and then fed him to her dog: police

A Russian-born newlywed butchered her German hubby — and fed strips of his flesh to her pooch, authorities said. Svetlana Batukova offered Horst Hans Henkels at their...

NYPOST.COM

Daily Mail
January 15, 2015

Mother charged with rape and sodomy of her son's 12-year-old friend



Mom, 30, 'raped and had oral sex with her son's 12-year-old friend'

Nicole Marie Smith, 30, (pictured) of St Charles County, Missouri, has been jailed after she allegedly targeted the 12-year-old boy at her home.

DAILYMAIL

April 4 at 4:48am



Female prison officers commit 90pc of sex assaults on male teens in US juvenile detention centres

Lawsuit in Idaho highlights the prevalence of sexual victimization of juvenile offenders.

BTIMES.CO.UK | BY NICOLE ROJAS

This mother filmed herself raping her own son and then sold it to a man for \$300. The courts just decide her fate. When you see what she got, you're going to be outraged.



Mother Who Filmed Herself Raping Her 1-Year-Old Son Receives Shocking Sentence

"...then used the money to buy herself a laptop..."

AMERICANNEWS.COM

In several countries or rather in several regions of the world, family system has collapsed, due to bad nature and naughty acts of women.

Particularly in Britain, and America, almost 50% people are alone, lonely, separated, divorced or failed marriages. In 2013, 48% children were born out of wedlock. **It was projected that by 2016, more than 51% children will be born, to unmarried mothers.** In these developed countries " paternity fraud " by women, are close to 20%. You can see several articles in the net, and in wikipedia etc. This means 1 out of 5 children are calling a wrong man as dad. The lonely, alone " mothers " are frustrated. They see the children as burden. Love in the Society in general is lost, long time ago. The types of " Mothers " and " Women " we have now

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This is the type of women we have in this world. These kind of women were also someones daughter



Mother Stabs Her Baby 90 Times With Scissors After He Bit Her While Breastfeeding Him!

Eight-month-old Xiao Bao was discovered by his uncle in a pool of blood. He needed 100 stitches after the incident; he is now recovering in hospital. Reports say his...

MOMMABUZZ.COM



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Not All Feminist Theory is Equal

Christina Hoff Sommers
Factual and Equity Feminist



"That is the corrosive paradox of gender feminism's misandrist stance: no group of women can wage war on men without at the same time denigrating the women who respect those men."

Andrea Dworkin
Radical and Gender Feminist



"Under patriarchy, every woman's son is her potential betrayer and also the inevitable rapist or exploiter of another woman"

ManKind
Equality Network

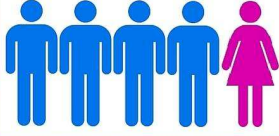
WILLFUL BLINDNESS

Ignoring the Majority of Victims Because of Their Gender

I am a feminist because it bothers me that a woman gets killed by her male partner every single week, and somehow that doesn't qualify as a tools-down national crisis even though if a man got killed by a shark every week we'd probably arrange to have the ocean drained.

Annabel Crabb

79% of Homicide victims worldwide are MALE
"Where is the tools-down national crisis?"



United Nations Office of Drugs and Alcohol. Global Study on Homicide (2013, p. 13).
https://en.wikipedia.org/wiki/Willful_blindness

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Not All Feminist Theory is Equal

Christina Hoff Sommers
Equity Feminist



Christina H. Sommers
@CH-Sommers
Want to close wage gap? Step one: Change your major from feminist dance therapy to electrical engineering.

Valerie Solanis
Radical and Gender Feminist



"To call a man an animal is to flatter him; he's a machine, a walking dildo"

n.b. Some Feminists

ManKind
Equality Network



Muslim Woman Caught RAPING Her Own Son - Gives Disgusting Excuse to Judge | John Hawkins' Right Wing News

RIGHTWINGNEWS.COM

By now if you have assumed that Indian women are not doing any crime then please become friends with MRA Guri <https://www.facebook.com/profile.php?id=100004138754180>

He has dedicated his life to expose Indian Criminals



Delhi Woman Who Tried To Rape An Auto Driver, While Her Friend Filmed The Act, Has Been Arrested

Men are raped too!

MENSXP.COM | BY NIKITA MUKHERJEE



Muslim mother, 43, jailed for sex offences against girl, nine

Raheelah Dar, 43, from Middlesbrough, has been jailed for seven years for carrying out a string of sex offences against a nine-year-old girl.

DAILYMAIL.CO.UK

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MailOnline

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Mother who had been forced into an arranged marriage is jailed for filming herself having sex with her 14-year-old son and sending the clips to relatives in Pakistan

- Vile mother filmed having sex with her teenage son in sick porn video
- Clips sent to cousin in Pakistan who allegedly asked her to make film
- She also sent her relative indecent images of her three-year-old daughter

By **ALEX MATTHEWS FOR MAILONLINE**

PUBLISHED: 12:44 GMT, 1 August 2016 | UPDATED: 11:23 GMT, 2 August 2016



Wife Stabs Husband And Runs Away After He Stops Her From Gambling

The husband said his wife had become a habitual gambler who was also addicted to liquor.

INDIATIMES.COM



Teacher learns fate for 6 months of sex with boy

(CBS8) — SAN DIEGO (CNS) — A Crawford High School teacher and coach who carried on a six-month sexual relationship with a 15-year-old male student was sentenced Friday to a two-year prison term. Toni Nicole Sutton, 38, pleaded guilty...

WIND.COM



Mom jailed for 40 years after body of daughter, 9, found in fridge

Amber Keyes, 37, was sentenced in the death of Ayahna Comb in Houston on Friday. Ayahna, who had cerebral palsy, had been in the fridge for six months...

DAILYMAIL.CO.UK

HURT FEMINISM BY DOING NOTHING

- ✗ DON'T HELP WOMEN
- ✗ DON'T FIX THINGS FOR WOMEN
- ✗ DON'T SUPPORT WOMEN'S ISSUES
- ✗ DON'T COME TO WOMEN'S DEFENSE¹
- ✗ DON'T SPEAK FOR WOMEN
- ✗ DON'T VALUE WOMEN'S FEELINGS
- ✗ DON'T PORTRAY WOMEN AS VICTIMS
- ✗ DON'T PROTECT WOMEN²

✓ WITHOUT WHITE KNIGHTS FEMINISM WOULD END TODAY

¹Don't even nawalt ("Not All Women Are Like That") ²for example from criticism or insults

How Society prioritize Men

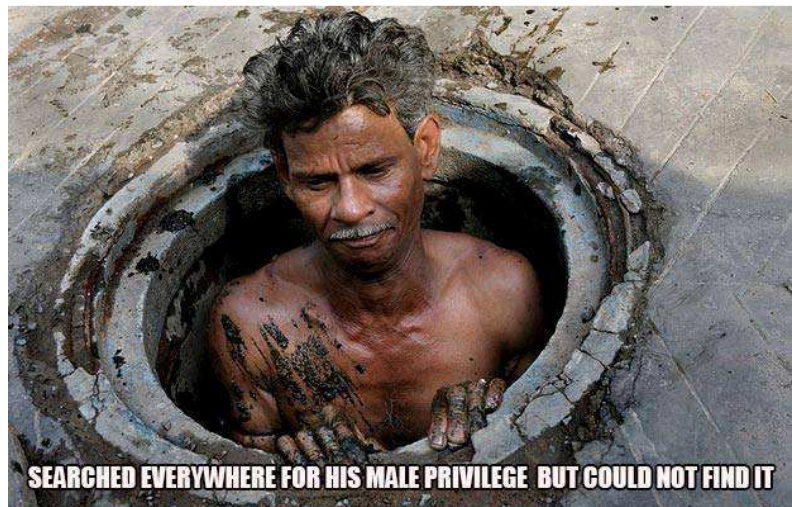
High Priority

Low Priority

Rich women		They can get away with murder.
Women		They get all the rights with no responsibility and Shelters for Homeless women.
Rich Men		They get tax bail outs and short prison sentence.
Girls		They get educational benefits but no violence against kids Act.
Boys		They have some support but don't have any education that fits boys.
Animals		They have animal rights and PETA.
Prisoners		They get conjugal visits and 3 squares and a roof.
Men		Paid slaves.
Poor Men		Nothing.

Who pays the most Taxes?
This is why MGTOW exist.

MGTOW



Professor Subhashish Chattopadhyay

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Preface for Physics

Professor H. C. Verma wrote amazing books in Physics. There are many other good books for IIT JEE and other exams. **Krishna's Guides**, Books by Professor **N. N. Ghosh**, Professor D. C. Pandey, GRB Publications Physics Guides etc are very good. For numericals the Irodov's books remain the King !

“Concepts of Physics” by Professor H C Verma have been available since 1991. (and did not change or updated since). Previous to that, past papers of IIT JEE, and other exams, were the source for preparation. I was in High School in 1980s. I had 6-7 Russian books apart from Irodov. All these were very good. Resnick and Halliday 's (Walker and Krane came in subsequently) book was also well known. There were too many “ uncles “ who used to advice that “ only Resnick and Halliday 's book was enough “ !

Well I agreed and disagreed. There were many IIT JEE questions which were ditto or verbatim picked-up from Resnick Halliday ! **But, something more was always needed.** Brilliant Tutorials, Agarwal Coaching etc, were famous those days. (1980s 90 s). They were giving **several new questions**, which enabled more practice. **People slowly realized that “ every type “ of questions are NOT there in Resnick & Halliday, or say Irodov.**

Uncles saying “ only Resnick and Halliday 's book was enough “ ! were wrong. “Concepts of Physics” by Professor H C Verma sold so much because of very good step by step explanations, new solved examples, new exercises. Several gaps were filled-up.

The word Physics is derived from Latin **physica**, from Greek (ta) **phusika**, (the things) of nature, from neuter plural of phusikos.

So, why am I writing “another book” in Physics ? (The description of nature)

I wish to answer this most important question, first !

There are many kind of Questions which are not covered in “ Concepts of Physics “ of Professor H. C. Verma. Also Irodov, in his books, **does not** explain or cover several kinds of Problems or Questions. The “ Coaching Institutes “ very rightly thrived on these gaps. Almost 100% students benefit more **with more examples**. **As Coaching Institutes discuss, cover and repeat several more examples in each chapter compared to School or Text books; explains the reason of their popularity.**

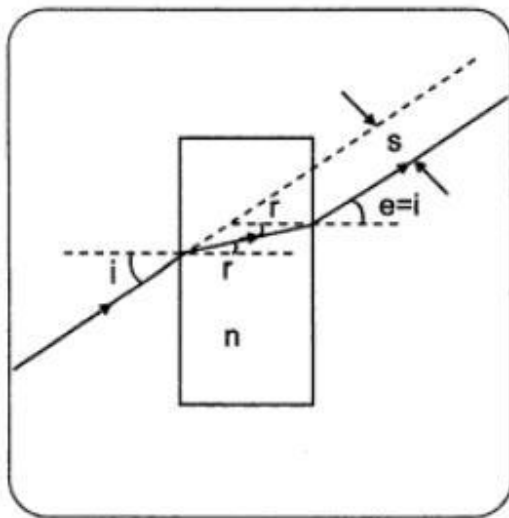
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Let me list a few examples to explain all this.

Optics - 1) The expression for deviation of a ray passing through a slab

Refraction through a transparent slab (lateral shift)

Consider a transparent slab of thickness t , and refractive index n . A monochromatic beam of light falls on one side at an angle of incidence i as shown in Fig. Emergent ray will be parallel to incident ray, but there will be a lateral shift S of the incident ray. At the first interface,



$1 \sin i = n \sin r$ and at the second interface

$n \sin r = 1 \sin e$

where, r is the angle of refraction at the first interface and e , the angle of refraction at the second interface. $\therefore e = i$

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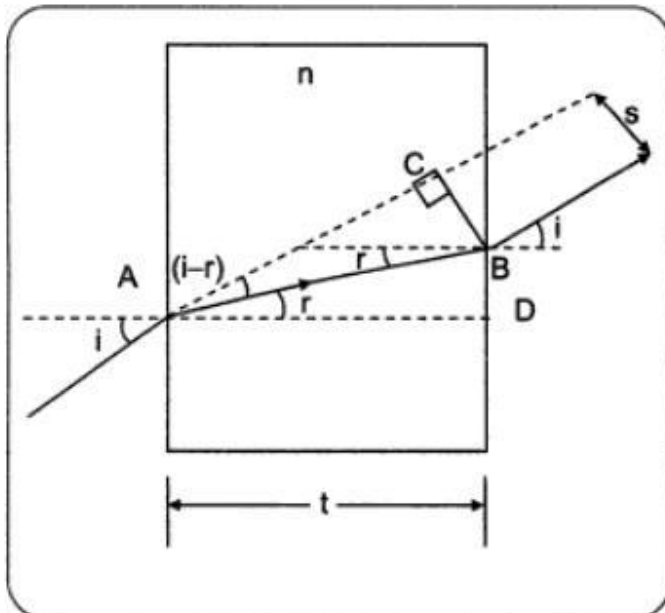
From Fig. lateral shift is calculated as follows:

$$AD = t; AB = \frac{AD}{\cos r} = \frac{t}{\cos r}$$

$$\text{Lateral shift } S = BC = AB \sin(i - r) = \frac{t \sin(i - r)}{\cos r}$$

$$\text{i.e., } S = \frac{t \sin(i - r)}{\cos r}$$

It may be noted that $S_{\max} = t$ for $i = 90^\circ$ (grazing incidence) and $S_{\min} = 0$ for $i = 0$ (normal incidence)



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Special case:

(i) **small i**

$$t \frac{\sin(i - r)}{\cos r} = \frac{t[\sin i \cos r - \cos i \sin r]}{\cos r}$$

$$[r \text{ small} \Rightarrow \cos r \approx 1] ; i \text{ small} \Rightarrow \cos i \approx 1]$$

$$\therefore S = t(\sin i - \sin r) = t \sin i \left[1 - \frac{\sin r}{\sin i} \right]$$

$$\Rightarrow S = t \sin i \left[1 - \frac{1}{n} \right] = t i \left(1 - \frac{1}{n} \right) [i \text{ small} \Rightarrow \sin i = i]$$

$$\Rightarrow S = t i \frac{(n - 1)}{n}$$

(Note: use formula $S = t \frac{\sin(i - r)}{\cos r}$ unless it is given that $i = \text{small}$)

(ii) When i is not small, it can be shown that

$$S = \frac{t \sin(i - r)}{\cos r} = t \sin i \left[1 - \frac{\cos i}{\sqrt{n^2 - \sin^2 i}} \right] \text{ or}$$

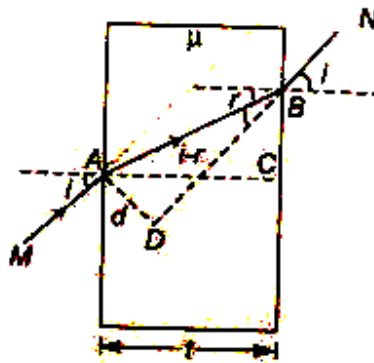
$$S = t \sin i \left[1 - \sqrt{\frac{1 - \sin^2 i}{n^2 - \sin^2 i}} \right]$$

See again

Lateral Shift

In the following figure, ray MA is parallel to ray BN . But the emergent ray is displaced laterally by a distance d which depends upon μ , t and i and its value is given by

$$d = t \left(1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}} \right) \sin i.$$



From the figure, $AB = \frac{AC}{\cos r} = \frac{t}{\cos r}$ (as, $AC = t$)

Since,

$$d = AB \sin(i - r)$$

$$= \frac{t}{\cos r} [\sin i \cos r - \cos i \sin r]$$

$$d = t [\sin i - \cos i \tan r]$$

Further,

$$\mu = \frac{\sin i}{\sin r} \text{ or } \sin r = \frac{\sin i}{\mu}$$

\therefore

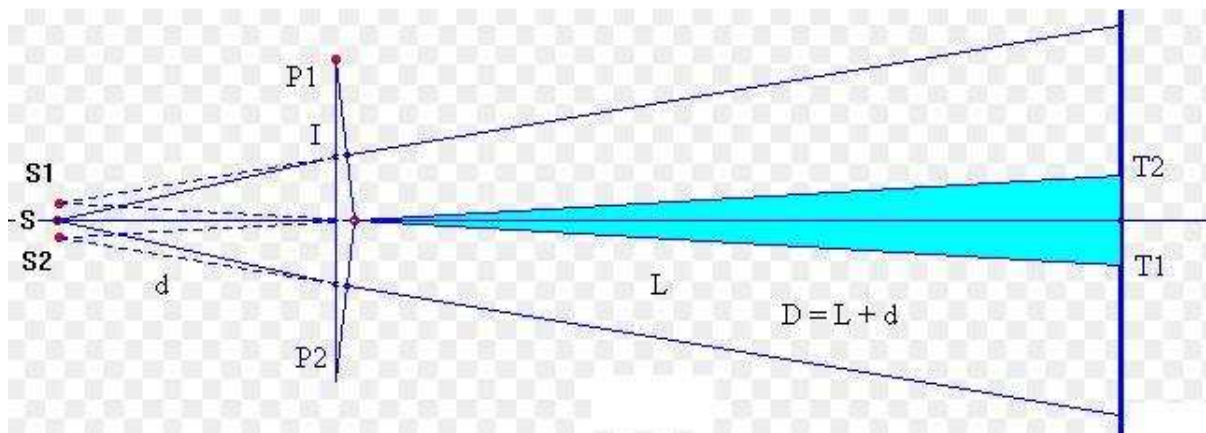
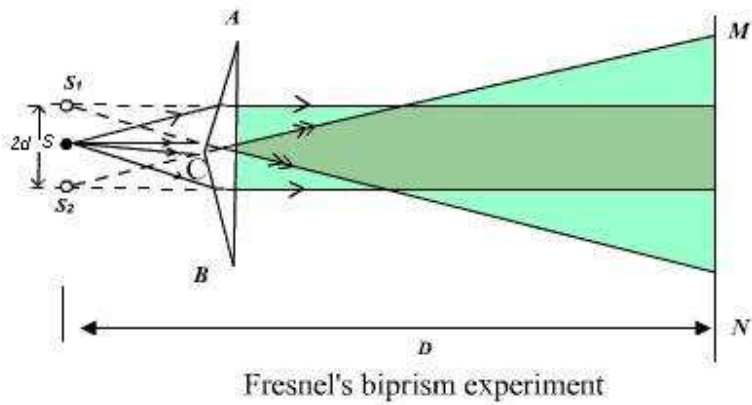
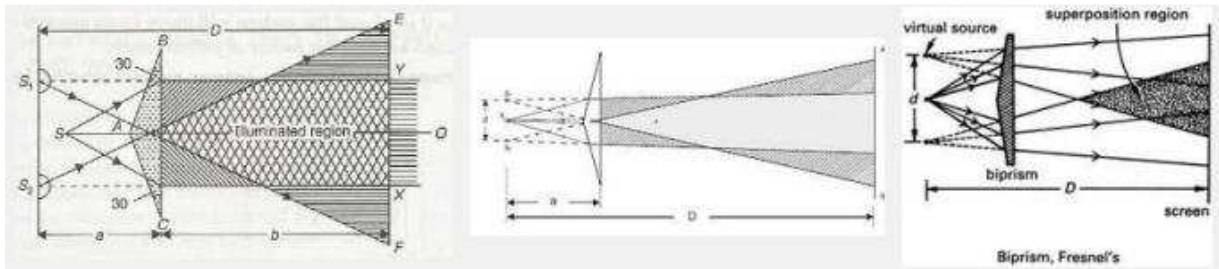
$$\tan r = \frac{\sin i}{\sqrt{\mu^2 - \sin^2 i}}$$

The expression for d now is

$$d = \left(\sqrt{1} - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}} \right) t \sin i$$

Note For small angles of incidence $d = t \left(\frac{\mu - 1}{\mu} \right)$

Optics - 2) Fresnel's Biprism



very small refracting angle α , is given by

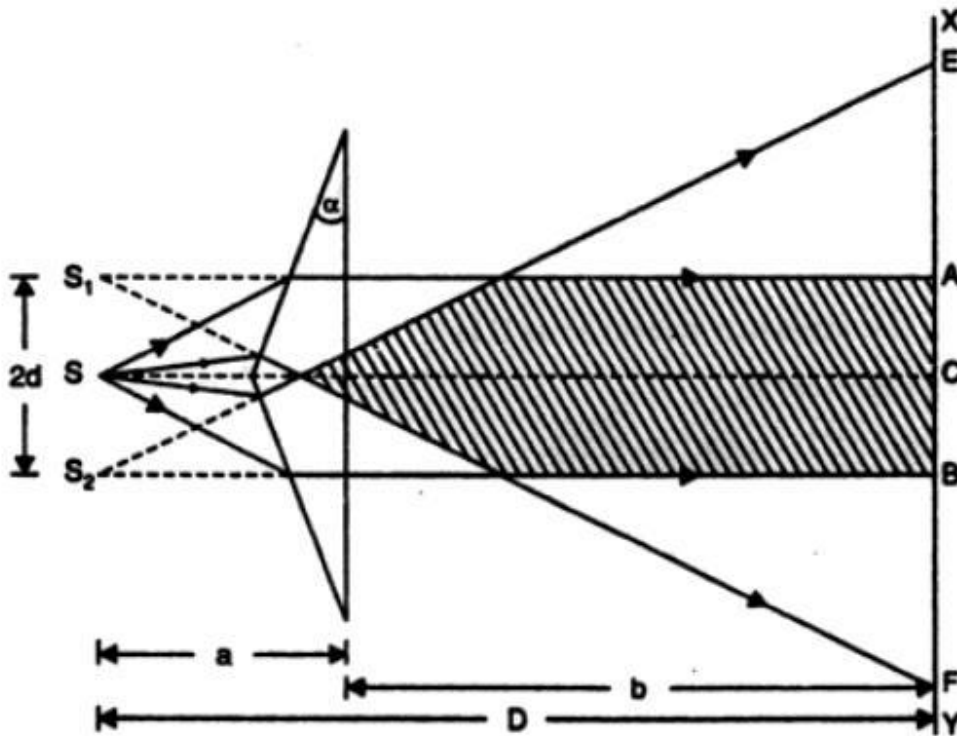
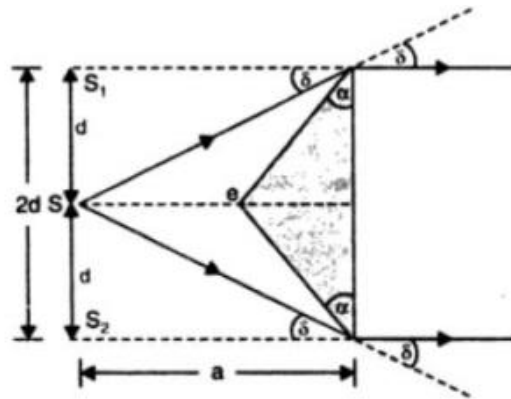
$$\delta = (\mu - 1)\alpha,$$
 where μ is the refractive index of the material of the prism. Note that α is in radian

It is clear from Fig. that

$$\delta = \frac{d}{a}$$

$$\therefore (\mu - 1)\alpha = \frac{d}{a} \quad \text{or} \quad d = (\mu - 1)\alpha a$$

$$\therefore \boxed{2d = 2(\mu - 1)\alpha a}$$



In a biprism experiment, the eye-piece was placed at a distance of 120 cm from the source. The distance between two virtual images was found equal to 0.075 cm. Find the wavelength of light of source if eye-piece is moved through a distance of 1.888 cm for 20 fringes to cross the field of view.

$$D = 120 \text{ cm,}$$

$$2d = 0.075 \text{ cm, } \lambda = ?$$

$$\beta = \frac{1.888}{20} \text{ cm}$$

$$\beta = \frac{\lambda D}{2d} \quad \text{or} \quad \lambda = \frac{\beta(2d)}{D} \text{ cm}$$

$$\lambda = \frac{1.888}{20} \times 0.075 \text{ cm}$$

$$= 5900 \times 10^{-8} \text{ cm} = 5900 \text{ \AA}$$

The inclined faces of a glass prism ($\mu = 1.5$) make an angle of 1° with the base of the prism. The slit is 10 cm from the biprism and is illuminated by light of $\lambda = 5900 \text{ \AA}$. Find the fringe width observed at a distance of 1m from the biprism.

Solution. $\alpha = 1^\circ = \frac{\pi}{180}$ radian,

$$\mu = 1.5,$$

$$D = 10 \text{ cm} + 100 \text{ cm} = 110 \text{ cm,}$$

$$\lambda = 5900 \times 10^{-8} \text{ cm}$$

$$\beta = \frac{D\lambda}{2d} = \frac{D\lambda}{2(\mu - 1) \alpha a}$$

or
$$\beta = \frac{110 \times 5900 \times 10^{-8} \times 7 \times 180}{2(1.5 - 1) 22 \times 10} \text{ cm}$$
$$= 0.037 \text{ cm.}$$

A biprism is placed 5 cm from a slit illuminated by sodium light ($\lambda = 5890 \text{ \AA}$). The width of the fringes obtained on a screen 75 cm from the biprism is $9.424 \times 10^{-2} \text{ cm}$. What is the distance between the two coherent sources ?

Solution. $D = 5 \text{ cm} + 75 \text{ cm} = 80 \text{ cm}$
 $\beta = 9.424 \times 10^{-2} \text{ cm}$
 $2d = ?$

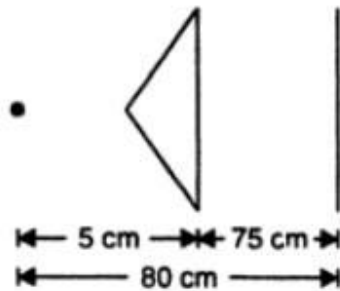


Fig. 2.25

$$\lambda = 5890 \text{ \AA} = 5890 \times 10^{-8} \text{ cm}$$

We know that $\beta = \frac{\lambda D}{2d}$

or
$$2d = \frac{\lambda D}{\beta} = \frac{5890 \times 10^{-8} \times 80}{9.424 \times 10^{-2}} \text{ cm}$$

$$= 0.05 \text{ cm.}$$

In a Fresnel's biprism experiment, the fringe width is observed to be 0.087 mm. What will it become if the slit to biprism distance is reduced to $\frac{3}{4}$ of the original distance ? (all else remaining unchanged).

Solution. $2d = 2(\mu - 1) \alpha a$... (1)

$$2d' = 2(\mu - 1) \alpha \left(\frac{3}{4} a \right) \quad \dots (2)$$

Dividing (2) by (1), $\frac{2d'}{2d} = \frac{3}{4}$

Again, we know that $\beta = \frac{D\lambda}{2d}$

$$\frac{\beta'}{\beta} = \frac{2d}{2d'} = \frac{4}{3}$$

or $\beta' = \frac{4}{3} \beta = \frac{4}{3} \times 0.087 \text{ mm} = \mathbf{0.116 \text{ mm}}$.

The inclined faces of biprism of refractive index 1.50 make angles of 2° with its base. A slit illuminated by monochromatic light is placed at a distance of 10 cm from the biprism. If

distance between two dark fringes observed at a distance of 1 metre from biprism is 0.18 mm, find the wavelength of light used.

Solution. $\mu = 1.50$,

$$\alpha = 2^\circ = 2 \times \frac{\pi}{180} = \frac{\pi}{90} \text{ radian,}$$

$$a = 10 \text{ cm, } b = 1 \text{ m} = 100 \text{ cm,}$$

$$\beta = 0.18 \text{ mm} = 0.018 \text{ cm, } \lambda = ?$$

We know that

$$\beta = \frac{D\lambda}{2d}, D = a + b \text{ and } 2d = 2(\mu - 1)\alpha a$$

$$\therefore \beta = \frac{\lambda(a + b)}{2(\mu - 1)\alpha a}$$

$$\therefore \lambda = \frac{2\beta(b - 1)\alpha a}{a + b}$$

$$\begin{aligned} &= \frac{2 \times 0.018 \times (1.50 - 1) \frac{\pi}{90} \times 10}{10 + 100} \text{ cm} \\ &= 5714 \times 10^{-8} \text{ cm} = \mathbf{5714 \text{ \AA}.} \end{aligned}$$

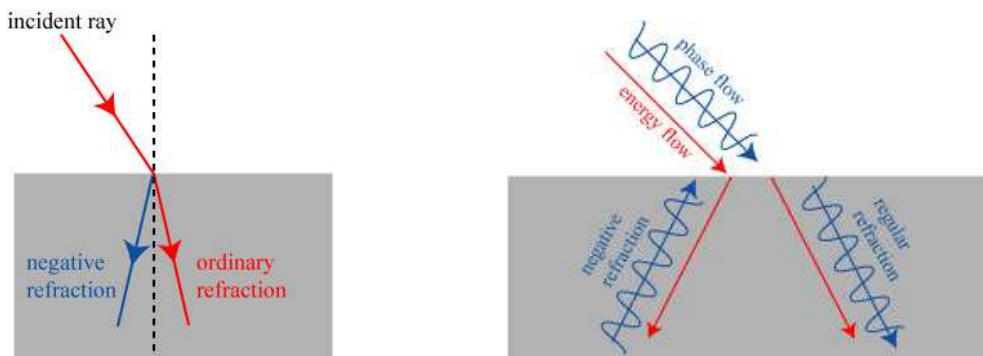
If Fresnel biprism is immersed in a liquid of refractive index μ' , then

$$\beta_{\text{new}} = \frac{\frac{\lambda}{\mu'}(a + b)}{2a\left(\frac{\mu}{\mu'} - 1\right)\alpha} = \frac{\lambda(a + b)}{2a(\mu - \mu')\alpha}$$

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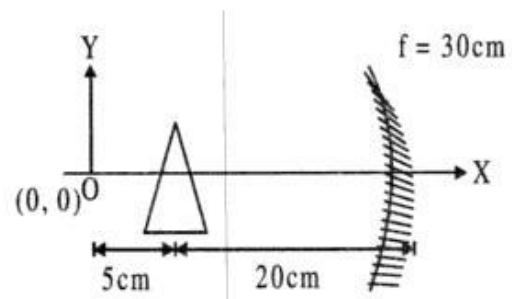
Optics - 3) Negative Refractive Index. For meta-materials we can have Negative Refractive index. So “ Refractive Index “ is a ‘ rare ‘ scalar which can be negative. [Recall most scalars are positive, such as volume, mass, pressure, viscosity, resistance, inductance, capacitance etc. **Can you think of a few scalars which can be negative also apart from charge or current ?**]

Negative refractive index question was asked in 2012 IIT JEE



Optics - 4) Combination of Prism and Mirror problems

Find the co-ordinates of image of the point object 'O' formed after reflection from concave mirror as shown in figure assuming prism to be thin and small in size of prism angle 2° . Refractive index of the prism material is $3/2$.

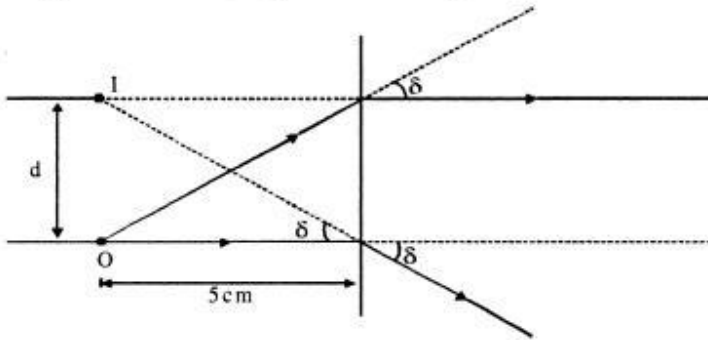


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Consider image formation through prism. All incident rays will be deviated by

$$\delta = (\mu - 1)A = \left(\frac{3}{2} - 1\right)2^\circ = 1^\circ = \frac{\pi}{180} \text{ rad}$$

As prism is thin, object and image will be in the same plane as shown in figure.



It is clear $\frac{d}{5} = \tan \delta \approx \delta$ ($\because \delta$ is very small) or $d = \frac{\pi}{36} \text{ cm}$

Now this image will act as an object for concave mirror.

$$u = -25 \text{ cm}, f = -30 \text{ cm}, \therefore v = \frac{uf}{u-f} = 150 \text{ cm}, \text{ Also, } m = \frac{-v}{u} = +6$$

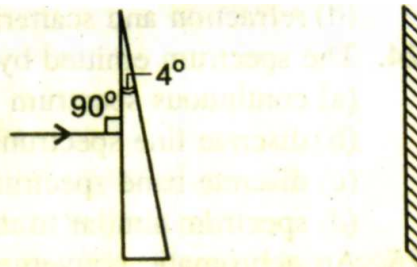
$$\therefore \text{Distance of image from principal axis} = \frac{\pi}{36} \times 6 = \frac{\pi}{6} \text{ cm}$$

Hence, co-ordinates of image formed after reflection from concave mirror are

$$\left(175 \text{ cm}, \frac{\pi}{6} \text{ cm}\right)$$

A prism having an apex

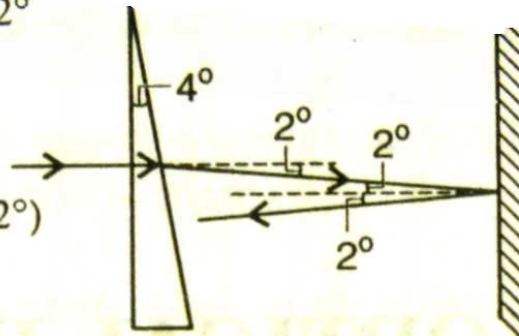
angle 4° and refractive index 1.5 is located in front of a vertical plane mirror as shown in figure. Through what total angle is the ray deviated after reflection from the mirror?



- (a) 176° (b) 4°
(c) 178° (d) 2°

$$\delta_{\text{prism}} = (\mu - 1)A = (1.5 - 1)4^\circ = 2^\circ$$

$$\begin{aligned} \therefore \delta_{\text{total}} &= \delta_{\text{prism}} + \delta_{\text{mirror}} \\ &= (\mu - 1)A + (180 - 2i) \\ &= (1.5 - 1)4^\circ + (180 - 2 \times 2^\circ) \\ &= 2^\circ + 176^\circ = 178^\circ \end{aligned}$$

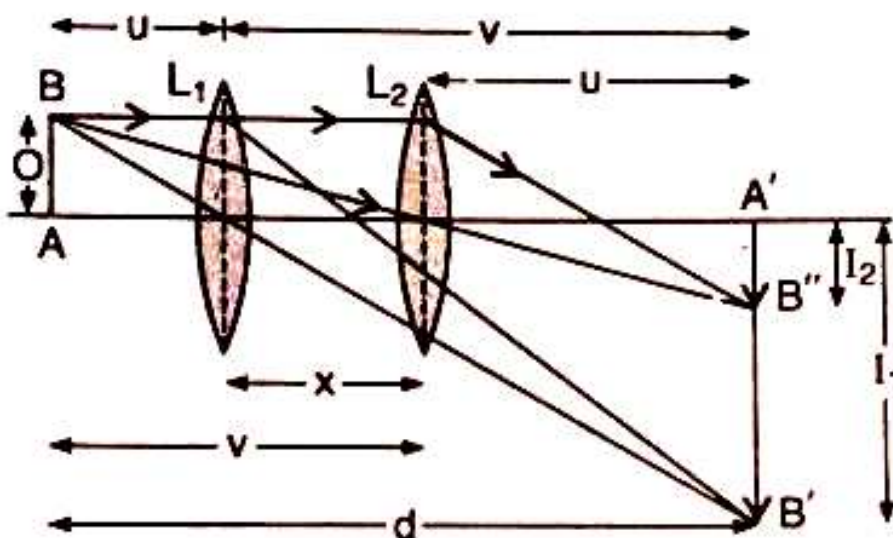


The ray diagram is shown in figure.

Optics - 5) How do we find focal length of a lens ?

Focal length of convex lens by displacement method:

- (i) When the distance between object and screen d , is greater than $4f$, then there are two positions of the lens for which the image of the object on the screen is distinct and clear. In these two positions of lens, the distances of object and image from the lens are interchanged.



- (ii) Here, I_1 and I_2 are the lengths of images in first and second position of lens L . O is the length of the object. In first position of lens,

$$m_1 = \frac{v}{u} = \frac{I_1}{O}$$

In second position, the magnification of the lens is

given by: $m_2 = \frac{u}{v} = \frac{I_2}{O}$

$$\therefore m_1 m_2 = \frac{I_1 I_2}{O^2} = 1$$

$$\therefore O = \sqrt{I_1 I_2}$$

(iii) Further, $\frac{m_1}{m_2} = \frac{v^2}{u^2}$

From figure, $u + x + u = d$ or $u = \frac{d-x}{2}$

According to sign convention, $u = -(d-x)/2$

Similarly, $v = d - u = (d+x)/2$

Using lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get;

$$f = \left(\frac{d^2 - x^2}{4d} \right)$$

In the displacement method, a convex lens is placed in between an object and a screen. If the magnifications in the two positions are m_1 and m_2 and the displacement of the lens between the two positions is x , then the focal length of the lens is:

(a) $\frac{x}{(m_1 + m_2)}$

(b) $\frac{x}{(m_1 - m_2)}$

(c) $\frac{x}{(m_1 + m_2)^2}$

(d) $\frac{x}{(m_1 - m_2)^2}$

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$$m_1 = \frac{v}{u}, m_2 = \frac{u}{v}$$

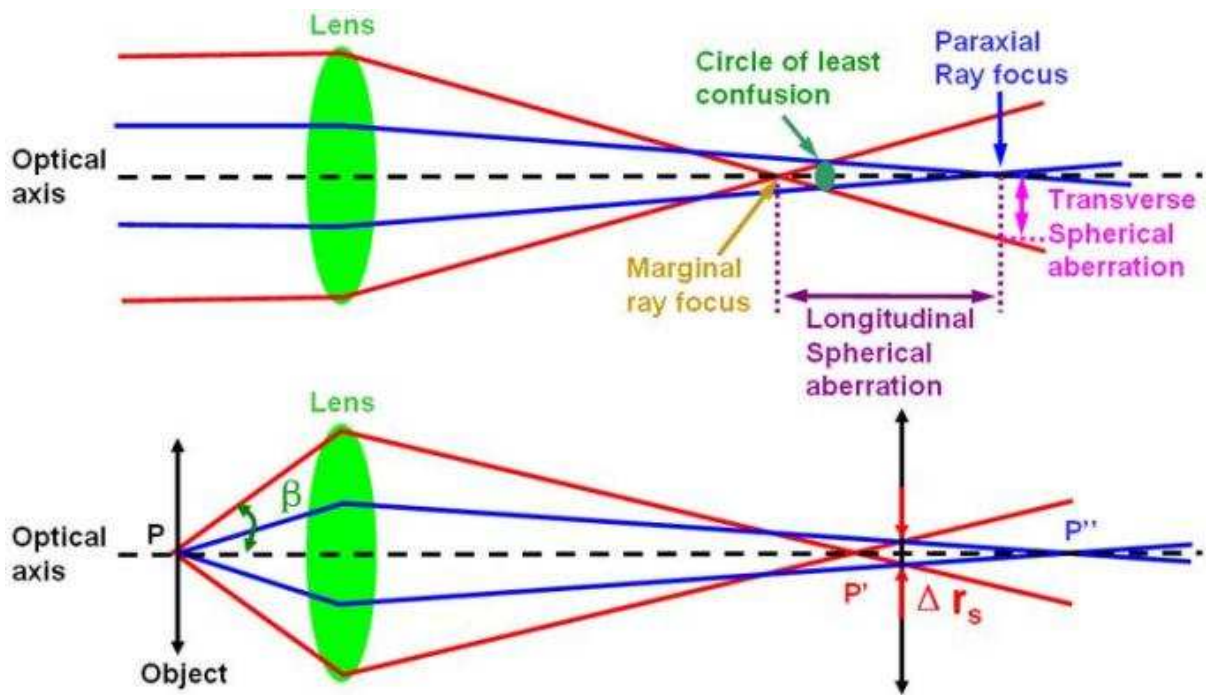
$$m_1 - m_2 = \frac{v}{u} - \frac{u}{v}$$

$$m_1 - m_2 = \frac{v^2 - u^2}{uv} = \frac{(v - u)(v + u)}{uv}$$

$$\text{Now } v - u = x, \frac{1}{f} = \frac{1}{v} + \frac{1}{u} \quad \text{or} \quad \frac{1}{f} = \frac{u + v}{uv}$$

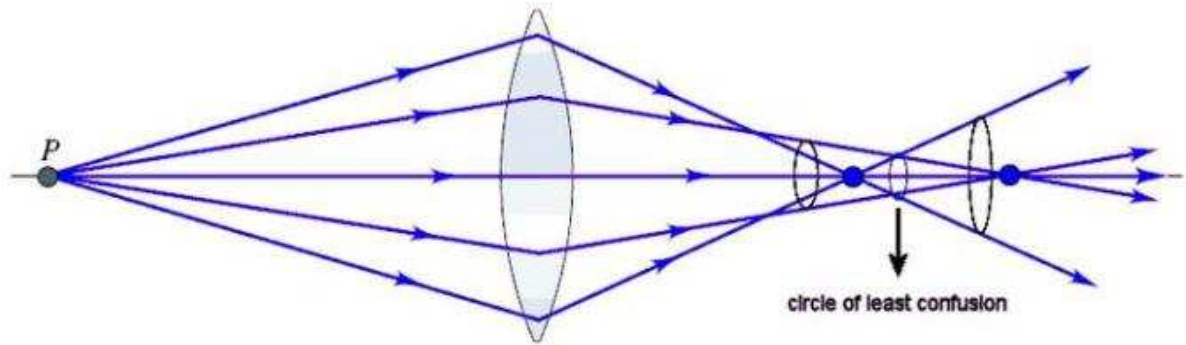
$$\therefore m_1 - m_2 = \frac{x}{f} \quad \text{or} \quad f = \frac{x}{m_1 - m_2}$$

Optics - 6) Circle of least confusion

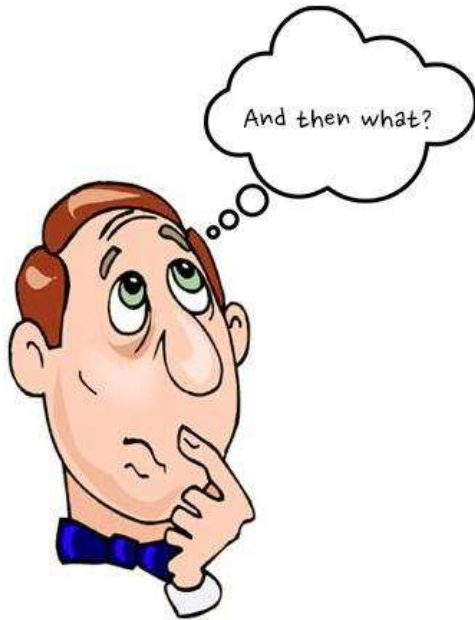


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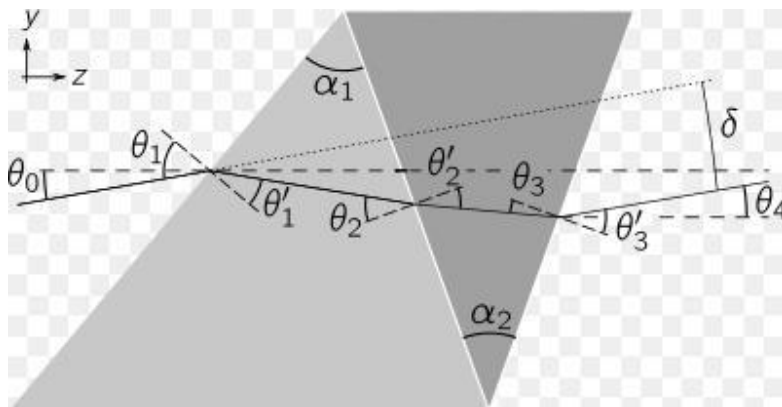
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Optics - 7)

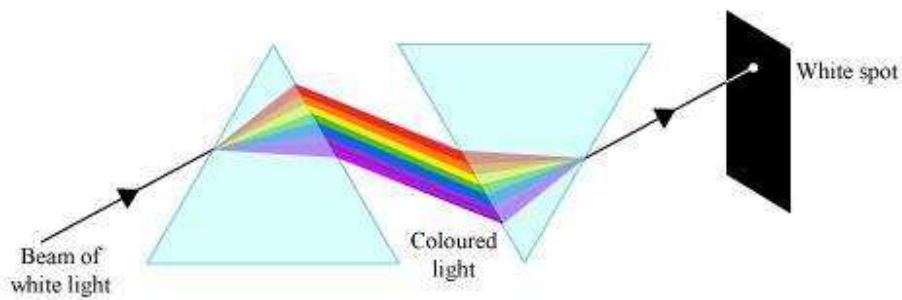
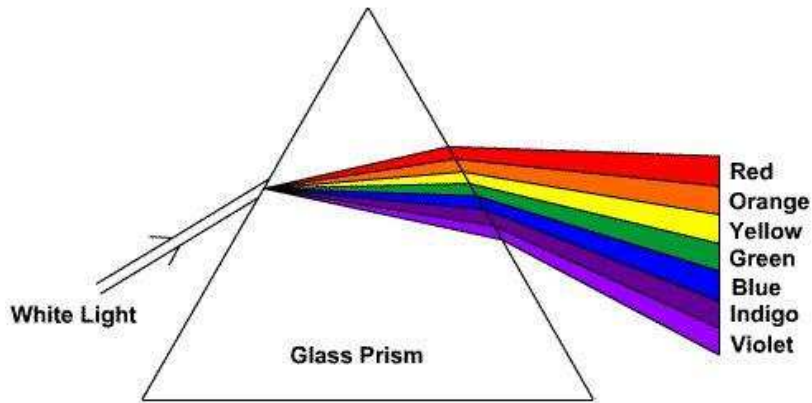


Deviation diagrams

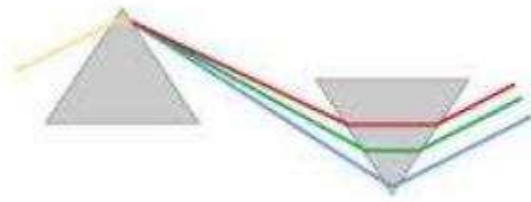


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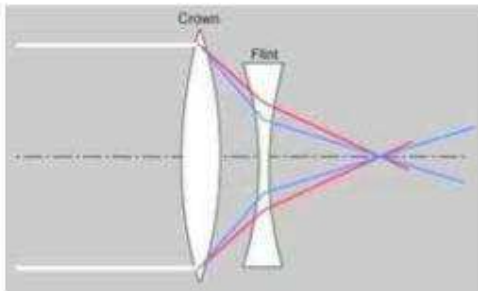
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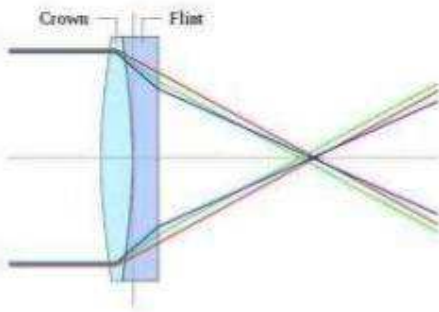
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Prisms with equal vertex angle (= light deviation power) and same glass type (= equal dispersion) can exactly cancel out color that is between them.



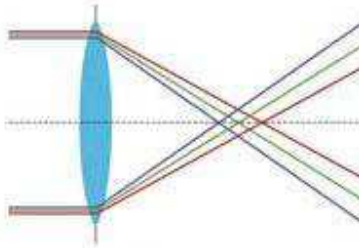
The color of a positive lens can be cancelled by an equal power negative lens of the same glass, but then the focal length of the lens pair would be zero, if they were in contact. Instead we want the negative lens to be a more dispersive glass than the positive lens, so that a weaker power negative lens can still cancel out the color and give a total power of the lens pair that is not zero. When the red and blue light rays come to the same focus primary color has been corrected.



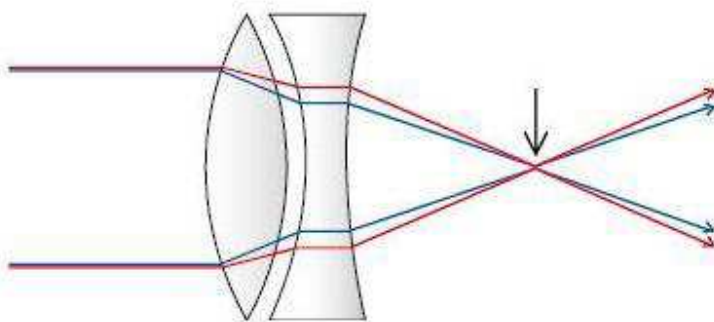
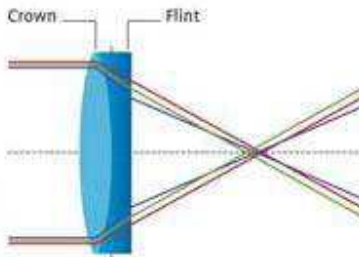
In a typical contact doublet the negative lens glass is about 1.5X to 2X more dispersive than the positive lens glass.

