

Mirrors Prisms Lens Slabs - Optics by Prof. Subhashish Chattopadhyay SKMClasses Bangalore Useful for IIT-JEE, CBSE, I.Sc. PU-II, Boards, CET, CEE, PET, IGCSE IB AP-Physics and other exams



## Spoon Feeding Mirrors Prisms Lens Slabs - Optics



Simplified Knowledge Management Classes Bangalore

My name is [Subhashish Chattopadhyay](#). I have been teaching for IIT-JEE, Various International Exams ( such as IMO [ International Mathematics Olympiad ], IPhO [ International Physics Olympiad ], IChO [ International Chemistry Olympiad ] ), IGCSE ( IB ), CBSE, I.Sc, Indian State Board exams such as WB-Board, Karnataka PU-II etc since 1989. As I write this book in 2016, it is my 27 th year of teaching. I was a Visiting Professor to BARC Mankhurd, Chembur, Mumbai, Homi Bhabha Centre for Science Education ( HBCSE ) Physics Olympics camp BARC Campus.

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I am Life Member of ...

- [IAPT \( Indian Association of Physics Teachers \)](#)
- [IPA \( Indian Physics Association \)](#)
- [AMTI \( Association of Mathematics Teachers of India \)](#)
- [National Human Rights Association](#)
- [Men's Rights Movement \( India and International \)](#)
- [MGTOW Movement \( India and International \)](#)

And also of

[IACT \( Indian Association of Chemistry Teachers \)](#)



The selection for National Camp ( for Official Science Olympiads - Physics, Chemistry, Biology, Astronomy ) happens in the following steps ....

1 ) NSEP ( National Standard Exam in Physics ) and NSEC ( National Standard Exam in Chemistry ) held around 24 rth November. Approx 35,000 students appear for these exams every year. The exam fees is Rs 100 each. Since 1998 the IIT JEE toppers have been topping these exams and they get to know their rank / performance ahead of others.

2 ) INPhO ( Indian National Physics Olympiad ) and INChO ( Indian National Chemistry Olympiad ). Around 300 students in each subject are allowed to take these exams. Students coming from outside cities are paid fair from the Govt of India.

3 ) The Top 35 students of each subject are invited at HBCSE ( Homi Bhabha Center for Science Education ) Mankhurd, near Chembur, BARC, Mumbai. After a 2-3 weeks camp the top 5 are selected to represent India. The flight tickets and many other expenses are taken care by Govt of India.

Since last 50 years there has been no dearth of “Good Books“. Those who are interested in studies have been always doing well. This e-Book does not intend to replace any standard text book. These topics are very old and already standardized.

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### **There are 3 kinds of Text Books**

- The thin Books - Good students who want more details are not happy with these. Average students who need more examples are not happy with these. Most students who want to “Cram” quickly and pass somehow find the thin books “good” as they have to read less !!

- The Thick Books - Most students do not like these, as they want to read as less as possible. Average students are “busy” with many other things and have no time to read all these.

- The Average sized Books - Good students do not get all details in any one book. Most bad students do not want to read books of “this much thickness” also !!

**We know there can be no shoe that’s fits in all.**

Printed books are not e-Books! Can’t be downloaded and kept in hard-disc for reading “later” .....

So if you read this book later, you will get all kinds of examples in a single place. This becomes a very good “Reference Material”. I sincerely wish that all find this “very useful”.

Students who do not practice lots of problems, do not do well. The rules of “doing well” had never changed .... Will never change !

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After 2016 CBSE Mathematics exam, lots of students complained that the paper was tough!

Updated 8:47 am Mar 22, 2016

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## CBSE assures remedial measures for tricky and tough Class XII Math paper

Posted on: 12:17 PM IST Mar 17, 2016 | Updated on: 12:20 pm, Mar 17, 2016 IST

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14 More+

After several students claimed that the Central Board of Secondary Education (CBSE) Class XII board Mathematics examination paper was 'tricky' and tough, the board has issued a clarification on remedial measures which are likely to be taken before evaluation.

The CBSE says that feedback received from various stakeholders like students, subject teachers and examiners will be put before the committee of subject experts.

close

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On 21 st May 2016 the CBSE standard 12 result was declared. I loved the headline

INDIATODAY.IN NEW DELHI, MAY 21, 2016 | UPDATED 16:40 IST

## CBSE Class 12 Results out: No leniency in Maths paper, high paper standard to be maintained in future

The CBSE Class 12 Mathematics board exam on March 14 reduced many students to tears as they found the paper quite lengthy and tough and many couldn't finish it on time. The results show an overall lowering of marks received in the Maths paper.



### RELATED STORIES

- CBSE Board result 2016 declared! Thiruvananthapuram obtains the highest part percentage, check how your region scored
- Meet CBSE topper Sukriti Gupta: Check her percentage here!
- CBSE Class 12 Boards 2016: Results announced ahead of time!
- CBSE results declared at [www.cbse.nic.in](http://www.cbse.nic.in): Steps to check online
- Exclusive! CBSE declares Class 12 Results at [www.cbseresults.nic.in](http://www.cbseresults.nic.in) and [cbse.nic.in](http://cbse.nic.in)

The CBSE (Central Board of Secondary Education) Class 12 Board exam results have been announced today, i.e on May 21, around 10:30 am ahead of time. Students may check their scores at the official website, [www.cbseresults.nic.in](http://www.cbseresults.nic.in). (Read: **CBSE Class 12 Boards 2016: Results announced ahead of time! Check your score at [cbseresults.nic.in](http://cbseresults.nic.in)**)

In 2015 also the same complain was there by many students

The screenshot shows a news article on the Zee News website. The header includes the Zee News logo, language options (Hindi, Marathi, Bangla), and mobile app icons (Apple, Android, Facebook). The navigation bar lists categories like India, States, World, S Asia, Biz, Sports, Cricket, Sci-Tech, Showbiz, Health, Blog, and Exclusive. The article title is "CBSE Class 12 exam: Issue of tough maths paper raised in Parliament". The sub-headline reads: "A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue 'seriously'." The article is dated "Last Updated: Thursday, March 19, 2015 - 14:41". It has 2547 shares (Facebook, Twitter, G+), 16 comments, and 33 comments. A "Follow @ZeeNews" button is also visible. The main text of the article is: "New Delhi: A senior Congress member on Thursday raised the issue of the tough **mathematics** question paper in the ongoing **CBSE** board examinations and asked the government to consider the issue 'seriously'."

So we see that by raising frivolous requests, even upto parliament, actually does not help. Many times requests from several quarters have been put to CBSE, or Parliament etc for easy Math Paper. These kinds of requests actually can-not be entertained, never will be.

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In March 2016, students of Karnataka PU-II also complained the same, regarding standard 12 ( PU-II Mathematics Exam ). Even though the Math Paper was identical to previous year, most students had not even solved the 2015 Question Paper.

Friday, March 25, 2016 - 13:28

The **NEWS** Minute

HOME NEWS ANDHRA KARNATAKA KERALA TAMIL NADU TELANGANA CULTURE MEDIA BLOG

Exams

## Online petition for lenient evaluation of K'taka II PU math paper gets over 8000 supporters

The campaign, which was launched on Monday, has garnered over 8000 supporters

TNM Staff | Wednesday, March 16, 2016 - 09:32

[Follow @thenewsminute](#)

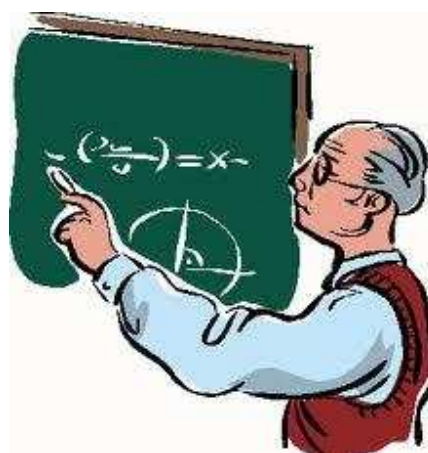
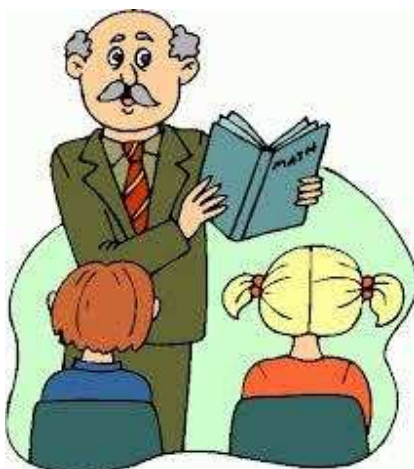
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Following a "very tough" math paper that left many II PU students in tears, Saket Ravindran a student launched an online campaign demanding lenient evaluation.

These complains are not new. In fact since last 40 years, ( since my childhood ), I always see this; every year the same setback, same complain!

In this e-Book I am trying to solve this problem. Those students who practice can learn.

**No one can help those who are not studying, or practicing.**



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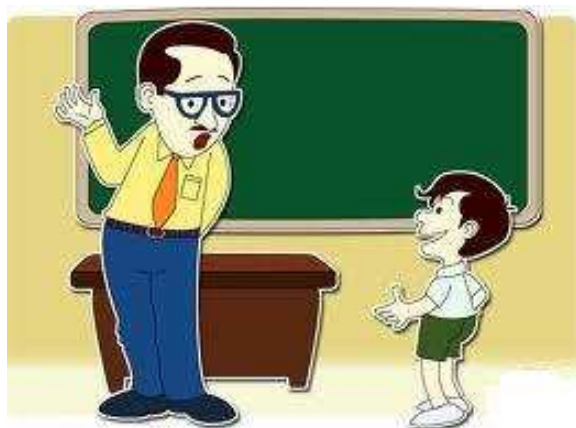
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Twitter - <https://twitter.com/ZookeeperPhy>

Facebook - <https://www.facebook.com/IIT.JEE.by.Prof.Subhashish/>

Blog - <http://skmclasses.kinja.com>



**A very polite request :**

I wish these e-Books are read only by Boys and Men. Girls and Women, better read something else; learn from somewhere else.

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### Preface

We all know that in the species “Homo Sapiens “, males are bigger than females. The reasons are explained in standard 10, or 11 ( high school ) Biology texts. **This shapes or size, influences all of our culture.** Before we recall / understand the reasons once again, let us see some random examples of the influence

#### Random - 1

If there is a Road rage, then who all fight ? ( generally ? ). Imagine two cars driven by adult drivers. Each car has a woman of similar age as that of the Man. The cars “ touch “ or “ some issue happens”. Who all comes out and fights ? Who all are most probable to drive the cars ?



( Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win )

#### Random - 2

Heavy metal music artists are all Men. Metallica, Black Sabbath, Motley Crue, Megadeth, Motorhead, AC/DC, Deep Purple, Slayer, Guns & Roses, Led Zeppelin, Aerosmith .... [the list can be in thousands](#). All these are grown-up Boys, known as Men.



( **Men strive for perfection. Men are eager to excel. Men work hard. Men want to win.**  )



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Random - 3

Apart from Marie Curie, only one more woman got Nobel Prize in Physics. ( Maria Goeppert Mayer - 1963 ). So, ... almost all are men.



( **Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.** )

Random - 4

The best Tabla Players are all Men.



( **Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.** )

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Random - 5

History is all about, [which all Kings ruled](#). Kings, their men, and Soldiers went for wars. [History is all about wars, fights, and killings by men](#). Who won, and who controlled !



**Boys start fighting from school days. Girls do not fight like this**



( [Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.](#) )

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Random - 6

The highest award in Mathematics, the “ Fields Medal “ is around since decades. Till date only one woman could get that. ( Maryam Mirzakhani - 2014 ). So, ... almost all are men.



( Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women. )

Random - 7

Actor is a gender neutral word. Could the movie like “ Top Gun “ be made with Female actors ? The best pilots, astronauts, Fighters are all Men.



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Random - 8

In my childhood had seen a movie named “ The Tower in Inferno “. In the movie when the tall tower is in fire, women were being saved first, as only one lift was working....



Many decades later another movie is made. A box office hit. “ The Titanic “. In this also .... As the ship is sinking women are being saved. Men are disposable. Men may get their turn later... ( never ) !!



Movies are not training programs. Movies do not teach people what to do, or not to do. Movies only reflect the prevalent culture. Men are disposable; is the culture in the society. Knowingly, unknowingly, the culture is depicted in Movies, Theaters, Stories, Poems, Rituals, etc. I or you can't write a story, or make a movie in which after a minor car accident the Male passengers keep seating in the back seat, while the both the women drivers come out of the car and start fighting very bitterly on the road. There has been no story in this world, or no movie made, where after an accident or calamity, Men are being helped for safety first, and women are told to wait.



Random - 9

Artists generally follow the prevalent culture of the Society. In paintings, sculptures, stories, poems, movies, cartoon, Caricatures, knowingly / unknowingly, “ **the prevalent Reality** “ is depicted. **The opposite will not go well with people.** If deliberately “ the opposite “ is shown then it may only become a special art, considered as a special mockery.

पत्नी (सल्लू से): मुझे नई साड़ी ला दो प्लीज।  
 सल्लू : पर तुम्हारी दो-दो अलमारियां साईं डियों से ही तो भरी है।  
 पत्नी - वह सारी तो पूरे मोहल्ले वालों ने देख रखी है।  
 सल्लू - तो साड़ी लेने के बजाए मोहल्ला बदल लेते हैं।



Random - 10

Men go to “girl / woman’s house” to marry / win, and bring her to his home. That is a sort of winning her. When a boy gets a “ Girl-Friend “, generally he and his friends consider that as an achievement. The boy who “ got / won “ a girl-friend feels proud. His male friends feel, jealous, competitive and envious. Millions of stories have been written on these themes. Lakhs of movies show this. Boys / Men go for “ bike race “, or say “ Car Race “, where the winner “ gets “ the most beautiful girl of the college.



( Men want to excel. Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win. )

Prithviraj Chauhan ‘ **went** ` to “ **pickup** “ or “ **abduct** “ or “ **win** “ or “ **bring** “ his love. There was a Hindi movie ( hit ) song ... “ **Pasand ho jaye, to ghar se utha laye** “. It is not other way round. Girls do not go to Boy’s house or man’s house to marry. Nor the girls go in a gang to “ pick-up “ the boy / man and bring him to their home / place / den.

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Random - 11

We have the word "ice cold". While, when it snows heavily, the cleaning of the roads is done by Men. Ice avalanche is cleared by Guns, by Men.



Can women do this please ?



Random - 12



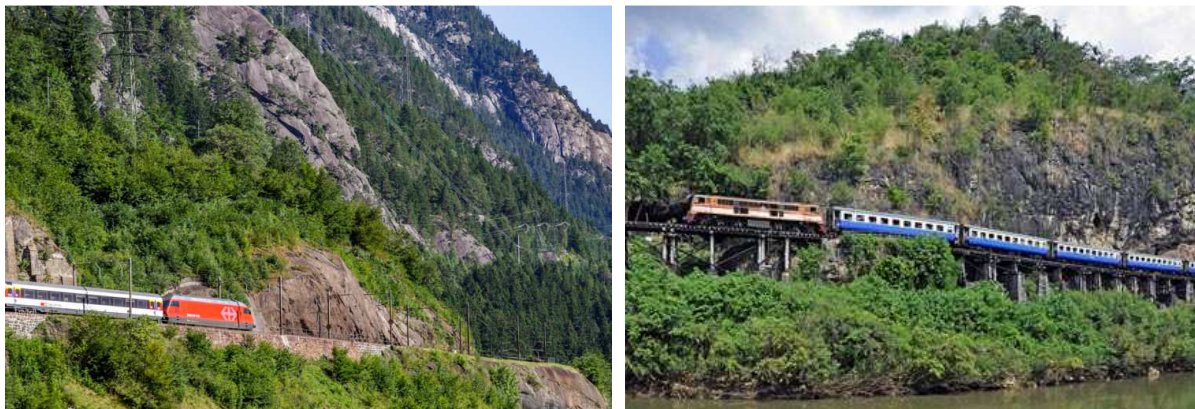
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There are many remote mines in this world which are connected by rails through Hilly regions. These railroads move through steep ups and downs. Optimum speed of the train has to be maintained so that the brakes do not burn out, but the next climbing can be done. Sudden braking is not possible as the load of the wagons will derail the train, and will mean huge loss and deaths. The **Drivers are Men** who risk their lives in every journey.

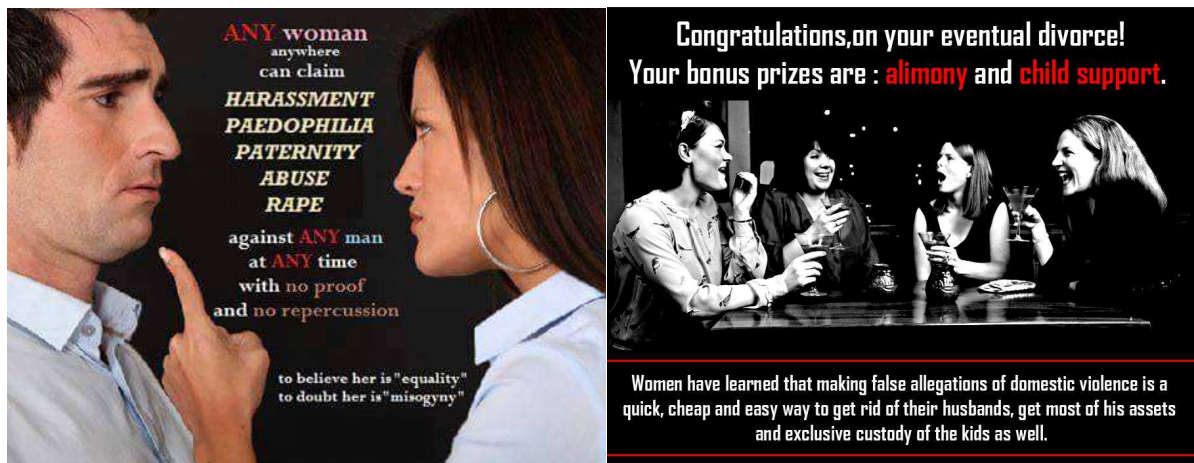


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Random - 13

Almost all of us are very biased. Instead of I asking some questions, see the following images



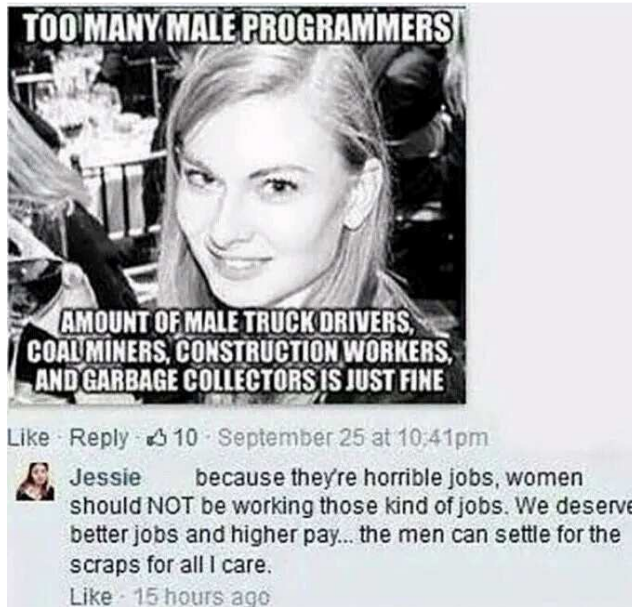




**All women are born  
evil. Some just realize  
their potential later in  
life than others.**

QUOTEHD.COM

Chad A. Gamble



Proof that girls are evil

First we state that girls require time and money.

$$\text{GIRLS} = \text{TIME} \times \text{MONEY}$$

And as we all know "time is money"

$$\text{TIME} = \text{MONEY}$$

Therefore:

$$\text{GIRLS} = \text{MONEY} \times \text{MONEY} = (\text{MONEY})^2$$

And because "money is the root of all evil":

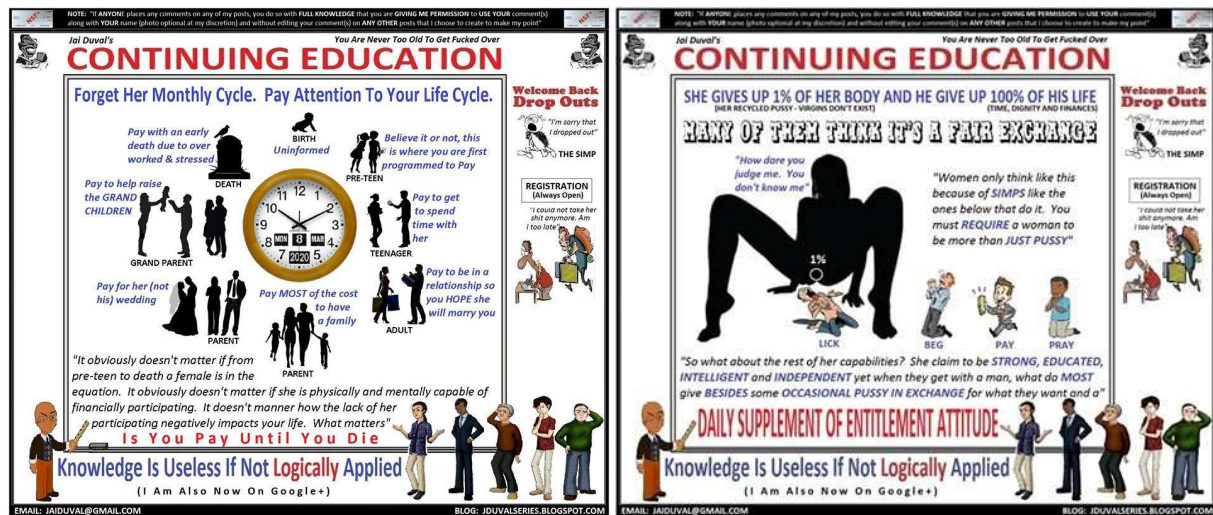
$$\text{MONEY} = \sqrt{\text{EVIL}}$$

Therefore:

$$\text{GIRLS} = (\sqrt{\text{EVIL}})^2$$

We are forced to conclude that:

$$\text{GIRLS} = \text{EVIL}$$





Random - 14

Rich people; often are very hard working. Successful business men, establish their business ( empire ), amass lot of wealth, with lot of difficulty. Lots of sacrifice, lots of hard work, gets into this. **Rich people's wives had no contribution in this wealth creation.** Women are smart, and successful upto the extent to choose the right/rich man to marry. So generally what happens in case of Divorces ? Search the net on " most costly divorces " and you will know. The women;( who had no contribution at all, in setting up the business / empire ), often gets in Billions, or several Millions in divorce settlements.

Number 1

### Rupert & Anna Murdoch -- \$1.7 billion

One of the richest men in the world, **Rupert Murdoch** developed his worldwide media empire when he inherited his father's Australian newspaper in 1952. He married Anna Murdoch in the '60s and they remained together for 32 years, springing off three children.

They split amicably in 1998 but soon Rupert forced Anna off the board of News Corp and the gloves came off. The divorce was finalized in June 1999 when Rupert agreed to let his ex-wife leave with \$1.7 billion worth of his assets, \$110 million of it in cash. Seventeen days later, Rupert married Wendi Deng, one of his employees.



### Ted Danson & Casey Coates -- \$30 million

Ted Danson's claim to fame is undoubtedly his decade-long stint as Sam Malone on NBC's celebrated sitcom Cheers. While he did other TV shows and movies, he will always be known as the bartender of that place where everybody knows your name. He met his future first bride Casey, a designer, in 1976 while doing Erhard Seminars Training.

Ten years his senior, she suffered a paralyzing stroke while giving birth to their first child in 1979. In order to nurse her back to health, Danson took a break from acting for six months. But after two children and 15 years of marriage, the infatuation fell to pieces. Danson had started seeing Whoopi Goldberg while filming the comedy, Made in America and this precipitated the 1992 divorce. Casey got \$30 million for her trouble.

See <https://zookeepersblog.wordpress.com/misandry-and-men-issues-a-short-summary-at-single-place/>

See <http://skmclasses.kinja.com/save-the-male-1761788732>

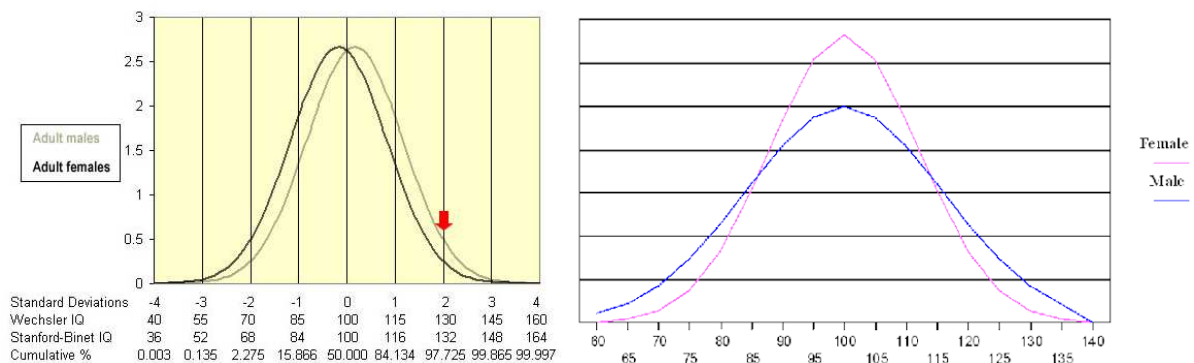
**It was Boys and Men, who brought the girls / women home. The Laws are biased, completely favoring women. The men are paying for their own mistakes.**

See <https://zookeepersblog.wordpress.com/biased-laws/>

( Man brings the Woman home. When she leaves, takes away her share of big fortune! )

Random - 15

A standardized test of Intelligence will never be possible. It never happened before, nor ever will happen in future. No IQ test results will be acceptable by all. **In the net there are thousands of charts which show that the intelligence scores of girls / women are lesser.** Debates of Trillion words, does not improve performance of Girls.



I am not wasting a single second debating or discussing with anyone, on this. **I am simply accepting ALL the results.** IQ is only one of the variables which is required for success in life. Thousands of books have been written on “ Networking Skills “, EQ ( Emotional Quotient ), Drive, Dedication, Focus, “ Tenacity towards the end goal “ ... etc. **In each criteria, and in all together, women ( in general ) do far worse than men.** Bangalore is known as “ ..... capital of India “. [ Fill in the blanks ]. The blanks are generally filled as “ Software Capital “, “ IT Capital “, “ Startup Capital “, etc. I am member in several startup eco-systems / groups.

**I have attended hundreds of meetings, regarding “ technology startups “, or “ idea startups “.** These meetings have very few women. ( Generally in most meetings there are no women at all ! ). Starting up new companies are all “ **Men’s Game** “ / “ **Men’s business** “. Only in Divorce settlements women will take their goodies, due to Biased laws. **There is no dedication, towards wealth creation, by women.** Women want easy money.

Random - 16

Many men, as fathers, very unfortunately treat their daughters as “ Princess “. Every “ non-performing “ woman / wife was “ princess daughter “ of some loving father. Pampering the girls, in name of “ equal opportunity “, or “ women empowerment “, have led to nothing.

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See <http://skmclasses.kinja.com/progressively-daughters-become-monsters-1764484338>

See <http://skmclasses.kinja.com/vivacious-vixens-1764483974>

There can be thousands of more such random examples, where “ Bigger Shape / size “ of males have influenced our culture, our Society. Let us recall the reasons, that we already learned in standard 10 - 11, Biology text Books. In humans, women have a long gestation period, and also spends many years ( almost a decade ) to grow, nourish, and stabilize the child. ( Million years of habit ) Due to survival instinct Males want to inseminate. Boys and Men fight for the “ facility ( of womb + care ) “ the girl / woman may provide. Bigger size for males, has a winning advantage. Whoever wins, gets the “ woman / womb / facility “. The male who is of “ Bigger Size “, has an advantage to win.... Leading to Natural selection over millions of years. In general “ Bigger Males “; the “ fighting instinct “ in men; have led to wars, and solving tough problems ( Mathematics, Physics, Technology, startups of new businesses, Wealth creation, Unreasonable attempts to make things [ such as planes ], Hard work .... )

So let us see the IIT-JEE results of girls. Statistics of several years show that there are around 17, ( or less than 20 ) girls in top 1000 ranks, at all India level. Some people will yet not understand the performance, till it is said that ... year after year we have around 980 boys in top 1000 ranks. Generally we see only 4 to 5 girls in top 500. **In last 50 years not once any girl topped in IIT-JEE advanced.** Forget about Single digit ranks, double digit ranks by girls have been extremely rare. It is all about “ good boys “, “ hard working “, “ focused “, “Bel-esprit “ boys.

In 2015, Only 2.6% of total candidates who qualified are girls ( upto around 12,000 rank ). while 20% of the Boys, amongst all candidates qualified. The Total number of students who appeared for the exam were around 1.4 million for IIT-JEE main. Subsequently 1.2 lakh ( around 120 thousands ) appeared for IIT-JEE advanced.

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IIT-JEE results and analysis, of many years is given at

<https://zookeepersblog.wordpress.com/iit-jee-iseet-main-and-advanced-results/>

**In Bangalore it is rare to see a girl with rank better than 1000 in IIT-JEE advanced. We hardly see 6-7 boys with rank better than 1000. Hardly 2-3 boys get a rank better than 500.**

See <http://skmclasses.weebly.com/everybody-knows-so-you-should-also-know.html>

### So what " some women " are doing ?

Thousands of people are exposing the heinous crimes that Motherly Women are doing, or Female Teachers are committing. See <https://www.facebook.com/WomenCriminals/>

Some Random Examples must be known by all

It is extremely unfortunate that the " woman empowerment " has created. This is the kind of society and women we have now. I and many other sensible Men hate such women. Be away from such women, be aware of reality.



Mother Admits On Facebook to Sleeping with 15 Yr Old Son, They Have a Baby Together - Alwaysturntup

Sometimes it hard to believe w From Alwaysturntup

ALWAYZTURNUP.ME



'Sex with my son is incredible - we're in love and we want a baby'

Ben Ford, who ditched his wife when he met his mother Kim West after 30 years, claims what the couple are doing 'isn't incest'

MIRROR.CO.UK

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Woman sent to jail for the rest of her life after raping her four grandchildren is described as the 'most evil person' the judge has ever seen

Edwina Louis rape...

[See More](#)



### Former Shelbyville ISD teacher who had sex with underage student gets 3 years in prison

After a two day break over the weekend, A Shelby County jury was back in the courtroom looking to conclude the trial of a former Shelbyville ISD teacher who had...

KLTV.COM | BY CALEB BEAMES

<http://www.thenativecanadian.com/.../eastern-ontario-teacher-...>



### Woman sent to jail for raping her four grandchildren

A Ohio grandmother has been sentenced to four consecutive life terms after being found guilty of the rape of her own grandchildren. Edwina Louis, 53, will spend the rest of her life behind bars.

DAILYMAIL.CO.UK



### The N.C. Chronicles.: Eastern Ontario teacher charged with 36 sexual offences

anti feminism, Child abuse, children's rights, Feminist hypocrisy,

THENATIVECANADIAN.COM | BY:BLACKWOLF



### Hyd woman kills newborn boy as she wanted daughter - Times of India

Having failed to bear a daughter for the third time, a shopkeeper's wife slit the throat of her 24day-old son with a shaving blade and left him to die in a street on Tuesday night.Pumima's first child was a stillborn boy, followed by another boy born five years ago.

TIMESOFINDIA.INDIATIMES.COM


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Montgomery's son, Alan Vonn Webb, took the stand and was a key witness in her conviction.

"I want to see her placed somewhere she can never do that to children

....

See More




**Woman sentenced to 40 years in prison for raping her children**

A Murfreesboro mother found guilty of raping her own children learned her fate on Wednesday.

WAFF.COM | BY DENNIS FERRIER

gentler sex? Violence against men.'s photo.



**Up to 64,000 women in UK are child-sex offenders**

~ The Guardian

**Women, the gentler sex? Violence against men.**

April 8 at 1:38am

Like Page

In fact, the past decade has seen a dramatic increase in the number of incidents of women raping and sexually assaulting boys and men. On May 2014, Jezebel repo...

In Facebook, and internet + whatsapp etc we have unending number of posts describing frustration of men / husbands on naughty unreasonable women. **Most women are very illogical, Punic, perfidious, treacherous, naughty, gamey bitches.**

We also see zillions of Jokes which basically describe how unreasonable women / girls are. How stupid they are, making life of Boys / Men / Husband a hell.

While each of these girls was someones daughter. Millions of foolish Dads are into Fathers rights movement, who want their daughter back for pampering.

**Most girls are being cockered, coddled, cosseted, mollycoddled, featherbedded, spoilt into brats.**

**Foolish fathers are breeding Monsters who are filing false rape cases. Enacting Biased Laws. Filing False domestic violence cases. Filing false sexual assault cases. Asking for alimony, and taking custody of the Daughter, not allowing the " monster " to meet dad. The cycle goes on and on and on.**

**Foolish men keep pampering future demons who make other Men's life a hell. ( Now read this again from beginning ). Every day we see the same posts of frustration.**

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**When I grow up  
I will beat my  
husband  
No one will care  
No one will stop me**



October is Domestic Violence Awareness Month

**53% of Domestic Violence  
Victims are Men  
Stop the Silence  
Stop the Violence**



**FUNNY.**

**NOT FUNNY.**

**DOUBLE STANDARDS HURT  
EVERYONE.**

<https://nicewomen.wordpress.com/>

Each women as described below was someone's Pampered Princess ...

End violence against women ...



**North Carolina Grandma Eats Her Daughter's New Born Baby After Smoking Bath Salts**

Henderson, North Carolina—A North Carolina grandmother of 4 and recovering drug addict, is now in custody after she allegedly ate her daughter's newborn baby....

AZ-365.TOP



**28-Year-Old Texas Teacher Accused of Sending Nude Picture to 14-Year-Old Former Student**

BREITBART.COM

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<http://latest.com/.../attractive-girl-gang-lured-men-alleywa.../>



### Attractive Girl Gang Lured Men Into Alleyways Where Female Body Builder Would Attack Them

A Mexican street gang made up entirely of women has been accused of using their feminine wiles to lure men into alleyways and then beating them up and...

LATEST.COM

<http://www.wfmj.com/.../youngstown-woman-convicted-of-rape-...>



### Youngstown woman convicted of raping a 1 year old is back in jail

A Youngstown woman who went to prison for raping a 1-year-old boy fifteen years ago is in trouble with the law again.

WFMJ.COM

End violence against women



### Women are raping boys and young men

Rape advocacy has been maligned and twisted into a political agenda controlled by radicalized activists. Tim Patten takes a razor keen and well supported look into the manufactured rape culture and...

AVOICEFORMEN.COM | BY TIM PATTEN

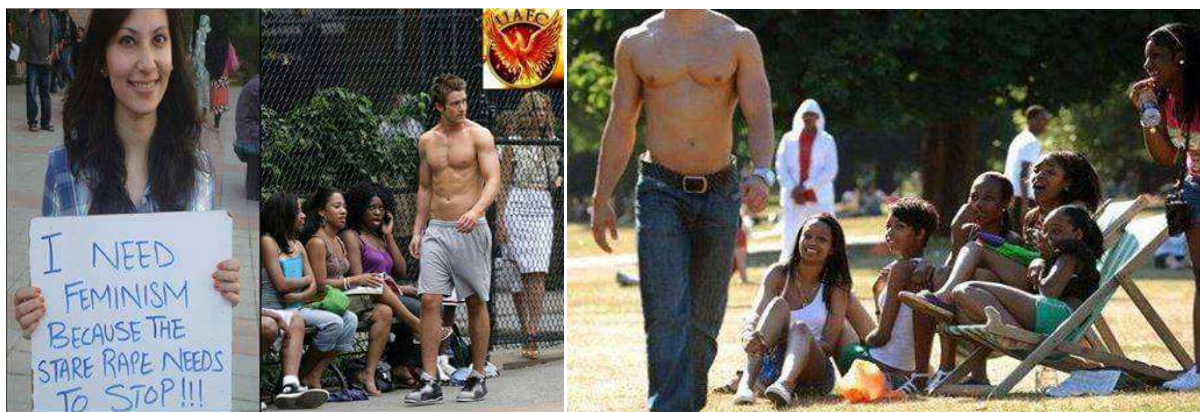


### Bronx Woman Convicted of Poisoning and Drowning Her Children

Lisette Barnenga researched methods on the Internet before she killed her son and daughter in 2012.

NYTIMES.COM | BY MARC SANTORA

Monster women have very easy and cozy life. Easy to demand anything and get law in favor !



**If the lawmakers submit to these strange demands of say ... " Stare Rape ! "; then we can easily see what kind of havoc that will create.**

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**55%**  
of Biological Parents  
Who Kill Their Children are  
**Mothers**

Homicidal Encounters: A Study of Homicide in Australia 1989-1999.  
Australian Institute of Criminology (Monash, 2000, p. 142, Fig. 74).

**ManKind**  
Equality Network



**Woman charged with killing baby also had previous infant die**

Woman charged with killing baby also had previous infant die

ABC7.COM | BY ROB MCMILLAN



**Female Sex Predators: A Crime Epidemic** shared a link.

Yesterday at 12:40am · 🌐



**WTVB.com | Woman pleads guilty to having sex with a dog**

PITTSBORO, Miss. (WTVB) -- A Calhoun County woman has pleaded guilty to charges she had unnatural intercourse with a dog. Sheriff Greg Poll

WTVB.COM

👍 Like    💬 Comment    ➦ Share

👍 🤔 🤢 Mhra Leander Pallat, Eric Antonio Alvarado and 31 others    Top Comments ▾



**Oklahoma Teacher Receives 15-Year Prison Sentence For Sex With 15-Year-Old Boy**

A former Oklahoma middle school teacher has pleaded guilty to 6 counts of rape, child enticement...

THREEPERCENTERNATION.COM



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A Russian-born newlywed slowly butchered her German husband — feeding strips of his flesh to their dog until he took his last breath. Svetlana Batukova, 46, was...

See More



**She killed her husband and then fed him to her dog: police**

A Russian-born newlywed butchered her German hubby — and fed strips of his flesh to her pooch, authorities said. Svetlana Batukova offered Horst Hans Henkels at their...

NYPOST.COM

**Daily Mail**  
January 15, 2015

Mother charged with rape and sodomy of her son's 12-year-old friend



**Mom, 30, 'raped and had oral sex with her son's 12-year-old friend'**

Nicole Marie Smith, 30, (pictured) of St Charles County, Missouri, has been jailed after she allegedly targeted the 12-year-old boy at her home.

DAILYMAIL

April 4 at 4:48am



**Female prison officers commit 90pc of sex assaults on male teens in US juvenile detention centres**

Lawsuit in Idaho highlights the prevalence of sexual victimization of juvenile offenders.

IBTIMES.CO.UK | BY NICOLE ROJAS

This mother filmed herself raping her own son and then sold it to a man for \$300. The courts just decide her fate. When you see what she got, you're going to be outraged.



**Mother Who Filmed Herself Raping Her 1-Year-Old Son Receives Shocking Sentence**

"...then used the money to buy herself a laptop..."

AMERICANNEWS.COM

**In several countries or rather in several regions of the world, family system has collapsed, due to bad nature and naughty acts of women.** Particularly in Britain, and America, almost 50% people are alone, lonely, separated, divorced or failed marriages. In 2013, 48% children were born out of wedlock. **It was projected that by 2016, more than 51% children will be born, to unmarried mothers.** In these developed countries " paternity fraud " by women, are close to 20%. You can see several articles in the net, and in wikipedia etc. This means 1 out of 5 children are calling a wrong man as dad. The lonely, alone " mothers " are frustrated. They see the children as burden. Love in the Society in general is lost, long time ago. The types of " Mothers " and " Women " we have now .....

This is the type of women we have in this world. These kind of women were also someones daughter



### Mother Stabs Her Baby 90 Times With Scissors After He Bit Her While Breastfeeding Him!

Eight-month-old Xiao Bao was discovered by his uncle in a pool of blood. He needed 100 stitches after the incident; he is now recovering in hospital. Reports say his...

MOMMABUZZ.COM





### Not All Feminist Theory is Equal

Christina Hoff Sommers  
Factual and Equity Feminist



"That is the corrosive paradox of gender feminism's misandrist stance: no group of women can wage war on men without at the same time denigrating the women who respect those men."

Andrea Dworkin  
Radical and Gender Feminist



"Under patriarchy, every woman's son is her potential betrayer and also the inevitable rapist or exploiter of another woman"

**ManKind**  
Equality Network

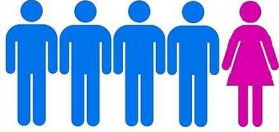
### WILLFUL BLINDNESS

Ignoring the Majority of Victims Because of Their Gender

I am a feminist because it bothers me that a woman gets killed by her male partner every single week, and somehow that doesn't qualify as a tools-down national crisis even though if a man got killed by a shark every week we'd probably arrange to have the ocean drained.

Annabel Crabb

79% of Homicide victims worldwide are MALE  
"Where is the tools-down national crisis?"



United Nations Office of Drugs and Alcohol. Global Study on Homicide (2013, p. 13).  
https://en.wikipedia.org/wiki/Willful\_blindness

**ManKind**  
Equality Network

### Not All Feminist Theory is Equal

Christina Hoff Sommers  
Equity Feminist



Christina H. Sommers  
@CH-Sommers  
Want to close wage gap? Step one: Change your major from feminist dance therapy to electrical engineering.

Valerie Solanis  
Radical and Gender Feminist



"To call a man an animal is to flatter him; he's a machine, a walking dildo"

n.b. Some Feminists

**ManKind**  
Equality Network



Muslim Woman Caught RAPING Her Own Son - Gives Disgusting Excuse to Judge | John Hawkins' Right Wing News

RIGHTWINGNEWS.COM

By now if you have assumed that Indian women are not doing any crime then please become friends with MRA Guri <https://www.facebook.com/profile.php?id=100004138754180>

He has dedicated his life to expose Indian Criminals



Delhi Woman Who Tried To Rape An Auto Driver, While Her Friend Filmed The Act, Has Been Arrested

Men are raped too!

MENSPX.COM | BY NIKITA MUKHERJEE




Muslim mother, 43, jailed for sex offences against girl, nine

Raheelah Dar, 43, from Middlesbrough, has been jailed for seven years for carrying out a string of sex offences against a nine-year-old girl.

DAILYMAIL.CO.UK



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## Mother who had been forced into an arranged marriage is jailed for filming herself having sex with her 14-year-old son and sending the clips to relatives in Pakistan

- Vile mother filmed having sex with her teenage son in sick porn video
- Clips sent to cousin in Pakistan who allegedly asked her to make film
- She also sent her relative indecent images of her three-year-old daughter

By **ALEX MATTHEWS FOR MAILONLINE**

PUBLISHED: 12:44 GMT, 1 August 2016 | UPDATED: 11:23 GMT, 2 August 2016



## Wife Stabs Husband And Runs Away After He Stops Her From Gambling

The husband said his wife had become a habitual gambler who was also addicted to liquor.

INDIATIMES.COM



## Teacher learns fate for 6 months of sex with boy

(CBS8) — SAN DIEGO (CNS) — A Crawford High School teacher and coach who carried on a six-month sexual relationship with a 15-year-old male student was sentenced Friday to a two-year prison term. Toni Nicole Sutton, 38, pleaded guilty...

WIND.COM



## Mom jailed for 40 years after body of daughter, 9, found in fridge

Amber Keyes, 37, was sentenced in the death of Ayahna Comb in Houston on Friday. Ayahna, who had cerebral palsy, had been in the fridge for six months...

DAILYMAIL.CO.UK






# HURT FEMINISM BY DOING NOTHING

- ✗ DON'T HELP WOMEN
- ✗ DON'T FIX THINGS FOR WOMEN
- ✗ DON'T SUPPORT WOMEN'S ISSUES
- ✗ DON'T COME TO WOMEN'S DEFENSE<sup>1</sup>
- ✗ DON'T SPEAK FOR WOMEN
- ✗ DON'T VALUE WOMEN'S FEELINGS
- ✗ DON'T PORTRAY WOMEN AS VICTIMS
- ✗ DON'T PROTECT WOMEN<sup>2</sup>


✓ WITHOUT WHITE KNIGHTS FEMINISM WOULD END TODAY

<sup>1</sup>Don't even nawalt ("Not All Women Are Like That")    <sup>2</sup>for example from criticism or insults

### How Society prioritize Men

High Priority	Rich women		They can get away with murder.
	Women		They get all the rights with no responsibility and Shelters for Homeless women.
	Rich Men		They get tax bail outs and short prison sentence.
	Girls		They get educational benefits but no violence against kids Act.
	Boys		They have some support but don't have any education that fits boys.
	Animals		They have animal rights and PETA.
	Prisoners		They get conjugal visits and 3 squares and a roof.
	Men		Paid slaves.
Low Priority	Poor Men		Nothing.

**Who pays the most Taxes?**  
**This is why MGTOW exist.**

 # MGTOW



Professor Subhashish Chattopadhyay



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### Preface for Physics

Professor H. C. Verma wrote amazing books in Physics. There are many other good books for IIT JEE and other exams. **Krishna's Guides**, Books by Professor **N. N. Ghosh**, Professor D. C. Pandey, GRB Publications Physics Guides etc are very good. For numericals the Irodov's books remain the King !

“Concepts of Physics” by Professor H C Verma have been available since 1991. ( and did not change or updated since ). Previous to that, past papers of IIT JEE, and other exams, were the source for preparation. I was in High School in 1980s. I had 6-7 Russian books apart from Irodov. All these were very good. Resnick and Halliday 's ( Walker and Krane came in subsequently ) book was also well known. There were too many “ uncles “ who used to advice that “ only Resnick and Halliday 's book was enough “ !

**Well I agreed and disagreed.** There were many IIT JEE questions which were ditto or verbatim picked-up from Resnick Halliday ! **But, something more was always needed.** Brilliant Tutorials, Agarwal Coaching etc, were famous those days. ( 1980s 90 s ). They were giving **several new questions**, which enabled more practice. **People slowly realized that “ every type “ of questions are NOT there in Resnick & Halliday, or say Irodov.**

Uncles saying “ only Resnick and Halliday 's book was enough “ ! were wrong. “Concepts of Physics” by Professor H C Verma sold so much because of very good step by step explanations, new solved examples, new exercises. Several gaps were filled-up.

The word Physics is derived from Latin **physica**, from Greek (ta) **phusika**, (the things) of nature, from neuter plural of phusikos.

**So, why am I writing “another book” in Physics ? ( The description of nature )**

I wish to answer this most important question, first !

There are many kind of Questions which are **not** covered in “ Concepts of Physics “ of Professor H. C. Verma. Also Irodov, in his books, **does not** explain or cover several kinds of Problems or Questions. The “ Coaching Institutes “ very rightly thrived on these gaps. Almost 100% students benefit more **with more examples**. **As Coaching Institutes discuss, cover and repeat several more examples in each chapter compared to School or Text books; explains the reason of their popularity.**

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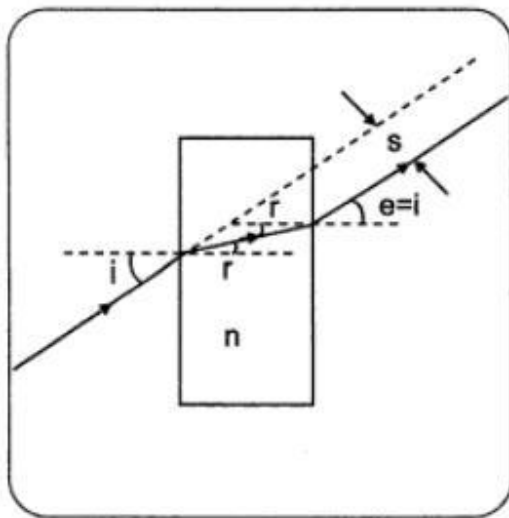


Let me list a few examples to explain all this.

Optics - 1 ) The expression for deviation of a ray passing through a slab

### ***Refraction through a transparent slab (lateral shift)***

Consider a transparent slab of thickness  $t$ , and refractive index  $n$ . A monochromatic beam of light falls on one side at an angle of incidence  $i$  as shown in Fig. Emergent ray will be parallel to incident ray, but there will be a lateral shift  $S$  of the incident ray. At the first interface,



$1 \sin i = n \sin r$  and at the second interface

$n \sin r = 1 \sin e$

where,  $r$  is the angle of refraction at the first interface and  $e$ , the angle of refraction at the second interface.  $\therefore e = i$

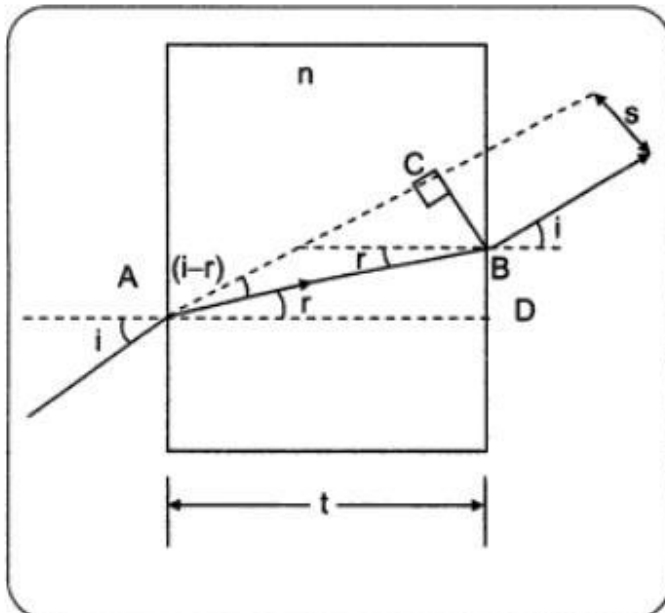
From Fig. lateral shift is calculated as follows:

$$AD = t; AB = \frac{AD}{\cos r} = \frac{t}{\cos r}$$

$$\text{Lateral shift } S = BC = AB \sin(i - r) = \frac{t \sin(i - r)}{\cos r}$$

$$\text{i.e., } S = \frac{t \sin(i - r)}{\cos r}$$

It may be noted that  $S_{\max} = t$  for  $i = 90^\circ$  (grazing incidence) and  $S_{\min} = 0$  for  $i = 0$  (normal incidence)



Special case:

(i) **small i**

$$t \frac{\sin(i - r)}{\cos r} = \frac{t[\sin i \cos r - \cos i \sin r]}{\cos r}$$

$$[r \text{ small} \Rightarrow \cos r \approx 1]; i \text{ small} \Rightarrow \cos i \approx 1]$$

$$\therefore S = t(\sin i - \sin r) = t \sin i \left[ 1 - \frac{\sin r}{\sin i} \right]$$

$$\Rightarrow S = t \sin i \left[ 1 - \frac{1}{n} \right] = t i \left( 1 - \frac{1}{n} \right) [i \text{ small} \Rightarrow \sin i = i]$$

$$\Rightarrow S = t i \frac{(n - 1)}{n}$$

(Note: use formula  $S = t \frac{\sin(i - r)}{\cos r}$  unless it is given that  $i = \text{small}$ )

(ii) When  $i$  is not small, it can be shown that

$$S = \frac{t \sin(i - r)}{\cos r} = t \sin i \left[ 1 - \frac{\cos i}{\sqrt{n^2 - \sin^2 i}} \right] \text{ or}$$

$$S = t \sin i \left[ 1 - \sqrt{\frac{1 - \sin^2 i}{n^2 - \sin^2 i}} \right]$$

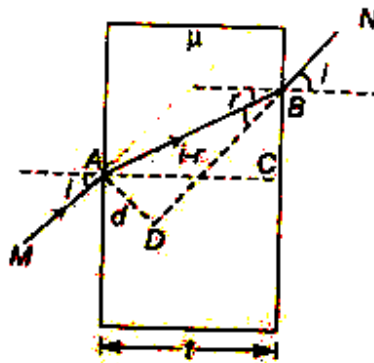


See again

### Lateral Shift

In the following figure, ray  $MA$  is parallel to ray  $BN$ . But the emergent ray is displaced laterally by a distance  $d$  which depends upon  $\mu$ ,  $t$  and  $i$  and its value is given by

$$d = t \left( 1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}} \right) \sin i.$$



From the figure,  $AB = \frac{AC}{\cos r} = \frac{t}{\cos r}$  (as,  $AC = t$ )

Since,

$$d = AB \sin(i - r)$$

$$= \frac{t}{\cos r} [\sin i \cos r - \cos i \sin r]$$

$$d = t [\sin i - \cos i \tan r]$$

Further,

$$\mu = \frac{\sin i}{\sin r} \text{ or } \sin r = \frac{\sin i}{\mu}$$

$\therefore$

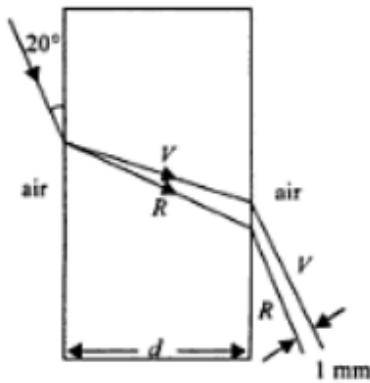
$$\tan r = \frac{\sin i}{\sqrt{\mu^2 - \sin^2 i}}$$

The expression for  $d$  now is

$$d = \left( \sqrt{1} - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}} \right) t \sin i$$

**Note** For small angles of incidence  $d = t \left( \frac{\mu - 1}{\mu} \right)$

A white light is incident at  $20^\circ$  on a material of silicate flint glass slab as shown.  $\mu_{\text{violet}} = 1.66$  and  $\mu_r = 1.6$ . For what value of  $d$  will the separation be 1 mm in red and violet rays.



- (a)  $\frac{5}{3}$  cm                      (b)  $\frac{10}{3}$  cm  
 (c) 5 cm                              (d)  $\frac{20}{3}$  cm

**Solution** (b)  $\sin r_1 = \frac{\sin 70}{1.66} = \frac{.9397}{1.66}$  or  $r_1 = 34^\circ 30'$

$$\sin r_2 = \frac{\sin 70}{1.6} = \frac{.9397}{1.6} \text{ or } r_2 = 36^\circ$$

Using  $y = \frac{t \sin(i-r)}{\cos r}$

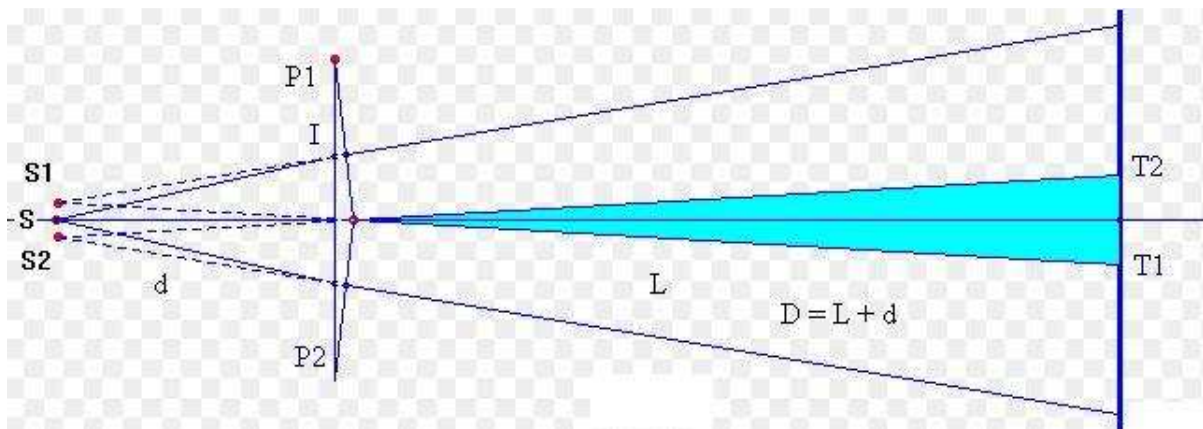
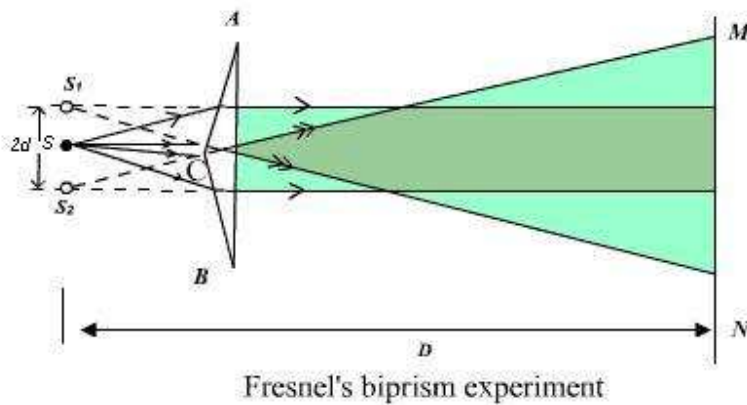
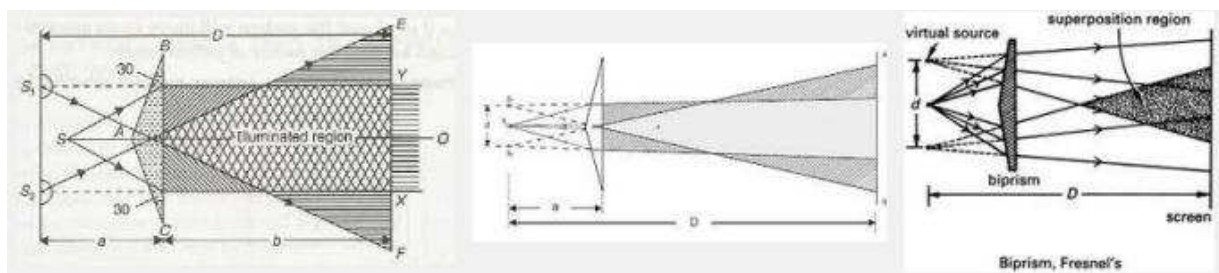
$$y_1 - y_2 = d \left[ \frac{\sin(i-r_1)}{\cos r_1} - \frac{\sin(i-r_2)}{\cos r_2} \right]$$

$$0.1 = d \left[ \frac{\sin 35^\circ 30'}{\cos 34^\circ 30'} - \frac{\sin 34^\circ}{\cos 36^\circ} \right]$$

$$\text{or } 0.1 = d \left[ \frac{0.5807}{0.8241} - \frac{0.5592}{0.8090} \right] = d[0.71 - 0.68]$$

$$\text{or } d = \frac{0.1}{0.03} = \frac{10}{3} \text{ cm}$$

### Optics - 2 ) Fresnel's Biprism





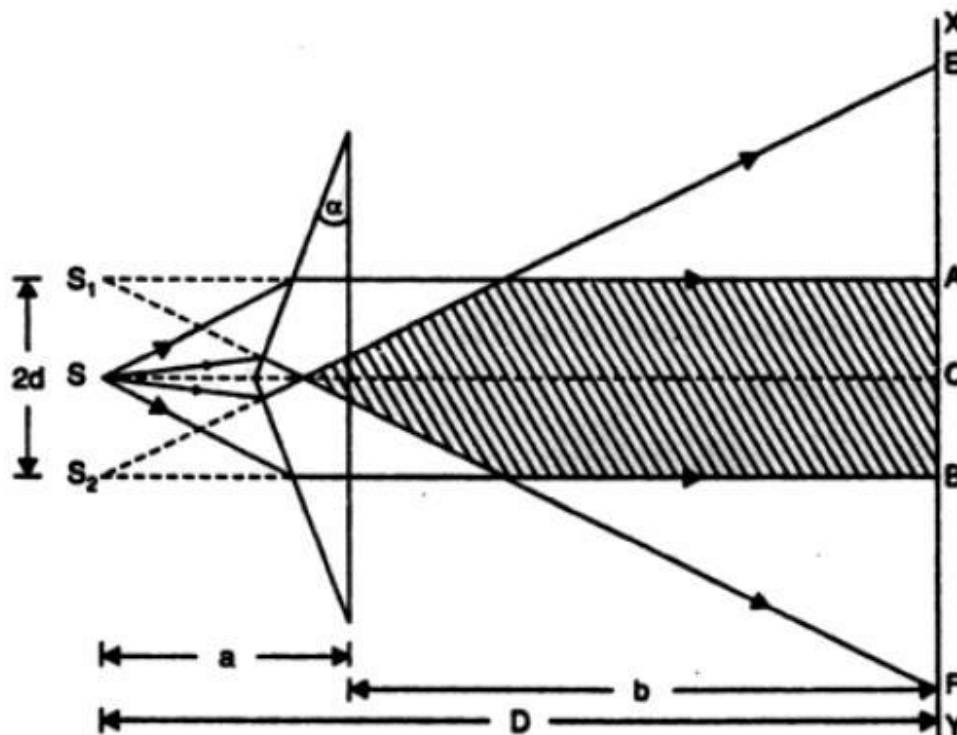
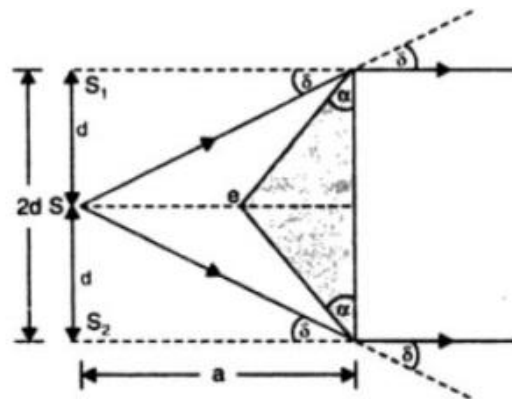
very small refracting angle  $\alpha$ , is given by  

$$\delta = (\mu - 1)\alpha,$$
 where  $\mu$  is the refractive index of the material of prism. Note that  $\alpha$  is in radian  
 It is clear from Fig. that

$$\delta = \frac{d}{a}$$

$$\therefore (\mu - 1)\alpha = \frac{d}{a} \quad \text{or} \quad d = (\mu - 1)\alpha a$$

$$\therefore \boxed{2d = 2(\mu - 1)\alpha a}$$



*In a biprism experiment, the eye-piece was placed at a distance of 120 cm from the source. The distance between two virtual images was found equal to 0.075 cm. Find the wavelength of light of source if eye-piece is moved through a distance of 1.888 cm for 20 fringes to cross the field of view.*

$$D = 120 \text{ cm,}$$

$$2d = 0.075 \text{ cm, } \lambda = ?$$

$$\beta = \frac{1.888}{20} \text{ cm}$$

$$\beta = \frac{\lambda D}{2d} \quad \text{or} \quad \lambda = \frac{\beta(2d)}{D} \text{ cm}$$

$$\lambda = \frac{1.888}{20} \times 0.075 \text{ cm}$$

$$= \frac{1.888 \times 0.075}{120} \text{ cm}$$

$$= 5900 \times 10^{-8} \text{ cm} = \mathbf{5900 \text{ \AA}}$$

*The inclined faces of a glass prism ( $\mu = 1.5$ ) make an angle of  $1^\circ$  with the base of the prism. The slit is 10 cm from the biprism and is illuminated by light of  $\lambda = 5900 \text{ \AA}$ . Find the fringe width observed at a distance of 1m from the biprism.*

**Solution.**  $\alpha = 1^\circ = \frac{\pi}{180}$  radian,

$$\mu = 1.5,$$

$$D = 10 \text{ cm} + 100 \text{ cm} = 110 \text{ cm,}$$

$$\lambda = 5900 \times 10^{-8} \text{ cm}$$

$$\beta = \frac{D\lambda}{2d} = \frac{D\lambda}{2(\mu - 1) \alpha a}$$

or 
$$\beta = \frac{110 \times 5900 \times 10^{-8} \times 7 \times 180}{2(1.5 - 1) 22 \times 10} \text{ cm}$$

$$= \mathbf{0.037 \text{ cm.}}$$

*A biprism is placed 5 cm from a slit illuminated by sodium light ( $\lambda = 5890 \text{ \AA}$ ). The width of the fringes obtained on a screen 75 cm from the biprism is  $9.424 \times 10^{-2} \text{ cm}$ . What is the distance between the two coherent sources ?*

**Solution.**  $D = 5 \text{ cm} + 75 \text{ cm} = 80 \text{ cm}$   
 $\beta = 9.424 \times 10^{-2} \text{ cm}$   
 $2d = ?$

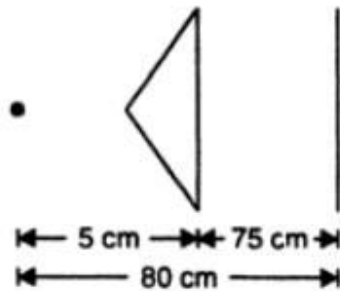


Fig. 2.25

$$\lambda = 5890 \text{ \AA} = 5890 \times 10^{-8} \text{ cm}$$

We know that  $\beta = \frac{\lambda D}{2d}$

or 
$$2d = \frac{\lambda D}{\beta} = \frac{5890 \times 10^{-8} \times 80}{9.424 \times 10^{-2}} \text{ cm}$$
  
$$= 0.05 \text{ cm.}$$



*In a Fresnel's biprism experiment, the fringe width is observed to be 0.087 mm. What will it become if the slit to biprism distance is reduced to  $\frac{3}{4}$  of the original distance ? (all else remaining unchanged).*

**Solution.**  $2d = 2(\mu - 1) \alpha a$  ... (1)

$$2d' = 2(\mu - 1) \alpha \left( \frac{3}{4} a \right) \quad \dots (2)$$

Dividing (2) by (1),  $\frac{2d'}{2d} = \frac{3}{4}$

Again, we know that  $\beta = \frac{D\lambda}{2d}$

$$\frac{\beta'}{\beta} = \frac{2d}{2d'} = \frac{4}{3}$$

or  $\beta' = \frac{4}{3} \beta = \frac{4}{3} \times 0.087 \text{ mm} = \mathbf{0.116 \text{ mm}}$ .

*The inclined faces of biprism of refractive index 1.50 make angles of  $2^\circ$  with its base. A slit illuminated by monochromatic light is placed at a distance of 10 cm from the biprism. If*

**distance between two dark fringes observed at a distance of 1 metre from biprism is 0.18 mm, find the wavelength of light used.**

**Solution.**  $\mu = 1.50,$

$$\alpha = 2^\circ = 2 \times \frac{\pi}{180} = \frac{\pi}{90} \text{ radian,}$$

$$a = 10 \text{ cm, } b = 1 \text{ m} = 100 \text{ cm,}$$

$$\beta = 0.18 \text{ mm} = 0.018 \text{ cm, } \lambda = ?$$

We know that

$$\beta = \frac{D\lambda}{2d}, \quad D = a + b \text{ and } 2d = 2(\mu - 1)\alpha a$$

$$\therefore \beta = \frac{\lambda(a + b)}{2(\mu - 1)\alpha a}$$

$$\therefore \lambda = \frac{2\beta(b - 1)\alpha a}{a + b}$$

$$\begin{aligned} &= \frac{2 \times 0.018 \times (1.50 - 1) \frac{\pi}{90} \times 10}{10 + 100} \text{ cm} \\ &= 5714 \times 10^{-8} \text{ cm} = \mathbf{5714 \text{ \AA}.} \end{aligned}$$

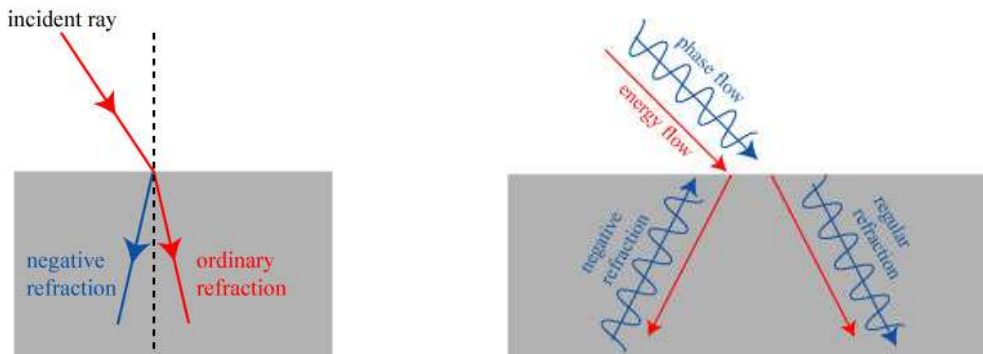
If Fresnel biprism is immersed in a liquid of refractive index  $\mu'$ , then

$$\beta_{\text{new}} = \frac{\frac{\lambda}{\mu'}(a + b)}{2a\left(\frac{\mu}{\mu'} - 1\right)\alpha} = \frac{\lambda(a + b)}{2a(\mu - \mu')\alpha}$$

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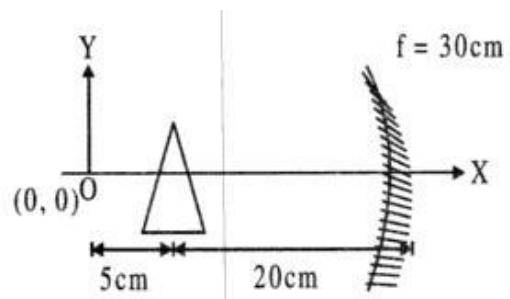
Optics - 3 ) Negative Refractive Index. For meta-materials we can have Negative Refractive Index. So “ Refractive Index “ is a ‘ rare ‘ scalar which can be negative. [ Recall most scalars are positive, such as volume, mass, pressure, viscosity, resistance, inductance, capacitance etc. **Can you think of a few scalars which can be negative also apart from charge or current ?** ]

Negative refractive index question was asked in 2012 IIT JEE



Optics - 4 ) Combination of Prism and Mirror problems

**Find the co-ordinates of image of the point object 'O' formed after reflection from concave mirror as shown in figure assuming prism to be thin and small in size of prism angle  $2^\circ$ . Refractive index of the prism material is  $3/2$ .**



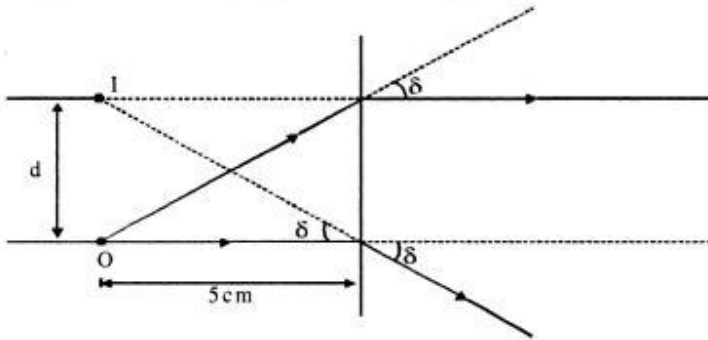
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Consider image formation through prism. All incident rays will be deviated by

$$\delta = (\mu - 1)A = \left(\frac{3}{2} - 1\right)2^\circ = 1^\circ = \frac{\pi}{180} \text{ rad}$$

As prism is thin, object and image will be in the same plane as shown in figure.



It is clear  $\frac{d}{5} = \tan \delta \approx \delta$  ( $\because \delta$  is very small) or  $d = \frac{\pi}{36} \text{ cm}$

Now this image will act as an object for concave mirror.

$$u = -25 \text{ cm}, f = -30 \text{ cm}, \therefore v = \frac{uf}{u-f} = 150 \text{ cm}, \text{ Also, } m = \frac{-v}{u} = +6$$

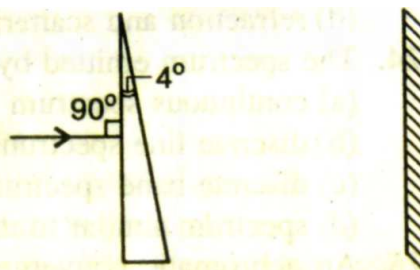
$$\therefore \text{Distance of image from principal axis} = \frac{\pi}{36} \times 6 = \frac{\pi}{6} \text{ cm}$$

Hence, co-ordinates of image formed after reflection from concave mirror are

$$\left(175 \text{ cm}, \frac{\pi}{6} \text{ cm}\right)$$

A prism having an apex

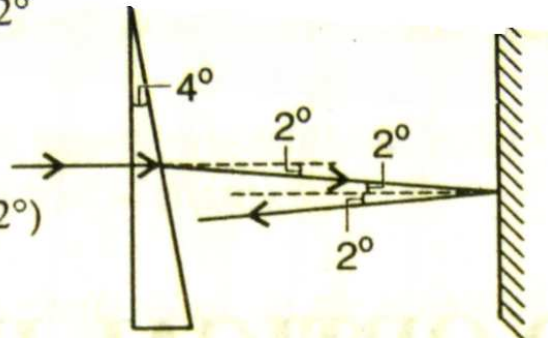
angle  $4^\circ$  and refractive index 1.5 is located in front of a vertical plane mirror as shown in figure. Through what total angle is the ray deviated after reflection from the mirror?



- (a)  $176^\circ$  (b)  $4^\circ$   
(c)  $178^\circ$  (d)  $2^\circ$

$$\delta_{\text{prism}} = (\mu - 1)A = (1.5 - 1)4^\circ = 2^\circ$$

$$\begin{aligned} \therefore \delta_{\text{total}} &= \delta_{\text{prism}} + \delta_{\text{mirror}} \\ &= (\mu - 1)A + (180 - 2i) \\ &= (1.5 - 1)4^\circ + (180 - 2 \times 2^\circ) \\ &= 2^\circ + 176^\circ = 178^\circ \end{aligned}$$

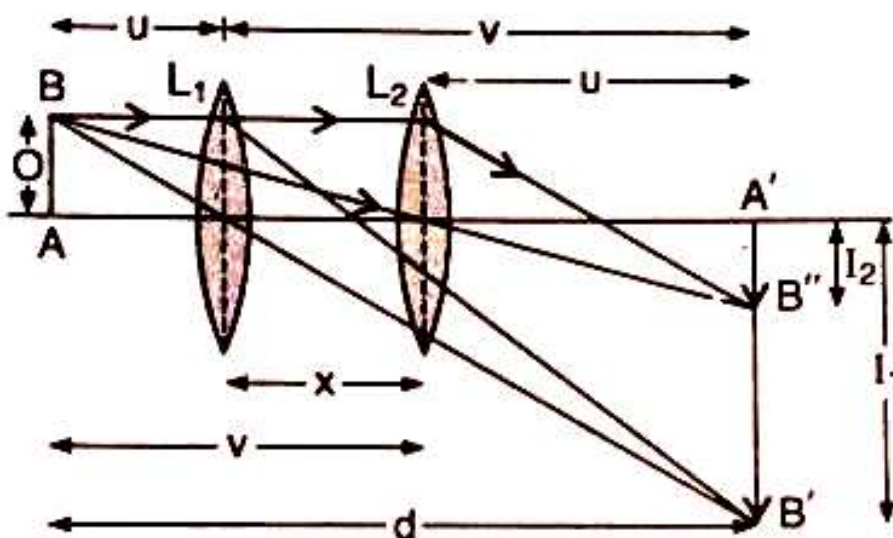


The ray diagram is shown in figure.

Optics - 5 ) How do we find focal length of a lens ?

### **Focal length of convex lens by displacement method:**

- (i) When the distance between object and screen  $d$ , is greater than  $4f$ , then there are two positions of the lens for which the image of the object on the screen is distinct and clear. In these two positions of lens, the distances of object and image from the lens are interchanged.



- (ii) Here,  $I_1$  and  $I_2$  are the lengths of images in first and second position of lens  $L$ .  $O$  is the length of the object. In first position of lens,

$$m_1 = \frac{v}{u} = \frac{I_1}{O}$$

In second position, the magnification of the lens is

given by:  $m_2 = \frac{u}{v} = \frac{I_2}{O}$

$$\therefore m_1 m_2 = \frac{I_1 I_2}{O^2} = 1$$

$$\therefore O = \sqrt{I_1 I_2}$$

(iii) Further,  $\frac{m_1}{m_2} = \frac{v^2}{u^2}$

From figure,  $u + x + u = d$  or  $u = \frac{d-x}{2}$

According to sign convention,  $u = -(d-x)/2$

Similarly,  $v = d - u = (d+x)/2$

Using lens formula,  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ , we get;

$$f = \left( \frac{d^2 - x^2}{4d} \right)$$

In the displacement method, a convex lens is placed in between an object and a screen. If the magnifications in the two positions are  $m_1$  and  $m_2$  and the displacement of the lens between the two positions is  $x$ , then the focal length of the lens is:

(a)  $\frac{x}{(m_1 + m_2)}$

(b)  $\frac{x}{(m_1 - m_2)}$

(c)  $\frac{x}{(m_1 + m_2)^2}$

(d)  $\frac{x}{(m_1 - m_2)^2}$



$$m_1 = \frac{v}{u}, m_2 = \frac{u}{v}$$

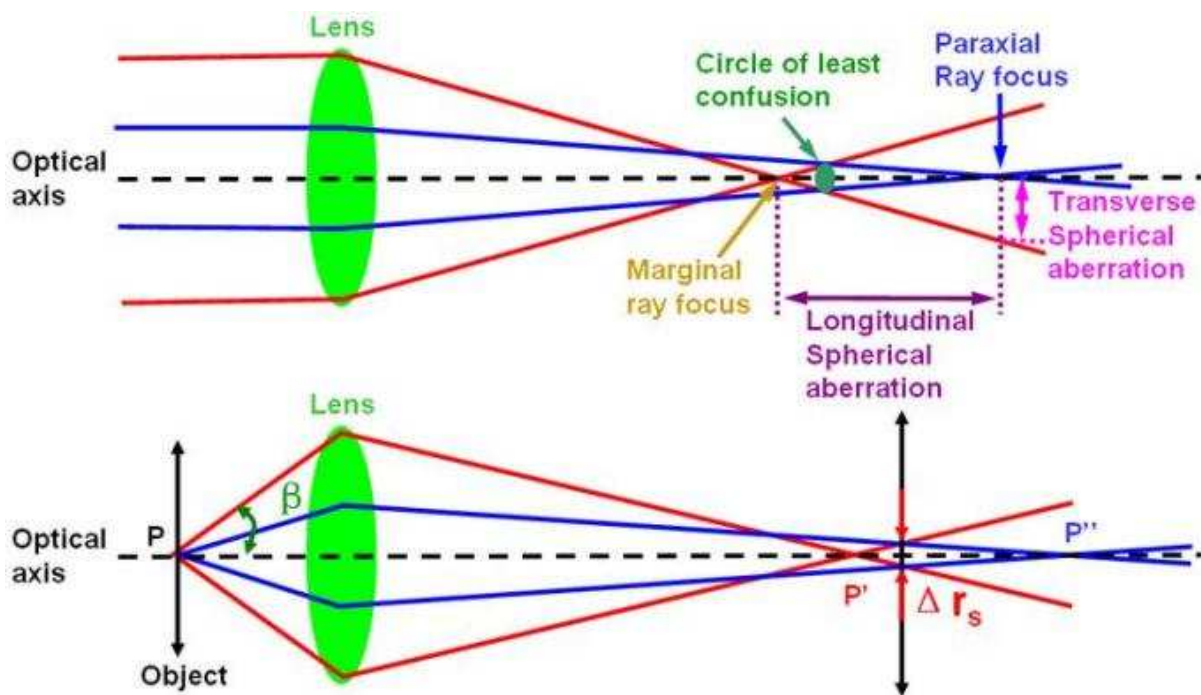
$$m_1 - m_2 = \frac{v}{u} - \frac{u}{v}$$

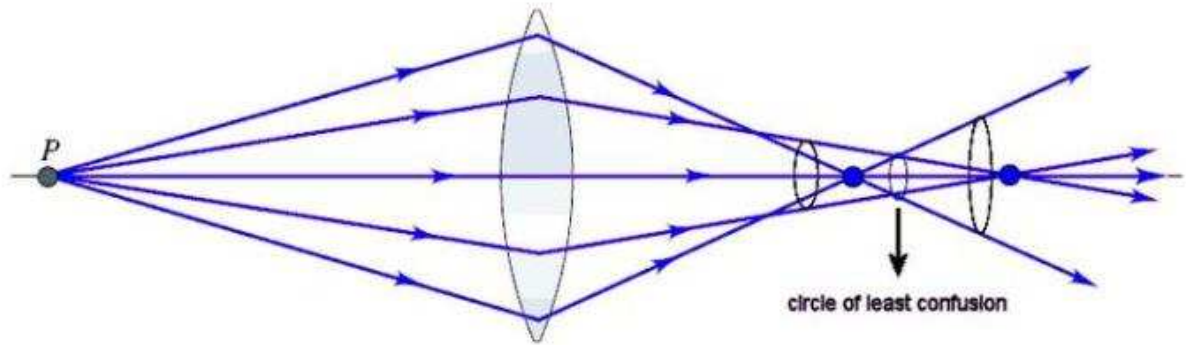
$$m_1 - m_2 = \frac{v^2 - u^2}{uv} = \frac{(v - u)(v + u)}{uv}$$

$$\text{Now } v - u = x, \frac{1}{f} = \frac{1}{v} + \frac{1}{u} \quad \text{or} \quad \frac{1}{f} = \frac{u + v}{uv}$$

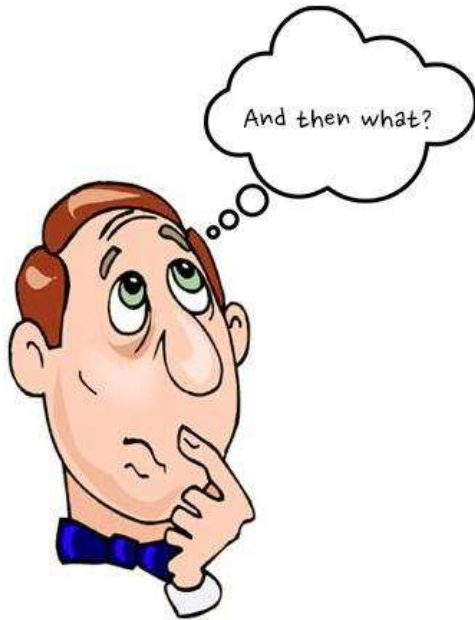
$$\therefore m_1 - m_2 = \frac{x}{f} \quad \text{or} \quad f = \frac{x}{m_1 - m_2}$$

Optics - 6 ) Circle of least confusion

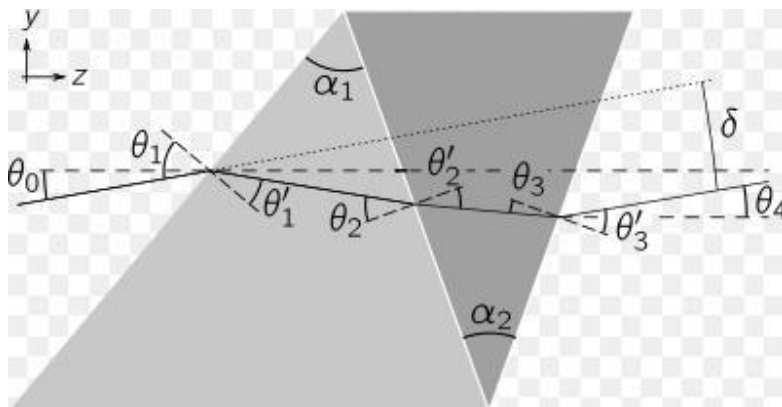




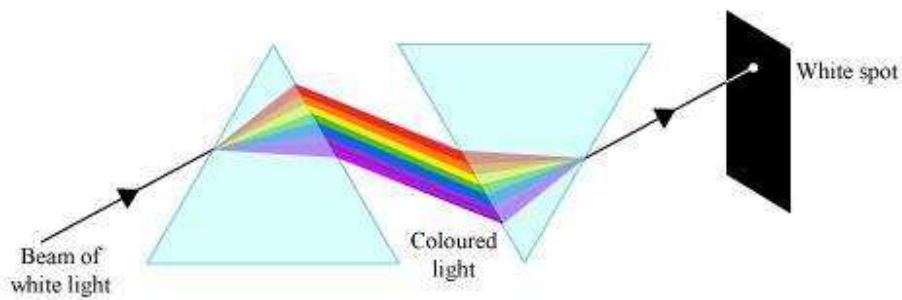
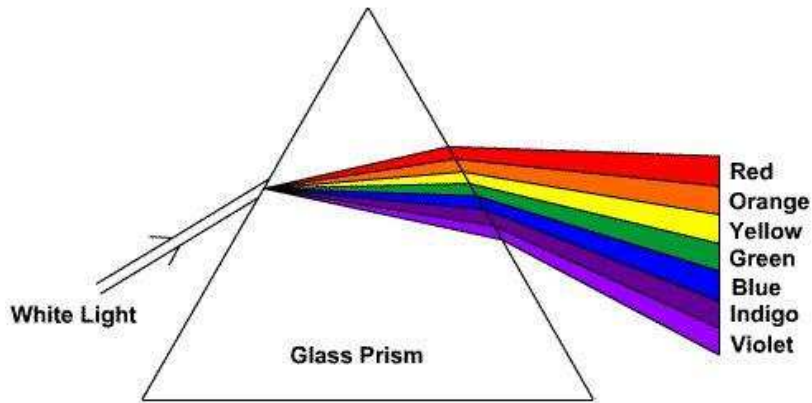
Optics - 7 )



Deviation diagrams

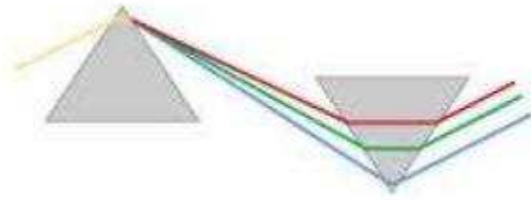


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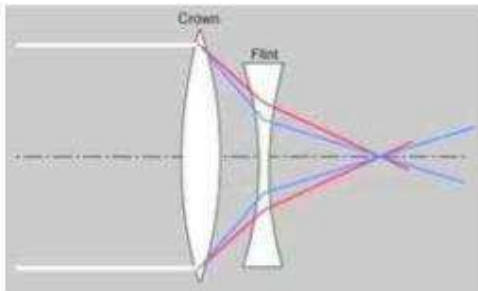


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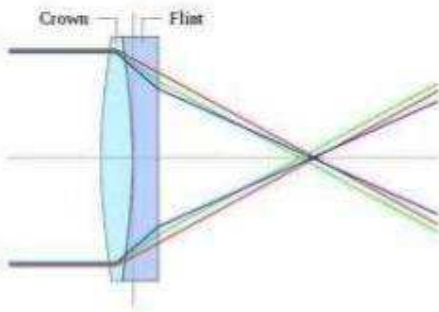




Prisms with equal vertex angle (= light deviation power) and same glass type (= equal dispersion) can exactly cancel out color that is between them.



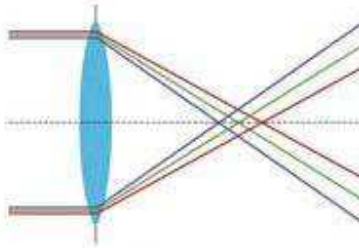
The color of a positive lens can be cancelled by an equal power negative lens of the same glass, but then the focal length of the lens pair would be zero, if they were in contact. Instead we want the negative lens to be a more dispersive glass than the positive lens, so that a weaker power negative lens can still cancel out the color and give a total power of the lens pair that is not zero. When the red and blue light rays come to the same focus primary color has been corrected.



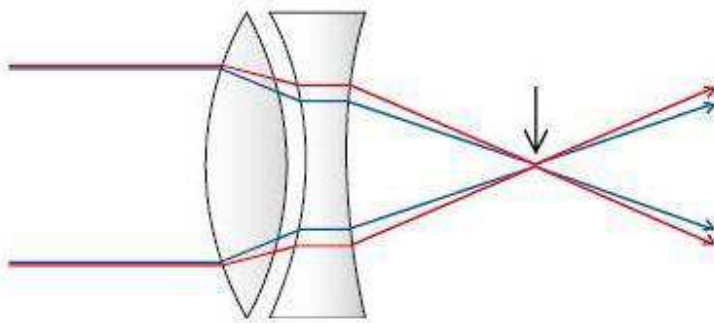
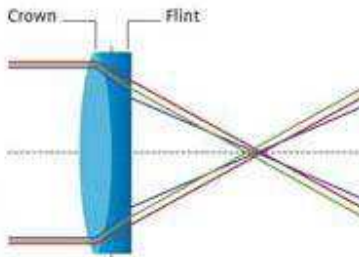
In a typical contact doublet the negative lens glass is about 1.5X to 2X more dispersive than the positive lens glass.

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**Chromatic aberration**

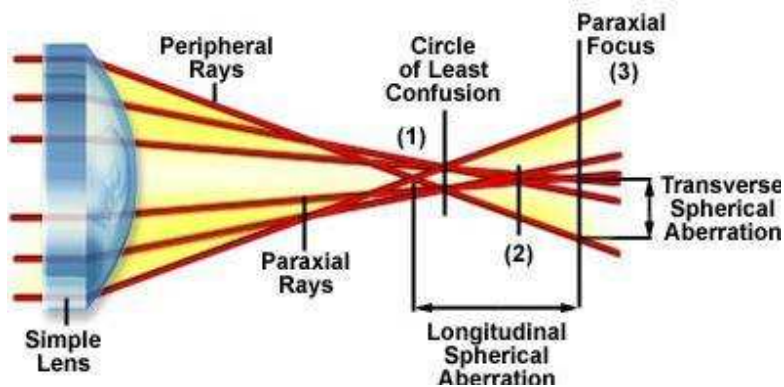


**An achromatic lens**



While this combination will also have a circle of least confusion

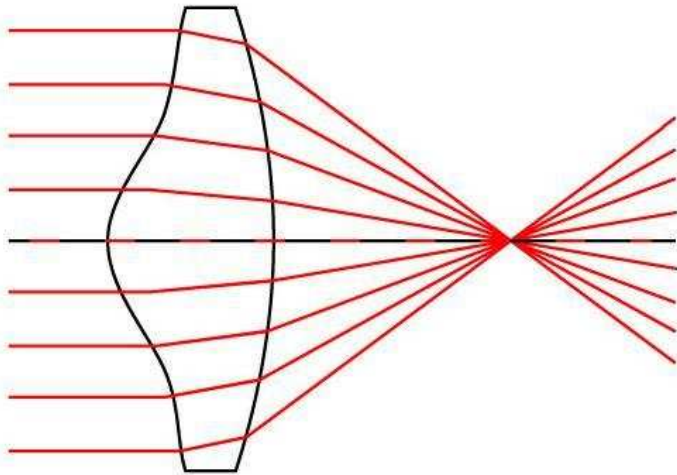
**Longitudinal and Transverse Spherical Aberration**



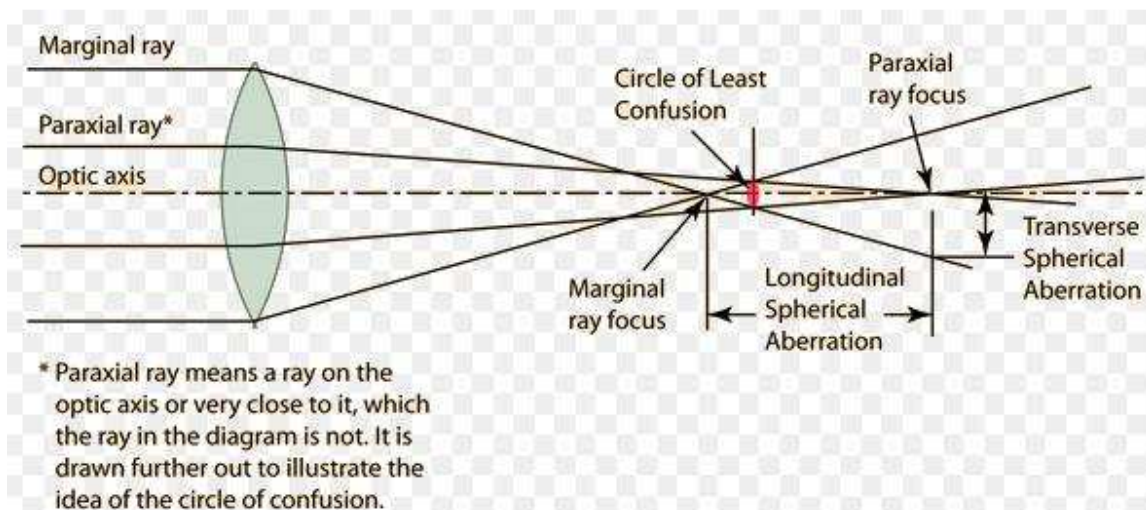
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Optics - 8 ) Aspherical lenses can be used to reduce axial spread ( of paraxial rays ), apart from stoppers or rather with combinations of stoppers.

## Aspherical Lens



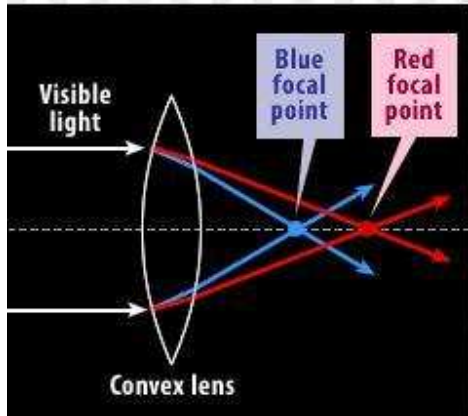
Remember more curved surface should face the light first. In plano-convex lens the convex part should face the light for better utilization of refraction properties. Also this minimizes the errors.