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Chromatic aberration

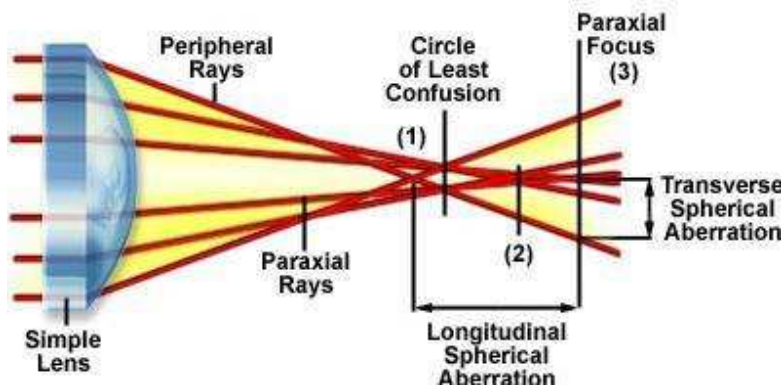


An achromatic lens



While this combination will also have a circle of least confusion

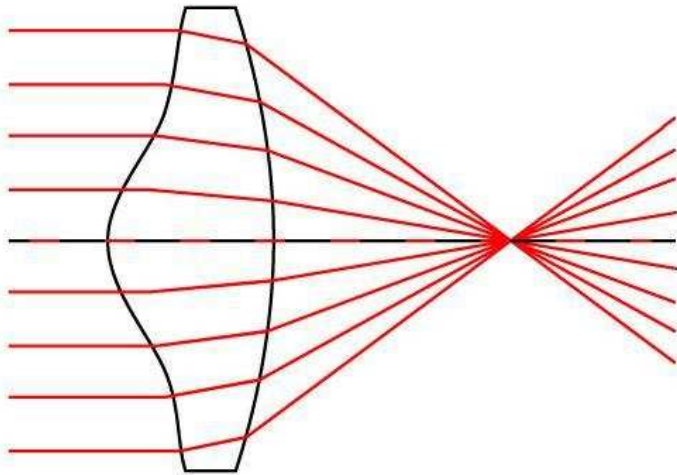
Longitudinal and Transverse Spherical Aberration



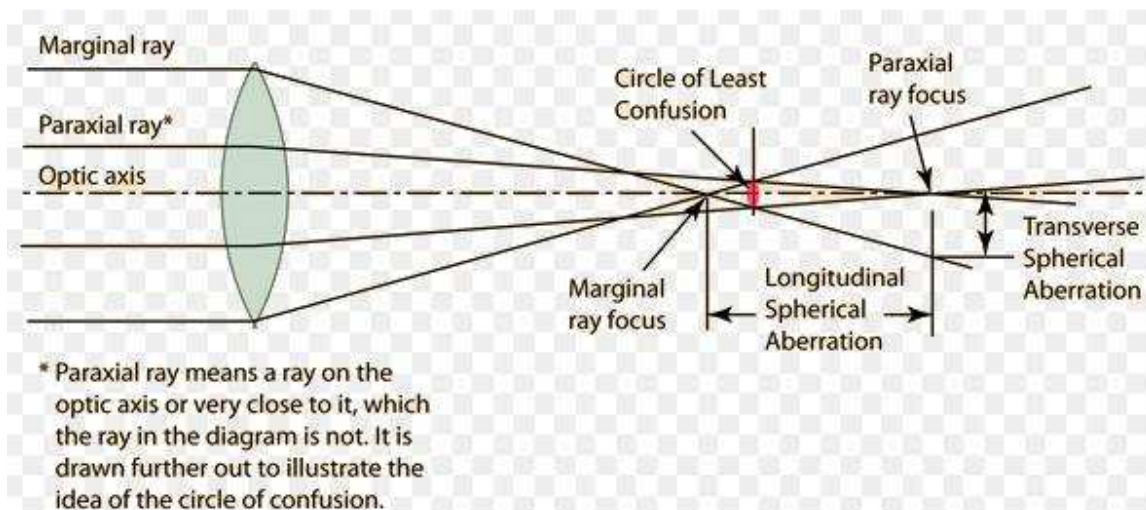
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Optics - 8) Aspherical lenses can be used to reduce axial spread (of paraxial rays), apart from stoppers or rather with combinations of stoppers.

Aspherical Lens

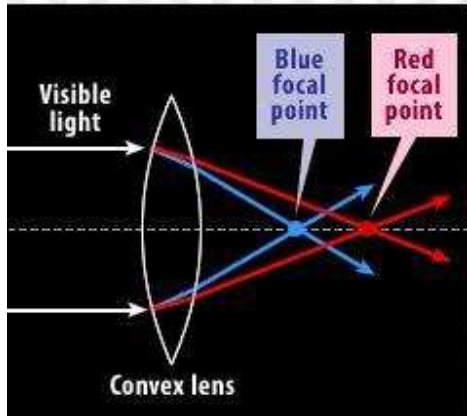


Remember more curved surface should face the light first. In plano-convex lens the convex part should face the light for better utilization of refraction properties. Also this minimizes the errors.

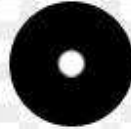


Optics - 9) The conical image of a point

Looking at only red and blue light:



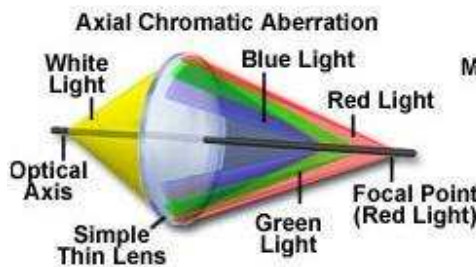
Result: A fringe of color may appear around bright objects seen through the lens:



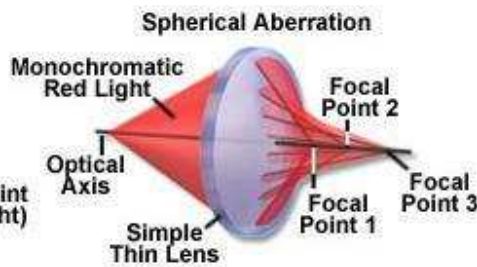
A star, as seen through a telescope without chromatic aberration



A star, as seen through a telescope with chromatic aberration (exaggerated)

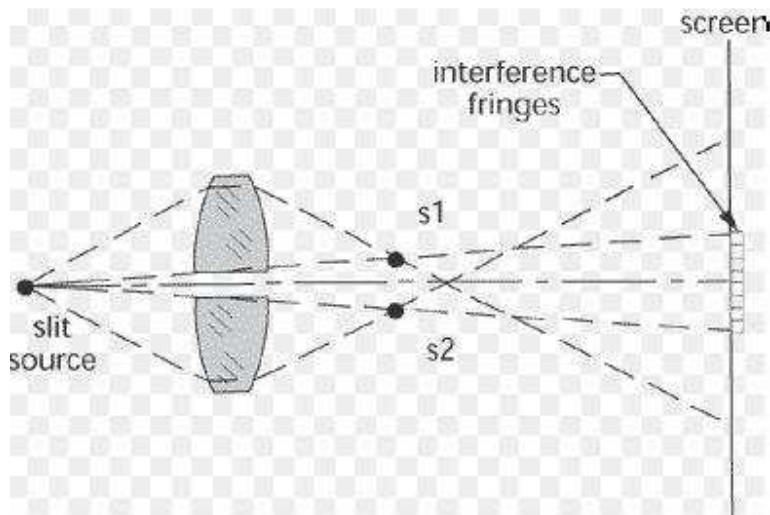


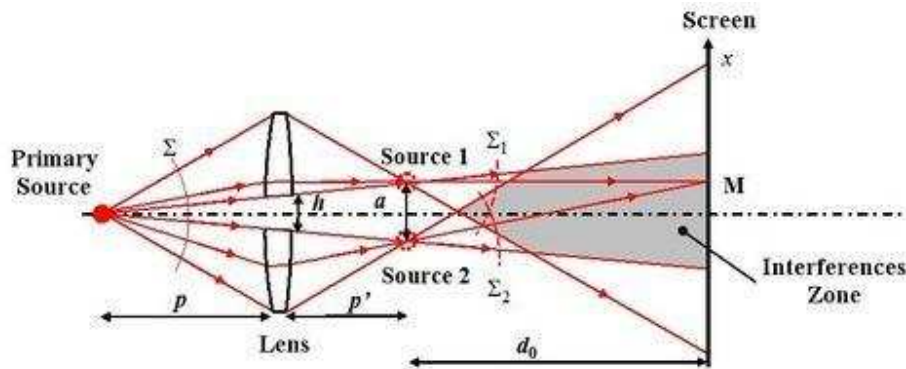
Axial Chromatic Aberration



Spherical Aberration

Optics - 10) Split lenses

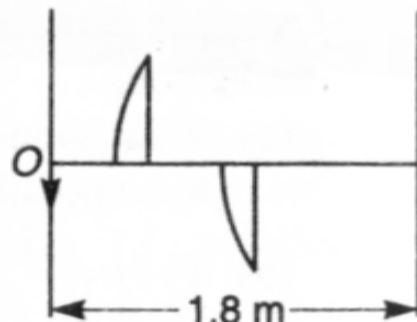




- Was asked in Physics Olympiad before being asked in IIT JEE

A thin plano-convex lens of focal length f is split into two halves. One of the halves is shifted along the optical axis. The

separation between object and image planes is 1.8 m. The magnification of the image formed by one of the half lens is 2. Find the focal length of the lens and separation between the halves. Draw the ray diagram for image formation. (1996, 5M)



Solution

For both the halves, position of object and image is same. Only difference is of magnification. Magnification for one of the halves is given as $2 (> 1)$. This can be for the first one, because for this, $|v| > |u|$. Therefore, magnification, $|m| = |v/u| > 1$

So, for the first half

$$|v/u| = 2 \quad \text{or} \quad |v| = 2|u|$$

Let $u = -x$ then $v = +2x$ and $|u| + |v| = 1.8\text{m}$

$$\text{i.e., } 3x = 1.8 \text{ m} \quad \text{or} \quad x = 0.6 \text{ m}$$

Hence, $u = -0.6 \text{ m}$ and $v = +1.2 \text{ m}$.

$$\text{Using, } \frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{1.2} - \frac{1}{-0.6} = \frac{1}{0.4}$$

$$\therefore f = 0.4 \text{ m}$$

For the second half

$$\frac{1}{f} = \frac{1}{1.2 - d} - \frac{1}{-(0.6 + d)}$$

$$\text{or } \frac{1}{0.4} = \frac{1}{1.2 - d} + \frac{1}{0.6 + d}$$

Solving this, we get $d = 0.6 \text{ m}$.

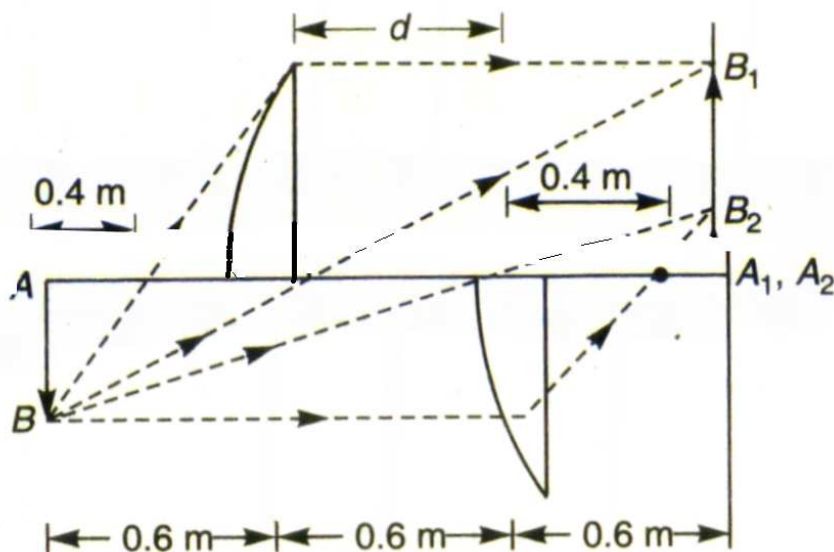
Magnification for the second half will be

$$m_2 = \frac{v}{u} = \frac{0.6}{-(1.2)} = -\frac{1}{2}$$

and magnification for the first half is

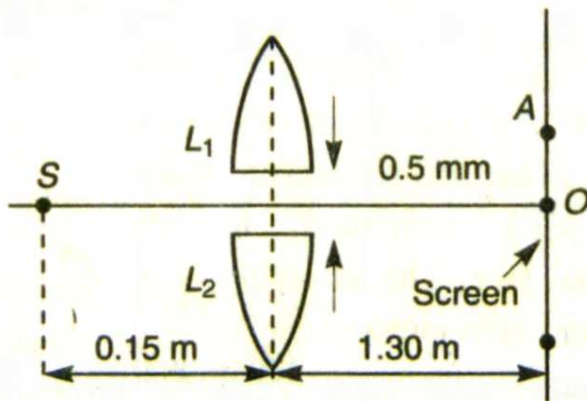
$$m_1 = \frac{v}{u} = \frac{1.2}{-(0.6)} = -2$$

The ray diagram is as follows :



In given figure, S is a monochromatic point source emitting light of wavelength $\lambda = 500\text{ nm}$. A thin lens of circular shape and focal length 0.10 m is cut into two identical halves L_1 and L_2 by a plane passing through a diameter. The two halves are placed symmetrically about the central axis SO with a gap of 0.5 mm . The distance along the axis from S to L_1 and L_2 is 0.15 m while that from L_1 and L_2 to O is 1.30 m . The screen at O is normal to SO . (1993, 5+1M)

Solution



If the third intensity maximum occurs at the point A on the screen, find the distance OA .

If the gap between L_1 and L_2 is reduced from its original value of 0.5 mm , will the distance OA increase, decrease, or remain the same.

Solution

(a) For the lens, $u = -0.15 \text{ m}$; $f = +0.10 \text{ m}$

Therefore, using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we have

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$$

$$= \frac{1}{(-0.15)} + \frac{1}{(0.10)}$$

or $v = 0.3 \text{ m}$

Linear magnification, $m = \frac{v}{u} = \frac{0.3}{-0.15} = -2$

Hence, two images S_1 and S_2 of S will be formed at 0.3 m from the lens as shown in figure. Image S_1 due to part 1

will be formed at 0.5 mm above its optic axis ($m = -2$). Similarly, S_2 due to part 2 is formed 0.5 mm below the optic axis of this part as shown.

Hence, $d =$ distance between S_1 and $S_2 = 1.5 \text{ mm}$

$$D = 1.30 - 0.30 = 1.0 \text{ m} = 10^3 \text{ mm}$$

$$\lambda = 500 \text{ nm} = 5 \times 10^{-4} \text{ mm}$$

Therefore, fringe width,

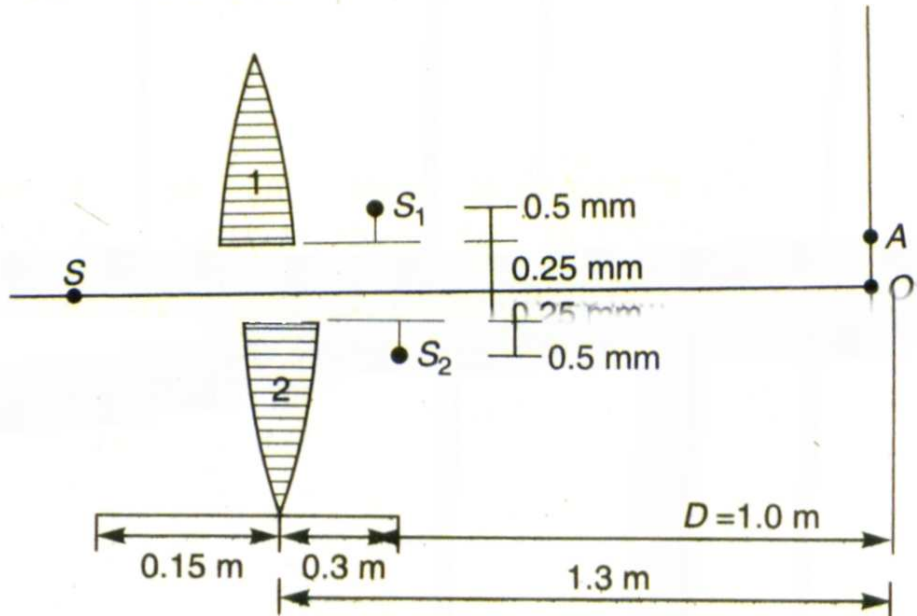
$$\omega = \frac{\lambda D}{d} = \frac{(5 \times 10^{-4})(10^3)}{(1.5)} = \frac{1}{3} \text{ mm}$$

Now, as the point A is at the third maxima

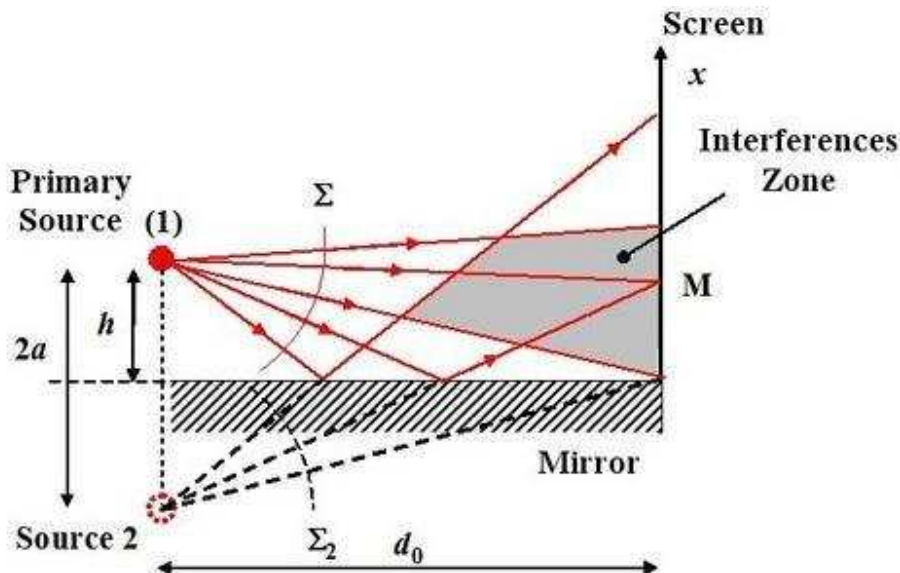
$$OA = 3\omega = 3(1/3) \text{ mm}$$

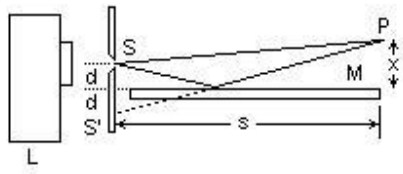
or $OA = 1 \text{ mm}$

(b) If the gap between L_1 and L_2 is reduced, d will decrease. Hence, the fringe width ω will increase or the distance OA will increase.

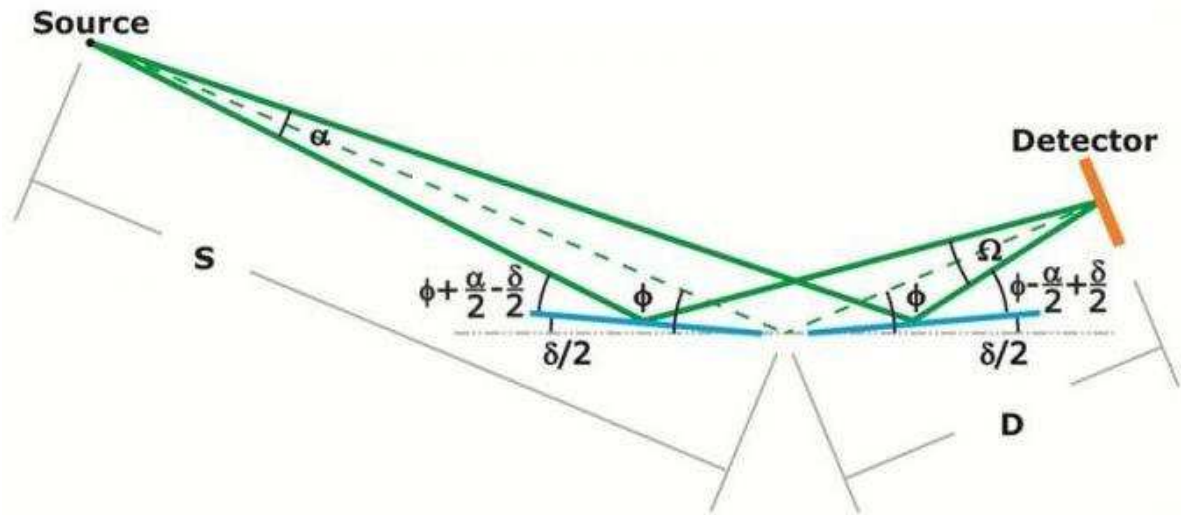


Optics - 11) Lloyd's Mirror

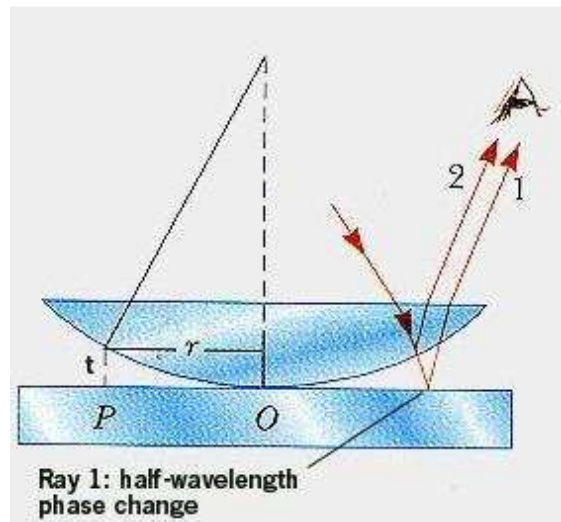
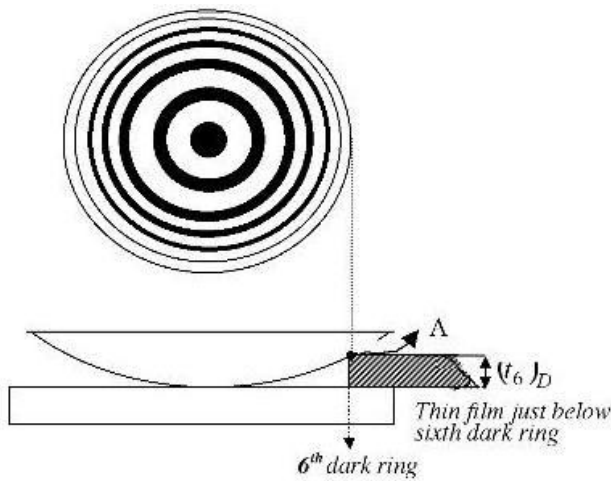




Lloyd's Mirror



Optics - 12) Newton's Rings



film thickness t .

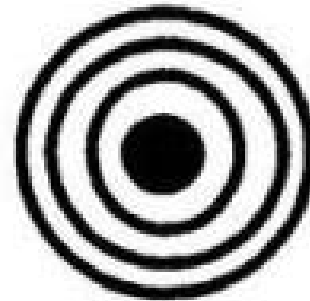
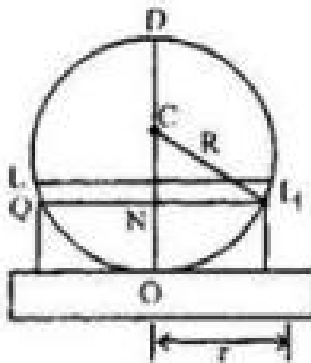


Fig. Newton Rings in Reflected Light

$$2t + \frac{\lambda}{2} = n\lambda$$

$$2t = \frac{(2n-1)\lambda}{2} \text{ for a bright ring } n = 1, 2, 3, \dots$$

&

$$2t = n\lambda \text{ for dark ring } n = 0, 1, 2, 3, \dots$$

From the property of the circle,

$$NP \times NQ = NO \times ND$$

Substituting values,

$$r \times r = t \times (2R - t) = 2Rt - t^2 \approx 2Rt \text{ approximately.}$$

\therefore

$$t = \frac{r^2}{2R}$$

Thus, for bright ring,

$$2 \cdot \frac{r^2}{2R} = \frac{(2n-1)\lambda}{2}$$

$$r = \frac{D}{2} \text{ where } D \text{ is diameter}$$

$$\frac{D^2}{4} = \frac{(2n-1)\lambda R}{2}$$

$$D_n = \sqrt{2(2n-1)\lambda R}$$

$$D_n \propto \sqrt{(2n-1)}$$

i.e., diameter of n^{th} bright ring is proportional to square root of odd natural number.

Optics - 13) Plano-Convex lens problems

The apparent thickness of a thick plano-convex lens is measured once with the plane face upward and then with the convex face upwards. The value will be :

- (a) More in the first case.
- (b) Same in the two cases
- (c) More in the II case
- (d) Can be any of the above depending on the value of its actual thickness

The apparent thickness in case (a)

$$OA' = \frac{\text{real } (OA)}{\mu} = \frac{t}{\mu}$$

In case (b) when the convex surface is placed down then refraction takes place through curved surface.

Object is in denser medium, then $\mu_2 = 1, \mu_1 = \mu$

$$\therefore \text{ by } \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{v} + \frac{\mu}{t} = \frac{1 - \mu}{-R}$$

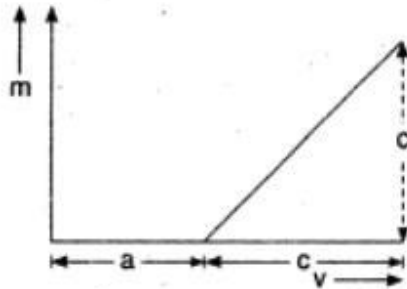
$$\frac{1}{v} = \frac{(\mu - 1)}{R} - \frac{\mu}{t}$$

$$v = \frac{Rt}{(\mu - 1)t - \mu R}$$

Clearly in the second case the apparent thickness is more.

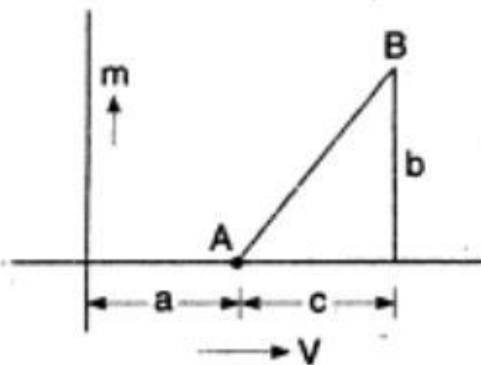
Optics - 14) Lens problems with graphs

The graph shows how the magnification m produced by a thin convex lens varies with image distance v . What was the focal length of the lens used ?



- (a) b/c (b) b/ca (c) bc/a (d) c/b

For point B, $m = b$ or $\frac{v}{u} = b$



$$\frac{(a+c)}{u} = b \text{ or } u = \left(\frac{a+c}{b}\right)$$

$$\frac{1}{f} = \frac{1}{(a+c)} + \frac{b}{(a+c)} = \frac{(1+b)}{(a+c)} \text{ or } f = \left(\frac{a+c}{1+b}\right) \dots(1)$$

Again for point A, $m = 0$

$$v = \infty, m = 0 = \frac{v}{u} = \frac{a}{u}$$

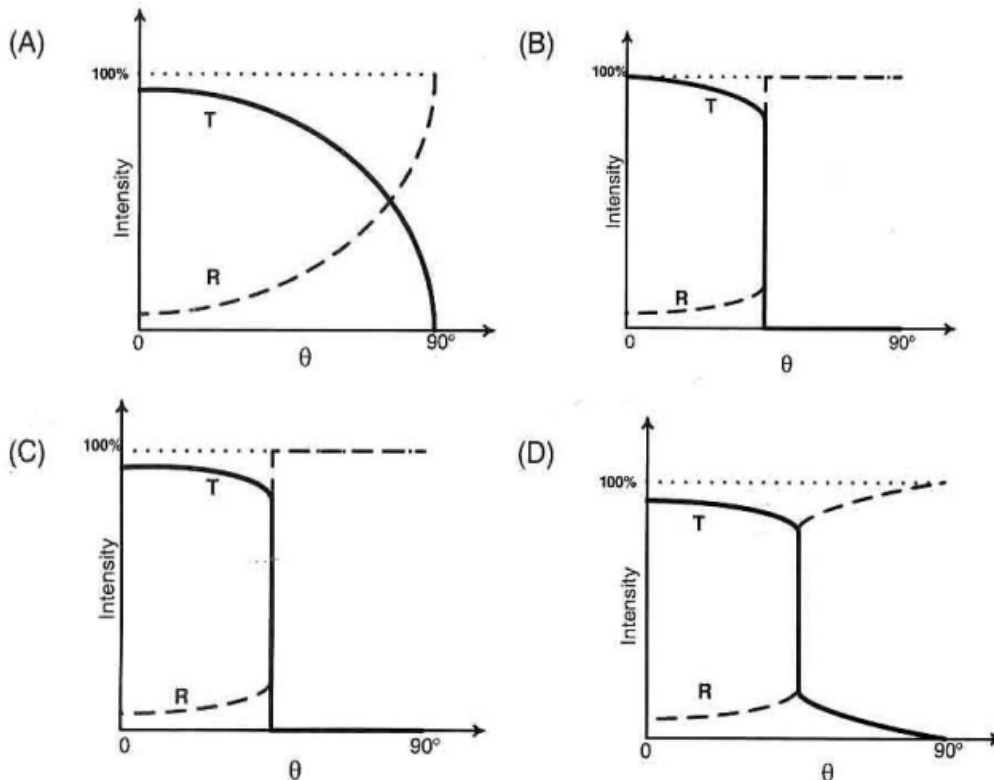
$$u = \infty \therefore v = a = f$$

Putting in (1)

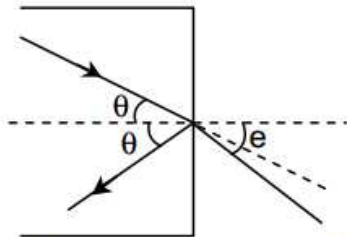
$$f = \frac{f+c}{1+b}, f+fb = f+c \text{ or } f = \frac{c}{b}$$

IIT JEE 2011

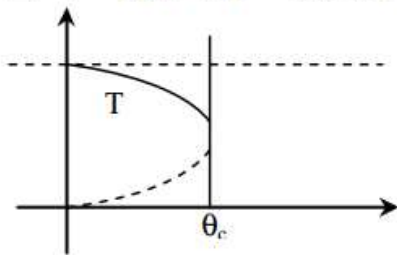
A light ray traveling in glass medium is incident on glass-air interface at an angle of incidence θ . The reflected (R) and transmitted (T) intensities, both as function of θ , are plotted. The correct sketch is



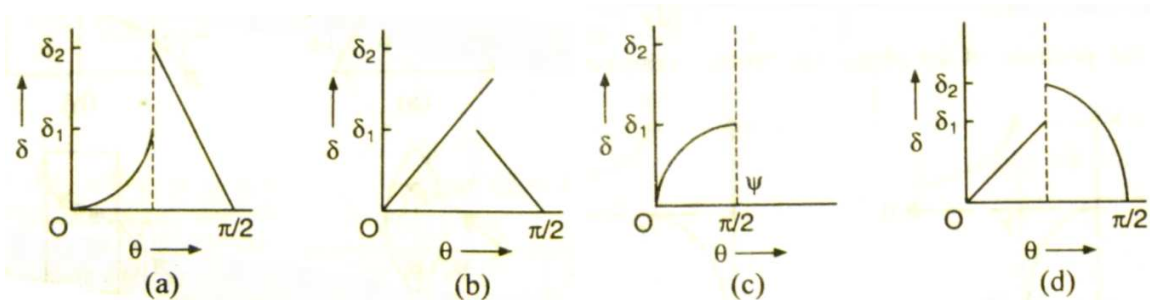
Answer [c]



When $\theta > \theta_c$, no ray will transmit
 $\Rightarrow T = 0, T + R = 100\%$ and $R > 0$



A ray of light travels from a medium of refractive index μ to air. Its angle of incidence in the medium is θ , measured from the normal to the boundary and its angle of deviation is δ . δ is plotted against θ which of the following best represents the resulting curve?



Answer (a)

In the above problem which of the following relations are correct

(a) $\psi = \sin^{-1} \left(\frac{1}{\mu} \right)$

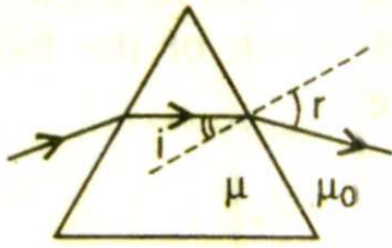
(b) $\psi = \frac{\pi}{2} - \sin^{-1} \left(\frac{1}{\mu} \right)$

(c) $\frac{\delta_2}{\delta_1} = \mu$

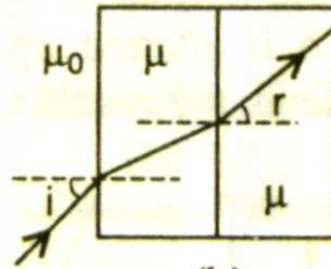
(d) $\frac{\delta_2}{\delta_1} = 2$

Answer (a) and (d)

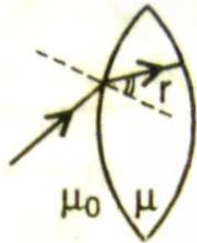
For which of the following cases $(\sin i/\sin r)$ is equal to (μ/μ_0) ?



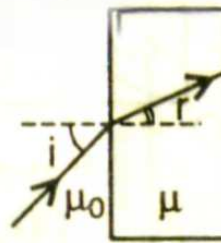
(a)



(b)



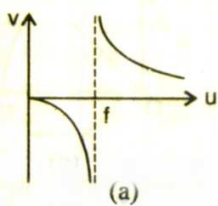
(c)



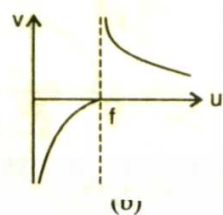
(d)

Answer - b, c, d

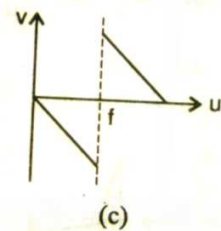
As the position of an object (u) from a concave mirror is varied, the position of the image (v) also varies. By letting u change from 0 to ∞ the graph between v and u will be ?



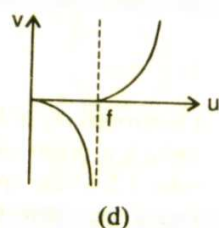
(a)



(b)



(c)

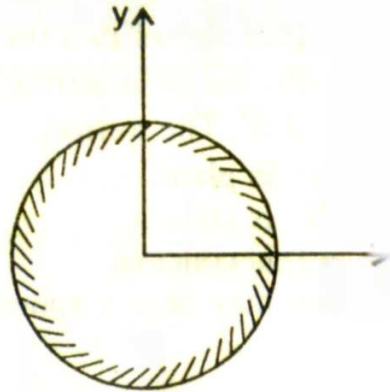


(d)

Answer - (a)

A reflecting surface is represented by the equation $x^2 + y^2 = a^2$. A ray travelling in negative x -direction is directed towards positive y -direction after reflection from the surface at some point P . Then the co-ordinates of point P are:

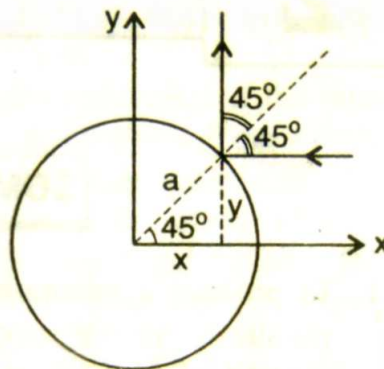
- (a) $(0.8a, 0.6a)$
- (b) $(0.6a, 0.8a)$
- (c) $(a, 0)$
- (d) none of the above



The ray diagram is shown in the figure.

$$x = \frac{a}{\sqrt{2}} \quad \text{and} \quad y = \frac{a}{\sqrt{2}}$$

$$\therefore P = \left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}} \right)$$



So Answer - (d)

Optics - 15) Lens immersed in a liquid

The focal length of lens of refractive index 1.5 in air is 30 cm. When it is immersed in a liquid of refractive index $\frac{4}{3}$, then its focal length in liquid will be

- (a) 30 cm
 - (b) 60 cm
 - (c) 120 cm
 - (d) 240 cm
- (BHU 2002)

We know that focal length in liquid

$$(f_m) = \left[\frac{\mu_g - 1}{(\mu_g / \mu_m) - 1} \right] \times f_a = \left[\frac{1.5 - 1}{(1.5 / 1.33) - 1} \right] \times 30$$

$$= \left[\frac{1.5 - 1}{1.125 - 1} \right] \times 30 = 120 \text{ cm.}$$

A bi-convex lens ($\mu = 1.5$) of focal length 0.2 m acts as a divergent lens of power one dioptre when immersed in a liquid. The refractive index of the liquid is :

- (a) 1.33 (b) 1.67 (c) 1.25 (d) 1.2

$$f_a = 20 \text{ cm, } f_w = -100 \text{ cm.}$$

$$\therefore \frac{f_w}{f_a} = \frac{(a\mu_g - 1)}{\left(\frac{a\mu_g}{a\mu_w} - 1 \right)} \quad \text{or} \quad -\frac{100}{20} = \frac{(1.5 - 1)}{\left(\frac{1.5}{a\mu_w} - 1 \right)}$$

$$\text{or} \quad \frac{1.5}{a\mu_w} - 1 = -\frac{0.5}{5} = -\frac{1}{10}$$

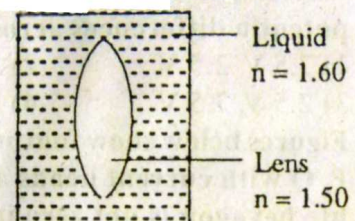
$$\therefore \frac{1.5}{a\mu_w} = 1 - \frac{1}{10} = \frac{9}{10}$$

$$a\mu_w = \frac{15}{9} = 1.67$$

Karnataka CET 1996 problem - Lens put in Slab with liquid

Shown in the figure is a convergent lens placed inside a cell filled with a liquid. The lens has a focal length +20 cm. when in air and its material has a refractive index 1.50. If the liquid has a refractive index 1.60, the focal length of the system is: (II-U-1-3)

- 1) -24 cm 2) -100 cm
3) +80 cm 4) -80 cm



$$\begin{aligned} \text{In air } \frac{1}{f} &= (n_g - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \\ \frac{1}{20} &= 0.5 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \rightarrow \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{1}{20 \times 0.5} = \frac{1}{10} \\ \text{In the liquid} \\ \frac{1}{f^l} &= \left(\frac{n_g}{n_e} - 1 \right) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \left(\frac{1.5}{1.6} - 1 \right) \times \frac{1}{10} \\ &= -\frac{1}{16} \times \frac{1}{10} = -\frac{1}{160} \quad \text{or} \quad f = -160 \text{ cm} \end{aligned}$$

If the formula was printed as +ve, then the absolute values of Radius will be taken.

Given ${}^a\mu_g = 3/2$ and ${}^a\mu_w = 4/3$. There is an equiconvex lens with radius of each surface equal to 20 cm. There is air in the object space and water in the image space. The focal length of lens is:

- (a) 80 cm (b) 40 cm (c) 20 cm (d) 10 cm

Solution :

$$\begin{aligned} \frac{{}^a\mu_w}{f} &= \frac{({}^a\mu_g - 1)}{R_1} - \frac{({}^a\mu_g - {}^a\mu_w)}{R_2} \\ &= \frac{\left(\frac{3}{2} - 1 \right)}{20} - \frac{\left(\frac{3}{2} - \frac{4}{3} \right)}{-20} = \frac{1}{40} + \frac{1}{120} = \frac{1}{30} \\ f &= \frac{4}{3} \times 30 = 40 \text{ cm} \end{aligned}$$

There can be problems with lens and different transparent materials on either side or both sides

A hollow double concave lens is made of very thin transparent material. It can be filled with air or either of two liquids L_1 or L_2 having refractive indices n_1 and n_2 respectively ($n_2 > n_1 > 1$). The lens will diverge a parallel beam of light if it is filled with: **(IIT 2000)**

- (a) air and placed in air (b) air and immersed in L_1
(c) L_1 and immersed in L_2 (d) L_2 and immersed in L_1

Solution : (d)

The lens maker's formula is:

$$\frac{1}{f} = \left(\frac{n_L}{n_m} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Where n_L = refractive index of lens material

n_m = refractive index of medium

In case of double concave lens R_1 is -ve and R_2 is +ve. Therefore

$\left(\frac{1}{R_1} - \frac{1}{R_2} \right)$ will be -ve.

For the lens to be diverging in nature, focal length f should be

negative or $\left(\frac{n_L}{n_m} - 1 \right)$ should be positive or $n_L > n_m$; but since

$n_2 > n_1$ (given), therefore the lens should be filled with L_2 and immersed in L_1 .

Optics - 16) Trick questions with distance of object, Image, focal length of lenses

The focal length of a convex lens is f . An object is placed at a distance x from its first focal point. The ratio of the size of the real image to that of the object is :

- (a) f/x^2 (b) x^2/f (c) f/x (d) x/f

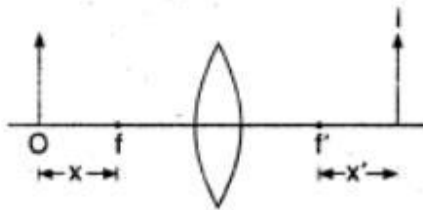
$$u = f + x, \frac{1}{f} = \frac{1}{v} - \frac{1}{-u} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{f+x} \quad \text{or} \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{f+x}$$

$$\frac{1}{v} = \frac{f+x-f}{f(f+x)} = \frac{x}{f(f+x)} \quad \text{or} \quad v = \frac{f(f+x)}{x}$$

$$\frac{v}{u} = \frac{f(f+x)}{x(f+x)} = \frac{f}{x}$$

An object is placed at a point distant x from the focus of a convex lens and its image is formed at I as shown in the figure. The distances x, x' satisfy the relation :



- (a) $\frac{x \times x'}{2} = f$ (b) $f^2 = xx'$
 (c) $x + x' = 2f$ (d) $x - x' = 2f$

the magnification is :

- (a) $\frac{f}{x+x'}$ (b) $\frac{x'}{x}$
 (c) $\frac{f}{x}$ (d) None of these

$$xx' = f^2, \text{ Newton's formula.}$$

$$u = f + x, v = f + x'$$

$$m = \frac{v}{u} = \frac{f + x'}{f + x}$$

$$x' = \frac{f^2}{x} \therefore m = \frac{f + f^2/x}{f + x}$$

$$m = \frac{f(x + f)}{x(x + f)} = \frac{f}{x}$$

A convex lens of focal length f is placed somewhere in between an object and a screen. The distance between the object and the screen is x . If the numerical value of the magnification produced by the lens is m , the focal length of the lens is:

(a) $\frac{mx}{(m+1)^2}$ (b) $\frac{mx}{(m-1)^2}$ (c) $\frac{(m+1)^2}{m}x$ (d) $\frac{(m-1)^2}{m}x$

Here,

$$x = u + v$$

$$m = \frac{f}{f+u} = \frac{(f-v)}{f}$$

For real image, m is -ve.

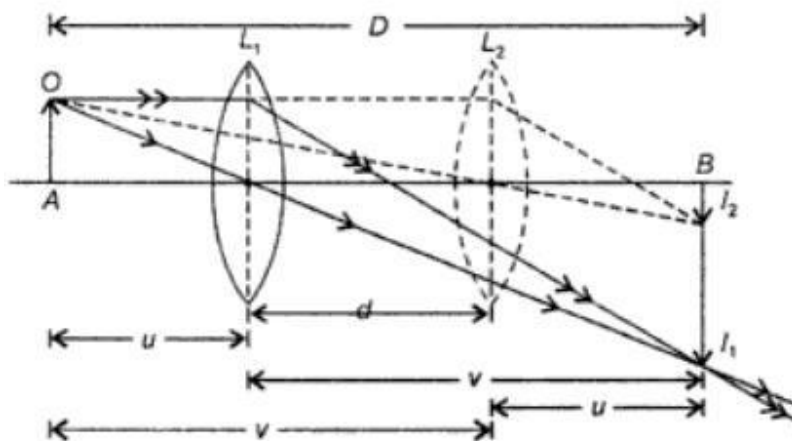
$$\therefore -m = f/(f+u) \quad \text{or} \quad u = \frac{-(m+1)}{m}f$$

$$\text{and} \quad -m = \frac{(f-v)}{f} \quad \text{or} \quad v = (m+1)f$$

$$\therefore x = (m+1)f + \frac{(m+1)}{m}f \quad \text{or} \quad f = \frac{mx}{(m+1)^2}$$

The distance between object and the screen is D . Real images of an object are formed on the screen for two positions of a lens separated by a distance d . The ratio between the sizes of two images will be:

- (a) D/d (b) D^2/d^2
 (c) $(D-d)^2/(D+d)^2$ (d) $\sqrt{(D/d)}$



Let O be the size of object held perpendicular to the principal axis of the lens. A real, inverted and magnified image of size I_1 is formed when the lens is at position L_1 . When the lens is shifted to position L_2 after moving to a distance d , a diminished image of size I_2 is formed.

The magnification produced by lens, when image size is I_1 .

$$m_1 = \frac{I_1}{O} = \frac{v}{u} \quad \dots(i)$$

The magnification produced by lens, when image size is I_2 .

$$m_2 = \frac{I_2}{O} = \frac{u}{v} \quad \dots(ii)$$

(By the principle of conjugate focii we can assume position of image as object position and vice-versa)

From equation (i) and (ii), we get

$$m_1 m_2 = \frac{I_1}{O} \times \frac{I_2}{O} = \frac{v}{u} \times \frac{u}{v}$$

or $m_1 m_2 = 1$

and $O = \sqrt{I_1 I_2}$

Again, from equation (i) and (ii)

$$\frac{m_1}{m_2} = \frac{I_1}{I_2} = \frac{v^2}{u^2}$$

From the figure, $D = u + v$

and $d = v - u$

Then $v = \frac{D+d}{2}$ and $u = \frac{D-d}{2}$

Hence, $\frac{m_1}{m_2} = \frac{I_1}{I_2} = \left(\frac{D+d}{D-d}\right)^2$

Using lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ and putting the value of

$$u = -\left(\frac{D-d}{2}\right) \text{ and } v = +\left(\frac{D+d}{2}\right), \text{ we get}$$

$$f = \frac{D^2 - d^2}{4D}$$

The focal length of lens can also be calculated by relation

$$f = \frac{d}{m_1 - m_2}$$

Thus

- (i) The minimum distance between the object and its real image is $4f$.
- (ii) If the distance between object and screen is greater than $4f$. There will be two positions separated by d for the lens which gives sharp image on the screen.
- (iii) As the lens is moved away from the source, the diminished image is formed.

(iv) Power of lens can also be calculated by the relation

$$P = \frac{4D}{D^2 - d^2}$$

A convex lens produces an image of real object

on a screen with magnification $\frac{1}{2}$ when the

lens is moved 30 cm towards the object, the

magnification of the image on the screen is 2.

Find the focal length of the lens.

Solution : Since, $f = \frac{d}{m_1 - m_2}$

Here $d = 30$ cm, $m_1 = 2$ and $m_2 = \frac{1}{2}$

So, $f = \frac{30}{2 - \frac{1}{2}} = 20$ cm.

A short linear object of length L lies on the axis of a spherical mirror of focal length f at a distance u from the mirror. Its image has an axial length L' equal to ?

(a) $L \left[\frac{f}{(u-f)} \right]^{1/2}$

(b) $L \left[\frac{(u+f)}{f} \right]^{1/2}$

(c) $L \left[\frac{(u-f)}{f} \right]^2$

(d) $L \left[\frac{f}{(u-f)} \right]^2$

Solution :

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad -\frac{dv}{v^2} - \frac{du}{u^2} = 0$$

i.e., $dv = -du[v/u]^2$

But $v = \frac{uf}{(u-f)}$ or $\frac{v}{u} = \frac{f}{(u-f)}$

So $dv = -du \left[\frac{f}{(u-f)} \right]^2$

Hence, $|dv| = L \left[\frac{f}{(u-f)} \right]^2$

A concave mirror of focal length f produces an image n times the size of the object. If the image is real, then the distance of the object from the mirror is:

- (a) $(n - 1)f$ (b) $[(n - 1)/n]f$
(c) $[(n + 1)f/n]$ (d) $(n + 1)f$

As the image is real it will be inverted and so

$$m = -(v/u) = -n, \quad \text{i.e.,} \quad v = nu$$

$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{1}{nu} + \frac{1}{u} = \frac{1}{-f}$$

$$\text{or} \quad \frac{(1+n)}{nu} = -\frac{1}{f} \quad \text{or} \quad u = -\frac{(n+1)}{n}f$$

i.e., object is in front of mirror at a distance $[(n + 1)f/n]$.

A convex mirror of focal length f produces an image $(1/n)$ th of the size of the object. The distance of the object from the mirror is:

- (a) nf (b) f/n (c) $(n + 1)f$ (d) $(n - 1)f$

Solution :

As the image formed by a convex mirror is always virtual or erect, so

$$m = -(v/u) = + (1/n) \quad \text{or} \quad v = -\frac{u}{n}$$

$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad -\frac{n}{u} + \frac{1}{u} = \frac{1}{+f}$$

$$\text{or} \quad \frac{-(n-1)}{u} = \frac{1}{f} \quad \text{or} \quad u = -(n-1)f$$

i.e., object is in front of mirror at a distance $(n - 1)f$.