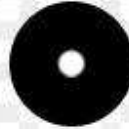


Optics - 9 ) The conical image of a point

Looking at only red and blue light:



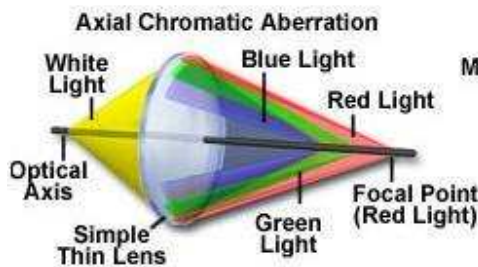
**Result:** A fringe of color may appear around bright objects seen through the lens:



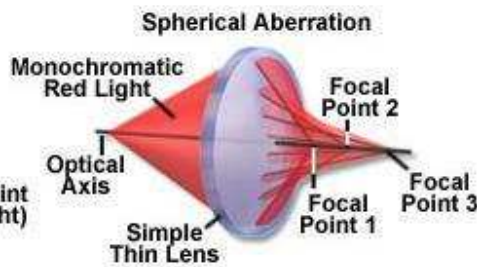
A star, as seen through a telescope without chromatic aberration



A star, as seen through a telescope with chromatic aberration (exaggerated)

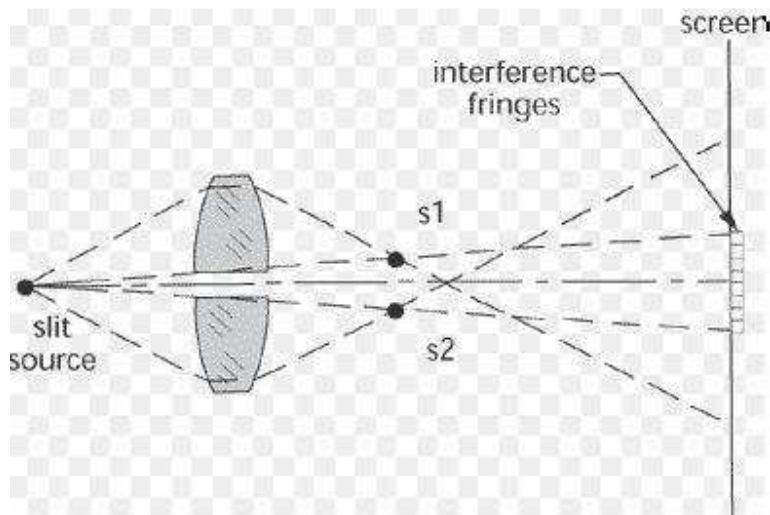


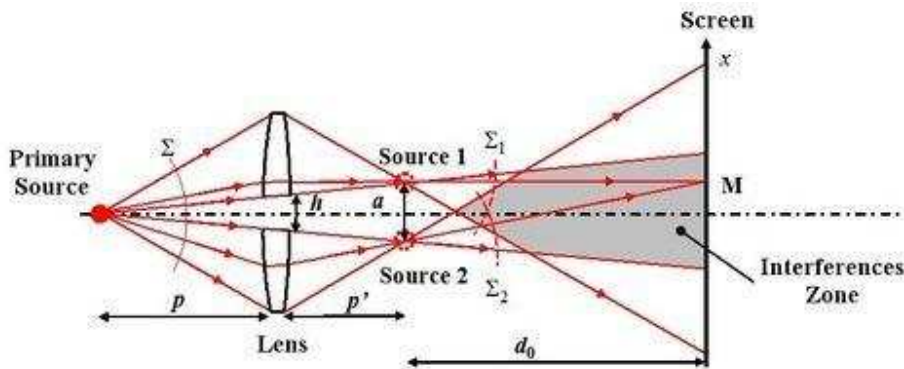
Axial Chromatic Aberration



Spherical Aberration

Optics - 10 ) Split lenses

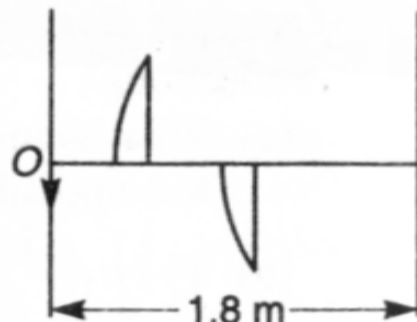




- Was asked in Physics Olympiad before being asked in IIT JEE

A thin plano-convex lens of focal length  $f$  is split into two halves. One of the halves is shifted along the optical axis. The

separation between object and image planes is 1.8 m. The magnification of the image formed by one of the half lens is 2. Find the focal length of the lens and separation between the halves. Draw the ray diagram for image formation. (1996, 5M)



Solution

For both the halves, position of object and image is same. Only difference is of magnification. Magnification for one of the halves is given as  $2 (> 1)$ . This can be for the first one, because for this,  $|v| > |u|$ . Therefore, magnification,  $|m| = |v/u| > 1$

So, for the first half

$$|v/u| = 2 \quad \text{or} \quad |v| = 2|u|$$

Let  $u = -x$  then  $v = +2x$  and  $|u| + |v| = 1.8\text{m}$

$$\text{i.e., } 3x = 1.8 \text{ m} \quad \text{or} \quad x = 0.6 \text{ m}$$

Hence,  $u = -0.6 \text{ m}$  and  $v = +1.2 \text{ m}$ .

$$\text{Using, } \frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{1.2} - \frac{1}{-0.6} = \frac{1}{0.4}$$

$$\therefore f = 0.4 \text{ m}$$

For the second half

$$\frac{1}{f} = \frac{1}{1.2 - d} - \frac{1}{-(0.6 + d)}$$

$$\text{or } \frac{1}{0.4} = \frac{1}{1.2 - d} + \frac{1}{0.6 + d}$$

Solving this, we get  $d = 0.6 \text{ m}$ .

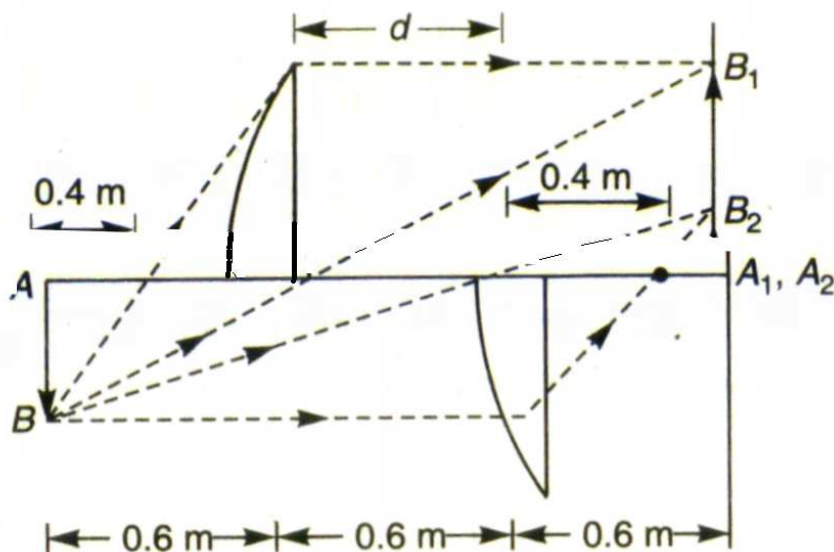
Magnification for the second half will be

$$m_2 = \frac{v}{u} = \frac{0.6}{-(1.2)} = -\frac{1}{2}$$

and magnification for the first half is

$$m_1 = \frac{v}{u} = \frac{1.2}{-(0.6)} = -2$$

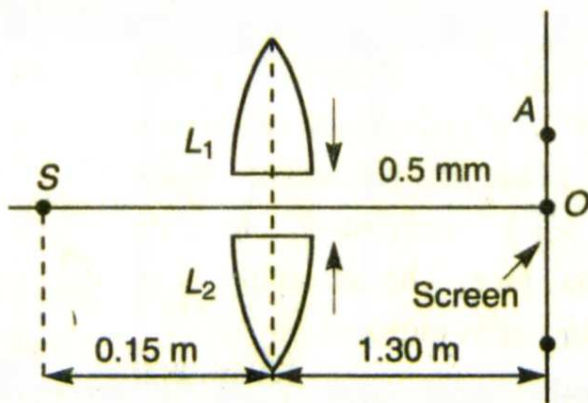
The ray diagram is as follows :





In given figure,  $S$  is a monochromatic point source emitting light of wavelength  $\lambda = 500\text{ nm}$ . A thin lens of circular shape and focal length  $0.10\text{ m}$  is cut into two identical halves  $L_1$  and  $L_2$  by a plane passing through a diameter. The two halves are placed symmetrically about the central axis  $SO$  with a gap of  $0.5\text{ mm}$ . The distance along the axis from  $S$  to  $L_1$  and  $L_2$  is  $0.15\text{ m}$  while that from  $L_1$  and  $L_2$  to  $O$  is  $1.30\text{ m}$ . The screen at  $O$  is normal to  $SO$ . (1993, 5+1M)

Solution



If the third intensity maximum occurs at the point  $A$  on the screen, find the distance  $OA$ .

If the gap between  $L_1$  and  $L_2$  is reduced from its original value of  $0.5\text{ mm}$ , will the distance  $OA$  increase, decrease, or remain the same.

(a) For the lens,  $u = -0.15 \text{ m}$ ;  $f = +0.10 \text{ m}$

Therefore, using  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$  we have

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$$

$$= \frac{1}{(-0.15)} + \frac{1}{(0.10)}$$

or  $v = 0.3 \text{ m}$

Linear magnification,  $m = \frac{v}{u} = \frac{0.3}{-0.15} = -2$

Hence, two images  $S_1$  and  $S_2$  of  $S$  will be formed at  $0.3 \text{ m}$  from the lens as shown in figure. Image  $S_1$  due to part 1

will be formed at  $0.5 \text{ mm}$  above its optic axis ( $m = -2$ ). Similarly,  $S_2$  due to part 2 is formed  $0.5 \text{ mm}$  below the optic axis of this part as shown.

Hence,  $d =$  distance between  $S_1$  and  $S_2 = 1.5 \text{ mm}$

$$D = 1.30 - 0.30 = 1.0 \text{ m} = 10^3 \text{ mm}$$

$$\lambda = 500 \text{ nm} = 5 \times 10^{-4} \text{ mm}$$

Therefore, fringe width,

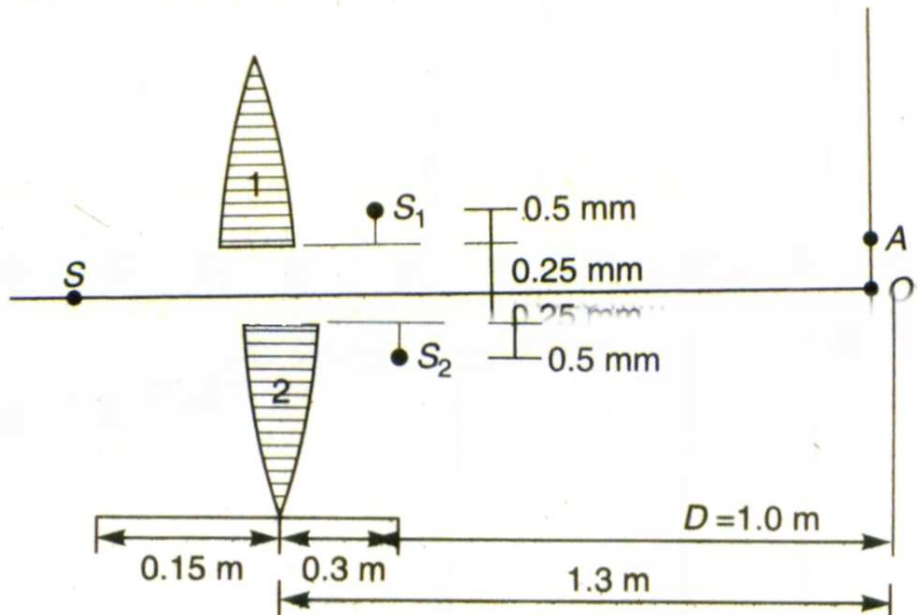
$$\omega = \frac{\lambda D}{d} = \frac{(5 \times 10^{-4})(10^3)}{(1.5)} = \frac{1}{3} \text{ mm}$$

Now, as the point  $A$  is at the third maxima

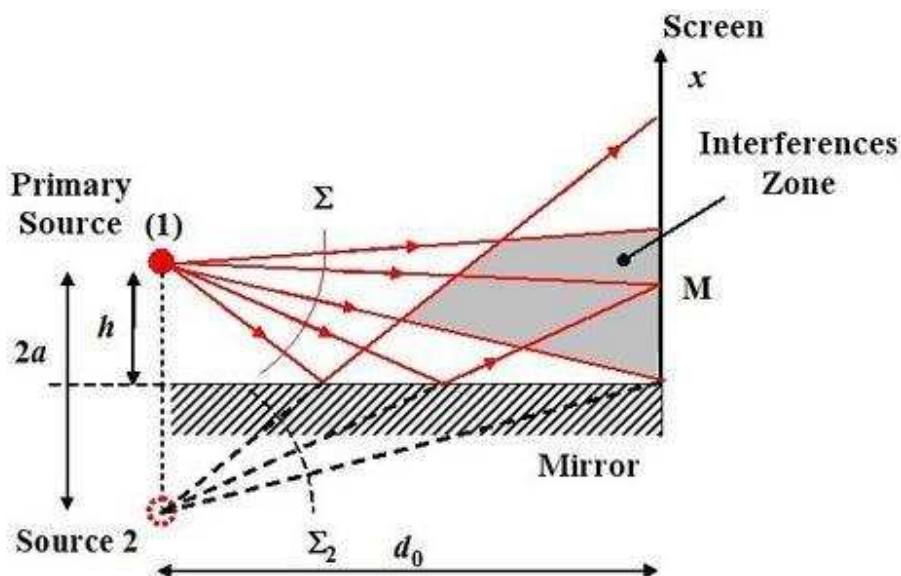
$$OA = 3\omega = 3(1/3) \text{ mm}$$

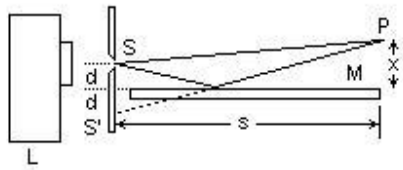
or  $OA = 1 \text{ mm}$

- (b) If the gap between  $L_1$  and  $L_2$  is reduced,  $d$  will decrease. Hence, the fringe width  $\omega$  will increase or the distance  $OA$  will increase.

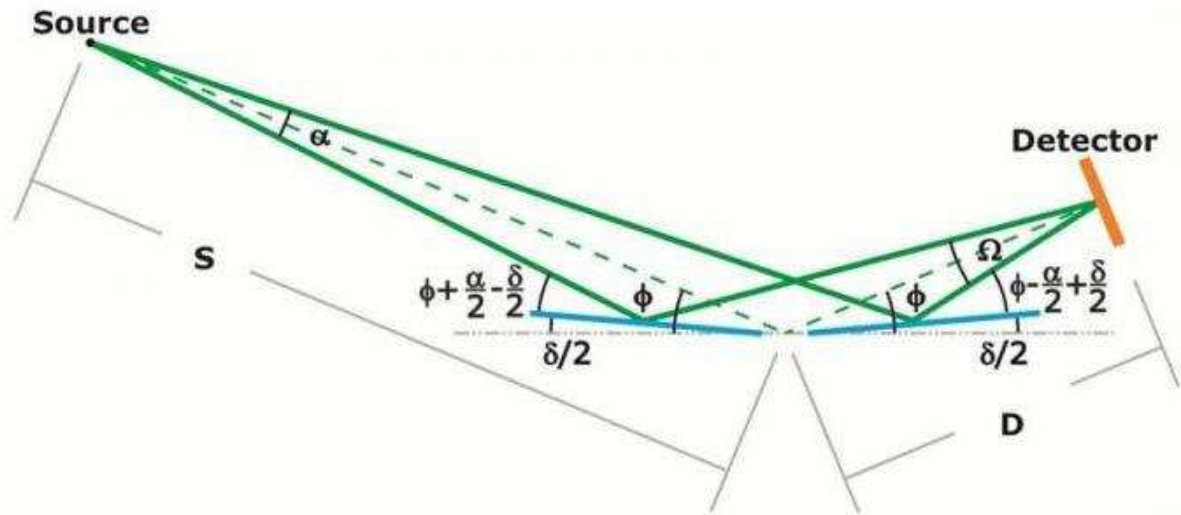


Optics - 11 ) Lloyd's Mirror

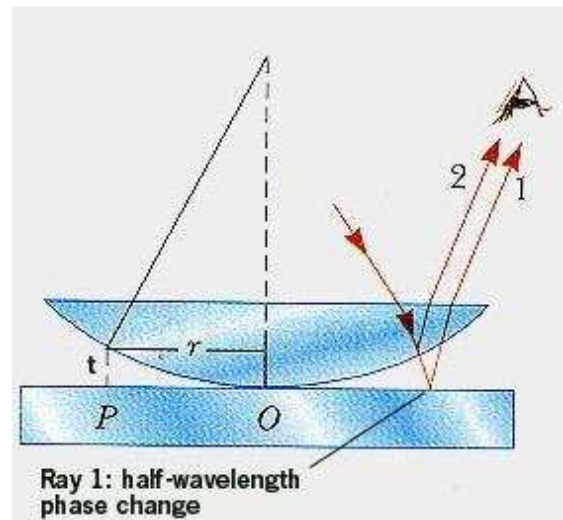
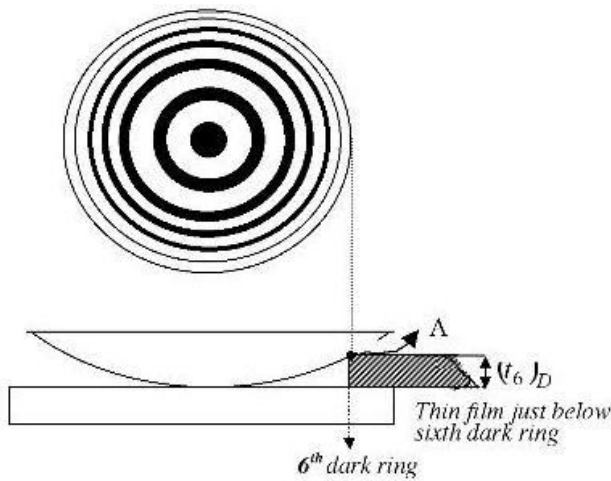




Lloyd's Mirror



Optics - 12 ) Newton's Rings





film thickness  $t$ .

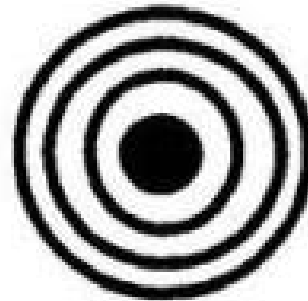
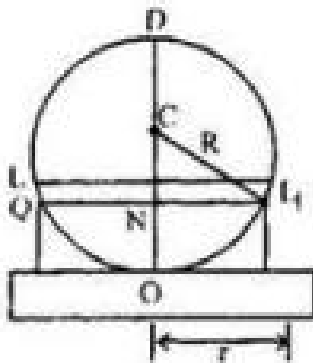


Fig. Newton Rings in Reflected Light

$$2t + \frac{\lambda}{2} = n\lambda$$

$$2t = \frac{(2n-1)\lambda}{2} \text{ for a bright ring } n = 1, 2, 3, \dots$$

&

$$2t = n\lambda \text{ for dark ring } n = 0, 1, 2, 3, \dots$$

From the property of the circle,

$$NP \times NQ = NO \times ND$$

Substituting values,

$$r \times r = t \times (2R - t) = 2Rt - t^2 \approx 2Rt \text{ approximately.}$$

$\therefore$

$$t = \frac{r^2}{2R}$$

Thus, for bright ring,

$$2 \cdot \frac{r^2}{2R} = \frac{(2n-1)\lambda}{2}$$

$$r = \frac{D}{2} \text{ where } D \text{ is diameter}$$

$$\frac{D^2}{4} = \frac{(2n-1)\lambda R}{2}$$

$$D_n = \sqrt{2(2n-1)\lambda R}$$

$$D_n \propto \sqrt{(2n-1)}$$

i.e., diameter of  $n^{\text{th}}$  bright ring is proportional to square root of odd natural number.

Optics - 13 ) Plano-Convex lens problems

The apparent thickness of a thick plano-convex lens is measured once with the plane face upward and then with the convex face upwards. The value will be :

- (a) More in the first case.
- (b) Same in the two cases
- (c) More in the II case
- (d) Can be any of the above depending on the value of its actual thickness

The apparent thickness in case (a)

$$OA' = \frac{\text{real } (OA)}{\mu} = \frac{t}{\mu}$$

In case (b) when the convex surface is placed down then refraction takes place through curved surface.

Object is in denser medium, then  $\mu_2 = 1, \mu_1 = \mu$

$$\therefore \text{by } \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{v} + \frac{\mu}{t} = \frac{1 - \mu}{-R}$$

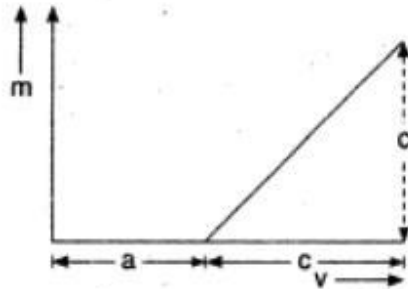
$$\frac{1}{v} = \frac{(\mu - 1)}{R} - \frac{\mu}{t}$$

$$v = \frac{Rt}{(\mu - 1)t - \mu R}$$

Clearly in the second case the apparent thickness is more.

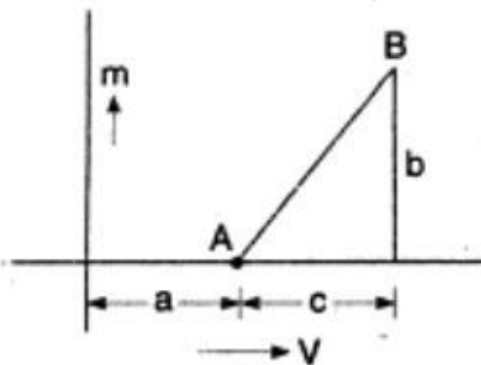
Optics - 14 ) Lens problems with graphs

The graph shows how the magnification  $m$  produced by a thin convex lens varies with image distance  $v$ . What was the focal length of the lens used ?



- (a)  $b/c$     (b)  $b/ca$     (c)  $bc/a$     (d)  $c/b$

For point B,  $m = b$  or  $\frac{v}{u} = b$



$$\frac{(a+c)}{u} = b \text{ or } u = \left( \frac{a+c}{b} \right)$$

$$\frac{1}{f} = \frac{1}{(a+c)} + \frac{b}{(a+c)} = \frac{(1+b)}{(a+c)} \text{ or } f = \left( \frac{a+c}{1+b} \right) \dots(1)$$

Again for point A,  $m = 0$

$$v = \infty, m = 0 = \frac{v}{u} = \frac{a}{u}$$

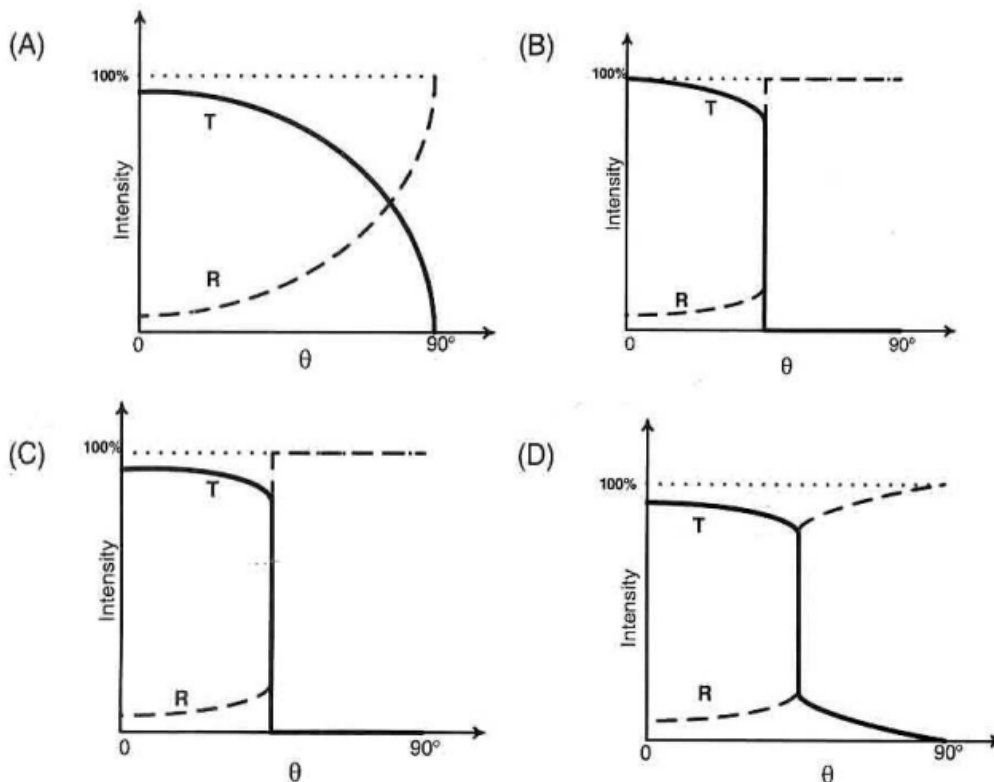
$$u = \infty \therefore v = a = f$$

Putting in (1)

$$f = \frac{f+c}{1+b}, f+fb = f+c \text{ or } f = \frac{c}{b}$$

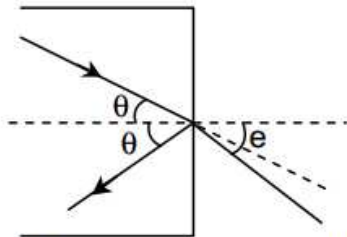
IIT JEE 2011

A light ray traveling in glass medium is incident on glass-air interface at an angle of incidence  $\theta$ . The reflected ( R ) and transmitted ( T ) intensities, both as function of  $\theta$ , are plotted. The correct sketch is

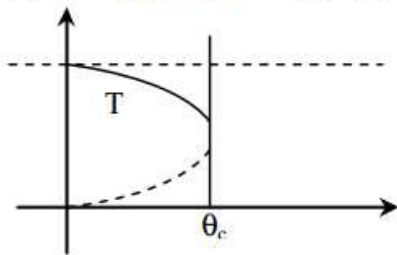


Answer [ c ]

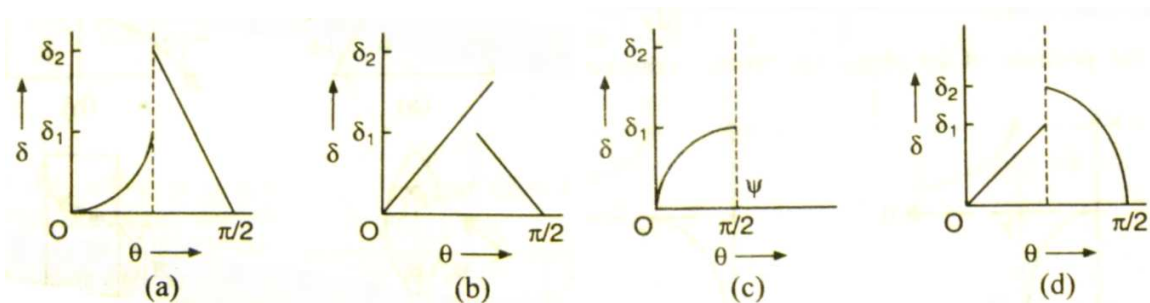




When  $\theta > \theta_c$ , no ray will transmit  
 $\Rightarrow T = 0, T + R = 100\%$  and  $R > 0$



A ray of light travels from a medium of refractive index  $\mu$  to air. Its angle of incidence in the medium is  $\theta$ , measured from the normal to the boundary and its angle of deviation is  $\delta$ .  $\delta$  is plotted against  $\theta$  which of the following best represents the resulting curve?



Answer ( a )

In the above problem which of the following relations are correct

(a)  $\psi = \sin^{-1} \left( \frac{1}{\mu} \right)$

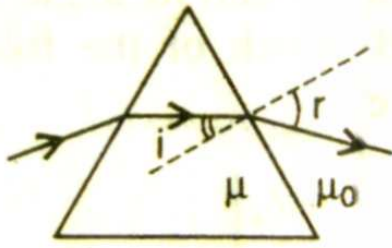
(b)  $\psi = \frac{\pi}{2} - \sin^{-1} \left( \frac{1}{\mu} \right)$

(c)  $\frac{\delta_2}{\delta_1} = \mu$

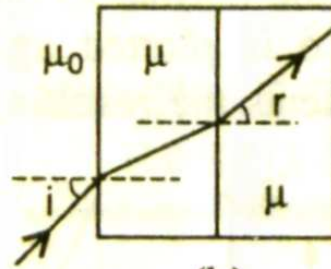
(d)  $\frac{\delta_2}{\delta_1} = 2$

Answer ( a ) and ( d )

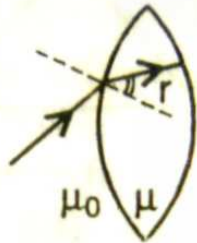
For which of the following cases  $(\sin i / \sin r)$  is equal to  $(\mu / \mu_0)$ ?



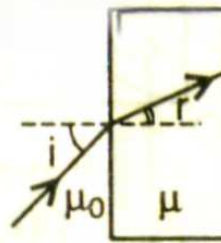
(a)



(b)



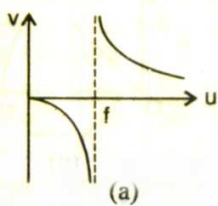
(c)



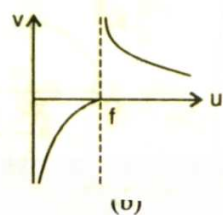
(d)

Answer - b, c, d

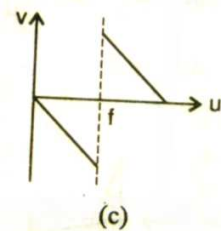
As the position of an object ( u ) from a concave mirror is varied, the position of the image ( v ) also varies. By letting u change from 0 to ∞ the graph between v and u will be ?



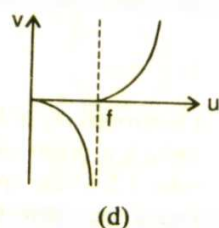
(a)



(b)



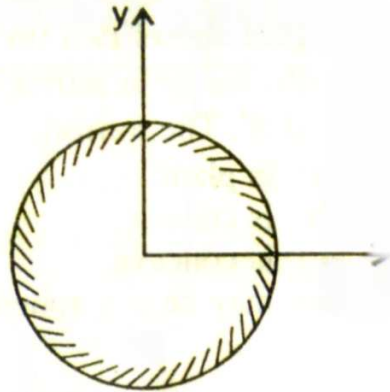
(c)



(d)

Answer - ( a )

A reflecting surface is represented by the equation  $x^2 + y^2 = a^2$ . A ray travelling in negative  $x$ -direction is directed towards positive  $y$ -direction after reflection from the surface at some point  $P$ . Then the co-ordinates of point  $P$  are:

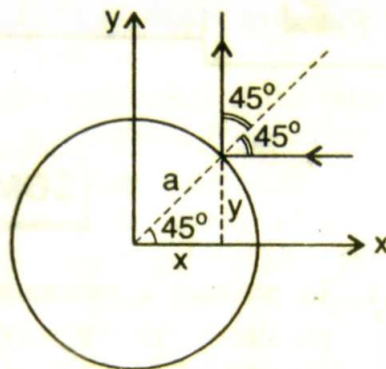


- (a)  $(0.8a, 0.6a)$
- (b)  $(0.6a, 0.8a)$
- (c)  $(a, 0)$
- (d) none of the above

The ray diagram is shown in the figure.

$$x = \frac{a}{\sqrt{2}} \quad \text{and} \quad y = \frac{a}{\sqrt{2}}$$

$$\therefore P = \left( \frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}} \right)$$



So Answer - ( d )

Optics - 15 ) Lens immersed in a liquid

The focal length of lens of refractive index 1.5 in air is 30 cm. When it is immersed in a liquid of refractive index  $\frac{4}{3}$ , then its focal length in liquid will be

- (a) 30 cm
  - (b) 60 cm
  - (c) 120 cm
  - (d) 240 cm
- (BHU 2002)

We know that focal length in liquid

$$(f_m) = \left[ \frac{\mu_g - 1}{(\mu_g / \mu_m) - 1} \right] \times f_a = \left[ \frac{1.5 - 1}{(1.5 / 1.33) - 1} \right] \times 30$$

$$= \left[ \frac{1.5 - 1}{1.125 - 1} \right] \times 30 = 120 \text{ cm.}$$

A bi-convex lens ( $\mu = 1.5$ ) of focal length 0.2 m acts as a divergent lens of power one dioptre when immersed in a liquid. The refractive index of the liquid is :

- (a) 1.33      (b) 1.67      (c) 1.25      (d) 1.2

$$f_a = 20 \text{ cm, } f_w = -100 \text{ cm.}$$

$$\therefore \frac{f_w}{f_a} = \frac{(a\mu_g - 1)}{\left( \frac{a\mu_g}{a\mu_w} - 1 \right)} \quad \text{or} \quad -\frac{100}{20} = \frac{(1.5 - 1)}{\left( \frac{1.5}{a\mu_w} - 1 \right)}$$

$$\text{or} \quad \frac{1.5}{a\mu_w} - 1 = -\frac{0.5}{5} = -\frac{1}{10}$$

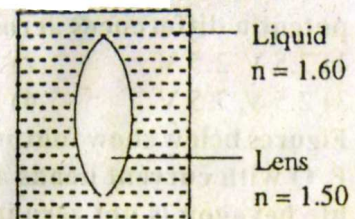
$$\therefore \frac{1.5}{a\mu_w} = 1 - \frac{1}{10} = \frac{9}{10}$$

$$a\mu_w = \frac{15}{9} = 1.67$$

Karnataka CET 1996 problem - Lens put in Slab with liquid

Shown in the figure is a convergent lens placed inside a cell filled with a liquid. The lens has a focal length +20 cm. when in air and its material has a refractive index 1.50. If the liquid has a refractive index 1.60, the focal length of the system is: (II-U-1-3)

- 1) -24 cm                      2) -100 cm  
3) +80 cm                      4) -80 cm





$$\text{In air } \frac{1}{f} = (n_g - 1) \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\frac{1}{20} = 0.5 \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \rightarrow \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{1}{20 \times 0.5} = \frac{1}{10}$$

In the liquid

$$\frac{1}{f^l} = \left( \frac{n_g}{n_e} - 1 \right) \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \left( \frac{1.5}{1.6} - 1 \right) \times \frac{1}{10}$$

$$= -\frac{1}{16} \times \frac{1}{10} = -\frac{1}{160} \quad \text{or} \quad f = -160 \text{ cm}$$

If the formula was printed as +ve, then the absolute values of Radius will be taken.

Given  ${}^a\mu_g = 3/2$  and  ${}^a\mu_w = 4/3$ . There is an equiconvex lens with radius of each surface equal to 20 cm. There is air in the object space and water in the image space. The focal length of lens is:

- (a) 80 cm      (b) 40 cm      (c) 20 cm      (d) 10 cm

Solution :

$$\frac{{}^a\mu_w}{f} = \frac{({}^a\mu_g - 1)}{R_1} - \frac{({}^a\mu_g - {}^a\mu_w)}{R_2}$$

$$= \frac{\left( \frac{3}{2} - 1 \right)}{20} - \frac{\left( \frac{3}{2} - \frac{4}{3} \right)}{-20} = \frac{1}{40} + \frac{1}{120} = \frac{1}{30}$$

$$f = \frac{4}{3} \times 30 = 40 \text{ cm}$$

There can be problems with lens and different transparent materials on either side or both sides

A hollow double concave lens is made of very thin transparent material. It can be filled with air or either of two liquids  $L_1$  or  $L_2$  having refractive indices  $n_1$  and  $n_2$  respectively ( $n_2 > n_1 > 1$ ). The lens will diverge a parallel beam of light if it is filled with: **(IIT 2000)**

- (a) air and placed in air      (b) air and immersed in  $L_1$   
(c)  $L_1$  and immersed in  $L_2$       (d)  $L_2$  and immersed in  $L_1$

Solution : ( d )

The lens maker's formula is:

$$\frac{1}{f} = \left( \frac{n_L}{n_m} - 1 \right) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

Where  $n_L$  = refractive index of lens material

$n_m$  = refractive index of medium

In case of double concave lens  $R_1$  is -ve and  $R_2$  is +ve. Therefore

$\left( \frac{1}{R_1} - \frac{1}{R_2} \right)$  will be -ve.

For the lens to be diverging in nature, focal length  $f$  should be

negative or  $\left( \frac{n_L}{n_m} - 1 \right)$  should be positive or  $n_L > n_m$ ; but since

$n_2 > n_1$  (given), therefore the lens should be filled with  $L_2$  and immersed in  $L_1$ .

Optics - 16 ) Trick questions with distance of object, Image, focal length of lenses

The focal length of a convex lens is  $f$ . An object is placed at a distance  $x$  from its first focal point. The ratio of the size of the real image to that of the object is :

- (a)  $f/x^2$     (b)  $x^2/f$     (c)  $f/x$     (d)  $x/f$

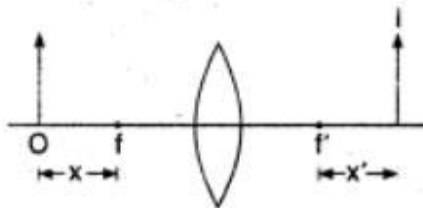
$$u = f + x, \frac{1}{f} = \frac{1}{v} - \frac{1}{-u} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{f+x} \quad \text{or} \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{f+x}$$

$$\frac{1}{v} = \frac{f+x-f}{f(f+x)} = \frac{x}{f(f+x)} \quad \text{or} \quad v = \frac{f(f+x)}{x}$$

$$\frac{v}{u} = \frac{f(f+x)}{x(f+x)} = \frac{f}{x}$$

An object is placed at a point distant  $x$  from the focus of a convex lens and its image is formed at  $I$  as shown in the figure. The distances  $x, x'$  satisfy the relation :



- (a)  $\frac{x \times x'}{2} = f$                       (b)  $f^2 = xx'$   
 (c)  $x + x' = 2f$                       (d)  $x - x' = 2f$

the magnification is :

- (a)  $\frac{f}{x+x'}$                       (b)  $\frac{x'}{x}$   
 (c)  $\frac{f}{x}$                       (d) None of these

$$xx' = f^2, \text{ Newton's formula.}$$

$$u = f + x, v = f + x'$$

$$m = \frac{v}{u} = \frac{f + x'}{f + x}$$

$$x' = \frac{f^2}{x} \therefore m = \frac{f + f^2/x}{f + x}$$

$$m = \frac{f(x + f)}{x(x + f)} = \frac{f}{x}$$

A convex lens of focal length  $f$  is placed somewhere in between an object and a screen. The distance between the object and the screen is  $x$ . If the numerical value of the magnification produced by the lens is  $m$ , the focal length of the lens is:

(a)  $\frac{mx}{(m+1)^2}$     (b)  $\frac{mx}{(m-1)^2}$     (c)  $\frac{(m+1)^2}{m}x$     (d)  $\frac{(m-1)^2}{m}x$

Here,

$$x = u + v$$

$$m = \frac{f}{f+u} = \frac{f-v}{f}$$

For real image,  $m$  is -ve.

$$\therefore -m = f/(f+u) \quad \text{or} \quad u = \frac{-(m+1)}{m}f$$

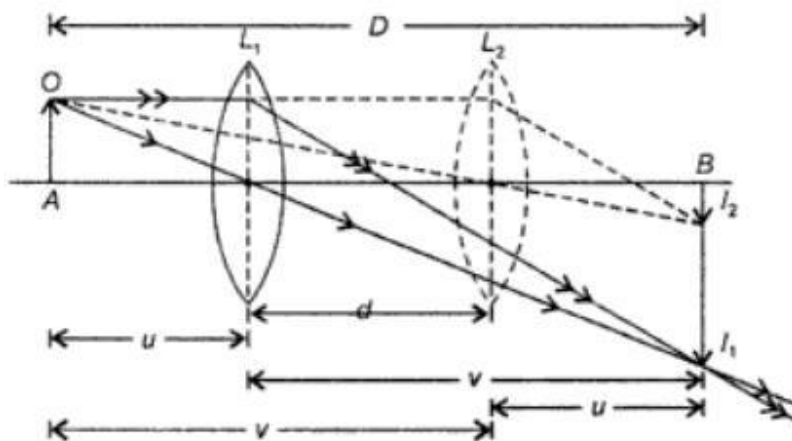
$$\text{and} \quad -m = \frac{f-v}{f} \quad \text{or} \quad v = (m+1)f$$

$$\therefore x = (m+1)f + \frac{(m+1)}{m}f \quad \text{or} \quad f = \frac{mx}{(m+1)^2}$$



The distance between object and the screen is  $D$ . Real images of an object are formed on the screen for two positions of a lens separated by a distance  $d$ . The ratio between the sizes of two images will be:

- (a)  $D/d$  (b)  $D^2/d^2$   
 (c)  $(D-d)^2/(D+d)^2$  (d)  $\sqrt{(D/d)}$



Let  $O$  be the size of object held perpendicular to the principal axis of the lens. A real, inverted and magnified image of size  $I_1$  is formed when the lens is at position  $L_1$ . When the lens is shifted to position  $L_2$  after moving to a distance  $d$ , a diminished image of size  $I_2$  is formed.

The magnification produced by lens, when image size is  $I_1$ .

$$m_1 = \frac{I_1}{O} = \frac{v}{u} \quad \dots(i)$$

The magnification produced by lens, when image size is  $I_2$ .

$$m_2 = \frac{I_2}{O} = \frac{u}{v} \quad \dots(ii)$$

(By the principle of conjugate focii we can assume position of image as object position and vice-versa)

From equation (i) and (ii), we get

$$m_1 m_2 = \frac{I_1}{O} \times \frac{I_2}{O} = \frac{v}{u} \times \frac{u}{v}$$

or  $m_1 m_2 = 1$

and  $O = \sqrt{I_1 I_2}$

Again, from equation (i) and (ii)

$$\frac{m_1}{m_2} = \frac{I_1}{I_2} = \frac{v^2}{u^2}$$

From the figure,  $D = u + v$

and  $d = v - u$

Then  $v = \frac{D + d}{2}$  and  $u = \frac{D - d}{2}$

Hence,  $\frac{m_1}{m_2} = \frac{I_1}{I_2} = \left(\frac{D + d}{D - d}\right)^2$

Using lens formula  $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$  and putting the value of

$u = -\left(\frac{D - d}{2}\right)$  and  $v = +\left(\frac{D + d}{2}\right)$ , we get

$$f = \frac{D^2 - d^2}{4D}$$

The focal length of lens can also be calculated by relation

$$f = \frac{d}{m_1 - m_2}$$

Thus

- (i) The minimum distance between the object and its real image is  $4f$ .
- (ii) If the distance between object and screen is greater than  $4f$ . There will be two positions separated by  $d$  for the lens which gives sharp image on the screen.
- (iii) As the lens is moved away from the source, the diminished image is formed.

(iv) Power of lens can also be calculated by the relation

$$P = \frac{4D}{D^2 - d^2}$$

A convex lens produces an image of real object

on a screen with magnification  $\frac{1}{2}$  when the

lens is moved 30 cm towards the object, the

magnification of the image on the screen is 2.

Find the focal length of the lens.

**Solution :** Since,  $f = \frac{d}{m_1 - m_2}$

Here  $d = 30$  cm,  $m_1 = 2$  and  $m_2 = \frac{1}{2}$

So,  $f = \frac{30}{2 - \frac{1}{2}} = 20$  cm.

A short linear object of length  $L$  lies on the axis of a spherical mirror of focal length  $f$  at a distance  $u$  from the mirror. Its image has an axial length  $L'$  equal to ?

(a)  $L \left[ \frac{f}{(u-f)} \right]^{1/2}$

(b)  $L \left[ \frac{(u+f)}{f} \right]^{1/2}$

(c)  $L \left[ \frac{(u-f)}{f} \right]^2$

(d)  $L \left[ \frac{f}{(u-f)} \right]^2$

**Solution :**

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad -\frac{dv}{v^2} - \frac{du}{u^2} = 0$$

i.e.,  $dv = -du[v/u]^2$

But  $v = \frac{uf}{(u-f)}$  or  $\frac{v}{u} = \frac{f}{(u-f)}$

So  $dv = -du \left[ \frac{f}{(u-f)} \right]^2$

Hence,  $|dv| = L \left[ \frac{f}{(u-f)} \right]^2$

A concave mirror of focal length  $f$  produces an image  $n$  times the size of the object. If the image is real, then the distance of the object from the mirror is:

- (a)  $(n - 1)f$                       (b)  $[(n - 1)/n]f$   
(c)  $[(n + 1)f/n]$                 (d)  $(n + 1)f$

As the image is real it will be inverted and so

$$m = -(v/u) = -n, \quad \text{i.e.,} \quad v = nu$$

$$\therefore \quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{1}{nu} + \frac{1}{u} = \frac{1}{-f}$$

$$\text{or} \quad \frac{(1 + n)}{nu} = -\frac{1}{f} \quad \text{or} \quad u = -\frac{(n + 1)}{n}f$$

i.e., object is in front of mirror at a distance  $[(n + 1)f/n]$ .

A convex mirror of focal length  $f$  produces an image  $(1/n)$ th of the size of the object. The distance of the object from the mirror is:

- (a)  $nf$                       (b)  $f/n$                       (c)  $(n + 1)f$                 (d)  $(n - 1)f$

Solution :

As the image formed by a convex mirror is always virtual or erect, so

$$m = -(v/u) = + (1/n) \quad \text{or} \quad v = -\frac{u}{n}$$

$$\therefore \quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad -\frac{n}{u} + \frac{1}{u} = \frac{1}{+f}$$

$$\text{or} \quad \frac{-(n - 1)}{u} = \frac{1}{f} \quad \text{or} \quad u = -(n - 1)f$$

i.e., object is in front of mirror at a distance  $(n - 1)f$ .



Optics - 17 ) Application of Geometry in sphere to understand a plano-convex lens problem

Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If the speed of light in the material of the lens is  $2 \times 10^8$  metres per sec, the focal length of the lens is :

- (a) 15 cm (b) 20 cm (c) 30 cm (d) 10 cm

Application of Sagitta Theorem

$$\text{R.I. of material of lens } \mu = \frac{c}{v} = \frac{3 \times 10^8}{2 \times 10^8} = 1.5$$

by Sagitta theorem

$$0.3(2R - 0.3) = 3 \times 3$$

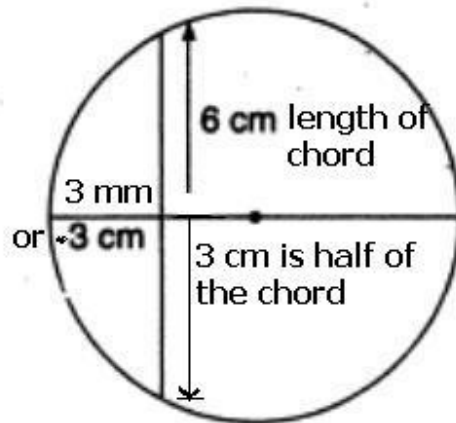
$$0.6R = 9 \quad (\text{neglecting } 0.09)$$

$$R = 15 \text{ cm}$$

$$\frac{1}{f} = (\mu - 1) \left( \frac{1}{R} + \frac{1}{\infty} \right)$$

$$\frac{1}{f} = (1.5 - 1) \left( \frac{1}{15} \right) = \frac{1}{30}$$

$$\text{or } f = 30 \text{ cm.}$$

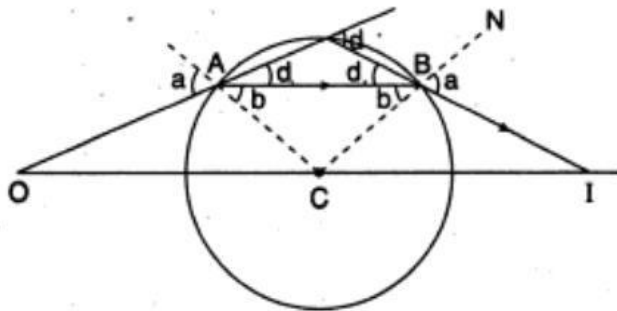


Optics - 18 ) Spherical lens

A ray of light falls on the surface of a spherical glass paper weight making an angle  $\alpha$  with the normal and is refracted in the medium at an angle  $\beta$ . The angle of deviation of the emergent ray from the direction of the incident ray is :

- (a)  $(\alpha - \beta)$  (b)  $2(\alpha - \beta)$   
 (c)  $(\alpha - \beta) / 2$  (d)  $(\beta - \alpha)$

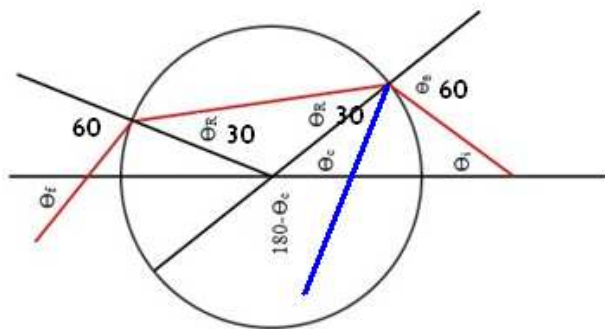
$\angle CAB = \angle CBA = \beta$  because in  $\Delta ABC$



$AC = BC = \text{radius}$   
 Naturally  $\angle IBN = \alpha$   
 $\delta_1 = \alpha - \beta, \delta_2 = \alpha - \beta$   
 $\therefore \delta = \delta_1 + \delta_2 = 2(\alpha - \beta)$

A ray is incident on a sphere, with incidence angle of  $60^\circ$ . Refractive Index of the sphere is  $\sqrt{3}$ . The ray is reflected and refracted on the further surface. The angle between the reflected and refracted surface is ?

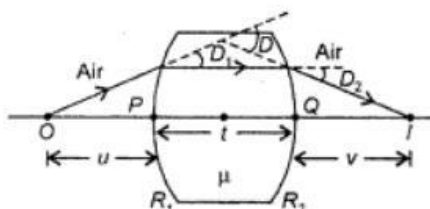
Answer  $90^\circ$



$\sin 60 / \sin r_1 = \sqrt{3} \Rightarrow \sin r_1 = 1/2 \Rightarrow r_1 = 30^\circ$   
 $\sin i_2 / \sin r_2 = \sqrt{3} \Rightarrow i_2 = 60^\circ$  as  $r_1 = r_2 = 30^\circ$   
 Angle of deviation  $180 - (r_2 + i_2) = 180 - 90 = 90$

### Optics - 19 ) Thick lenses

#### Refraction through Thick Lens



- The focal length of thick lens,

$$\frac{1}{f} = (\mu - 1) \left[ \frac{1}{R_1} - \frac{1}{R_2} + \frac{(\mu - 1)t}{\mu R_1 R_2} \right]$$

- Power of thick lens,

$$P = P_1 + P_2 - \frac{P_1 P_2 t}{\mu}$$

Where,  $P_1$  = Power of first refracting surface

$$P_1 = \frac{\mu - 1}{R_1}$$

and  $P_2$  = Power of second refracting surface

$$P_2 = \frac{1 - \mu}{R_2}$$

A convergent thick lens has radii of curvature 10.0 cm and  $-6.0$  cm,  $\mu = 1.60$  and thickness  $t = 5.0$  cm. Deduce its focal length.

**Solution :** Focal length of a lens of thickness  $t$  is given by

$$\frac{1}{f} = (\mu - 1) \left[ \frac{1}{R_1} - \frac{1}{R_2} + \frac{(\mu - 1)t}{\mu R_1 R_2} \right]$$

Here,  $\mu = 1.60$ ,  $R_1 = +10.0$  cm,  $R_2 = -6.0$  cm and  $t = 5.0$  cm.

$$\therefore \frac{1}{f} = (1.60 - 1) \left[ \frac{1}{10.0} + \frac{1}{6.0} + \frac{(1.60 - 1) \times 5.0}{1.60 \times 10.0 \times (-6.0)} \right]$$

$$\text{or} \quad \frac{1}{f} = 0.60 \left[ \frac{1}{10} + \frac{1}{6} - \frac{1}{32} \right]$$

$$\Rightarrow f = +7.14 \text{ cm.}$$

Optics - 20 ) Cauchy's formula for Refractive Index

$$n_{25^\circ\text{C}} = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$$

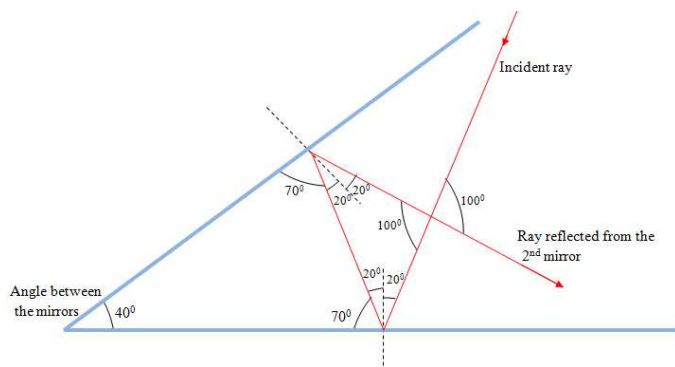
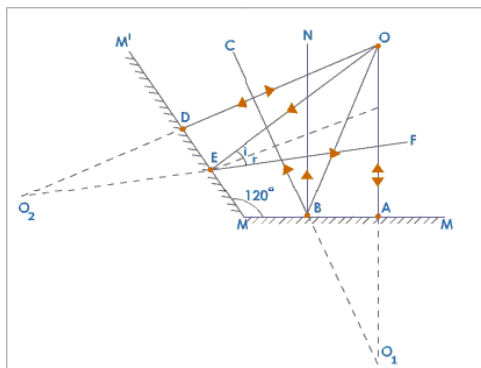
Cauchy's formula for  $\mu$

$$n(\lambda) = B + \frac{C}{\lambda^2},$$

Material	B	C ( $\mu\text{m}^2$ )
Fused silica	1.4580	0.00354
Borosilicate glass BK7	1.5046	0.00420
Hard crown glass K5	1.5220	0.00459
Barium crown glass BaK4	1.5690	0.00531
Barium flint glass BaF10	1.6700	0.00743
Dense flint glass SF10	1.7280	0.01342

-

Optics - 21 ) Reflection images in inclined mirrors



Number of images is given as greatest integer of  $[(360/\theta) - 1]$

-



Optics - 22 ) Optics problems with vectors, 3D imagination

The x - y plane is boundary between two transparent media. Medium-1 with  $z \geq 0$  has a refractive index  $\sqrt{2}$  and medium 2 with  $z \leq 0$  has refractive index  $\sqrt{3}$ . A ray of light in medium-1 given by vector  $\vec{A} = 6\sqrt{3} \hat{i} + 8\sqrt{3} \hat{j} - 10\hat{k}$  is incident on the plane of separation, find the unit vector in the direction of the refracted ray in medium-2.

Solution: Let refracted ray be  $\vec{r} = a\hat{i} + b\hat{j} - c\hat{k}$

$$\text{Normal to plane of incident and normal} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6\sqrt{3} & 8\sqrt{3} & -10 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= 8\sqrt{3} \hat{i} - 6\sqrt{3} \hat{j}$$

it must also be normal to refracted ray

$$\begin{aligned} \therefore \vec{r} \cdot \vec{n} &= 0 \\ \Rightarrow 8\sqrt{3} a - 6\sqrt{3} b &= 0 \Rightarrow 4a = 3b \\ \Rightarrow b &= \frac{4a}{3} \end{aligned}$$

$$\begin{aligned} \therefore \cos(\pi - i) &= \frac{(6\sqrt{3} \hat{i} + 8\sqrt{3} \hat{j} - 10\hat{k}) \cdot \hat{k}}{\left| 6\sqrt{3} \hat{i} + 8\sqrt{3} \hat{j} - 10\hat{k} \right| \left| \hat{k} \right|} \\ &= \frac{-10}{20} = \cos 120^\circ \\ \therefore i &= 60^\circ \end{aligned}$$

$$\sqrt{3} \sin r = \sqrt{2} \sin i = \sqrt{2} \times \frac{\sqrt{3}}{2} \Rightarrow \sin r = \frac{1}{\sqrt{2}}$$

$$r = 45^\circ$$

Now since angle between refracted ray and Normal =  $45^\circ$

$$\therefore \cos 45^\circ = \frac{(a\hat{i} + b\hat{j} + c\hat{k}) \cdot \hat{k}}{\sqrt{a^2 + b^2 + c^2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sqrt{2} c = \sqrt{a^2 + b^2 + c^2}$$

$$\Rightarrow c^2 = a^2 + b^2 = a^2 + \frac{16a^2}{a} = \frac{25a^2}{a}$$

$$\Rightarrow c = \frac{\pm 5a}{3}$$

$$\therefore c = \frac{-5a}{3}$$

$$\therefore r = a\hat{i} + \frac{4a}{3}\hat{j} - \frac{5a}{3}\hat{k} = \frac{a}{3}(3\hat{i} + 4\hat{j} - 5\hat{k})$$

$$y = \frac{3\hat{i} + 4\hat{j} - 5\hat{k}}{\sqrt{50}} = \frac{1}{5\sqrt{2}}(3\hat{i} + 4\hat{j} - 5\hat{k})$$

Optics - 23 ) Problems with continuously varying refractive index ( First asked in IPhO and then in IIT JEE )

A ray of light in air is incident at grazing angle ( $i = 90^\circ$ ) on a long rectangular slab of a transparent medium of thickness  $t = 1.0$  m. The point of incidence is the origin  $A(0, 0)$ .

The medium has a variable index of refraction  $n(y)$  given by  $n(y) = \sqrt{ky^{3/2} + 1}$  where  $k = 1.0 \text{ m}^{-3/2}$ .

The refractive index of air is 1. (i) Obtain a relation between the slope of the trajectory of the ray at a point  $B(x, y)$  in the point. (ii) Obtain an equation for trajectory  $y(x)$  of the ray in

the point. (iii) Determine the co-ordinates  $(x, y_1)$  of the point P where the ray intersects the upper surface of the slab-air boundary. (d) Indicate the path of the ray subsequently.

Solution:

Taking on arbitrary point P(x, y) refractive index at this point  $n = \left(y^{3/2} + 1\right)^{1/2}$

from Snell's law  $n \sin \theta = \text{constant}$  applying this for initial pt. (when ray is entering medium B) and at point.

$$1 \times \sin 90^\circ = \sqrt{\left(y^{3/2} + 1\right)} \sin i$$

$$\Rightarrow \sin i = \frac{1}{\sqrt{y^{3/2} + 1}} \quad \text{it can be seen that } i = \frac{\pi}{2} - \theta$$

$$\therefore \text{Slope} = \tan \theta = \cot i = \frac{dy}{dx}$$

$$(ii) \frac{dy}{dx} = \cot i = \frac{y^{3/4}}{1}$$

$$\Rightarrow \int y^{-3/4} dy = \int dx$$

$$\Rightarrow x = 4y^{1/4} + C$$

it passes through origin  $\therefore C = 0$

$\therefore x = 4y^{1/4}$  is the equation of trajectory

when ray comes out of the mediums

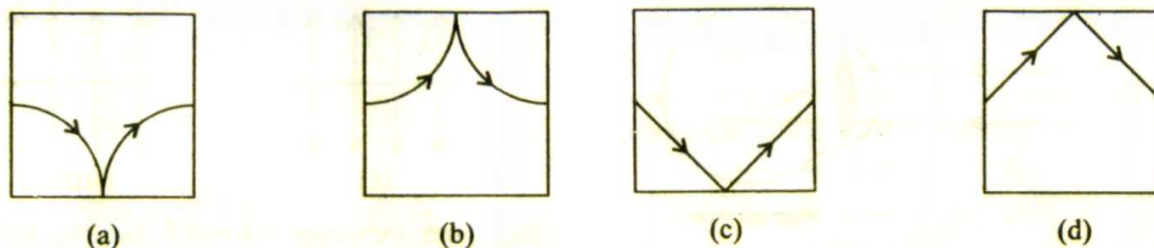
$$\text{then } x = 4 \times 1 = 4$$

$\therefore$  Co-ordinate of pt- is (4, 1)

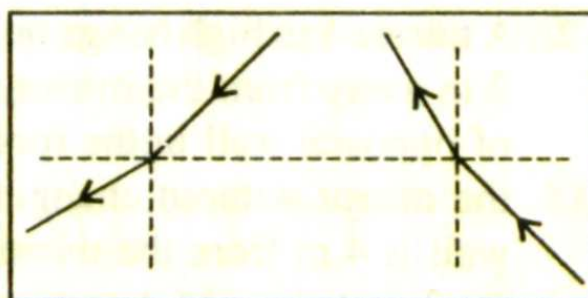
If medium on both sides are same, then angle with which the ray enters the medium = angle with which the ray comes out.

$\therefore$  Ray will be parallel to x-axis.

A cubic container is filled with a liquid whose refractive index increases linearly from top to bottom. Which of the following represents the path of a ray of light inside the liquid?



Since the refractive index is changing, the light cannot travel in a straight line in the liquid as shown in options (c) and (d). Initially, it will bend towards normal and after reflecting from the bottom it will bend away from the normal as shown in the figure.

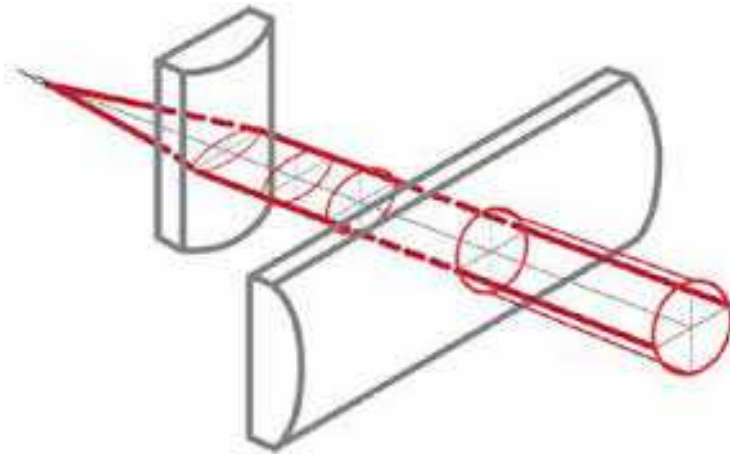


Optics - 24 ) Cylindrical lens ( IIT JEE 1999 )

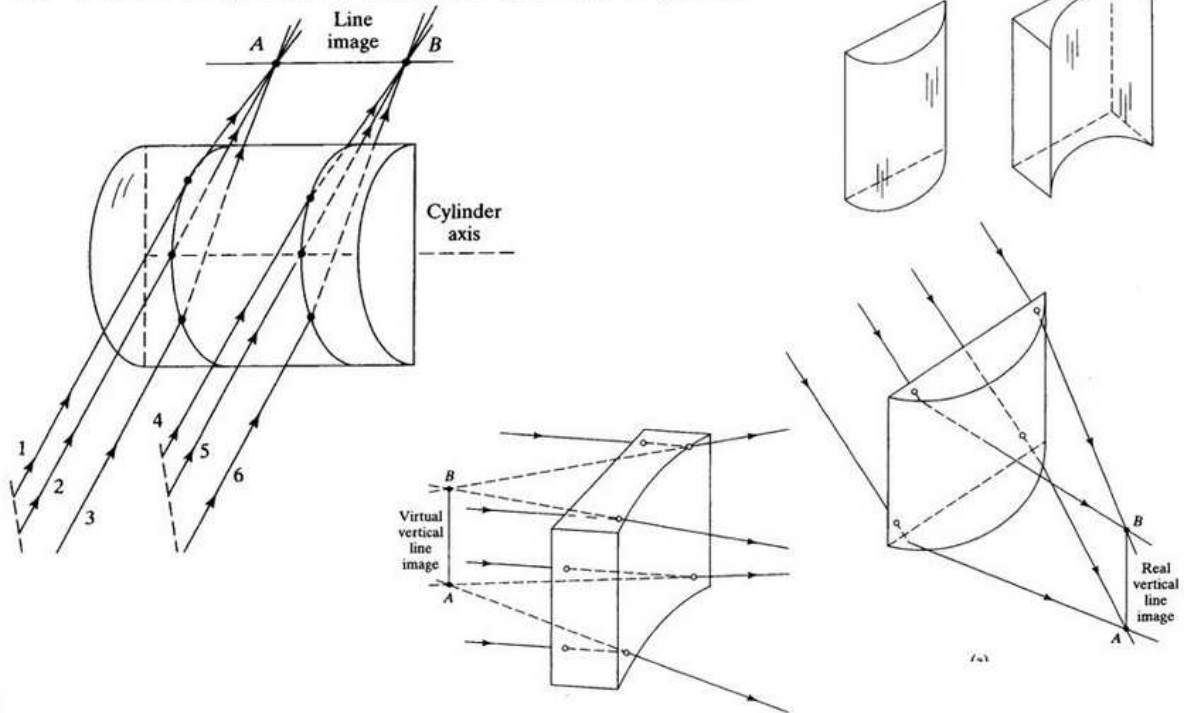
A thin slice is cut out of a glass cylinder along a plane parallel to its axis. The slice is placed on a flat plate . The observed interference fringes from this combination shall be

1. Straight
2. Circular
3. Equally spaced
4. Having fringe spacing which increases as we go outwards



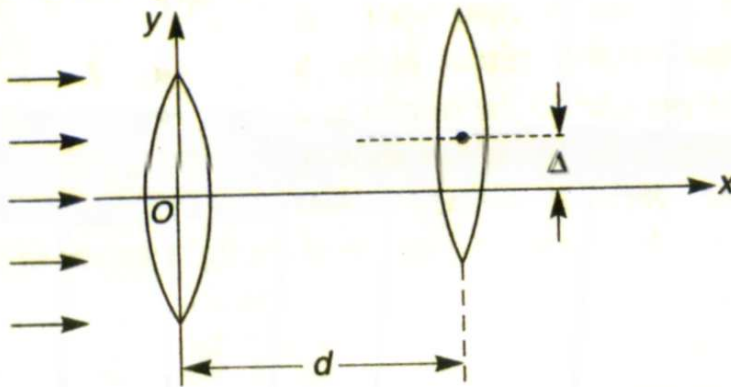


**Cylindrical Lens:** Cylindrical lens is a section of a cylindrical rod. One surface is cylindrical while the opposite is plane.



Optics - 25 ) Two lenses or mirrors whose axis is not coinciding ( IIT JEE 1993 ) Shifted lenses or mirrors

Two thin convex lenses of focal lengths  $f_1$  and  $f_2$  are separated by a horizontal distance  $d$  (where  $d < f_1, d < f_2$ ) and their centres are displaced by a vertical separation  $\Delta$  as shown in the figure. (1993)



Taking the origin of coordinates,  $O$ , at the centre of the first lens, the  $x$  and  $y$ -coordinates of the focal point of this lens system, for a parallel beam of rays coming from the left, are given by (1993; 2M)

(a)  $x = \frac{f_1 f_2}{f_1 + f_2}, y = \Delta$

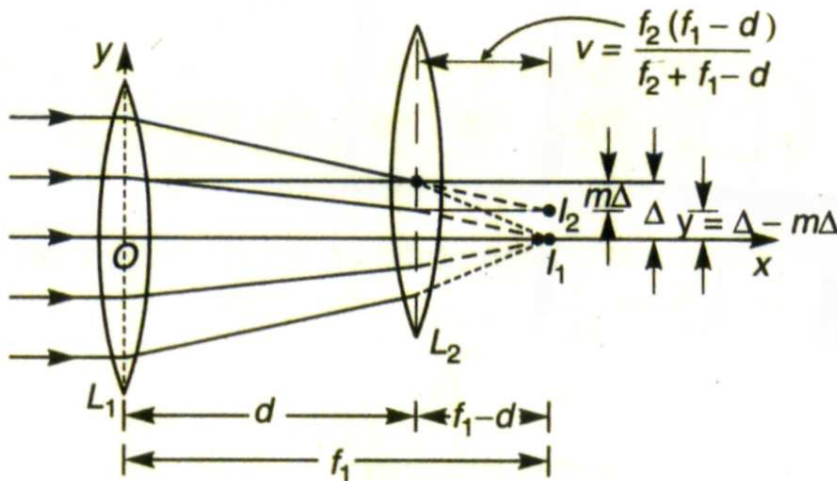
(b)  $x = \frac{f_1(f_2 + d)}{f_1 + f_2 - d}, y = \frac{\Delta}{f_1 + f_2}$

(c)  $x = \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}, y = \frac{\Delta(f_1 - d)}{f_1 + f_2 - d}$

(d)  $x = \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}, y = 0$

Solution

From the first lens parallel beam of light is focused at its focus *i.e.*, at a distance  $f_1$  from it. This image  $I_1$  acts as virtual object for second lens  $L_2$ . Therefore, for  $L_2$

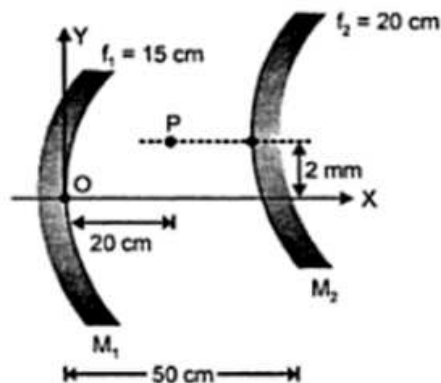


$$u = +(f_1 - d), f = +f_2$$

$$\therefore \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{f_2} + \frac{1}{f_1 - d}$$

<https://archive.org/details/IITJEE1993OpticsInterestingShiftedLensImageMagnificationAndPosition>

**Find the co-ordinates of image of point object P formed after two successive reflection in situation as shown in fig. considering first reflection at concave mirror and then at convex mirror.**



So  $f_1 = -15$  cm

$$v_1 = \frac{u \cdot f_1}{u - f_1} = \frac{(-20)(-15)}{-20 + 15}$$

or

$$v_1 = -60 \text{ cm}$$

$$\text{Magnification } (m_1) = -\frac{v_1}{u} = -\frac{-60}{-20} = -3 \quad (\text{Inverted})$$

$$\therefore A'P' = m_1(AP) = 3 \times 2 = 6 \text{ mm}$$

For reflection at convex mirror  $M_2$

$$u = +10 \text{ cm}$$

$$f_2 = +20 \text{ cm}$$

$$v_2 = \frac{u \cdot f_2}{u - f_2} = \frac{(10)(20)}{10 - 20} = -20 \text{ cm}$$

$$\text{Magnification } m_2 = -\frac{v_2}{u} \Rightarrow -\frac{-20}{10} \Rightarrow 2$$

$$\therefore C^*P^* = m_2(C'P') = 2 \times 8 = 16 \text{ mm}$$

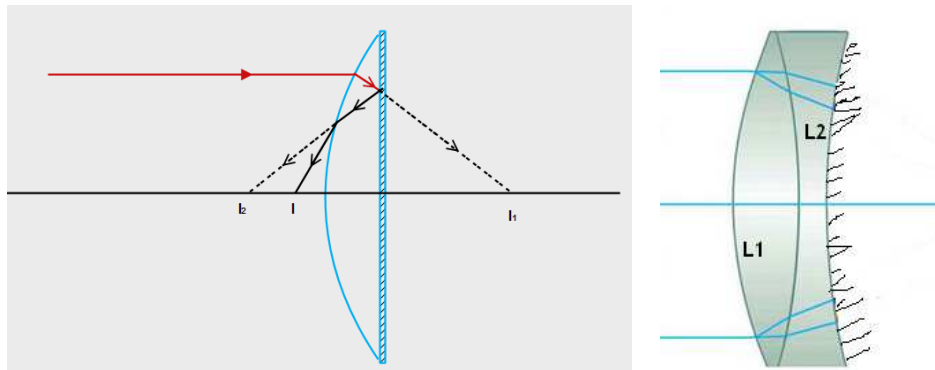
So, the co-ordinate of image of point object P (30 cm, -14 mm).

Optics - 26 ) Painted lens or Combination of lenses where the last one is painted ( silvered )

If I am recalling correctly IIT JEE and other exams ( till 2016 ) had more than 10 questions of this kind. Most books do not discuss the easy formula of  $-1/F = 2/f_{L1} + 2/f_{L2} - 1/f_m$

( In 1990 I had derived this formula of my own for quick solving of this kind of problems )





$F_m$  is focal length of the mirror as  $R/2$  +ve or -ve as per conditions

The plane face of a plano-convex lens is silvered. If  $\mu$  be the refractive index and  $R$ , the radius of curvature of curved surface, then the system will behave like a concave mirror of radius of curvature:

- (a)  $\mu R$  (b)  $R/(\mu - 1)$   
 (c)  $R^2/\mu$  (d)  $[(\mu + 1)/(\mu - 1)]R$

Solution :

Focal length of planar side is  $f_m = R/2 = -\infty$

$\frac{1}{f_l} = (\mu - 1) \left( \frac{1}{R} \right)$  by lens makers formula.  $R$  is positive because center of curvature is on right side

Use  $-1/F = 2/f_{L1} + 1/f_m$  or  $1/F = -\frac{2(\mu - 1)}{R}$  or  $F = \frac{-R}{2(\mu - 1)}$

$R$  ( equivalent ) =  $2F = \frac{-R}{(\mu - 1)}$

We don't have to use the formula  $-1/F = 2/f_{L1} + 2/f_{L2} - 1/f_m$  for every problem

See a Karnataka CET problem of 2004 ( Was also asked in IIT JEE and solved in "Concepts of Physics by Professor H C Verma )

A thin plano-convex lens acts like a concave mirror of focal length 0.2 m, when silvered on its plane surface. The refractive index of the material of lens is 1.5. The radius of curvature of the convex surface of the lens will be:

[CET (Karnataka) 2004]

- (a) 0.1 m      (b) 0.2 m      (c) 0.4 m      (d) 0.8 m

Solution :

Given focal length of mirror when its plane surface is silvered ( $f_m$ ) = 0.2 m. Radius of curvature of curved surface ( $R_1$ ) =  $R$ ; radius of curvature of plane side ( $R_2$ ) =  $\infty$ ; refractive index of the material of lens ( $\mu$ ) = 1.5.

Since a thin plano-convex lens acts like a concave mirror when silvered on its plane surface, therefore focal length of lens ( $f$ ) =  $2 \times f_m = 2 \times 0.2 = 0.4$  m.

We know that

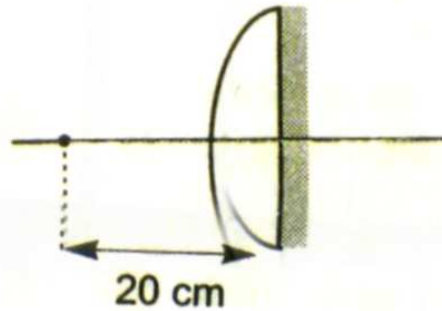
$$\frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

or 
$$\frac{1}{0.4} = (1.5 - 1) \left( \frac{1}{R} - \frac{1}{\infty} \right) = \frac{0.5}{R}$$

$\therefore R = 0.2 \text{ m}$

IIT JEE 2006

A point object is placed at a distance of 20 cm from a thin planoconvex lens of focal length 15 cm. The plane surface of the lens is now silvered. The image created by the system is at (2006, 3M)



- (a) 60 cm to the left of the system
- (b) 60 cm to the right of the system
- (c) 12 cm to the left of the system
- (d) 12 cm to the right of the system

Solution :

Long method

$$\text{Refraction from lens : } \frac{1}{v_1} - \frac{1}{-20} = \frac{1}{15}$$

$$\therefore v = 60 \text{ cm} \quad \xrightarrow{\text{+ ve direction}}$$

ie, first image is formed at 60 cm to the right of lens system.

**Reflection from mirror**

After reflection from the mirror, the second image will be formed at a distance of 60 cm to the left of lens system.

**Refraction from lens**

$$\frac{1}{v_3} - \frac{1}{60} = \frac{1}{15} \quad \longleftarrow \text{+ ve direction}$$

or  $v_3 = 12 \text{ cm}$

Therefore, the final image is formed at 12 cm to the left of the lens system.

Shorter Method

$$\frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\text{Use } F = \frac{-R}{2(\mu - 1)} \quad \text{and} \quad (1.5 - 1) \left( \frac{1}{R} - \frac{1}{\infty} \right) = \frac{0.5}{R} = 1/(2R)$$

$$1/15 = 1/2R \Rightarrow 15 = 2R \Rightarrow R = 7.5 \text{ cm}$$

$$F = -7.5 / (2 \times 0.5) = -7.5 = -15/2$$

Using  $1/v + 1/u = 1/F$  for equivalent mirror

$$1/v + 1/(-20) = 1/(-7.5)$$

$$\Rightarrow 1/v = 1/20 - 2/15 = (3 - 8)/60 = -5/60 = -1/12$$

$$\Rightarrow v = -12 \text{ cm}$$

Even more shorter method

If I am appearing for an exam I would have done  $-1/F = 2/f_{L1} - 1/f_m$

$$\text{So } -1/F = 2/(15) - 1/(\infty) = 1/7.5 - 0 \Rightarrow F = -7.5 \text{ cm}$$

Then Using  $1/v + 1/u = 1/F$  for equivalent mirror

$$1/v + 1/(-20) = 1/(-7.5)$$

$$\Rightarrow 1/v = 1/20 - 2/15 = (3 - 8)/60 = -5/60 = -1/12$$

$$\Rightarrow v = -12 \text{ cm}$$

-

IIT JEE 1978

**A pin is placed 10 cm in front of a convex lens of focal length 20 cm and made of a material of refractive index 1.5. The convex surface of the lens farther away from the pin is silvered and has a radius of curvature of 22 cm. Determine the position of the final image. Is the image real or virtual ? (1978)**



Let us use  $-1/F = 2/f_{L1} - 1/f_m$

And  $\frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$  giving  $1/20 = 0.5 ( 1/R_1 - 1/(-22) )$  or  $R_1 = 55/3$

$R_1$  actually is not required. We can find  $f_m$  as  $R_2 / 2 = - 11$  cm

So  $- 1/F = 2/20 - 1/(- 11 ) = 1/10 + 1/11 = 21/110$

or  $F = - 110/21$  ( not required !  $1/F = -21/110$  is enough )

Using mirror formula  $1/v + 1/u = 1/F$

So  $1/v + 1 / ( -10 ) = -21/110$

$\Rightarrow 1/v = 1/10 - 21/110 = ( 11 - 21 )/110 = -10/110 = -1/11$

$\Rightarrow v = -11$  cm

virtual image on left at 11 cm

( Now do you guys see that even though we got problems of this kind since 1978 and before, but yet the formula is not there in every book ! )

-

IIT JEE 1979

**The radius of curvature of the convex face of a planoconvex lens is 12 cm and its  $\mu = 1.5$ .**

**(a) Find the focal length of the lens. The plane face of the lens is now silvered.**

**(b) At what distance from the lens will parallel rays incident on the convex surface converge ?**

**(c) Sketch the ray diagram to locate the image, when a point object is placed on the axis 20 cm from the lens.**

**(d) Calculate the image distance when the object is placed as in (c)**

**(1979)**

Now you know that this problem can be solved by 3 different ways.

The longest method being successive image method. Meaning find the first image due to lens, then 2<sup>nd</sup> image due to silvered surface as mirror. The 3<sup>rd</sup> and final image is due to light travelling from right to left through the lens again.

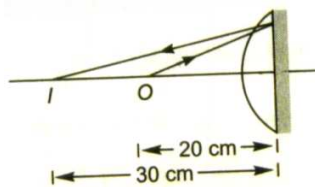
I will discuss the shorter methods

$$\begin{aligned} \text{(a)} \quad \frac{1}{f} &= (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \\ &= (1.5 - 1) \left( \frac{1}{12} - \frac{1}{\infty} \right) \\ &= \frac{1}{24} \\ \therefore f &= +24 \text{ cm} \end{aligned}$$

(b) use  $-1/F = 2/f_L$  so  $F = -12$  cm

The system will act as a concave mirror of focal length 12 cm. The parallel rays will converge at 12 cm left of this silvered lens.

(c)



(d)

Using mirror formula

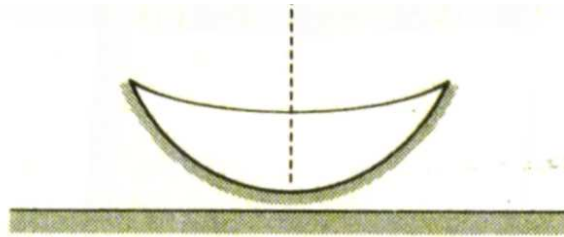
$$\frac{1}{v} - \frac{1}{20} = \frac{-1}{12}$$

Solving we get  $v = -30$  cm.

Therefore the image will be formed at a distance of 30 cm to the left of system.

IIT JEE 1981

The convex surface of a thin concavo-convex lens of glass of refractive index 1.5 has a radius of curvature



20 cm. The concave surface has a radius of curvature 60 cm. The convex side is silvered and placed on a horizontal surface. **(1981, 2M)**

- (a) Where should a pin be placed on the optic axis such that its image is formed at the same place?
- (b) If the concave part is filled with water of refractive index  $\frac{4}{3}$ , find the distance through which the pin should be moved, so that the image of the pin again coincides with the pin.

I will prefer to solve this by  $-1/F = 2/f_{L1} + 2/f_{L2} - 1/f_m$  ( note it was a 2 marks problem )

While for practice and to know how successive image method of solving works see ...

Image of object will coincide with it if ray of light after refraction from the concave surface fall normally on concave mirror so formed by silvering the convex surface. Or image after refraction from concave surface should form at centre of curvature of concave mirror or at a distance of 20 cm on same side of the combination. Let  $x$  be the distance of pin from the given optical system.

Applying.

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

With proper signs

$$\frac{1.5}{-20} - \frac{1}{-x} = \frac{1.5 - 1}{-60}$$

or 
$$\frac{1}{x} = \frac{3}{40} - \frac{1}{120} = \frac{8}{120}$$

$\therefore x = \frac{120}{8} = 15 \text{ cm}$

(b)

Now, before striking with the concave surface, the ray is first refracted from a plane surface. So, let  $x$  be the distance of pin, then the plane surface will form its image at a distance  $\frac{4}{3}x$  ( $h_{\text{app.}} = \mu h$ ) from it.

Now, using  $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$  with proper signs,

we have 
$$\frac{1.5}{-20} - \frac{4/3}{-4x} = \frac{1.5 - 4/3}{-60}$$

or 
$$\frac{1}{x} = \frac{3}{40} - \frac{1}{360}$$

or 
$$x = 13.84 \text{ cm}$$

$\therefore \Delta x = x_1 - x_2$   
 $= 15 \text{ cm} - 13.84 \text{ cm}$   
 $= 1.16 \text{ cm} \quad \text{(downwards)}$

Now can you guys check the results using  $-1/F = 2/f_{L1} + 2/f_{L2} - 1/f_m$



A plano-convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now this lens has been used to form the image of an object. At what distance from this lens an object be placed in order to have a real image of the size of the object? (AIEEE 2004)

- (a) 20 cm                      (b) 30 cm  
(c) 60 cm                      (d) 80 cm

Solution :

To obtain the real image of the size of the object, the object must be placed at the centre of curvature of the equivalent mirror formed as a result of silvering

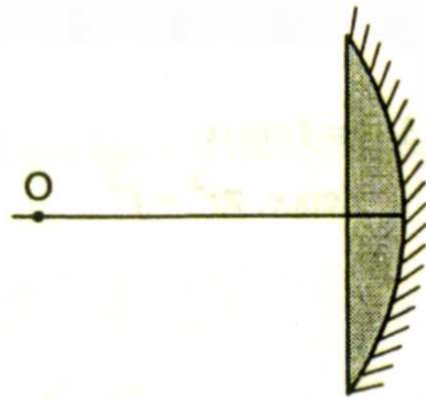
$$\frac{1}{F} = \frac{2}{f_l} + \frac{1}{f_m}$$

$$\text{and } \frac{1}{f_l} = (1.5 - 1) \left( \frac{1}{\infty} - \frac{1}{-30} \right) = \frac{1}{60}$$

$$\text{and } f_m = 15 \text{ cm}$$

$$\therefore F = 10 \text{ cm}$$

Hence, object should be placed at 20 cm from the lens because radius of curvature of equivalent mirror =  $2F = 2 \times 10 = 20$  cm. Hence, option (a) is correct.



Video explanations of Painted or Silvered lenses

<https://archive.org/details/PaintedLensIITJEEProblemImageNeedsToCoincideWithObjectHCVPof.HCVermaPart1>

Optics - 27 ) Image speed when object is moving as seen from various mirrors and lenses ( concave, convex, silvered etc )

Mirror formula (  $1/v + 1/u = 1/f$  ) or Lens formula (  $1/v - 1/u = 1/f$  ) have to be differentiated to find  $du/dt$  or  $dv/dt$

A luminous point is moving at speed  $v_0$  towards a spherical mirror, along its axis. Then the speed at which the image of this point object is moving is given by: (with  $R$  = radius of curvature and  $u$  = object distance)

$$(a) v_i = -v_0 \qquad (b) v_i = -v_0 \left( \frac{R}{2u - R} \right)$$

$$(c) v_i = -v_0 \left( \frac{2u - R}{R} \right) \qquad (d) v_i = -v_0 \left( \frac{R}{2u - R} \right)^2$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad -\frac{1}{v^2} \frac{dv}{dt} - \frac{1}{u^2} \frac{du}{dt} = 0$$

$$\therefore \frac{dv}{dt} = v_i = -\left(\frac{v}{u}\right)^2 \frac{du}{dt} = -\left(\frac{v}{u}\right)^2 v_0$$

$$\text{Now, } \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{2}{R} - \frac{1}{u} = \frac{2u - R}{Ru}$$

$$\therefore v = \frac{uR}{2u - R}$$

$$\therefore v_i = -\left(\frac{v}{u}\right)^2 v_0 = -v_0 \left(\frac{R}{2u - R}\right)^2$$

Optics - 28 ) Slab with a hole or gap, then may be filled with liquid etc

Given  ${}^a\mu_g = 3/2$  and  ${}^a\mu_w = 4/3$ . There is an equiconvex lens with radius of each surface equal to 20 cm. There is air in the object space and water in the image space. The focal length of lens is:

- (a) 80 cm      (b) 40 cm      (c) 20 cm      (d) 10 cm

Solution :

$$\begin{aligned}\frac{{}^a\mu_w}{f} &= \frac{({}^a\mu_g - 1)}{R_1} - \frac{({}^a\mu_g - {}^a\mu_w)}{R_2} \\ &= \frac{\left(\frac{3}{2} - 1\right)}{20} - \frac{\left(\frac{3}{2} - \frac{4}{3}\right)}{-20} = \frac{1}{40} + \frac{1}{120} = \frac{1}{30} \\ f &= \frac{4}{3} \times 30 = 40 \text{ cm}\end{aligned}$$

Optics - 29 ) Constraint in interference conditions

Two identical coherent sources are placed on a diameter of a circle of radius  $R$  at separation  $x$  ( $\ll R$ ) symmetrically about the centre of the circle. The sources emit identical wavelength  $\lambda$  each. The number of points on the circle with maximum intensity is: ( $x = 5\lambda$ )

- (a) 20      (b) 22      (c) 24      (d) 26

Solution :

Path difference at  $P$  is

$$\Delta x = 2 \left( \frac{x}{2} \cos \theta \right) = x \cos \theta$$

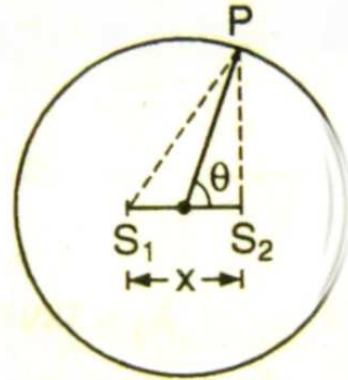
For intensity to be maximum,

$$\Delta x = n\lambda$$

$$(n = 0, 1, 2, \dots)$$

$$\therefore x \cos \theta = n\lambda$$

$$\cos \theta = \frac{n\lambda}{x}$$



$$\cos \theta \neq 1$$

$$\therefore \frac{n\lambda}{x} \neq 1$$

$$\therefore n \neq \frac{x}{\lambda}$$

$$\text{Putting } x = 5\lambda, \quad n \neq 5$$

$$\text{or } n = 1, 2, 3, 4, 5$$

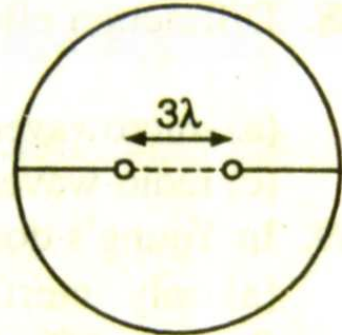
Therefore in all four quadrants there can be 20 maximas. There are

more maximas at  $\theta = 0^\circ$  and  $\theta = 180^\circ$ .

But  $n = 5$  corresponds to  $\theta = 90^\circ$  and  $\theta = 270^\circ$  which are coming only twice while we have multiplied it four times. Therefore, total number of maximas are still 20, *i.e.*,  $n = 1$  to 4 in four quadrants (total 16) plus four more at  $\theta = 0^\circ, 90^\circ, 180^\circ$  and  $270^\circ$ .



If two coherent sources are placed at a distance  $3\lambda$  from each other symmetric to the centre of the circle shown in the figure, then number of fringes shown on the screen placed along the circumference is: **(UPSEAT 2002)**



- (a) 16
- (b) 12
- (c) 8
- (d) 4

Answer ( b ) See above Solution

White light is used to illuminate the two slits in a Young's double slit experiment. The separation between the slits is  $b$  and the screen is at a distance  $d (\gg b)$  from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing. Some of these missing wavelengths are: **[CET (J&K) 2003; PET (Kerala) 2006]**

- (a)  $\lambda = 3b^2/d$
- (b)  $\lambda = 2b^2/d$
- (c)  $\lambda = b^2/3d$
- (d)  $\lambda = 2b^2/3d$

Solution :

$$\text{Path difference} = (S_2P - S_1P)$$

$$\text{From figure, } (S_2P)^2 - (S_1P)^2 = b^2$$

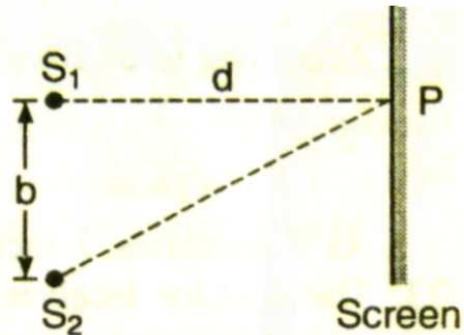
$$\text{or } (S_2P - S_1P)(S_2P + S_1P) = b^2$$

$$\text{or } (S_2P - S_1P) = \frac{b^2}{2d}$$

$$\text{For dark fringes, } \frac{b^2}{2d} = (2n + 1) \frac{\lambda}{2}$$

$$\text{For } n = 0, \quad \frac{b^2}{2d} = \frac{\lambda}{2} \quad \text{or} \quad \lambda = \frac{b^2}{d}$$

$$\text{For } n = 1, \quad \frac{b^2}{2d} = \frac{3\lambda}{2} \quad \text{or} \quad \lambda = \frac{b^2}{3d}$$



Optics - 30 ) Silvered Prisms or Painted Prisms

If one face of a prism of prism angle  $30^\circ$  and  $\mu = \sqrt{2}$  is silvered, the incident ray retraces its initial path. The angle of incidence is:

- (a)  $60^\circ$       (b)  $30^\circ$       (c)  $45^\circ$       (d)  $90^\circ$

Solution : ( c )

It is clear from the figure that the ray will retrace the path when the refracted ray  $QR$  is incident normally on the polished surface  $AC$ . Thus, angle of refraction  $r = 30^\circ$ .