Optics - 17) Application of Geometry in sphere to understand a plano-convex lens problem

Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If the speed of light in the material of the lens is 2×10^8 metres per sec, the focal length of the lens is :

- (a) 15 cm (b) 20 cm (c) 30 cm (d) 10 cm

Application of Sagitta Theorem

$$\text{R.I. of material of lens } \mu = \frac{c}{v} = \frac{3 \times 10^8}{2 \times 10^8} = 1.5$$

by Sagitta theorem

$$0.3(2R - 0.3) = 3 \times 3$$

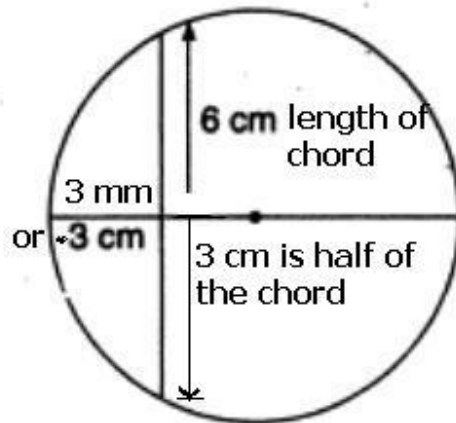
$$0.6R = 9 \quad (\text{neglecting } 0.09)$$

$$R = 15 \text{ cm}$$

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R} + \frac{1}{\infty} \right)$$

$$\frac{1}{f} = (1.5 - 1) \left(\frac{1}{15} \right) = \frac{1}{30}$$

$$\text{or } f = 30 \text{ cm.}$$



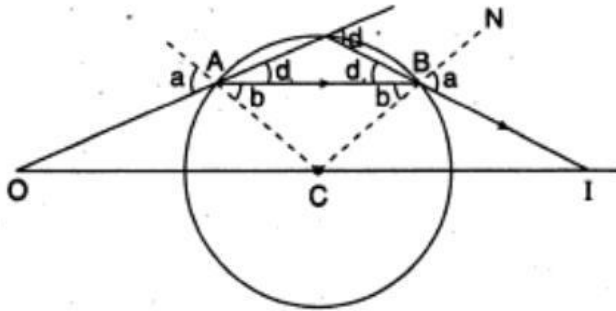
Optics - 18) Spherical lens

A ray of light falls on the surface of a spherical glass paper weight making an angle α with the normal and is refracted in the medium at an angle β . The angle of deviation of the emergent ray from the direction of the incident ray is :

- (a) $(\alpha - \beta)$ (b) $2(\alpha - \beta)$
 (c) $(\alpha - \beta) / 2$ (d) $(\beta - \alpha)$

Ray or Geometrical Optics by Prof. Subhashish Chattopadhyay SKMClasses Bangalore Useful for IIT-JEE, CBSE, I.Sc. PU-II, Boards, CET, CEE, PET, IGCSE IB AP-Physics and other exams

$\angle CAB = \angle CBA = \beta$ because in ΔABC



$AC = BC = \text{radius}$

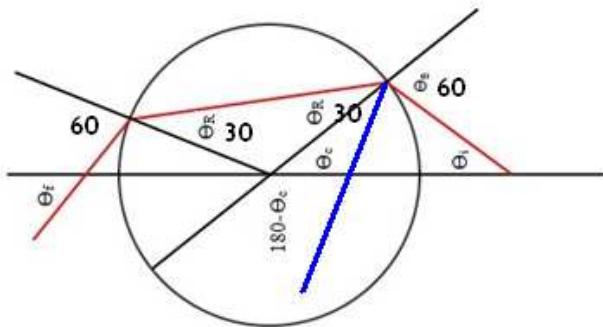
Naturally $\angle IBN = \alpha$

$\delta_1 = \alpha - \beta, \delta_2 = \alpha - \beta$

$\therefore \delta = \delta_1 + \delta_2 = 2(\alpha - \beta)$

A ray is incident on a sphere, with incidence angle of 60° . Refractive Index of the sphere is $\sqrt{3}$. The ray is reflected and refracted on the further surface. The angle between the reflected and refracted surface is ?

Answer 90°



$$\sin 60 / \sin r_1 = \sqrt{3} \Rightarrow \sin r_1 = 1/2 \Rightarrow r_1 = 30^\circ$$

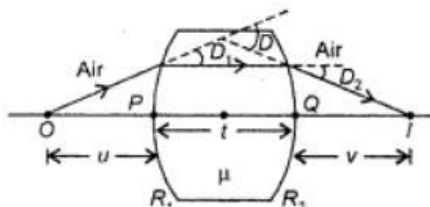
$$\sin i_2 / \sin r_2 = \sqrt{3} \Rightarrow i_2 = 60^\circ \text{ as } r_1 = r_2 = 30^\circ$$

$$\text{Angle of deviation } 180 - (r_2 + i_2) = 180 - 90 = 90$$

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Optics - 19) Thick lenses

Refraction through Thick Lens



- The focal length of thick lens,

$$\frac{1}{f} = (\mu - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(\mu - 1)t}{\mu R_1 R_2} \right]$$

- Power of thick lens,

$$P = P_1 + P_2 - \frac{P_1 P_2 t}{\mu}$$

Where, P_1 = Power of first refracting surface

$$P_1 = \frac{\mu - 1}{R_1}$$

and P_2 = Power of second refracting surface

$$P_2 = \frac{1 - \mu}{R_2}$$

A convergent thick lens has radii of curvature 10.0 cm and -6.0 cm, $\mu = 1.60$ and thickness $t = 5.0$ cm. Deduce its focal length.

Solution : Focal length of a lens of thickness t is given by

$$\frac{1}{f} = (\mu - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(\mu - 1)t}{\mu R_1 R_2} \right]$$

Here, $\mu = 1.60$, $R_1 = +10.0$ cm, $R_2 = -6.0$ cm and $t = 5.0$ cm.

$$\therefore \frac{1}{f} = (1.60 - 1) \left[\frac{1}{10.0} + \frac{1}{6.0} + \frac{(1.60 - 1) \times 5.0}{1.60 \times 10.0 \times (-6.0)} \right]$$

$$\text{or} \quad \frac{1}{f} = 0.60 \left[\frac{1}{10} + \frac{1}{6} - \frac{1}{32} \right]$$

$$\Rightarrow f = +7.14 \text{ cm.}$$

Optics - 20) Cauchy's formula for Refractive Index

$$n_{25^\circ\text{C}} = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$$

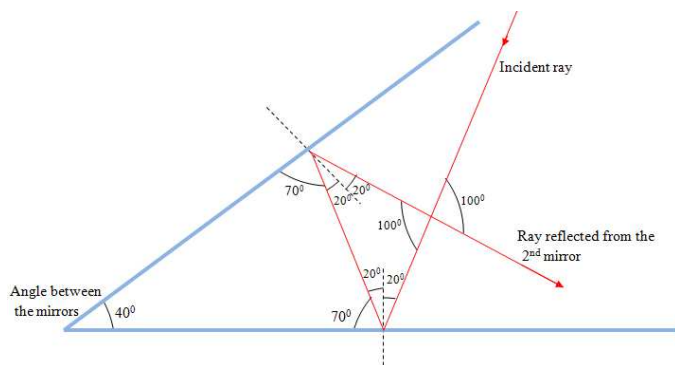
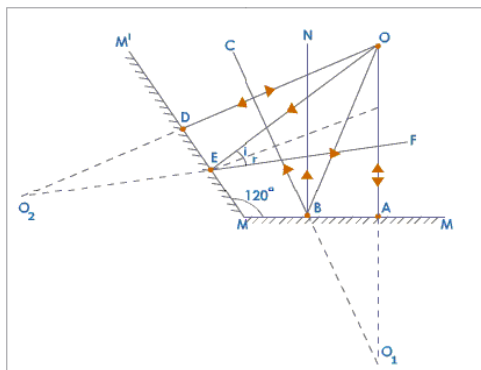
Cauchy's formula for μ

$$n(\lambda) = B + \frac{C}{\lambda^2},$$

Material	B	C (μm^2)
Fused silica	1.4580	0.00354
Borosilicate glass BK7	1.5046	0.00420
Hard crown glass K5	1.5220	0.00459
Barium crown glass BaK4	1.5690	0.00531
Barium flint glass BaF10	1.6700	0.00743
Dense flint glass SF10	1.7280	0.01342

-

Optics - 21) Reflection images in inclined mirrors



Number of images is given as greatest integer of $[(360/\theta) - 1]$

-

Optics - 22) Optics problems with vectors, 3D imagination

The x - y plane is boundary between two transparent media. Medium-1 with $z \geq 0$ has a refractive index $\sqrt{2}$ and medium 2 with $z \leq 0$ has refractive index $\sqrt{3}$. A ray of light in medium-1 given by vector $\vec{A} = 6\sqrt{3} \hat{i} + 8\sqrt{3} \hat{j} - 10\hat{k}$ is incident on the plane of separation, find the unit vector in the direction of the refracted ray in medium-2.

Solution: Let refracted ray be $\vec{r} = a\hat{i} + b\hat{j} - c\hat{k}$

$$\text{Normal to plane of incident and normal} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6\sqrt{3} & 8\sqrt{3} & -10 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= 8\sqrt{3} \hat{i} - 6\sqrt{3} \hat{j}$$

it must also be normal to refracted ray

$$\therefore \vec{r} \cdot \vec{n} = 0$$

$$\Rightarrow 8\sqrt{3} a - 6\sqrt{3} b = 0 \Rightarrow 4a = 3b$$

$$\Rightarrow b = \frac{4a}{3}$$

$$\therefore \cos(\pi - i) = \frac{(6\sqrt{3} \hat{i} + 8\sqrt{3} \hat{j} - 10\hat{k}) \cdot \hat{k}}{\left| 6\sqrt{3} \hat{i} + 8\sqrt{3} \hat{j} - 10\hat{k} \right| \left| \hat{k} \right|}$$

$$= \frac{-10}{20} = \frac{-1}{2} = \cos 120^\circ$$

$$\therefore i = 60^\circ$$

$$\sqrt{3} \sin r = \sqrt{2} \sin i = \sqrt{2} \times \frac{\sqrt{3}}{2} \Rightarrow \sin r = \frac{1}{\sqrt{2}}$$

$$r = 45^\circ$$

Now since angle between refracted ray and Normal = 45°

$$\therefore \cos 45^\circ = \frac{(a\hat{i} + b\hat{j} + c\hat{k}) \cdot \hat{k}}{\sqrt{a^2 + b^2 + c^2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sqrt{2} c = \sqrt{a^2 + b^2 + c^2}$$

$$\Rightarrow c^2 = a^2 + b^2 = a^2 + \frac{16a^2}{a} = \frac{25a^2}{a}$$

$$\Rightarrow c = \frac{\pm 5a}{3}$$

$$\therefore c = \frac{-5a}{3}$$

$$\therefore r = a\hat{i} + \frac{4a}{3}\hat{j} - \frac{5a}{3}\hat{k} = \frac{a}{3}(3\hat{i} + 4\hat{j} - 5\hat{k})$$

$$y = \frac{3\hat{i} + 4\hat{j} - 5\hat{k}}{\sqrt{50}} = \frac{1}{5\sqrt{2}}(3\hat{i} + 4\hat{j} - 5\hat{k})$$

Optics - 23) Problems with continuously varying refractive index (First asked in IPhO and then in IIT JEE)

A ray of light in air is incident at grazing angle ($i = 90^\circ$) on a long rectangular slab of a transparent medium of thickness $t = 1.0$ m. The point of incidence is the origin $A(0, 0)$.

The medium has a variable index of refraction $n(y)$ given by $n(y) = \sqrt{ky^{3/2} + 1}$ where $k = 1.0 \text{ m}^{-3/2}$.

The refractive index of air is 1. (i) Obtain a relation between the slope of the trajectory of the ray at a point $B(x, y)$ in the point. (ii) Obtain an equation for trajectory $y(x)$ of the ray in

the point. (iii) Determine the co-ordinates (x, y_1) of the point P where the ray intersects the upper surface of the slab-air boundary. (d) Indicate the path of the ray subsequently.

Solution:

Taking on arbitrary point P(x, y) refractive index at this point $n = \left(y^{3/2} + 1\right)^{1/2}$

from Snell's law $n \sin \theta = \text{constant}$ applying this for initial pt. (when ray is entering medium B) and at point.

$$1 \times \sin 90^\circ = \sqrt{\left(y^{3/2} + 1\right)} \sin i$$

$$\Rightarrow \sin i = \frac{1}{\sqrt{y^{3/2} + 1}} \quad \text{it can be seen that } i = \frac{\pi}{2} - \theta$$

$$\therefore \text{Slope} = \tan \theta = \cot i = \frac{dy}{dx}$$

$$(ii) \frac{dy}{dx} = \cot i = \frac{y^{3/4}}{1}$$

$$\Rightarrow \int y^{-3/4} dy = \int dx$$

$$\Rightarrow x = 4y^{1/4} + C$$

it passes through origin $\therefore C = 0$

$\therefore x = 4y^{1/4}$ is the equation of trajectory

when ray comes out of the mediums

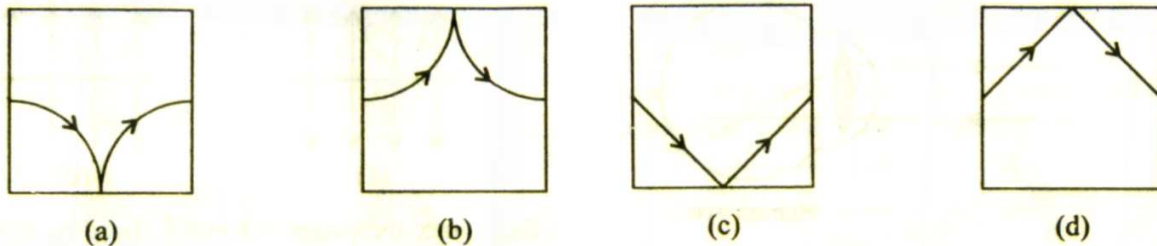
$$\text{then } x = 4 \times 1 = 4$$

\therefore Co-ordinate of pt- is (4, 1)

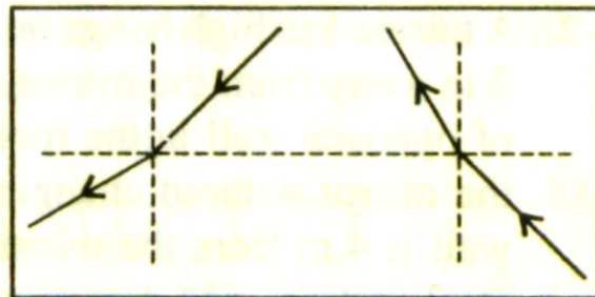
If medium on both sides are same, then angle with which the ray enters the medium = angle with which the ray comes out.

\therefore Ray will be parallel to x-axis.

A cubic container is filled with a liquid whose refractive index increases linearly from top to bottom. Which of the following represents the path of a ray of light inside the liquid?



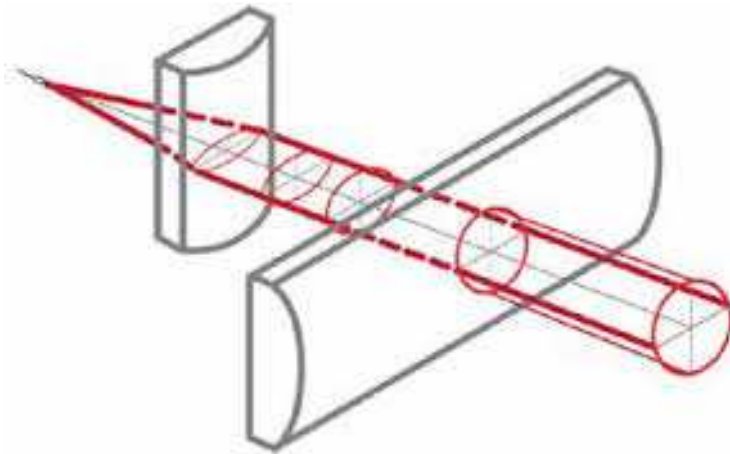
Since the refractive index is changing, the light cannot travel in a straight line in the liquid as shown in options (c) and (d). Initially, it will bend towards normal and after reflecting from the bottom it will bend away from the normal as shown in the figure.



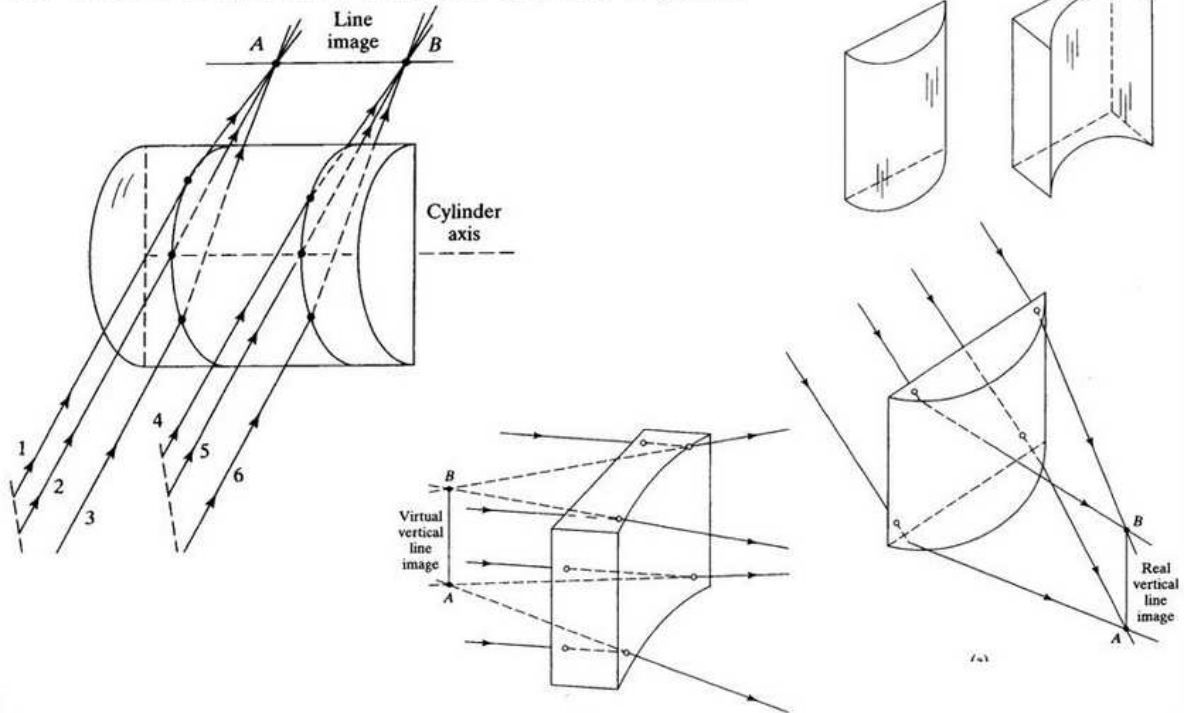
Optics - 24) Cylindrical lens (IIT JEE 1999)

A thin slice is cut out of a glass cylinder along a plane parallel to its axis. The slice is placed on a flat plate . The observed interference fringes from this combination shall be

1. Straight
2. Circular
3. Equally spaced
4. Having fringe spacing which increases as we go outwards

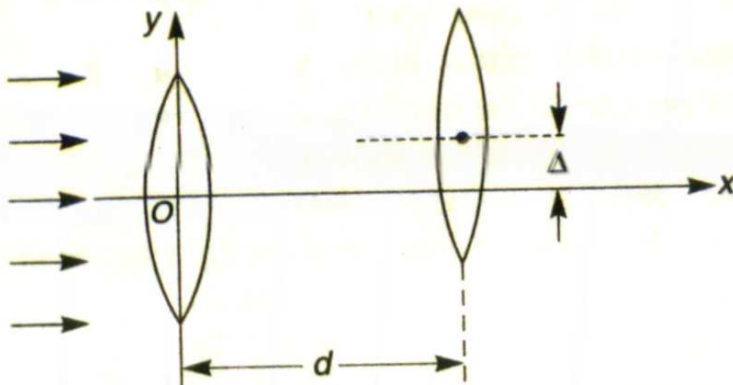


Cylindrical Lens: Cylindrical lens is a section of a cylindrical rod. One surface is cylindrical while the opposite is plane.



Optics - 25) Two lenses whose axis is not coinciding (IIT JEE 1993)

Two thin convex lenses of focal lengths f_1 and f_2 are separated by a horizontal distance d (where $d < f_1, d < f_2$) and their centres are displaced by a vertical separation Δ as shown in the figure. (1993)



Taking the origin of coordinates, O , at the centre of the first lens, the x and y -coordinates of the focal point of this lens system, for a parallel beam of rays coming from the left, are given by (1993; 2M)

(a) $x = \frac{f_1 f_2}{f_1 + f_2}, y = \Delta$

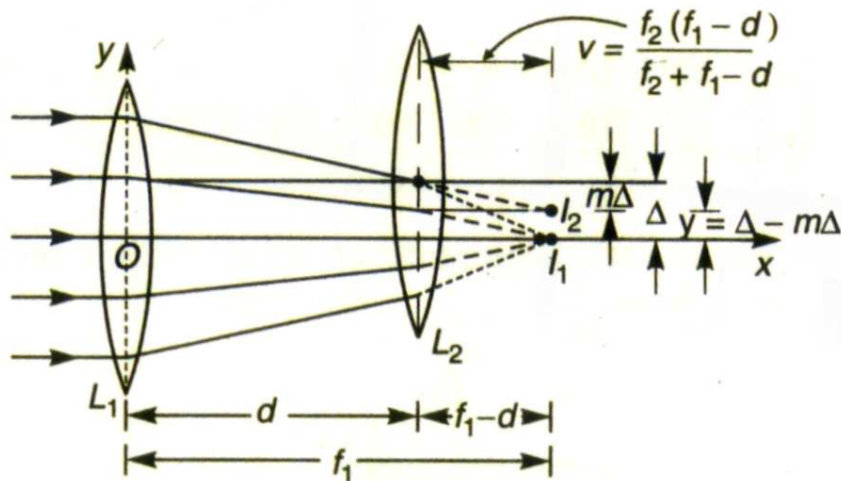
(b) $x = \frac{f_1(f_2 + d)}{f_1 + f_2 - d}, y = \frac{\Delta}{f_1 + f_2}$

(c) $x = \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}, y = \frac{\Delta(f_1 - d)}{f_1 + f_2 - d}$

(d) $x = \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}, y = 0$

Solution

From the first lens parallel beam of light is focused at its focus *i.e.*, at a distance f_1 from it. This image I_1 acts as virtual object for second lens L_2 . Therefore, for L_2



$$u = + (f_1 - d), f = + f_2$$

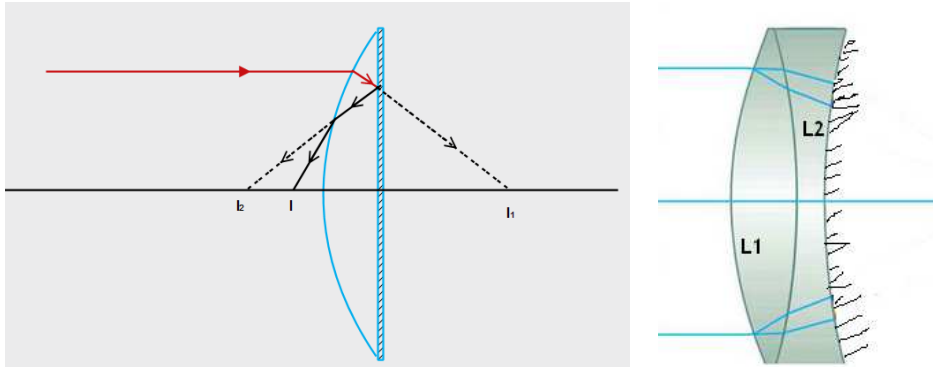
$$\therefore \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{f_2} + \frac{1}{f_1 - d}$$

<https://archive.org/details/IITJEE1993OpticsInterestingShiftedLensImageMagnificationAndPosition>

Optics - 26) Painted lens or Combination of lenses where the last one is painted (silvered)

If I am recalling correctly IIT JEE and other exams (till 2016) had more than 10 questions of this kind. Most books do not discuss the easy formula of $-1/F = 2/f_{L1} + 2/f_{L2} - 1/f_m$

(In 1990 I had derived this formula of my own for quick solving of this kind of problems)



F_m is focal length of the mirror as $R/2$ +ve or -ve as per conditions

The plane face of a plano-convex lens is silvered. If μ be the refractive index and R , the radius of curvature of curved surface, then the system will behave like a concave mirror of radius of curvature:

- (a) μR (b) $R/(\mu - 1)$
 (c) R^2/μ (d) $[(\mu + 1)/(\mu - 1)]R$

Solution :

Focal length of planar side is $f_m = R/2 = -\infty$

$$\frac{1}{f_l} = (\mu - 1) \left(\frac{1}{R} \right)$$

by lens makers formula. R is positive because center of curvature is on right side

$$\text{Use } -1/F = 2/f_{L1} - 1/f_m \quad \text{or } 1/F = -\frac{2(\mu - 1)}{R} \quad \text{or } F = \frac{-R}{2(\mu - 1)}$$

$$R \text{ (equivalent) } = 2F = \frac{-R}{(\mu - 1)}$$

We don't have to use the formula $-1/F = 2/f_{L1} + 2/f_{L2} - 1/f_m$ for every problem

See a Karnataka CET problem of 2004 (Was also asked in IIT JEE and solved in "Concepts of Physics by Professor H C Verma)

A thin plano-convex lens acts like a concave mirror of focal length 0.2 m, when silvered on its plane surface. The refractive index of the material of lens is 1.5. The radius of curvature of the convex surface of the lens will be:

[CET (Karnataka) 2004]

- (a) 0.1 m (b) 0.2 m (c) 0.4 m (d) 0.8 m

Solution :

Given focal length of mirror when its plane surface is silvered (f_m) = 0.2 m. Radius of curvature of curved surface (R_1) = R ; radius of curvature of plane side (R_2) = ∞ ; refractive index of the material of lens (μ) = 1.5.

Since a thin plano-convex lens acts like a concave mirror when silvered on its plane surface, therefore focal length of lens (f) = $2 \times f_m = 2 \times 0.2 = 0.4$ m.

We know that

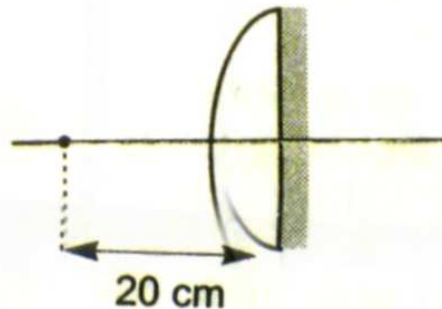
$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

or
$$\frac{1}{0.4} = (1.5 - 1) \left(\frac{1}{R} - \frac{1}{\infty} \right) = \frac{0.5}{R}$$

$\therefore R = 0.2 \text{ m}$

IIT JEE 2006

A point object is placed at a distance of 20 cm from a thin planoconvex lens of focal length 15 cm. The plane surface of the lens is now silvered. The image created by the system is at (2006, 3M)



- (a) 60 cm to the left of the system
- (b) 60 cm to the right of the system
- (c) 12 cm to the left of the system
- (d) 12 cm to the right of the system

Solution :

Long method

$$\text{Refraction from lens : } \frac{1}{v_1} - \frac{1}{-20} = \frac{1}{15}$$

$$\therefore v = 60 \text{ cm} \quad \xrightarrow{\text{+ ve direction}}$$

ie, first image is formed at 60 cm to the right of lens system.

Reflection from mirror

After reflection from the mirror, the second image will be formed at a distance of 60 cm to the left of lens system.

Refraction from lens

$$\frac{1}{v_3} - \frac{1}{60} = \frac{1}{15} \quad \longleftarrow \text{+ ve direction}$$

or $v_3 = 12 \text{ cm}$

Therefore, the final image is formed at 12 cm to the left of the lens system.

Shorter Method

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\text{Use } F = \frac{-R}{2(\mu - 1)} \quad \text{and} \quad (1.5 - 1) \left(\frac{1}{R} - \frac{1}{\infty} \right) = \frac{0.5}{R} = 1/(2R)$$

$$1/15 = 1/2R \Rightarrow 15 = 2R \Rightarrow R = 7.5 \text{ cm}$$

$$F = -7.5 / (2 \times 0.5) = -7.5 = -15/2$$

Using $1/v + 1/u = 1/F$ for equivalent mirror

$$1/v + 1/(-20) = 1/(-7.5)$$

$$\Rightarrow 1/v = 1/20 - 2/15 = (3 - 8)/60 = -5/60 = -1/12$$

$$\Rightarrow v = -12 \text{ cm}$$

Even more shorter method

If I am appearing for an exam I would have done $-1/F = 2/f_{L1} - 1/f_m$

$$\text{So } -1/F = 2/(15) - 1/(\infty) = 1/7.5 - 0 \Rightarrow F = -7.5 \text{ cm}$$

Then Using $1/v + 1/u = 1/F$ for equivalent mirror

$$1/v + 1/(-20) = 1/(-7.5)$$

$$\Rightarrow 1/v = 1/20 - 2/15 = (3 - 8)/60 = -5/60 = -1/12$$

$$\Rightarrow v = -12 \text{ cm}$$

-

IIT JEE 1978

A pin is placed 10 cm in front of a convex lens of focal length 20 cm and made of a material of refractive index 1.5. The convex surface of the lens farther away from the pin is silvered and has a radius of curvature of 22 cm. Determine the position of the final image. Is the image real or virtual? (1978)

Let us use $-1/F = 2/f_{L1} - 1/f_m$

And $\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$ giving $1/20 = 0.5 (1/R_1 - 1/(-22))$ or $R_1 = 55/3$

R_1 actually is not required. We can find f_m as $R_2 / 2 = -11$ cm

So $-1/F = 2/20 - 1/(-11) = 1/10 + 1/11 = 21/110$

or $F = -110/21$ (not required ! $1/F = -21/110$ is enough)

Using mirror formula $1/v + 1/u = 1/F$

So $1/v + 1/(-10) = -21/110$

$\Rightarrow 1/v = 1/10 - 21/110 = (11 - 21)/110 = -10/110 = -1/11$

$\Rightarrow v = -11$ cm

virtual image on left at 11 cm

(Now do you guys see that even though we got problems of this kind since 1978 and before, but yet the formula is not there in every book !)

-

IIT JEE 1979

The radius of curvature of the convex face of a planoconvex lens is 12 cm and its $\mu = 1.5$.

(a) Find the focal length of the lens. The plane face of the lens is now silvered.

(b) At what distance from the lens will parallel rays incident on the convex surface converge ?

(c) Sketch the ray diagram to locate the image, when a point object is placed on the axis 20 cm from the lens.

(d) Calculate the image distance when the object is placed as in (c)

(1979)

Now you know that this problem can be solved by 3 different ways.

The longest method being successive image method. Meaning find the first image due to lens, then 2nd image due to silvered surface as mirror. The 3rd and final image is due to light travelling from right to left through the lens again.

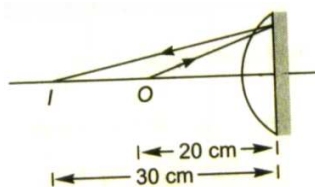
I will discuss the shorter methods

$$\begin{aligned}
 \text{(a)} \quad \frac{1}{f} &= (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \\
 &= (1.5 - 1) \left(\frac{1}{12} - \frac{1}{\infty} \right) \\
 &= \frac{1}{24} \\
 \therefore f &= +24 \text{ cm}
 \end{aligned}$$

(b) use $-1/F = 2/f_L$ so $F = -12 \text{ cm}$

The system will act as a concave mirror of focal length 12 cm. The parallel rays will converge at 12 cm left of this silvered lens.

(c)



(d)

Using mirror formula

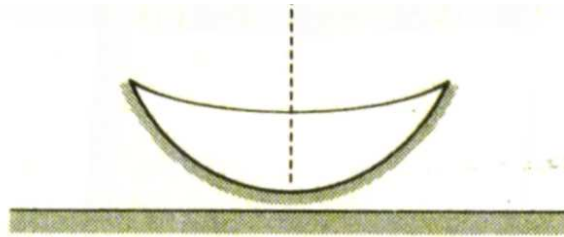
$$\frac{1}{v} - \frac{1}{20} = \frac{-1}{12}$$

Solving we get $v = -30 \text{ cm}$.

Therefore the image will be formed at a distance of 30 cm to the left of system.

IIT JEE 1981

The convex surface of a thin concavo-convex lens of glass of refractive index 1.5 has a radius of curvature



20 cm. The concave surface has a radius of curvature 60 cm. The convex side is silvered and placed on a horizontal surface. (1981, 2M)

- Where should a pin be placed on the optic axis such that its image is formed at the same place?
- If the concave part is filled with water of refractive index $\frac{4}{3}$, find the distance through which the pin should be moved, so that the image of the pin again coincides with the pin.

I will prefer to solve this by $-1/F = 2/f_{L1} + 2/f_{L2} - 1/f_m$ (note it was a 2 marks problem)

While for practice and to know how successive image method of solving works see ...

Image of object will coincide with it if ray of light after refraction from the concave surface fall normally on concave mirror so formed by silvering the convex surface. Or image after refraction from concave surface should form at centre of curvature of concave mirror or at a distance of 20 cm on same side of the combination. Let x be the distance of pin from the given optical system.

Applying.

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

With proper signs

$$\frac{1.5}{-20} - \frac{1}{-x} = \frac{1.5 - 1}{-60}$$

or
$$\frac{1}{x} = \frac{3}{40} - \frac{1}{120} = \frac{8}{120}$$

$\therefore x = \frac{120}{8} = 15 \text{ cm}$

(b)

Now, before striking with the concave surface, the ray is first refracted from a plane surface. So, let x be the distance of pin, then the plane surface will form its image at a distance $\frac{4}{3}x$ ($h_{\text{app.}} = \mu h$) from it.

Now, using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ with proper signs,

we have
$$\frac{1.5}{-20} - \frac{4/3}{-4x} = \frac{1.5 - 4/3}{-60}$$

or
$$\frac{1}{x} = \frac{3}{40} - \frac{1}{360}$$

or
$$x = 13.84 \text{ cm}$$

$\therefore \Delta x = x_1 - x_2$
 $= 15 \text{ cm} - 13.84 \text{ cm}$
 $= 1.16 \text{ cm} \quad \text{(downwards)}$

Now can you guys check the results using $-1/F = 2/f_{L1} + 2/f_{L2} - 1/f_m$

A plano-convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now this lens has been used to form the image of an object. At what distance from this lens an object be placed in order to have a real image of the size of the object? (AIEEE 2004)

- (a) 20 cm (b) 30 cm
(c) 60 cm (d) 80 cm

Solution :

To obtain the real image of the size of the object, the object must be placed at the centre of curvature of the equivalent mirror formed as a result of silvering

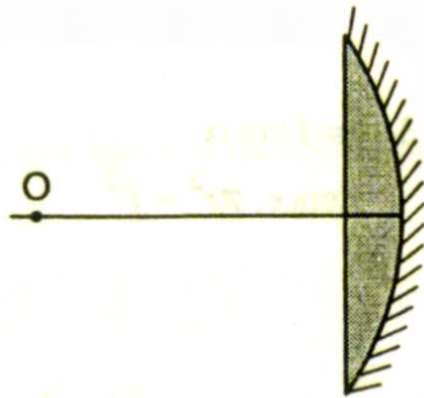
$$\frac{1}{F} = \frac{2}{f_l} + \frac{1}{f_m}$$

$$\text{and } \frac{1}{f_l} = (1.5 - 1) \left(\frac{1}{\infty} - \frac{1}{-30} \right) = \frac{1}{60}$$

$$\text{and } f_m = 15 \text{ cm}$$

$$\therefore F = 10 \text{ cm}$$

Hence, object should be placed at 20 cm from the lens because radius of curvature of equivalent mirror = $2F = 2 \times 10 = 20$ cm. Hence, option (a) is correct.



Video explanations of Painted or Silvered lenses

<https://archive.org/details/PaintedLensIITJEEProblemImageNeedsToCoincideWithObjectHCVermaPart1>

Optics - 27) Image speed when object is moving as seen from various mirrors and lenses (concave, convex, silvered etc)

Mirror formula ($1/v + 1/u = 1/f$) or Lens formula ($1/v - 1/u = 1/f$) have to be differentiated to find du/dt or dv/dt

A luminous point is moving at speed v_0 towards a spherical mirror, along its axis. Then the speed at which the image of this point object is moving is given by: (with R = radius of curvature and u = object distance)

(a) $v_i = -v_0$ (b) $v_i = -v_0 \left(\frac{R}{2u - R} \right)$
 (c) $v_i = -v_0 \left(\frac{2u - R}{R} \right)$ (d) $v_i = -v_0 \left(\frac{R}{2u - R} \right)^2$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad -\frac{1}{v^2} \frac{dv}{dt} - \frac{1}{u^2} \frac{du}{dt} = 0$$

$$\therefore \frac{dv}{dt} = v_i = -\left(\frac{v}{u}\right)^2 \frac{du}{dt} = -\left(\frac{v}{u}\right)^2 v_0$$

Now, $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{2}{R} - \frac{1}{u} = \frac{2u - R}{Ru}$

$$\therefore v = \frac{uR}{2u - R}$$

$$\therefore v_i = -\left(\frac{v}{u}\right)^2 v_0 = -v_0 \left(\frac{R}{2u - R}\right)^2$$

Optics - 28) Slab with a hole or gap, then may be filled with liquid etc

Given ${}^a\mu_g = 3/2$ and ${}^a\mu_w = 4/3$. There is an equiconvex lens with radius of each surface equal to 20 cm. There is air in the object space and water in the image space. The focal length of lens is:

- (a) 80 cm (b) 40 cm (c) 20 cm (d) 10 cm

Solution :

$$\begin{aligned}\frac{{}^a\mu_w}{f} &= \frac{({}^a\mu_g - 1)}{R_1} - \frac{({}^a\mu_g - {}^a\mu_w)}{R_2} \\ &= \frac{\left(\frac{3}{2} - 1\right)}{20} - \frac{\left(\frac{3}{2} - \frac{4}{3}\right)}{-20} = \frac{1}{40} + \frac{1}{120} = \frac{1}{30} \\ f &= \frac{4}{3} \times 30 = 40 \text{ cm}\end{aligned}$$

Optics - 29) Constraint in interference conditions

Two identical coherent sources are placed on a diameter of a circle of radius R at separation x ($\ll R$) symmetrically about the centre of the circle. The sources emit identical wavelength λ each. The number of points on the circle with maximum intensity is: ($x = 5\lambda$)

- (a) 20 (b) 22 (c) 24 (d) 26

Solution :

Path difference at P is

$$\Delta x = 2 \left(\frac{x}{2} \cos \theta \right) = x \cos \theta$$

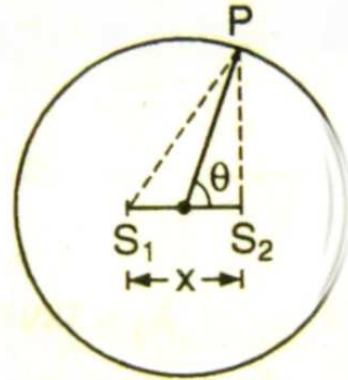
For intensity to be maximum,

$$\Delta x = n\lambda$$

$$(n = 0, 1, 2, \dots)$$

$$\therefore x \cos \theta = n\lambda$$

$$\cos \theta = \frac{n\lambda}{x}$$



$$\cos \theta \neq 1$$

$$\therefore \frac{n\lambda}{x} \neq 1$$

$$\therefore n \neq \frac{x}{\lambda}$$

$$\text{Putting } x = 5\lambda, \quad n \neq 5$$

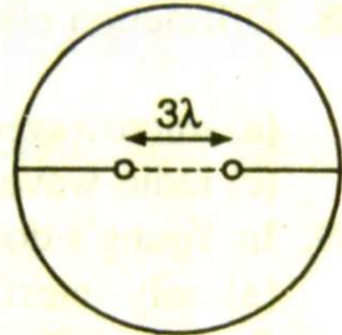
$$\text{or} \quad n = 1, 2, 3, 4, 5$$

Therefore in all four quadrants there can be 20 maximas. There are

more maximas at $\theta = 0^\circ$ and $\theta = 180^\circ$.

But $n = 5$ corresponds to $\theta = 90^\circ$ and $\theta = 270^\circ$ which are coming only twice while we have multiplied it four times. Therefore, total number of maximas are still 20, *i.e.*, $n = 1$ to 4 in four quadrants (total 16) plus four more at $\theta = 0^\circ, 90^\circ, 180^\circ$ and 270° .

If two coherent sources are placed at a distance 3λ from each other symmetric to the centre of the circle shown in the figure, then number of fringes shown on the screen placed along the circumference is: **(UPSEAT 2002)**



- (a) 16
- (b) 12
- (c) 8
- (d) 4

Answer (b) See above Solution

White light is used to illuminate the two slits in a Young's double slit experiment. The separation between the slits is b and the screen is at a distance d ($\gg b$) from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing. Some of these missing wavelengths are: **[CET (J&K) 2003; PET (Kerala) 2006]**

- (a) $\lambda = 3b^2/d$
- (b) $\lambda = 2b^2/d$
- (c) $\lambda = b^2/3d$
- (d) $\lambda = 2b^2/3d$

Solution :

$$\text{Path difference} = (S_2P - S_1P)$$

$$\text{From figure, } (S_2P)^2 - (S_1P)^2 = b^2$$

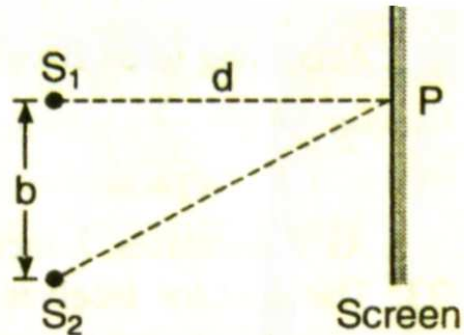
$$\text{or } (S_2P - S_1P)(S_2P + S_1P) = b^2$$

$$\text{or } (S_2P - S_1P) = \frac{b^2}{2d}$$

$$\text{For dark fringes, } \frac{b^2}{2d} = (2n + 1) \frac{\lambda}{2}$$

$$\text{For } n = 0, \quad \frac{b^2}{2d} = \frac{\lambda}{2} \quad \text{or} \quad \lambda = \frac{b^2}{d}$$

$$\text{For } n = 1, \quad \frac{b^2}{2d} = \frac{3\lambda}{2} \quad \text{or} \quad \lambda = \frac{b^2}{3d}$$



Optics - 30) Silvered Prisms or Painted Prisms

If one face of a prism of prism angle 30° and $\mu = \sqrt{2}$ is silvered, the incident ray retraces its initial path. The angle of incidence is:

- (a) 60° (b) 30° (c) 45° (d) 90°

Solution : (c)

It is clear from the figure that the ray will retrace the path when the refracted ray QR is incident normally on the polished surface AC . Thus, angle of refraction $r = 30^\circ$.

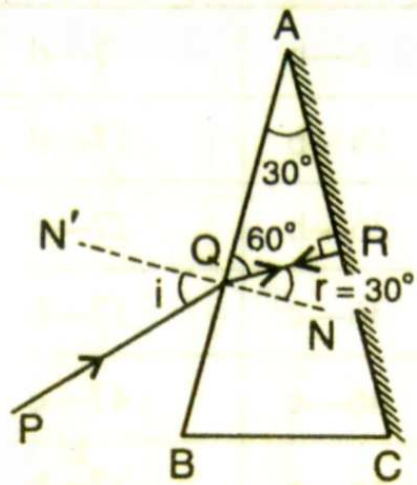
We know that $\mu = \sin i / \sin r$

$$\therefore \sin i = \mu \sin r$$

$$= \sqrt{2} \times \sin 30^\circ = \sqrt{2} \times \frac{1}{2}$$

$$= (1/\sqrt{2})$$

$$\therefore i = 45^\circ$$

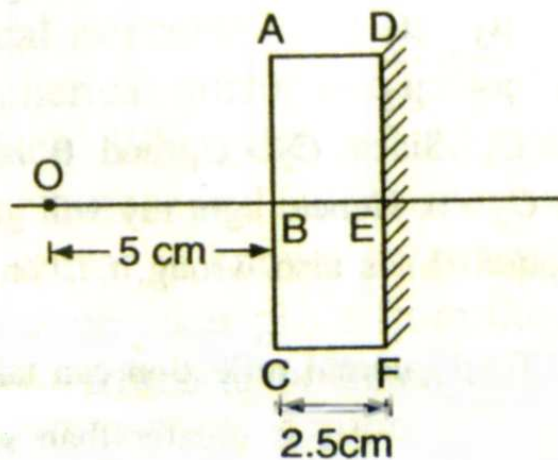


Optics - 31) A slab is silvered on one side or Painted on one side

A plane mirror is made of a glass slab ($\mu_g = 1.5$) 2.5 cm thick and silvered on its back. A point object is placed 5 cm in front of the unsilvered face of the mirror. What will be the position of the final image?

- (a) 12 cm from unsilvered face
- (b) 14.6 cm from unsilvered face
- (c) 5.67 cm from unsilvered face
- (d) 8.33 cm from unsilvered face

Solution : (d)



Let I_1 , I_2 and I_3 be the images formed by

- (i) refraction from ABC
- (ii) reflection from DEF and
- (iii) again refraction from ABC

Then $BI_1 = (5)\mu_g = 5 \times 1.5 = 7.5 \text{ cm}$

Now $EI_1 = 7.5 + 2.5 = 10 \text{ cm}$

$\therefore EI_2 = 10 \text{ cm}$ behind the mirror

Now, $BI_2 = (10 + 2.5) = 12.5 \text{ cm}$

$\therefore BI_3 = \frac{12.5}{\mu_g} = \frac{12.5}{1.5} = 8.33 \text{ cm}$

Real and apparent depth:

- (i) When one looks into a pool of water, it does not appear to be as deep as it really is. Also when one looks into a slab of glass, the material does not appear to be as thick as it really is. This all happens due to refraction of light.
- ii) If a beaker is filled with water and a point lying at its bottom is observed by someone located in air, then the bottom point appears raised. The apparent depth t_{ap} is less than the actual depth t_{ac} . It can be shown that

$$\text{apparent depth } (t_{ap}) = \frac{\text{actual depth } (t_{ac})}{\text{refractive index } (n)}$$

iii)

If there is an ink spot at the bottom of a glass slab, it appears to be raised by a distance

$$d = t_{ac} - t_{ap} = t - \frac{t}{n} = t \left(1 - \frac{1}{n} \right)$$

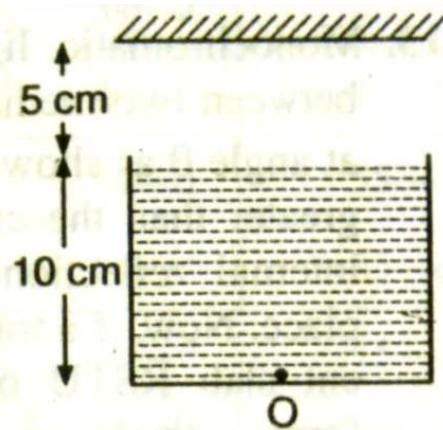
where t is the thickness of the glass slab and n is its refractive index.

iv)

If a beaker is filled with immiscible transparent liquids of refractive indices n_1, n_2, n_3 and individual depth d_1, d_2, d_3 respectively, then the apparent depth of the beaker is found to be:

$$t_{\text{ap}} = \frac{d_1}{n_1} + \frac{d_2}{n_2} + \frac{d_3}{n_3}$$

Consider the situation shown in figure. Water ($\mu_w = 4/3$) is filled in a beaker upto a height of 10 cm. A plane mirror is fixed at a height of 5 cm from the surface of water. Distance of image from the mirror after reflection from it of an object O at the bottom of the beaker is:



- (a) 15 cm (b) 12.5 cm
(c) 7.5 cm (d) 10 cm

Solution : (b)

Distance of first image (I_1) formed after refraction from the plane surface of water is = $\frac{10}{4/3} = 7.5$ cm from water surface

$$\left[\because d_{\text{app}} = \frac{d_{\text{actual}}}{\mu} \right]$$

Now distance of this image is $5 + 7.5 = 12.5$ cm from the plane mirror. Therefore, distance of second image (I_2) will also be equal to 12.5 cm from the mirror.