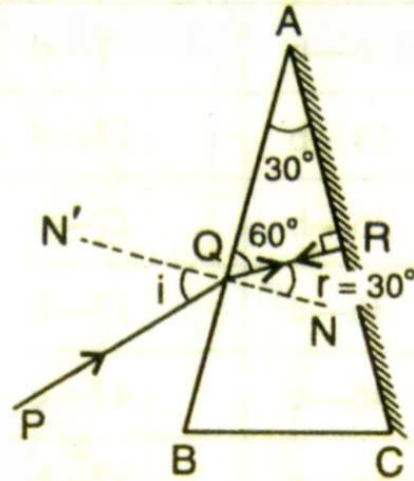
We know that  $\mu = \sin i / \sin r$

$$\therefore \sin i = \mu \sin r$$

$$= \sqrt{2} \times \sin 30^\circ = \sqrt{2} \times \frac{1}{2}$$

$$= (1/\sqrt{2})$$

$$\therefore i = 45^\circ$$

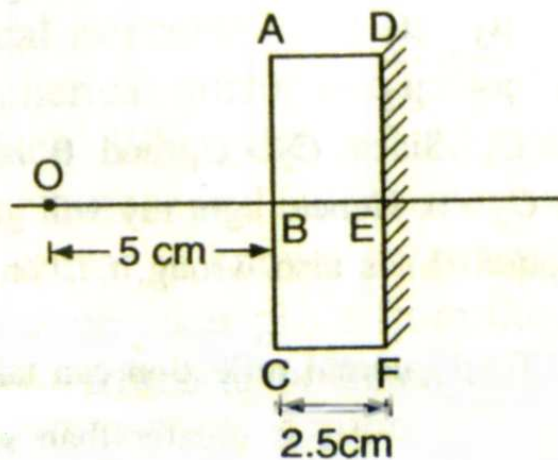


Optics - 31 ) A slab is silvered on one side or Painted on one side

A plane mirror is made of a glass slab ( $\mu_g = 1.5$ ) 2.5 cm thick and silvered on its back. A point object is placed 5 cm in front of the unsilvered face of the mirror. What will be the position of the final image?

- (a) 12 cm from unsilvered face
- (b) 14.6 cm from unsilvered face
- (c) 5.67 cm from unsilvered face
- (d) 8.33 cm from unsilvered face

Solution : ( d )



Let  $I_1$ ,  $I_2$  and  $I_3$  be the images formed by

- (i) refraction from  $ABC$
- (ii) reflection from  $DEF$  and
- (iii) again refraction from  $ABC$

Then  $BI_1 = (5)\mu_g = 5 \times 1.5 = 7.5 \text{ cm}$

Now  $EI_1 = 7.5 + 2.5 = 10 \text{ cm}$

$\therefore EI_2 = 10 \text{ cm}$  behind the mirror

Now,  $BI_2 = (10 + 2.5) = 12.5 \text{ cm}$

$\therefore BI_3 = \frac{12.5}{\mu_g} = \frac{12.5}{1.5} = 8.33 \text{ cm}$



### Real and apparent depth:

- (i) When one looks into a pool of water, it does not appear to be as deep as it really is. Also when one looks into a slab of glass, the material does not appear to be as thick as it really is. This all happens due to refraction of light.
- ii) If a beaker is filled with water and a point lying at its bottom is observed by someone located in air, then the bottom point appears raised. The apparent depth  $t_{ap}$  is less than the actual depth  $t_{ac}$ . It can be shown that

$$\text{apparent depth } (t_{ap}) = \frac{\text{actual depth } (t_{ac})}{\text{refractive index } (n)}$$

iii )

If there is an ink spot at the bottom of a glass slab, it appears to be raised by a distance

$$d = t_{ac} - t_{ap} = t - \frac{t}{n} = t \left( 1 - \frac{1}{n} \right)$$

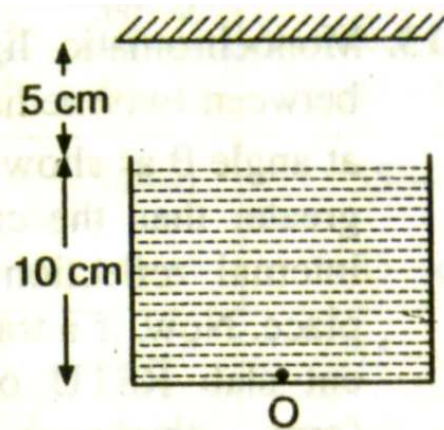
where  $t$  is the thickness of the glass slab and  $n$  is its refractive index.

iv )

If a beaker is filled with immisible transparent liquids of refractive indices  $n_1, n_2, n_3$  and individual depth  $d_1, d_2, d_3$  respectively, then the apparent depth of the beaker is found to be:

$$t_{\text{ap}} = \frac{d_1}{n_1} + \frac{d_2}{n_2} + \frac{d_3}{n_3}$$

Consider the situation shown in figure. Water ( $\mu_w = 4/3$ ) is filled in a beaker upto a height of 10 cm. A plane mirror is fixed at a height of 5 cm from the surface of water. Distance of image from the mirror after reflection from it of an object  $O$  at the bottom of the beaker is:



- (a) 15 cm
- (b) 12.5 cm
- (c) 7.5 cm
- (d) 10 cm

Solution : ( b )

Distance of first image ( $I_1$ ) formed after refraction from the plane surface of water is  $= \frac{10}{4/3} = 7.5$  cm from water surface

$$\left[ \because d_{\text{app}} = \frac{d_{\text{actual}}}{\mu} \right]$$

Now distance of this image is  $5 + 7.5 = 12.5$  cm from the plane mirror. Therefore, distance of second image ( $I_2$ ) will also be equal to 12.5 cm from the mirror.



A beaker containing liquid is placed on a table, underneath a microscope which can be moved along a vertical scale. The microscope is focussed through the liquid on to a mark on

the table when the reading on the scale is  $a$ . It is next focussed on the upper surface of the liquid and the reading is  $b$ . More liquid is added and the observations are repeated, the corresponding readings are  $c$  and  $d$ . The refractive index of the liquid is:

- (a)  $\frac{d-b}{d-c-b+a}$                       (b)  $\frac{b-d}{d-c-b+a}$   
(c)  $\frac{d-c-b+a}{d-b}$                       (d)  $\frac{d-b}{a+b-c-d}$

Solution : ( a )

The real depth = R.I.  $\times$  apparent depth

In first case,

The real depth  $h_1 = n(b - a)$

Similarly, in the second case, the real depth  $h_2 = n(d - c)$

Since,  $h_2 > h_1$ , the difference of real depths

$$= h_2 - h_1 = n(d - c - b + a)$$

Since the liquid is added in second case,

$$h_2 - h_1 = d - b$$

$$\therefore n = \frac{d - b}{d - c - b + a}$$

Optics - 32 ) In YDSE experiment the light falls at an angle on 2 slits

**Example:** Recalculate the angular spread to the above problem if the incidence is at an angle of  $15^\circ$  with the normal to the plane of the slit.

**Solution.** (a) Let us first consider a point P (above centre O of the screen) on the screen as shown in Fig. From B, drop a perpendicular  $BN'$ . From A, drop a perpendicular  $AN$  on BP. If first minimum is formed at P, then the corresponding path difference is given by

$$BN - AN' = \lambda$$

$$BN - AN' = \lambda$$

or  $d \sin \theta_1 - d \sin 15^\circ = \lambda$

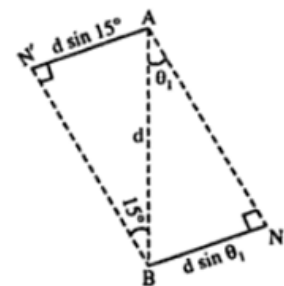
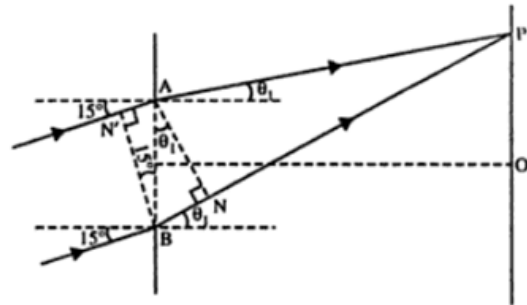
or  $\sin \theta_1 - \sin 15^\circ = \frac{\lambda}{d} = \frac{2 \text{ cm}}{5 \text{ cm}} = 0.4$

or  $\sin \theta_1 = 0.4 + \sin 15^\circ = 0.4 + 0.2588 = 0.6588$

or  $\theta_1 = \sin^{-1}(0.6588) = 41^\circ 13'$  (from tables of natural sines)

(b) Let us now consider a point P' below O. Let the first minimum be at P'. Then, the corresponding path difference is given by

$$N'A + AN = \lambda \quad \text{or} \quad d \sin 15^\circ + d \sin \theta_2 = \lambda$$



Optics - 33 ) Diffraction Grating

**Example** A diffraction grating one cm wide has 1000 lines and is used in third order. What are the diffraction angles for violet and orange light? What is the angular size of the diffraction maximum for monochromatic light? The wavelengths for violet and orange are 400 nm and 600 nm respectively.

**Solution.** For third order,  $n = 3$ ,  $\theta_o = \frac{3 \times 4 \times 10^{-7}}{10^{-5}} \text{ rad} = 12 \times 10^{-2} \text{ rad} = 6.9^\circ$

$$\theta_o = 18 \times 10^{-2} \text{ rad} = 10.3^\circ$$

The spectrum is thus spread over an angle of nearly  $3.4^\circ$ .

At a maximum, we have  $\theta = \frac{3\lambda}{d}$

The path difference between the first and the last slit in the grating is an integral number of wavelengths. Let us increase  $\theta$  so that an extra path difference of  $\lambda$  is introduced across the width  $w$ . The change in  $\theta$  required to do this is denoted by  $\Delta\theta$ .

$$\Delta\theta = \frac{\lambda}{w}$$

Because of the  $360^\circ$  extra phase across the grating, we can again divide it into two halves so that there is a  $180^\circ$  phase difference between slits separated by  $w/2$ . So, we get zero intensity at

$$\Delta\theta = \frac{\lambda}{w} = \frac{4 \times 10^{-7}}{10^{-2}} \text{ rad} = 4 \times 10^{-5} \text{ rad} = 2.3 \times 10^{-3} \text{ degrees for violet light}$$

The maximum is sufficiently sharp



Optics - 34 ) Interference with equations

**Two coherent waves are described by the expressions.**

$$E_1 = E_{0\sin}\left(\frac{2\pi x_1}{\lambda} - 2\pi ft + \frac{\pi}{6}\right) ; \quad E_2 = E_{0\sin}\left(\frac{2\pi x_2}{\lambda} - 2\pi ft + \frac{\pi}{8}\right)$$

**Determine the relationship between  $x_1$  and  $x_2$  that produces constructive interference when the two waves are superposed?**

**Sol.** In interference,  $E_r = E_1 + E_2$  (by superposition principle)

$$\phi_1 = \frac{2\pi x_1}{\lambda} - 2\pi ft + \frac{\pi}{6} \quad ; \quad \phi_2 = \frac{2\pi x_2}{\lambda} - 2\pi ft + \frac{\pi}{8}$$

$$\text{Phase difference at } t=0, \quad \Delta\phi = \left(\frac{2\pi x_1}{\lambda} + \frac{\pi}{6}\right) - \left(\frac{2\pi x_2}{\lambda} + \frac{\pi}{8}\right)$$

$$\text{For constructive interference, } \Delta\phi = \pm 2n\pi \quad (\text{where } n = 0, 1, 2, 3 \dots\dots)$$

$$\Rightarrow \pm 2n\pi = \frac{2\pi}{\lambda}(x_1 - x_2) + \frac{\pi}{24} \Rightarrow \pm \left(n - \frac{1}{48}\right)\lambda = (x_1 - x_2)$$

$$[\text{Ans. } \left(n - \frac{1}{48}\right)\lambda = x_1 - x_2]$$

Optics - 35 ) f number of a camera

Focal number of the lens of a camera is  $5f$  and that of another is  $2.5f$ . The time of exposure for the second

is..... if that for the first is  $\frac{1}{200}$  s

$$\left( \text{Given } f = \frac{\text{focal length}}{\text{aperature}} \right)$$

(a)  $\frac{1}{200}$  s

(b)  $\frac{1}{800}$  s

(c)  $\frac{1}{3200}$  s

(d)  $\frac{1}{6400}$  s

[BHU 2005]

**Solution** (b)  $f$  number decreases by 2  $\therefore$  time of exposure should decrease by  $(2^2)$ .

$$\therefore t_{\text{new}} = \frac{1}{4} \times \frac{1}{200} = \frac{1}{800} \text{ s.}$$

Modern Physics 1 ) Spallation reactions ( MP-PET-2002 Madhya Pradesh Pre Engineering Test )

See <http://skmclasses.weebly.com/spallation-reaction.html>

Modern Physics 2 ) Ruby LASER ( asked in COMED-K Karnataka )

See <http://skmclasses.weebly.com/ruby-laser.html>

Modern Physics 3 ) Various details in Particle Physics ( asked in several state exams, including Karnataka CET and COMED-K )

See <http://skmclasses.weebly.com/particle-physics.html>

Modern Physics 4 ) “Magic Numbers” and “Doubly Magic Numbers” in Nuclear Isotope Stability

See <http://skmclasses.weebly.com/magic-number-doubly-magic-in-nucleus.html>

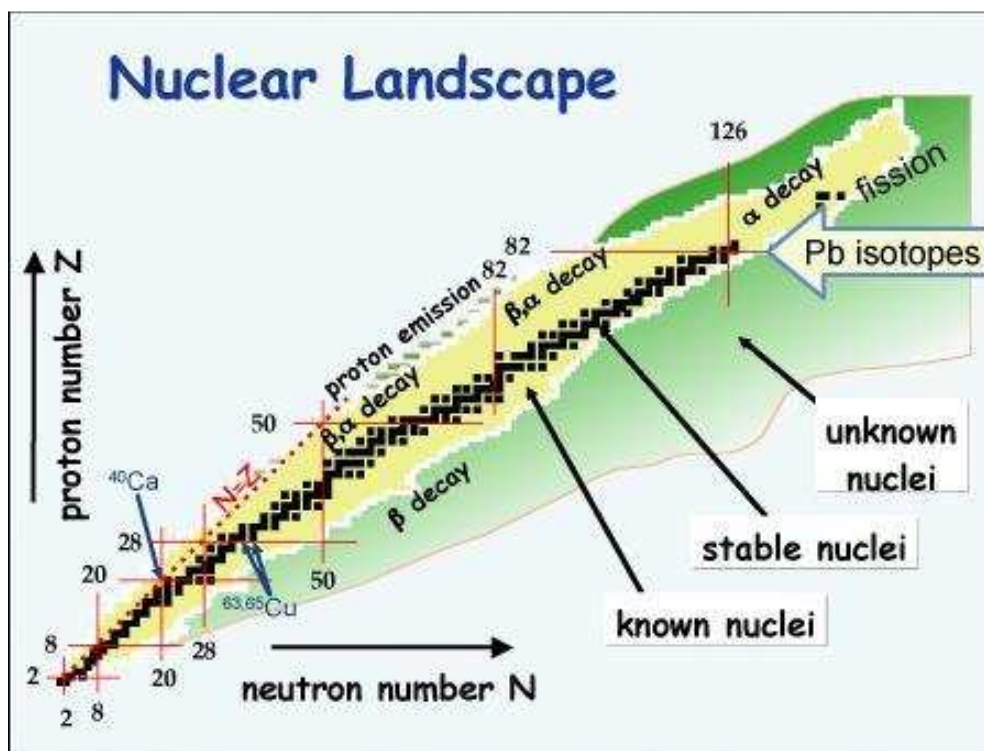
Modern Physics 5 ) Every Alpha (  $\alpha$  ) decay produces an isodiapher. Meaning isodiaphers are extremely common. There was AIEEE question on isodiaphers. Also asked in many other

exams. Even though every book talks of  $\alpha$ ,  $\beta$ , and  $\gamma$  decay; most do not talk about isodiaphers, and positron decay. I find this very strange or rather weird !

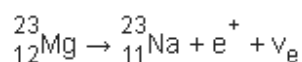
In nuclear physics, **isodiaphers** refers to nuclides which have different atomic numbers and mass numbers but the same neutron excess, which is the difference between numbers of neutrons and protons in the nucleus. For example, for both  ${}_{90}^{234}\text{Th}$  and  ${}_{92}^{238}\text{U}$  the difference between the neutron number ( $N$ ) and proton number ( $Z$ ) is  $N - Z = 54$ .

One large family of isodiaphers has zero neutron excess,  $N = Z$ . It contains many primordial isotopes of elements up to calcium. It includes ubiquitous  ${}_{6}^{12}\text{C}$ ,  ${}_{8}^{16}\text{O}$ , and  ${}_{7}^{14}\text{N}$ .

The daughter nuclide of an alpha decay is an isodiapher of the original nucleus. Similarly, beta decays (and other weak-force-involving decays) produce isobars.



An example of positron emission ( $\beta^+$  decay) is shown with Magnesium 23 decaying into Sodium 23

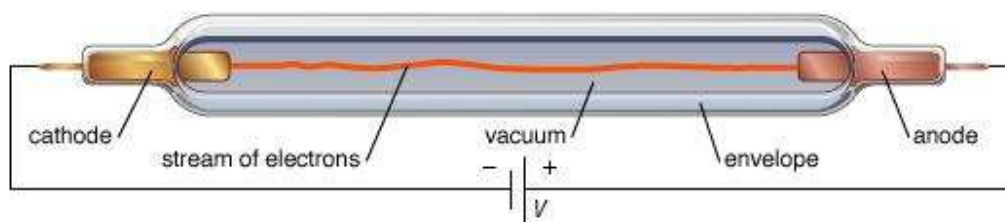


With a positron emission a Proton changes to Neutron. So Mass number remains the same. In 1934 Frederic and Irene Joliot Curie bombarded aluminium with alpha particles to effect the

nuclear reaction  ${}^4_2\text{He} + {}^{27}_{13}\text{Al} \rightarrow {}^{30}_{15}\text{P} + {}^1_0\text{n}$ , and observed that the product isotope  ${}^{30}_{15}\text{P}$  emits a positron identical to those found in cosmic rays by Carl David Anderson in 1932. Meaning it is surely not so new or modern phenomena that “ Modern Physics “ chapter of Modern Books are not covering this ! 3 year back a IIT JEE question with Positron is also not changing the taboo !

Once again I will say “ So strange is this World ! “

Modern Physics 6 ) Relativistic correction for mass when electrons are flying at very high speed due to very high voltage.



If the voltage is 10KV then what will be the speed of the electrons ?

We know Charge X Voltage = Energy =  $\frac{1}{2} mv^2$

Well so far so good. Substitute the values ....

Charge of electron  $e = 1.6 \times 10^{-19}$  Coulomb and mass of electron  $m = 9.1 \times 10^{-31}$  kg or 0.511 MeV For sake of this discussion let us approximate electron mass as  $0.5 \text{ MeV}/c^2$

So  $e ( 10^4 ) V = 10^4 \text{ eV} = \frac{1}{2} mv^2 = ( \frac{1}{2} ) ( \frac{1}{2} \text{ MeV} ) (v/c)^2 = ( \text{MeV}/4 ) (v/c)^2$

$4 \times 10^4 = 10^6 (v/c)^2 \Rightarrow 4/100 = (v/c)^2 \Rightarrow v/c = 1/5 \Rightarrow v = c/5$

Upto speed of around  $c/5$  we do not take relativistic corrections.

Now what would be the speed of the electrons if the voltage was 1MV ?

A wrong calculation and thus wrong answer would be

X  $e ( 10^6 ) V = \frac{1}{2} mv^2 = ( \frac{1}{2} ) ( \frac{1}{2} \text{ MeV} ) (v/c)^2 = ( \text{MeV}/4 ) (v/c)^2$

X  $4 = (v/c)^2$

X  $v/c = 2 \Rightarrow v = 2c$

Students should know that particles can't move at speed more than  $c$

**An 1 mark question in Karnataka CET had an option close to 98% of  $c$ . Student can guess this and tick.** While the calculation will be as follows



Let  $k = \sqrt{1 - \frac{v^2}{c^2}}$

We will have  $e ( 10^6 ) V = \frac{1}{2} (m/k)v^2 = ( \frac{1}{2} ) ( \frac{1}{2} \text{ MeV}/k)(v/c)^2 = ( \text{ MeV}/4k ) (v/c)^2$

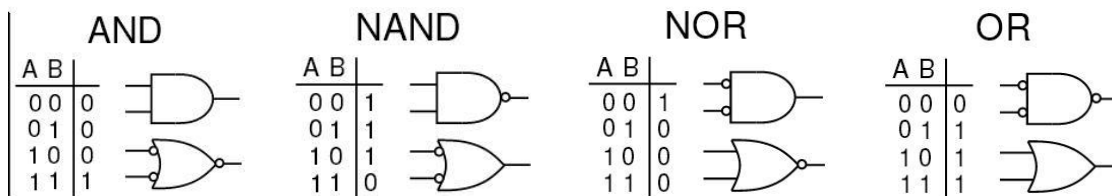
So  $4k = (v/c)^2$  put  $v/c = x$  we get  $4 \sqrt{1 - x^2} = x^2$  put  $x^2 = y$  so  $4\sqrt{1 - y} = y$

Or  $16 ( 1 - y ) = y^2 \Rightarrow y^2 + 16y - 16 = 0$  Solve the quadratic to get  $y = 0.95$

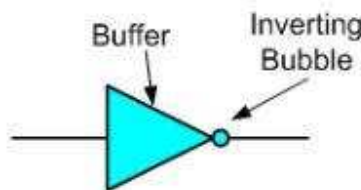
So  $x^2 = 0.95$  or  $x = \sqrt{0.95} = 0.975 \Rightarrow v/c = 0.975$  or  $v = 97.5\%$  of light speed

-

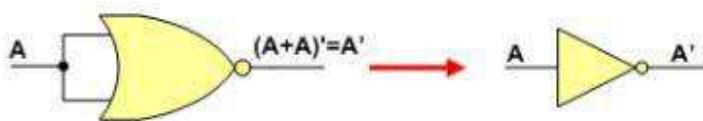
Electronics 1 )



The small circle (bubble) at the output of the graphic symbol of a NOT gate is formally called a negation indicator and designates the logical complement.

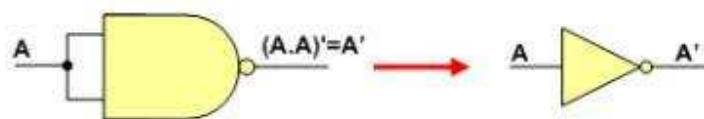


NOT gate can be implemented by NOR Gate. All the pins have to be connected to same signal.

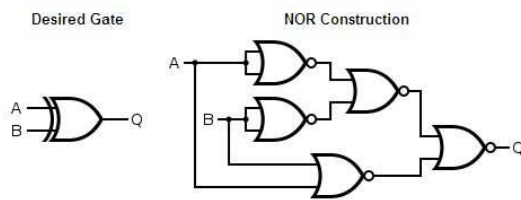


Similarly NOT gate can be implemented with NAND gates

All NAND input pins connect to the input signal A gives an output A'.



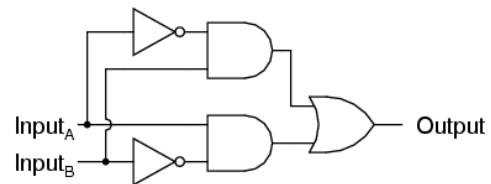
XOR ( exclusive OR ) gate can be implemented with other gates. In various exams the connections are asked.



Truth Table

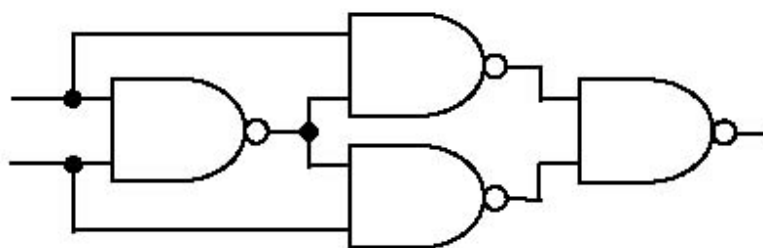
Input A	Input B	Output Q
0	0	0
0	1	1
1	0	1
1	1	0

Exclusive-OR equivalent circuit



A	B	Output
0	0	0
0	1	1
1	0	1
1	1	0

### Exclusive OR (XOR)



A	B	Output
0	0	0
0	1	1
1	0	1
1	1	0

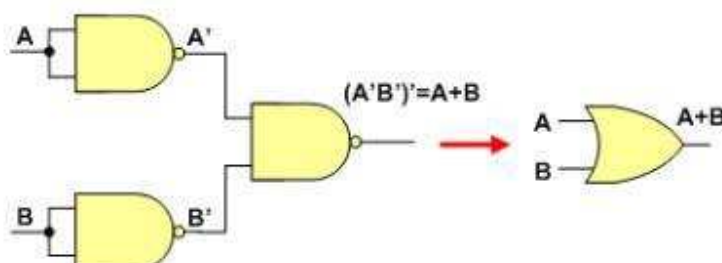
To design the logic circuits the following laws of Boolean algebra are commonly used: commutativity, associativity, distributivity, and De Morgan's laws. Note that distributivity of disjunction over conjunction and both De Morgan's laws do not have their counterparts in ordinary algebra of real numbers.

Property	For conjunction	For disjunction
Commutativity	$A \cdot B = B \cdot A$	$A + B = B + A$
Associativity	$A \cdot (B \cdot C) = (A \cdot B) \cdot C$	$A + (B + C) = (A + B) + C$
Distributivity	$A \cdot (B + C) = A \cdot B + A \cdot C$	$A + B \cdot C = (A + B) \cdot (A + C)$
De Morgan's laws	$\overline{A \cdot B \cdot \dots} = \overline{A} + \overline{B} + \dots$	$\overline{A + B + \dots} = \overline{A} \cdot \overline{B} \cdot \dots$
Basic identities	$A \cdot 0 = 0$ $A \cdot 1 = A$ $A \cdot A = A$ $A \cdot \overline{A} = 0$	$A + 1 = 1$ $A + 0 = A$ $A + A = A$ $A + \overline{A} = 1$
Additional identities	$A \cdot (A + B) = A$ $A + \overline{A} \cdot B = A + B$ $(A + B) \cdot (\overline{A} + \overline{B}) = \overline{A \cdot B}$	$A + A \cdot B = A + B$ $A \cdot (\overline{A} + B) = A \cdot B$ $A \cdot B + \overline{A} \cdot B = B$

Principal identities and laws of Boolean algebra.

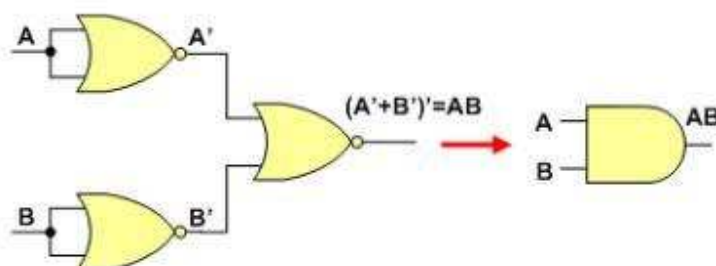
### Implementing OR Gate with NAND gates

An **OR gate** can be replaced by NAND gates as shown in the figure (The OR gate is replaced by a NAND gate with all its inputs complemented by NAND gate inverters).



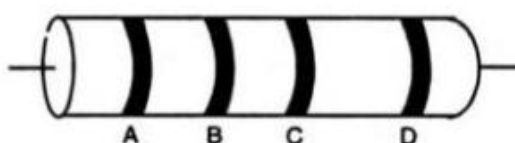
### Implementing AND gate with NOR gates

An **AND gate** can be replaced by NOR gates as shown in the figure (The AND gate is replaced by a NOR gate with all its inputs complemented by NOR gate inverters).



### Colour Code for Carbon Resistors

Since a carbon resistor is physically quite small, it is more convenient to use a *colour code* indicating the resistance value than to imprint the numerical value on the case. In this scheme, there are generally four colour bands *A*, *B*, *C* and *D* printed on the body of the resistor as shown in Fig. The first three colour bands (*A*, *B* and *C*) give the value of the resistance while the fourth



band (D) tells about the \*tolerance in percentage. The table below shows the colour code for resistance values and colour code for tolerance.

Colour Code for Resistance Values				Colour Code for Tolerance	
Black	0	Green	5	Gold	$\pm 5\%$
Brown	1	Blue	6	Silver	$\pm 10\%$
Red	2	Violet	7	No colour	$\pm 20\%$
Orange	3	Grey	8		
Yellow	4	White	9		

(i) To read the resistance value, we refer to the first three colour bands (A, B and C). The first two colour bands (A, B) specify the first two digits of the resistance value and the third colour band (C) gives the number of zeros that follow the first two digits. Suppose the first three colour bands (A, B, C) on the resistor are red, brown, orange respectively. Then value of the resistance is 21,000  $\Omega$ .

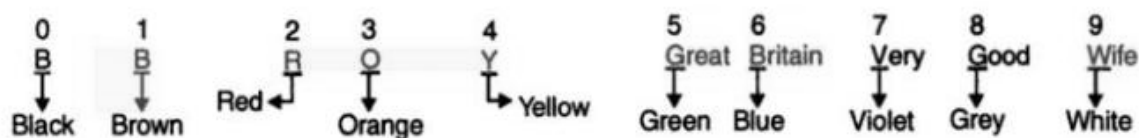
Red : 2

Brown : 1

Orange : 000

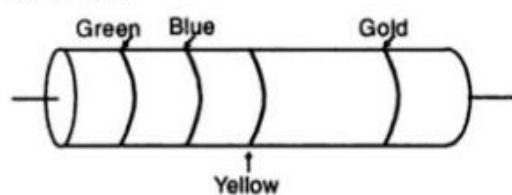
$\therefore$  Value = 21,000  $\Omega$

(ii) The fourth band D gives the value of tolerance in percentage. If colour of the fourth band is gold, tolerance is  $\pm 5$  per cent and if silver, then tolerance is  $\pm 10$  per cent. If the fourth band is omitted, the tolerance is assumed to be  $\pm 20$  per cent.

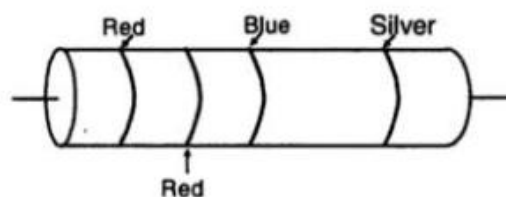


**Note.** In order to remember the colour code, the above sentence may be helpful.

**Example** The colour coded carbon resistors are shown in Fig. Find their resistance values.



(i)



(ii)



**Solution.** The first colour represents the digit 5. The second colour represents the digit 6. The third colour represents the digit 4, *i.e.*, four zeros. Therefore, the value of the resistance is 56,0000  $\Omega$ . The fourth gold strip indicates  $\pm 5\%$  tolerance. Hence, resistance specification of the resistor is

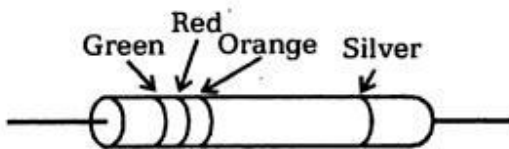
$$560000 \Omega ; \pm 5\%$$

(ii) Refer to Fig. resistor is

Following above procedure, the resistance specification of this

$$22,000000 \Omega ; \pm 10\%$$

- Due to manufacturing variations, the resistance value may not be the same as indicated by colour code. Thus, a resistor marked 100  $\Omega$ ,  $\pm 10\%$  tolerance means that resistance value is between 90  $\Omega$  and 110  $\Omega$ .



"Carbon resistor colour code"

The value of the above resistor as shown in the fig. is

- The first ring Green - 5
- The second ring Red - 2
- The third ring Orange ring corresponds to -  $10^3$
- The silver ring represents 10% tolerance
- $\therefore$  Total resistance is
- $52 \times 10^3 \pm 10\%$  (or)  $52\text{k}\Omega$ , 10%

Varactor diode

$\text{---} \triangleright \text{---}$  is the symbol of

- (a) a capacitor
- (b) photo diode
- (c) varactor diode
- (d) tunnel diode

**Ans: (c)**

## Common emitter

**In a common emitter configuration the base-emitter voltage is  $3 \times 10^{-2}$  V. If the base current is  $30 \mu\text{A}$ , the input impedance is**

- (a)  $1 \text{ k}\Omega$                       (b)  $3 \text{ k}\Omega$   
 (c)  $100 \Omega$                       (d)  $2 \text{ k}\Omega$

**Ans: (a)**

**Solution:**

**Given data:-**

$$(V_{BE}) = \text{Base emitter voltage} = 3 \times 10^{-2} \text{V}$$

$$\text{Base current } (I_B) = 30 \times 10^{-6} \text{ A}$$

$$\text{Input impedance } Z_i = \left( \frac{\Delta V_{BE}}{\Delta I_B} \right)_{V_{CE}}$$

$$Z_i = \frac{3 \times 10^{-2}}{30 \times 10^{-6}}$$

$$Z_i = Z_i = \frac{3 \times 10^{-2}}{10 \times 10^{-6}} = 10^{-2-1+6}$$

$$Z_i = 10^3 \Omega$$

$$Z_i = 1 \text{ k}\Omega$$

## Common base

**In a common base configuration, the collector current is  $0.95 \text{ mA}$  and base current is  $0.05 \text{ mA}$ , then the value of current gain is**

- (a)  $0.89$                       (b)  $0.9$   
 (c)  $0.95$                       (d)  $0.99$

**Ans: (c)**

**Given data:-**

$$\text{Collector current } I_C = 0.95 \times 10^{-3} \text{A}$$

$$\text{Base current } I_B = 0.05 \text{ mA} = 0.05 \times 10^{-3} \text{A}$$

**Solution:**

$$\text{Current gain } \alpha = \left( \frac{I_C}{I_E} \right)$$

$$I_E = \text{Emitter current} = I_C + I_B$$

$$= (0.95 + 0.05) \times 10^{-3} \text{ A}$$

$$= 1 \times 10^{-3} \text{ A} = 1 \text{ mA}$$

$$\alpha = \frac{0.95 \times 10^{-3} \text{ A}}{1 \times 10^{-3} \text{ A}} = 0.95$$

The current gain is  $0.95$

### Common emitter

**In a common emitter amplifier, the output resistance is  $5000 \Omega$  and the input resistance is  $2000 \Omega$ . If the peak value of the signal voltage is  $10 \text{ mV}$  and  $\beta = 50$ , then the peak value of the output voltage is**

- (a)  $5 \times 10^{-6} \text{ V}$                       (b)  $1.25 \text{ V}$   
(c)  $125 \text{ V}$                                 (d)  $2.5 \times 10^{-4} \text{ V}$

**Ans: (b)**

**Given data:-**

$$R_L = 5000 \Omega$$

$$R_i = 2000 \Omega$$

$$\beta = 50$$

**Solution:**

The ac voltage gain is given by

$$\beta \times \frac{R_L}{R_i} = \frac{50 \times 5000}{2000} = 125$$

$$\begin{aligned} \therefore \text{peak output voltage} &= \text{voltage gain} \times \text{signal voltage} \\ &= 125 \times 10 \text{ mV} = 1250 \text{ mV} = 1.25 \text{ V} \end{aligned}$$

### Common base

**In a common base amplifier circuit, calculate the change in base current if that in the collector current is  $2 \text{ mA}$  and  $\alpha = 0.98$**

- (a)  $0.04 \text{ mA}$                               (b)  $1.96 \text{ mA}$   
(c)  $980 \text{ mA}$                                 (d)  $2 \text{ mA}$

**Ans: (a)**

**Solution:**

$$\beta = \frac{\alpha}{1 - \alpha} = \frac{0.98}{1 - 0.98} = 49$$

$$\text{Now } \Delta I_c / \Delta I_b = 49$$

$$\text{or } \Delta I_b = \Delta I_c / 49$$

$$\begin{aligned} \therefore \Delta I_b &= 2 \text{ mA} / 49 \\ &= 0.04 \text{ mA} \end{aligned}$$

### Common base

**In a common base circuit of a transistor, current amplification factor is 0.95. Calculate the base current when emitter current is 2 mA.**

- (a) 0.1 mA                      (b) 1 mA  
(c) 0.01 mA                    (d) none of these

**Ans: (a)**

**Solution:**

$$\alpha = \frac{I_C}{I_E}$$

$$0.95 = \frac{I_C}{2 \times 10^{-3}}$$

$$I_C = 1.90 \times 10^{-3} \text{ A} = 1.9 \text{ mA}$$

$$\text{Now } I_B = I_E - I_C = 0.1 \text{ mA}$$

### Common emitter

**A transistor is connected in common emitter (CE) configuration. The collector supply is 8V and the voltage drop across a resistor of  $800\Omega$  in the collector circuit is 0.5V. If the current gain factor ( $\alpha$ ) is 0.96. Find the base current.**

- (a) 20  $\mu\text{A}$                       (b) 26  $\mu\text{A}$   
(c) 30  $\mu\text{A}$                       (d) none of these

**Ans: (b)**

**Solution:**

$$\text{Collector current } I_C = \frac{0.5}{800} \text{ A}$$

$$\text{Current gain } \beta = \frac{I_C}{I_B}$$

$$\frac{\alpha}{1 - \alpha} = \frac{0.96}{800} \text{ A}$$

$$I_B = \frac{I_C}{24} = \frac{0.5}{800 \times 24}$$
$$= 26 \mu\text{A}$$



### Conductivity

Conductivity is defined as the current density per unit applied electric field. If  $J$  is the current density due to an applied electric field  $\epsilon$ , then the conductivity ( $\sigma$ ) is given by,

$$\sigma = \frac{J}{\epsilon} \quad \text{.....(1)}$$

In S.I.,  $\sigma$  is given in Siemens/meter or mho/meter as 1 siemen = 1 mho

For a cylindrical semiconductor, the current density is given by,

$$J = ne v \quad \text{.....(2)}$$

where  $n$  is the number of charge carriers in the semiconductor  $e$  is the electronic charge and  $v$  is the drift velocity of the electron.

Also, we have  $v = \mu \epsilon$  .....(3)

where  $\mu$  is the mobility of the charge carrier and  $\epsilon$  is the applied electric field.

Then, equation (2) can be written,

$$J = ne \mu \epsilon$$

then, equation (1) becomes,

$$\sigma = ne \mu \quad \text{.....(4)}$$

Now, if the conductivity of a semiconductor is due to electron then it is denoted by  $\sigma_n$ , and equation (4), can be written as

$$\sigma_n = ne \mu_n \quad \text{.....(5)}$$

where  $n$  is the number of electron and  $\mu_n$  is the mobility of electron.

Similarly, the conductivity of a semiconductor due to the holes is given by,

$$\sigma_p = pe \mu_p \quad \text{.....(6)}$$

where  $p$  is the hole concentration and  $\mu_p$  is the hole mobility.

Hence, the overall conductivity of the semiconductor containing electrons and holes is given by,

$$\sigma = \sigma_n + \sigma_p = e(n\mu_n + p\mu_p) \quad \text{.....(7)}$$

For an intrinsic semiconductor,  $n = p = n_i$

Therefore, the conductivity of an intrinsic semiconductor,

$$\sigma_{\text{int}} = n_i e(\mu_n + \mu_p) \quad \text{.....(8)}$$

For an  $n$ -type semiconductor,  $n \gg p$ , then

$$\sigma_n \approx ne \mu_n \quad \text{.....(9)}$$

Similarly, for a  $p$ -type semiconductor

$$\sigma_p = pe \mu_p \quad \text{.....(10)}$$

These equations shows that conductivity  $\sigma$  has the same temperature dependence as  $\mu_e$  or  $\mu_n$ .

Mobility is a more useful property for characterizing a semiconductor than conductivity. Conductivity,  $\sigma$  depends on carrier concentration *i.e.*, on doping level but mobility  $\mu$  does not depend. Thus, mobility is the property of semiconductor itself.

**Problem 1 :** At 300 K, the intrinsic carrier concentration of silicon is  $1.5 \times 10^{16} \text{ m}^{-3}$ . If the electron and the hole mobilities are 0.13 and 0.05  $\text{m}^2/\text{sec-V}$  respectively. Determine the conductivity and resistivity of silicon.

**Solution :** The electrical conductivity of intrinsic semiconductor is given by,

$$\sigma_i = n_i e (\mu_n + \mu_p)$$

Here,  $n_i = 1.5 \times 10^{16} \text{ m}^{-3}$ ,  $\mu_n = 0.13 \text{ m}^2/\text{sec-V}$ ,  $\mu_p = 0.05 \text{ m}^2/\text{sec-V}$  and  $e = 1.6 \times 10^{-19} \text{ Coulomb}$

$$\begin{aligned} \therefore \sigma_i &= 1.5 \times 10^{16} \times 1.6 \times 10^{-19} \times (0.13 + 0.05) \\ &= 2.4 \times 10^{-3} \times 0.18 = 0.432 \times 10^{-3} \\ &= 4.32 \times 10^{-4} \text{ mho/m} \end{aligned}$$

Hence, the resistivity  $\rho_i$  is given by

$$\rho_i = \frac{1}{\sigma_i} = \frac{1}{4.32 \times 10^{-4}} = 2.31 \times 10^3 \text{ ohm-m}$$

**Problem 2 :** The resistivity of pure silicon at room temperature is 3000 ohm-m. Calculate the intrinsic carrier concentration. Given that :  $\mu_n = 0.14 \text{ m}^2/\text{sec-V}$  and  $\mu_p = 0.05 \text{ m}^2/\text{sec-V}$ .

**Solution :** In pure silicon, electrons and holes (the intrinsic charge carriers) are equal in numbers. The conductivity of pure semiconductor is given by

$$\begin{aligned} \sigma &= n_i e (\mu_n + \mu_p) \quad \text{or} \quad n_i = \frac{\sigma}{e(\mu_n + \mu_p)} = \frac{1}{\rho e (\mu_n + \mu_p)} \quad \left[ \because \rho = \frac{1}{\sigma} \right] \\ \therefore n_i &= \frac{1}{(0.14 + 0.05) \times 3000 \times 1.602 \times 10^{-19}} = 1.095 \times 10^{16} \text{ m}^{-3}. \end{aligned}$$

**The band gap of a specimen of gallium arsenide phosphide is 1.98 eV. Determine the wavelength of the radiation that is emitted when electron jumps from conduction band to the valence band to recombine with a hole.**

**Solution :** The wavelength of emitted radiation is given by,

$$\lambda = \frac{hc}{E_g}$$

Here,  $h$  = Planck's constant =  $6.62 \times 10^{-34} \text{ J.s}$ ,  $c$  = velocity of light =  $3 \times 10^8 \text{ m/s}$  and

$E_g$  = Energy band gap =  $1.98 \text{ eV} = 1.98 \times 1.6 \times 10^{-19} \text{ J}$ .

$$\begin{aligned} \therefore \lambda &= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{1.98 \times 1.6 \times 10^{-19}} = 6.269 \times 10^{-7} \text{ m} \\ &= 6269 \text{ \AA} \end{aligned}$$

Since this wave-length is in the visible range, so the colour of the emitted radiation will be red.

**Q. An  $n$ -type semiconductor crystal has more free electrons than holes. Is it then negatively charged ?**

**Ans :** An  $n$ -type semiconductor has free electrons as charge carriers. These are donated by pentavalent impurity atoms which becomes positively charged. Although there are some thermally generated electron-hole pairs, but the number of these holes is negligibly small in comparison to the total number of electrons. Thus,  $n$ -type semiconductor mainly consists of negatively charged free electrons and nearly equal number of positively charged donor ions. Hence, the material as a whole is electrically neutral.

**Q. *p*-type semiconductor crystal has more holes than electrons. Is it then positively charged ?**

**Ans :** A *p*-type semiconductor has holes as charge carriers. These holes are due to trivalent impurity atoms which become negatively charged by accepting the electrons from the neighbouring Ge atom. Although there are some thermally generated electron-hole pairs, but the number of these electrons is negligibly small in comparison to the total number of holes. Thus, *p*-type semiconductor mainly consists of positively charged holes and nearly equal number of negatively charged acceptor ions. Hence, the material as a whole is electrically neutral.

**Q. Why does the width of depletion region increase when a *p-n* junction is reverse biased ?**

**Ans :** In reverse bias, negative terminal of the battery is connected to *p*-side and positive terminal to *n*-side of *p-n* junction. So, the electrons are attracted towards positive terminal and holes towards negative terminal of the battery. Thus, holes and free electrons move away from the junction. Therefore, the depletion layer gets wider. The width of the layer increases with increasing reverse voltage.

**Q. The small current flowing through a reverse biased junction diode is called the reverse saturation current, why ?**

**Ans :** The reverse current is due to the thermally generated minority carriers. We cannot increase the number of these minority carriers by applying and increasing the reverse voltage. So, it is termed as saturation current. This current flows in the opposite direction with respect to forward bias, so it is called reverse. Due to above both factors it is called reverse saturation current.

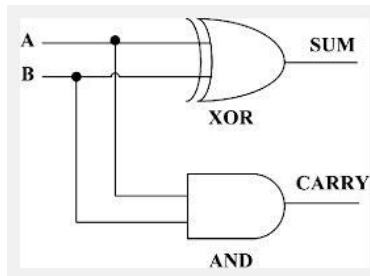
**Q. The reverse saturation current of a Si diode is much smaller than a Ge diode of the same size, why ?**

**Ans :** The barrier potential of Si is 0.7 eV while that of Ge is 0.3 V. Hence, less number of thermally generated minority carriers cross the junction in Si diode than that in Ge diode of the same size. Therefore, the reverse current in Si diode is smaller than the Ge diode at the same temperature and for the same size.

**Q. Differentiate between Avalanche and Zener breakdown.**

**Ans :** **Avalanche breakdown :** For a simple *p-n* junction, if we apply a reverse bias to the junction, a very small current due to minority carriers flows through the junction. On increasing the reverse voltage the minority carriers (electrons) may attain sufficient kinetic energy to knock out valency electron from the covalent bonds. As a result more electron-hole pairs are generated. Due to the high reverse bias voltage, these new carriers are also accelerated and collide with other covalent bonds. This process will continue until an avalanche of electrons is formed and a very large current flows through the junction diode. This breakdown is known as Avalanche breakdown. This breakdown occurs at very high voltage.

**Zener breakdown :** If the *p-n* junction is heavily doped then the electric field across the depletion layer becomes large enough. When we apply a reverse bias to this junction then this electric field becomes so large even at low voltage that it may cause rupture of the covalent bonds and breakdown the junction. This breakdown is known as Zener breakdown and this diode is known as Zener diode. This breakdown occurs at lower voltage than avalanche breakdown.



Half Adder

**Q. : What is mass-action law for the carrier concentrations in a semiconductor ? What is its significance ?**

**Ans :** The law of mass-action states that in any type of semiconductor ( $p$  or  $n$  type), the product of free electrons concentration,  $n$  and hole concentration,  $p$  is a constant and equal to  $n_i^2$  where  $n_i$  is the intrinsic carrier concentration *i.e.*,

$$np = n_i^2$$

The intrinsic carrier concentration  $n_i$  is a function of temperature. At a given temperature if electron concentration is increased by doping, the corresponding hole concentration ( $p$ ) must decrease (or vice-versa) to keep  $np$  a constant ( $= n_i^2$ ) at a particular temperature.

**Q. : Explain why an extrinsic semiconductor at high temperature behaves like an intrinsic one.**

**Ans :** At very high temperature, the concentration of thermally generated free electrons from the valence band becomes much larger than concentration of free electrons contributed by donors (as donor atoms are already ionized). In this condition, the hole and electron concentrations will be nearly equal and semiconductor will behave like an intrinsic one. Due to the same reason  $p$ -type semiconductor will also behave like an intrinsic semiconductor at very temperatures. So, we can say that an extrinsic semiconductor changes to an intrinsic one at very high temperatures.

**Q. : What do you mean by the term "doping" and "dopant" . Name some dopant materials ?**

**Ans :** The addition of a small percentage of impurity atoms to a semiconductor is called "doping" and the impurity, which is added, is referred to as "dopant". In Ge or Si, the elements of V group like phosphorous (P) antimony (Sb) and arsenic (As) and the elements of III group like aluminium (Al). Indium (In) boron (B) and gallium (Ga) are dopant.

**Q. Write diode equation and with the help of this equation describe the volt-ampere characteristics of the diode.**

**Ans :** The diode equation is written as,

$$I = I_0 (\exp \frac{eV}{kT} - 1) \quad \dots\dots(1)$$

where  $I$  is current at applied voltage,  $V$



$I_0$  is constant and known as reverse saturation current  
 $e$  is electronic charge  
 $k$  is Boltzmann constant  
 and  $T$  is absolute temperature  
 With the help of this equation we can describe the volt ampere characteristics as shown in fig 1.23

If  $V$  is positive *i.e.*, for a forward bias

then,  $\exp \frac{eV}{kT} \gg 1$

So, equation (1) can be written as,

$$I = I_0 \exp \frac{eV}{kT}$$

Hence, for a forward bias, current increases exponentially as shown in fig. 1.24.

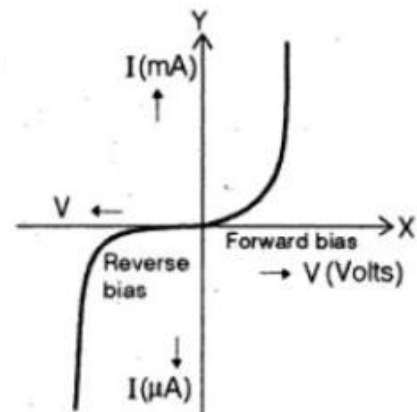
Similarly, if  $V$  is negative *i.e.*, for a reverse bias then,

$$\exp^{-\frac{eV}{kT}} \ll 1$$

So, equation (1) can be written as,

$$I = -I_0$$

Hence, for a reverse bias current is constant in reverse direction as shown in fig.



**Q.** How reverse current depends upon the temperature of the junction ?

**Ans :** The reverse current in a  $p-n$  junction diode depends on the temperature  $T$ . The rise in temperature increases the generation of electron hole pairs in semiconductors and increases their conductivity as a result the current through junction diode increases with temperature. For practical diodes it is found that reverse saturation current  $I_0$  will just about double in magnitude for every  $10^\circ\text{C}$  increase in temperature. Typical values of  $I_0$  for silicon are much lower than that of Germanium for similar power and current levels. The result is that silicon junction diodes are more preferred than Ge for rectifiers and have higher breakdown voltage.

**Q. :** What do you mean by tunnel diode ?

**Ans :** Tunnel diode is very high doped ( $\approx 10^{25}/\text{m}^3$ )  $p-n$  junction in both  $p$  and  $n$  region. Since, the depletion layer of this diode becomes very thin, so, on applying forward bias many carriers can tunnel through the depletion layer and the process is known as tunnelling. Hence, the diode is known as tunnel diode.

Heat or Thermodynamics 1 ) So many exams including IIT JEE had questions on Polytropic processes. Apart from Professor N. N. Ghosh's books, hardly this is covered in Physics Books

I am surprised and **amused** to see so many coaching Institutes making errors in Polytropic Process Problems. In most cases the teachers are avoiding it, and in rare cases when it is being covered there are errors.

Let us do it here.

We assume ideal gas for Thermodynamics process problems. So  $PV = nRT$  is taken as true regardless the process gas is taken through. So Isothermal ( meaning constant Temperature ), Isobaric ( meaning const Pressure ), Isochoric ( meaning constant Volume ) or even  $PV^z = \text{Const}$  ( P into V to the power z is constant ) where z is a constant of the polytropic process, the expression  $PV = nRT$  is taken as true. We do substitute that to exchange the variables in many problems.

Work done by system on boundary is:

$$W = \int_{V_1}^{V_2} p \, dV$$

P | can be a  
polytropic exp

This form is used for expansion and contraction of gases  
Ideal Gases

### **Ideal (Perfect) Gas Law**

$$pV = nRT$$

$$R = 8314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

If the gas expands ( often due to supply of heat ) the work done by the gas is taken as positive.

Work done expression in Isothermal ( or isotropic as some people say it ) is given by

### **Isothermal (Constant Temp) Process or Isothermal process**

- For a constant temperature process in a closed system (i.e. mass is constant) –  $pV = mRT = C$ . Where C is a constant. Note C can be written as  $p_1V_1$  or as  $p_2V_2$ .

$$\bullet \quad W = \int_{V_1}^{V_2} \frac{C}{V} \, dV = C \ln\left(\frac{V_2}{V_1}\right) = p_1V_1 \ln\left(\frac{V_2}{V_1}\right) = nRT \ln\left(\frac{V_f}{V_i}\right) = nRT \ln\left(\frac{P_i}{P_f}\right)$$

**Polytropic process -  $pV^n=C$  where C is a constant.**

these occur in ideal gases for various processes and the value of  $n$  changes depending on the type of process (e.g.  $n = 1$  is a isotropic process).

Note that  $p_1V_1^n=C \rightarrow p_1V_1=\frac{CV_1}{V_1^n}=CV_1^{1-n}$  This also holds for  $p_2V_2$ .

$$W = \int_{V_1}^{V_2} \frac{C}{V^n} dV = \frac{C}{1-n} (V_2^{1-n} - V_1^{1-n}) = \frac{p_2V_2 - p_1V_1}{1-n}$$

In case of adiabatic process ( where no heat exchange takes place ),  $n$  is  $\gamma$  ( gamma ), so in the above expression replace  $n$  as  $\gamma$

$$pV^\gamma = p_1V_1^\gamma = p_2V_2^\gamma = k$$

Thus, 
$$p = \frac{k}{V^\gamma}$$

The work done by the gas in the process is

$$W = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} \frac{k}{V^\gamma} dV = \frac{1}{1-\gamma} \left[ \frac{k}{V_2^{\gamma-1}} - \frac{k}{V_1^{\gamma-1}} \right]$$

From equation (i),

$$\frac{k}{V_2^\gamma} = p_2 \text{ and } \frac{k}{V_1^\gamma} = p_1$$

Thus, 
$$W = -\frac{1}{\gamma-1} (p_2V_2 - p_1V_1) = \frac{p_1V_1 - p_2V_2}{\gamma-1}$$

There are other expressions which are handy ( given for 1 mole of gas ), for Heat supplied in Polytropic Process

$$\Delta H = C_P^0(T_2 - T_1) = \frac{\gamma R}{\gamma-1} (T_2 - T_1) = \frac{\gamma}{\gamma-1} (P_2V_2 - P_1V_1) = \frac{\gamma P_1V_1}{\gamma-1} \left[ \left( \frac{P_2}{P_1} \right)^{1-\frac{1}{n}} - 1 \right]$$

Heat Supplied in a process at constant Pressure is  $\Delta H = C_P^0(T_2 - T_1)$

Process	Work Done: (W)	Heat Exchanged ( $\Delta Q$ )
Isothermal process	$W = 2.303 nRT \log_{10} \frac{V_2}{V_1}$	$\Delta Q = 2.30 nRT \log \frac{V_2}{V_1}$
Adiabatic process	$W = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1}$ $= \frac{nR(T_2 - T_1)}{\gamma - 1}$	$\Delta Q = 0$
Isochoric process	$W = 0$	$\Delta Q = nC_v \Delta T$ (use definition of $C_v$ )
Isobaric process	$W = p\Delta V = p(V_2 - V_1)$ $W = nR(T_2 - T_1)$	$\Delta Q = nC_p \Delta T$ (use definition of $C_p$ )

VdP expression in polytropic process

For a polytropic process  $P_1 V_1^n = P V^n$

$$V = \left( \frac{P_1 V_1^n}{P} \right)^{\frac{1}{n}} = \left( \frac{P_1}{P} \right)^{\frac{1}{n}} V_1$$

$$\int V dP = P_1^{\frac{1}{n}} V_1 \int \frac{1}{P^{\frac{1}{n}}} dP$$

$$\int V dP = \frac{P_1^{\frac{1}{n}} V_1}{1 - \frac{1}{n}} \left( P_2^{1 - \frac{1}{n}} - P_1^{1 - \frac{1}{n}} \right)$$

$$\int V dP = \frac{n V_1}{n - 1} \left( P_1^{\frac{1}{n}} P_2^{1 - \frac{1}{n}} - P_1 \right)$$

$$P_2 \int_{P_1}^2 V dP = \frac{n P_1 V_1}{n - 1} \left[ \left( \frac{P_2}{P_1} \right)^{1 - \frac{1}{n}} - 1 \right]$$

$$-\int_1^2 V dP = \frac{n P_1 V_1}{n - 1} \left[ 1 - \left( \frac{P_2}{P_1} \right)^{1 - \frac{1}{n}} \right]$$



Specific heat in case of Polytropic process and  $C_v$  in terms of gamma

$$C = \frac{R}{\gamma-1} - \frac{R}{k-1}$$
$$C_v = \frac{R}{\gamma-1}$$

Example

One mole of Argon is heated using  $PV^{3/2} = \text{const}$ . Find the amount of heat obtained by the process when the temperature changes by  $\Delta T = -26$  K.

**Solution**

Let  $p$  be the number of moles here  $p = 1$

$$\text{then } C = \frac{R}{\gamma-1} - \frac{R}{\eta-1} = \frac{R}{\frac{5}{3}-1} - \frac{R}{\frac{3}{2}-1}$$

$$\Delta Q = pC\Delta T = 1 \left( \frac{3}{2}R - 2R \right) (-26)$$

$$= +26 \left( \frac{8.314}{2} \right) = 108 \text{ J}$$

You can also write  $+ R / (1-k)$  in Specific heat expression so see an example

An ideal gas expands according to the law  $PV^{3/2} =$  constant. We conclude

- (a) The adiabatic exponent of the gas  $K = 1.5$
- (b) The molar heat capacity  $C = C_v - 2R$
- (c) Temperature increases during the process
- (d) Such a process is not feasible

Ans - (b) Molar heat capacity

$$C = C_v + \frac{R}{1-K} = C_v + \frac{R}{1-\frac{3}{2}} = C_v - 2R$$

IIT JEE 1995 Polytrropic Thermodynamics Process Problem

3 moles of a gas mixture having volume  $V$  and temperature  $T$  is compressed to 1/5th of the initial volume. Find the change in its adiabatic compressibility if the gas obeys  $PV^{19/13} = \text{constant}$  [ $R = 8.3 \text{ J/mol - K}$ ]

[IIT 1995]

$$\text{Bulk modulus } B = \gamma P$$

$$\text{Compressibility } C = \left( \frac{1}{B} \right) = \frac{1}{\gamma P}$$

and  $\Delta C = C - C$

or  $\Delta C = \frac{1}{\gamma} \left[ \frac{1}{P'} - \frac{1}{P} \right]$

$$PV^\gamma = P' \left( \frac{V}{5} \right)^\gamma$$

$$\text{With } \gamma = \frac{19}{13} \text{ and } P' = 5^\gamma P, 11$$

$$\Delta C = \frac{1}{\gamma P} \left[ \frac{1}{5^\gamma} - \frac{1}{1} \right] = \frac{13 \times 0.905}{19P}$$

$$\text{But } PV = nRT \text{ or } P = \frac{nRT}{V}$$

$$\Delta C = \frac{13(.905)V}{19 \times 3 \times 8.317T} = \frac{-0.0248V}{T}$$

An ideal gas with adiabatic exponent  $\gamma$ , is expanded according to the law

$$P = aV$$

where  $a$  is a constant. The initial volume of the gas is  $V_0$ . As a result volume increases  $\eta$  times. Find the increment in internal energy and work done.

**Solution -** Let  $k$  be number of moles

$$P = aV \text{ or } PV^{-1} = a$$

The process is polytropic with index  $n = -1$

$$\therefore V_{\text{initial}} = V_0, V_{\text{final}} = \eta V_0$$

$$\text{and } P_{\text{initial}} = aV_0; P_{\text{final}} = a\eta V_0$$

$$\Delta U = \frac{kR}{\gamma - 1} (T_{\text{final}} - T_{\text{initial}}), P_{\text{final}} V_{\text{final}} - P_{\text{initial}} V_{\text{initial}}$$

Work done,

$$W = \frac{P_{\text{initial}} V_{\text{initial}} - P_{\text{final}} V_{\text{final}}}{n - 1} = \frac{\alpha V_0^2 [\eta^2 - 1]}{2}$$

**In a polytropic process an ideal gas ( $\gamma = 1.40$ ) was compressed from volume  $V_1 = 10$  litres to  $V_2 = 5$  litres. The pressure increased from  $p_1 = 10^5$  Pa to  $p_2 = 5 \times 10^5$  Pa. Determine: (a) the polytropic exponent  $n$ , (b) the molar heat capacity of the gas for the process.**

**Solution.**

In a polytropic process  $pV^n = k$  (a constant)

$$\therefore p_1 V_1^n = p_2 V_2^n \quad \text{or} \quad \left(\frac{V_1}{V_2}\right)^n = \frac{p_2}{p_1}$$

$$\text{or} \quad n = \frac{\ln p_2/p_1}{\ln V_1/V_2}$$

$$\text{Here} \quad n = \frac{\ln 5}{\ln 2} = \frac{1.6094}{0.6931} = 2.32$$

In a polytropic process

$$C = \frac{R}{\gamma - 1} - \frac{R}{n - 1} = \frac{R}{1.4 - 1} - \frac{R}{2.32 - 1} = 1.74 R$$

*An ideal gas expands according to the law  $pV^2 = \text{constant}$  (a) Is it heated or cooled? (b) What is the molar heat capacity in this process?*

**Solution.**

This is a polytropic process of exponent  $n = 2$ . To find whether it is heated or cooled we have to examine whether  $\Delta Q$  is +ve or -ve or whether  $T$  increases or decreases.

$$pV^2 = \text{constant.} \quad \text{But } pV = RT \quad (\text{always})$$

$$\therefore \frac{pV^2}{pV} = \frac{\text{constant}}{RT} \quad \text{or} \quad V \propto \frac{1}{T}$$

Thus when volume increases  $T$  decreases. Here the gas is cooled.

$$(b) \quad C = \frac{R}{\gamma - 1} - \frac{R}{n - 1} = C_V - R$$



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Heat or Thermodynamics 2 ) Formula for equivalent gamma in mixture of gases.  $n_1$  moles of gas with  $\gamma_1$  and  $n_2$  mole of gas with  $\gamma_2$  are mixed, then what is equivalent gamma ?

Why  $C_v = R / (\gamma - 1)$

Specific heat of a polytropic process. Derivation of work done in polytropic process.

$$\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

Equivalent  $\gamma$  of a mixture of gas.  
 $n_1$  moles of  $\gamma_1$  and  $n_2$  moles of  $\gamma_2$  are mixed

$$C_v = \frac{R}{\gamma - 1}$$

we have  $\gamma = \frac{C_p}{C_v} = \frac{C_v + R}{C_v}$

$$= 1 + \frac{R}{C_v}$$

$$PV^\gamma = \text{const}$$
  
 Polytropic Process  $\Rightarrow \frac{R}{C_v} = \gamma - 1 \Rightarrow C_v = \frac{R}{\gamma - 1}$

$$C = \frac{R}{\gamma_1 - 1} + \frac{R}{1 - \gamma} = C_v + \frac{R}{1 - \gamma}$$

Derivation of Work done in a polytropic Process.

$$\Delta Q = n C \Delta T$$

$$= n \left( C_v + \frac{R}{1 - \gamma} \right) (T_2 - T_1)$$

$$= n C_v \Delta T + \frac{n R \Delta T}{1 - \gamma}$$

$$= \Delta U + \Delta W$$

So  $\Delta W = \text{Work done} = \frac{n R \Delta T}{1 - \gamma}$

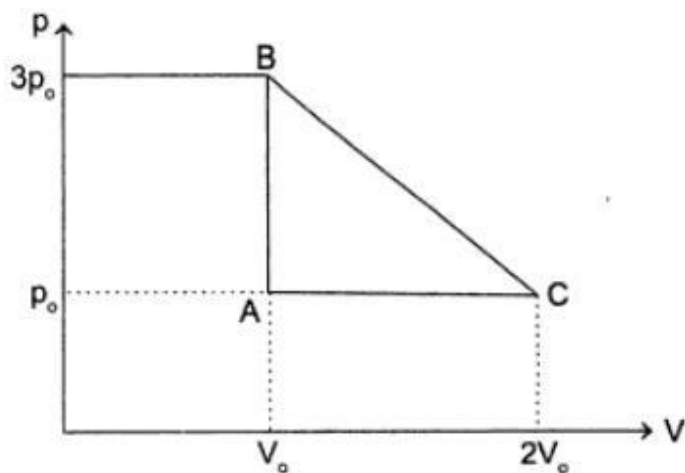
where Polytropic Process is  $PV^\gamma = \text{const}$

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Heat or Thermodynamics 3 ) Work done calculations in various situations

**One mole of an ideal gas is taken round the cyclic process  $ABCA$  as shown in the figure. Calculate:**

- (i) **The work done by the gas.**
- (ii) **The heat rejected by the gas in the path  $CA$  and the absorbed by the gas in the path  $BC$ .**
- (iii) **The net heat absorbed by the gas in the path  $BC$ .**
- (iv) **The maximum temperature attained by the gas during the cycle.**



**Solution**

- (i) Work done by the gas during a cyclic process is equal to the area enclosed by its P-V diagram. In the present case,

$$\begin{aligned}W &= \text{area of } \triangle ABC \\&= \frac{1}{2}(AC)(AB) \\&= \frac{1}{2}(2V_0 - V_0)(3p_0 - p_0) \\&= p_0 V_0\end{aligned}$$

- (ii) The path CA is an isobaric compression of one mole of an ideal gas from volume  $2V_0$  to  $V_0$ . The heat released in this path is

$$\begin{aligned}Q_1 &= n C_p \Delta T \\&= \left(\frac{3}{2}R\right)\left(\frac{p_0 \Delta V}{R}\right) \\&= \left(\frac{5}{2}p_0\right)(V_0 - 2V_0) = -\frac{5}{2}p_0 V_0\end{aligned}$$

The path AB is an isochoric expansion of one mole of an ideal gas from pressure  $p_0$  to  $3p_0$ . The heat released in this process is

$$\begin{aligned}Q_2 &= n C_v \Delta T \\&= \left(\frac{5}{2}R\right)\left(\frac{V_0 \Delta p}{R}\right) \\&= \left(\frac{3}{2}V_0\right)(3p_0 - p_0) = 3p_0 V_0\end{aligned}$$

(iii) In a cyclic process, the change in internal energy is zero. Hence

$$Q_{CA} + Q_{AB} + Q_{BC} = W$$

$$-\frac{5}{2}p_0V_0 + 3p_0V_0 + Q_{BC} = p_0V_0$$

$$\text{This gives } Q_{BC} = \frac{1}{2}p_0V_0$$

(iv) The path BC is a straight line path. It is represented by the expression

$$p - p_0 = \left( \frac{3p_0 - p_0}{V_0 - 2V_0} \right) (V - 2V_0)$$

$$= \left( \frac{-2p_0}{V_0} \right) (V - 2V_0)$$

$$\text{or } p = \frac{-2p_0}{V_0}V + 5p_0$$

Replacing  $p = \frac{RT}{V}$ , we get

$$T = -2\frac{p_0}{V_0R}V^2 + \frac{5V_0}{R}V$$

To determine  $T_{\max}$ , we set  $\frac{\partial T}{\partial V} = 0$

$$\text{i.e., } 0 = -\frac{2p_0}{V_0R}(2V) + \frac{5p_0}{R}$$



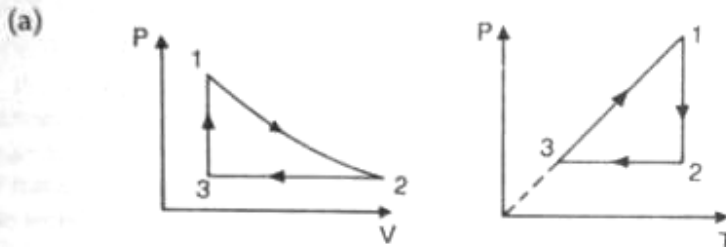
which gives  $V = \frac{5}{4} V_0$ .

With this  $T_{\max}$  is given by

$$\begin{aligned} T_{\max} &= -\frac{2p_0}{V_0 R} \left( \frac{5}{4} V_0 \right)^2 + \left( \frac{5p_0}{R} \right) \left( \frac{5}{4} V_0 \right) \\ &= \frac{p_0 V_0}{R} \left[ -\frac{25}{8} + \frac{25}{4} \right] \\ &= \frac{25 p_0 V_0}{8 R} \end{aligned}$$

Three moles of an ideal gas ( $C_p = \frac{7}{2} R$ ) at pressure  $p_A$  and temperature  $T_A$  is isothermally expanded to twice its initial volume. It is then compressed at constant pressure to its original volume. Finally the gas is compressed at constant volume to its original pressure  $p_A$ . (i) Sketch  $p$ - $V$  and  $p$ - $T$  diagrams for the complete process. (b) Calculate the net work done by the gas and net heat supplied to the gas during the complete process. [IIT 1991]

**Solution.**



(b) In the process  $1 \rightarrow 2$  the state changes from  $(p_A, V, T_A)$  to  $(p_2, 2V, T_A)$ .

$$\text{Hence } p_2 = \frac{p_A}{2}$$

$$\text{Here } \Delta U = 0 \quad \Delta W = \int_V^{2V} p dV = 3RT_A \ln 2, \quad \Delta Q = \Delta U + \Delta W = \Delta W$$

In the process  $2 \rightarrow 3$  the state changes from  $\left( \frac{p_A}{2}, 2V, T_A \right)$  to  $(p_A/2, V, T_3)$  so that  $\frac{p_A}{2} \times \frac{2V}{T_A} = \frac{p_A/2 \times V}{T_3}$  or  $T_3 = T_A/2$

$$\gamma = \frac{C_p}{C_v} = \frac{C_p}{C_p - R} = \frac{\frac{7}{2}R}{\frac{7}{2}R - R} = \frac{7}{5}$$

$$\therefore \Delta U = -\frac{3RT_A}{\left(\frac{7}{5}-1\right) \times 2} = -\frac{15RT_A}{4}$$

$$\Delta W = \int_{2V}^V p dV = \frac{p_A}{2} (V - 2V) = -\frac{p_A V}{2} = -\frac{3RT_A}{2}$$

$$\therefore \Delta Q = \Delta U + \Delta W = -\frac{15}{4} RT_A - \frac{3}{2} RT_A = -\frac{21RT_A}{4}$$

In the process 3  $\rightarrow$  1, the state changes from  $\left(\frac{p_A}{2}, V, \frac{T_A}{2}\right)$  to  $(p_A, V, T)$  that

$$\frac{p_A/2 \times V}{T_A/2} = \frac{p_A V}{T} \quad \text{or } T = T_A$$

$$\Delta U = 3C_v \left(T_A - \frac{T_A}{2}\right) = \frac{3R}{\frac{7}{5}-1} \times \frac{T_A}{2} = \frac{15}{4} RT_A$$

$$\Delta W = 0$$

$$\therefore \Delta Q = \Delta U = \frac{15}{4} RT_A$$

$$\therefore \text{Net } \Delta W = 3RT_A \ln 2 - \frac{3}{2} RT_A + 0 = 3RT_A \left(\ln 2 - \frac{1}{2}\right)$$

$$\text{Net } \Delta Q = 3RT_A \ln 2 - \frac{21RT_A}{4} + \frac{15RT_A}{4} = 3RT_A \left(\ln 2 - \frac{1}{2}\right)$$

A certain volume of a gas (diatomic) expands isothermally at  $20^\circ\text{C}$  until its volume is doubled and then adiabatically until its volume is again doubled. Find the final temperature of the gas, given  $\gamma = 1.4$  and that there is 0.1 mole of the gas. Also calculate the work done in the two cases.  $R = 8.3 \text{ J mole}^{-1} \text{ K}^{-1}$ .

**Solution.**

We require  $T$ - $V$  relation to calculate the final temperature.

We have  $TV^{\gamma-1} = \text{constant} \therefore (273 + 20) \cdot V^{\gamma-1} = (273 + t)(2V)^{\gamma-1}$

or  $273 + t = \frac{293}{2^{1.4-1}} = \frac{293}{2^{0.4}}$

$\log(273 + t) = \log 293 - 0.4 \log 2 = \log 293 - 0.4 \times 0.3010$   
 $= 2.4669 - 0.1204$

or  $\log(273 + t) = 2.3465$

or  $273 + t = \text{antilog } 2.3465$

or  $273 + t = 222.1$

$\therefore t = -50.9^\circ\text{C}$

(i) Work done in isothermal process

$$= nRT \log_e \frac{V_2}{V_1} = \frac{8.3}{10} \times 293 \log_e \frac{2V}{V} \quad \left( \because n = \frac{1}{10} \right)$$

$$= 0.83 \times 293 \times 2.3 \log_{10} 2 \quad (\because \log_e x = 2.3 \log_{10} x)$$

$$= 0.83 \times 293 \times 2.3 \times 0.3010 = 1.684 \times 10^2 \text{ J}$$

(ii) Work done in adiabatic process =  $\frac{nR(T - T')}{\gamma - 1} = \frac{0.83(293 - 222.1)}{1.4 - 1}$

$$= \frac{0.83 \times 70.9}{0.4} = 1.47 \times 10^2 \text{ J}$$

*The volume of one mole of an ideal gas with the adiabatic exponent  $\gamma$  is changed according to the relation  $V = aT^\gamma$ , where  $a$  is a constant. Find the amount of heat absorbed by the gas in the process if the temperature is increased by  $\Delta T$ .*

**Solution.**

We have  $\Delta W = \int p dV$  and  $\Delta U = \int C_V dT$ , for an ideal gas  $pV = RT$ ,

$$\therefore \Delta W = \int_T^{T+\Delta T} \frac{RT}{V} dV = \int_T^{T+\Delta T} \frac{RT^2}{a} \left( -\frac{a}{T^2} dT \right) = -R\Delta T$$

$$\Delta U = \int_T^{T+\Delta T} \frac{R}{\gamma-1} dT = \frac{R\Delta T}{\gamma-1}$$

$$\therefore \Delta Q = \Delta U + \Delta W = \frac{R\Delta T}{\gamma-1} + (-R\Delta T) = \frac{(2-\gamma)R\Delta T}{\gamma-1}$$

**Two moles an ideal mono-atomic gas initially at pressure  $p_1$  and volume  $V_1$  undergo an adiabatic compression until its volume is  $V_2$ . Then, the gas is given heat  $Q$  at constant volume  $V_2$ .**

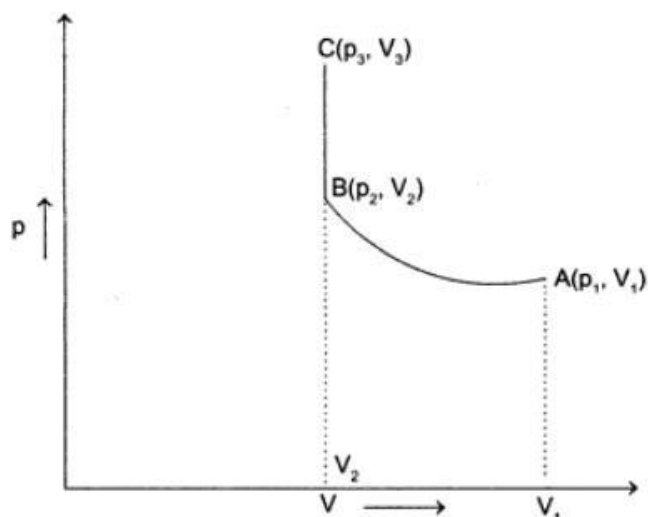
- (i) Sketch the complete process on a  $p$ - $V$  diagram.
- (ii) Find the total work done by the gas, total change in its internal energy and the final temperature of the gas.

[Give your answer in terms of  $p_1$ ,  $V_1$ ,  $V_2$ ,  $Q$  and  $R$ ].



**Solution**

(i) Figure displays the  $p$ - $V$  diagram of the gas undergone the given two processes.



The curve A to B represents the adiabatic compression of the gas from the volume  $V_1$  to  $V_2$ . In this process the pressure of the gas increases  $p_1$  to  $p_2$ .

The line B to C represents increase in pressure of the gas as a result of giving here  $Q$  to the gas at constant volume. In this process, the pressure of the gas increases from  $p_2$  to  $p_3$ .

(ii) (a) **Total work done by the gas**

Work done by the gas in adiabatic compression.

In an adiabatic process, since  $Q = 0$ , therefore from the first law of thermodynamics,

$$\Delta U_1 = -W_1$$

$$\text{or } W_1 = \Delta U_1 = -C_v \Delta T$$

$$= -C_v (T_2 - T_1)$$

$$= -C_v \left( \frac{p_2 V_2}{nR} - \frac{p_1 V_1}{nR} \right)$$

$$= \frac{C_{v,m}}{R} (p_2 V_2 - p_1 V_1) \quad \dots(i)$$

For a gas underdoes adiabatic process

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\text{where } \gamma = \frac{C_{p,m}}{C_{v,m}}$$

From equation (i),

$$W_1 = \frac{C_{v,m}}{R} \left[ \frac{p_1 V_1^\gamma}{V_2^\gamma} V_2 - p_1 V_1 \right]$$

$$= \frac{C_{v,m}}{R} p_1 V_1 \left[ \left( \frac{V_1}{V_2} \right)^{\gamma-1} - 1 \right]$$

For a mono-atomic gas,

$$C_{v,m} = \frac{3}{2}R, \text{ and } C_{p,m} = \frac{5}{2}R$$

$$\therefore \gamma = \frac{5}{3}$$

$$\text{Hence, } W_1 = -\frac{3p_1 V_1}{2} \left[ \left( \frac{V_1}{V_2} \right)^{\frac{2}{3}} - 1 \right]$$

Since the volume is held constant, work done by the gas on heating at constant volume, therefore

$$W_2 = 0$$

Total work don by the gas,

$$W = W_1 = W_2$$

$$= -\frac{3p_1V_1}{2} \left[ \left( \frac{V_1}{V_2} \right)^{\frac{2}{3}} - 1 \right]$$

**(b) Total change in internal Energy**

Change in internal energy in adiabatic compression, as derived above,

$$\Delta U_1 = \frac{3p_1V_1}{2} \left[ \left( \frac{V_1}{V_2} \right)^{\frac{2}{3}} - 1 \right]$$

Change in internal energy on heating the gas at constant volume

$$\Delta U_2 = Q$$

Total change in the internal energy of the gas

$$\Delta U = \Delta U_1 + \Delta U_2$$

$$= \frac{3p_1V_1}{2} \left[ \left( \frac{V_1}{V_2} \right)^{\frac{2}{3}} - 1 \right] + Q.$$

**(c) Final temperature of the gas**

Change in temperature in adiabatic compression.

Since,  $\Delta U = C_v \Delta T$

$$\text{therefore, } \Delta T = \frac{\Delta U_1}{C_v}$$

$$\text{or } T_2 - T_1 = \frac{3p_1V_1}{2C_v} \left[ \left( \frac{V_1}{V_2} \right)^{2/3} - 1 \right]$$

$$T_2 - T_1 + \frac{3p_1V_1}{2\left(\frac{3}{2}nR\right)} \left[ \left( \frac{V_1}{V_2} \right)^{2/3} - 1 \right]$$

$$= \frac{p_1V_1}{nR} + \frac{p_1V_1}{nR} \left[ \left( \frac{V_1}{V_2} \right)^{2/3} - 1 \right]$$

$$= \frac{p_1V_1}{nR} \left( \frac{V_1}{V_2} \right)^{2/3}$$

Change in temperature on heating the gas

$$Q = C_v \Delta T = C_v (T_3 - T_2)$$

$$\text{or } T_3 = \frac{Q}{C_v} + T_2 = \frac{Q}{\left(\frac{3}{2}\right)nR} + \frac{p_1V_1}{nR} \left( \frac{V_1}{V_2} \right)^{2/3}$$

Since  $n = 2$ , therefore

$$T_3 = \frac{Q}{(3 \text{ mole})R} + \frac{p_1V_1}{(2 \text{ mole})R} \left( \frac{V_1}{V_2} \right)^{2/3}.$$



**Two moles of helium gas ( $\gamma = 5/3$ ) are initially at temperature  $27^\circ\text{C}$  and occupy a volume of 20 litres. The gas is expanded**

**at constant pressure until the volume is doubled. Then, it undergoes an adiabatic change until the temperature returns to its initial value.**

- (i) Sketch the process on a p-V diagram.
- (ii) What are the final volume and pressure of the gas?
- (iii) What is the work done by the gas?

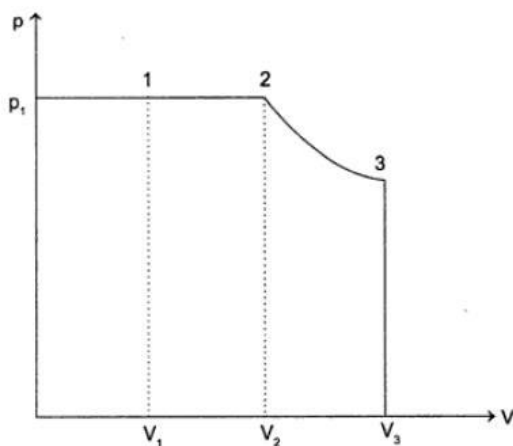
**Solution**

(i)  $V_1 = 20 \times 10^{-3} \text{ m}^3$

$T_1 = 300 \text{ K}$

$n = 2 \text{ moles}$

$\gamma = \frac{5}{3}$



Process 1  $\rightarrow$  2 is isobaric expansion

$$p_1 V_1 = nRT_1$$

$$\therefore p_1 = \frac{nRT_1}{V_1}$$

$$= \frac{2 \times 8.3 \times 300}{20 \times 10^{-3}} = 2.49 \times 10^5 \text{ Nm}^{-2}$$

Now,  $V \propto T$

$$\therefore \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\text{or } T_2 = T_1 \times \frac{V_2}{V_1} = 300 \times \frac{2V_1}{V_1}$$

$$\therefore T_2 = 600 \text{ K}$$

$$V_2 = 40 \times 10^{-3} \text{ m}^3$$

Work done during process 1  $\rightarrow$  2,

$$(W)_{1-2} = p \times \Delta V$$

$$= 2.49 \times 10^5 \times (40 - 20) \times 10^{-3}$$
$$= 4980 \text{ J}$$

Process 2  $\rightarrow$  3 is adiabatic expansion

$$T_2 = 600 \text{ K}$$

$$p_2 = p_1 = 2.48 \times 10^5 \text{ N/m}^2$$

$$V_2 = 40 \times 10^{-3} \text{ m}^3$$

Given,  $T_2 V_2^{\gamma-1} = T_3 V_3^{\gamma-1}$ ,  $T_3 = T_1$

$$\therefore \left( \frac{V_3}{V_2} \right)^{\frac{5}{3}-1} = \frac{T_2}{T_1} = \frac{600}{300} = 2$$

$$\therefore V_3 = V_2 \times (2)^{3/2}$$
$$= 40 \times 10^{-3} (2)^{3/2}$$
$$= 113.14 \times 10^{-3} \text{ m}^3$$

$$\begin{aligned} \text{Now, } p_2 V_2^\gamma &= p_3 V_3^\gamma \\ \therefore p_3 &= p_2 \left( \frac{V_2}{V_3} \right)^\gamma \\ &= 2.48 \times 10^5 \left( \frac{40}{113.14} \right)^{5/3} = 0.44 \times 10^5 \text{ N/m}^2 \\ (W)_{2-3} &= \frac{p_2 V_2 - p_3 V_3}{\gamma - 1} \\ &= \frac{(2.49 \times 10^5)(40 \times 10^{-3}) - (0.44 \times 10^5)(113.14 \times 10^{-3})}{(5/3) - 1} \\ &= \mathbf{7472.8 \text{ J.}} \end{aligned}$$

(ii) Final volume,  $V_3 = 113.14 \times 10^{-3} \text{ m}^3$

Final pressure,  $p_3 = 0.44 \times 10^5 \text{ Nm}^{-2}$

(iii) Total work done by the gas =  $W =$

$$\begin{aligned} &(W)_{1-2} + (W)_{2-3} \\ &= 4980 + 7472.8 = \mathbf{12452.8 \text{ J.}} \end{aligned}$$

Work done example in Isothermal expansion

A gram mole of a gas at  $127^\circ \text{C}$  expands isothermally until its volume is doubled. Find the amount of work done.

(a) 238 cal

(b) 548 cal

(c) 548 J

(d) 238 J

$$\begin{aligned} \text{(b) } W &= 2.303 RT \log_{10} \left( \frac{V_2}{V_1} \right) \\ &= 2.303 \times 8.311 \times 400 \times \log_{10} 2 \\ &= 2310.1 \text{ J} = 548 \text{ cal.} \end{aligned}$$

### Example in Isothermal Expansion

**How much work is done by an ideal gas in expanding isothermally from an initial volume of 3 litres of 20 atm to a final volume of 24 litres?**

**Solution** In isothermal process at temperature T,

$$W = 2.303nRT \log \frac{V_2}{V_1}$$

or  $W = 2.303(p_1 V_1) \log \frac{V_2}{V_1}$   
(using  $p_1 V_1 = nRT$ )

$$= 2.303 (20 \times 3) \log \frac{P_1}{P_2} \text{ lt. atm}$$

$$= 2.303 \times 60 \log 8 (101) \text{ J}$$

$$= \mathbf{1.26 \times 10^4 \text{ J}}$$

Work done by the gas

The ratio of work done by an ideal diatomic gas to the heat supplied by the gas in an isobaric process is

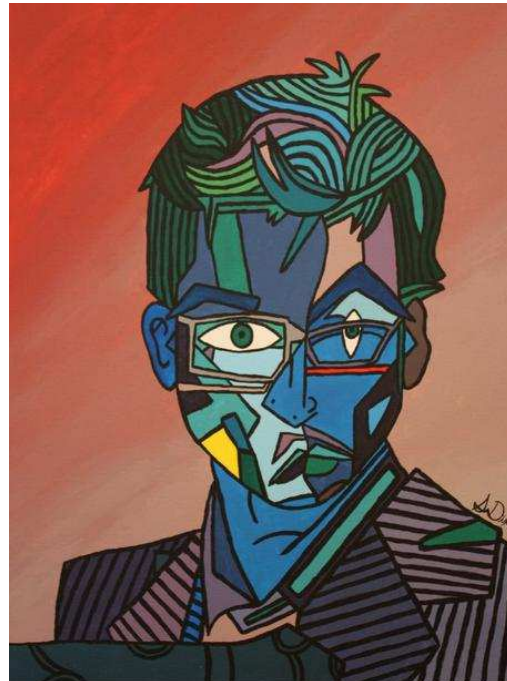
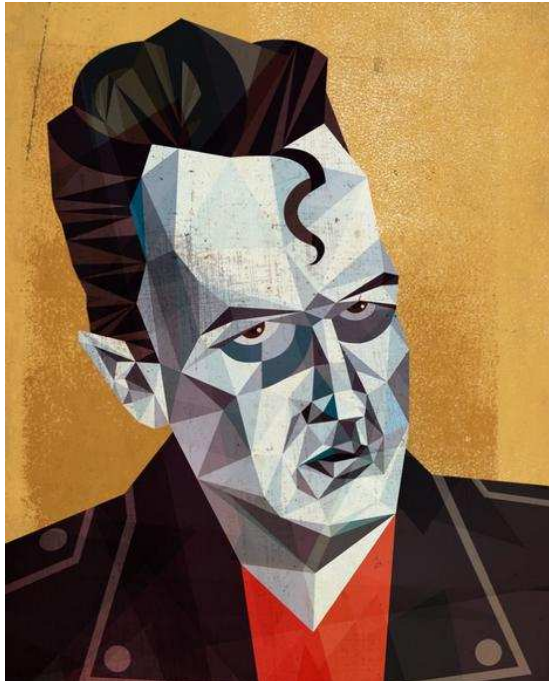
- (a)  $\frac{5}{7}$  (b)  $\frac{3}{5}$   
(c)  $\frac{2}{7}$  (d)  $\frac{5}{3}$

Ans - (c)  $\Delta U = nC_v\Delta T = n \frac{5}{2} R\Delta T$

$$\Delta Q = nC_p\Delta T = n \frac{7}{2} R\Delta T$$

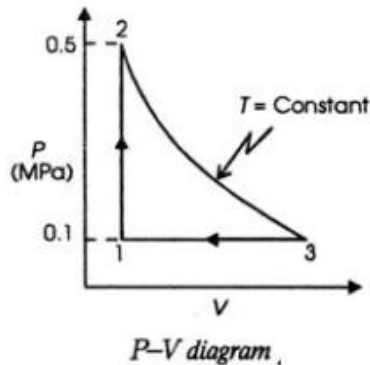
$$W = \Delta Q - \Delta U = \frac{n7}{2} R\Delta T - nR\Delta T$$

$$\frac{W}{Q} = \frac{2}{7}$$





One mole of a gas which obeys the relation  $Pv = RT$ , where  $R = 8.314 \text{ J/mol K}$  is initially at  $300 \text{ K}$  and  $0.1 \text{ MPa}$ . The gas is heated at constant volume till the pressure rises to  $0.5 \text{ MPa}$  and then allowed to expand at constant temperature till the pressure reduces to  $0.1 \text{ MPa}$ . Finally the gas is returned to its original state by compressing at constant pressure. Calculate the work done by the gas in each of the processes and also estimate the net work done by the gas.



**Solution** The process followed by the gas is shown in Fig.2.12. Work done by the gas during process 1–2 is given by

$$W_{1-2} = \int_1^2 Pdv = 0 \quad (\text{since } dv = 0)$$

We know  $P_1v_1 = RT_1$  and  $P_2v_2 = RT_2$ . Therefore

$$\frac{T_2}{T_1} = \frac{P_2v_2}{P_1v_1} = \frac{P_2}{P_1} = \frac{0.5 \times 10^6}{0.1 \times 10^6} = 5 \quad (\text{since } v_2 = v_1)$$

or  $T_2 = 5T_1 = 5 \times 300 = 1500 \text{ K}$

Work done by the gas during process 2–3 is given by

$$W_{2-3} = \int_2^3 Pdv = \int_2^3 \frac{RT}{v} dv = RT_2 \ln \frac{v_3}{v_2}$$

We know  $P_2v_2 = P_3v_3$  (since  $T_2 = T_3$ ). Therefore

$$\frac{v_3}{v_2} = \frac{P_2}{P_3} = \frac{0.5 \times 10^6}{0.1 \times 10^6} = 5$$

Hence  $W_{2-3} = RT_2 \ln 5 = 8.314 \times 1500 \times \ln 5 = 20.071 \text{ kJ}$ . Work done during process 3–1 is given by

$$W_{3-1} = \int_3^1 Pdv = P_1(v_1 - v_3) = P_1v_1 \left(1 - \frac{v_3}{v_1}\right) = RT_1 \left(1 - \frac{v_3}{v_1}\right)$$

We know  $P_1v_1 = RT_1$  and  $P_3v_3 = RT_3$

$$\text{or } \frac{v_3}{v_1} = \frac{RT_3}{P_3} \cdot \frac{P_1}{RT_1} = \frac{T_3}{T_1} \quad (\text{since } P_1 = P_3). \text{ Therefore}$$

$$W_{3-1} = RT_1 \left(1 - \frac{v_3}{v_1}\right) = RT_1 \left(1 - \frac{T_3}{T_1}\right) = 8.314 \times 300 \left(1 - \frac{1500}{300}\right) = -9.977 \text{ kJ}$$

$$\text{Net work done by the gas, } W = W_{1-2} + W_{2-3} + W_{3-1} = 0 + 20.071 - 9.977 = 10.094 \text{ kJ}$$

Work done by the gas

**A sample of ideal gas ( $\gamma = 1.4$ ) is heated at constant pressure. If an amount of 140 J of heat is supplied to the gas, find:**

- (i) **The change in internal energy of the gas.**
- (ii) **The work done by the gas.**

**Solution** Suppose, the sample contains  $n$  moles. Also, suppose the volume changes from  $V_1$  to  $V_2$  and the temperature changes from  $T_1$  to  $T_2$ .

The heat supplied is given by

$$\Delta Q = nC_p(T_2 - T_1)$$

- (i) Change in internal energy

$$\Delta U = nC_v(T_2 - T_1)$$

$$= \frac{C_v}{C_p} nC_p(T_2 - T_1)$$

$$= \frac{C_v}{C_p} \Delta Q = \frac{140 \text{ J}}{1.4} = \mathbf{100 \text{ J}}$$

- (ii) Work done by gas

$$\Delta W = \Delta Q - \Delta U$$

$$= 140 \text{ J} - 100 \text{ J} = \mathbf{40 \text{ J}}$$

work done by the gas

**A sample of gas ( $\gamma = 1.5$ ) is taken through an adiabatic process in which the volume is compressed from  $1600 \text{ cm}^3$  to  $400 \text{ cm}^3$ . If the initial pressure is  $150 \text{ kPa}$ ,**

- (i) **What is the final pressure?**
- (ii) **How much work is done by the gas in the process?**

**Solution**

- (i) For an adiabatic process

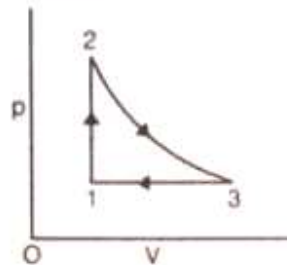
$$p_1 V_1^\gamma = p_2 V_2^\gamma$$

$$\begin{aligned}\text{Thus, } p_2 &= p_1 \left( \frac{V_1}{V_2} \right)^\gamma \\ &= (150 \text{ kPa}) \left( \frac{1600}{400} \right)^{\frac{3}{2}} \\ &= \mathbf{1200 \text{ kPa}}\end{aligned}$$

- (ii) Work done by the gas in an adiabatic process

$$\begin{aligned}W &= \frac{p_1 V_1 - p_2 V_2}{\gamma - 1} \\ &= \frac{(150 \text{ kPa})(1600 \text{ cm}^3) - (1200 \text{ kPa})(400 \text{ cm}^3)}{1.5 - 1} \\ &= \frac{240 \text{ J} - 480 \text{ J}}{0.5} = \mathbf{-480 \text{ J}}\end{aligned}$$

A cyclic process for an ideal monatomic gas ( $C_V = 12.5 \text{ J mol}^{-1} \text{ K}^{-1}$ ) is represented in the figure. The temperatures at 1, 2 and 3 are 300 K, 600 K and 455 K, respectively. Compute the values of  $\Delta Q$ ,  $\Delta U$  and  $\Delta W$  for each of the processes. The process from 2 to 3 is adiabatic.