A beaker containing liquid is placed on a table, underneath a microscope which can be moved along a vertical scale. The microscope is focussed through the liquid on to a mark on

the table when the reading on the scale is a . It is next focussed on the upper surface of the liquid and the reading is b . More liquid is added and the observations are repeated, the corresponding readings are c and d . The refractive index of the liquid is:

- (a) $\frac{d-b}{d-c-b+a}$ (b) $\frac{b-d}{d-c-b+a}$
(c) $\frac{d-c-b+a}{d-b}$ (d) $\frac{d-b}{a+b-c-d}$

Solution : (a)

The real depth = R.I. \times apparent depth

In first case,

The real depth $h_1 = n(b - a)$

Similarly, in the second case, the real depth $h_2 = n(d - c)$

Since, $h_2 > h_1$, the difference of real depths

$$= h_2 - h_1 = n(d - c - b + a)$$

Since the liquid is added in second case,

$$h_2 - h_1 = d - b$$

$$\therefore n = \frac{d - b}{d - c - b + a}$$

Ray or Geometrical Optics by Prof. Subhashish Chattopadhyay SKMClasses Bangalore Useful for IIT-JEE, CBSE, I.Sc. PU-II, Boards, CET, CEE, PET, IGCSE IB AP-Physics and other exams

Modern Physics 1) Spallation reactions (MP-PET-2002 Madhya Pradesh Pre Engineering Test)

See <http://skmclasses.weebly.com/spallation-reaction.html>

Modern Physics 2) Ruby LASER (asked in COMED-K Karnataka)

See <http://skmclasses.weebly.com/ruby-laser.html>

Modern Physics 3) Various details in Particle Physics (asked in several state exams, including Karnataka CET and COMED-K)

See <http://skmclasses.weebly.com/particle-physics.html>

Modern Physics 4) “Magic Numbers” and “Doubly Magic Numbers” in Nuclear Isotope Stability

See <http://skmclasses.weebly.com/magic-number-doubly-magic-in-nucleus.html>

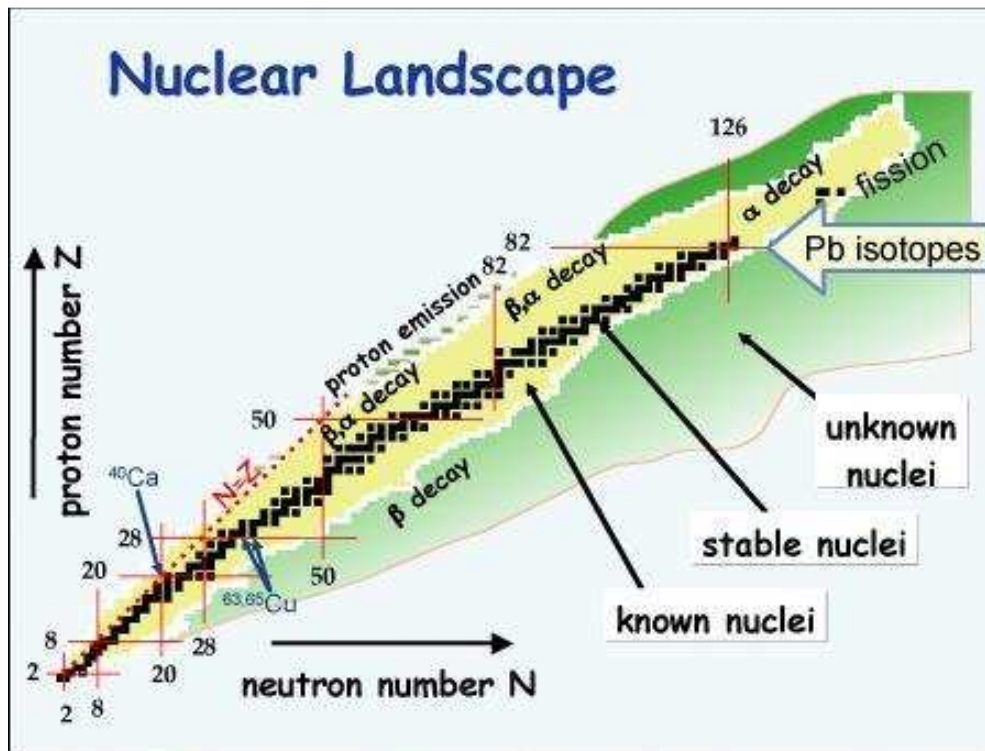
Modern Physics 5) Every Alpha (α) decay produces an isodiapher. Meaning isodiaphers are extremely common. There was AIEEE question on isodiaphers. Also asked in many other exams. Even though every book talks of α , β , and γ decay; most do not talk about isodiaphers, and positron decay. I find this very strange or rather weird !

In nuclear physics, **isodiaphers** refers to nuclides which have different atomic numbers and mass numbers but the same neutron excess, which is the difference between numbers of neutrons and protons in the nucleus. For example, for both ${}_{90}^{234}\text{Th}$ and ${}_{92}^{238}\text{U}$ the difference between the neutron number (N) and proton number (Z) is $N - Z = 54$.

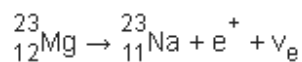
One large family of isodiaphers has zero neutron excess, $N = Z$. It contains many primordial isotopes of elements up to calcium. It includes ubiquitous ${}_{6}^{12}\text{C}$, ${}_{8}^{16}\text{O}$, and ${}_{7}^{14}\text{N}$.

The daughter nuclide of an alpha decay is an isodiapher of the original nucleus. Similarly, beta decays (and other weak-force-involving decays) produce isobars.

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An example of positron emission (β^+ decay) is shown with Magnesium 23 decaying into Sodium 23

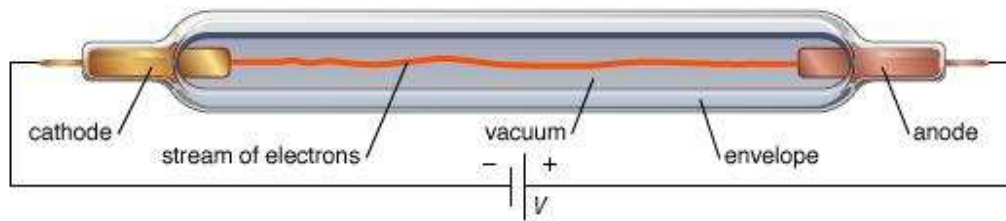


With a positron emission a Proton changes to Neutron. So Mass number remains the same. In 1934 Frederic and Irene Joliot Curie bombarded aluminium with alpha particles to effect the

nuclear reaction ${}^4_2\text{He} + {}^{27}_{13}\text{Al} \rightarrow {}^{30}_{15}\text{P} + {}^1_0\text{n}$, and observed that the product isotope ${}^{30}_{15}\text{P}$ emits a positron identical to those found in cosmic rays by Carl David Anderson in 1932. Meaning it is surely not so new or modern phenomena that "Modern Physics" chapter of Modern Books are not covering this ! 3 year back a IIT JEE question with Positron is also not changing the taboo !

Once again I will say " So strange is this World ! "

Modern Physics 6) Relativistic correction for mass when electrons are flying at very high speed due to very high voltage.



If the voltage is 10KV then what will be the speed of the electrons ?

We know Charge X Voltage = Energy = $\frac{1}{2} mv^2$

Well so far so good. Substitute the values

Charge of electron $e = 1.6 \times 10^{-19}$ Coulomb and mass of electron $m = 9.1 \times 10^{-31}$ kg or 0.511 MeV For sake of this discussion let us approximate electron mass as $0.5 \text{ MeV}/c^2$

So $e (10^4) V = 10^4 \text{ eV} = \frac{1}{2} mv^2 = (\frac{1}{2}) (\frac{1}{2} \text{ MeV})(v/c)^2 = (\text{MeV}/4) (v/c)^2$

$4 \times 10^4 = 10^6 (v/c)^2 \Rightarrow 4/100 = (v/c)^2 \Rightarrow v/c = 1/5 \Rightarrow v = c/5$

Upto speed of around $c/5$ we do not take relativistic corrections.

Now what would be the speed of the electrons if the voltage was 1MV ?

A wrong calculation and thus wrong answer would be

X $e (10^6) V = \frac{1}{2} mv^2 = (\frac{1}{2}) (\frac{1}{2} \text{ MeV})(v/c)^2 = (\text{MeV}/4) (v/c)^2$

X $4 = (v/c)^2$

X $v/c = 2 \Rightarrow v = 2c$

Students should know that particles can't move at speed more than c

An 1 mark question in Karnataka CET had an option close to 98% of c . Student can guess this and tick. While the calculation will be as follows

$$\text{Let } k = \sqrt{1 - \frac{v^2}{c^2}}$$

We will have $e (10^6) V = \frac{1}{2} (m/k)v^2 = (\frac{1}{2}) (\frac{1}{2} \text{ MeV}/k)(v/c)^2 = (\text{MeV}/4k) (v/c)^2$

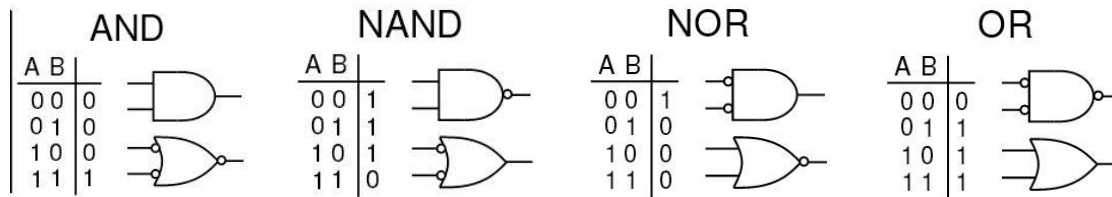
So $4k = (v/c)^2$ put $v/c = x$ we get $4\sqrt{1 - x^2} = x^2$ put $x^2 = y$ so $4\sqrt{1 - y} = y$

Or $16(1 - y) = y^2 \Rightarrow y^2 + 16y - 16 = 0$ Solve the quadratic to get $y = 0.95$

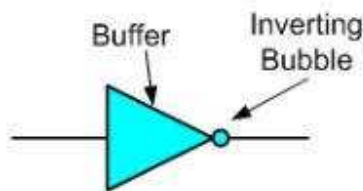
So $x^2 = 0.95$ or $x = \sqrt{0.95} = 0.975 \Rightarrow v/c = 0.975$ or $v = 97.5\%$ of light speed

-

Electronics 1)



The small circle (bubble) at the output of the graphic symbol of a NOT gate is formally called a negation indicator and designates the logical complement.

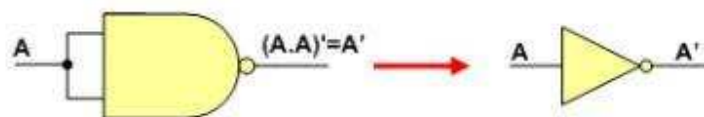


NOT gate can be implemented by NOR Gate. All the pins have to be connected to same signal.

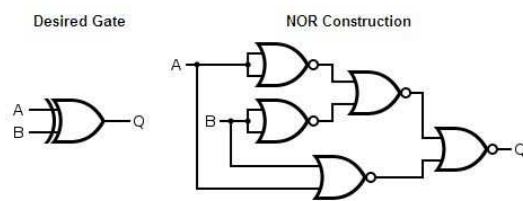


Similarly NOT gate can be implemented with NAND gates

All NAND input pins connect to the input signal **A** gives an output **A'**.



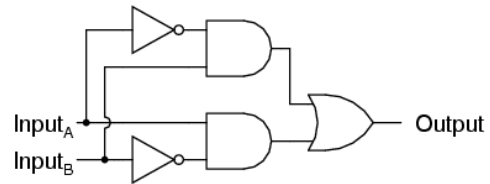
XOR (exclusive OR) gate can be implemented with other gates. In various exams the connections are asked.



Truth Table

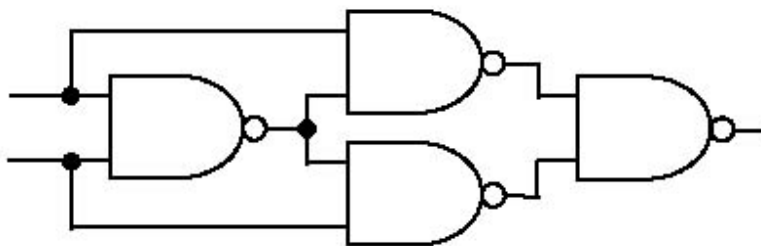
Input A	Input B	Output Q
0	0	0
0	1	1
1	0	1
1	1	0

Exclusive-OR equivalent circuit



A	B	Output
0	0	0
0	1	1
1	0	1
1	1	0

Exclusive OR (XOR)



A	B	Output
0	0	0
0	1	1
1	0	1
1	1	0

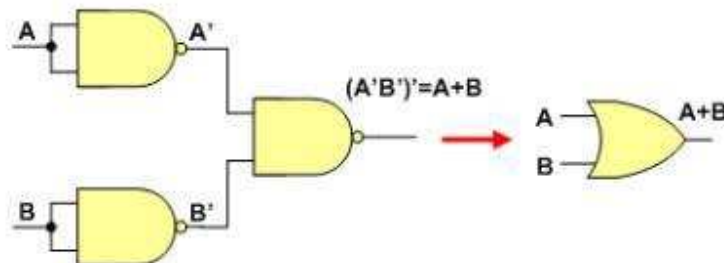
To design the logic circuits the following laws of Boolean algebra are commonly used: commutativity, associativity, distributivity, and De Morgan's laws. Note that distributivity of disjunction over conjunction and both De Morgan's laws do not have their counterparts in ordinary algebra of real numbers.

Property	For conjunction	For disjunction
Commutativity	$A \cdot B = B \cdot A$	$A + B = B + A$
Associativity	$A \cdot (B \cdot C) = (A \cdot B) \cdot C$	$A + (B + C) = (A + B) + C$
Distributivity	$A \cdot (B + C) = A \cdot B + A \cdot C$	$A + B \cdot C = (A + B) \cdot (A + C)$
De Morgan's laws	$\overline{A \cdot B \cdot \dots} = \overline{A} + \overline{B} + \dots$	$\overline{A + B + \dots} = \overline{A} \cdot \overline{B} \cdot \dots$
Basic identities	$A \cdot 0 = 0$ $A \cdot 1 = A$ $A \cdot A = A$ $A \cdot \overline{A} = 0$	$A + 1 = 1$ $A + 0 = A$ $A + A = A$ $A + \overline{A} = 1$
Additional identities	$A \cdot (A + B) = A$ $A + \overline{A} \cdot B = A + B$ $(A + B) \cdot (\overline{A} + \overline{B}) = \overline{A \cdot B}$	$A + A \cdot B = A + B$ $A \cdot (\overline{A} + B) = A \cdot B$ $A \cdot B + \overline{A} \cdot B = B$

Principal identities and laws of Boolean algebra.

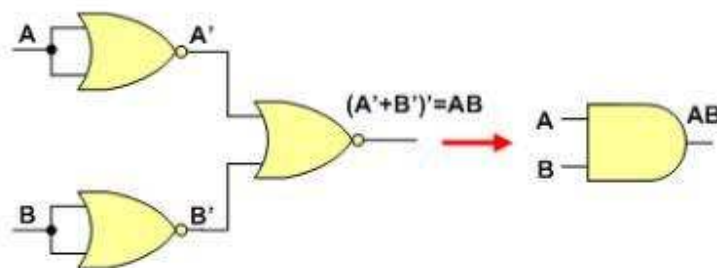
Implementing OR Gate with NAND gates

An **OR gate** can be replaced by NAND gates as shown in the figure (The OR gate is replaced by a NAND gate with all its inputs complemented by NAND gate inverters).



Implementing AND gate with NOR gates

An **AND gate** can be replaced by NOR gates as shown in the figure (The AND gate is replaced by a NOR gate with all its inputs complemented by NOR gate inverters).



Colour Code for Carbon Resistors

Since a carbon resistor is physically quite small, it is more convenient to use a *colour code* indicating the resistance value than to imprint the numerical value on the case. In this scheme, there are generally four colour bands *A*, *B*, *C* and *D* printed on the body of the resistor as shown in Fig. The first three colour bands (*A*, *B* and *C*) give the value of the resistance while the fourth



band (D) tells about the *tolerance in percentage. The table below shows the colour code for resistance values and colour code for tolerance.

Colour Code for Resistance Values				Colour Code for Tolerance	
Black	0	Green	5	Gold	$\pm 5\%$
Brown	1	Blue	6	Silver	$\pm 10\%$
Red	2	Violet	7	No colour	$\pm 20\%$
Orange	3	Grey	8		
Yellow	4	White	9		

(i) To read the resistance value, we refer to the first three colour bands (A, B and C). The first two colour bands (A, B) specify the first two digits of the resistance value and the third colour band (C) gives the number of zeros that follow the first two digits. Suppose the first three colour bands (A, B, C) on the resistor are red, brown, orange respectively. Then value of the resistance is 21,000 Ω .

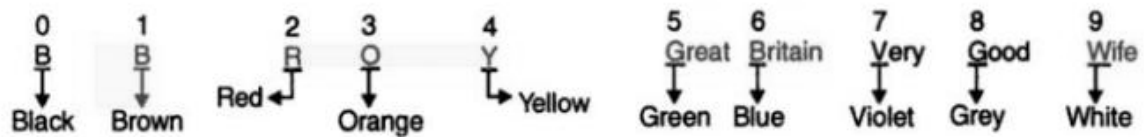
Red : 2

Brown : 1

Orange : 000

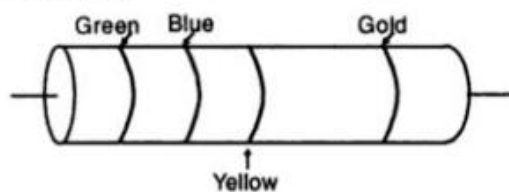
\therefore Value = 21,000 Ω

(ii) The fourth band D gives the value of tolerance in percentage. If colour of the fourth band is gold, tolerance is ± 5 per cent and if silver, then tolerance is ± 10 per cent. If the fourth band is omitted, the tolerance is assumed to be ± 20 per cent.

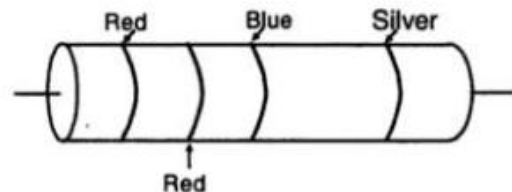


Note. In order to remember the colour code, the above sentence may be helpful.

Example The colour coded carbon resistors are shown in Fig. Find their resistance values.



(i)



(ii)

Solution. The first colour represents the digit 5. The second colour represents the digit 6. The third colour represents the digit 4, *i.e.*, four zeros. Therefore, the value of the resistance is 56,0000 Ω . The fourth gold strip indicates $\pm 5\%$ tolerance. Hence, resistance specification of the resistor is

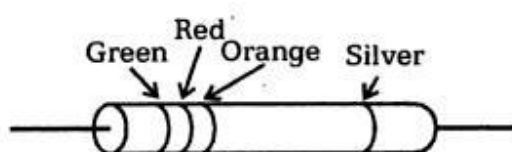
$$560000 \Omega ; \pm 5\%$$

(ii) Refer to Fig. resistor is

Following above procedure, the resistance specification of this

$$22,000000 \Omega ; \pm 10\%$$

- Due to manufacturing variations, the resistance value may not be the same as indicated by colour code. Thus, a resistor marked 100 Ω , $\pm 10\%$ tolerance means that resistance value is between 90 Ω and 110 Ω .



"Carbon resistor colour code"

The value of the above resistor as shown in the fig. is

- The first ring Green - 5
- The second ring Red - 2
- The third ring Orange ring corresponds to - 10^3
- The silver ring represents 10% tolerance
- \therefore Total resistance is
- $52 \times 10^3 \pm 10\%$ (or) $52\text{k}\Omega$, 10%

Varactor diode

$\text{---} \triangleright \text{---}$ is the symbol of

- (a) a capacitor
- (b) photo diode
- (c) varactor diode
- (d) tunnel diode

Ans: (c)

Common emitter

In a common emitter configuration the base - emitter voltage is 3×10^{-2} V. If the base current is $30 \mu\text{A}$, the input impedance is

- (a) $1 \text{ k}\Omega$ (b) $3 \text{ k}\Omega$
(c) 100Ω (d) $2 \text{ k}\Omega$

Ans: (a)

Solution:

Given data:-

$$(V_{BE}) = \text{Base emitter voltage} = 3 \times 10^{-2} \text{V}$$

$$\text{Base current } (I_B) = 30 \times 10^{-6} \text{ A}$$

$$\text{Input impedance } Z_i = \left(\frac{\Delta V_{BE}}{\Delta I_B} \right)_{V_{CE}}$$

$$Z_i = \frac{3 \times 10^{-2}}{30 \times 10^{-6}}$$

$$Z_i = Z_i = \frac{3 \times 10^{-2}}{10 \times 10^{-6}} = 10^{-2-1+6}$$

$$Z_i = 10^3 \Omega$$

$$Z_i = 1 \text{ k}\Omega$$

Common base

In a common base configuration, the collector current is 0.95 mA and base current is 0.05 mA , then the value of current gain is

- (a) 0.89 (b) 0.9
(c) 0.95 (d) 0.99

Ans: (c)

Given data:-

$$\text{Collector current } I_C = 0.95 \times 10^{-3} \text{A}$$

$$\text{Base current } I_B = 0.05 \text{ mA} = 0.05 \times 10^{-3} \text{A}$$

Solution:

$$\text{Current gain } \alpha = \left(\frac{I_C}{I_E} \right)$$

$$I_E = \text{Emitter current} = I_C + I_B$$

$$= (0.95 + 0.05) \times 10^{-3} \text{ A}$$

$$= 1 \times 10^{-3} \text{ A} = 1 \text{ mA}$$

$$\alpha = \frac{0.95 \times 10^{-3} \text{ A}}{1 \times 10^{-3} \text{ A}} = 0.95$$

The current gain is 0.95

Common emitter

In a common emitter amplifier, the output resistance is 5000Ω and the input resistance is 2000Ω . If the peak value of the signal voltage is 10 mV and $\beta = 50$, then the peak value of the output voltage is

- (a) $5 \times 10^{-6} \text{ V}$ (b) 1.25 V
(c) 125 V (d) $2.5 \times 10^{-4} \text{ V}$

Ans: (b)

Given data:-

$$R_L = 5000 \Omega$$

$$R_i = 2000 \Omega$$

$$\beta = 50$$

Solution:

The ac voltage gain is given by

$$\beta \times \frac{R_L}{R_i} = \frac{50 \times 5000}{2000} = 125$$

$$\begin{aligned} \therefore \text{peak output voltage} &= \text{voltage gain} \times \text{signal voltage} \\ &= 125 \times 10 \text{ mV} = 1250 \text{ mV} = 1.25 \text{ V} \end{aligned}$$

Common base

In a common base amplifier circuit, calculate the change in base current if that in the collector current is 2 mA and $\alpha = 0.98$

- (a) 0.04 mA (b) 1.96 mA
(c) 980 mA (d) 2 mA

Ans: (a)

Solution:

$$\beta = \frac{\alpha}{1 - \alpha} = \frac{0.98}{1 - 0.98} = 49$$

$$\text{Now } \Delta I_c / \Delta I_b = 49$$

$$\text{or } \Delta I_b = \Delta I_c / 49$$

$$\begin{aligned} \therefore \Delta I_b &= 2 \text{ mA} / 49 \\ &= 0.04 \text{ mA} \end{aligned}$$

Common base

In a common base circuit of a transistor, current amplification factor is 0.95. Calculate the base current when emitter current is 2 mA.

- (a) 0.1 mA (b) 1 mA
(c) 0.01 mA (d) none of these

Ans: (a)

Solution:

$$\alpha = \frac{I_C}{I_E}$$

$$0.95 = \frac{I_C}{2 \times 10^{-3}}$$

$$I_C = 1.90 \times 10^{-3} \text{ A} = 1.9 \text{ mA}$$

$$\text{Now } I_B = I_E - I_C = 0.1 \text{ mA}$$

Common emitter

A transistor is connected in common emitter (CE) configuration. The collector supply is 8V and the voltage drop across a resistor of 800Ω in the collector circuit is 0.5V. If the current gain factor (α) is 0.96. Find the base current.

- (a) 20 μA (b) 26 μA
(c) 30 μA (d) none of these

Ans: (b)

Solution:

$$\text{Collector current } I_C = \frac{0.5}{800} \text{ A}$$

$$\text{Current gain } \beta = \frac{I_C}{I_B}$$

$$\frac{\alpha}{1 - \alpha} = \frac{0.96}{800} \text{ A}$$

$$I_B = \frac{I_C}{24} = \frac{0.5}{800 \times 24}$$
$$= 26 \mu\text{A}$$

Conductivity

Conductivity is defined as the current density per unit applied electric field. If J is the current density due to an applied electric field ϵ , then the conductivity (σ) is given by,

$$\sigma = \frac{J}{\epsilon} \quad \text{.....(1)}$$

In S.I., σ is given in Siemens/meter or mho/meter as 1 siemen = 1 mho

For a cylindrical semiconductor, the current density is given by,

$$J = ne v \quad \text{.....(2)}$$

where n is the number of charge carriers in the semiconductor e is the electronic charge and v is the drift velocity of the electron.

Also, we have $v = \mu \epsilon$ (3)

where μ is the mobility of the charge carrier and ϵ is the applied electric field.

Then, equation (2) can be written,

$$J = ne \mu \epsilon$$

then, equation (1) becomes,

$$\sigma = ne \mu \quad \text{.....(4)}$$

Now, if the conductivity of a semiconductor is due to electron then it is denoted by σ_n , and equation (4), can be written as

$$\sigma_n = ne \mu_n \quad \text{.....(5)}$$

where n is the number of electron and μ_n is the mobility of electron.

Similarly, the conductivity of a semiconductor due to the holes is given by,

$$\sigma_p = pe \mu_p \quad \text{.....(6)}$$

where p is the hole concentration and μ_p is the hole mobility.

Hence, the overall conductivity of the semiconductor containing electrons and holes is given by,

$$\sigma = \sigma_n + \sigma_p = e(n \mu_n + p \mu_p) \quad \text{.....(7)}$$

For an intrinsic semiconductor, $n = p = n_i$

Therefore, the conductivity of an intrinsic semiconductor,

$$\sigma_{\text{int}} = n_i e(\mu_n + \mu_p) \quad \text{.....(8)}$$

For an n -type semiconductor, $n \gg p$, then

$$\sigma_n \approx ne \mu_n \quad \text{.....(9)}$$

Similarly, for a p -type semiconductor

$$\sigma_p = pe \mu_p \quad \text{.....(10)}$$

These equations shows that conductivity σ has the same temperature dependence as μ_e or μ_n .

Mobility is a more useful property for characterizing a semiconductor than conductivity. Conductivity, σ depends on carrier concentration *i.e.*, on doping level but mobility μ does not depend. Thus, mobility is the property of semiconductor itself.

Problem 1 : At 300 K, the intrinsic carrier concentration of silicon is $1.5 \times 10^{16} \text{ m}^{-3}$. If the electron and the hole mobilities are 0.13 and 0.05 $\text{m}^2/\text{sec-V}$ respectively. Determine the conductivity and resistivity of silicon.

Solution : The electrical conductivity of intrinsic semiconductor is given by,

$$\sigma_i = n_i e (\mu_n + \mu_p)$$

Here, $n_i = 1.5 \times 10^{16} \text{ m}^{-3}$, $\mu_n = 0.13 \text{ m}^2/\text{sec-V}$, $\mu_p = 0.05 \text{ m}^2/\text{sec-V}$ and $e = 1.6 \times 10^{-19} \text{ Coulomb}$

$$\begin{aligned} \therefore \sigma_i &= 1.5 \times 10^{16} \times 1.6 \times 10^{-19} \times (0.13 + 0.05) \\ &= 2.4 \times 10^{-3} \times 0.18 = 0.432 \times 10^{-3} \\ &= 4.32 \times 10^{-4} \text{ mho/m} \end{aligned}$$

Hence, the resistivity ρ_i is given by

$$\rho_i = \frac{1}{\sigma_i} = \frac{1}{4.32 \times 10^{-4}} = 2.31 \times 10^3 \text{ ohm-m}$$

Problem 2 : The resistivity of pure silicon at room temperature is 3000 ohm-m. Calculate the intrinsic carrier concentration. Given that : $\mu_n = 0.14 \text{ m}^2/\text{sec-V}$ and $\mu_p = 0.05 \text{ m}^2/\text{sec-V}$.

Solution : In pure silicon, electrons and holes (the intrinsic charge carriers) are equal in numbers. The conductivity of pure semiconductor is given by

$$\begin{aligned} \sigma &= n_i e (\mu_n + \mu_p) \quad \text{or} \quad n_i = \frac{\sigma}{e(\mu_n + \mu_p)} = \frac{1}{\rho e (\mu_n + \mu_p)} \quad \left[\because \rho = \frac{1}{\sigma} \right] \\ \therefore n_i &= \frac{1}{(0.14 + 0.05) \times 3000 \times 1.602 \times 10^{-19}} = 1.095 \times 10^{16} \text{ m}^{-3}. \end{aligned}$$

The band gap of a specimen of gallium arsenide phosphide is 1.98 eV. Determine the wavelength of the radiation that is emitted when electron jumps from conduction band to the valence band to recombine with a hole.

Solution : The wavelength of emitted radiation is given by,

$$\lambda = \frac{hc}{E_g}$$

Here, h = Planck's constant = $6.62 \times 10^{-34} \text{ J.s}$, c = velocity of light = $3 \times 10^8 \text{ m/s}$ and

E_g = Energy band gap = $1.98 \text{ eV} = 1.98 \times 1.6 \times 10^{-19} \text{ J}$.

$$\begin{aligned} \therefore \lambda &= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{1.98 \times 1.6 \times 10^{-19}} = 6.269 \times 10^{-7} \text{ m} \\ &= 6269 \text{ \AA} \end{aligned}$$

Since this wave-length is in the visible range, so the colour of the emitted radiation will be red.

Q. An n-type semiconductor crystal has more free electrons than holes. Is it then negatively charged ?

Ans : An n-type semiconductor has free electrons as charge carriers. These are donated by pentavalent impurity atoms which becomes positively charged. Although there are some thermally generated electron-hole pairs, but the number of these holes is negligibly small in comparison to the total number of electrons. Thus, n-type semiconductor mainly consists of negatively charged free electrons and nearly equal number of positively charged donor ions. Hence, the material as a whole is electrically neutral.

Q. *p*-type semiconductor crystal has more holes than electrons. Is it then positively charged ?

Ans : A *p*-type semiconductor has holes as charge carriers. These holes are due to trivalent impurity atoms which become negatively charged by accepting the electrons from the neighbouring Ge atom. Although there are some thermally generated electron-hole pairs, but the number of these electrons is negligibly small in comparison to the total number of holes. Thus, *p*-type semiconductor mainly consists of positively charged holes and nearly equal number of negatively charged acceptor ions. Hence, the material as a whole is electrically neutral.

Q. Why does the width of depletion region increase when a *p-n* junction is reverse biased ?

Ans : In reverse bias, negative terminal of the battery is connected to *p*-side and positive terminal to *n*-side of *p-n* junction. So, the electrons are attracted towards positive terminal and holes towards negative terminal of the battery. Thus, holes and free electrons move away from the junction. Therefore, the depletion layer gets wider. The width of the layer increases with increasing reverse voltage.

Q. The small current flowing through a reverse biased junction diode is called the reverse saturation current, why ?

Ans : The reverse current is due to the thermally generated minority carriers. We cannot increase the number of these minority carriers by applying and increasing the reverse voltage. So, it is termed as saturation current. This current flows in the opposite direction with respect to forward bias, so it is called reverse. Due to above both factors it is called reverse saturation current.

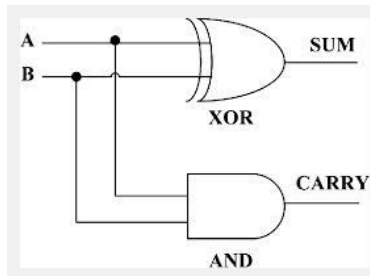
Q. The reverse saturation current of a Si diode is much smaller than a Ge diode of the same size, why ?

Ans : The barrier potential of Si is 0.7 eV while that of Ge is 0.3 V. Hence, less number of thermally generated minority carriers cross the junction in Si diode than that in Ge diode of the same size. Therefore, the reverse current in Si diode is smaller than the Ge diode at the same temperature and for the same size.

Q. Differentiate between Avalanche and Zener breakdown.

Ans : **Avalanche breakdown :** For a simple *p-n* junction, if we apply a reverse bias to the junction, a very small current due to minority carriers flows through the junction. On increasing the reverse voltage the minority carriers (electrons) may attain sufficient kinetic energy to knock out valency electron from the covalent bonds. As a result more electron-hole pairs are generated. Due to the high reverse bias voltage, these new carriers are also accelerated and collide with other covalent bonds. This process will continue until an avalanche of electrons is formed and a very large current flows through the junction diode. This breakdown is known as Avalanche breakdown. This breakdown occurs at very high voltage.

Zener breakdown : If the *p-n* junction is heavily doped then the electric field across the depletion layer becomes large enough. When we apply a reverse bias to this junction then this electric field becomes so large even at low voltage that it may cause rupture of the covalent bonds and breakdown the junction. This breakdown is known as Zener breakdown and this diode is known as Zener diode. This breakdown occurs at lower voltage than avalanche breakdown.



Half Adder

Q. : What is mass-action law for the carrier concentrations in a semiconductor ? What is its significance ?

Ans : The law of mass-action states that in any type of semiconductor (p or n type), the product of free electrons concentration, n and hole concentration, p is a constant and equal to n_i^2 where n_i is the intrinsic carrier concentration *i.e.*,

$$np = n_i^2$$

The intrinsic carrier concentration n_i is a function of temperature. At a given temperature if electron concentration is increased by doping, the corresponding hole concentration (p) must decrease (or vice-versa) to keep np a constant ($= n_i^2$) at a particular temperature.

Q. : Explain why an extrinsic semiconductor at high temperature behaves like an intrinsic one.

Ans : At very high temperature, the concentration of thermally generated free electrons from the valence band becomes much larger than concentration of free electrons contributed by donors (as donor atoms are already ionized). In this condition, the hole and electron concentrations will be nearly equal and semiconductor will behave like an intrinsic one. Due to the same reason p -type semiconductor will also behave like an intrinsic semiconductor at very temperatures. So, we can say that an extrinsic semiconductor changes to an intrinsic one at very high temperatures.

Q. : What do you mean by the term "doping" and "dopant" . Name some dopant materials ?

Ans : The addition of a small percentage of impurity atoms to a semiconductor is called "doping" and the impurity, which is added, is referred to as "dopant". In Ge or Si, the elements of V group like phosphorous (P) antimony (Sb) and arsenic (As) and the elements of III group like aluminium (Al). Indium (In) boron (B) and gallium (Ga) are dopant.

Q. Write diode equation and with the help of this equation describe the volt-ampere characteristics of the diode.

Ans : The diode equation is written as,

$$I = I_0 (\exp \frac{eV}{kT} - 1) \quad \dots\dots(1)$$

where I is current at applied voltage, V

I_0 is constant and known as reverse saturation current
 e is electronic charge
 k is Boltzmann constant
 and T is absolute temperature
 With the help of this equation we can describe the volt ampere characteristics as shown in fig 1.23

If V is positive *i.e.*, for a forward bias

then, $\exp \frac{eV}{kT} \gg 1$

So, equation (1) can be written as,

$$I = I_0 \exp \frac{eV}{kT}$$

Hence, for a forward bias, current increases exponentially as shown in fig. 1.24.

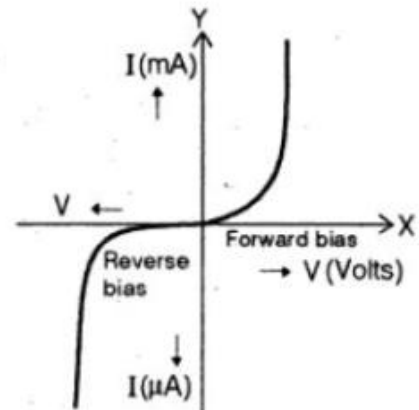
Similarly, if V is negative *i.e.*, for a reverse bias then,

$$\exp^{-\frac{eV}{kT}} \ll 1$$

So, equation (1) can be written as,

$$I = -I_0$$

Hence, for a reverse bias current is constant in reverse direction as shown in fig.



Q. How reverse current depends upon the temperature of the junction ?

Ans : The reverse current in a $p-n$ junction diode depends on the temperature T . The rise in temperature increases the generation of electron hole pairs in semiconductors and increases their conductivity as a result the current through junction diode increases with temperature. For practical diodes it is found that reverse saturation current I_0 will just about double in magnitude for every 10°C increase in temperature. Typical values of I_0 for silicon are much lower than that of Germanium for similar power and current levels. The result is that silicon junction diodes are more preferred than Ge for rectifiers and have higher breakdown voltage.

Q. : What do you mean by tunnel diode ?

Ans : Tunnel diode is very high doped ($\approx 10^{25}/\text{m}^3$) $p-n$ junction in both p and n region. Since, the depletion layer of this diode becomes very thin, so, on applying forward bias many carriers can tunnel through the depletion layer and the process is known as tunnelling. Hence, the diode is known as tunnel diode.

Heat or Thermodynamics 1) So many exams including IIT JEE had questions on Polytropic processes. Apart from Professor N. N. Ghosh's books, hardly this is covered in Physics Books

I am surprised and **amused** to see so many coaching Institutes making errors in Polytropic Process Problems. In most cases the teachers are avoiding it, and in rare cases when it is being covered there are errors.

Let us do it here.

We assume ideal gas for Thermodynamics process problems. So $PV = nRT$ is taken as true regardless the process gas is taken through. So Isothermal (meaning constant Temperature), Isobaric (meaning const Pressure), Isochoric (meaning constant Volume) or even $PV^z = \text{Const}$ (P into V to the power z is constant) where z is a constant of the polytropic process, the expression $PV = nRT$ is taken as true. We do substitute that to exchange the variables in many problems.

Work done by system on boundary is:

$$W = \int_{V_1}^{V_2} p \, dV$$

P can be a polytropic exp

This form is used for expansion and contraction of gases
Ideal Gases

Ideal (Perfect) Gas Law

$$pV = nRT$$

$$R = 8314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

If the gas expands (often due to supply of heat) the work done by the gas is taken as positive.

Work done expression in Isothermal (or isotropic as some people say it) is given by

Isotropic (Constant Temp) Process or Isothermal process

- For a constant temperature process in a closed system (i.e. mass is constant) – $pV = mRT = C$. Where C is a constant. Note C can be written as p_1V_1 or as p_2V_2 .

$$\bullet \quad W = \int_{V_1}^{V_2} \frac{C}{V} \, dV = C \ln \left(\frac{V_2}{V_1} \right) = p_1V_1 \ln \left(\frac{V_2}{V_1} \right) = nRT \ln \left(\frac{V_f}{V_i} \right) = nRT \ln \left(\frac{P_i}{P_f} \right)$$

Polytropic process - $pV^n=C$ where C is a constant.

these occur in ideal gases for various processes and the value of n changes depending on the type of process (e.g. $n = 1$ is a isotropic process).

Note that $p_1V_1^n=C \rightarrow p_1V_1=\frac{CV_1}{V_1^n}=CV_1^{1-n}$ This also holds for p_2V_2 .

$$W = \int_{V_1}^{V_2} \frac{C}{V^n} dV = \frac{C}{1-n} (V_2^{1-n} - V_1^{1-n}) = \frac{p_2V_2 - p_1V_1}{1-n}$$

In case of adiabatic process (where no heat exchange takes place), n is γ (gamma), so in the above expression replace n as γ

$$pV^\gamma = p_1V_1^\gamma = p_2V_2^\gamma = k$$

Thus,
$$p = \frac{k}{V^\gamma}$$

The work done by the gas in the process is

$$W = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} \frac{k}{V^\gamma} dV = \frac{1}{1-\gamma} \left[\frac{k}{V_2^{\gamma-1}} - \frac{k}{V_1^{\gamma-1}} \right]$$

From equation (i),

$$\frac{k}{V_2^\gamma} = p_2 \text{ and } \frac{k}{V_1^\gamma} = p_1$$

Thus,
$$W = -\frac{1}{\gamma-1} (p_2V_2 - p_1V_1) = \frac{p_1V_1 - p_2V_2}{\gamma-1}$$

There are other expressions which are handy (given for 1 mole of gas), for Heat supplied in Polytropic Process

$$\Delta H = C_P^0(T_2 - T_1) = \frac{\gamma R}{\gamma-1} (T_2 - T_1) = \frac{\gamma}{\gamma-1} (P_2V_2 - P_1V_1) = \frac{\gamma P_1V_1}{\gamma-1} \left[\left(\frac{P_2}{P_1} \right)^{1-\frac{1}{n}} - 1 \right]$$

Heat Supplied in a process at constant Pressure is $\Delta H = C_P^0(T_2 - T_1)$

Process	Work Done: (W)	Heat Exchanged (ΔQ)
Isothermal process	$W = 2.303 nRT \log_{10} \frac{V_2}{V_1}$	$\Delta Q = 2.30 nRT \log \frac{V_2}{V_1}$
Adiabatic process	$W = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1}$ $= \frac{nR(T_2 - T_1)}{\gamma - 1}$	$\Delta Q = 0$
Isochoric process	$W = 0$	$\Delta Q = nC_v \Delta T$ (use definition of C_v)
Isobaric process	$W = p\Delta V = p(V_2 - V_1)$ $W = nR(T_2 - T_1)$	$\Delta Q = nC_p \Delta T$ (use definition of C_p)

VdP expression in polytropic process

For a polytropic process $P_1 V_1^n = P V^n$

$$V = \left(\frac{P_1 V_1^n}{P} \right)^{\frac{1}{n}} = \left(\frac{P_1}{P} \right)^{\frac{1}{n}} V_1$$

$$\int V dP = P_1^{\frac{1}{n}} V_1 \int \frac{1}{P^{\frac{1}{n}}} dP$$

$$\int V dP = \frac{P_1^{\frac{1}{n}} V_1}{1 - \frac{1}{n}} \left(P_2^{1 - \frac{1}{n}} - P_1^{1 - \frac{1}{n}} \right)$$

$$\int V dP = \frac{n V_1}{n - 1} \left(P_1^{\frac{1}{n}} P_2^{1 - \frac{1}{n}} - P_1 \right)$$

$$P \int_{P_1}^{P_2} V dP = \frac{n P_1 V_1}{n - 1} \left[\left(\frac{P_2}{P_1} \right)^{1 - \frac{1}{n}} - 1 \right]$$

$$- \int_1^2 V dP = \frac{n P_1 V_1}{n - 1} \left[1 - \left(\frac{P_2}{P_1} \right)^{1 - \frac{1}{n}} \right]$$

Specific heat in case of Polytropic process and C_v in terms of γ

$$C = \frac{R}{\gamma-1} - \frac{R}{k-1}$$
$$C_v = \frac{R}{\gamma-1}$$

Example

One mole of Argon is heated using $PV^{3/2} = \text{const}$. Find the amount of heat obtained by the process when the temperature changes by $\Delta T = -26$ K.

Solution

Let p be the number of moles here $p = 1$

$$\text{then } C = \frac{R}{\gamma-1} - \frac{R}{\eta-1} = \frac{R}{\frac{5}{3}-1} - \frac{R}{\frac{3}{2}-1}$$

$$\Delta Q = pC\Delta T = 1 \left(\frac{3}{2}R - 2R \right) (-26)$$

$$= +26 \left(\frac{8.314}{2} \right) = 108 \text{ J}$$

You can also write $+ R / (1-k)$ in Specific heat expression so see an example

An ideal gas expands according to the law $PV^{3/2} =$ constant. We conclude

- (a) The adiabatic exponent of the gas $K = 1.5$
- (b) The molar heat capacity $C = C_v - 2R$
- (c) Temperature increases during the process
- (d) Such a process is not feasible

Ans - (b) Molar heat capacity

$$C = C_v + \frac{R}{1-K} = C_v + \frac{R}{1-\frac{3}{2}} = C_v - 2R$$

IIT JEE 1995 Polytrropic Thermodynamics Process Problem

3 moles of a gas mixture having volume V and temperature T is compressed to 1/5th of the initial volume. Find the change in its adiabatic compressibility if the gas obeys $PV^{19/13} = \text{constant}$ [$R = 8.3 \text{ J/mol - K}$]

[IIT 1995]

$$\text{Bulk modulus } B = \gamma P$$

$$\text{Compressibility } C = \left(\frac{1}{B} \right) = \frac{1}{\gamma P}$$

and $\Delta C = C - C$

or $\Delta C = \frac{1}{\gamma} \left[\frac{1}{P'} - \frac{1}{P} \right]$

$$PV^\gamma = P' \left(\frac{V}{5} \right)^\gamma$$

$$\text{With } \gamma = \frac{19}{13} \text{ and } P' = 5^{\gamma} P, 11$$

$$\Delta C = \frac{1}{\gamma P} \left[\frac{1}{5^{\gamma}} - \frac{1}{1} \right] = \frac{13 \times 0.905}{19P}$$

$$\text{But } PV = nRT \text{ or } P = \frac{nRT}{V}$$

$$\Delta C = \frac{13(.905)V}{19 \times 3 \times 8.317T} = \frac{-0.0248V}{T}$$

An ideal gas with adiabatic exponent γ , is expanded according to the law

$$P = aV$$

where a is a constant. The initial volume of the gas is V_0 . As a result volume increases η times. Find the increment in internal energy and work done.

Solution - Let k be number of moles

$$P = aV \text{ or } PV^{-1} = a$$

The process is polytropic with index $n = -1$

$$\therefore V_{\text{initial}} = V_0, V_{\text{final}} = \eta V_0$$

$$\text{and } P_{\text{initial}} = aV_0; P_{\text{final}} = a\eta V_0$$

$$\Delta U = \frac{kR}{\gamma - 1} (T_{\text{final}} - T_{\text{initial}}), P_{\text{final}} V_{\text{final}} - P_{\text{initial}} V_{\text{initial}}$$

Work done,

$$W = \frac{P_{\text{initial}} V_{\text{initial}} - P_{\text{final}} V_{\text{final}}}{n - 1} = \frac{\alpha V_0^2 [\eta^2 - 1]}{2}$$

In a polytropic process an ideal gas ($\gamma = 1.40$) was compressed from volume $V_1 = 10$ litres to $V_2 = 5$ litres. The pressure increased from $p_1 = 10^5$ Pa to $p_2 = 5 \times 10^5$ Pa. Determine: (a) the polytropic exponent n , (b) the molar heat capacity of the gas for the process.

Solution.

In a polytropic process $pV^n = k$ (a constant)

$$\therefore p_1 V_1^n = p_2 V_2^n \quad \text{or} \quad \left(\frac{V_1}{V_2}\right)^n = \frac{p_2}{p_1}$$

$$\text{or} \quad n = \frac{\ln p_2/p_1}{\ln V_1/V_2}$$

$$\text{Here} \quad n = \frac{\ln 5}{\ln 2} = \frac{1.6094}{0.6931} = 2.32$$

In a polytropic process

$$C = \frac{R}{\gamma - 1} - \frac{R}{n - 1} = \frac{R}{1.4 - 1} - \frac{R}{2.32 - 1} = 1.74 R$$

An ideal gas expands according to the law $pV^2 = \text{constant}$ (a) Is it heated or cooled? (b) What is the molar heat capacity in this process?

Solution.

This is a polytropic process of exponent $n = 2$. To find whether it is heated or cooled we have to examine whether ΔQ is +ve or -ve or whether T increases or decreases.

$$pV^2 = \text{constant.} \quad \text{But } pV = RT \quad (\text{always})$$

$$\therefore \frac{pV^2}{pV} = \frac{\text{constant}}{RT} \quad \text{or} \quad V \propto \frac{1}{T}$$

Thus when volume increases T decreases. Here the gas is cooled.

$$(b) \quad C = \frac{R}{\gamma - 1} - \frac{R}{n - 1} = C_V - R$$

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Heat or Thermodynamics 2) Formula for equivalent gamma in mixture of gases. n_1 moles of gas with γ_1 and n_2 mole of gas with γ_2 are mixed, then what is equivalent gamma ?

Why $C_v = R / (\gamma - 1)$

Specific heat of a polytropic process. Derivation of work done in polytropic process.

$$\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

Equivalent γ of a mixture of gas.
 n_1 moles of γ_1 and n_2 moles of γ_2 are mixed

$$C_v = \frac{R}{\gamma - 1}$$

we have $\gamma = \frac{C_p}{C_v} = \frac{C_v + R}{C_v}$

$$= 1 + \frac{R}{C_v}$$

$$PV^\gamma = \text{const}$$

 Polytropic Process $\Rightarrow \frac{R}{C_v} = \gamma - 1 \Rightarrow C_v = \frac{R}{\gamma - 1}$

$$C = \frac{R}{\gamma_1 - 1} + \frac{R}{1 - \gamma} = C_v + \frac{R}{1 - \gamma}$$

Derivation of Work done in a polytropic Process.

$$\Delta Q = n C_{\text{Poly}} \Delta T$$

$$= n \left(C_v + \frac{R}{1 - \gamma} \right) (T_2 - T_1)$$

$$= n C_v \Delta T + \frac{n R \Delta T}{1 - \gamma}$$

$$= \Delta U + \Delta W$$

So $\Delta W = \text{Work done} = \frac{n R \Delta T}{1 - \gamma}$

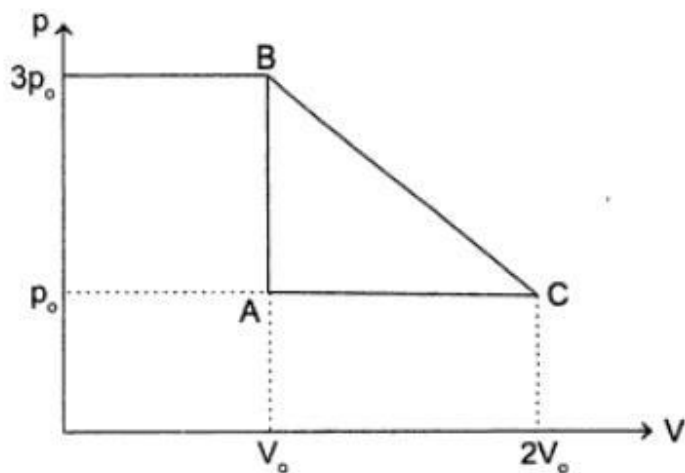
where Polytropic Process is $PV^\gamma = \text{const}$

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Heat or Thermodynamics 3) Work done calculations in various situations

One mole of an ideal gas is taken round the cyclic process $ABCA$ as shown in the figure. Calculate:

- (i) **The work done by the gas.**
- (ii) **The heat rejected by the gas in the path CA and the absorbed by the gas in the path BC .**
- (iii) **The net heat absorbed by the gas in the path BC .**
- (iv) **The maximum temperature attained by the gas during the cycle.**



Solution

- (i) Work done by the gas during a cyclic process is equal to the area enclosed by its P-V diagram. In the present case,

$$\begin{aligned}W &= \text{area of } \triangle ABC \\&= \frac{1}{2}(AC)(AB) \\&= \frac{1}{2}(2V_0 - V_0)(3p_0 - p_0) \\&= p_0 V_0\end{aligned}$$

- (ii) The path CA is an isobaric compression of one mole of an ideal gas from volume $2V_0$ to V_0 . The heat released in this path is

$$\begin{aligned}Q_1 &= n C_p \Delta T \\&= \left(\frac{3}{2}R\right)\left(\frac{p_0 \Delta V}{R}\right) \\&= \left(\frac{5}{2}p_0\right)(V_0 - 2V_0) = -\frac{5}{2}p_0 V_0\end{aligned}$$

The path AB is an isochoric expansion of one mole of an ideal gas from pressure p_0 to $3p_0$. The heat released in this process is

$$\begin{aligned}Q_2 &= n C_v \Delta T \\&= \left(\frac{5}{2}R\right)\left(\frac{V_0 \Delta p}{R}\right) \\&= \left(\frac{3}{2}V_0\right)(3p_0 - p_0) = 3p_0 V_0\end{aligned}$$

(iii) In a cyclic process, the change in internal energy is zero. Hence

$$Q_{CA} + Q_{AB} + Q_{BC} = W$$

$$-\frac{5}{2}p_0V_0 + 3p_0V_0 + Q_{BC} = p_0V_0$$

This gives $Q_{BC} = \frac{1}{2}p_0V_0$

(iv) The path BC is a straight line path. It is represented by the expression

$$p - p_0 = \left(\frac{3p_0 - p_0}{V_0 - 2V_0} \right) (V - 2V_0)$$

$$= \left(\frac{-2p_0}{V_0} \right) (V - 2V_0)$$

or $p = \frac{-2p_0}{V_0}V + 5p_0$

Replacing $p = \frac{RT}{V}$, we get

$$T = -2\frac{p_0}{V_0R}V^2 + \frac{5V_0}{R}V$$

To determine T_{\max} , we set $\frac{\partial T}{\partial V} = 0$

i.e., $0 = -\frac{2p_0}{V_0R}(2V) + \frac{5p_0}{R}$

which gives $V = \frac{5}{4} V_0$.