**Solution.**

In the process from 1 to 2

$$\Delta W = \int p dV = 0 \quad (\text{volume remains constant})$$

$$\begin{aligned} \Delta Q &= \int_{T_1}^{T_2} C_V dT = C_V(T_2 - T_1) \\ &= 12.5(600 - 300) = 3750 \text{ joules} \end{aligned}$$

By the first law of thermodynamics

$$\begin{aligned} \Delta Q &= \Delta U + \Delta W \quad \text{or} \quad \Delta U = \Delta Q - \Delta W \\ &= 3750 - 0 = 3750 \text{ joules} \end{aligned}$$

In the process 2 to 3  $\Delta Q = 0$   
(since the process is adiabatic)

$$\begin{aligned}\Delta W &= \frac{R(T_2 - T_3)}{\gamma - 1} \\ &= C_V(T_2 - T_3) \quad \left( \because C_V = \frac{R}{\gamma - 1} \right) \\ &= 12.5(600 - 455) = 12.5 \times 145 = 1812.5 \text{ joules}\end{aligned}$$

$$\therefore \Delta U = \Delta Q - \Delta W = 0 - 1812.5 = -1812.5 \text{ joules}$$

$$\text{In the process from 3 to 1, } \Delta W = \int_V^{V_1} p dV = p(V_1 - V_3) = pV_1 - pV_3$$

$$\begin{aligned}\text{or } \Delta W &= R(T_1 - T_3) \quad (\because pV = RT) \\ &= 8.31(300 - 455) = -1288 \text{ joules}\end{aligned}$$

$$\begin{aligned}\Delta Q &= \int_{T_3}^{T_1} C_p dT = C_p(T_1 - T_3) = 1.67 \times 12.5 \times (300 - 455) \quad \left( \because \gamma = \frac{C_p}{C_v} \right) \\ &= -3235.6 \text{ joules.}\end{aligned}$$

By the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

$$\therefore \Delta U = \Delta Q - \Delta W = (-3235.6) - (-1288) = 1989.1 \text{ joules}$$

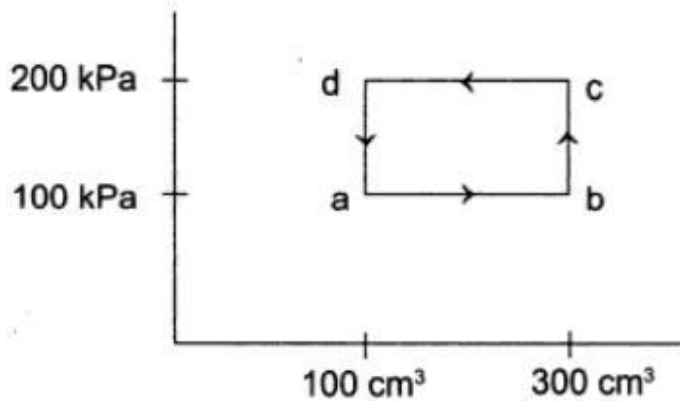
Question on Total Heat rejected

**A thermodynamic system is taken through the cycle  $a b c d a$ .**

**(i) Calculate the work done by the gas during the parts  $ab$ ,  $bc$ ,  $cd$  and  $da$ .**



(ii) Find the total heat rejected by the gas during the process.



**Solution**

(i) Work done during the part  $ab = \int_a^b p dV$

$$\begin{aligned}
 &= (100 \times \text{Pa}) \int_a^b dV \\
 &= (100 \text{ kPa}) (300 \text{ cm}^3 - 100 \text{ cm}^3) \\
 &= 20 \text{ J}
 \end{aligned}$$

The work done during  $bc$  is zero as the volume does not change. The work done during  $cd$ ,

$$\begin{aligned}
 &= \int_d^c p dV \\
 &= (200 \text{ kPa}) (100 \text{ cm}^3 - 300 \text{ cm}^3) \\
 &= -40 \text{ J}
 \end{aligned}$$

The work done during  $da$  is zero as the volume does not change.

(ii) Total work done by the system during the cycle  $a b c d a$ .

$$\Delta W = 20 \text{ J} - 40 \text{ J}$$

$$= -20 \text{ J}$$

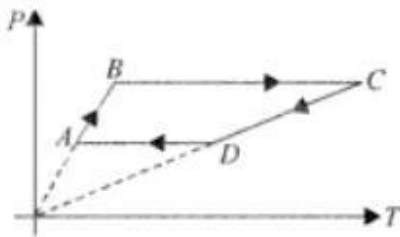
Change in the internal energy,  $\Delta U = 0$ , as the initial state is the same as the final state.

$$\begin{aligned} \text{Thus, } \Delta Q &= \Delta U + \Delta W \\ &= -20 \text{ J} \end{aligned}$$

So, the system rejects **20 J** of heat during the cycle.

- Question with P T diagram

3 moles of an ideal monoatomic gas perform a cycle shown in Fig. The gas temperatures  $T_A = 400 \text{ K}$ ,  $T_B = 800 \text{ K}$ ,  $T_C = 2400 \text{ K}$ ,  $T_D = 1200 \text{ K}$ . Find the work done by the gas.



Solution :

$$W_{BC} = 3R(T_C - T_B)$$

$$W_{AB} = W_{CD} = 0$$

because the processes are isochoric

$$W_{DA} = 3R(T_A - T_D)$$

Total work done

$$\begin{aligned} W_{BC} + W_{DA} &= 3R(T_A + T_C - T_B - T_D) \\ &= 3R(400 + 2400 - 800 - 1200) \\ &= 2400R = 20 \text{ kJ} \end{aligned}$$

Work done by the gas

**Two moles of Helium gas ( $\gamma = \frac{5}{3}$ ) are initially at  $27^\circ \text{C}$  and occupy a volume of 20 litres. The gas is first expanded at constant pressure until the volume is doubled. Then it undergoes an adiabatic change until the temperature returns to its initial value.**

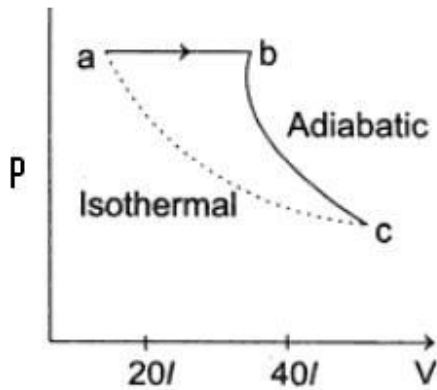
- (i) Sketch the process in a p–V diagram.
- (ii) What is the final volume and pressure of the gas?
- (iii) What is the work done by the gas?

**Solution**

- (i) The process is shown in the figure. During the part *ab*, since the pressure is constant, we have

$$\frac{P_a V_a}{T_a} = \frac{P_b V_b}{T_b}$$

$$\begin{aligned} \text{or } T_b &= \frac{V_b}{V_a} T_a \\ &= 2T_a = \mathbf{600 \text{ K}} \end{aligned}$$



During the part  $bc$ , the gas is adiabatically returned to the temperature  $T_a$ . The point  $a$  and point  $c$  are on the same isothermal. Thus, we draw an adiabatic curve  $bc$  and an isothermal from  $a$  and look for the point of intersection  $c$ . That is the final state.

(ii) From the isothermal  $ac$ ,

$$p_a V_a = p_b V_b \quad \dots(i)$$

And from the adiabatic curve  $bc$ ,

$$p_b V_b^\gamma = p_c V_c^\gamma$$

$$\text{or } p_a (2V_a)^\gamma = p_c V_c^\gamma$$

Dividing equation (ii) by equation (i), we get

$$2^\gamma (V_a)^{\gamma-1} = (V_c)^{\gamma-1}$$

$$\begin{aligned} \text{or } V_c &= 2^{\frac{\gamma}{\gamma-1}} V_a 4\sqrt{2} V_a \\ &= 113 \text{ litres} \end{aligned}$$

From equation (i),

$$p_c = \frac{p_a V_a}{V_c} = \frac{nRT}{V_c}$$
$$= \frac{2 \text{ mol} \times (8.3 \text{ J/mol} - k)(300 \text{ K})}{113 \times 10^{-3} \text{ m}^3}$$
$$= 4.4 \times 10^4 \text{ Pa}$$

(iii) Work done by the gas in the part *ab*

$$= p_a (V_b - V_a)$$
$$= p_a V_b - p_a V_a = nRT_2 - nRT_1$$
$$= 2 \text{ mole} \times (8.3 \text{ J/mol} - \text{K}) \times (600 \text{ K} - 300 \text{ K})$$
$$= 4980 \text{ J}$$

Work done in the adiabatic part *bc*

$$= \frac{p_b V_b - p_c V_c}{\gamma - 1}$$
$$= \frac{nR(T_2 - T_1)}{\gamma - 1} = \frac{4980}{\frac{5}{3} - 1} = 7470 \text{ J}$$

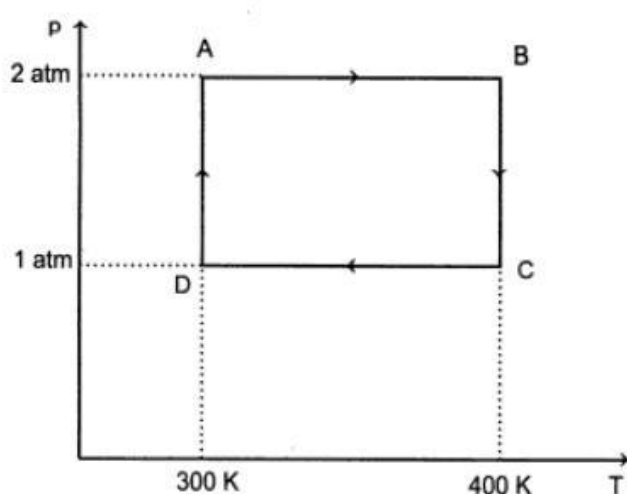
Net work done by the gas = 4980 J +  
7470 J = **12450 J.**

Example of cycle given P T diagram



**Two moles of helium gas undergo a cyclic process as shown in the figure. Assuming the gas to be ideal, calculate the following quantities in this process:**

- (i) **The net change in the heat energy.**
  - (ii) **The net work done.**
  - (iii) **The net change in internal energy.**
- [ $R = 8.32 \text{ J mol}^{-1}$ ]**



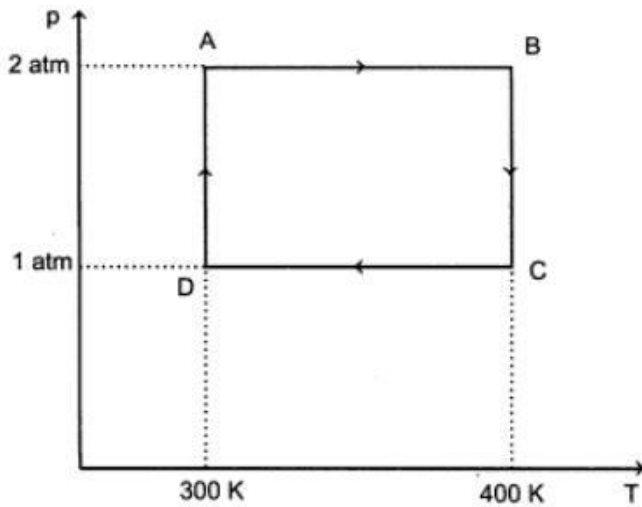
**Solution** Number of moles,  $n = 2$   
Helium is a mono-atomic gas.

$$\therefore C_V = \frac{3}{2}R$$

$$C_P = \frac{5}{2}R$$

The gas undergoes cyclic process.

Since, internal energy is property of the system, the net change in internal energy during the cyclic process is zero.



Hence, according to the first law of thermodynamics, the net change in the heat energy is equal to the net work done.

$$(i) (\Delta Q)_{Net} = (\Delta Q)_{AB} + (\Delta Q)_{BC} + (\Delta Q)_{DA}$$

$$\begin{aligned} (\Delta Q)_{AB} &= n \times C_p \times (T_B - T_A) \\ &= 2 \times \frac{5}{2} \times 8.32(400 - 300) = 4160 \text{ J} \end{aligned}$$

Since Process BC is isothermal, therefore  $\Delta U = 0$

$$\begin{aligned} (\Delta Q)_{BC} &= (\Delta W)_{BC} \\ &= nRT \ln\left(\frac{V_C}{V_B}\right) = nRT \ln\left(\frac{P_B}{P_C}\right) \\ &= 2 \times 8.32 \times 400 \ln\left(\frac{2}{1}\right) = 4613.6 \text{ J} \end{aligned}$$

$$\begin{aligned} (\Delta Q)_{DA} &= nRT \ln\left(\frac{P_D}{P_A}\right) \\ &= 2 \times 8.32 \times 300 \ln\left(\frac{2}{1}\right) = -3460.2 \text{ J} \end{aligned}$$



Efficiency of Refrigerator is given by

### Efficiency of Refrigerator

$$\eta = 1 - \frac{T_c}{T_H}$$

So in this case efficiency  $\eta = 1 - (273 / 303) = 0.099 \approx (\text{approx}) 0.1$  or 10%

Refrigerator Problem

**A refrigerator works between 0°C and 27°C. Heat is to be removed from the refrigerated space at the rate of 50 kcal/minute, the power of the motor of the refrigerator is :**

- (a) 0.346 kW                      (b) 3.46 kW  
(c) 34.6 kW                        (d) 346 kW

Ans : a )

$$\begin{aligned} \frac{T_2}{T_1 - T_2} &= \frac{Q_2}{W} \\ \frac{273}{300 - 273} &= \frac{50,000}{W} \\ W &= \frac{27 \times 50,000}{273} \text{ cal / min} \\ P &= \frac{W}{t} = \frac{4.2 \times 27 \times 50,000}{60 \times 273} \text{ Joule / sec} \\ &= 346 \text{ watt} = 0.346 \text{ kW} \end{aligned}$$

### Efficiency of Refrigerator

$$\eta = 1 - \frac{T_c}{T_H}$$

So in this case efficiency  $\eta = 1 - (273 / 300) = 0.09 \approx$  or 9%

### Refrigerator Problem

An ideal refrigerator has a freezer at a temperature of  $-13\text{ }^{\circ}\text{C}$ . The coefficient of performance of the engine is 5. The temperature of the air (to which heat is rejected) is :

- (a)  $320\text{ }^{\circ}\text{C}$    (b)  $39\text{ }^{\circ}\text{C}$    (c)  $325\text{ K}$    (d)  $325\text{ }^{\circ}\text{C}$

Ans : b )

$$T_2 = 273 - 13 = 260, K = \frac{T_2}{T_1 - T_2}$$

$$5 = \frac{260}{T_1 - 260} \quad \text{or} \quad T_1 - 260 = 52$$

$$T_1 = 312\text{ K}, T_2 = 312 - 273 = 39\text{ }^{\circ}\text{C}$$

### Efficiency of Refrigerator

$$\eta = 1 - \frac{T_c}{T_H}$$

So in this case efficiency  $\eta = 1 - (260 / 312) = 0.16666 \approx (\text{approx}) 0.16667$  or  $16.67\%$

### Refrigerator Problem

A Carnot's engine works as a refrigerator between  $250\text{ K}$  and  $300\text{ K}$ . If it receives  $750$  calories of heat from the reservoir at the lower temperature, the amount of heat rejected at the higher temperature is :

- (a)  $900$  calories                      (b)  $625$  calories  
(c)  $750$  calories                      (d)  $1000$  calories

Ans : a )

$$\frac{750}{W} = \frac{250}{300 - 250}$$

$$\text{Heat rejected} = 750 + 150 = 900\text{ cal.}$$



### Efficiency of Refrigerator

$$\eta = 1 - \frac{T_c}{T_H}$$

So in this case efficiency  $\eta = 1 - (250 / 300) = 0.166666 \approx (\text{approx}) 0.16667$  or 16.67%

### Refrigerator Problem

A refrigerator having a coefficient of performance of 5 is run by an electric motor of power 1.2 kW. How much is the mass of ice formed from water at 0°C per hour by the refrigerator ?

- (a) nearly 6 kg                      (b) nearly 60 kg  
(c) nearly 25.2 kg                (d) 252 kg

Ans : b )

$$5 = \frac{Q_2}{Pt} \quad \text{or} \quad Q_2 = 5 \times 1.2 \times 1000 \times 3600 \text{ J}$$

$$Q = 216 \times 10^5 \text{ J} = 5142857 \text{ cal.}$$

$$\therefore Q = mL \quad \text{or} \quad m = Q/L = 64.2 \text{ kg} \quad \therefore m \approx 60 \text{ kg}$$

### Refrigerator

<b>Given :</b>	$T_H$	1200 °C	$T_C$	200 °C	
	$T_H$	1473.15 K	$T_C$	473.15 K	
<b>Find :</b>	$\eta$	?	$COP_R$	?	$COP_{HP}$ ?

**Part a.)** The thermal efficiency of a Carnot Cycle depends only on the temperatures of the reservoirs with which it interacts. The equation that defines this relationship is :

$$\eta = 1 - \frac{T_C}{T_H} \quad \text{Eqn 1}$$

Just be sure to use absolute temperature in Eqn 1 ! In this case, convert to Kelvin. Temperatures in Rankine will work also.

$$\eta = 67.9\%$$

**part b.)** The coefficient of performance of a Carnot Refrigeration Cycle also depends only on the temperatures of the reservoirs with which it interacts. The equation that defines this relationship is :

$$COP_R = \frac{1}{\frac{T_H}{T_C} - 1} = \frac{T_C}{T_H - T_C} \quad \text{Eqn 2}$$

Using **T** in Kelvin yields :

$$COP_R = 0.4732$$

This is an exceptionally **BAD**  $COP_R$  because it is less than 1. This isn't terribly surprising when you consider that the refrigerator must reject heat to a reservoir at **1200°C** !!

Carnot engine efficiency is covered in every book. But efficiency of refrigerator and Coefficient of Performance is rarely discussed.

Two engines are working in such a way that sink of one is source of the other. Their efficiencies are equal. Find the temperature of the sink of first if its source temperature is 927°C and temperature of sink of the second is 27°C.

- |           |                   |
|-----------|-------------------|
| (a) 327 K | (b) 327°C         |
| (c) 600°C | (d) none of these |

**Solution** (b)  $\eta = 1 - T_2/T_1 = 1 - T_3/T_2$  or  $T_2^2 = T_1 T_3$

or  $T_2 = \sqrt{1200 \times 300} = 600 \text{ K} = 327^\circ\text{C}$

Heat or Thermodynamics 5 ) Concept of “free expansion”

**Free expansion:**

If a system (a gas), expands in such a way that no heat enters or leaves the system (adiabatic process) and also no work is done by or on the system, then the expansion is called the free expansion.

Consider an adiabatic vessel with rigid walls divided into two parts. One containing a gas and the other evacuated. When the partition is suddenly broken, the gas rushes into the vacuum and expands freely.

∴ Net change in internal energy  
 $U_f - U_i = \Delta Q - W$  as  $\Delta Q = 0$  and  $W = 0$   
 $U_i = U_f$

The initial and final internal energies are equal in free expansion.

*One mole of an ideal diatomic gas underwent an adiabatic expansion from 298 K, 15.00 atm, and 5.25 L to 2.50 atm against a constant external pressure of 1.00 atm. What is the final temperature of the system?*

**Plan** This is an isobaric adiabatic expansion against constant external pressure, but overall pressure decreases (volume increases, gas expands). Final temperature  $T_2$  is given by P-V-T relation as:

$$T_2 = T_1 \left( \frac{C_V + P_{\text{ext}} \frac{R}{P_1}}{C_V + P_{\text{ext}} \frac{R}{P_2}} \right)$$

**Solution** For diatomic gas  $C_V = \frac{5}{2}R$ ,  $T_1 = 298 \text{ K}$ ,  $T_2 = ?$ ,

$P_2 = 2.50 \text{ atm}$ ,  $P_1 = 15.00 \text{ atm}$ ,  $P_{\text{ext}} = 1.00 \text{ atm}$

$$\begin{aligned} \therefore T_2 &= 298 \left( \frac{\frac{5}{2}R + \frac{R}{15}}{\frac{5}{2}R + \frac{R}{2.5}} \right) \\ &= 263.7 \text{ K} \end{aligned}$$

**One mole of a gas is put under a weightless piston of a vertical cylinder at temperature  $T$ . The space over the piston is atmosphere. How much work should be performed to increase isothermally the**

**volume under the piston to twice the volume (neglect friction of piston).**

**Solution** Let  $A$  be the area of piston, therefore

$$F + pA = p_0 A$$

or 
$$F = (p_0 - p) A$$

Work done by agent is given by

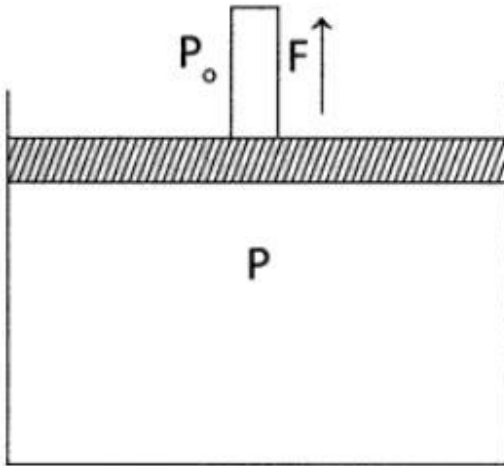
$$W = \int_v^{\eta v} (p_0 - p) A dx$$

$$= \int_v^{\eta v} (p_0 - p) dV$$

$$= \int_v^{\eta v} p_0 dV - \int_v^{\eta v} p dV$$

$$= p_0(\eta - 1)V - \int_v^{\eta v} nRT \frac{dV}{V}$$

(since  $pV = nRT$ )



$$= p_0(\eta - 1)V - nRT \log_e \eta$$

$$= nRT [(\eta - 1) \log_e \eta]$$

where,  $\eta = 2$  and  $n = 1$

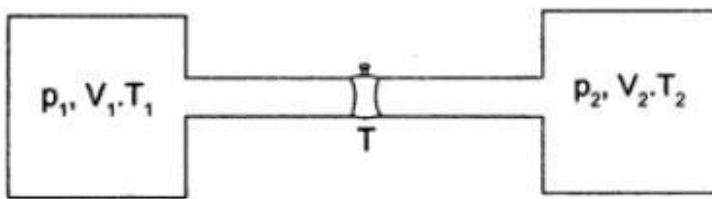
$$\mathbf{W = RT [1 - \log_e 2]}$$



### Adiabatic free expansion

**Two vessels of volume  $V_1$  and  $V_2$  contain the same ideal gas. The pressure in the vessels are  $p_1$  and  $p_2$  and the temperatures are  $T_1$  and  $T_2$  respectively. The two vessels are now connected to each other through a narrow tube. Assuming that no heat is exchanged between the surroundings and the vessels, find the common pressure and temperature attained after the connection.**

#### Solution



The amount of gas in vessel 1 is

$$n_1 = \frac{p_1 V_1}{RT_1}$$

If  $p'$  and  $T'$  are the common pressure and temperature after the connection is made, the amount are

$$n'_1 = \frac{p'V_1}{RT'}$$

and  $n'_2 = \frac{p'V_2}{RT'}$

We have,  $n_1 + n_2 = n'_1 + n'_2$

or  $\frac{p_1V_1}{RT_1} + \frac{p_2V_2}{RT_2} = \frac{p'V_1}{RT'} + \frac{p'V_2}{RT'}$

or  $\frac{p'}{T'} = \frac{1}{V_1 + V_2} \left( \frac{p_1V_1}{T_1} + \frac{p_2V_2}{T_2} \right)$

or  $\frac{T'}{P'} = \frac{T_1T_2(V_1 + V_2)}{p_1V_1T_2 + p_2V_2T_1}$

As the vessels have fixed volume, no work done by the gas plus the vessels system. Also, no heat is exchanged with the surroundings.

Thus, the internal energy of the total system remains constant. The internal energy of an ideal gas is

$$U = nC_vT = C_v \frac{pV}{R}$$

Internal energy of the gases before the connection

$$= \frac{C_v p_1 V_1}{R} + \frac{C_v p_2 V_2}{R}$$

And Internal energy of the gas after the connection

$$= \frac{C_v p' (V_1 + V_2)}{R}$$

Neglecting the change in internal energy of the vessels (the heat capacity of the vessels is assumed negligible).

$$\frac{C_v p_1 V_1}{R} + \frac{C_v p_2 V_2}{R} = \frac{C_v p' (V_1 + V_2)}{R}$$

or 
$$p' = \frac{p_1 V_1 + p_2 V_2}{V_1 + V_2}$$

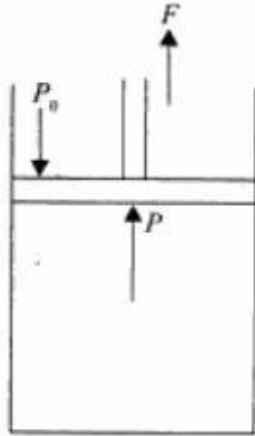
From equation (i), 
$$T' = \frac{T_1 T_2 p_1 V_1 + p_2 V_2}{p_1 V_1 T_2 + p_2 V_2 T_1}$$

Question on work done

One mole of an ideal gas is contained under a weightless piston of a vertical cylinder at a temperature  $T$ . The space over the piston opens into the atmosphere. What work has to be performed in order to increase isothermally the gas volume under the piston  $\eta$  times by slowly raising the piston? Neglect friction.

Solution :

Let A be the area of cross section



$$F + PA = P_0 A$$

$$F = (P_0 - P) A$$

Work done by the agent

$$W = \int_V^{\eta V} F dx = \int_V^{\eta V} (P_0 - P) A dx$$

$$= \int_V^{\eta V} (P_0 - P) dV$$

$$= P_0(\eta - 1) V - \int_V^{\eta V} nRT \frac{dV}{V}$$

$$= RT [(\eta - 1) - n \log_e \eta]$$

### Heat or Thermodynamics 6 ) Ingen Housz's experiment of identical rods

**Ingen-Housz's experiment** Ingen Housz showed that if a number of identical rods of different metals are coated with wax and one of their ends is put in boiling water, then in steady state, the square of length of the bar over which wax melts is directly proportional to the thermal conductivity of the metal. That is,

$$\frac{K}{L^2} = \text{constant}$$

### Heat or Thermodynamics 7 ) Concept of Internal Energy at Room temperature

*Find the internal energy of air in a room of volume  $40 \text{ m}^3$  at 1 standard atmospheric pressure.*

**Solution.**

We have  $U = \frac{pV}{\gamma - 1}$  for a perfect gas

Air is diatomic and therefore its  $\gamma$  is 1.4.

$$\therefore U = \frac{10^5 \times 40}{1.4 - 1} \quad (p = 1 \text{ atm} = 10^5 \text{ Nm}^{-2}) = 10^7 \text{ joules.}$$

### Question in Internal Energy

**The internal energy of a mono-atomic ideal gas is  $1.5 nRT$ . One mole of helium is kept in a cylinder of cross-section  $8.5 \text{ cm}^2$ . The cylinder is closed by a light frictionless piston. The gas is heated slowly in a process during which a total**



**of 42 J heat is given to the gas. If the temperature rised through 2° C, find the distance moved by piston. Atmospheric pressure = 100 kPa.**

**Solution** Change in internal energy of the gas

$$\begin{aligned}\Delta U &= 1.5 nRT \\ &= 1.5 (1 \text{ mole}) (8.3 \text{ J/mol} - \text{K}) (2\text{K}) \\ &= 24.9 \text{ J}\end{aligned}$$

Heat given to the gas = 43 J

$$\begin{aligned}\text{Work done by the gas is } \Delta W &= \Delta Q - \Delta U \\ &= 42 \text{ J} - 24.9 \text{ J} = 17.1 \text{ J}\end{aligned}$$

If the distance moved by the piston is  $x$ , then the work done

$$\begin{aligned}\Delta W &= (100 \text{ kPa}) (8.5 \text{ cm}^2)x \\ &= 17.1 \text{ J}\end{aligned}$$

$$\text{Thus, } (10^5 \text{ N/m}^2) (8.5 \times 10^{-4} \text{ m}^2) x = 17.1 \text{ J}$$

$$\text{or } x = 0.2 \text{ m} = \mathbf{20 \text{ cm}}$$

Heat or Thermodynamics 8 ) Saturated vapor pressure problems

*A saturated water vapour ( $M = 18$ ) is contained in a vessel fitted with a piston at a temperature  $t = 100^\circ\text{C}$ . As a result of slow introduction of the piston a small fraction of the vapour  $\Delta m = 1 \text{ g}$  gets condensed. What amount of work is done over the gas?*

**Solution.**

Work done = decrease in internal energy of the gas

$$= U_i - U_f = \frac{m_i RT}{M} - \frac{m_f RT}{M} = \frac{\Delta m RT}{M}$$

$$\therefore \text{ Here, } W = \frac{10^{-3} \times 8.3 \times (273 + 100)}{18 \times 10^{-3}} = 172 \text{ J.}$$

Water of mass  $m = 1$  kg and  $M$  (mol. mass) = 18 turns completely into saturated vapour at standard atmospheric pressure. Assuming the saturated vapour to be an ideal gas find increment of internal energy of the system. Specific latent heat of steam is  $L = 2250$  kJ/kg.

**Solution.**

$$\Delta Q = \text{heat added to the system} = mL$$

$$\Delta W = \text{work done by the system} = p_0 (V_v - V_w)$$

$$\approx p_0 V_v = \frac{m}{M} RT$$

By the first law ( $\Delta Q = \Delta U + \Delta W$ ),

$$\Delta U = mL - \frac{m}{M} RT = m \left( L - \frac{RT}{M} \right)$$

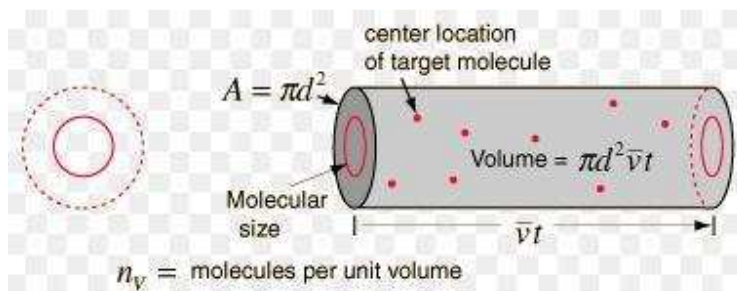
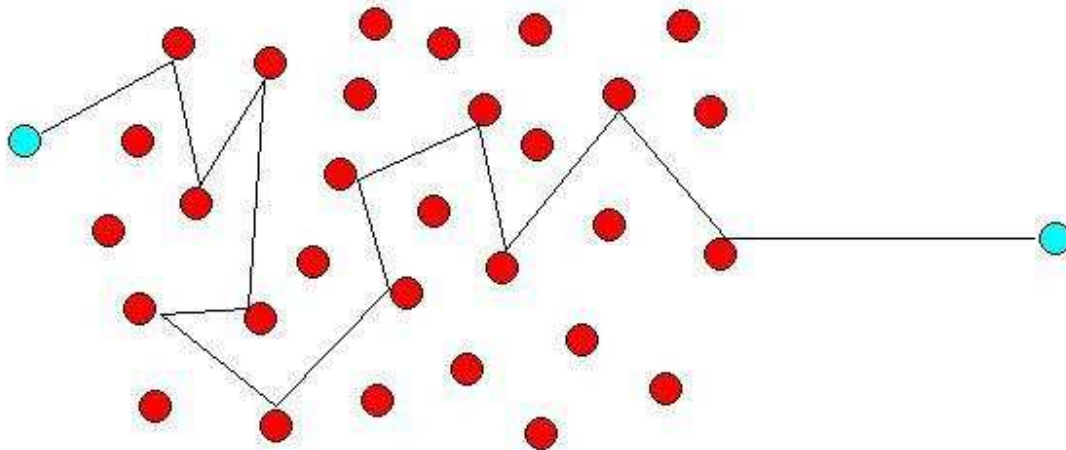
$$\Delta U = 1 \left( 2250 \times 10^3 - \frac{8.3 \times 373}{18 \times 10^{-3}} \right) = 2.078 \times 10^6 \text{ J}$$

Heat or Thermodynamics 9 ) Mean free path

## Mean free path of a gas molecule between 2 collisions

### Mean Free Path

all particles, including photons, suffer from collisions with other particles such that their path through space is very short the higher the densities. This typical path length is called the mean free path.



mean free path  $\lambda$  (the average distance travelled by a particle between collisions) to determine the best values for number of particles  $N$ , rms velocity  $V_{rms}$ , and box length  $L$ ,

$$\lambda = \frac{k_B T}{\sqrt{2} \pi d^2 p}$$

where  $d$  is the diameter of the particle and  $p$  is the pressure.  
which I can easily turn into:

$$\lambda = \frac{m v_{rms}^2}{2 \sqrt{2} \pi d^2 p}$$

**The average distance a particle can travel before colliding with another particle.**

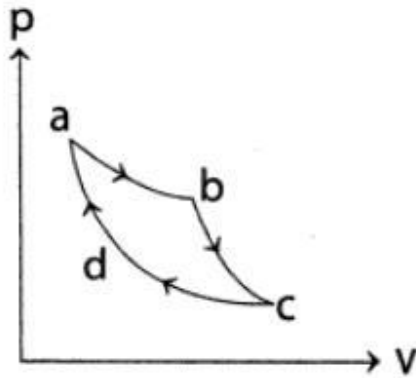
$$\lambda = \frac{1}{n\sigma}$$

**Effect of pressure:**  $\lambda \propto \frac{1}{p}$

Heat or Thermodynamics 10 ) Questions on efficiency of cycle

**Suppose 0.2 mole of an ideal di-atomic gas ( $\gamma = 1.4$ ) undergoes cycle with temperature  $T_H = 400$  K and  $T_C = 300$  K. The initial pressure is  $p_a = 10 \times 10^5$  Pa and during isothermal expansion at temperature  $T_H$  the volume doubles.**

- (i) Find  $Q$ ,  $W$  and  $\Delta U$  from each step in the cycle.**
- (ii) Find the efficiency.**



**Solution**

$$(i) \quad V_a = \frac{nRT_H}{p_a}$$

$$= \frac{0.2 \times 8.314 \times 400}{10 \times 10^5} = 6.65 \times 10^{-4} \text{ m}^3$$

For isothermal expansion  $a \rightarrow b$

$$p_a V_a = p_b V_b$$

$$\text{or} \quad p_b = \frac{p_a V_a}{V_b} = 5 \times 10^5 p_a$$

For adiabatic expansion  $b \rightarrow c$

$$T_H V_b^{\gamma-1} = T_c V_c^{\gamma-1}$$

$$\therefore V_c = V_b \left( \frac{T_H}{T_c} \right)^{\frac{1}{\gamma-1}}$$

$$= 13.3 \times 10^{-4} \times \left( \frac{4}{3} \right)^{2.5} = 27.3 \times 10^{-4} \text{ m}^3$$

$$p_c = \frac{nRT_c}{V_c} = \frac{0.2 \times 8.314 \times 300}{27.3 \times 10^{-4}}$$

$$= 1.83 \times 10^5 \text{ Pa}$$

For adiabatic compression  $d \rightarrow a$

$$T_c V_d^{\gamma-1} = T_H V_a^{\gamma-1}$$



$$\begin{aligned}V_d &= V_a \left( \frac{T_H}{T_C} \right)^{\frac{1}{\gamma-1}} = 6.65 \times 10^{-4} \times \left( \frac{4}{3} \right)^{2.5} \\ &= 13.65 \times 10^{-4} \quad p_d = \frac{nRT_c}{V_d} \\ &= \frac{0.2 \times 8.314 \times 300}{13.65 \times 10^{-4}} = 3.65 \times 10^5 \text{ Pa}\end{aligned}$$

For isothermal expansion  $a \rightarrow b$

$$\Delta U = 0$$

$$\begin{aligned}\therefore W &= Q_H = nRT_H \cdot \log_e \frac{V_b}{V_a} \\ &= 0.2 \times 8.314 \times 400 \log_e 2 \\ &= \mathbf{461 \text{ J}}\end{aligned}$$

For adiabatic expansion  $b \rightarrow c$

$$Q = 0$$

$$\begin{aligned}\therefore W &= -\Delta U = nC_v(T_H - T_C) \\ &= 0.2 \times 20.78 \times (400 - 300)\end{aligned}$$

$$= 415.7 \text{ J}$$

For isothermal compression  $c \rightarrow d$

$$\Delta U = 0$$

$$\begin{aligned} \therefore W = Q_c &= nRT_c \log_e \frac{V_d}{V_c} \\ &= 0.2 \times 8.314 \times 300 \log_e \frac{13.65 \times 10^{-4}}{27.3 \times 10^{-4}} \\ &= -345.8 \text{ J} \end{aligned}$$

For adiabatic expansion  $d \rightarrow a$

$$Q = 0$$

$$\begin{aligned} \therefore W &= -U \\ &= nC_v(T_c - T_H) \\ &= 0.2 \times 20.78 \times (300 - 400) \\ &= -415.7 \text{ J} \end{aligned}$$

The results may be tabulated as follows:

	<b>Q</b>	<b>W</b>	<b><math>\Delta U</math></b>
$a \rightarrow b$	461 J	461 J	0 J
$b \rightarrow c$	0 J	415.7 J	-415.7 J
$c \rightarrow d$	-345.8 J	-345.8 J	0 J
$d \rightarrow a$	0 J	-415.7 J	415.7 J
<b>Total</b>	<b>115.2 J</b>	<b>115.2 J</b>	<b>0 J</b>

(ii) For entire cycle,  $Q = W$

$$\Delta U = 0$$

$$\text{Total work done} = 115.2 \text{ J}$$

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$$Q_H = 461 \text{ J}$$
$$\therefore \eta = \frac{W}{Q_H} = \frac{115.2}{461} = \mathbf{0.25}$$

Efficiency of cycle example

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**One mole of a di-atomic ideal gas ( $\gamma = 1.4$ ) is taken through a cyclic process starting from point  $A$ . The process  $A \rightarrow B$  is an adiabatic compression,  $B \rightarrow C$  isobaric expansion,  $C \rightarrow D$  is an adiabatic expansion and  $D \rightarrow A$  isochoric expansion. The volume ratios are  $\frac{V_A}{V_B} = 16$  and  $\frac{V_C}{V_B} = 2$  and the temperature at  $A$  is  $T_A = 300$  K. Calculate the temperature of gas at the points  $B$  and  $D$  and find the efficiency of the cycle.**

**Solution** For an ideal gas undergoing adiabatic expansion or compression, we have

$$TV^{\gamma-1} = \text{Constant}$$

For the expansion at constant pressure, we have

$$\frac{V}{T} = \text{Constant}$$

With this information, temperature of the gas at different stages of the cyclic process may be determined as follows:

(i) Adiabatic compression from  $A$  to  $B$

$$T_B V_B^{\gamma-1} = T_A V_A^{\gamma-1}$$

$$\begin{aligned} \text{or } T_B &= \left(\frac{V_A}{V_B}\right)^{\gamma-1} T_A = (16)^{1.4-1} (300) \\ &= (3.03) (300 \text{ K}) = \mathbf{909 \text{ K}} \end{aligned}$$

(ii) Isobaric expansion from  $B$  to  $C$

$$\frac{V_C}{T_C} = \frac{V_B}{T_B}$$

$$\text{or } T_C = \left( \frac{V_C}{V_B} \right) T_B = 2(909) = \mathbf{1818 \text{ K}}$$

(iii) Adiabatic expansion from C to D

$$T_D V_D^{\gamma-1} = T_C V_C^{\gamma-1}$$

$$\text{or } T_D = \left( \frac{V_C}{V_D} \right)^{\gamma-1} T_C$$

Since, D  $\rightarrow$  A is isochoric process, therefore

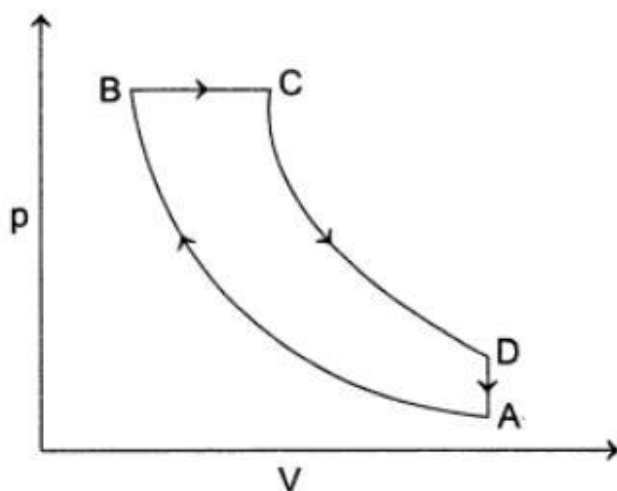
$$V_D = V_A$$

Hence,

$$\begin{aligned} T_D &= \left( \frac{V_C}{V_D} \right)^{\gamma-1} T_C = \left( \frac{V_C}{16 V_B} \right)^{\gamma-1} T_C \\ &= \left( \frac{2}{16} \right)^{1.4-1} (1818 \text{ K}) \\ &= (0.4353) (1818 \text{ K}) = 791.4 \text{ K} \end{aligned}$$

The given cyclic process is show in the figure.





Efficiency of the cycle is defined as

$$\eta = \frac{\text{Work obtained in one cycle}}{\text{Heat absorbed in the process } B \rightarrow C}$$

Now, the work obtained in one cycle is equal to the area within the cycle ABCDA. This

work is given as

$$\begin{aligned} W &= |W_{B \rightarrow C}| + |W_{C \rightarrow D}| + |W_{B \rightarrow A}| \\ &= RT_B + C_v(T_C - T_D) - C_v(T_B - T_A) \end{aligned}$$

For a di-atomic gas,

$$C_v = \frac{5}{2}R \text{ and } C_p = \frac{7}{2}R.$$

Hence,

$$\begin{aligned} W &= R \left[ T_B + \frac{5}{2}(T_C - T_D - T_B - T_A) \right] \\ &= (8.314 \text{ JK}^{-1} \text{ mol}^{-1}) \end{aligned}$$

$$\left[ (909 \text{ K}) + \frac{5}{2}(1818 - 791.4 - 909 + 300)\text{K} \right]$$

= 16237.2 Kelvin per mole

Heat absorbed in the process  $B \rightarrow C$  is given as

$$\begin{aligned} Q &= C_p (T_C - T_B) \\ &= \left(\frac{7}{2}R\right)(T_C - T_B) \\ &= \frac{7}{2} \times (8.314 \text{ JK}^{-1} \text{ mole}^{-1}) \\ &\quad (1818 \text{ K} - 909 \text{ K}) \\ &= 26451.0 \text{ J mole}^{-1} \end{aligned}$$

Hence, the efficiency of the cycle is

$$\eta = \frac{W}{Q} = \frac{16237.2}{26451.0} = \mathbf{0.614}$$

Example of Efficiency of a cycle

**An ideal gas is taken through a cycle thermodynamic process through four steps. The amount of heat involved in these steps are  $Q_1 = 5960 \text{ J}$ ,  $Q_2 = -5585 \text{ J}$ ,**

$Q_3 = -2980 \text{ J}$  and  $Q_4 = 3645 \text{ J}$  respectively.  
The corresponding worked involved are  
 $W_1 = 2200 \text{ J}$ ,  $W_2 = -825 \text{ J}$ ,  $W_3 = -1100 \text{ J}$   
and  $W_4$  respectively.

- (i) Find the value of  $W_4$ .  
(ii) What is the efficiency of the cycle?

**Solution** For a cyclic process

$$\Delta U = 0$$

(i) Cyclic  $\int dQ = \int dW$

i.e.,  $Q_1 + Q_2 + Q_3 + Q_4$   
 $= W_1 + W_2 + W_3 + W_4$

or  $5960 - 5585 - 2980 + 3645$   
 $= 2200 - 825 - 1100 + W_4$

or  $W_4 = 765 \text{ J}$

- (ii) Efficiency of the cycle,

$$\eta = \frac{\text{Net work output}}{\text{Total heat input}}$$

Net work output =  $5960 - 5585$   
 $- 2980 + 3645$   
 $= 1040 \text{ J}$

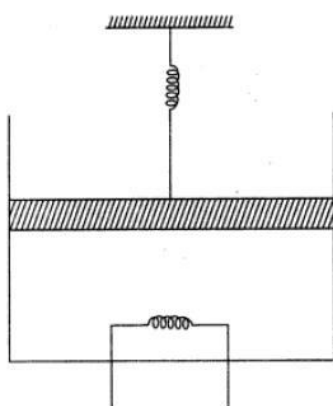
Total heat input =  $Q_1 + Q_4$   
 $= 5960 + 3645 = 9605 \text{ J}$

$$\therefore \eta = \frac{1040}{9605} \times 100 = 10.83\%$$

Question on Cycle

**2.00 mole of a mono-atomic ideal gas ( $U = 1.5 nRT$ ) is enclosed in an adiabatic, vertical cylinder fitted with a smooth light adiabatic piston. The piston is connected to a vertical spring of spring constant  $200 \text{ N/m}$  as shown in the figure. The area of cross-section of the cylinder is  $20.0 \text{ cm}^2$ . Initially, the spring is at its natural length and the temperature of the gas is  $300 \text{ K}$ . The atmospheric pressure is  $100 \text{ kPa}$ . The gas is heated slowly for some time by means of an electric heater so as to move the piston up through  $10 \text{ cm}$ . Find:**

- (i) **The work done by the gas.**
- (ii) **The final temperature of the gas.**
- (iii) **The heat supplied by the heater.**



Solution

(i) Force by the gas on the piston is

$$F = p_0 A + kx$$

where,  $P_0 = 100 \text{ kPa}$  is the atmospheric pressure.

$A = 20 \text{ cm}^2$  is the area of the cross-section,

$k = 200 \text{ N/m}$  is the spring constant, and

$x =$  the compression of spring.