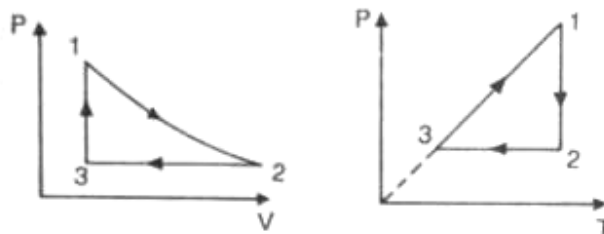
With this T_{\max} is given by

$$\begin{aligned} T_{\max} &= -\frac{2p_0}{V_0 R} \left(\frac{5}{4} V_0 \right)^2 + \left(\frac{5p_0}{R} \right) \left(\frac{5}{4} V_0 \right) \\ &= \frac{p_0 V_0}{R} \left[-\frac{25}{8} + \frac{25}{4} \right] \\ &= \frac{25 p_0 V_0}{8 R} \end{aligned}$$

Three moles of an ideal gas ($C_p = \frac{7}{2} R$) at pressure p_A and temperature T_A is isothermally expanded to twice its initial volume. It is then compressed at constant pressure to its original volume. Finally the gas is compressed at constant volume to its original pressure p_A . (i) Sketch p - V and p - T diagrams for the complete process. (b) Calculate the net work done by the gas and net heat supplied to the gas during the complete process. [IIT 1991]

Solution.

(a)



(b) In the process $1 \rightarrow 2$ the state changes from (p_A, V, T_A) to $(p_2, 2V, T_A)$.

Hence $p_2 = \frac{p_A}{2}$

Here $\Delta U = 0$ $\Delta W = \int_V^{2V} p dV = 3RT_A \ln 2$, $\Delta Q = \Delta U + \Delta W = \Delta W$

In the process $2 \rightarrow 3$ the state changes from $\left(\frac{p_A}{2}, 2V, T_A \right)$ to $(p_A/2, V, T_3)$ so that $\frac{p_A}{2} \times \frac{2V}{T_A} = \frac{p_A/2 \times V}{T_3}$ or $T_3 = T_A/2$

$$\gamma = \frac{C_p}{C_v} = \frac{C_p}{C_p - R} = \frac{\frac{7}{2}R}{\frac{7}{2}R - R} = \frac{7}{5}$$

$$\therefore \Delta U = -\frac{3RT_A}{\left(\frac{7}{5}-1\right) \times 2} = -\frac{15RT_A}{4}$$

$$\Delta W = \int_{2V}^V p dV = \frac{p_A}{2} (V - 2V) = -\frac{p_A V}{2} = -\frac{3RT_A}{2}$$

$$\therefore \Delta Q = \Delta U + \Delta W = -\frac{15}{4} RT_A - \frac{3}{2} RT_A = -\frac{21RT_A}{4}$$

In the process 3 \rightarrow 1, the state changes from $\left(\frac{p_A}{2}, V, \frac{T_A}{2}\right)$ to (p_A, V, T) that

$$\frac{p_A/2 \times V}{T_A/2} = \frac{p_A V}{T} \quad \text{or } T = T_A$$

$$\Delta U = 3C_v \left(T_A - \frac{T_A}{2}\right) = \frac{3R}{\frac{7}{5}-1} \times \frac{T_A}{2} = \frac{15}{4} RT_A$$

$$\Delta W = 0$$

$$\therefore \Delta Q = \Delta U = \frac{15}{4} RT_A$$

$$\therefore \text{Net } \Delta W = 3RT_A \ln 2 - \frac{3}{2} RT_A + 0 = 3RT_A \left(\ln 2 - \frac{1}{2}\right)$$

$$\text{Net } \Delta Q = 3RT_A \ln 2 - \frac{21RT_A}{4} + \frac{15RT_A}{4} = 3RT_A \left(\ln 2 - \frac{1}{2}\right)$$

A certain volume of a gas (diatomic) expands isothermally at 20°C until its volume is doubled and then adiabatically until its volume is again doubled. Find the final temperature of the gas, given $\gamma = 1.4$ and that there is 0.1 mole of the gas. Also calculate the work done in the two cases. $R = 8.3 \text{ J mole}^{-1} \text{ K}^{-1}$.

Solution.

We require T - V relation to calculate the final temperature.

We have $TV^{\gamma-1} = \text{constant} \therefore (273 + 20) \cdot V^{\gamma-1} = (273 + t)(2V)^{\gamma-1}$

or $273 + t = \frac{293}{2^{1.4-1}} = \frac{293}{2^{0.4}}$

$$\begin{aligned} \log(273 + t) &= \log 293 - 0.4 \log 2 = \log 293 - 0.4 \times 0.3010 \\ &= 2.4669 - 0.1204 \end{aligned}$$

or $\log(273 + t) = 2.3465$

or $273 + t = \text{antilog } 2.3465$

or $273 + t = 222.1$

$\therefore t = -50.9^\circ\text{C}$

(i) Work done in isothermal process

$$\begin{aligned} &= nRT \log_e \frac{V_2}{V_1} = \frac{8.3}{10} \times 293 \log_e \frac{2V}{V} \quad \left(\because n = \frac{1}{10} \right) \\ &= 0.83 \times 293 \times 2.3 \log_{10} 2 \quad (\because \log_e x = 2.3 \log_{10} x) \\ &= 0.83 \times 293 \times 2.3 \times 0.3010 = 1.684 \times 10^2 \text{ J} \end{aligned}$$

$$\begin{aligned} \text{(ii) Work done in adiabatic process} &= \frac{nR(T - T')}{\gamma - 1} = \frac{0.83(293 - 222.1)}{1.4 - 1} \\ &= \frac{0.83 \times 70.9}{0.4} = 1.47 \times 10^2 \text{ J} \end{aligned}$$

The volume of one mole of an ideal gas with the adiabatic exponent γ is changed according to the relation $V = aT^\gamma$, where a is a constant. Find the amount of heat absorbed by the gas in the process if the temperature is increased by ΔT .

Solution.

We have $\Delta W = \int p dV$ and $\Delta U = \int C_V dT$, for an ideal gas $pV = RT$,

$$\therefore \Delta W = \int_T^{T+\Delta T} \frac{RT}{V} dV = \int_T^{T+\Delta T} \frac{RT^2}{a} \left(-\frac{a}{T^2} dT \right) = -R\Delta T$$

$$\Delta U = \int_T^{T+\Delta T} \frac{R}{\gamma-1} dT = \frac{R\Delta T}{\gamma-1}$$

$$\therefore \Delta Q = \Delta U + \Delta W = \frac{R\Delta T}{\gamma-1} + (-R\Delta T) = \frac{(2-\gamma)R\Delta T}{\gamma-1}$$

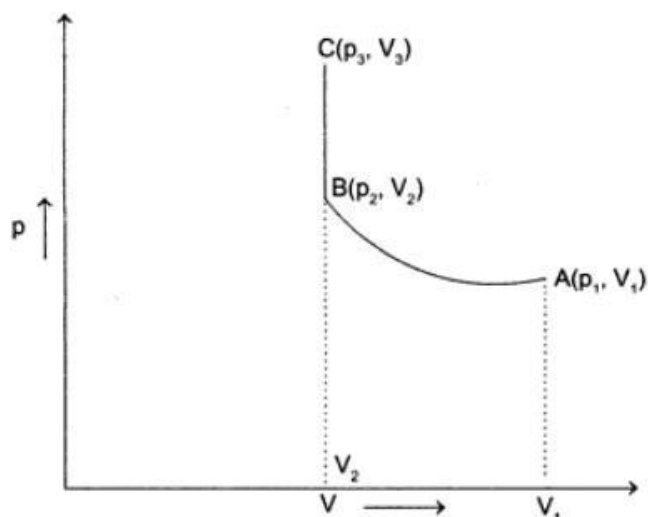
Two moles an ideal mono-atomic gas initially at pressure p_1 and volume V_1 undergo an adiabatic compression until its volume is V_2 . Then, the gas is given heat Q at constant volume V_2 .

- (i) Sketch the complete process on a p - V diagram.
- (ii) Find the total work done by the gas, total change in its internal energy and the final temperature of the gas.

[Give your answer in terms of p_1 , V_1 , V_2 , Q and R].

Solution

- (i) Figure displays the p - V diagram of the gas undergone the given two processes.



The curve A to B represents the adiabatic compression of the gas from the volume V_1 to V_2 . In this process the pressure of the gas increases p_1 to p_2 .

The line B to C represents increase in pressure of the gas as a result of giving here Q to the gas at constant volume. In this process, the pressure of the gas increases from p_2 to p_3 .

- (ii) (a) **Total work done by the gas**

Work done by the gas in adiabatic compression.

In an adiabatic process, since $Q = 0$, therefore from the first law of thermodynamics,

$$\Delta U_1 = -W_1$$

$$\text{or } W_1 = \Delta U_1 = -C_v \Delta T$$

$$= -C_v (T_2 - T_1)$$

$$= -C_v \left(\frac{p_2 V_2}{nR} - \frac{p_1 V_1}{nR} \right)$$

$$= \frac{C_{v,m}}{R} (p_2 V_2 - p_1 V_1) \quad \dots(i)$$

For a gas underdoes adiabatic process

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\text{where } \gamma = \frac{C_{p,m}}{C_{v,m}}$$

From equation (i),

$$W_1 = \frac{C_{v,m}}{R} \left[\frac{p_1 V_1^\gamma}{V_2^\gamma} V_2 - p_1 V_1 \right]$$

$$= \frac{C_{v,m}}{R} p_1 V_1 \left[\left(\frac{V_1}{V_2} \right)^{\gamma-1} - 1 \right]$$

For a mono-atomic gas,

$$C_{v,m} = \frac{3}{2}R, \text{ and } C_{p,m} = \frac{5}{2}R$$

$$\therefore \gamma = \frac{5}{3}$$

$$\text{Hence, } W_1 = -\frac{3p_1 V_1}{2} \left[\left(\frac{V_1}{V_2} \right)^{\frac{2}{3}} - 1 \right]$$

Since the volume is held constant, work done by the gas on heating at constant volume, therefore

$$W_2 = 0$$

Total work don by the gas,

$$W = W_1 = W_2$$

$$= -\frac{3p_1V_1}{2} \left[\left(\frac{V_1}{V_2} \right)^{\frac{2}{3}} - 1 \right]$$

(b) Total change in internal Energy

Change in internal energy in adiabatic compression, as derived above,

$$\Delta U_1 = \frac{3p_1V_1}{2} \left[\left(\frac{V_1}{V_2} \right)^{\frac{2}{3}} - 1 \right]$$

Change in internal energy on heating the gas at constant volume

$$\Delta U_2 = Q$$

Total change in the internal energy of the gas

$$\Delta U = \Delta U_1 + \Delta U_2$$

$$= \frac{3p_1V_1}{2} \left[\left(\frac{V_1}{V_2} \right)^{\frac{2}{3}} - 1 \right] + Q.$$

(c) Final temperature of the gas

Change in temperature in adiabatic compression.

Since, $\Delta U = C_v \Delta T$

$$\text{therefore, } \Delta T = \frac{\Delta U_1}{C_v}$$

$$\text{or } T_2 - T_1 = \frac{3p_1V_1}{2C_v} \left[\left(\frac{V_1}{V_2} \right)^{2/3} - 1 \right]$$

$$T_2 - T_1 + \frac{3p_1V_1}{2 \left(\frac{3}{2}nR \right)} \left[\left(\frac{V_1}{V_2} \right)^{2/3} - 1 \right]$$

$$= \frac{p_1V_1}{nR} + \frac{p_1V_1}{nR} \left[\left(\frac{V_1}{V_2} \right)^{2/3} - 1 \right]$$

$$= \frac{p_1V_1}{nR} \left(\frac{V_1}{V_2} \right)^{2/3}$$

Change in temperature on heating the gas

$$Q = C_v \Delta T = C_v (T_3 - T_2)$$

$$\text{or } T_3 = \frac{Q}{C_v} + T_2 = \frac{Q}{\left(\frac{3}{2} \right) nR} + \frac{p_1V_1}{nR} \left(\frac{V_1}{V_2} \right)^{2/3}$$

Since $n = 2$, therefore

$$T_3 = \frac{Q}{(3 \text{ mole})R} + \frac{p_1V_1}{(2 \text{ mole})R} \left(\frac{V_1}{V_2} \right)^{2/3}.$$

Two moles of helium gas ($\gamma = 5/3$) are initially at temperature 27°C and occupy a volume of 20 litres. The gas is expanded

at constant pressure until the volume is doubled. Then, it undergoes an adiabatic change until the temperature returns to its initial value.

- (i) Sketch the process on a p-V diagram.
- (ii) What are the final volume and pressure of the gas?
- (iii) What is the work done by the gas?

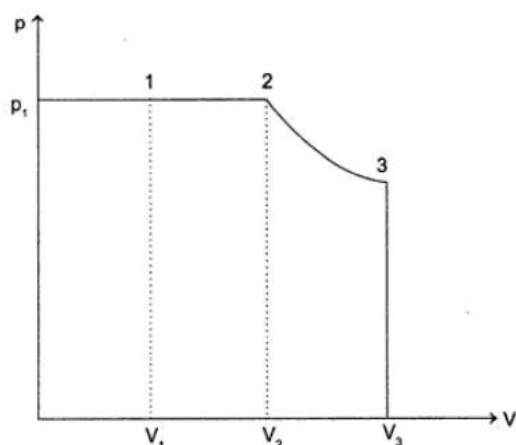
Solution

(i) $V_1 = 20 \times 10^{-3} \text{ m}^3$

$T_1 = 300 \text{ K}$

$n = 2 \text{ moles}$

$\gamma = \frac{5}{3}$



Process 1 \rightarrow 2 is isobaric expansion

$$p_1 V_1 = nRT_1$$

$$\begin{aligned}\therefore p_1 &= \frac{nRT_1}{V_1} \\ &= \frac{2 \times 8.3 \times 300}{20 \times 10^{-3}} = 2.49 \times 10^5 \text{ Nm}^{-2}\end{aligned}$$

Now, $V \propto T$

$$\therefore \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\text{or } T_2 = T_1 \times \frac{V_2}{V_1} = 300 \times \frac{2V_1}{V_1}$$

$$\therefore T_2 = 600 \text{ K}$$

$$V_2 = 40 \times 10^{-3} \text{ m}^3$$

Work done during process 1 \rightarrow 2,

$$\begin{aligned}(W)_{1-2} &= p \times \Delta V \\ &= 2.49 \times 10^5 \times (40 - 20) \times 10^{-3} \\ &= 4980 \text{ J}\end{aligned}$$

Process 2 \rightarrow 3 is adiabatic expansion

$$T_2 = 600 \text{ K}$$

$$p_2 = p_1 = 2.48 \times 10^5 \text{ N/m}^2$$

$$V_2 = 40 \times 10^{-3} \text{ m}^3$$

Given, $T_2 V_2^{\gamma-1} = T_3 V_3^{\gamma-1}$, $T_3 = T_1$

$$\therefore \left(\frac{V_3}{V_2} \right)^{\frac{5}{3}-1} = \frac{T_2}{T_1} = \frac{600}{300} = 2$$

$$\begin{aligned}\therefore V_3 &= V_2 \times (2)^{3/2} \\ &= 40 \times 10^{-3} (2)^{3/2} \\ &= 113.14 \times 10^{-3} \text{ m}^3\end{aligned}$$

$$\begin{aligned} \text{Now, } p_2 V_2^\gamma &= p_3 V_3^\gamma \\ \therefore p_3 &= p_2 \left(\frac{V_2}{V_3} \right)^\gamma \\ &= 2.48 \times 10^5 \left(\frac{40}{113.14} \right)^{5/3} = 0.44 \times 10^5 \text{ N/m}^2 \\ (W)_{2-3} &= \frac{p_2 V_2 - p_3 V_3}{\gamma - 1} \\ &= \frac{(2.49 \times 10^5)(40 \times 10^{-3}) - (0.44 \times 10^5)(113.14 \times 10^{-3})}{(5/3) - 1} \\ &= \mathbf{7472.8 \text{ J.}} \end{aligned}$$

(ii) Final volume, $V_3 = 113.14 \times 10^{-3} \text{ m}^3$

Final pressure, $p_3 = 0.44 \times 10^5 \text{ Nm}^{-2}$

(iii) Total work done by the gas = $W =$

$$\begin{aligned} &(W)_{1-2} + (W)_{2-3} \\ &= 4980 + 7472.8 = \mathbf{12452.8 \text{ J.}} \end{aligned}$$

Work done example in Isothermal expansion

A gram mole of a gas at 127°C expands isothermally until its volume is doubled. Find the amount of work done.

- (a) 238 cal (b) 548 cal
(c) 548 J (d) 238 J

$$\begin{aligned} \text{(b) } W &= 2.303 RT \log_{10} \left(\frac{V_2}{V_1} \right) \\ &= 2.303 \times 8.311 \times 400 \times \log_{10} 2 \\ &= 2310.1 \text{ J} = 548 \text{ cal.} \end{aligned}$$

Example in Isothermal Expansion

How much work is done by an ideal gas in expanding isothermally from an initial volume of 3 litres of 20 atm to a final volume of 24 litres?

Solution In isothermal process at temperature T,

$$W = 2.303nRT \log \frac{V_2}{V_1}$$

or $W = 2.303(p_1 V_1) \log \frac{V_2}{V_1}$
(using $p_1 V_1 = nRT$)

$$= 2.303 (20 \times 3) \log \frac{P_1}{P_2} \text{ lt. atm}$$

$$= 2.303 \times 60 \log 8 (101) \text{ J}$$

$$= \mathbf{1.26 \times 10^4 \text{ J}}$$

Work done by the gas

The ratio of work done by an ideal diatomic gas to the heat supplied by the gas in an isobaric process is

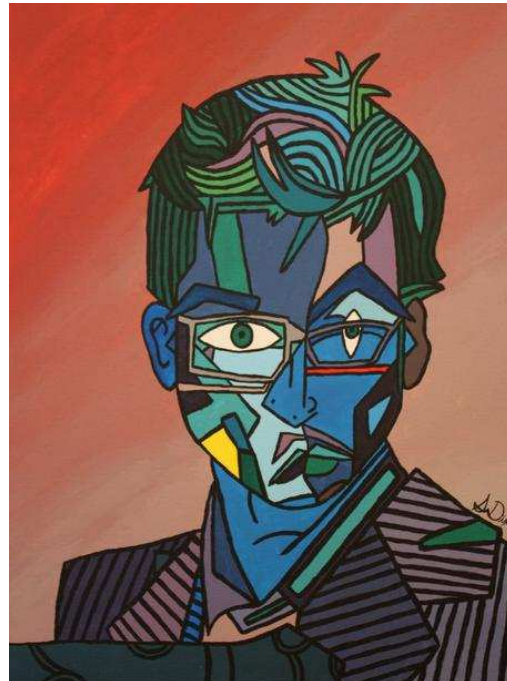
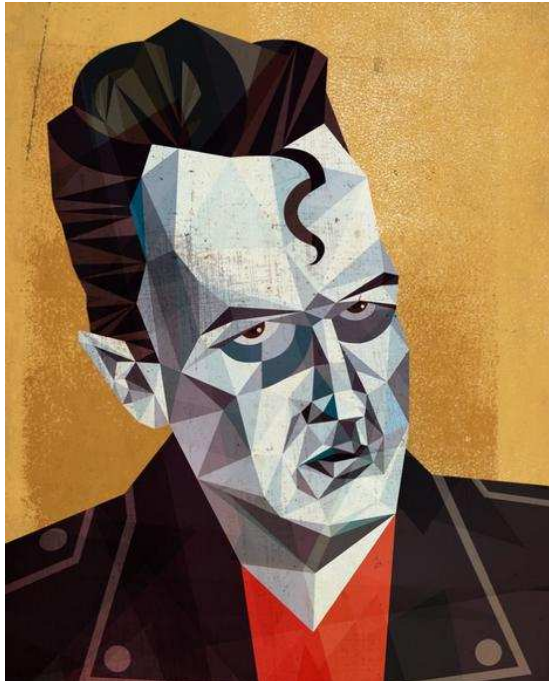
- (a) $\frac{5}{7}$ (b) $\frac{3}{5}$
(c) $\frac{2}{7}$ (d) $\frac{5}{3}$

Ans - (c) $\Delta U = nC_v\Delta T = n \frac{5}{2} R\Delta T$

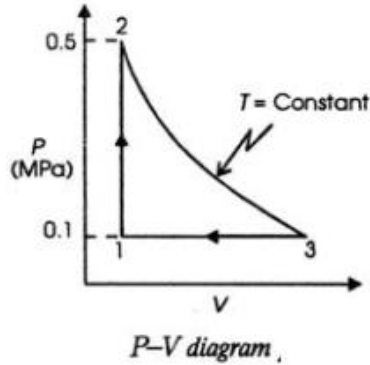
$$\Delta Q = nC_p\Delta T = n \frac{7}{2} R\Delta T$$

$$W = \Delta Q - \Delta U = \frac{n7}{2} R\Delta T - nR\Delta T$$

$$\frac{W}{Q} = \frac{2}{7}$$



One mole of a gas which obeys the relation $Pv = RT$, where $R = 8.314 \text{ J/mol K}$ is initially at 300 K and 0.1 MPa . The gas is heated at constant volume till the pressure rises to 0.5 MPa and then allowed to expand at constant temperature till the pressure reduces to 0.1 MPa . Finally the gas is returned to its original state by compressing at constant pressure. Calculate the work done by the gas in each of the processes and also estimate the net work done by the gas.



Solution The process followed by the gas is shown in Fig.2.12. Work done by the gas during process 1–2 is given by

$$W_{1-2} = \int_1^2 Pdv = 0 \quad (\text{since } dv = 0)$$

We know $P_1v_1 = RT_1$ and $P_2v_2 = RT_2$. Therefore

$$\frac{T_2}{T_1} = \frac{P_2v_2}{P_1v_1} = \frac{P_2}{P_1} = \frac{0.5 \times 10^6}{0.1 \times 10^6} = 5 \quad (\text{since } v_2 = v_1)$$

or $T_2 = 5T_1 = 5 \times 300 = 1500 \text{ K}$

Work done by the gas during process 2–3 is given by

$$W_{2-3} = \int_2^3 Pdv = \int_2^3 \frac{RT}{v} dv = RT_2 \ln \frac{v_3}{v_2}$$

We know $P_2v_2 = P_3v_3$ (since $T_2 = T_3$). Therefore

$$\frac{v_3}{v_2} = \frac{P_2}{P_3} = \frac{0.5 \times 10^6}{0.1 \times 10^6} = 5$$

Hence $W_{2-3} = RT_2 \ln 5 = 8.314 \times 1500 \times \ln 5 = 20.071 \text{ kJ}$. Work done during process 3–1 is given by

$$W_{3-1} = \int_3^1 Pdv = P_1(v_1 - v_3) = P_1v_1 \left(1 - \frac{v_3}{v_1}\right) = RT_1 \left(1 - \frac{v_3}{v_1}\right)$$

We know $P_1v_1 = RT_1$ and $P_3v_3 = RT_3$

$$\text{or } \frac{v_3}{v_1} = \frac{RT_3}{P_3} \cdot \frac{P_1}{RT_1} = \frac{T_3}{T_1} \quad (\text{since } P_1 = P_3). \text{ Therefore}$$

$$W_{3-1} = RT_1 \left(1 - \frac{v_3}{v_1}\right) = RT_1 \left(1 - \frac{T_3}{T_1}\right) = 8.314 \times 300 \left(1 - \frac{1500}{300}\right) = -9.977 \text{ kJ}$$

$$\text{Net work done by the gas, } W = W_{1-2} + W_{2-3} + W_{3-1} = 0 + 20.071 - 9.977 = 10.094 \text{ kJ}$$

Work done by the gas

A sample of ideal gas ($\gamma = 1.4$) is heated at constant pressure. If an amount of 140 J of heat is supplied to the gas, find:

- (i) **The change in internal energy of the gas.**
- (ii) **The work done by the gas.**

Solution Suppose, the sample contains n moles. Also, suppose the volume changes from V_1 to V_2 and the temperature changes from T_1 to T_2 .

The heat supplied is given by

$$\Delta Q = nC_p(T_2 - T_1)$$

- (i) Change in internal energy

$$\Delta U = nC_v(T_2 - T_1)$$

$$= \frac{C_v}{C_p} nC_p(T_2 - T_1)$$

$$= \frac{C_v}{C_p} \Delta Q = \frac{140 \text{ J}}{1.4} = \mathbf{100 \text{ J}}$$

- (ii) Work done by gas

$$\Delta W = \Delta Q - \Delta U$$

$$= 140 \text{ J} - 100 \text{ J} = \mathbf{40 \text{ J}}$$

work done by the gas

A sample of gas ($\gamma = 1.5$) is taken through an adiabatic process in which the volume is compressed from 1600 cm^3 to 400 cm^3 . If the initial pressure is 150 kPa ,

- (i) **What is the final pressure?**
- (ii) **How much work is done by the gas in the process?**

Solution

- (i) For an adiabatic process

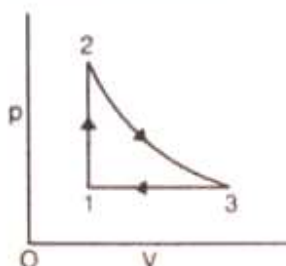
$$p_1 V_1^\gamma = p_2 V_2^\gamma$$

$$\begin{aligned}\text{Thus, } p_2 &= p_1 \left(\frac{V_1}{V_2} \right)^\gamma \\ &= (150 \text{ kPa}) \left(\frac{1600}{400} \right)^{\frac{3}{2}} \\ &= \mathbf{1200 \text{ kPa}}\end{aligned}$$

- (ii) Work done by the gas in an adiabatic process

$$\begin{aligned}W &= \frac{p_1 V_1 - p_2 V_2}{\gamma - 1} \\ &= \frac{(150 \text{ kPa})(1600 \text{ cm}^3) - (1200 \text{ kPa})(400 \text{ cm}^3)}{1.5 - 1} \\ &= \frac{240 \text{ J} - 480 \text{ J}}{0.5} = \mathbf{-480 \text{ J}}\end{aligned}$$

A cyclic process for an ideal monatomic gas ($C_V = 12.5 \text{ J mol}^{-1} \text{ K}^{-1}$) is represented in the figure. The temperatures at 1, 2 and 3 are 300 K, 600 K and 455 K, respectively. Compute the values of ΔQ , ΔU and ΔW for each of the processes. The process from 2 to 3 is adiabatic.



Solution.

In the process from 1 to 2

$$\Delta W = \int p dV = 0 \quad (\text{volume remains constant})$$

$$\begin{aligned} \Delta Q &= \int_{T_1}^{T_2} C_V dT = C_V(T_2 - T_1) \\ &= 12.5(600 - 300) = 3750 \text{ joules} \end{aligned}$$

By the first law of thermodynamics

$$\begin{aligned} \Delta Q &= \Delta U + \Delta W \quad \text{or} \quad \Delta U = \Delta Q - \Delta W \\ &= 3750 - 0 = 3750 \text{ joules} \end{aligned}$$

In the process 2 to 3 $\Delta Q = 0$
(since the process is adiabatic)

$$\begin{aligned}\Delta W &= \frac{R(T_2 - T_3)}{\gamma - 1} \\ &= C_V(T_2 - T_3) \quad \left(\because C_V = \frac{R}{\gamma - 1} \right) \\ &= 12.5(600 - 455) = 12.5 \times 145 = 1812.5 \text{ joules}\end{aligned}$$

$$\therefore \Delta U = \Delta Q - \Delta W = 0 - 1812.5 = -1812.5 \text{ joules}$$

$$\text{In the process from 3 to 1, } \Delta W = \int_{V_3}^{V_1} p dV = p(V_1 - V_3) = pV_1 - pV_3$$

$$\begin{aligned}\text{or } \Delta W &= R(T_1 - T_3) \quad (\because pV = RT) \\ &= 8.31(300 - 455) = -1288 \text{ joules}\end{aligned}$$

$$\begin{aligned}\Delta Q &= \int_{T_3}^{T_1} C_p dT = C_p(T_1 - T_3) = 1.67 \times 12.5 \times (300 - 455) \quad \left(\because \gamma = \frac{C_p}{C_v} \right) \\ &= -3235.6 \text{ joules.}\end{aligned}$$

By the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

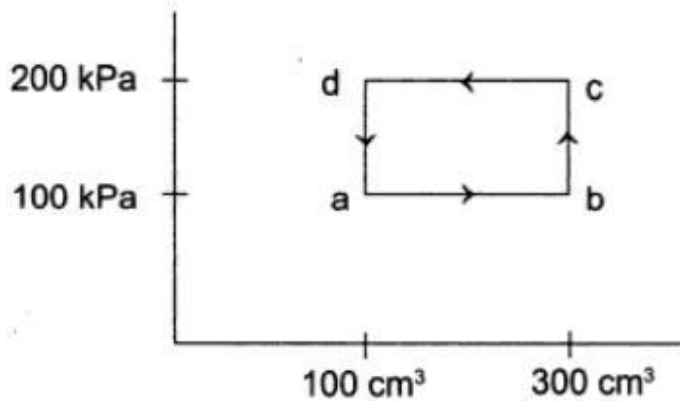
$$\therefore \Delta U = \Delta Q - \Delta W = (-3235.6) - (-1288) = -1947.6 \text{ joules}$$

Question on Total Heat rejected

A thermodynamic system is taken through the cycle $a b c d a$.

(i) Calculate the work done by the gas during the parts ab , bc , cd and da .

(ii) Find the total heat rejected by the gas during the process.



Solution

(i) Work done during the part $ab = \int_a^b p dV$

$$\begin{aligned} &= (100 \times \text{Pa}) \int_a^b dV \\ &= (100 \text{ kPa}) (300 \text{ cm}^3 - 100 \text{ cm}^3) \\ &= 20 \text{ J} \end{aligned}$$

The work done during bc is zero as the volume does not change. The work done during cd ,

$$\begin{aligned} &= \int_d^c p dV \\ &= (200 \text{ kPa}) (100 \text{ cm}^3 - 300 \text{ cm}^3) \\ &= -40 \text{ J} \end{aligned}$$

The work done during da is zero as the volume does not change.

(ii) Total work done by the system during the cycle $a b c d a$.

$$\Delta W = 20 \text{ J} - 40 \text{ J}$$

$$= -20 \text{ J}$$

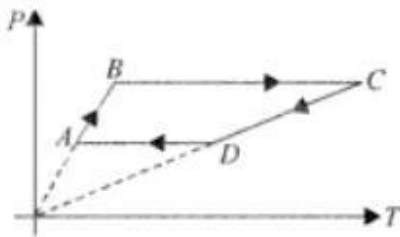
Change in the internal energy, $\Delta U = 0$, as the initial state is the same as the final state.

$$\begin{aligned} \text{Thus, } \Delta Q &= \Delta U + \Delta W \\ &= -20 \text{ J} \end{aligned}$$

So, the system rejects **20 J** of heat during the cycle.

- Question with P T diagram

3 moles of an ideal monoatomic gas perform a cycle shown in Fig. The gas temperatures $T_A = 400 \text{ K}$, $T_B = 800 \text{ K}$, $T_C = 2400 \text{ K}$, $T_D = 1200 \text{ K}$. Find the work done by the gas.



Solution :

$$W_{BC} = 3R(T_C - T_B)$$

$$W_{AB} = W_{CD} = 0$$

because the processes are isochoric

$$W_{DA} = 3R(T_A - T_D)$$

Total work done

$$\begin{aligned} W_{BC} + W_{DA} &= 3R(T_A + T_C - T_B - T_D) \\ &= 3R(400 + 2400 - 800 - 1200) \\ &= 2400R = 20 \text{ kJ} \end{aligned}$$

Work done by the gas

Two moles of Helium gas ($\gamma = \frac{5}{3}$) are initially at 27°C and occupy a volume of 20 litres. The gas is first expanded at constant pressure until the volume is doubled. Then it undergoes an adiabatic change until the temperature returns to its initial value.

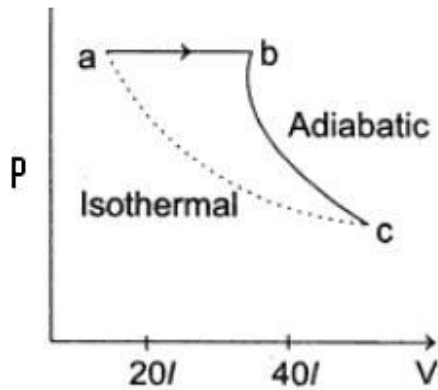
- (i) Sketch the process in a p–V diagram.
- (ii) What is the final volume and pressure of the gas?
- (iii) What is the work done by the gas?

Solution

- (i) The process is shown in the figure. During the part *ab*, since the pressure is constant, we have

$$\frac{P_a V_a}{T_a} = \frac{P_b V_b}{T_b}$$

$$\begin{aligned} \text{or } T_b &= \frac{V_b}{V_a} T_a \\ &= 2T_a = \mathbf{600 \text{ K}} \end{aligned}$$



During the part bc , the gas is adiabatically returned to the temperature T_a . The point a and point c are on the same isothermal. Thus, we draw an adiabatic curve bc and an isothermal from a and look for the point of intersection c . That is the final state.

(ii) From the isothermal ac ,

$$p_a V_a = p_b V_b \quad \dots(i)$$

And from the adiabatic curve bc ,

$$p_b V_b^\gamma = p_c V_c^\gamma$$

$$\text{or } p_a (2V_a)^\gamma = p_c V_c^\gamma$$

Dividing equation (ii) by equation (i), we get

$$2^\gamma (V_a)^{\gamma-1} = (V_c)^{\gamma-1}$$

$$\begin{aligned} \text{or } V_c &= 2^{\frac{\gamma}{\gamma-1}} V_a 4\sqrt{2} V_a \\ &= 113 \text{ litres} \end{aligned}$$

From equation (i),

$$\begin{aligned} p_c &= \frac{p_a V_a}{V_c} = \frac{nRT}{V_c} \\ &= \frac{2 \text{ mol} \times (8.3 \text{ J/mol-K})(300 \text{ K})}{113 \times 10^{-3} \text{ m}^3} \\ &= 4.4 \times 10^4 \text{ Pa} \end{aligned}$$

(iii) Work done by the gas in the part *ab*

$$\begin{aligned} &= p_a (V_b - V_a) \\ &= p_a V_b - p_a V_a = nRT_2 - nRT_1 \\ &= 2 \text{ mole} \times (8.3 \text{ J/mol-K}) \times (600 \text{ K} - 300 \text{ K}) \\ &= 4980 \text{ J} \end{aligned}$$

Work done in the adiabatic part *bc*

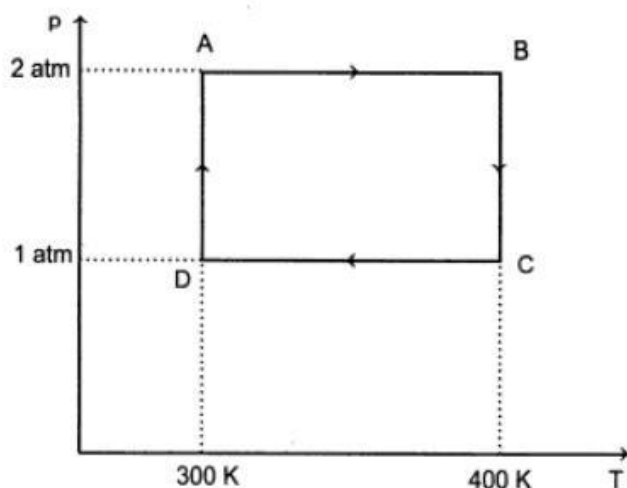
$$\begin{aligned} &= \frac{p_b V_b - p_c V_c}{\gamma - 1} \\ &= \frac{nR(T_2 - T_1)}{\gamma - 1} = \frac{4980}{\frac{5}{3} - 1} = 7470 \text{ J} \end{aligned}$$

Net work done by the gas = 4980 J + 7470 J = **12450 J.**

Example of cycle given P T diagram

Two moles of helium gas undergo a cyclic process as shown in the figure. Assuming the gas to be ideal, calculate the following quantities in this process:

- (i) **The net change in the heat energy.**
 - (ii) **The net work done.**
 - (iii) **The net change in internal energy.**
- [$R = 8.32 \text{ J mol}^{-1}$]**



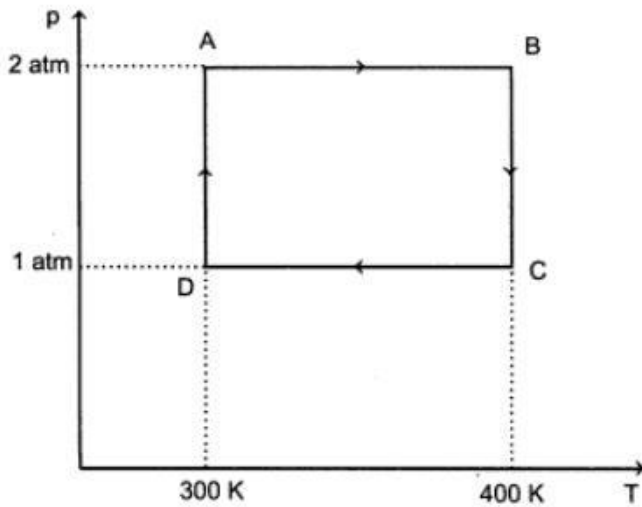
Solution Number of moles, $n = 2$
Helium is a mono-atomic gas.

$$\therefore C_V = \frac{3}{2}R$$

$$C_P = \frac{5}{2}R$$

The gas undergoes cyclic process.

Since, internal energy is property of the system, the net change in internal energy during the cyclic process is zero.



Hence, according to the first law of thermodynamics, the net change in the heat energy is equal to the net work done.

$$(i) (\Delta Q)_{Net} = (\Delta Q)_{AB} + (\Delta Q)_{BC} + (\Delta Q)_{DA}$$

$$\begin{aligned} (\Delta Q)_{AB} &= n \times C_p \times (T_B - T_A) \\ &= 2 \times \frac{5}{2} \times 8.32(400 - 300) = 4160 \text{ J} \end{aligned}$$

Since Process BC is isothermal, therefore $\Delta U = 0$

$$\begin{aligned} (\Delta Q)_{BC} &= (\Delta W)_{BC} \\ &= nRT \ln\left(\frac{V_C}{V_B}\right) = nRT \ln\left(\frac{P_B}{P_C}\right) \\ &= 2 \times 8.32 \times 400 \ln\left(\frac{2}{1}\right) = 4613.6 \text{ J} \end{aligned}$$

$$\begin{aligned} (\Delta Q)_{DA} &= nRT \ln\left(\frac{P_D}{P_A}\right) \\ &= 2 \times 8.32 \times 300 \ln\left(\frac{2}{1}\right) = -3460.2 \text{ J} \end{aligned}$$

$$\begin{aligned} \therefore (\Delta W)_{\text{Net}} &= 4160 + 4613.6 - 4160 \\ &\quad - 3460.2 \\ &= \mathbf{1153.4 \text{ J}} \end{aligned}$$

$$\begin{aligned} \text{(ii) } (\Delta W)_{\text{Net}} &= (\Delta Q)_{\text{Net}} \\ &= \mathbf{1153.4 \text{ J}} \end{aligned}$$

$$\text{(iii) } (\Delta U)_{\text{Net}} = \mathbf{0}$$

-

Heat or Thermodynamics 4) Efficiency of Refrigerator and Refrigeration constant

Coefficient of Performance of a Refrigerator

$$\begin{aligned} \beta &= \frac{\text{Heat absorbed from cold reservoir}}{\text{Work done on refrigerator}} \\ &= \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{1}{\frac{Q_1}{Q_2} - 1} \\ &= \frac{1}{\frac{T_1}{T_2} - 1} = \frac{T_2}{T_1 - T_2} \end{aligned}$$

Coefficient of performance of refrigerator working between temperatures 30 and 0 deg centigrade

What is the approximate coefficient of performance of a Carnot refrigerator working between 30°C and 0°C ?

- (a) 0 (b) 1
(c) 9 (d) 10.

Ans : c)

Coefficient of performance,

$$\beta = \frac{T_2}{T_1 - T_2} = \frac{273 + 0}{(273 + 30) - 273} = \frac{273}{30} = 9$$

Efficiency of Refrigerator is given by

Efficiency of Refrigerator

$$\eta = 1 - \frac{T_c}{T_H}$$

So in this case efficiency $\eta = 1 - (273 / 303) = 0.099 \approx (\text{approx}) 0.1$ or 10%

Refrigerator Problem

A refrigerator works between 0°C and 27°C. Heat is to be removed from the refrigerated space at the rate of 50 kcal/minute, the power of the motor of the refrigerator is :

- (a) 0.346 kW (b) 3.46 kW
(c) 34.6 kW (d) 346 kW

Ans : a)

$$\begin{aligned} \frac{T_2}{T_1 - T_2} &= \frac{Q_2}{W} \\ \frac{273}{300 - 273} &= \frac{50,000}{W} \\ W &= \frac{27 \times 50,000}{273} \text{ cal / min} \\ P &= \frac{W}{t} = \frac{4.2 \times 27 \times 50,000}{60 \times 273} \text{ Joule / sec} \\ &= 346 \text{ watt} = 0.346 \text{ kW} \end{aligned}$$

Efficiency of Refrigerator

$$\eta = 1 - \frac{T_c}{T_H}$$

So in this case efficiency $\eta = 1 - (273 / 300) = 0.09 \approx$ or 9%

Refrigerator Problem

An ideal refrigerator has a freezer at a temperature of -13°C . The coefficient of performance of the engine is 5. The temperature of the air (to which heat is rejected) is :

- (a) 320°C (b) 39°C (c) 325 K (d) 325°C

Ans : b)

$$T_2 = 273 - 13 = 260, K = \frac{T_2}{T_1 - T_2}$$

$$5 = \frac{260}{T_1 - 260} \text{ or } T_1 - 260 = 52$$

$$T_1 = 312\text{ K}, T_2 = 312 - 273 = 39^{\circ}\text{C}$$

Efficiency of Refrigerator

$$\eta = 1 - \frac{T_c}{T_H}$$

So in this case efficiency $\eta = 1 - (260 / 312) = 0.16666 \approx (\text{approx}) 0.16667$ or 16.67%

Refrigerator Problem

A Carnot's engine works as a refrigerator between 250 K and 300 K . If it receives 750 calories of heat from the reservoir at the lower temperature, the amount of heat rejected at the higher temperature is :

- (a) 900 calories (b) 625 calories
(c) 750 calories (d) 1000 calories

Ans : a)

$$\frac{750}{W} = \frac{250}{300 - 250}$$

$$\text{Heat rejected} = 750 + 150 = 900 \text{ cal.}$$

Efficiency of Refrigerator

$$\eta = 1 - \frac{T_c}{T_H}$$

So in this case efficiency $\eta = 1 - (250 / 300) = 0.1666666 \approx (\text{approx}) 0.16667$ or 16.67%

Refrigerator Problem

A refrigerator having a coefficient of performance of 5 is run by an electric motor of power 1.2 kW. How much is the mass of ice formed from water at 0°C per hour by the refrigerator ?

- (a) nearly 6 kg (b) nearly 60 kg
(c) nearly 25.2 kg (d) 252 kg

Ans : b)

$$5 = \frac{Q_2}{Pt} \quad \text{or} \quad Q_2 = 5 \times 1.2 \times 1000 \times 3600 \text{ J}$$

$$Q = 216 \times 10^5 \text{ J} = 5142857 \text{ cal.}$$

$$\therefore Q = mL \quad \text{or} \quad m = Q/L = 64.2 \text{ kg} \quad \therefore m \approx 60 \text{ kg}$$

Refrigerator

Given :	T_H	1200 °C	T_C	200 °C	
	T_H	1473.15 K	T_C	473.15 K	
Find :	η	?	COP_R	?	COP_{HP} ?

Part a.) The thermal efficiency of a **Carnot Cycle** depends only on the temperatures of the reservoirs with which it interacts. The equation that defines this relationship is :

$$\eta = 1 - \frac{T_C}{T_H} \quad \text{Eqn 1}$$

Just be sure to use absolute temperature in Eqn 1 ! In this case, convert to **Kelvin**.
Temperatures in Rankine will work also.

$$\eta = 67.9\%$$

part b.) The coefficient of performance of a **Carnot Refrigeration Cycle** also depends only on the temperatures of the reservoirs with which it interacts. The equation that defines this relationship is :

$$COP_R = \frac{1}{\frac{T_H}{T_C} - 1} = \frac{T_C}{T_H - T_C} \quad \text{Eqn 2}$$

Using **T** in Kelvin yields :

$$COP_R = 0.4732$$

This is an exceptionally **BAD COP_R** because it is less than 1. This isn't terribly surprising when you consider that the refrigerator must reject heat to a reservoir at **1200°C** !!

Carnot engine efficiency is covered in every book. But efficiency of refrigerator and Coefficient of Performance is rarely discussed.

Two engines are working in such a way that sink of one is source of the other. Their efficiencies are equal. Find the temperature of the sink of first if its source temperature is 927°C and temperature of sink of the second is 27°C.

- (a) 327 K (b) 327°C
(c) 600°C (d) none of these

Solution (b) $\eta = 1 - T_2/T_1 = 1 - T_3/T_2$ or $T_2^2 = T_1 T_3$

or $T_2 = \sqrt{1200 \times 300} = 600 \text{ K} = 327^\circ\text{C}$

Heat or Thermodynamics 5) Concept of “free expansion”

Free expansion:

If a system (a gas), expands in such a way that no heat enters or leaves the system (adiabatic process) and also no work is done by or on the system, then the expansion is called the free expansion.

Consider an adiabatic vessel with rigid walls divided into two parts. One containing a gas and the other evacuated. When the partition is suddenly broken, the gas rushes into the vacuum and expands freely.

∴ Net change in internal energy
 $U_f - U_i = \Delta Q - W$ as $\Delta Q = 0$ and $W = 0$
 ∴ $U_i = U_f$

The initial and final internal energies are equal in free expansion.

One mole of an ideal diatomic gas underwent an adiabatic expansion from 298 K, 15.00 atm, and 5.25 L to 2.50 atm against a constant external pressure of 1.00 atm. What is the final temperature of the system?

Plan This is an isobaric adiabatic expansion against constant external pressure, but overall pressure decreases (volume increases, gas expands). Final temperature T_2 is given by P-V-T relation as:

$$T_2 = T_1 \left(\frac{C_V + P_{\text{ext}} \frac{R}{P_1}}{C_V + P_{\text{ext}} \frac{R}{P_2}} \right)$$

Solution For diatomic gas $C_V = \frac{5}{2}R$, $T_1 = 298 \text{ K}$, $T_2 = ?$,

$P_2 = 2.50 \text{ atm}$, $P_1 = 15.00 \text{ atm}$, $P_{\text{ext}} = 1.00 \text{ atm}$

$$\begin{aligned} \therefore T_2 &= 298 \left(\frac{\frac{5}{2}R + \frac{R}{15}}{\frac{5}{2}R + \frac{R}{2.5}} \right) \\ &= 263.7 \text{ K} \end{aligned}$$

One mole of a gas is put under a weightless piston of a vertical cylinder at temperature T . The space over the piston is atmosphere. How much work should be performed to increase isothermally the

volume under the piston to twice the volume (neglect friction of piston).

Solution Let A be the area of piston, therefore

$$F + pA = p_0 A$$

or $F = (p_0 - p) A$

Work done by agent is given by

$$W = \int_v^{\eta v} (p_0 - p) A dx$$

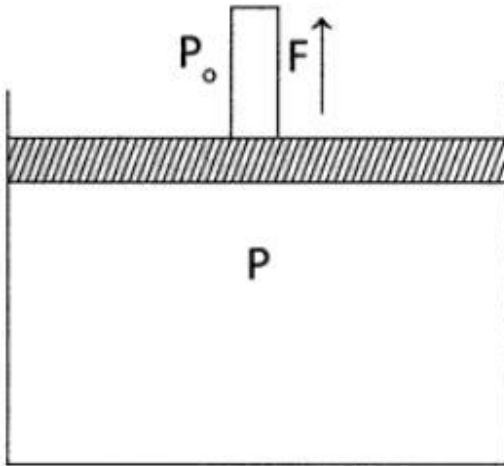
$$= \int_v^{\eta v} (p_0 - p) dV$$

$$= \int_v^{\eta v} p_0 dV - \int_v^{\eta v} p dV$$

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$$= p_0(\eta - 1)V - \int_v^{\eta v} nRT \frac{dV}{V}$$

(since $pV = nRT$)



$$= p_0(\eta - 1)V - nRT \log_e \eta$$

$$= nRT [(\eta - 1) \log_e \eta]$$

where, $\eta = 2$ and $n = 1$

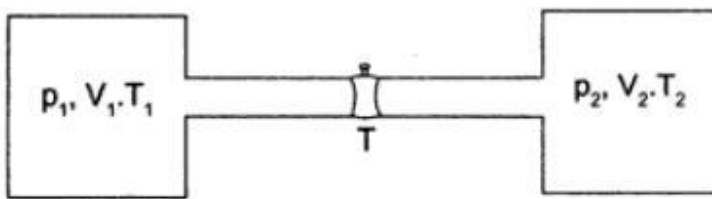
$$\mathbf{W = RT [1 - \log_e 2]}$$

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Adiabatic free expansion

Two vessels of volume V_1 and V_2 contain the same ideal gas. The pressure in the vessels are p_1 and p_2 and the temperatures are T_1 and T_2 respectively. The two vessels are now connected to each other through a narrow tube. Assuming that no heat is exchanged between the surroundings and the vessels, find the common pressure and temperature attained after the connection.

Solution



The amount of gas in vessel 1 is

$$n_1 = \frac{p_1 V_1}{RT_1}$$

If p' and T' are the common pressure and temperature after the connection is made, the amount are

$$n'_1 = \frac{p'V_1}{RT'}$$

and $n'_2 = \frac{p'V_2}{RT'}$

We have, $n_1 + n_2 = n'_1 + n'_2$

or $\frac{p_1V_1}{RT_1} + \frac{p_2V_2}{RT_2} = \frac{p'V_1}{RT'} + \frac{p'V_2}{RT'}$

or $\frac{p'}{T'} = \frac{1}{V_1 + V_2} \left(\frac{p_1V_1}{T_1} + \frac{p_2V_2}{T_2} \right)$

or $\frac{T'}{P'} = \frac{T_1T_2(V_1 + V_2)}{p_1V_1T_2 + p_2V_2T_1}$

As the vessels have fixed volume, no work done by the gas plus the vessels system. Also, no heat is exchanged with the surroundings.

Thus, the internal energy of the total system remains constant. The internal energy of an ideal gas is

$$U = nC_vT = C_v \frac{pV}{R}$$

Internal energy of the gases before the connection

$$= \frac{C_v p_1 V_1}{R} + \frac{C_v p_2 V_2}{R}$$

And Internal energy of the gas after the connection

$$= \frac{C_v p' (V_1 + V_2)}{R}$$

Neglecting the change in internal energy of the vessels (the heat capacity of the vessels is assumed negligible).

$$\frac{C_v p_1 V_1}{R} + \frac{C_v p_2 V_2}{R} = \frac{C_v p' (V_1 + V_2)}{R}$$

or
$$p' = \frac{p_1 V_1 + p_2 V_2}{V_1 + V_2}$$

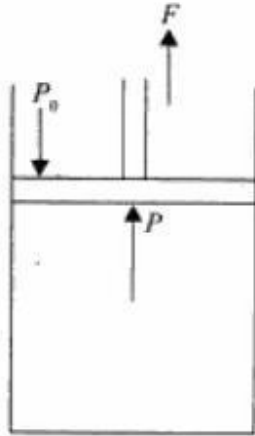
From equation (i),
$$T' = \frac{T_1 T_2 p_1 V_1 + p_2 V_2}{p_1 V_1 T_2 + p_2 V_2 T_1}$$

Question on work done

One mole of an ideal gas is contained under a weightless piston of a vertical cylinder at a temperature T . The space over the piston opens into the atmosphere. What work has to be performed in order to increase isothermally the gas volume under the piston η times by slowly raising the piston? Neglect friction.

Solution :

Let A be the area of cross section



$$F + PA = P_0 A$$

$$F = (P_0 - P) A$$

Work done by the agent

$$W = \int_V^{\eta V} F dx = \int_V^{\eta V} (P_0 - P) A dx$$

$$= \int_V^{\eta V} (P_0 - P) dV$$

$$= P_0(\eta - 1) V - \int_V^{\eta V} nRT \frac{dV}{V}$$

$$= RT [(\eta - 1) - n \log_e \eta]$$

Heat or Thermodynamics 6) Ingen Housz's experiment of identical rods

Ingen-Housz's experiment Ingen Housz showed that if a number of identical rods of different metals are coated with wax and one of their ends is put in boiling water, then in steady state, the square of length of the bar over which wax melts is directly proportional to the thermal conductivity of the metal. That is,

$$\frac{K}{L^2} = \text{constant}$$

Heat or Thermodynamics 7) Concept of Internal Energy at Room temperature

Find the internal energy of air in a room of volume 40 m^3 at 1 standard atmospheric pressure.

Solution.

We have $U = \frac{pV}{\gamma - 1}$ for a perfect gas

Air is diatomic and therefore its γ is 1.4.

$$\therefore U = \frac{10^5 \times 40}{1.4 - 1} \quad (p = 1 \text{ atm} = 10^5 \text{ Nm}^{-2}) = 10^7 \text{ joules.}$$

Question in Internal Energy

The internal energy of a mono-atomic ideal gas is $1.5 nRT$. One mole of helium is kept in a cylinder of cross-section 8.5 cm^2 . The cylinder is closed by a light frictionless piston. The gas is heated slowly in a process during which a total

of 42 J heat is given to the gas. If the temperature rised through 2° C, find the distance moved by piston. Atmospheric pressure = 100 kPa.

Solution Change in internal energy of the gas

$$\begin{aligned}\Delta U &= 1.5 nRT \\ &= 1.5 (1 \text{ mole}) (8.3 \text{ J/mol} - \text{K}) (2\text{K}) \\ &= 24.9 \text{ J}\end{aligned}$$

Heat given to the gas = 43 J

$$\begin{aligned}\text{Work done by the gas is } \Delta W &= \Delta Q - \Delta U \\ &= 42 \text{ J} - 24.9 \text{ J} = 17.1 \text{ J}\end{aligned}$$

If the distance moved by the piston is x , then the work done

$$\begin{aligned}\Delta W &= (100 \text{ kPa}) (8.5 \text{ cm}^2)x \\ &= 17.1 \text{ J}\end{aligned}$$

$$\text{Thus, } (10^5 \text{ N/m}^2) (8.5 \times 10^{-4} \text{ m}^2) x = 17.1 \text{ J}$$

$$\text{or } x = 0.2 \text{ m} = \mathbf{20 \text{ cm}}$$

Heat or Thermodynamics 8) Saturated vapor pressure problems

A saturated water vapour ($M = 18$) is contained in a vessel fitted with a piston at a temperature $t = 100^\circ\text{C}$. As a result of slow introduction of the piston a small fraction of the vapour $\Delta m = 1 \text{ g}$ gets condensed. What amount of work is done over the gas?

Solution.

Work done = decrease in internal energy of the gas

$$\begin{aligned} &= U_i - U_f = \frac{m_i RT}{M} - \frac{m_f RT}{M} = \frac{\Delta m RT}{M} \\ \therefore \text{ Here, } W &= \frac{10^{-3} \times 8.3 \times (273 + 100)}{18 \times 10^{-3}} = 172 \text{ J.}\end{aligned}$$

Water of mass $m = 1$ kg and M (mol. mass) = 18 turns completely into saturated vapour at standard atmospheric pressure. Assuming the saturated vapour to be an ideal gas find increment of internal energy of the system. Specific latent heat of steam is $L = 2250$ kJ/kg.

Solution.

$$\Delta Q = \text{heat added to the system} = mL$$

$$\Delta W = \text{work done by the system} = p_0 (V_v - V_w)$$

$$\approx p_0 V_v = \frac{m}{M} RT$$

By the first law ($\Delta Q = \Delta U + \Delta W$),

$$\Delta U = mL - \frac{m}{M} RT = m \left(L - \frac{RT}{M} \right)$$

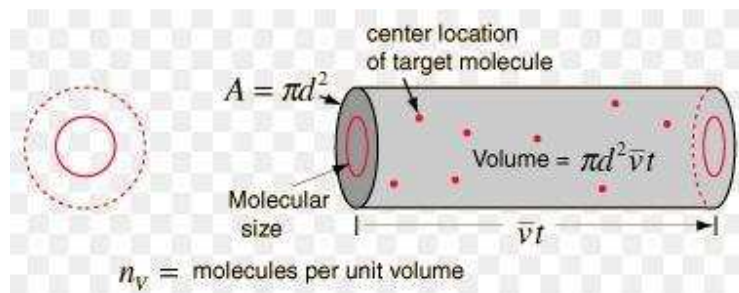
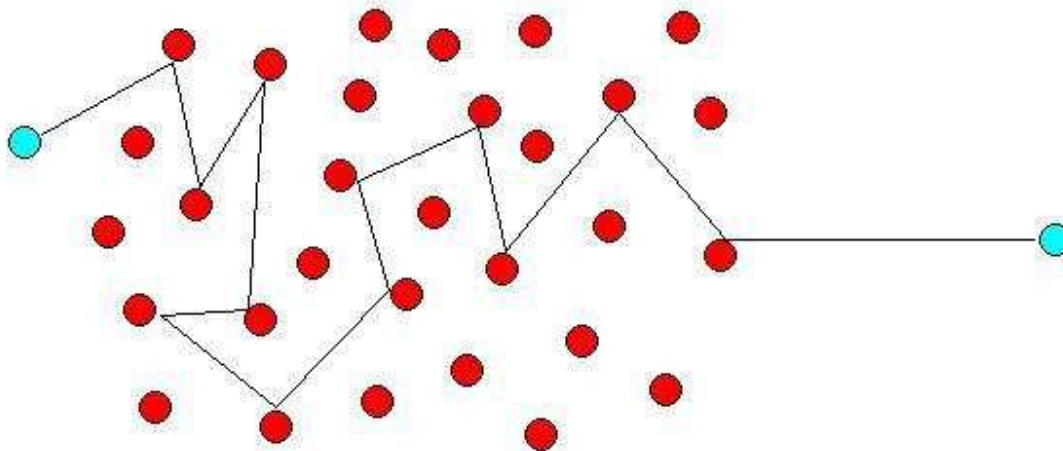
$$\Delta U = 1 \left(2250 \times 10^3 - \frac{8.3 \times 373}{18 \times 10^{-3}} \right) = 2.078 \times 10^6 \text{ J}$$

Heat or Thermodynamics 9) Mean free path

Mean free path of a gas molecule between 2 collisions

Mean Free Path

all particles, including photons, suffer from collisions with other particles such that their path through space is very short the higher the densities. This typical path length is called the mean free path.



mean free path λ (the average distance travelled by a particle between collisions) to determine the best values for number of particles N , rms velocity V_{rms} , and box length L ,

$$\lambda = \frac{k_B T}{\sqrt{2} \pi d^2 p}$$

where d is the diameter of the particle and p is the pressure. which I can easily turn into:

$$\lambda = \frac{m v_{rms}^2}{2 \sqrt{2} \pi d^2 p}$$

The average distance a particle can travel before colliding with another particle.

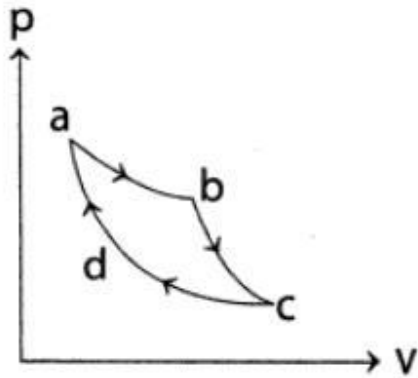
$$\lambda = \frac{1}{n\sigma}$$

Effect of pressure: $\lambda \propto \frac{1}{p}$

Heat or Thermodynamics 10) Questions on efficiency of cycle

Suppose 0.2 mole of an ideal di-atomic gas ($\gamma = 1.4$) undergoes cycle with temperature $T_H = 400$ K and $T_C = 300$ K. The initial pressure is $p_a = 10 \times 10^5$ Pa and during isothermal expansion at temperature T_H the volume doubles.

- (i) Find Q , W and ΔU from each step in the cycle.**
- (ii) Find the efficiency.**



Solution

$$(i) \quad V_a = \frac{nRT_H}{pa}$$
$$= \frac{0.2 \times 8.314 \times 400}{10 \times 10^5} = 6.65 \times 10^{-4} \text{ m}^3$$

For isothermal expansion $a \rightarrow b$

$$p_a V_a = p_b V_b$$

$$\text{or } p_b = \frac{p_a V_a}{V_b} = 5 \times 10^5 p_a$$

For adiabatic expansion $b \rightarrow c$

$$T_H V_b^{\gamma-1} = T_c V_c^{\gamma-1}$$

$$\therefore V_c = V_b \left(\frac{T_H}{T_c} \right)^{\frac{1}{\gamma-1}}$$

$$= 13.3 \times 10^{-4} \times \left(\frac{4}{3} \right)^{2.5} = 27.3 \times 10^{-4} \text{ m}^3$$

$$p_c = \frac{nRT_c}{V_c} = \frac{0.2 \times 8.314 \times 300}{27.3 \times 10^{-4}}$$

$$= 1.83 \times 10^5 \text{ Pa}$$

For adiabatic compression $d \rightarrow a$

$$T_c V_d^{\gamma-1} = T_H V_a^{\gamma-1}$$

$$V_d = V_a \left(\frac{T_H}{T_c} \right)^{\frac{1}{\gamma-1}} = 6.65 \times 10^{-4} \times \left(\frac{4}{3} \right)^{2.5}$$

$$= 13.65 \times 10^{-4} \quad p_d = \frac{nRT_c}{V_d}$$
$$= \frac{0.2 \times 8.314 \times 300}{13.65 \times 10^{-4}} = 3.65 \times 10^5 \text{ Pa}$$

For isothermal expansion $a \rightarrow b$

$$\Delta U = 0$$

$$\therefore W = Q_H = nRT_H \cdot \log_e \frac{V_b}{V_a}$$
$$= 0.2 \times 8.314 \times 400 \log_e 2$$
$$= \mathbf{461 \text{ J}}$$

For adiabatic expansion $b \rightarrow c$

$$Q = 0$$

$$\therefore W = -\Delta U = nC_V(T_H - T_C)$$
$$= 0.2 \times 20.78 \times (400 - 300)$$
$$= \mathbf{415.7 \text{ J}}$$

For isothermal compression $c \rightarrow d$

$$\Delta U = 0$$

$$\therefore W = Q_c = nRT_c \log_e \frac{V_d}{V_c}$$
$$= 0.2 \times 8.314 \times 300 \log_e \frac{13.65 \times 10^{-4}}{27.3 \times 10^{-4}}$$
$$= \mathbf{-345.8 \text{ J}}$$

For adiabatic expansion $d \rightarrow a$

$$Q = 0$$

$$\therefore W = -U$$
$$= nC_V(T_C - T_H)$$
$$= 0.2 \times 20.78 \times (300 - 400)$$
$$= \mathbf{-415.7 \text{ J}}$$

The results may be tabulated as follows:

	Q	W	ΔU
$a \rightarrow b$	461 J	461 J	0 J
$b \rightarrow c$	0 J	415.7 J	- 415.7 J
$c \rightarrow d$	- 345.8 J	- 345.8 J	0 J
$d \rightarrow a$	0 J	- 415.7 J	415.7 J
Total	115.2 J	115.2 J	0 J

(ii) For entire cycle, $Q = W$

$$\Delta U = 0$$

$$\text{Total work done} = 115.2 \text{ J}$$

$$Q_H = 461 \text{ J}$$

$$\therefore \eta = \frac{W}{Q_H} = \frac{115.2}{461} = \mathbf{0.25}$$

Efficiency of cycle example

One mole of a di-atomic ideal gas ($\gamma = 1.4$) is taken through a cyclic process starting from point A . The process $A \rightarrow B$ is an adiabatic compression, $B \rightarrow C$ isobaric expansion, $C \rightarrow D$ is an adiabatic expansion and $D \rightarrow A$ isochoric expansion. The volume ratios are $\frac{V_A}{V_B} = 16$ and $\frac{V_C}{V_B} = 2$ and the temperature at A is $T_A = 300$ K. Calculate the temperature of gas at the points B and D and find the efficiency of the cycle.

Solution For an ideal gas undergoing adiabatic expansion or compression, we have

$$TV^{\gamma-1} = \text{Constant}$$

For the expansion at constant pressure, we have

$$\frac{V}{T} = \text{Constant}$$

With this information, temperature of the gas at different stages of the cyclic process may be determined as follows:

(i) Adiabatic compression from A to B

$$T_B V_B^{\gamma-1} = T_A V_A^{\gamma-1}$$

$$\begin{aligned} \text{or } T_B &= \left(\frac{V_A}{V_B}\right)^{\gamma-1} T_A = (16)^{1.4-1} (300) \\ &= (3.03) (300 \text{ K}) = \mathbf{909 \text{ K}} \end{aligned}$$

(ii) Isobaric expansion from B to C

$$\frac{V_C}{T_C} = \frac{V_B}{T_B}$$

$$\text{or } T_C = \left(\frac{V_C}{V_B}\right) T_B = 2(909) = \mathbf{1818 \text{ K}}$$

(iii) Adiabatic expansion from C to D

$$T_D V_D^{\gamma-1} = T_C V_C^{\gamma-1}$$

$$\text{or } T_D = \left(\frac{V_C}{V_D}\right)^{\gamma-1} T_C$$

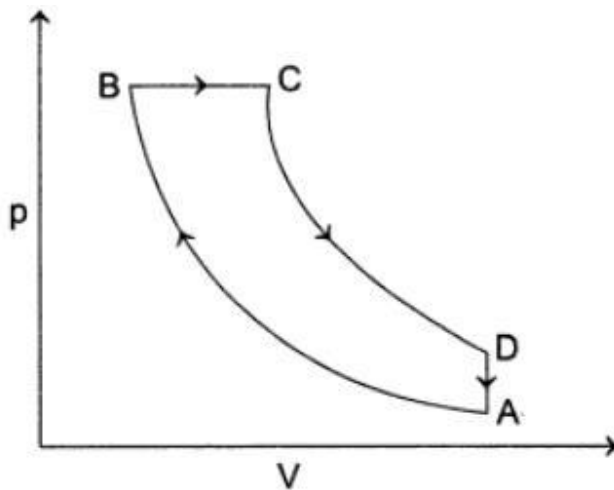
Since, $D \rightarrow A$ is isochoric process, therefore

$$V_D = V_A$$

Hence,

$$\begin{aligned}T_D &= \left(\frac{V_C}{V_D}\right)^{\gamma-1} T_C = \left(\frac{V_C}{16 V_B}\right)^{\gamma-1} T_C \\&= \left(\frac{2}{16}\right)^{1.4-1} (1818 \text{ K}) \\&= (0.4353) (1818 \text{ K}) = 791.4 \text{ K}\end{aligned}$$

The given cyclic process is shown in the figure.



Efficiency of the cycle is defined as

$$\eta = \frac{\text{Work obtained in one cycle}}{\text{Heat absorbed in the process } B \rightarrow C}$$

Now, the work obtained in one cycle is equal to the area within the cycle ABCDA. This

work is given as

$$W = |W_{B \rightarrow C}| + |W_{C \rightarrow D}| + |W_{B \rightarrow A}|$$

$$= RT_B + C_V(T_C - T_D) - C_V(T_B - T_A)$$

For a di-atomic gas,

$$C_V = \frac{5}{2} \text{ and } C_P = \frac{7}{2}R.$$

Hence,

$$W = R \left[T_B + \frac{5}{2}(T_C - T_D - T_B - T_A) \right]$$

$$= (8.314 \text{ JK}^{-1} \text{ mol}^{-1})$$

$$\left[(909 \text{ K}) + \frac{5}{2}(1818 - 791.4 - 909 + 300)\text{K} \right]$$

= 16237.2 Kelvin per mole

Heat absorbed in the process B → C is given as

$$Q = C_P(T_C - T_B)$$

$$= \left(\frac{7}{2}R \right) (T_C - T_B)$$

$$= \frac{7}{2} \times (8.314 \text{ JK}^{-1} \text{ mole}^{-1})$$

$$(1818 \text{ K} - 909 \text{ K})$$

$$= 26451.0 \text{ J mole}^{-1}$$

Hence, the efficiency of the cycle is

$$\eta = \frac{W}{Q} = \frac{16237.2}{26451.0} = \mathbf{0.614}$$

Example of Efficiency of a cycle

An ideal gas is taken through a cycle thermodynamic process through four steps. The amount of heat involved in these steps are $Q_1 = 5960 \text{ J}$, $Q_2 = -5585 \text{ J}$,

$Q_3 = -2980 \text{ J}$ and $Q_4 = 3645 \text{ J}$ respectively. The corresponding worked involved are $W_1 = 2200 \text{ J}$, $W_2 = -825 \text{ J}$, $W_3 = -1100 \text{ J}$ and W_4 respectively.

- (i) Find the value of W_4 .
- (ii) What is the efficiency of the cycle?

Solution For a cyclic process

$$\Delta U = 0$$

(i) Cyclic $\int dQ = \int dW$

$$\begin{aligned} \text{i.e., } Q_1 + Q_2 + Q_3 + Q_4 \\ = W_1 + W_2 + W_3 + W_4 \end{aligned}$$

$$\begin{aligned} \text{or } 5960 - 5585 - 2980 + 3645 \\ = 2200 - 825 - 1100 + W_4 \end{aligned}$$

$$\text{or } W_4 = 765 \text{ J}$$

(ii) Efficiency of the cycle,

$$\eta = \frac{\text{Net work output}}{\text{Total heat input}}$$

$$\begin{aligned}\text{Net work output} &= 5960 - 5585 \\ &\quad - 2980 + 3645 \\ &= 1040 \text{ J}\end{aligned}$$

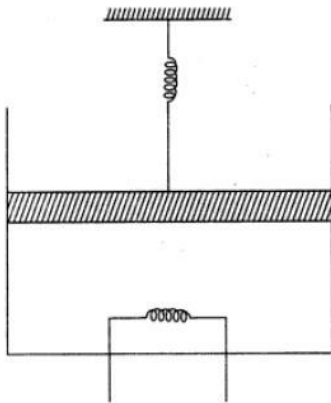
$$\begin{aligned}\text{Total heat input} &= Q_1 + Q_4 \\ &= 5960 + 3645 = 9605 \text{ J}\end{aligned}$$

$$\therefore \eta = \frac{1040}{9605} \times 100 = 10.83\%$$

Question on Cycle

2.00 mole of a mono-atomic ideal gas ($U = 1.5 nRT$) is enclosed in an adiabatic, vertical cylinder fitted with a smooth light adiabatic piston. The piston is connected to a vertical spring of spring constant 200 N/m as shown in the figure. The area of cross-section of the cylinder is 20.0 cm^2 . Initially, the spring is at its natural length and the temperature of the gas is 300 K . The atmospheric pressure is 100 kPa . The gas is heated slowly for some time by means of an electric heater so as to move the piston up through 10 cm . Find:

- (i) **The work done by the gas.**
- (ii) **The final temperature of the gas.**
- (iii) **The heat supplied by the heater.**



Solution

(i) Force by the gas on the piston is

$$F = p_0 A + kx$$

where, $P_0 = 100 \text{ kPa}$ is the atmospheric pressure.

$A = 20 \text{ cm}^2$ is the area of the cross-section,

$k = 200 \text{ N/m}$ is the spring constant, and

$x =$ the compression of spring.

Work done by the gas if the piston moves through $l = 10 \text{ cm}$ is

$$W = \int_0^l F dx$$

$$= P_0 A l + \frac{1}{2} k l^2$$

$$= (100 \times 10^3 \text{ Pa})$$

$$(20 \times 10^{-4} \text{ m}^2) \times (10 \times 10^{-2} \text{ m})$$

$$+ \frac{1}{2}(200 \text{ N/m})(100 \times 10^{-4} \text{ m}^2)$$

$$= 20 + 1 \text{ J} = \mathbf{21 \text{ J}}$$

- (ii) Initial temperature, $T_1 = 300 \text{ K}$. Let the final temperature by T_2 , then

$$nRT_1 = p_0 V_0$$

$$nRT = pV_2 = \left(p_0 + \frac{kl}{A} \right) (V_0 + Al)$$

$$= nRT_1 + p_0 Al + kl^2 + \frac{kl nRT_1}{AP_0}$$

or $T_2 = T_1 + \frac{p_0 Al + kl^2}{nR} + \frac{k l T_1}{A p_0}$

$$300 + \frac{20 \text{ J} + 2 \text{ J}}{20 \times 8.3} + \frac{200 \times 10 \times 10^{-2} \times 300}{20 \times 10^{-4} \times 100 \times 10^3 \text{ Pa}}$$

$$= 300 \text{ K} + 1.325 \text{ K} + 30 \text{ K}$$

$$= \mathbf{331 \text{ K}}$$

- (iii) Internal energy, $U = 1.5 nRT$

$$\therefore \Delta U = 1.5 nR\Delta T$$

$$= 1.5 \times 2.00 \times 8.3 \times 31$$

$$= 772 \text{ J}$$

From the first law,

$$\Delta Q = \Delta U + \Delta W$$

$$= (772 + 21) \text{ J} = \mathbf{793 \text{ J}}$$

Example where 2 vessels are connected

Two vessels contain in each of them one mole of mono-atomic gas. The initial volume of each vessel is $8.3 \times 10^{-3} \text{ m}^3$. Equal amount of heat is supplied to each vessel. In one vessel, the volume of gas is doubled without change in its internal energy whereas the volume of the gas is held constant in second vessel. The vessels are now connected to allow free mixing. Find the final temperature and pressure of the combined system.

Solution According to the first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

For the first vessel: $\Delta U = 0$, (Since, no change in temperature)

$$\Delta Q = \Delta W$$

$$Q = \int_{V_1}^{V_2} p dV$$

$$= \int_{V_1}^{V_2} nRT \frac{dV}{V} \quad (\text{since, } pV = nRT)$$

Since $V_2 = 2 V_1$, therefore

$$Q = nRT \log_e 2, \quad \dots(i)$$

For the second vessel: $\Delta W = 0$, (volume is constant)

$$Q = nC_v \Delta T = n \left(\frac{3}{2} R \right) \Delta T \quad \dots(ii)$$

Since, for mono-atomic gas $C_v = \frac{3R}{2}$

From equations (i) and (ii), we get

$$nRT \log_e 2 = n \left(\frac{3}{2} R \right) \Delta T$$

or $\Delta T = \frac{2}{3} \times 300 \times 0.693 = 138.6 \text{ K}$

It is the change in temperature of the second vessel.

Now, temperature of the gas in second vessel

$$= T + \Delta T$$

$$= 300 + 138.6 = 438.6 \text{ K}$$

Let after mixing T_f and p_f be the final tempera-

ture and pressure, therefore

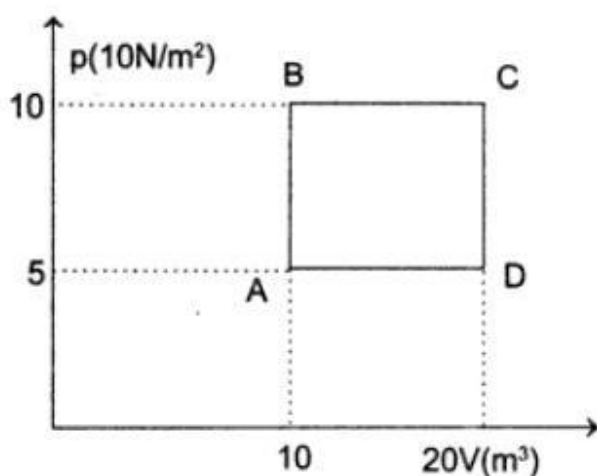
$$\begin{aligned} T_f &= \frac{T + (T + \Delta T)}{2} \\ &= \frac{300 + 438.6}{2} = 369.3 \text{ K} \end{aligned}$$

From the gas equation,

$$\begin{aligned} p_f V_f &= nRT_f \\ p_f &= \frac{nRT_f}{V_f} \\ &= \frac{2 \times 8.3 \times 369.3}{2 \times 8.3 \times 10^3 + 8.3 \times 10^{-3}} \\ &= 2.46 \times 10^5 \text{ N/m}^2 \end{aligned}$$

A sample of 2 kg of mono-atomic Helium (assumed ideal) is taken through the process *ABC* and another sample of 2 kg of the same gas is taken through the process *ADC*. Given relative molecular weight of Helium = 4.

- (i) **What is the temperature of Helium in each of the states A, B, C and D?**
- (ii) **Is there any way of telling afterwards which sample of Helium went through the process *ABC* and which went through the process *ADC*? Write yes or no.**
- (iii) **How much heat is evolved in each of the processes *ABC* and *ADC*?**



Solution

Amount of helium

$$= \frac{m}{M} = \frac{2 \times 10^3}{4 \text{ g mol}^{-1}} = 500 \text{ mole}$$

- (i) **The temperature of gas at the states A, B, C and D are**

$$\begin{aligned}T_A &= \frac{pV}{nR} \\&= \frac{(5 \times 10^4 \text{ N/m}^2)(10 \text{ m}^3)}{(500 \text{ mole})(8.314 \text{ JK}^{-1} \text{ mole}^{-1})} \\&= \mathbf{120.28 \text{ K}}\end{aligned}$$

$$\begin{aligned}T_B &= \frac{(10 \times 10^4 \text{ N/m}^2)(10 \text{ m}^3)}{(500 \text{ mole})(8.314 \text{ JK}^{-1} \text{ mole}^{-1})} \\&= \mathbf{240.56 \text{ K}}\end{aligned}$$

$$\begin{aligned}T_C &= \frac{(10 \times 10^4 \text{ N/m}^2)(10 \text{ m}^3)}{(500 \text{ mole})(8.314 \text{ JK}^{-1} \text{ mole}^{-1})} \\&= \mathbf{481.12 \text{ K}}\end{aligned}$$

$$\begin{aligned}T_D &= \frac{(5 \times 10^4 \text{ N/m}^2)(20 \text{ m}^3)}{(500 \text{ mole})(8.314 \text{ JK}^{-1} \text{ mole}^{-1})} \\&= \mathbf{240.50 \text{ K}}\end{aligned}$$

(ii) No.

(iii) For the process ABC, we have

$$\begin{aligned}Q_{AB} &= nC_V \Delta T \\&= (500 \text{ mole}) \left(\frac{3}{2} \times 8.314 \text{ JK}^{-1} \text{ mole}^{-1} \right) \\&\quad (240.56 \text{ K} - 120.28 \text{ K})\end{aligned}$$

$$= 7.5 \times 10^5 \text{ J}$$

$$Q_{BC} = nC_p \Delta T$$

$$= (500 \text{ mole}) \left(\frac{5}{2} \times 8.314 \text{ JK}^{-1} \text{ mole}^{-1} \right)$$

$$(481.12 \text{ K} - 240.56 \text{ K})$$

$$= 2.5 \times 10^6 \text{ J}$$

$$Q_{ABC} = Q_{AB} + Q_{BC}$$

$$= (7.5 \times 10^5 \text{ J} + 2.5 \times 10^6 \text{ J}) = \mathbf{3.25 \times 10^6 \text{ J}}$$

For the process ADC, we have

$$Q_{AD} = nC_p \Delta T$$

$$= (500 \text{ mole}) \left(\frac{5}{2} \times 8.314 \text{ JK}^{-1} \text{ mole}^{-1} \right)$$

$$(240.56 \text{ K} - 120.28 \text{ K})$$

$$= 1.25 \times 10^6 \text{ J}$$

$$Q_{DC} = nC_v \Delta T$$

$$= (500 \text{ mole}) \left(\frac{3}{2} \times 8.314 \text{ JK}^{-1} \text{ mole}^{-1} \right)$$

$$(481.12 \text{ K} - 240.56 \text{ K})$$

$$= 1.5 \times 10^6 \text{ J}$$

$$Q_{ADC} = Q_{AD} + Q_{DC}$$

$$= (1.25 \times 10^6 \text{ J} + 1.5 \times 10^6 \text{ J})$$

$$= \mathbf{2.75 \times 10^6 \text{ J}}$$

More example in Heat and Thermodynamics

A 1.00 mole sample of an ideal mono-atomic gas originally at a pressure of 1.00 atmosphere undergoes a three-step process:

- (i) **It is expanded adiabatically from $T_1 = 550$ K and $T_2 = 389$ K.**
- (ii) **It is compressed at constant pressure until its temperature reaches T_3 .**
- (iii) **It then returns to its original pressure and temperature by a constant-volume process.**
 - (a) Plot these processes on a p - V diagram.
 - (b) Determine T_3 .
 - (c) Calculate the change in integral energy the workdone by the gas, and heat added to gas for each process.
 - (d) For the complete cycle.

Solution *First step Adiabatic Expansion*

$$Q_1 = 0$$

$$\begin{aligned}W_1 &= n_1 C_v (T_2 - T_1) \\&= (1.00 \text{ mol}) \left(\frac{3}{2} \times 8.314 \text{ JK}^{-1} \text{ mole}^{-1} \right) \\&\quad \times (389 \text{ K} - 550 \text{ K}) \\&= -2007.8 \text{ J}\end{aligned}$$

For adiabatic expansion of an ideal gas

$$p_2 T_2^{(-C_p/R)_g} = p_1 T_1^{(-C_p/R)_g}$$

$$\text{Hence, } p_2 = p_1 \left(\frac{T_1}{T_2} \right)^{\frac{C_p}{R}} = (1.00 \text{ atm}) \left(\frac{389}{550} \right)^{\frac{5}{2}}$$

$$= 0.421 \text{ atm.}$$

$$V_2 = \frac{nRT_2}{p_2}$$

$$= \frac{(1.0 \text{ mole})(8.314 \text{ JK}^{-1} \text{ mole}^{-1})(550 \text{ K})}{(1.0 \times 101.325 \text{ kPa})}$$

$$= 45.1 \text{ dm}^3$$

$$\Delta U_1 = W_1 = -2007.8 \text{ J}$$

Second step compression at constant pressure:

The final volume in this process will be V_1 as in the third step, the system returns to the original state by constant volume process. Hence, in the second step,

$$T_2 = (389 \text{ K}) \text{ changes to } T_3$$

$$V_2 = (75.8 \text{ dm}^3) \text{ changes to } V_1$$

$$= 45.1 \text{ dm}^3$$

$$p_2 = \text{remains constant.}$$

Workdone in the process

$$W_2 = -p_2 (V_1 - V_2)$$

$$= -(0.421 \times 101.325 \text{ kPa}) (45.1 \text{ dm}^3 - 75.8 \text{ dm}^3)$$

$$= 1309.6 \text{ J}$$

$$T_3 = \left(\frac{V_1}{V_2}\right) T_2 = \left(\frac{45.1}{75.8}\right) (389 \text{ K}) = \mathbf{231.4 \text{ K}}$$

$$\begin{aligned} Q_2 &= n C_p (T_3 - T_2) \\ &= \left(\frac{5}{2} \times 8.314 \text{ JK}^{-1}\right) (231.4 \text{ K} - 389 \text{ K}) \\ &= -3275.7 \text{ J} \\ \Delta U_2 &= Q_2 + W_2 \\ &= -3275.7 \text{ J} + 1309.6 \text{ J} = \mathbf{-1966.1 \text{ J}} \end{aligned}$$

Third step compression at constant volume in this process:

$$W_3 = 0$$

$$V_1 = (45.1 \text{ dm}^3) \text{ remains constant}$$

$$Q_3 = n C_v (T_1 - T_3)$$

$$= \left(\frac{3}{2} \times 8.314 \text{ JK}^{-1}\right) (550 \text{ K} - 231.4 \text{ K})$$

$$\Delta U = \mathbf{3973.3 \text{ J}}$$

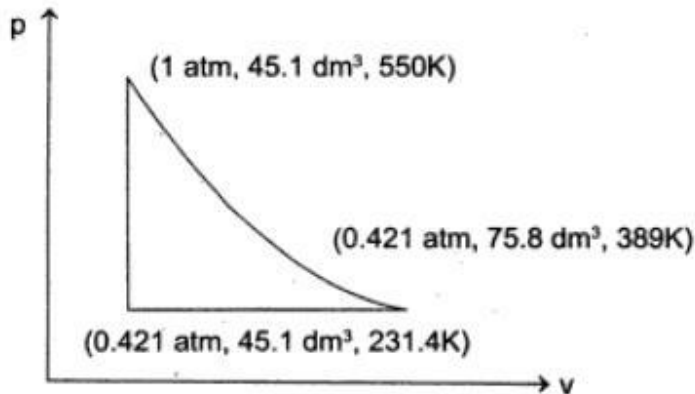
Since, the system return to its original state, we will have

$$\Delta U = Q + W = 0$$

$$\begin{aligned} \text{Now, } W &= W_1 + W_2 + W_3 \\ &= -2007.8 \text{ J} + 1309.6 \text{ J} + 0 \\ &= \mathbf{-698.2 \text{ J}} \end{aligned}$$

$$\therefore Q = -W = \mathbf{698.6 \text{ J}}$$

The p - V plot of the given process is shown in the figure:



In the complete cycle

$$\Delta U = 0$$

$$Q = Q_1 + Q_2 + Q_3$$

$$= 0 - 3275.7 \text{ J} + 3973.3 \text{ J} = 697.6 \text{ J}$$

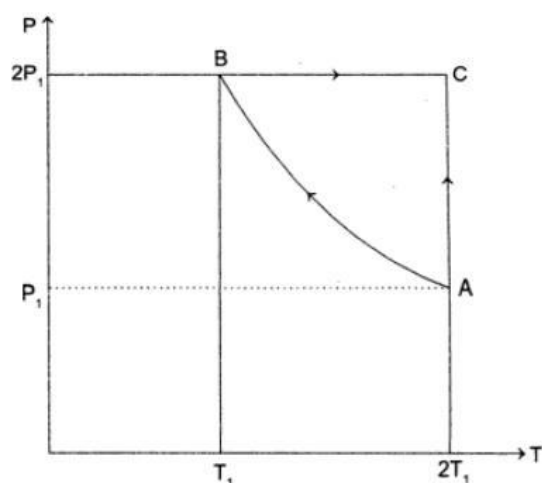
$$W = -Q = 697.6 \text{ J} (= W_1 + W_2 + W_3)$$

$$= -2007.8 + 1309.6 \text{ J} + 0 = \mathbf{698.2 \text{ J}}$$

Two mole of an ideal mono-atomic gas is taken through a cycle $ABCA$ as shown in the $p - T$ diagram. During this process AB , pressure and temperature of the gas vary such that $pT = \text{constant}$. If $T_1 = 300 \text{ K}$, calculate:

- (i) **The work done on the gas in the process AB .**
- (ii) **The heat absorbed or released by the gas in each of the process.**

Give answers in terms of the gas constant R .



Solution The volumes of the gas at three states A, B and C are as follows:

$$V_A = \frac{nRT_A}{P_A} = \frac{nR(2T_1)}{P_1} = \frac{2nRT_1}{P_1} \dots(i)$$

$$V_B = \frac{nRT_B}{P_B} = \frac{nR(2T_1)}{P_1} = \frac{1}{2} \frac{nRT_1}{P_1} \dots(ii)$$

$$V_C = \frac{nRT_C}{P_C} = \frac{nR(2T_1)}{2P_1} = \frac{nRT_1}{P_1} \dots(iii)$$

It is given that during the process AB,

$$pT = K \dots(iv)$$

where, K is constant and is given as

$$K = p_A T_A = (p_1) (2T_1) = 2 p_1 T_1 \dots(v)$$

In the process AB, we will have

$$= \sqrt{nR}K \left[2\sqrt{V_B} - 2\sqrt{V_A} \right]$$

Using equations (i), (ii) and (v), we get

$$\begin{aligned} W_{AB} &= \sqrt{nR(2p_1T_1)} \left[2\sqrt{\frac{nRT_1}{2p_1}} - 2\sqrt{\frac{2nRT_1}{p_1}} \right] \\ &= (\sqrt{2} nRT_1)(2) \left[\frac{1}{2} - \sqrt{2} \right] \\ &= -2nT_1R \\ &= -2(2 \text{ mole})(200 \text{ K})R \\ &= -(1200 \text{ mole K})R \end{aligned}$$

The negative sign implies that the work is done on the gas.

Hence, work done on the gas
= **(1200 mole K) R**

(ii) Change in energy of the gas in the process AB is

$$\begin{aligned} \Delta U_{AB} &= nC_v\Delta T \\ &= (2 \text{ mole}) \left(\frac{3}{2}R \right) (T_1 - 2T_1) \\ &= -(3 \text{ mole})T_1R \\ &= -(3 \text{ mole})(300 \text{ K})R \\ &= -(900 \text{ mole K})R \end{aligned}$$

Now, from the first law of thermodynamics,

$$\begin{aligned} Q_{AB} &= \Delta U_{AB} + W_{AB} \\ &= -(1200 \text{ mole K}) \\ &\quad - (900 \text{ mole K})R \\ &= -(2100 \text{ mole K})R \end{aligned}$$

The negative sign implies that the heat is released in the process AB. The proc-

Process BC takes place at constant pressure.

Hence,

$$\begin{aligned}W_{BC} &= p \Delta V \\&= (2 p_1) (V_C - V_B) \\&= (2 p_1) \left[\frac{nRT_1}{P_1} - \frac{nRT_1}{2 P_1} \right] \\&= nRT_1 \\&= (2 \text{ mole}) (300 \text{ K}) R \\&= (600 \text{ mole K}) R\end{aligned}$$

Now, $\Delta U_{BC} = nC_v \Delta T$

$$= (2 \text{ mole}) \left(\frac{3}{2} R \right) (T_C - T_B)$$

$$\begin{aligned}&= (3 \text{ mole}) (R) (2 T_1 - T_1) \\&= (3 \text{ mole}) (R) (300 \text{ K}) \\&= (900 \text{ mole K}) R\end{aligned}$$

$$\begin{aligned}Q_{BC} &= \Delta U_{BC} + W_{BC} \\&= (900 \text{ mole K}) R + (600 \text{ mole K}) R \\&= (1500 \text{ mole K}) R\end{aligned}$$

The positive sign implies that the heat is absorbed in the process BC.

The process CA takes place at constant temperature. Hence,

$$\begin{aligned}W_{CA} &= \int_{V_A}^{V_B} p dV \\&= \int_{V_A}^{V_B} \frac{nRT}{V} dV\end{aligned}$$

$$= nRT \ln \frac{V_A}{V_C} \dots (\text{where, } T = 2 T_1)$$

$$= (2 \text{ mole}) (R) (2 \times 300 \text{ K}) \ln 2$$

$$= (1200 \text{ mole K}) R \ln 2$$

$$\Delta U_{CA} = 0$$

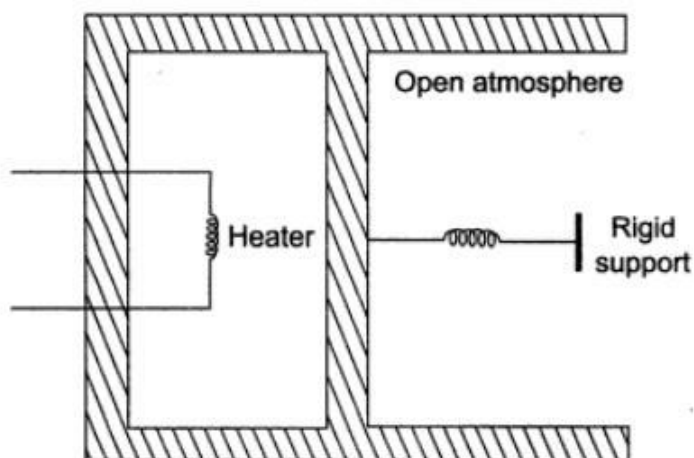
$$Q_{CA} = \Delta U_{CA} + W_{CA}$$

$$= 0 + (1200 \text{ mole K}) R \ln 2$$

The positive sign implies that the heat is absorbed in the process CA.

An ideal mono-atomic is confined in a cylinder by a spring-loaded piston of cross-section $8 \times 10^{-3} \text{ m}^2$. Initially, the gas is at 300 K and occupies a volume of $2.4 \times 10^{-3} \text{ m}^3$ and the spring is on its relaxed (unstretched, uncompressed) state as shown the figure. The gas is heated by a small electric heater until the piston moves out slowly by 0.1 m. Calculate the final temperature of the gas and the heat supplied (in joules) by the heater. The force constant of the spring is 8000 Nm^{-1} and atmospheric pressure is $1 \times 10^5 \text{ Nm}^2$. The cylinder and the piston are thermally insulated. The piston is massless and there is no friction between the piston and cylinder.

Neglect heat loss through the lead wires of the heater. The heat capacity of the heater coil is negligible. [Assume the spring to be massless].



Solution Let p_0 be the atmospheric pressure. Initially for the equilibrium of the piston, $p_L = p_R = p_0$ where p_L and p_R are the pressures on the left hand and right hand side of the piston.

Force exerted by the spring on the piston when it moves

$$F = kx = 8000 \times 0.1 \\ = 800 \text{ N}$$

\therefore Pressure exerted on the piston by the spring

$$p_s = \frac{F}{A} = \frac{800 \text{ N}}{8 \times 10^{-3} \text{ m}^2} = 1 \times 10^5 \text{ Nm}^{-2}$$

\therefore Total pressure acting on the right hand side

$$p'_R = p_0 + p_s \\ = 2 \times 10^5 \text{ N/m}^2$$

Under equilibrium $p'_L = p'_R$

$$\begin{aligned}\text{or } \frac{p_L V_L}{T_L} &= \frac{p'_L V'_L}{T'_L} \\ &= \frac{1 \times 10^5 \times 2.4 \times 10^{-3}}{300} \\ &= \frac{2 \times 10^5 \times 3.2 \times 10^{-3}}{T'_L}\end{aligned}$$

$$T'_L = 800 \text{ K}$$

$$\Delta U = n C_v \Delta T$$

$$\begin{aligned}\text{where, } n &= \frac{p_L V_L}{R T_L} = \frac{1 \times 10^5 \times 2.4 \times 10^{-3}}{8.3 \times 300} \\ &= 0.09638 \text{ mole}\end{aligned}$$

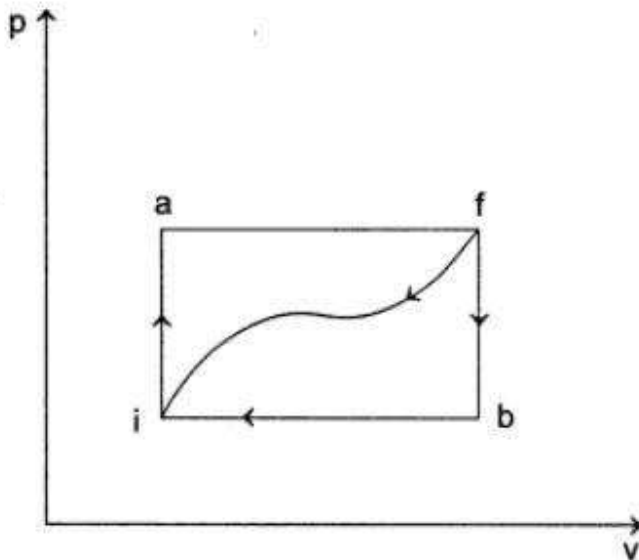
$$\therefore \Delta U = 0.09638 \times \frac{3}{2} \times 8.3 \times (800 - 300) = 600 \text{ J}$$

$$\begin{aligned}\Delta W &= \frac{1}{2} k \cdot x^2 + p_0 \cdot \Delta V \\ &= \frac{1}{2} \times 800 \times (0.1)^2 + 1 \times 10^5 \times 8 \times 10^{-4} \\ &= 120 \text{ J}\end{aligned}$$

$$\Delta Q = \Delta U + \Delta W = 600 + 120 = \mathbf{720 \text{ J.}}$$

A system is taken from state i to the state f (refer to the figure). Along path “ iaf ”, it is found that $\Delta Q = 50$ cal. $\Delta W = 20$ cal. Along the path “ ibf ”, $\Delta Q = 36$ cal. Calculate:

- (i) ΔW along the path “ ibf ”.
- (ii) If $\Delta W = -13$ cal for the curved path “ fi ”, what is the ΔQ for this path?
- (iii) Taking $U_i = 10$ cal, what is U_f ?
- (iv) If $U_b = 22$ cal, what is ΔQ for the process “ ib ” and the process “ bf ”?



Solution Path "iaf" $\Delta Q = 50 \text{ cal}$

$$\Delta W = 20 \text{ cal}$$

$$\Rightarrow \Delta U = \Delta Q - \Delta W \\ = 50 - 20 = 30 \text{ cal}$$

$$\Rightarrow U_f - U_i = 30 \text{ cal}$$

As internal energy change is a state function.

ΔU will be same for any path from i to f .

$$\begin{aligned} \text{(i) Path "ibf"} \Delta W &= \Delta Q - \Delta U \\ &= 36 - (U_f - U_i) \\ &= 36 - 30 = \mathbf{6 \text{ cal.}} \end{aligned}$$

$$\begin{aligned} \text{(ii) Path "fi"} \Delta Q &= \Delta U + \Delta W \\ &= (U_f - U_i) + \Delta W \\ &= (-30) + (-13) \\ &= \mathbf{-43 \text{ cal}} \end{aligned}$$

$$\begin{aligned} \text{(iii) } U_f - U_i &= 30 \text{ cal} \\ U_f &= U_i + 30 \quad \therefore = \mathbf{40 \text{ cal.}} \end{aligned}$$

$$\begin{aligned} \text{(iv) Process "ib"} \Delta Q &= \Delta U + \Delta W \\ &= (U_b - U_i) + (\Delta W)_{ibf} \end{aligned}$$

$$(\Delta W)_{ib} = (\Delta W)_{ibf}$$

Because $(\Delta W)_{bf} = 0$

$$\begin{aligned} \Delta Q &= (22 - 10) + 6 \\ &= \mathbf{18 \text{ cal.}} \end{aligned}$$

$$\begin{aligned} \text{Process "bf"} \Delta Q &= \Delta U + \Delta W \\ &= (U_f - U_b) + 0 \\ &= (40 - 22) \\ &= \mathbf{18 \text{ cal.}} \end{aligned}$$

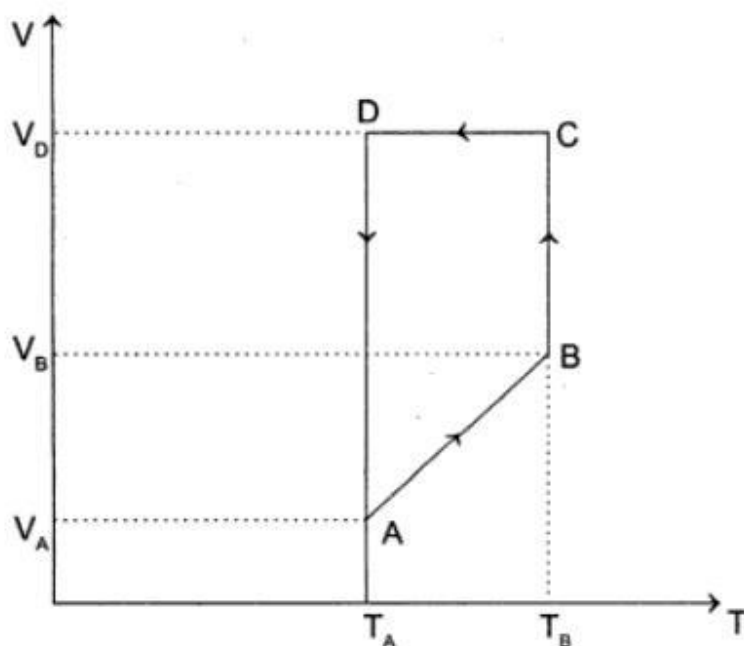
A mono-atomic ideal gas of two moles is taken through a cyclic process starting from A as shown in the figure. The

volume ratios are $\frac{V_B}{V_A} = 2$ and $\frac{V_D}{V_A} = 4$.

If the temperature T_A at A is 27°C , calculate:

- (i) **The temperature of the gas at point B .**
- (ii) **Heat absorbed or released by the gas in each process.**
- (iii) **The total work done by the gas during complete cycle.**

Express your answer in terms of the gas constant R .



Solution

Given: $\frac{V_B}{V_A} = 2$ and $\frac{V_D}{V_A} = 4$
 $T_A = 27^\circ\text{C}$

- (i) The process $A \rightarrow B$ in which the plot of V verse T is linear occurs at constant pressure condition.

Hence $\frac{V_A}{T_A} = \frac{V_B}{T_B}$

or $T_B = \left(\frac{V_B}{V_A}\right)T_A = (2)(300 \text{ K})$
 $= 600 \text{ K}$

- (ii) The process $A \rightarrow B$ occurs at constant pressure. Hence,

$$Q_{A \rightarrow B} = n C_p (T_B - T_A)$$
$$= (2 \text{ mole}) \left(\frac{5}{2}R\right) (600 \text{ K} - 300 \text{ K})$$
$$= (1500 \text{ mole K}) R.$$

The process $B \rightarrow C$ occurs at constant temperature. From first law of thermodynamics

$$dU = dQ - dW$$

Since, the internal energy of an ideal gas depends only on temperature, therefore

$$dU = 0 \text{ and } dQ = dW$$

$$Q_{B \rightarrow C} = W_{B \rightarrow C}$$

$$= \int p dV = nRT_B \int \frac{dV}{V}$$

$$= nR T_B \ln \frac{V_C}{V_B}$$

$$= nR T_B \ln \frac{V_D}{V_B} \dots \dots (\text{as } V_C = V_D)$$

$$= nR T_B \ln \left(\frac{V_D}{V_A} \frac{V_A}{V_B} \right)$$

$$= (2 \text{ mole}) (R) (600 \text{ K}) \ln \left(\frac{4}{2} \right)$$

$$= (1200 \text{ mole K}) R \ln 2$$

The process $C \rightarrow D$ occurs at constant volume. Hence,

$$Q_{C \rightarrow D} = nC_v (T_A - T_B)$$

$$= (2 \text{ mole}) \left(\frac{3}{2} R \right) (300 \text{ K} - 600 \text{ K})$$

$$= - (900 \text{ mole K}) R$$

The process $D \rightarrow A$ occurs at constant temperature. Hence,

$$\begin{aligned}Q_{D \rightarrow A} &= W_{D \rightarrow A} = nRT_A \ln \frac{V_A}{V_D} \\&= (2 \text{ mole}) (R) (300 \text{ K}) \ln \left(\frac{1}{4} \right) \\&= - (1200 \text{ mole K}) R \ln 2.\end{aligned}$$

(iii) Since, the process ABCDA is a cyclic process, therefore

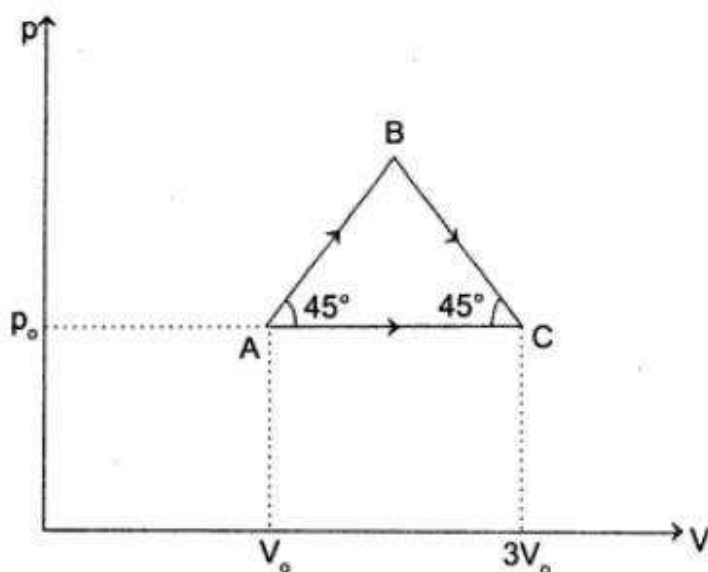
$$U = 0, W = Q$$

where,

$$\begin{aligned}Q &= Q_{A \rightarrow B} + Q_{B \rightarrow C} + Q_{C \rightarrow D} + Q_{D \rightarrow A} \\&= (1500 \text{ mole K}) R + (1200 \text{ mole K}) R \ln 2 - (900 \text{ mole K}) R \\&\quad - (1200 \text{ mole K}) R \ln 2 \\&= (600 \text{ mole K}) R.\end{aligned}$$

An ideal gas expands from a volume $V_0 = 1$ litre and pressure $p_0 = 1$ bar to volume 3 litre along two different paths ABC and AC as shown in figure. The heat added to the gas along the path ABC is 600 J.

- (i) **Sketch the process on $p - T$ diagram.**
- (ii) **Find the work done by the gas along the paths ABC and AC .**
- (iii) **Find the heat transfer in the process along the path AC .**



Solution

(i) Equation of line AB,

$$p - p_0 = \tan 45^\circ (V - V_0)$$

Hence for ideal gas, $p = V$

Now $pV = KT$

$$\Rightarrow p^2 = KT \text{ (parabola)}$$

.....(where K is constant.)

At B $V_B = 2V_0$ and $p_B = 2p_0$

Equation of line BC, $p - 2p_0 = -\tan 45^\circ (V - 2V_0)$

$$\therefore p = -V + 4$$

$$\Rightarrow p = -\frac{KT}{p} + 4$$

$$\therefore p^2 - 4p = -KT \text{ (Parabola)}$$

(ii) Workdone along path AC = $(\Delta W)_{AC}$

$$= p_0 (3V_0 - V_0)$$

$$= 2p_0 V_0$$

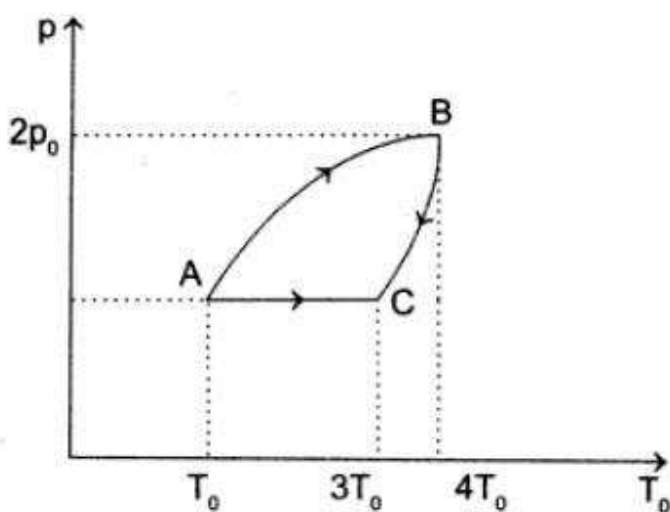
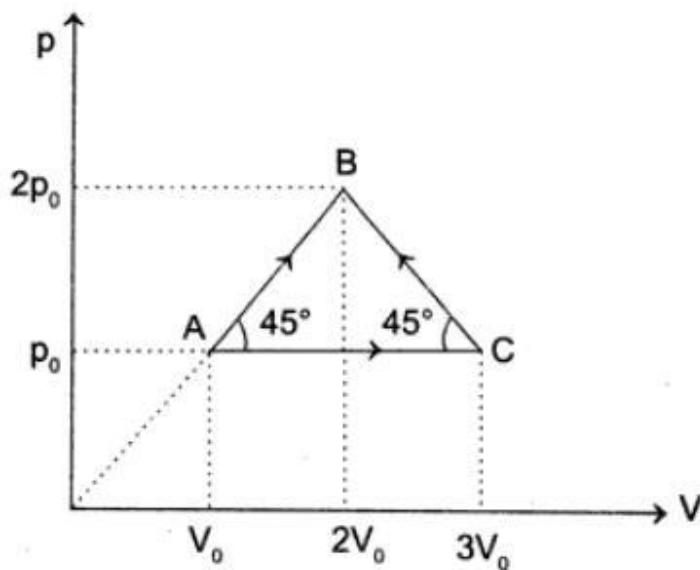
$$= 2 \times 1 \times 10^5 \times 1 \times 10^{-3}$$

$$= \mathbf{200 \text{ J.}}$$

(iii) For path ABC $(\Delta Q)_{ABC} = (\Delta U)_{AC} + (\Delta W)_{ABC}$
 $\Rightarrow (\Delta U)_{AC} = 600 - 300 = 300 \text{ J.}$

Heat transfer in the process along path AC,

$$(Q)_{AC} = (\Delta U)_{AC} + (\Delta W)_{AC} = 300 + 200 = \mathbf{500 \text{ J.}}$$



A monatomic ideal gas, initially at temperature T_1 is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 are the lengths of the gas column before and after expansion respectively, then T_1/T_2 is given by

(a) $\left(\frac{L_1}{L_2}\right)^{2/3}$

(b) $\frac{L_1}{L_2}$

(c) $\frac{L_2}{L_1}$

(d) $\left(\frac{L_2}{L_1}\right)^{2/3}$

$$TV^{\gamma-1} = \text{constant}$$

For monatomic gas $\gamma = \frac{5}{3}$

$$\Rightarrow TV^{2/3} = \text{constant}$$

Since volume is proportional to length, therefore,

$$\frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{2/3}$$

Hence (d) is correct.

Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V . The mass of gas contained in A is m_A and that in B is m_B . The gas in each cylinder is now allowed to expand isothermally to the same final volume $2V$. The change in the pressure in A and B are found to be Δp and $1.5 \Delta p$ respectively. Then

- (a) $4m_A = 9m_B$ (b) $2m_A = 3m_B$
(c) $3m_A = 2m_B$ (d) $9m_A = 4m_B$

$$\text{For gas in A, } p_1 = \left(\frac{m_A}{M}\right) \frac{RT}{V_1}$$

$$p_2 = \left(\frac{m_A}{M}\right) \frac{RT}{V_2}$$

$$\therefore \Delta p = p_2 - p_1 = \left(\frac{RT}{M}\right) m_A \left(\frac{1}{V_1} - \frac{1}{V_2}\right)$$

Putting $V_1 = V$ and $V_2 = 2V$, we get

$$\Delta p = \left(\frac{RT}{M}\right) \frac{m_A}{2V}$$

$$\text{Similarly for Gas in B, } 1.5 \Delta p = \left(\frac{RT}{M}\right) \frac{m_B}{2V}$$

From equation (i) and (ii) we get

$$2m_B = 3m_A$$

Hence (c) is the correct.

Two insulating cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at temperature 300 K . The piston A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K . Then the rise in temperature of the gas in B is

- (a) 30 K (b) 18 K
(c) 50 K (d) 42 K

For cylinder A For cylinder B

$$dQ = n C_p dT_1 \quad dQ = n C_V dT_2$$
$$= n (C_V + R) dT_1$$

$$\therefore n C_V dT_2 = n (C_V + R) 30$$

$$\therefore dT_2 = \frac{(C_V + R) 30}{C_V}$$

For diatomic gas $C_V = \frac{5}{2}R$

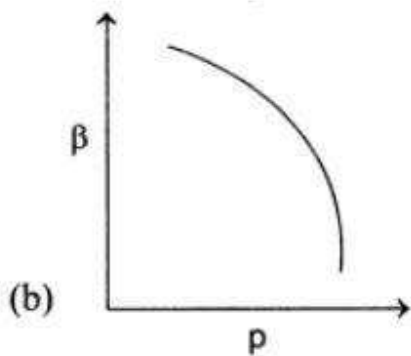
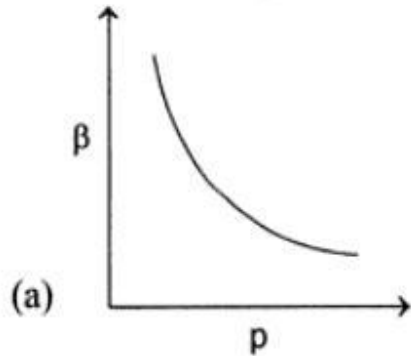
$$\therefore dT_2 = 42\text{ K}$$

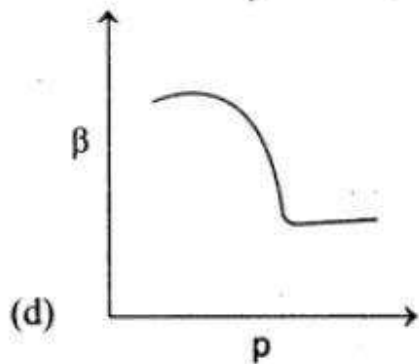
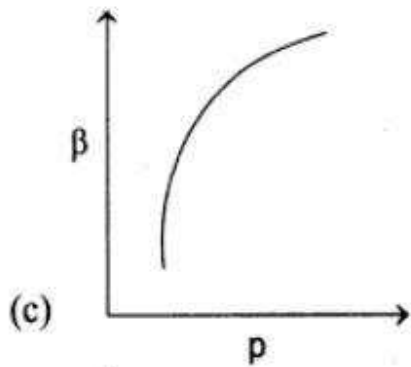
Hence (d) is correct.

Which of the following graph correctly represents the variation of

$$\hat{a} = -\left(\frac{dV}{dP}\right)/V \text{ with } p \text{ for an ideal gas}$$

at constant temperature?





As temperature is constant,

$$pV = \text{constant}$$

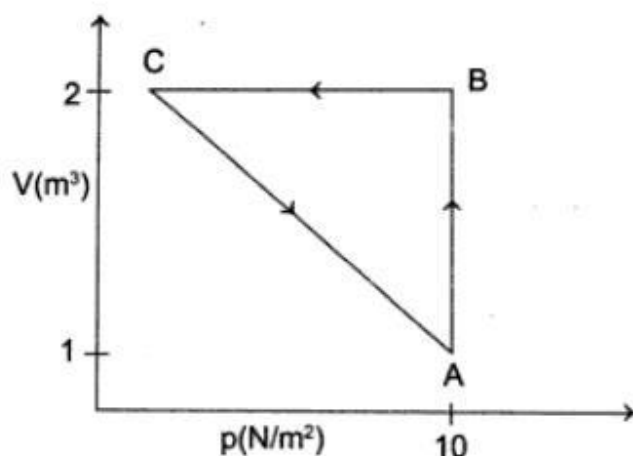
$$\Rightarrow p dV + V dp = 0$$

$$\Rightarrow -\frac{(dV / dp)}{V} = \frac{1}{p}$$

$$\Rightarrow \beta = \frac{1}{p}$$

An ideal gas is taken through the cycle $A \rightarrow B \rightarrow C \rightarrow A$, as shown in the gas in the cycle is 5J, the work done by the gas in the process $C \rightarrow A$ is

- (a) $-5J$ (b) $-10J$
(c) $-15J$ (d) $-20J$



For the cyclic process $\Delta U = 0$

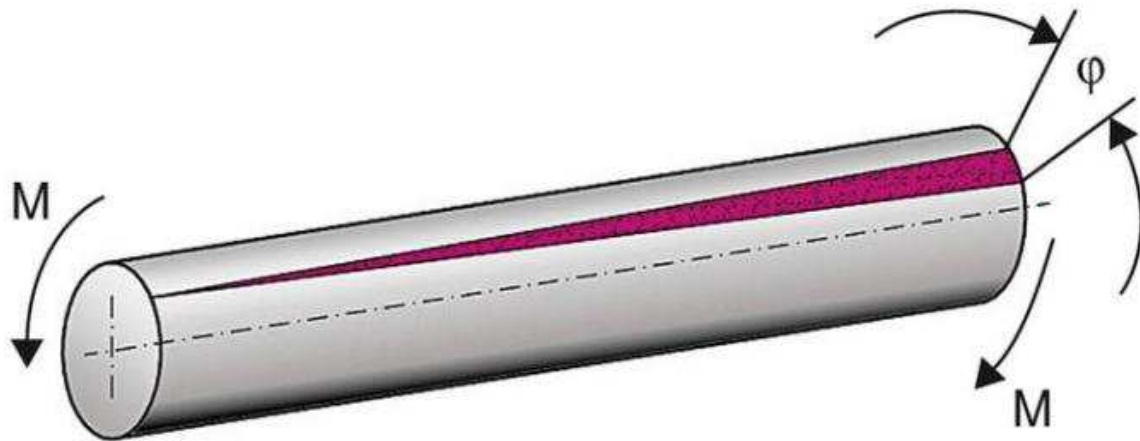
$$\begin{aligned}\Delta W &= W_{AB} + W_{BC} + W_{CA} \\ &= (10 + 0 + W_{CA}) J\end{aligned}$$

Given: $\Delta Q = 5J$

From first law of thermodynamics

$$\begin{aligned}5 &= 10 + 0 + W_{CA} \\ \Rightarrow W_{CA} &= -5J\end{aligned}$$

Properties of Material 1) Torsional Torque per unit twist



The torque T can be equated to the sum of the moments of the tangential stresses on the element $2\pi r \delta r$

$$\therefore T = \int s (2\pi r dr) r$$

$$T = \int \frac{C\theta}{l} (2\pi r^3) dr$$

$$= \frac{C\theta}{l} \pi \frac{r^4}{2}$$

Properties of Material 2) Torsion of a cylinder

TORSION OF A CYLINDER/TWISTING WIRE

Let, l = length of cylinder

r = radius of cylinder

ϕ = angle of twist

θ = angle of shear

η = modulus of rigidity

τ = restoring torque developed in the cylinder twisting

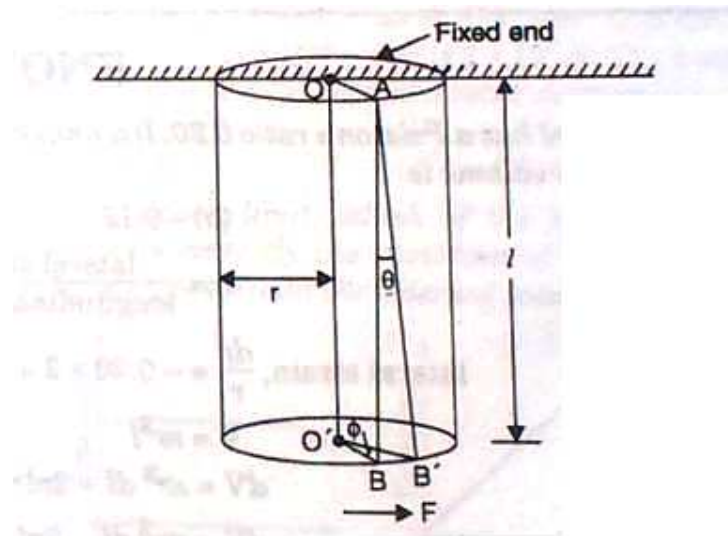
c = restoring couple per unit twist

F = tangential force applied at the free end.

(i) Relation between angles of shear and twist

$$BB' = l\theta = r\phi \text{ or}$$

$$\theta = \frac{r}{l} \phi$$



$$(ii) \tau = \frac{\pi \eta r^4}{2l} \phi$$

$$\tau = c\phi$$

(iii) Work done in twisting through angle ϕ is

$$\frac{1}{2} c\phi^2, \text{ i.e., } \frac{\pi \eta r^4}{4l} \phi^2$$

Properties of Material 3) Coefficient of Resilience

3 kinds of Coefficient of Resilience

The amount of energy absorbed per unit volume of the body. This is affected by the class of deformation whether axial, bending, or torsional; hence there are three kinds of coefficients of resilience.

Some Authors refer Coefficient of Restitution as Coefficient of resilience.

If a ball falls from a height falling vertically, and just before hitting the ground, it has a speed of v_1 . Then after hitting the ground it jumps upward with a vertical upward speed of v_2 .

Then the coefficient of restitution $e = \text{mod of } (v_2 / v_1)$

If a ball is moving at u_1 and another is moving at u_2 , they collide. After collision if these move at v_1 and v_2 then $e = \text{mod of } (v_2 - v_1) / (u_2 - u_1)$

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$$e = \frac{\text{Velocity of Separation}}{\text{Velocity of approach}}$$

ie. $e = \frac{V_2 - V_1}{U_2 - U_1} \dots(2)$

Properties of Material 4) Relations between various Elastic constants

RELATIONS CONNECTING THE ELASTIC CONSTANTS

$$1. K = \frac{Y}{3(1 - 2\sigma)} \quad 3. \frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K}$$

$$2. \eta = \frac{Y}{2(1 + \sigma)} \quad 4. \sigma = \frac{3K - 2\eta}{2(3K + \eta)}$$

Write many times to memorize

Relations between Elastic Constants Y , η , K and σ

$$(i) \eta = \frac{Y}{2(1 + \sigma)} \quad (ii) K = \frac{Y}{3(1 - 2\sigma)}$$

$$(iii) \frac{3}{Y} = \frac{1}{3K} + \frac{1}{\eta} \quad (iv) \sigma = \frac{3K - 2\eta}{2\eta + 6K}$$

There is a mistake in the formula below. Y/η should be $2(1 + \sigma)$

Note $\beta = \frac{Y}{3(1 - 2\sigma)}$, $\frac{Y}{\eta} = 2(1 + \sigma)$, $Y = \frac{\sigma\eta\beta}{\eta + 3\beta}$,

Torsional rigidity $C = \frac{k\eta r^4}{2l}$

Torsional couple (Torque) $G = C\theta$. If tangential stress is T then $\frac{T}{r} = \eta$ where ϕ is shear angle. $\phi = \frac{x\theta}{l}$ where θ is angle of twist.

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Poisson's ratio cannot exceed

- (a) 0.25 (b) 1.0
(c) 0.75 (d) 0.5

(d) $B = \frac{Y}{3(1-2\sigma)}$ if $B = \infty$ $1 - 2\sigma \rightarrow 0$ or
 $\sigma_{\max} = \frac{1}{2}$.

A copper wire of cross-section A is under a tension T . Find the decrease in the cross-section area. Young's modulus is Y and Poisson's ratio is σ .

- (a) $\frac{\sigma T}{2AY}$ (b) $\frac{\sigma T}{AY}$
(c) $\frac{2\sigma T}{AY}$ (d) $\frac{4\sigma T}{AY}$

$$\frac{\Delta r}{r} = \sigma \frac{\Delta l}{l} \text{ and } \frac{\Delta l}{l} = \frac{T}{AY}$$

$$\frac{\Delta A}{A} = \frac{2\Delta r}{r} = \frac{2\sigma T}{AY}$$

Properties of Material 5) Bending of the Beam

Depression of Beam at center

The Depression of a Beam at its Centre

The depression at the centre of a beam is given by

$$= \frac{MgL^3}{4bd^3Y}$$

M = Suspended Mass, L = Length of the beam, b = Bread of the beam,

Y = Young's modulus and d = Thickness of the beam

***SUPPORTED BEAM, CENTRALLY LOADED,**
[Assumption : Weight of the beam is ineffective.]

(i) If the beam is of circular cross-section, then depression y is given by :

$$y = \frac{WL^3}{12Y\pi r^4}$$

where W is the load suspended at the middle of the beam, L is the length of the beam between two supported points, Y is Young's modulus of elasticity and r is the radius of the circular cross-section of the beam.

(ii) If the beam is of rectangular cross-section of breadth b and depth d , then depression at the middle is given by

$$y = \frac{WL^3}{4Ybd^3}$$

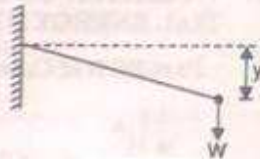
THE CANTILEVER—DEPRESSION OF ITS LOADED END

[Assumption : Weight of cantilever is ineffective]

$$Y = \frac{WL^3}{3YI}$$

For a beam of rectangular cross-section of breadth b and depth d , $I = \frac{bd^3}{12}$

$$\therefore y = \frac{WL^3 \times 12}{3Y \times bd^3} = \frac{4WL^3}{Ybd^3}$$



If the cross-section is square in shape, then $b = d$.

$$\therefore I = \frac{b^4}{12}$$

$$\therefore y = \frac{WL^3 \times 12}{3Yb^4} = \frac{4WL^3}{Yb^4}$$

For the beam of circular cross-section of radius r ,

$$I = \frac{\pi r^4}{4}$$

$$\therefore y = \frac{WL^3}{3Y \left[\frac{\pi r^4}{4} \right]} = \frac{4WL^3}{3Y\pi r^4}$$

For the same cross-sectional area and for given load, the ratio of depression for the beam of a square cross-section and circular cross-section is :

- (a) 3 : π
(c) 1 : π

- (b) π : 3
(d) π : 1.

Sol. $y_1 = \frac{4WL^3}{Yb^4}$, $y_2 = \frac{4WL^3}{3Y\pi r^4}$

$$\frac{y_1}{y_2} = \frac{4WL^3}{Yb^4} \times \frac{3Y\pi r^4}{4WL^3} = \frac{3\pi r^4}{b^4} = \frac{3\pi r^4}{(\pi r^2)^2}$$

[$\because b^2 = \pi r^2$]

$$= \frac{3}{\pi}$$

So, (a) is the right choice.

Properties of Material 6) Measurement of Radius of Curvature

To measure the radius of curvature with a spherometer, we use the formula

(a) $R = \frac{h^2}{6} + \frac{l}{2}$

(b) $R = \frac{l^2}{6h} + \frac{h}{2}$

(c) $R = \frac{h^2}{2l} + \frac{l}{h}$

(d) $R = \frac{2l^2}{h} + \frac{6}{l}$

A spherometer (Fig. 11) is used to determine the radius of curvature of a spherical surface. The theory of the method is briefly described below:

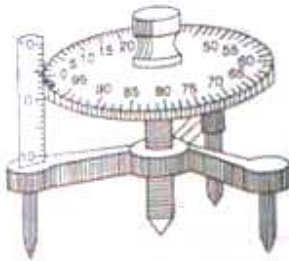


Fig. 11

In the Fig. 12, $r^2 = h(2R - h)$

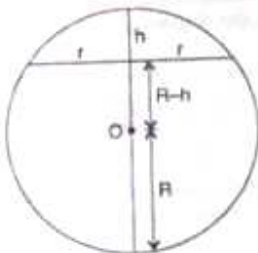


Fig. 12

On simplification,

$$R = \frac{r^2}{2h} + \frac{h}{2}$$

But

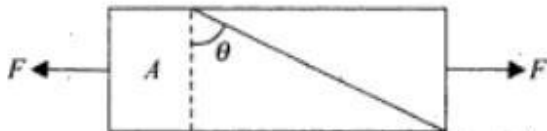
$$r = \frac{l}{\sqrt{3}}$$

[Think of an equilateral triangle of side l .]

$$\therefore R = \frac{l^2}{6h} + \frac{h}{2}$$

Properties of Material 7) Shear stress

A bar of cross-section A is subjected to equal and opposite tensile forces F at its ends. Consider a plane through the bar making an angle θ with a plane at right angles to the bar. Then shearing stress will be maximum if θ



- (a) 0°
- (b) 30°
- (c) 45°
- (d) 60°
- (e) 90°

(c) Shear stress = $\frac{F \sin \theta}{A / \cos \theta} = \frac{F \sin 2\theta}{2A}$

Shear stress will be maximum if $\sin 2\theta = 1$ or $2\theta = 90^\circ$ i.e. $\theta = 45^\circ$.

Properties of Material 8) Thermal stress and force

Thermal Stress

(i) The thermal stress set up in the rod which is not free to expand or contract is given by,

$$\text{Stress in the rod} = \frac{F}{A} = Y \alpha (\theta_2 - \theta_1).$$

Y = Young's modulus, α = Linear coefficient of expansion and $(\theta_2 - \theta_1)$ = Temperature difference.

(ii) Thermal force = $F = YA \alpha (\theta_2 - \theta_1)$

(iii) Two different rods of different materials are joined end to end and the composite rod is fixed between the two supports. The temperature difference is $(\theta_2 - \theta_1)$. Then force is given by

$$F = \frac{L_1 \alpha_1 (\theta_2 - \theta_1) + L_2 \alpha_2 (\theta_2 - \theta_1)}{\frac{L_1}{A_1 Y_1} + \frac{L_2}{A_2 Y_2}}$$

Properties of Material 9) Proof Resilience

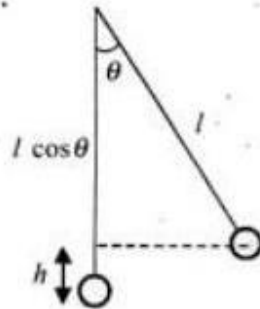
Proof resilience is related to

- (a) PE stored in an elastic body.
- (b) stiffness of a beam.
- (c) elastic fatigue.
- (d) elastic relaxation.

Ans : (a)

Properties of Material 10) Elongation in a Pendulum

A sphere of mass M kg is suspended by a metal wire of length L and diameter d . When in equilibrium, there is a gap of Δl between the sphere and the floor. The sphere is gently pushed aside so that it makes an angle θ with the vertical. Find θ_{\max} so that sphere fails to rub the Floor. Young's modulus of the wire is Y .



- (a) $\sin^{-1} \left(1 - \frac{Y\pi d^2 \Delta l}{8MgL} \right)$ (b) $\tan^{-1} \left(1 - \frac{Y\pi d^2 \Delta l}{8MgL} \right)$
 (c) $\cos^{-1} \left(1 - \frac{Y\pi d^2 \Delta l}{8MgL} \right)$ (d) none

Ans : (c) $Y = \frac{Fl}{A\Delta l} = \frac{2Mg(1 - \cos \theta)L}{\pi \frac{d^2}{4} \Delta l}$

or $1 - \cos \theta = \frac{Y\pi d^2 \Delta l}{8Mgl}$ or $\cos \theta = 1 - \frac{Y\pi d^2 \Delta l}{8Mgl}$

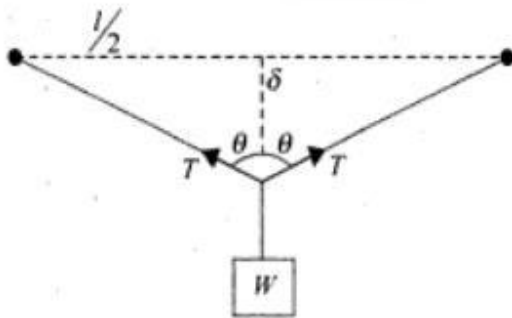
$$\frac{mv^2}{2} = mgl(1 - \cos \theta)$$

or $\frac{mv^2}{l} = 2mg(1 - \cos \theta)$

$$\theta = \cos^{-1} \left(1 - \frac{Y\pi d^2 \Delta l}{8MgL} \right)$$

Properties of Material 11) Depression at center of rod

A wire of length L is clamped at two ends so that it lies horizontally and without tension. A weight W is suspended from the middle point of the wire. The vertical depression is δ . Young's modulus is Y .



- (a) $\sqrt{\frac{2Tl^2}{4AY} + \frac{T^2l^2}{4A^2Y^2}}$ (b) $\sqrt{\frac{2Tl^2}{4AY} - \frac{T^2l^2}{4A^2Y^2}}$
 (c) $\sqrt{\frac{2Tl^2}{4AY}}$ (d) $\frac{Tl}{2AY}$

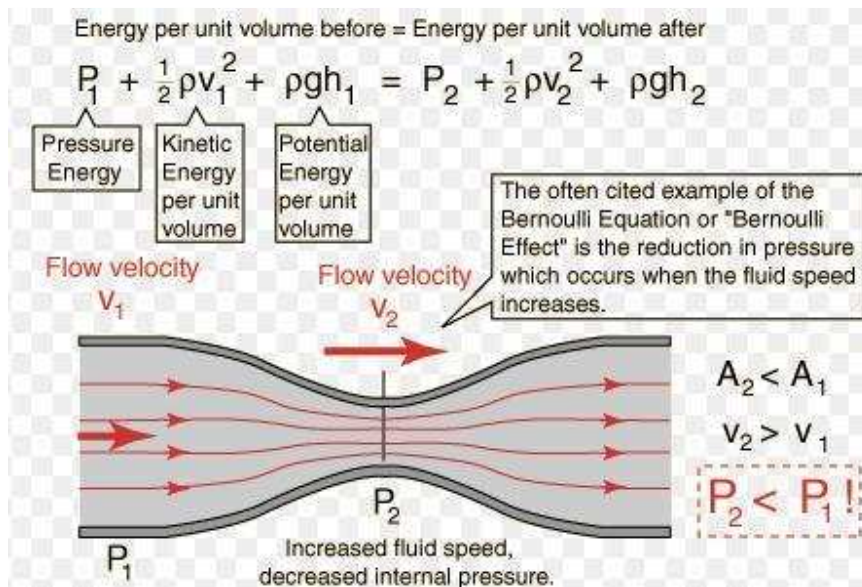
(a) $2T \cos \theta = W$

or $T = \frac{W}{2 \cos \theta}$

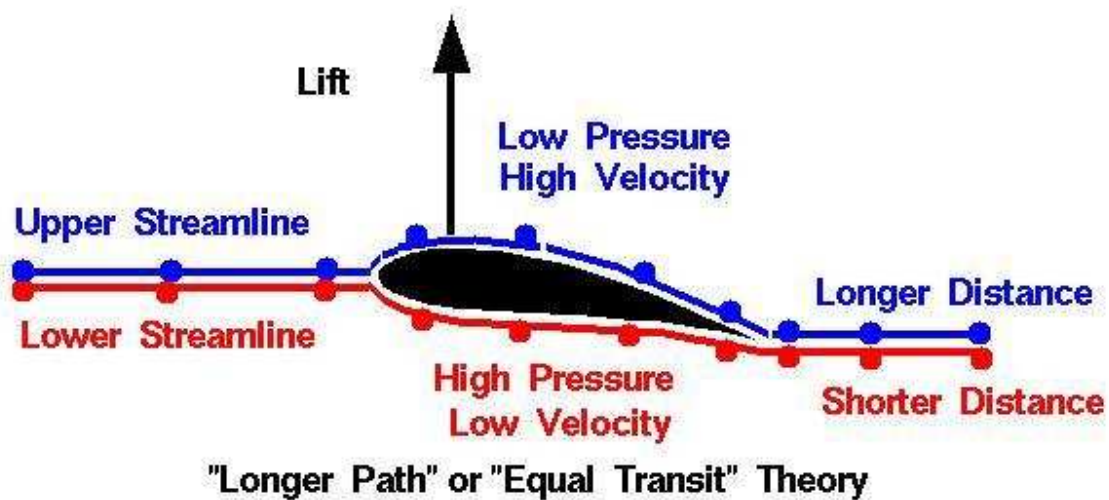
$\Delta l = \frac{Tl}{2AY}$ $\delta = \sqrt{\left(\frac{l}{2} + \Delta l\right)^2 - \frac{l^2}{4}}$

or $\delta = \sqrt{\left(\frac{l}{2} + \frac{Tl}{2AY}\right)^2 - \frac{l^2}{4}} = \sqrt{\frac{2Tl^2}{4AY} + \frac{T^2l^2}{4A^2Y^2}}$

Fluid 1) Bernoulli's Principle and Application



Differential velocity at top and bottom of an aircraft wing, for uplift



Dynamic lift in aircraft

Aeroplanes get the dynamic lift because of the shape of their wings. The upper surface of the wing is made more curved than the lower surface; air flows with greater speed above the wing; pressure above the wing is less. The wing gets dynamic lift upwards.

$$\text{Dynamic lift} = (P_2 - P_1)A = \frac{1}{2}\rho(v_1^2 - v_2^2)A$$

Where ρ is the density of air, A is the area of the wing, v_1 and v_2 are the speeds of air above and below the wing and P_1 and P_2 are pressures above and below the wing.

Air is streaming past a horizontal air plane wing such that its speed is 90 m s^{-1} at the lower surface and 120 m s^{-1} over the upper surface. If the wing is 10 m long and has an average width of 2 m, the difference of pressure on the two sides and the gross lift on the wing are [Density of air = 1.3 kg m^{-3}]

(a) 5 Pa, 900 N

(b) 95 Pa, 900 N

(c) 4095 Pa, 900 N

(d) 4095 Pa, 81900 N.

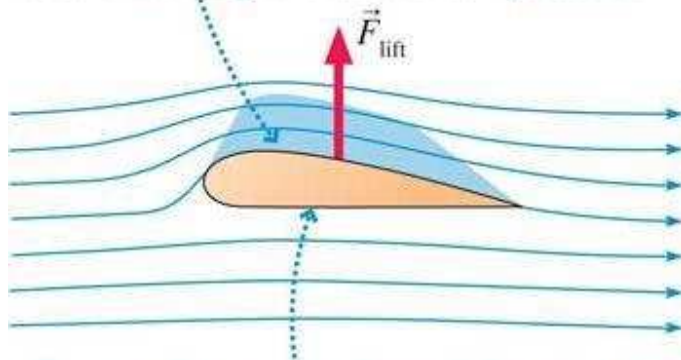
Ans :

Pressure Difference = $\Delta P = \frac{1}{2}(\rho)v^2$

$$(d) P_2 - P_1 = \frac{1}{2} \times 1.3 [120^2 - 90^2] = 4095 \text{ Pa}$$

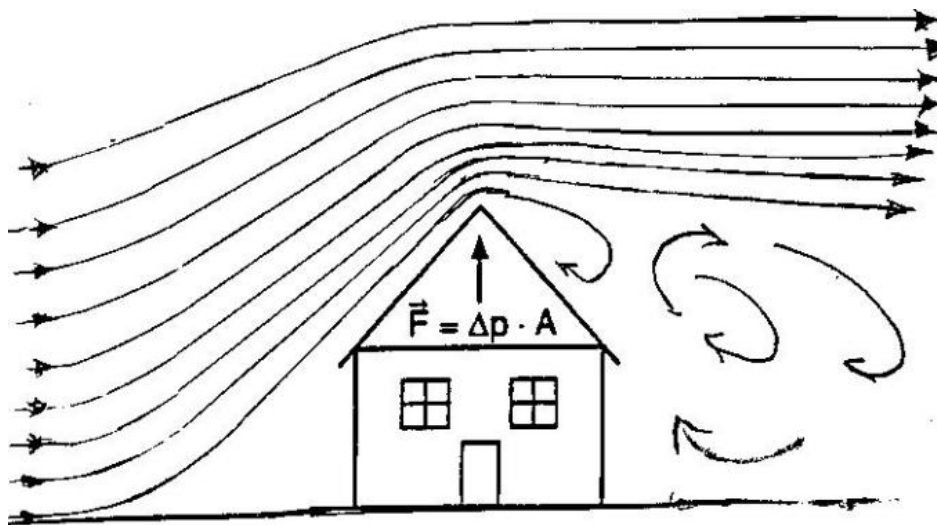
$$\text{Lift} = 4095 \times 2 \times 10 \text{ N} = 81900 \text{ N}$$

A pressure gradient is needed to accelerate the air around the curved upper surface of the wing. Thus the air just above the wing is a zone of low pressure.

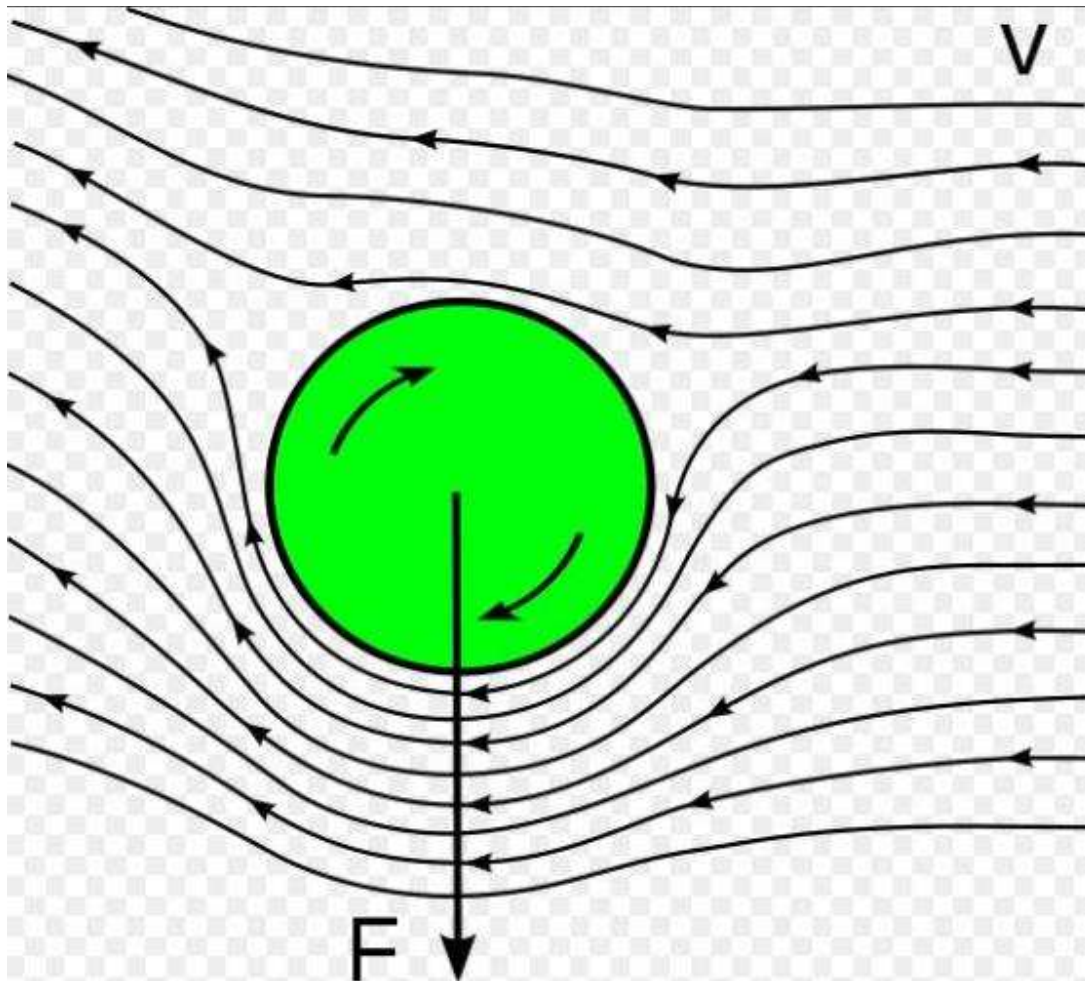


Because the pressure beneath the wing is higher than the pressure above, there's a net upward force on the wing. This is lift.

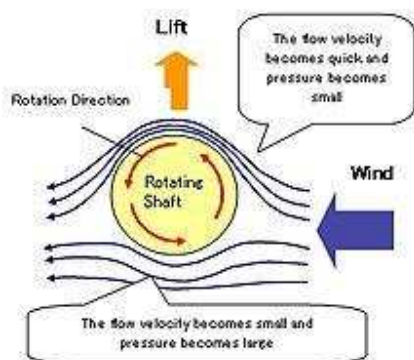
roof of hut being flown off due to strong wind



Fluid 2) Magnus Effect Top Spin



Magnus Effect lift



Fluid 3) Reynold's Number

$$N_{Re} = \frac{D V C}{\eta}$$

D = inside pipe diameter

V = average velocity

C = density

η = absolute viscosity

Fluid 4) Surface Tension Formula

Work done = energy = Area \times Surface tension

Energy for film = 2(Area \times Surface tension)

Absorbed energy when drop of radius R splits into n identical drops of radius r , is

$$= 4\pi R^2(n^{1/3} - 1)T = 4\pi r^2 n^{2/3} (n^{1/3} - 1)T$$

Excess pressure inside the soap bubble = $\frac{4T}{r}$

Excess pressure inside the liquid drop = $\frac{2T}{r}$

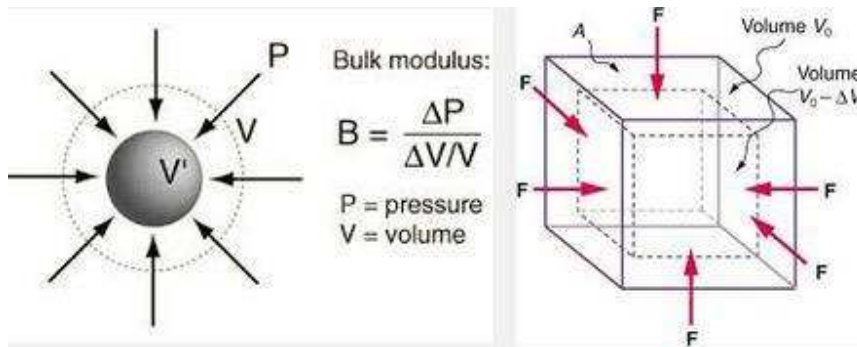
Difference between convex concave side is

$$p = T \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

When two drops of radii r_1, r_2 coalesce to form a new drop of radius R under isothermal condition, then $R = \sqrt{r_1^2 + r_2^2}$

When a soap bubble of radius r_1 and another of radius r_2 are brought together the radius of the common interface is $\frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2}$

Fluid 5) Bulk Modulus and Compression of liquid



$$\beta = -\frac{\delta V}{V \delta p} = \text{Compressibility}$$

$$-\frac{\delta V}{V} = \frac{\delta p}{K} \quad K = \rho \frac{dp}{d\rho}$$

$$V = \frac{1}{\rho} \quad K = \text{Bulk Modulus}$$

$\rho = \text{Density,}$

$V = \text{Volume, } p = \text{Pressure,}$

Find the density of water 2 km deep in a sea. Bulk modulus = 2×10^9 Pa.

- (a) 10^3 kgm^{-3} (b) 1010 kgm^{-3}
 (c) 1100 kgm^{-3} (d) 1040 kgm^{-3}

$$(b) \frac{\Delta V}{V} = \frac{P}{B} = \frac{2 \times 10^3 \times 10^3 \times 10}{2 \times 10^9} = .01$$

$$\frac{\Delta V}{V} = \frac{\Delta \delta}{\delta} \text{ or } \Delta \delta = 10 \text{ kg/m}^3.$$

density of water = 1010 kg m^{-3}

The average depth of Indian Ocean is about 3000 m. Bulk modulus of water is $2.2 \times 10^9 \text{ Nm}^{-2}$, $g = 10 \text{ ms}^{-2}$, then fractional compression $\left(\frac{\Delta V}{V}\right)$ of water at the bottom of the Indian Ocean is

- (a) 1.36% (b) 20.6% (c) 13.9% (d) 0.52%

Interpret (a) The pressure exerted by a 3000 m column of water on the bottom layer

$$p = h\rho g = 3000 \times 1000 \times 10 \\ = 3 \times 10^7 \text{ kg m}^{-1} \text{ s}^{-2} = 3 \times 10^7 \text{ Nm}^{-2}$$

Fractional compression $\left(\frac{\Delta V}{V}\right)$

$$= \frac{\text{Stress}}{B} = \frac{(3 \times 10^7 \text{ Nm}^{-2})}{(2.2 \times 10^9)} = 1.36 \times 10^{-2}$$

$$\frac{\Delta V}{V} = 1.36\%$$

Find the volume density of elastic energy of fresh water at a depth of 1000 m

- (a) 2.5 kJm^{-3} (b) 25 kJm^{-3}
 (c) 0.25 kJm^{-3} (d) none

$$(b) \frac{dW}{V} = \frac{1}{2} P \frac{\Delta V}{V} = \frac{1}{2} P \left(\frac{P}{B}\right) \\ = \frac{(\rho gh)^2}{2 \times 2 \times 10^9} = \frac{(10^3 \times 10 \times 10^3)^2}{2 \times 2 \times 10^9} = 2.5 \times 10^4 \text{ J/m}^3.$$

A diver at a depth of 45 m exhales a bubble of air that is 1.0 cm in radius. Assuming ideal gas behaviour, what will be the radius of this bubble as it breaks the surface of water?

Plan Inside water $P_{\text{Total}} = \text{atmospheric pressure} + \rho gh$

Using $P_1 V_1 = P_2 V_2$, V_2 at the surface of water is calculated (V_2 is the volume of bubble at the surface), thus, r can be calculated.

Solution Atmospheric pressure = 1 atm.

Pressure due to depth of 45 m = ρgh

where ρ = density of water = $1 \text{ g cm}^{-3} = 1000 \text{ kg m}^{-3}$,
 $g = 9.81 \text{ m s}^{-2}$, $h = 45 \text{ m}$

$$\rho gh = 1000 \times 9.81 \times 45 \text{ N m}^{-2}$$

$$= \frac{1000 \times 9.81 \times 45}{101325} \text{ atm} = 4.36 \text{ atm}$$

$$(\because 1 \text{ atm} = 1.01325 \times 10^5 \text{ N m}^{-2})$$

$$\therefore P_1 = \text{atmospheric pressure} + \rho gh = 1 + 4.36 = 5.36 \text{ atm}$$

$$P_2 = 1 \text{ atm}$$

$$V_1 = \frac{4}{3} \pi r^3 = \frac{4}{3} \times \pi \times (1)^3 \text{ cm}^3$$

$$V_2 = \frac{4}{3} \pi r^3 = \text{volume of bubble at } P_2 \text{ (at the surface)}$$

using

$$P_1 V_1 = P_2 V_2$$

$$V_2 = \frac{P_1 V_1}{P_2}$$

$$\frac{4}{3} \pi r^3 = \frac{5.36 \times \frac{4}{3} \pi (1)^3}{1}$$

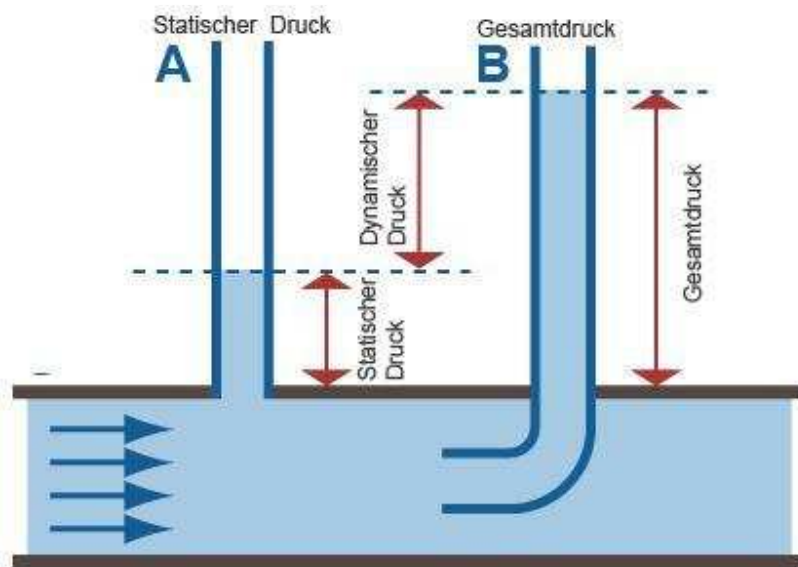
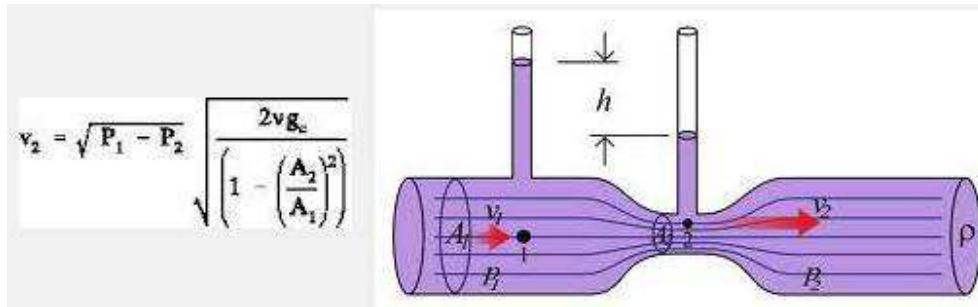
$$r^3 = 5.36 \text{ cm}^3$$

$$r = 1.75 \text{ cm}$$

Fluid 6) Time taken for water to go from h_1 to h_2

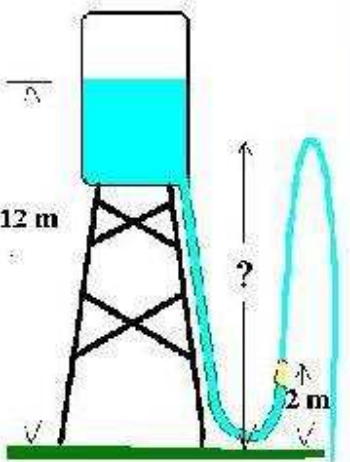
A cylindrical vessel of area of cross-section A has a hole of area of cross-section ' a ' in its bottom. Time taken for the water level to decrease from h_1 to h_2 as water flows out from the hole is

$$t = \frac{A}{a} \sqrt{\frac{2}{g}} (\sqrt{h_1} - \sqrt{h_2})$$



Application of Bernoulli's Equation in Siphon

Application of Bernoulli's Equation



Water level is 12 m above ground. Hose nozzle is 2.0 m above ground.

- 1 What is (gauge) pressure in hose at nozzle?
- 2 What is water speed at nozzle?
- 3 How high does water squirt?

Magnetic Properties of Materials 1) Diamagnetic, Paramagnetic, Ferrimagnetic, Antiferromagnetic

Magnetic Properties : Solids can be classified into different types depending upon their behaviour towards applied magnetic field.

a. Diamagnetic Substances : Which are weakly repelled by magnetic field. They have paired electrons.

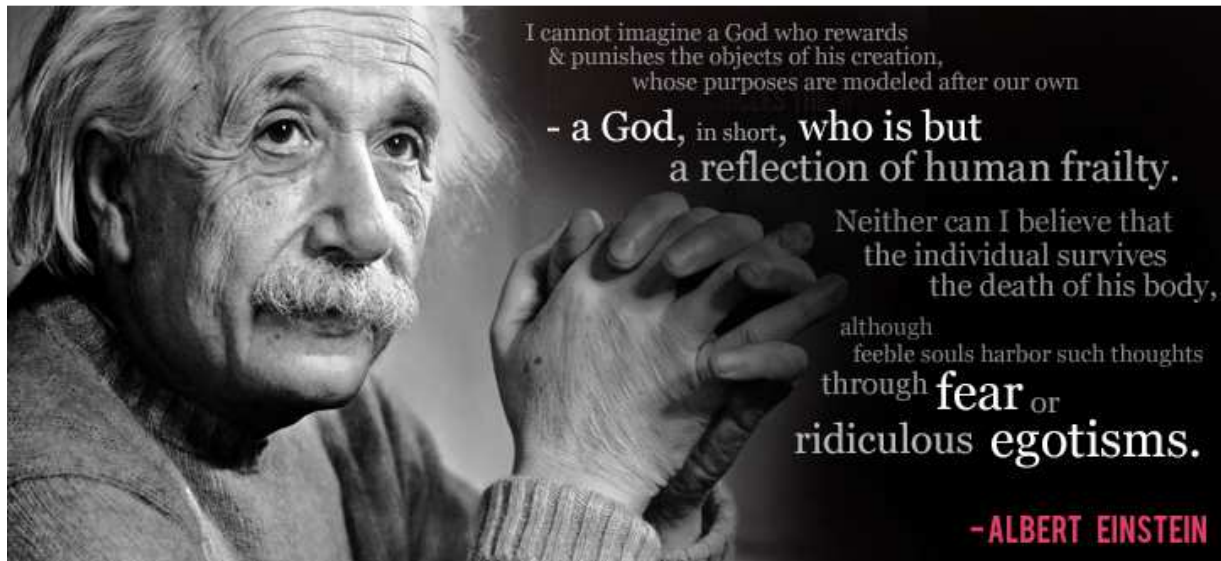
NaCl, V_2O_5 , TiO_2 .

b. Paramagnetic Substances : Which are weakly attracted by magnetic field. They have permanent dipoles due to presence of unpaired electrons. They lose their magnetism on removal of magnetic field. TiO , Ti_2O_3 , VO, VO_2 , CuO.

c. Ferromagnetic Substances : Spontaneous alignment of magnetic dipoles of ions or atoms in same direction. It changes into paramagnetic substances at higher temperature. Fe, Co, Ni, CrO_2 .

d. Ferrimagnetic Substances : Alignment of magnetic dipoles of ions or atoms in such a way so that there is some net magnetic moment due to unequal number of parallel and anti-parallel magnetic dipoles. It also changes into paramagnetic substances at higher temperature. Fe_3O_4 .

e. Anti Ferromagnetic Substances : Alignment of magnetic dipoles of ions or atoms in such a way so that there is no net magnetic moment (i.e. zero magnetic moment) due to equal number of parallel and anti-parallel magnetic dipoles. V_2O_3 , Cr_2O_3 , MnO, Mn_2O_3 , MnO_2 , FeO, Fe_2O_3 , CoO, NiO.

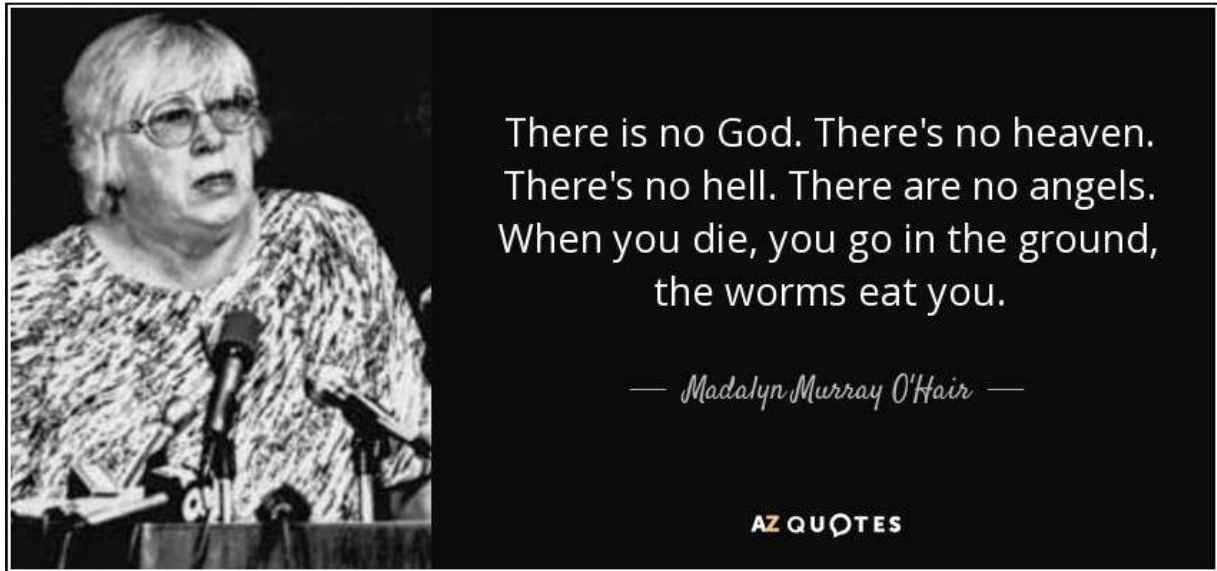


(Apart from Millions of smart people) Several Nobel Laureates were Atheists.

Some famous examples have been [Albert Einstein](#) (1921), [Richard Feynman](#) (1965), [Erwin Schrödinger](#) (1933), [Paul Dirac](#) (1933), [Lawrence M. Krauss](#) (2011), [Niels Bohr](#) (1922), [Peter Higgs](#) (2013), [John Bardeen](#) (**The only person receiving the Physics Nobel prize twice. 1956, 1972**), [Frederick Sanger](#) (**The only person receiving the Chemistry prize twice. 1958, 1980**), [Marie Curie](#) (1903, 1911), [Frédéric Joliot-Curie](#) and [Irène Joliot-Curie](#) (1935), [Milton Friedman](#) (1976), [John Harsanyi](#) (1994), [Friedrich Hayek](#) (1974), [John Forbes Nash, Jr.](#) (1994), [Amartya Sen](#) (1998), [Subrahmanyan Chandrasekhar](#) (1983), [Enrico Fermi](#) (1938), [C. V. Raman](#) (1930), [Eugene Wigner](#) (1963), [Steven Weinberg](#) (1979), [Chen-Ning Yang](#) (1957) etc

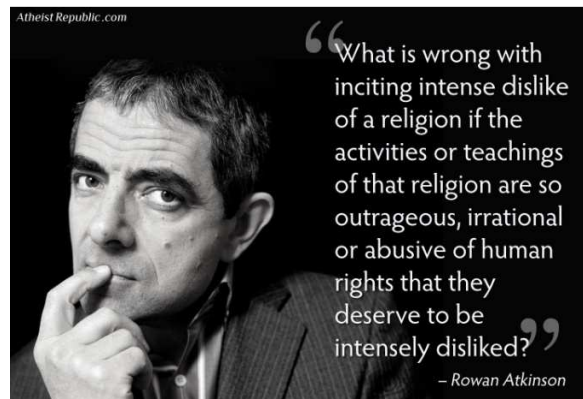
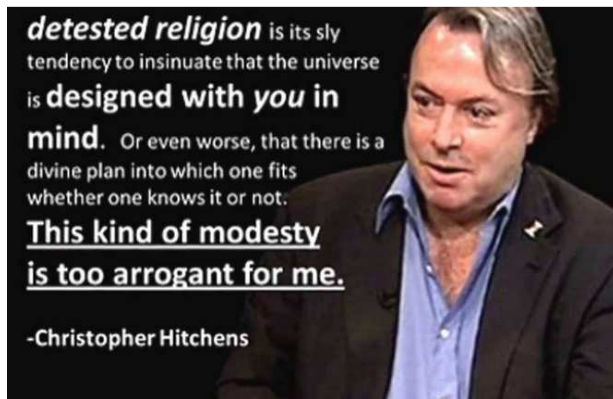
A bigger (incomplete) list can be seen at

https://en.wikipedia.org/wiki/List_of_nonreligious_Nobel_laureates



(When the body is burnt, oxides are the ash. The gases and water vapor spread in the air)

My **personal favorites** (among these Atheists) are **Richard Feynman**, **Peter Higgs**, **Lawrence Krauss**.



Richard Feynman openly laughed (Publicly and in class) about Gods, Fairies etc. see <https://www.youtube.com/watch?v=j3mhkYbznBk>

and https://www.youtube.com/results?search_query=Richard+Feynman

Peter Higgs was very unhappy about " Higgs Boson " being called "G..(I don't want to name this) Particle". **Stupid Journalists**, Media, and dumb people kept repeating that word, and Peter requested to refrain from using this word.

Lawrence Krauss openly laughs and ridicules the Theists or any non-Atheists. The crap of Agnosticism does not work with me or Krauss.

Ray or Geometrical Optics by Prof. Subhashish Chattopadhyay SKMClasses Bangalore Useful for IIT-JEE, CBSE, I.Sc. PU-II, Boards, CET, CEE, PET, IGCSE IB AP-Physics and other exams

We are in Modern Times. I am lucky to learn the correct things quite early in my life, in a so "peaceful" society. When I was in standard 9, (in early 1980s), I was writing a book on Atheism. I was convinced to understand, learn, and imbibe the correct approach and knowledge.

But that was not the case previously. Copernicus used to discuss and explain people widely and randomly, that Earth is rotating around the Sun, and it is not a Geocentric" universe. Nicolaus Copernicus had to waste lot of time arguing, fighting and convincing the stupids.

Measuring something, which is very slow; is very difficult. I have asked lot of "educated / engineer / Software or IT (senior position) Parents" that " How do we know that Earth is moving around the Sun in 365 days or say 365.242196 days " ? **Believe me I never got an answer.** The Modern iPad / smartphone **community in general does not know how 365.24 days was measured almost thousand years ago !**

A metal triangle was set at top of buildings (Mosques or churches) and the position of the shadow was marked at a particular time. Say 8 AM everyday. The position of the shadow varied everyday. It was seen that after 365 days the shadow matched the position but after sometime, not exactly at 8 AM but after a few hours (approx 6 hours) so at around 2 PM or slightly before.

See details of this at <http://blog.world-mysteries.com/science/ancient-timekeepers-part4-calendars/>

See the video https://www.youtube.com/watch?v=IhqzW97_47w

It took many centuries to introduce the leap year corrections. A century is a leap year only if divisible by 400 and not the rule of divisible by 4. Year 1900 was not a Leap year. But year 2000 was. I have met computer Science guys who are aware that Microsoft Database SQL-server do not accept some old dates, while Oracle database does not accept some specific dates of the past. But none whom I met knew the detailed or actual reasons.

See <https://zookeepersblog.wordpress.com/everyone-must-know-about-the-calendar/>

“ How do you prove that day and night is happening due to rotation of Earth around its own axis in contrast to Sun is rotating around Earth “ ?

No student from Bangalore, whom I met, answered this. Though conservation of Angular Momentum is in course. (I am being polite) Hardly met any parent who knew the explanation. See https://www.youtube.com/watch?v=iqpV1236_Q0

And <https://www.youtube.com/results?q=Foucault%27s+pendulum>

Tyco Brahe took the boldest step to create the " Foundation of Science ". Experiments or "Double blind experimental observations " are the supreme. The **Theory follows the experimental verification.** There are some universities who award M.Sc in Psychology. A psychologist may **guess** something **But that is not reality or truth.** Till something is

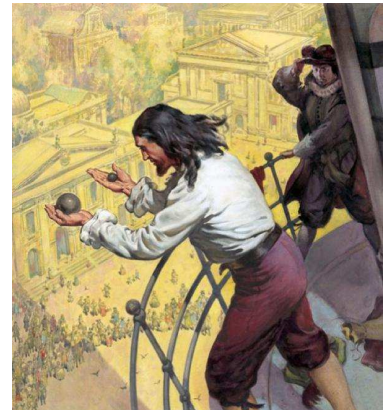
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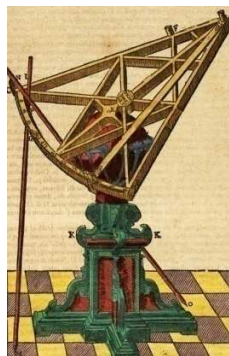
experimentally verified it remains as a Perception. Truth is known only after experiments. Because the subject Psychology; completely stands of experimental verification; so the Master in Science degree.



Galileo Galilei (1564-1642)

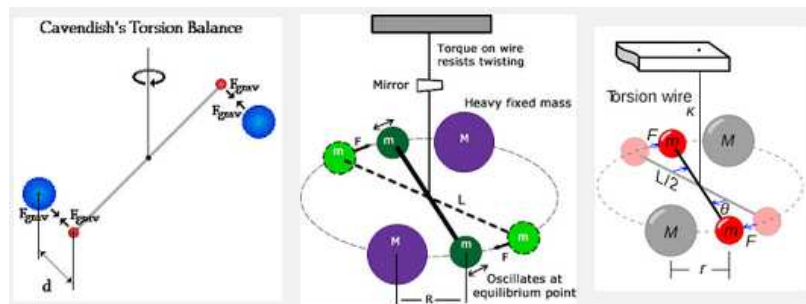
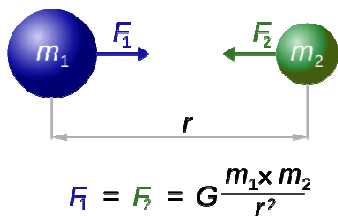


Galileo was the first person who wanted to experimentally verify the speed of light.



Tycho decided to observe the skies (around 1573). In those days sky was synonymous to God. He had the courage to go to the King to ask for donations to make an observatory. He said to the king that "he wants to observe the Gods and take conclusions ". Salute to Tycho's paradigm that even Gods can be observed and conclusions can be drawn.

Amazing leap to start Science.



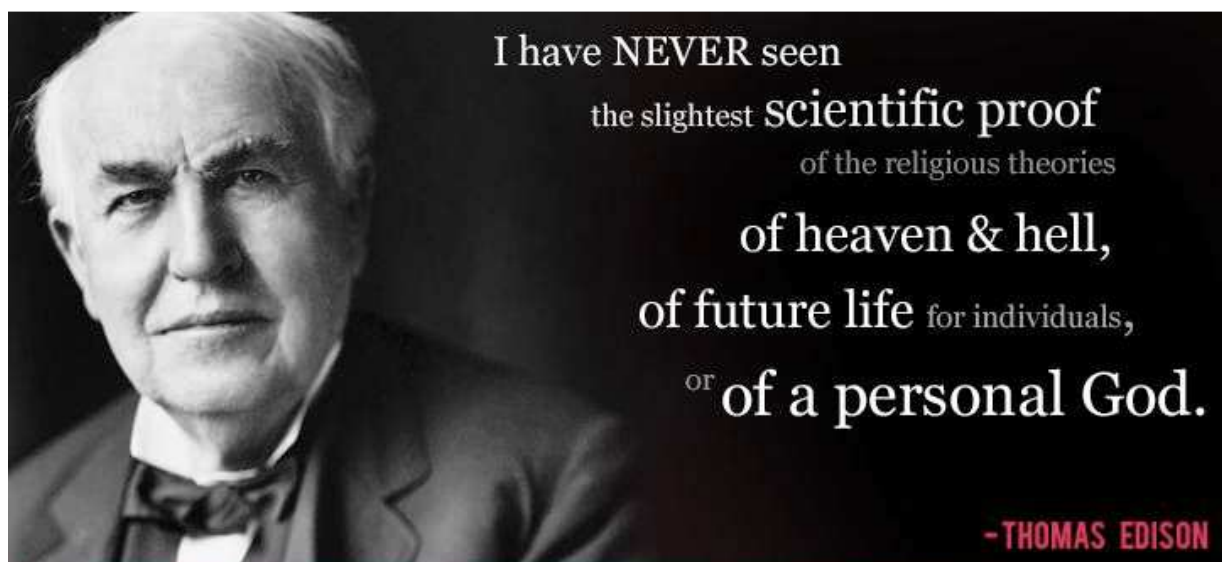
Ray or Geometrical Optics by Prof. Subhashish Chattopadhyay SKMClasses Bangalore Useful for IIT-JEE, CBSE, I.Sc. PU-II, Boards, CET, CEE, PET, IGCSE IB AP-Physics and other exams

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Since those days till now we observed and concluded about Kepler's Laws, Gravitation Laws, Cavendish measuring value of G, measuring speed of light, X-Ray, [Electromagnetism / Maxwell's equations](#), Radioactivity, No Aether was "observed" in Michelson Morley's experiments, Protons, Neutrons, General Theory of Relativity, Slowing of clocks at high speed, Bending of space, [Bending of light and gravitational lens](#), YDSE, Quantum Mechanics, Ernst Ruska designed and built the first [electron microscope](#), [Casimir Forces](#), Virtual particles and more than 400 kinds of particles, Quarks, Unruh effect (an accelerating thermometer shows higher temperature), Negative Kelvin Temperature, Bose-Einstein condensates, Superconductivity, [Solution to EPR paradox by John Stewart Bell](#), Violation of Parity in certain situations - Madam Wu, Yang and Lee, Quantum entanglement in Alain Aspect's Experiments, Black holes, mass of Neutrinos, Caesium Atomic Clocks, Dark Matter, Dark energy, Magnetic Monopole, Gravitational Waves, Nano Materials, Meta Materials, Quantum Computers

[No God was observed](#), or [no role of God was observed](#). There is no conspiracy theory going around in Science. Those who want to verify God have to die waiting

... Nothing ever will be reported regarding this illusion.



(As I write these words { 2016 } [GUT \[General Unified Theory \]](#) is being modified to introduce a 5th fundamental force, because some heavy particles have been observed at [CERN](#) and various other experiments and [Producing Gravitational waves at will, without mass](#))



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Learn Science from <https://www.youtube.com/user/cassiopeiaproject/videos>

Some easy Physics (much easier than IIT-JEE)

https://www.youtube.com/channel/UCLiSRiiRVQuDfgxl_QN_Fmw/videos

(Pradeep Kshetrapal Sir's Videos are at -

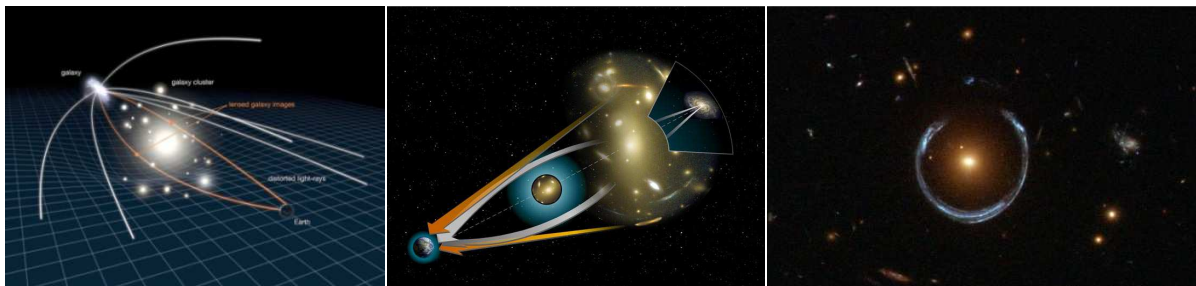
<https://www.youtube.com/user/PradeepKshetrapal/videos>)

IIT-JEE is extremely tough for most humans. A productive PhD in Physics, or actually contributing to growth of the subject is much more tougher (than IIT JEE). { I personally know quite a few IIT-JEE single or double digit rankers, joining for PhD and then dropped out due to performance }. **Most people have an illusion that they can argue with Scientists and imagine to ask some " smart " questions which the Scientists will not able to answer, so the argument is won, and existence of God is proved.** As if Scientist are eagerly sitting or waiting to answer every crap asked. I can only say; that most scientists (since more than 100 years) have stopped wasting their time arguing or convincing fools. I am not a Scientist. Even being a simple teacher, I do not try to teach fools, or argue with anyone.

[For History of Physics I recommend

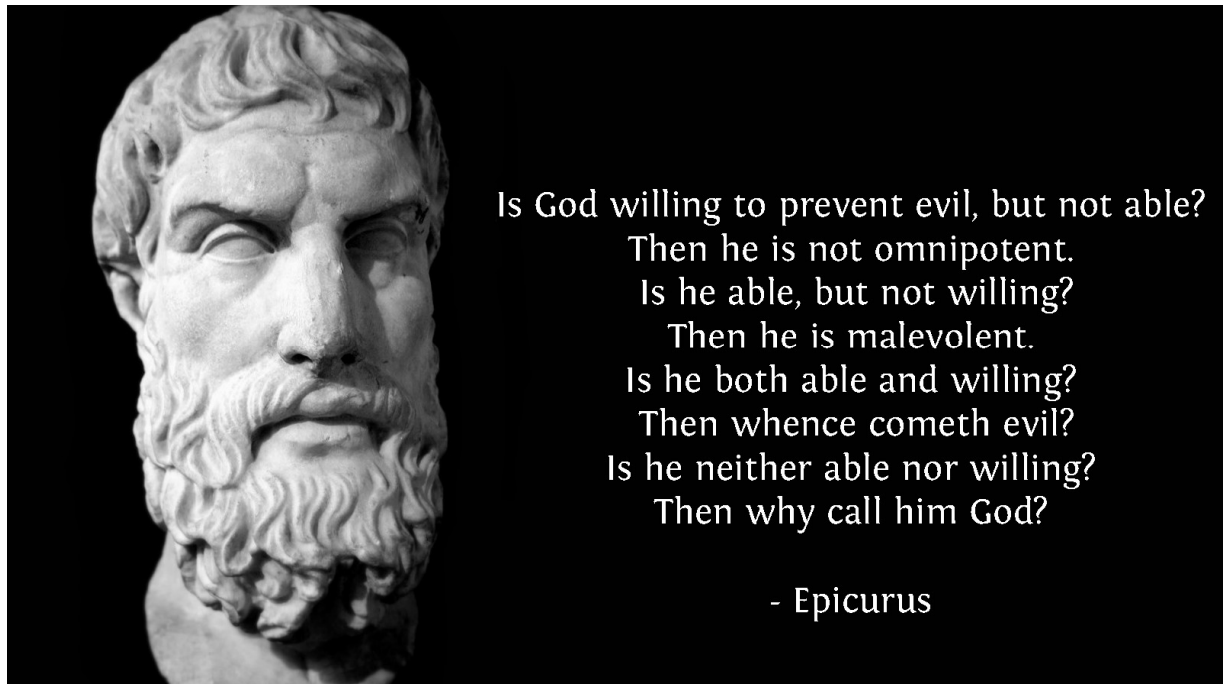
<http://www.historyworld.net/wrldhis/PlainTextHistories.asp?ParagraphID=kqq>]

[Gravitational lens and Einstein ring due to bending of light by mass]



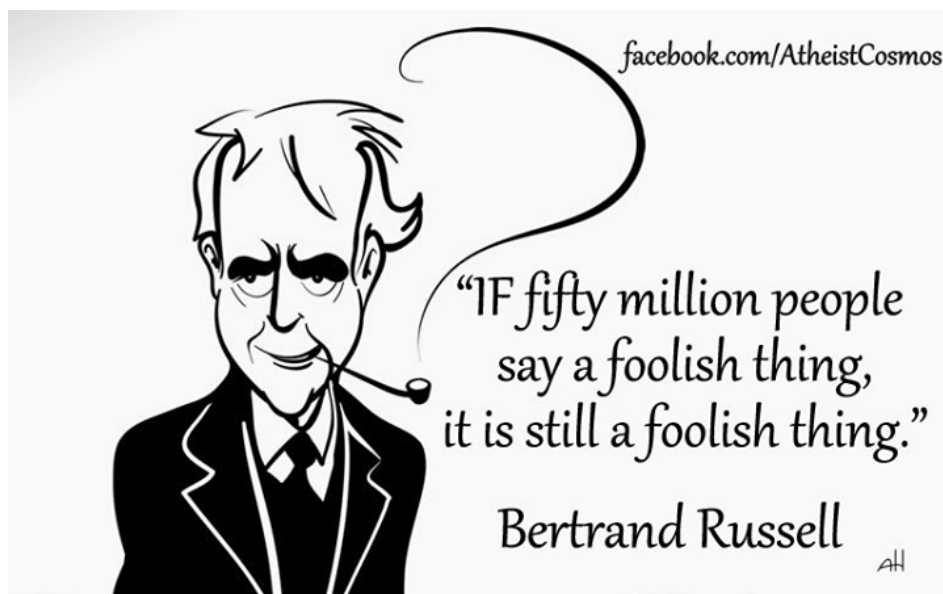
Recall what I said at the beginning of the book " **Someone will learn only by his hard work, his desire to learn.** " No arguments or no 'time wasting' with fools. There is too much of good material (data, books, videos etc) out and free in this world. If someone wants to learn, can learn; instead of wasting time arguing. Since centuries stupid and/or fools are being eliminated in various exams. Entrance exam, is a misnomer. These are elimination tests. The society has systems of Interviews, Peer reviews, appraisals, Thesis evaluation etc... to eliminate crap, foolish things, and nonsense.

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Religion and/or " war between religions " mostly to decide whose God is better; have killed millions. Instead of fighting and killing; to decide which custom to follow; how to dress; what rituals to do on a daily basis; better to spend time experimenting and developing new things, new technologies, new ideas. Scientists (**the men**) are busy; and **always will be busy!** Rather, in war; **with new frontiers of knowledge;** not in arguments, verbal wars, or physical wars. **Atheism is the most peaceful Doctrine.**

"Bertrand Arthur William Russell" the famous Philosopher, Mathematician, Logician, received 1950 Nobel Prize for Literature.



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So those who want to learn can continue learning ...

See https://www.youtube.com/results?search_query=History+of+science

See

https://www.youtube.com/results?search_query=history+of+science+the+complete+full+documentary+

I will choose only two extreme examples of what Human beings have “**seen**” by now ...

For far and big) Very powerful cameras ready with video recording facilities were scanning the sky. Coincidentally the “place or region” a camera was looking had an event (many million years back though) of a black hole devouring a star.

<https://www.youtube.com/watch?v=O3Z5AS3TTS4>

<https://www.youtube.com/watch?v=x7ZX10UbMus>

For small) Photographs of molecules and subsequently atoms

<https://www.youtube.com/watch?v=yqLlglaz1L0>

<https://www.youtube.com/watch?v=ofp-OHlq6Wo>

<https://www.youtube.com/watch?v=oSCX78-8-q0>

<https://www.youtube.com/watch?v=RTLeWlqynW4>

<https://www.youtube.com/watch?v=J3xLuZnKhLY>

<https://www.youtube.com/watch?v=SMgi2j9Ks9k>

https://www.youtube.com/watch?v=V0KjXsGRvoA&list=PLC3E0tG-9im_kuMwYIM7-NZR62VyWZ6rl

Enjoy

-

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Spoon Feeding Series - Ray or Geometrical Optics

History of Optics can be seen at

<http://www.angelfire.com/ga/astronomyclubaugusta/History/hist.htm>

And <http://www.benvr.net/technology/historyofoptics/>

The important formulae of Lenses

(i) When a lens is kept in a medium other than air, then

$$\frac{1}{f_m} = \left(\frac{\mu_g}{\mu_m} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

where μ_m is the refractive index of the medium in which lens is placed.

Hence,
$$\frac{f_m}{f_a} = \frac{(\mu_g - 1)}{\left(\frac{\mu_g}{\mu_m} - 1 \right)}$$

where f_m is the focal length of the lens in medium and f_a is the focal length of lens in air.

- (a) When a lens is placed in a medium for which μ is less than that of the lens, its focal length increases and power decreases. The nature of the lens remains unchanged.
 - (b) When the lens is placed in a medium for which μ is equal to that of the lens, the focal length of the lens becomes infinity and power becomes zero. The lens behaves like a plane glass plate.
 - (c) When the lens is placed in a medium for which μ is greater than that of the lens, the nature of the lens changes. (Air bubble in water behaves as a divergent lens.)
-
- (ii) When a lens of focal length f is cut into two equal halves perpendicular to principal axis, then each part of the lens has a focal length $2f$.
 - (iii) When a lens of focal length f is cut into two equal halves parallel to principal axis, then each part has a focal length f . The intensity of image is decreased.
 - (iv) (a) If a plano-convex lens of focal length f is silvered on the plane surface, it behaves like a concave mirror of focal length $f/2$.
 - (b) When a plano-convex lens of focal length f is silvered at convex surface, it behaves like a convex mirror.

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Level 1 Videos -

<https://archive.org/details/1Day213VariousNewThingsThatULearnInOpticsStd1112IITJEEPhysics>

Level 2 Videos -

<https://archive.org/details/2DiscussionFormulaeOfPhotometryLuxCandelaPhotLumenIlluminanceIlluminocity>

Level 3 Videos -

<https://archive.org/details/3Day312IITJEEOpticsPaintedLensImageSuperimposedOnObjectPhy>

More Level 3 Videos -

<https://archive.org/details/3DefectsOfEyeIscCBSEStd1112SchoolPhysicsOptics>

Level 4 Videos -

<https://archive.org/details/4DeriveTheFormulaMu2ByVMu1ByUMu2Mu1ByROpticsIITJEE>

Level 5 Videos -

<https://archive.org/details/5Day5DDxOfXnSpeedOfImageInConvexMirrorObjectMovingConstSpeedIITJEE>

Level 6 Videos -

<https://archive.org/details/6HelpToRememberVariousMagnificationFormulaeInMicroscopeTelescopesOpticsPhy>

More Optics Videos -

<https://archive.org/details/OpticsPart2ChromaticAberrationInSphericalLensIntroductionFormulaCBSEIscIITJEE>

Day 1 Videos -

<https://archive.org/details/1Day1OpticsBasicsMuVGTmuRVioletBendsMoreThanRed1Physics>

Day 2 Videos -

https://archive.org/details/1Day213VariousNewThingsThatULearnInOpticsStd1112IITJEEPhysics_201608

Day 3 Videos -

https://archive.org/details/3Day312IITJEEOpticsPaintedLensImageSuperimposedOnObjectPhy_201608

Day 4 Videos -

<https://archive.org/details/1Day4MuBiggerDistanceBySmallerDistanceSlabOpticsPhysicsStd1112CETIITJEE>

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Day 5 Videos -

<https://archive.org/details/2Day5ShiftBySlabT11ByMulsOnlyWayOutIfTheObjectIsTouchingCurvedSurfaceIIT>

Discussion on Formulae of Photometry Lux Candela Phot Lumen Illuminance Illuminocity

https://archive.org/details/2DiscussionFormulaeOfPhotometryLuxCandelaPhotLumenIlluminancelluminocity_201608

Video Solutions to many IIT JEE Questions from 1980 till 2008

<https://archive.org/details/IITJEE2006AngularMagnificationByASingleLensImageInFocalPlaneRKF2>

HCV pg 412, 413, 416, 417 Optics Solutions (“Concepts of Physics “ by Professor H C Verma)

<https://archive.org/details/4HCVPg417Pr79ImageOfObjectFallingAcceleratingFrameDAlembertsForceN3rdLawIIT>

Pradeep Kshetrapal Sir’s Optics Video playlist

https://www.youtube.com/playlist?list=PLJZk2__oyAljbDI6gMYXH28b3o2JlaUmR

:-{D

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To recall standard integrals

$f(x)$	$\int f(x)dx$	$f(x)$	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1} \quad (n \neq -1)$	$[g(x)]^n g'(x)$	$\frac{[g(x)]^{n+1}}{n+1} \quad (n \neq -1)$
$\frac{1}{x}$	$\ln x $	$\frac{g'(x)}{g(x)}$	$\ln g(x) $
e^x	e^x	a^x	$\frac{a^x}{\ln a} \quad (a > 0)$
$\sin x$	$-\cos x$	$\sinh x$	$\cosh x$
$\cos x$	$\sin x$	$\cosh x$	$\sinh x$
$\tan x$	$-\ln \cos x $	$\tanh x$	$\ln \cosh x$
$\operatorname{cosec} x$	$\ln \tan \frac{x}{2} $	$\operatorname{cosech} x$	$\ln \tanh \frac{x}{2} $
$\sec x$	$\ln \sec x + \tan x $	$\operatorname{sech} x$	$2 \tan^{-1} e^x$
$\sec^2 x$	$\tan x$	$\operatorname{sech}^2 x$	$\tanh x$
$\cot x$	$\ln \sin x $	$\operatorname{coth} x$	$\ln \sinh x $
$\sin^2 x$	$\frac{x}{2} - \frac{\sin 2x}{4}$	$\sinh^2 x$	$\frac{\sinh 2x}{4} - \frac{x}{2}$
$\cos^2 x$	$\frac{x}{2} + \frac{\sin 2x}{4}$	$\cosh^2 x$	$\frac{\sinh 2x}{4} + \frac{x}{2}$

$f(x)$	$\int f(x) dx$	$f(x)$	$\int f(x) dx$
$\frac{1}{a^2+x^2}$	$\frac{1}{a} \tan^{-1} \frac{x}{a}$ $(a > 0)$	$\frac{1}{a^2-x^2}$	$\frac{1}{2a} \ln \left \frac{a+x}{a-x} \right \quad (0 < x < a)$ $\frac{1}{2a} \ln \left \frac{x-a}{x+a} \right \quad (x > a > 0)$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1} \frac{x}{a}$ $(-a < x < a)$	$\frac{1}{\sqrt{a^2+x^2}}$	$\ln \left \frac{x+\sqrt{a^2+x^2}}{a} \right \quad (a > 0)$ $\ln \left \frac{x+\sqrt{x^2-a^2}}{a} \right \quad (x > a > 0)$
$\sqrt{a^2-x^2}$	$\frac{a^2}{2} \left[\sin^{-1} \left(\frac{x}{a} \right) + \frac{x\sqrt{a^2-x^2}}{a^2} \right]$	$\sqrt{a^2+x^2}$	$\frac{a^2}{2} \left[\sinh^{-1} \left(\frac{x}{a} \right) + \frac{x\sqrt{a^2+x^2}}{a^2} \right]$ $\sqrt{x^2-a^2}$ $\frac{a^2}{2} \left[-\cosh^{-1} \left(\frac{x}{a} \right) + \frac{x\sqrt{x^2-a^2}}{a^2} \right]$

Some series Expansions -

$$\frac{\pi}{2} = \left(\frac{2}{1} \frac{2}{3}\right) \left(\frac{4}{3} \frac{4}{5}\right) \left(\frac{6}{5} \frac{6}{7}\right) \left(\frac{8}{7} \frac{8}{9}\right) \dots$$

$$\pi = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \frac{4}{13} - \dots$$

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots$$

$$\pi = \sqrt{12} \left(1 - \frac{1}{3 \cdot 3} + \frac{1}{5 \cdot 3^2} - \frac{1}{7 \cdot 3^3} + \dots\right)$$

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\int_0^{\pi/2} \log \sin x \, dx = -\frac{\pi}{2} \log 2 = \frac{\pi}{2} \log \frac{1}{2}$$

Solve a series problem

If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$ upto $\infty = \frac{\pi^2}{6}$, then value of

$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ up to ∞ is

- (a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{6}$ (c) $\frac{\pi^2}{8}$ (d) $\frac{\pi^2}{12}$

Ans. (c)

Solution We have $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ upto ∞

$$= \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \dots \text{ upto } \infty$$

$$- \frac{1}{2^2} \left[1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots\right]$$

$$= \frac{\pi^2}{6} - \frac{1}{4} \left(\frac{\pi^2}{6}\right) = \frac{\pi^2}{8}$$

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \dots \infty = \frac{\pi^2}{12}$$

$$\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \dots \infty = \frac{\pi^2}{24}$$

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$$\frac{\sin \sqrt{x}}{\sqrt{x}} = 1 - \frac{x}{3!} + \frac{x^2}{5!} - \frac{x^3}{7!} + \frac{x^4}{9!} - \frac{x^5}{11!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots = \sum_{k=0}^n \frac{(-1)^k x^{2k}}{(2k)!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots = \sum_{k=0}^n \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots = \sum_{k=0}^n \frac{x^{2k}}{(2k)!}$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots = \sum_{k=0}^n \frac{x^{2k+1}}{(2k+1)!}$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \quad (-1 \leq x < 1)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \dots + \frac{2^{2n} (2^{2n} - 1) B_n x^{2n-1}}{(2n)!} + \dots \quad |x| < \frac{\pi}{2}$$

$$\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \dots + \frac{E_n x^{2n}}{(2n)!} + \dots \quad |x| < \frac{\pi}{2}$$

$$\csc x = \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^5}{15120} + \dots + \frac{2(2^{2n-1} - 1) B_n x^{2n-1}}{(2n)!} + \dots \quad 0 < |x| < \pi$$

$$\cot x = \frac{1}{x} - \frac{x}{3} + \frac{x^3}{45} - \frac{2x^5}{945} - \dots - \frac{2^{2n} B_n x^{2n-1}}{(2n)!} - \dots \quad 0 < |x| < \pi$$

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$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \dots$$

$$\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{4} + \dots$$

$$\log(\cos x) = -\frac{x^2}{2} - \frac{2x^4}{4} - \dots$$

$$\log(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \dots$$

$$\sin^{-1} x = x + \frac{1}{2} \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \frac{x^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{x^7}{7} + \dots \quad |x| < 1$$

$$\begin{aligned} \cos^{-1} x &= \frac{\pi}{2} - \sin^{-1} x \\ &= \frac{\pi}{2} - \left(x + \frac{1}{2} \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \frac{x^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{x^7}{7} + \dots \right) \quad |x| < 1 \end{aligned}$$

$$\tan^{-1} x = \begin{cases} x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots & |x| < 1 \\ \pm \frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \dots & \begin{cases} + \text{if } x \geq 1 \\ - \text{if } x \leq -1 \end{cases} \end{cases}$$

$$\begin{aligned} \sec^{-1} x &= \cos^{-1} \left(\frac{1}{x} \right) \\ &= \frac{\pi}{2} - \left(\frac{1}{x} + \frac{1}{2 \cdot 3x^3} + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5x^5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7x^7} + \dots \right) \quad |x| > 1 \end{aligned}$$

$$\begin{aligned} \csc^{-1} x &= \sin^{-1}(1/x) \\ &= \frac{1}{x} + \frac{1}{2 \cdot 3x^3} + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5x^5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7x^7} + \dots \quad |x| > 1 \end{aligned}$$

$$\begin{aligned} \cot^{-1} x &= \frac{\pi}{2} - \tan^{-1} x \\ &= \begin{cases} \frac{\pi}{2} - \left(x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \right) & |x| < 1 \\ p\pi + \frac{1}{x} - \frac{1}{3x^3} + \frac{1}{5x^5} + \dots & \begin{cases} p = 0 \text{ if } x \geq 1 \\ p = 1 \text{ if } x \leq -1 \end{cases} \end{cases} \end{aligned}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\ln x = 2 \left[\frac{x-1}{x+1} + \frac{1}{3} \left(\frac{x-1}{x+1} \right)^3 + \frac{1}{5} \left(\frac{x-1}{x+1} \right)^5 + \dots \right]$$

$$= 2 \sum_{n=1}^{\infty} \frac{1}{2n-1} \left(\frac{x-1}{x+1} \right)^{2n-1} \quad (x > 0)$$

$$\ln x = \frac{x-1}{x} + \frac{1}{2} \left(\frac{x-1}{x} \right)^2 + \frac{1}{3} \left(\frac{x-1}{x} \right)^3 + \dots$$

$$= \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{x-1}{x} \right)^n \quad \left(x > \frac{1}{2} \right)$$

$$\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \dots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} (x-1)^n \quad (0 < x \leq 2)$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \dots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} x^n \quad (|x| < 1)$$

$$\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots \infty \quad (-1 \leq x < 1)$$

$$\log_e(1+x) - \log_e(1-x) =$$

$$\log_e \frac{1+x}{1-x} = 2 \left(x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \infty \right) \quad (-1 < x < 1)$$

$$\log_e \left(1 + \frac{1}{n} \right) = \log_e \frac{n+1}{n} = 2 \left[\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \dots \infty \right]$$

$$\log_e(1+x) + \log_e(1-x) = \log_e(1-x^2) = -2 \left(\frac{x^2}{2} + \frac{x^4}{4} + \dots \infty \right) \quad (-1 < x < 1)$$

$$\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots = \frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \dots$$

Important Results

$$(i) (a) \int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cos^n x}{\sin^n x + \cos^n x} dx$$

$$(b) \int_0^{\pi/2} \frac{\tan^n x}{1 + \tan^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{dx}{1 + \tan^n x}$$

$$(c) \int_0^{\pi/2} \frac{dx}{1 + \cot^n x} = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{1 + \cot^n x} dx$$

$$(d) \int_0^{\pi/2} \frac{\tan^n x}{\tan^n x + \cot^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{\tan^n x + \cot^n x} dx$$

$$(e) \int_0^{\pi/2} \frac{\sec^n x}{\sec^n x + \operatorname{cosec}^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\operatorname{cosec}^n x}{\sec^n x + \operatorname{cosec}^n x} dx \text{ where, } n \in R$$

$$(ii) \int_0^{\pi/2} \frac{a^{\sin x}}{a^{\sin x} + a^{\cos x}} dx = \int_0^{\pi/2} \frac{a^{\cos x}}{a^{\sin x} + a^{\cos x}} dx = \frac{\pi}{4}$$

$$(iii) (a) \int_0^{\pi/2} \log \sin x dx = \int_0^{\pi/2} \log \cos x dx = -\frac{\pi}{2} \log 2$$

$$(b) \int_0^{\pi/2} \log \tan x dx = \int_0^{\pi/2} \log \cot x dx = 0$$

$$(c) \int_0^{\pi/2} \log \sec x dx = \int_0^{\pi/2} \log \operatorname{cosec} x dx = \frac{\pi}{2} \log 2$$

$$(iv) (a) \int_0^{\infty} e^{-ax} \sin bx dx = \frac{b}{a^2 + b^2}$$

$$(b) \int_0^{\infty} e^{-ax} \cos bx dx = \frac{a}{a^2 + b^2}$$

$$(c) \int_0^{\infty} e^{-ax} x^n dx = \frac{n!}{a^{n+1}}$$

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$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln \left(x + \sqrt{x^2 - a^2} \right) + C$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln \left(x + \sqrt{x^2 + a^2} \right) + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left(\frac{x - a}{x + a} \right) + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left(\frac{a + x}{a - x} \right) + C$$

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) + C$$

$$\int \sqrt{a^2 + x^2} dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \sinh^{-1} \left(\frac{x}{a} \right) + C$$

$$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \cosh^{-1} \left(\frac{x}{a} \right) + C$$

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