Work done by the gas if the piston moves through  $l = 10 \text{ cm}$  is

$$W = \int_0^l F dx$$

$$= P_0 A l + \frac{1}{2} k l^2$$

$$= (100 \times 10^3 \text{ Pa})$$

$$(20 \times 10^{-4} \text{ m}^2) \times (10 \times 10^{-2} \text{ m})$$

$$+ \frac{1}{2} (200 \text{ N/m}) (100 \times 10^{-4} \text{ m}^2)$$

$$= 20 + 1 \text{ J} = \mathbf{21 \text{ J}}$$

(ii) Initial temperature,  $T_1 = 300 \text{ K}$ . Let the final temperature by  $T_2$ , then

$$nRT_1 = p_0 V_0$$

$$nRT = pV_2 = \left( p_0 + \frac{kl}{A} \right) (V_0 + Al)$$

$$= nRT_1 + p_0 Al + kl^2 + \frac{kl nRT_1}{AP_0}$$

$$\text{or } T_2 = T_1 + \frac{p_0 Al + kl^2}{nR} + \frac{k l T_1}{A P_0}$$



$$\begin{aligned} 300 + \frac{20 \text{ J} + 2 \text{ J}}{20 \times 8.3} + \frac{200 \times 10 \times 10^{-2} \times 300}{20 \times 10^{-4} \times 100 \times 10^3 \text{ Pa}} \\ = 300 \text{ K} + 1.325 \text{ K} + 30 \text{ K} \\ = \mathbf{331 \text{ K}} \end{aligned}$$

(iii) Internal energy,  $U = 1.5 nRT$

$$\begin{aligned} \therefore \Delta U &= 1.5 nR\Delta T \\ &= 1.5 \times 2.00 \times 8.3 \times 31 \\ &= 772 \text{ J} \end{aligned}$$

From the first law,

$$\begin{aligned} \Delta Q &= \Delta U + \Delta W \\ &= (772 + 21) \text{ J} = \mathbf{793 \text{ J}} \end{aligned}$$

Example where 2 vessels are connected

**Two vessels contain in each of them one mole of mono-atomic gas. The initial volume of each vessel is  $8.3 \times 10^{-3} \text{ m}^3$ . Equal amount of heat is supplied to each vessel. In one vessel, the volume of gas is doubled without change in its internal energy whereas the volume of the gas is held constant in second vessel. The vessels are now connected to allow free mixing. Find the final temperature and pressure of the combined system.**

**Solution** According to the first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

For the first vessel:  $\Delta U = 0$ , (Since, no change in temperature)

$$\Delta Q = \Delta W$$

$$Q = \int_{V_1}^{V_2} p dV$$

$$= \int_{V_1}^{V_2} nRT \frac{dV}{V} \quad (\text{since, } pV = nRT)$$

Since  $V_2 = 2 V_1$ , therefore

$$Q = nRT \log_e 2, \quad \dots(i)$$

For the second vessel:  $\Delta W = 0$ , (volume is constant)

$$Q = nC_v \Delta T = n \left( \frac{3}{2} R \right) \Delta T \quad \dots(ii)$$

Since, for mono-atomic gas  $C_v = \frac{3R}{2}$

From equations (i) and (ii), we get

$$nRT \log_e 2 = n \left( \frac{3}{2} R \right) \Delta T$$

$$\text{or } \Delta T = \frac{2}{3} \times 300 \times 0.693 = 138.6 \text{ K}$$

It is the change in temperature of the second vessel.

Now, temperature of the gas in second vessel

$$= T + \Delta T$$

$$= 300 + 138.6 = 438.6 \text{ K}$$

Let after mixing  $T_f$  and  $p_f$  be the final tempera-

ture and pressure, therefore

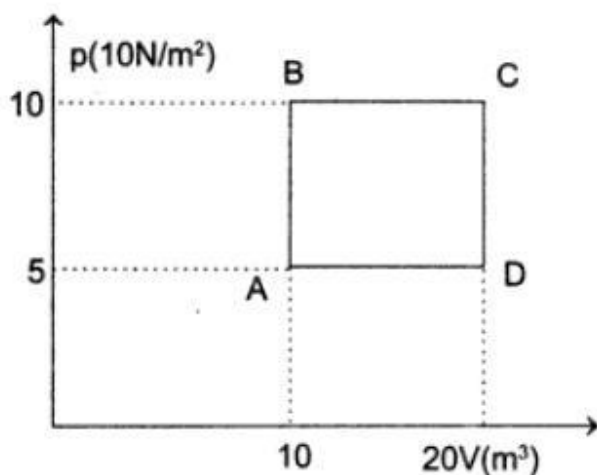
$$\begin{aligned}T_f &= \frac{T + (T + \Delta T)}{2} \\ &= \frac{300 + 438.6}{2} = 369.3 \text{ K}\end{aligned}$$

From the gas equation,

$$\begin{aligned}p_f V_f &= nRT_f \\ p_f &= \frac{nRT_f}{V_f} \\ &= \frac{2 \times 8.3 \times 369.3}{2 \times 8.3 \times 10^3 + 8.3 \times 10^{-3}} \\ &= 2.46 \times 10^5 \text{ N/m}^2\end{aligned}$$

**A sample of 2 kg of mono-atomic Helium (assumed ideal) is taken through the process ABC and another sample of 2 kg of the same gas is taken through the process ADC. Given relative molecular weight of Helium = 4.**

- (i) **What is the temperature of Helium in each of the states A, B, C and D?**
- (ii) **Is there any way of telling afterwards which sample of Helium went through the process ABC and which went through the process ADC? Write yes or no.**
- (iii) **How much heat is evolved in each of the processes ABC and ADC?**



### Solution

Amount of helium

$$= \frac{m}{M} = \frac{2 \times 10^3}{4 \text{ g mol}^{-1}} = 500 \text{ mole}$$

- (i) The temperature of gas at the states A, B, C and D are

$$\begin{aligned} T_A &= \frac{pV}{nR} \\ &= \frac{(5 \times 10^4 \text{ N/m}^2)(10 \text{ m}^3)}{(500 \text{ mole})(8.314 \text{ JK}^{-1} \text{ mole}^{-1})} \\ &= \mathbf{120.28 \text{ K}} \end{aligned}$$

$$\begin{aligned} T_B &= \frac{(10 \times 10^4 \text{ N/m}^2)(10 \text{ m}^3)}{(500 \text{ mole})(8.314 \text{ JK}^{-1} \text{ mole}^{-1})} \\ &= \mathbf{240.56 \text{ K}} \end{aligned}$$

$$T_c = \frac{(10 \times 10^4 \text{ N/m}^2)(10 \text{ m}^3)}{(500 \text{ mole})(8.314 \text{ JK}^{-1} \text{ mole}^{-1})}$$
$$= \mathbf{481.12 \text{ K}}$$

$$T_D = \frac{(5 \times 10^4 \text{ N/m}^2)(20 \text{ m}^3)}{(500 \text{ mole})(8.314 \text{ JK}^{-1} \text{ mole}^{-1})}$$
$$= \mathbf{240.50 \text{ K}}$$

(ii) No.

(iii) For the process ABC, we have

$$Q_{AB} = nC_v \Delta T$$
$$= (500 \text{ mole}) \left( \frac{3}{2} \times 8.314 \text{ JK}^{-1} \text{ mole}^{-1} \right)$$
$$(240.56 \text{ K} - 120.28 \text{ K})$$

$$= 7.5 \times 10^5 \text{ J}$$

$$Q_{BC} = nC_p \Delta T$$
$$= (500 \text{ mole}) \left( \frac{5}{2} \times 8.314 \text{ JK}^{-1} \text{ mole}^{-1} \right)$$
$$(481.12 \text{ K} - 240.56 \text{ K})$$

$$= 2.5 \times 10^6 \text{ J}$$

$$Q_{ABC} = Q_{AB} + Q_{BC}$$
$$= (7.5 \times 10^5 \text{ J} + 2.5 \times 10^6 \text{ J}) = \mathbf{3.25 \times 10^6 \text{ J}}$$

For the process ADC, we have

$$Q_{AD} = nC_p \Delta T$$
$$= (500 \text{ mole}) \left( \frac{5}{2} \times 8.314 \text{ JK}^{-1} \text{ mole}^{-1} \right)$$
$$(240.56 \text{ K} - 120.28 \text{ K})$$



$$= 1.25 \times 10^6 \text{ J}$$

$$Q_{DC} = nC_V \Delta T$$

$$= (500 \text{ mole}) \left( \frac{3}{2} \times 8.314 \text{ JK}^{-1} \text{ mole}^{-1} \right) \\ (481.12 \text{ K} - 240.56 \text{ K})$$

$$= 1.5 \times 10^6 \text{ J}$$

$$Q_{ADC} = Q_{AD} + Q_{DC}$$

$$= (1.25 \times 10^6 \text{ J} + 1.5 \times 10^6 \text{ J})$$

$$= 2.75 \times 10^6 \text{ J}$$

More example in Heat and Thermodynamics

**A 1.00 mole sample of an ideal mono-atomic gas originally at a pressure of 1.00 atmosphere undergoes a three-step process:**

- (i) **It is expanded adiabatically from  $T_1 = 550 \text{ K}$  and  $T_2 = 389 \text{ K}$ .**
- (ii) **It is compressed at constant pressure until its temperature reaches  $T_3$ .**
- (iii) **It then returns to its original pressure and temperature by a constant-volume process.**
  - (a) Plot these processes on a  $p$ - $V$  diagram.
  - (b) Determine  $T_3$ .
  - (c) Calculate the change in integral

energy the workdone by the gas,  
and heat added to gas for each process.

(d) For the complete cycle.

**Solution** *First step Adiabatic Expansion*

$$Q_1 = 0$$

$$\begin{aligned} W_1 &= n_1 C_v (T_2 - T_1) \\ &= (1.00 \text{ mol}) \left( \frac{3}{2} \times 8.314 \text{ JK}^{-1} \text{ mole}^{-1} \right) \\ &\quad \times (389 \text{ K} - 550 \text{ K}) \\ &= -2007.8 \text{ J} \end{aligned}$$

For adiabatic expansion of an ideal gas

$$p_2 T_2^{(-C_p/R)} = p_1 T_1^{(-C_p/R)}$$

$$\begin{aligned} \text{Hence, } p_2 &= p_1 \left( \frac{T_1}{T_2} \right)^{\frac{C_p}{R}} = (1.00 \text{ atm}) \left( \frac{389}{550} \right)^{\frac{5}{2}} \\ &= 0.421 \text{ atm.} \end{aligned}$$

$$\begin{aligned} V_2 &= \frac{nRT_2}{p_2} \\ &= \frac{(1.0 \text{ mole})(8.314 \text{ JK}^{-1} \text{ mole}^{-1})(550 \text{ K})}{(1.0 \times 101.325 \text{ KPa})} \\ &= 45.1 \text{ dm}^3 \end{aligned}$$

$$\Delta U_1 = W_1 = -2007.8 \text{ J}$$

*Second step compression at constant pressure:*

The final volume in this process will be  $V_1$  as in the third step, the system returns to the original state by constant volume process. Hence, in the second step,

$$T_2 = (389 \text{ K}) \text{ changes to } T_3$$

$$V_2 = (75.8 \text{ dm}^3) \text{ changes to } V_1 \\ = 45.1 \text{ dm}^3$$

$$p_2 = \text{remains constant.}$$

Workdone in the process

$$W_2 = -p_2 (V_1 - V_2) \\ = -(0.421 \times 101.325 \text{ kPa}) (45.1 \text{ dm}^3 - 75.8 \text{ dm}^3) \\ = 1309.6 \text{ J}$$

$$T_3 = \left( \frac{V_1}{V_2} \right) T_2 = \left( \frac{45.1}{75.8} \right) (389 \text{ K}) = \mathbf{231.4 \text{ K}}$$

$$Q_2 = n C_p (T_3 - T_2)$$

$$= \left( \frac{5}{2} \times 8.314 \text{ JK}^{-1} \right) (231.4 \text{ K} - 389 \text{ K})$$

$$= -3275.7 \text{ J}$$

$$\Delta U_2 = Q_2 + W_2$$

$$= -3275.7 \text{ J} + 1309.6 \text{ J} = \mathbf{-1966.1 \text{ J}}$$

*Third step compression at constant volume in this process:*

$$W_3 = 0$$

$$V_1 = (45.1 \text{ dm}^3) \text{ remains constant}$$

$$Q_3 = n C_v (T_1 - T_3)$$

$$= \left( \frac{3}{2} \times 8.314 \text{ JK}^{-1} \right) (550 \text{ K} - 231.4 \text{ K})$$

$$\Delta U = 3973.3 \text{ J}$$

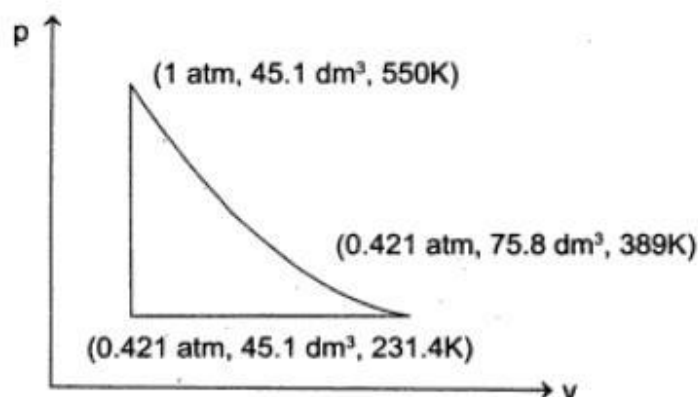
Since, the system return to its original state, we will have

$$\Delta U = Q + W = 0$$

$$\begin{aligned} \text{Now, } W &= W_1 + W_2 + W_3 \\ &= -2007.8 \text{ J} + 1309.6 \text{ J} + 0 \\ &= -698.2 \text{ J} \end{aligned}$$

$$\therefore Q = -W = 698.6 \text{ J}$$

The  $p$ - $V$  plot of the given process is shown in the figure:



In the complete cycle

$$\Delta U = 0$$

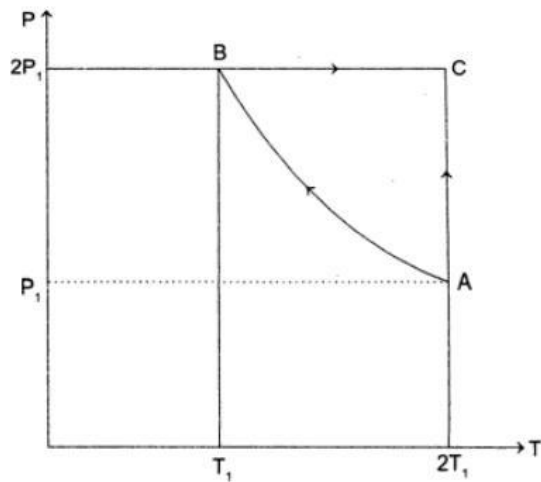
$$\begin{aligned} Q &= Q_1 + Q_2 + Q_3 \\ &= 0 - 3275.7 \text{ J} + 3973.3 \text{ J} = 697.6 \text{ J} \end{aligned}$$

$$\begin{aligned} W &= -Q = 697.6 \text{ J} (= W_1 + W_2 + W_3) \\ &= -2007.8 + 1309.6 \text{ J} + 0 = 698.2 \text{ J} \end{aligned}$$

**Two mole of an ideal mono-atomic gas is taken through a cycle  $ABCA$  as shown in the  $p - T$  diagram. During this process  $AB$ , pressure and temperature of the gas vary such that  $pT = \text{constant}$ . If  $T_1 = 300$  K, calculate:**

- (i) The work done on the gas in the process  $AB$ .**
- (ii) The heat absorbed or released by the gas in each of the process.**

Give answers in terms of the gas constant  $R$ .





**Solution** The volumes of the gas at three states A, B and C are as follows:

$$V_A = \frac{nRT_A}{P_A} = \frac{nR(2T_1)}{P_1} = \frac{2nRT_1}{P_1} \dots(i)$$

$$V_B = \frac{nRT_B}{P_B} = \frac{nR(2T_1)}{P_1} = \frac{1}{2} \frac{nRT_1}{P_1} \dots(ii)$$

$$V_C = \frac{nRT_C}{P_C} = \frac{nR(2T_1)}{2P_1} = \frac{nRT_1}{P_1} \dots(iii)$$

It is given that during the process AB,

$$pT = K \dots(iv)$$

where, K is constant and is given as

$$K = p_A T_A = (p_1) (2T_1) = 2 p_1 T_1 \dots(v)$$

In the process AB, we will have

$$= \sqrt{nRk} \left[ 2\sqrt{V_B} - 2\sqrt{V_A} \right]$$

Using equations (i), (ii) and (v), we get

$$\begin{aligned} W_{AB} &= \sqrt{nR(2p_1T_1)} \left[ 2\sqrt{\frac{nRT_1}{2p_1}} - 2\sqrt{\frac{2nRT_1}{p_1}} \right] \\ &= (\sqrt{2} nRT_1)(2) \left[ \frac{1}{2} - \sqrt{2} \right] \\ &= -2n T_1 R \\ &= -2 (2 \text{ mole}) (200 \text{ K}) R \\ &= - (1200 \text{ mole K}) R \end{aligned}$$

The negative sign implies that the work is done on the gas.

Hence, work done on the gas  
= **(1200 mole K) R**

(ii) Change in energy of the gas in the process AB is

$$\begin{aligned}\Delta U_{AB} &= nC_v \Delta T \\ &= (2 \text{ mole}) \left( \frac{3}{2} R \right) (T_1 - 2 T_1) \\ &= - (3 \text{ mole}) T_1 R \\ &= - (3 \text{ mole}) (300 \text{ K}) R \\ &= - (900 \text{ mole K}) R\end{aligned}$$

Now, from the first law of thermodynamics,

$$\begin{aligned}Q_{AB} &= \Delta U_{AB} + W_{AB} \\ &= - (1200 \text{ mole K}) R \\ &= - (900 \text{ mole K}) R \\ &= - (2100 \text{ mole K}) R\end{aligned}$$

The negative sign implies that the heat is released in the process AB. The process

BC takes place at constant pressure.

Hence,

$$\begin{aligned}W_{BC} &= p \Delta V \\ &= (2 p_1) (V_C - V_B) \\ &= (2 p_1) \left[ \frac{nRT_1}{P_1} - \frac{nRT_1}{2 P_1} \right] \\ &= nRT_1 \\ &= (2 \text{ mole}) (300 \text{ K}) R \\ &= (600 \text{ mole K}) R\end{aligned}$$

Now,  $\Delta U_{BC} = nC_v \Delta T$

$$= (2 \text{ mole}) \left( \frac{3}{2} R \right) (T_C - T_B)$$

$$\begin{aligned} &= (3 \text{ mole}) (R) (2 T_1 - T_1) \\ &= (3 \text{ mole}) (R) (300 \text{ K}) \\ &= (900 \text{ mole K}) R \\ Q_{BC} &= \Delta U_{BC} + W_{BC} \\ &= (900 \text{ mole K}) R + (600 \text{ mole K}) R \\ &= \mathbf{(1500 \text{ mole K}) R} \end{aligned}$$

The positive sign implies that the heat is absorbed in the process BC.

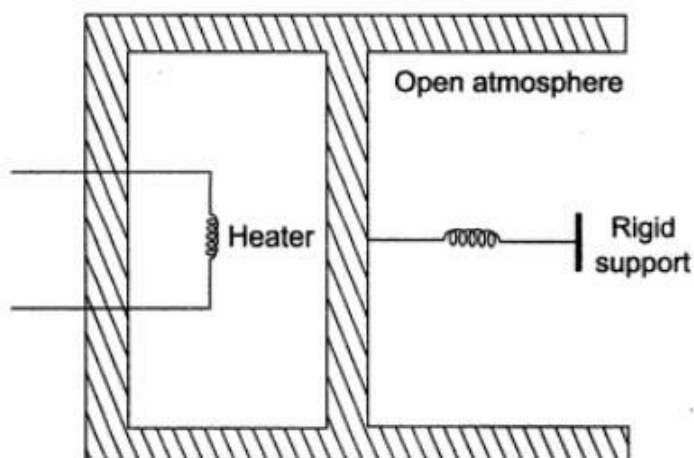
The process CA takes place at constant temperature. Hence,

$$\begin{aligned} W_{CA} &= \int_{V_A}^{V_B} p dV \\ &= \int_{V_A}^{V_B} \frac{nRT}{V} dV \\ &= nRT \ln \frac{V_A}{V_C} \dots \text{(where, } T = 2 T_1) \\ &= (2 \text{ mole}) (R) (2 \times 300 \text{ K}) \ln 2 \\ &= (1200 \text{ mole K}) R \ln 2 \end{aligned}$$

$$\begin{aligned} \Delta U_{CA} &= 0 \\ Q_{CA} &= \Delta U_{CA} + W_{CA} \\ &= 0 + (1200 \text{ mole K}) R \ln 2 \end{aligned}$$

The positive sign implies that the heat is absorbed in the process CA.

**An ideal mono-atomic is confined in a cylinder by a spring-loaded piston of cross-section  $8 \times 10^{-3} \text{ m}^2$ . Initially, the gas is at 300 K and occupies a volume of  $2.4 \times 10^{-3} \text{ m}^3$  and the spring is on its relaxed (unstretched, uncompressed) state as shown the figure. The gas is heated by a small electric heater until the piston moves out slowly by 0.1 m. Calculate the final temperature of the gas and the heat supplied (in joules) by the heater. The force constant of the spring is  $8000 \text{ Nm}^{-1}$  and atmospheric pressure is  $1 \times 10^5 \text{ Nm}^2$ . The cylinder and the piston are thermally insulated. The piston is massless and there is no friction between the piston and cylinder. Neglect heat loss through the lead wires of the heater. The heat capacity of the heater coil is negligible. [Assume the spring to be massless].**



**Solution** Let  $p_0$  be the atmospheric pressure. Initially for the equilibrium of the piston,  $p_L = p_R = p_0$  where  $p_L$  and  $p_R$  are the pressures on the left hand and right hand side of the piston.

Force exerted by the spring on the piston when it moves

$$F = kx = 8000 \times 0.1 \\ = 800 \text{ N}$$

$\therefore$  Pressure exerted on the piston by the spring

$$p_s = \frac{F}{A} = \frac{800 \text{ N}}{8 \times 10^{-3} \text{ m}^2} = 1 \times 10^5 \text{ Nm}^{-2}$$

$\therefore$  Total pressure acting on the right hand side

$$p'_R = p_0 + p_s \\ = 2 \times 10^5 \text{ N/m}^2$$

Under equilibrium  $p'_L = p'_R$



$$\text{or } \frac{p_L V_L}{T_L} = \frac{p'_L V'_L}{T'_L}$$

$$= \frac{1 \times 10^5 \times 2.4 \times 10^{-3}}{300}$$

$$= \frac{2 \times 10^5 \times 3.2 \times 10^{-3}}{T'_L}$$

$$T'_L = 800 \text{ K}$$

$$\Delta U = n C_v \Delta T$$

$$\text{where, } n = \frac{p_L V_L}{R T_L} = \frac{1 \times 10^5 \times 2.4 \times 10^{-3}}{8.3 \times 300}$$

$$= 0.09638 \text{ mole}$$

$$\therefore \Delta U = 0.09638 \times \frac{3}{2} \times 8.3 \times (800 - 300) = 600 \text{ J}$$

$$\Delta W = \frac{1}{2} k \cdot x^2 + p_0 \cdot \Delta V$$

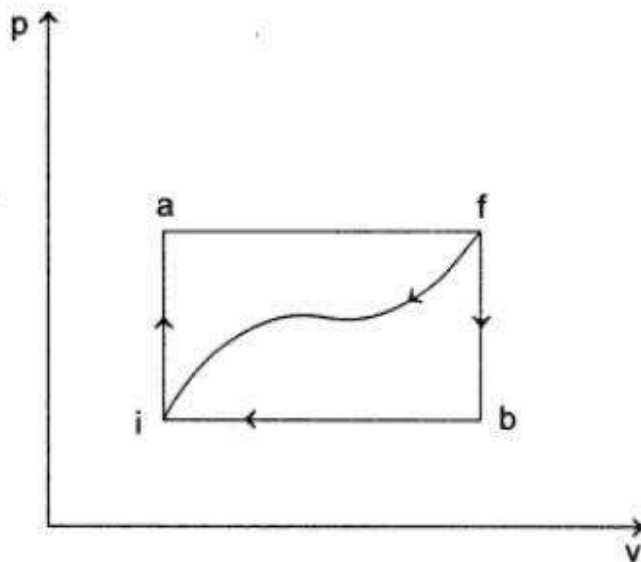
$$= \frac{1}{2} \times 800 \times (0.1)^2 + 1 \times 10^5 \times 8 \times 10^{-4}$$

$$= 120 \text{ J}$$

$$\Delta Q = \Delta U + \Delta W = 600 + 120 = \mathbf{720 \text{ J.}}$$

**A system is taken from state  $i$  to the state  $f$  (refer to the figure). Along path “ $iaf$ ”, it is found that  $\Delta Q = 50$  cal.  $\Delta W = 20$  cal. Along the path “ $ibf$ ”,  $\Delta Q = 36$  cal. Calculate:**

- (i)  $\Delta W$  along the path “ $ibf$ ”.
- (ii) If  $\Delta W = -13$  cal for the curved path “ $fi$ ”, what is the  $\Delta Q$  for this path?
- (iii) Taking  $U_i = 10$  cal, what is  $U_f$ ?
- (iv) If  $U_b = 22$  cal, what is  $\Delta Q$  for the process “ $ib$ ” and the process “ $bf$ ”?



**Solution** Path "iaf"  $\Delta Q = 50 \text{ cal}$

$$\Delta W = 20 \text{ cal}$$

$$\Rightarrow \Delta U = \Delta Q - \Delta W \\ = 50 - 20 = 30 \text{ cal}$$

$$\Rightarrow U_f - U_i = 30 \text{ cal}$$

As internal energy change is a state function.

$\Delta U$  will be same for any path from  $i$  to  $f$ .

$$\begin{aligned} \text{(i) Path "ibf"} \Delta W &= \Delta Q - \Delta U \\ &= 36 - (U_f - U_i) \\ &= 36 - 30 = \mathbf{6 \text{ cal.}} \end{aligned}$$

$$\begin{aligned} \text{(ii) Path "fi"} \Delta Q &= \Delta U + \Delta W \\ &= (U_f - U_i) + \Delta W \\ &= (-30) + (-13) \\ &= \mathbf{-43 \text{ cal}} \end{aligned}$$

$$\begin{aligned} \text{(iii) } U_f - U_i &= 30 \text{ cal} \\ U_f &= U_i + 30 \quad \therefore = \mathbf{40 \text{ cal.}} \end{aligned}$$

$$\begin{aligned} \text{(iv) Process "ib"} \Delta Q &= \Delta U + \Delta W \\ &= (U_b - U_i) + (\Delta W)_{ibf} \end{aligned}$$

$$(\Delta W)_{ib} = (\Delta W)_{ibf}$$

$$\text{Because } (\Delta W)_{bf} = 0$$

$$\begin{aligned} \Delta Q &= (22 - 10) + 6 \\ &= \mathbf{18 \text{ cal.}} \end{aligned}$$

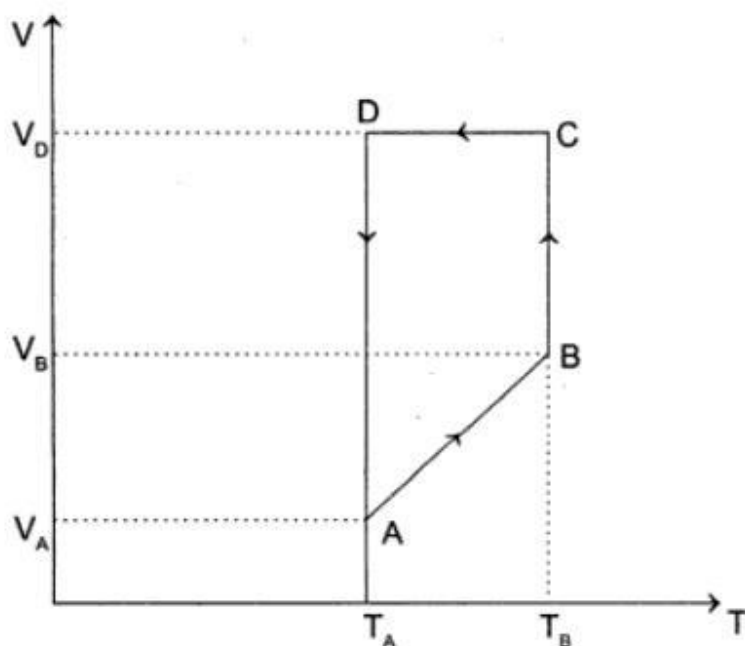
$$\begin{aligned} \text{Process "bf"} \Delta Q &= \Delta U + \Delta W \\ &= (U_f - U_b) + 0 \\ &= (40 - 22) \\ &= \mathbf{18 \text{ cal.}} \end{aligned}$$

**A mono-atomic ideal gas of two moles is taken through a cyclic process starting from  $A$  as shown in the figure. The volume ratios are  $\frac{V_B}{V_A} = 2$  and  $\frac{V_D}{V_A} = 4$ .**

**If the temperature  $T_A$  at  $A$  is  $27^\circ\text{C}$ , calculate:**

- (i) The temperature of the gas at point  $B$ .**
- (ii) Heat absorbed or released by the gas in each process.**
- (iii) The total work done by the gas during complete cycle.**

**Express your answer in terms of the gas constant  $R$ .**



### **Solution**

Given:  $\frac{V_B}{V_A} = 2$  and  $\frac{V_D}{V_A} = 4$

$$T_A = 27^\circ\text{C}$$

- (i) The process  $A \rightarrow B$  in which the plot of  $V$  verse  $T$  is linear occurs at constant pressure condition.

$$\text{Hence } \frac{V_A}{T_A} = \frac{V_B}{T_B}$$

$$\text{or } T_B = \left( \frac{V_B}{V_A} \right) T_A = (2)(300 \text{ K})$$

$$= \mathbf{600 \text{ K}}$$

- (ii) The process  $A \rightarrow B$  occurs at constant pressure. Hence,

$$Q_{A \rightarrow B} = n C_p (T_B - T_A)$$

$$= (2 \text{ mole}) \left( \frac{5}{2} R \right) (600 \text{ K} - 300 \text{ K})$$

$$= (1500 \text{ mole K}) R.$$

The process  $B \rightarrow C$  occurs at constant temperature. From first law of thermodynamics

$$dU = dQ - dW$$

Since, the internal energy of an ideal gas depends only on temperature, therefore

$$dU = 0 \text{ and } dQ = dW$$

$$Q_{B \rightarrow C} = W_{B \rightarrow C}$$

$$= \int p dV = nRT_B \int \frac{dV}{V}$$

$$= nR T_B \ln \frac{V_C}{V_B}$$

$$= nR T_B \ln \frac{V_D}{V_B} \dots \dots (\text{as } V_C = V_D)$$

$$\begin{aligned}
 &= nR T_B \ln\left(\frac{V_D}{V_A} \frac{V_A}{V_B}\right) \\
 &= (2 \text{ mole}) (R) (600 \text{ K}) \ln\left(\frac{4}{2}\right) \\
 &= (1200 \text{ mole K}) R \ln 2
 \end{aligned}$$

The process C  $\rightarrow$  D occurs at constant volume. Hence,

$$\begin{aligned}
 Q_{C \rightarrow D} &= nC_v (T_A - T_B) \\
 &= (2 \text{ mole}) \left(\frac{3}{2}R\right) (300 \text{ K} - 600 \text{ K}) \\
 &= - (900 \text{ mole K}) R
 \end{aligned}$$

The process D  $\rightarrow$  A occurs at constant temperature. Hence,

$$\begin{aligned}
 Q_{D \rightarrow A} &= W_{D \rightarrow A} = nRT_A \ln \frac{V_A}{V_D} \\
 &= (2 \text{ mole}) (R) (300 \text{ K}) \ln\left(\frac{1}{4}\right) \\
 &= - (1200 \text{ mole K}) R \ln 2.
 \end{aligned}$$

(iii) Since, the process ABCDA is a cyclic process, therefore

$$U = 0, W = Q$$

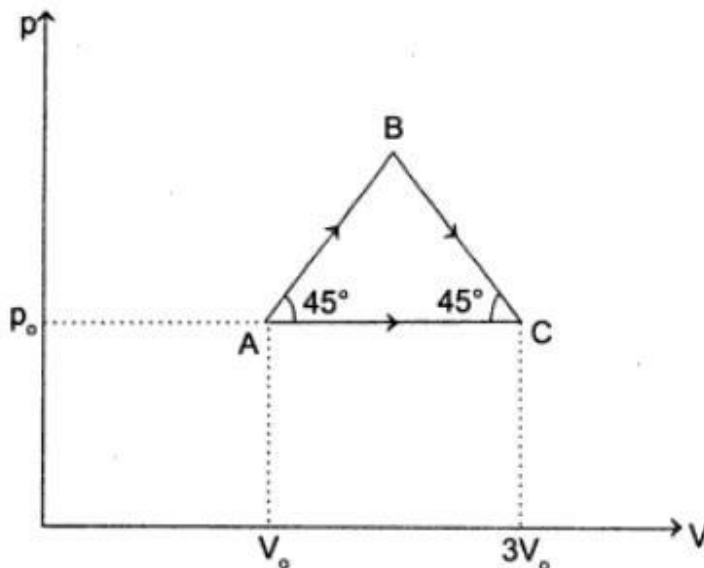
where,

$$\begin{aligned}
 Q &= Q_{A \rightarrow B} + Q_{B \rightarrow C} + Q_{C \rightarrow D} + Q_{D \rightarrow A} \\
 &= (1500 \text{ mole K}) R + (1200 \text{ mole K}) R \ln 2 - (900 \text{ mole K}) \\
 &\quad R - (1200 \text{ mole K}) R \ln 2 \\
 &= (600 \text{ mole K}) R.
 \end{aligned}$$



**An ideal gas expands from a volume  $V_0 = 1$  litre and pressure  $p_0 = 1$  bar to volume 3 litre along two different paths  $ABC$  and  $AC$  as shown in figure. The heat added to the gas along the path  $ABC$  is 600 J.**

- (i) Sketch the process on  $p - T$  diagram.
- (ii) Find the work done by the gas along the paths  $ABC$  and  $AC$ .
- (iii) Find the heat transfer in the process along the path  $AC$ .



**Solution**

- (i) Equation of line AB,

$$p - p_0 = \tan 45^\circ (V - V_0)$$

Hence for ideal gas,  $p = V$

Now  $pV = KT$

$$\Rightarrow p^2 = KT \text{ (parabola)}$$

.....(where K is constant.)

At B  $V_B = 2 V_0$  and  $p_B = 2 p_0$   
Equation of line BC,  $p - 2 p_0 = -\tan 45^\circ$   
( $V - 2 V_0$ )

$$\therefore p = -V + 4$$

$$\Rightarrow p = -\frac{KT}{P} + 4$$

$$\therefore P^2 - 4p = -KT \text{ (Parabola)}$$

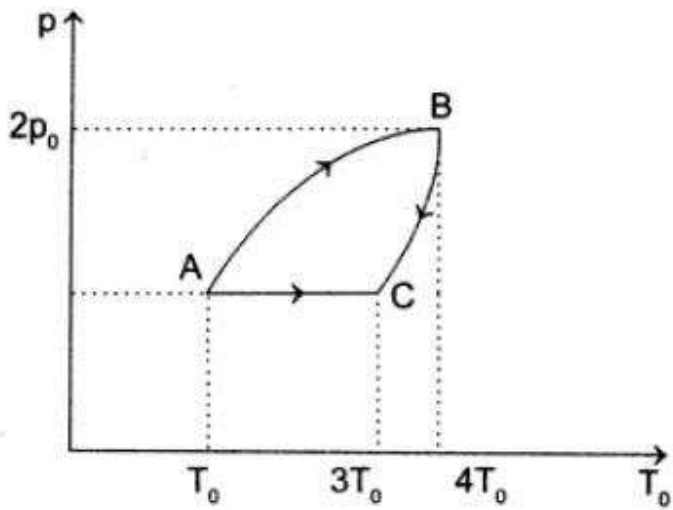
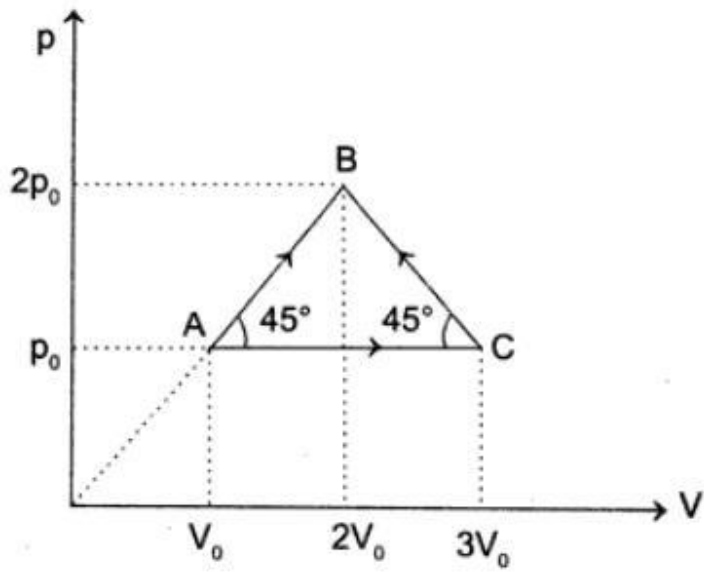
(ii) Workdone along path AC =  $(\Delta W)_{AC}$   
 $= p_0 (3 V_0 - V_0)$   
 $= 2 p_0 V_0$   
 $= 2 \times 1 \times 10^5 \times 1 \times 10^{-3}$   
 $= \mathbf{200 \text{ J.}}$

(iii) For path ABC  $(\Delta Q)_{ABC} = (\Delta U)_{AC} +$   
 $(\Delta W)_{ABC}$   
 $\Rightarrow (\Delta U)_{AC} = 600 - 300$   
 $= 300 \text{ J.}$

Heat transfer in the process along path AC,

$$(Q)_{AC} = (\Delta U)_{AC} + (\Delta W)_{AC}$$
$$= 300 + 200 = \mathbf{500 \text{ J.}}$$

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**A monatomic ideal gas, initially at temperature  $T_1$  is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature  $T_2$  by releasing the piston suddenly. If  $L_1$  and  $L_2$  are the lengths of the gas column before and after expansion respectively, then  $T_1/T_2$  is given by**

(a)  $\left(\frac{L_1}{L_2}\right)^{2/3}$

(b)  $\frac{L_1}{L_2}$

(c)  $\frac{L_2}{L_1}$

(d)  $\left(\frac{L_2}{L_1}\right)^{2/3}$

$$TV^{\gamma-1} = \text{constant}$$

For monatomic gas  $\gamma = \frac{5}{3}$

$$\Rightarrow TV^{2/3} = \text{constant}$$

Since volume is proportional to length, therefore,

$$\frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{2/3}$$

Hence (d) is correct.

**Two identical containers  $A$  and  $B$  with frictionless pistons contain the same ideal gas at the same temperature and the same volume  $V$ . The mass of gas contained in  $A$  is  $m_A$  and that in  $B$  is  $m_B$ . The gas in each cylinder is now allowed to expand isothermally to the same final volume  $2V$ . The change in the pressure in  $A$  and  $B$  are found to be  $\Delta p$  and  $1.5 \Delta p$  respectively. Then**

- (a)  $4m_A = 9m_B$                       (b)  $2m_A = 3m_B$   
(c)  $3m_A = 2m_B$                       (d)  $9m_A = 4m_B$

$$\text{For gas in A, } p_1 = \left(\frac{m_A}{M}\right) \frac{RT}{V_1}$$

$$p_2 = \left(\frac{m_A}{M}\right) \frac{RT}{V_2}$$

$$\therefore \Delta p = p_2 - p_1 = \left(\frac{RT}{M}\right) m_A \left(\frac{1}{V_1} - \frac{1}{V_2}\right)$$

Putting  $V_1 = V$  and  $V_2 = 2V$ , we get

$$\Delta p = \left(\frac{RT}{M}\right) \frac{m_A}{2V}$$

$$\text{Similarly for Gas in B, } 1.5 \Delta p = \left(\frac{RT}{M}\right) \frac{m_B}{2V}$$

From equation (i) and (ii) we get

$$2m_B = 3m_A$$

Hence (c) is the correct.

**Two insulating cylinders  $A$  and  $B$  fitted with pistons contain equal amounts of an ideal diatomic gas at temperature 300 K. The piston  $A$  is free to move, while that of  $B$  is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in  $A$  is 30 K. Then the rise in temperature of the gas in  $B$  is**

- (a) 30 K                      (b) 18 K  
(c) 50 K                      (d) 42 K

For cylinder A    For cylinder B

$$dQ = n C_p dT_1 \quad dQ = n C_V dT_2$$
$$= n (C_V + R) dT_1$$

$$\therefore n C_V dT_2 = n (C_V + R) 30$$

$$\therefore dT_2 = \frac{(C_V + R) 30}{C_V}$$

For diatomic gas  $C_V = \frac{5}{2}R$

$$\therefore dT_2 = 42 \text{ K}$$

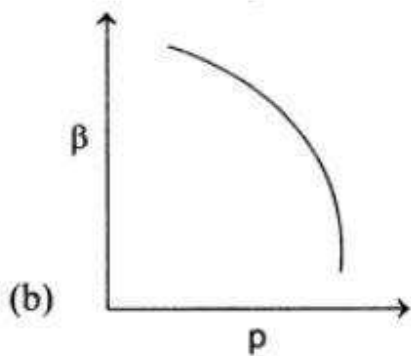
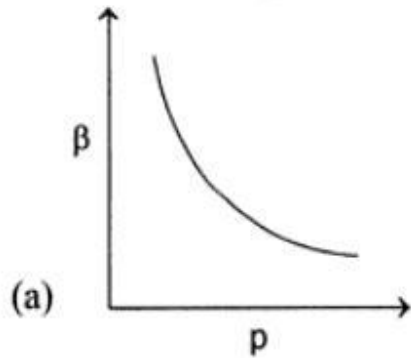
Hence (d) is correct.

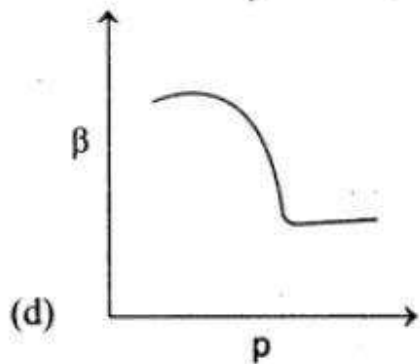
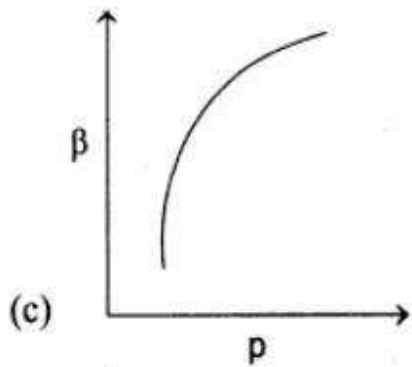


Which of the following graph correctly represents the variation of

$$\hat{a} = -\left(\frac{dV}{dP}\right)/V \text{ with } p \text{ for an ideal gas}$$

at constant temperature?





As temperature is constant,

$$pV = \text{constant}$$

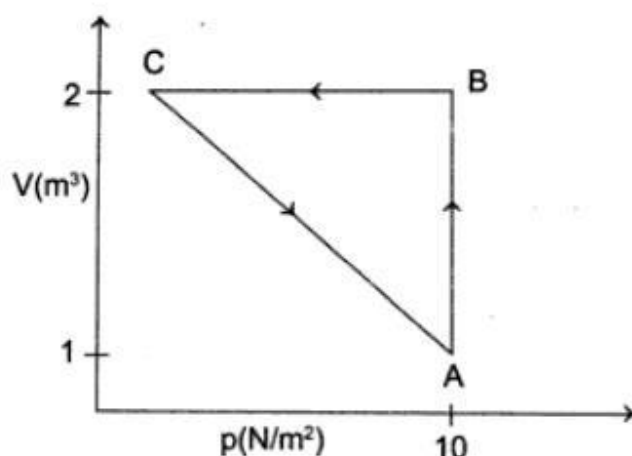
$$\Rightarrow pdV + Vdp = 0$$

$$\Rightarrow -\frac{(dV/dp)}{V} = \frac{1}{p}$$

$$\Rightarrow \beta = \frac{1}{p}$$

**An ideal gas is taken through the cycle  $A \rightarrow B \rightarrow C \rightarrow A$ , as shown in the gas in the cycle is 5J, the work done by the gas in the process  $C \rightarrow A$  is**

- (a)  $-5\text{J}$                       (b)  $-10\text{J}$   
(c)  $-15\text{J}$                       (d)  $-20\text{J}$



For the cyclic process  $\Delta U = 0$

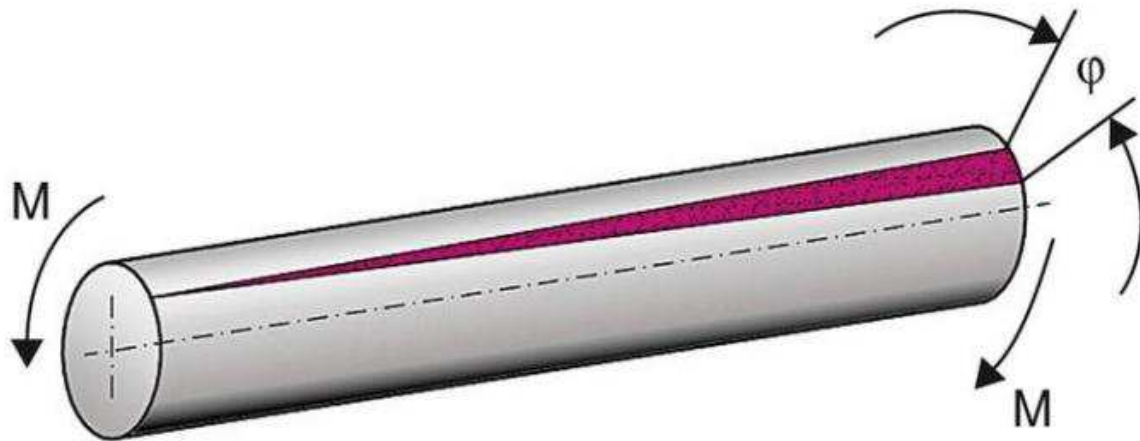
$$\begin{aligned}\Delta W &= W_{AB} + W_{BC} + W_{CA} \\ &= (10 + 0 + W_{CA}) \text{ J}\end{aligned}$$

Given:  $\Delta Q = 5\text{J}$

From first law of thermodynamics

$$\begin{aligned}5 &= 10 + 0 + W_{CA} \\ \Rightarrow W_{CA} &= -5\text{J}\end{aligned}$$

Properties of Material 1 ) Torsional Torque per unit twist



The torque  $T$  can be equated to the sum of the moments of the tangential stresses on the element  $2\pi r \delta r$

$$\therefore T = \int s (2\pi r dr) r$$

$$T = \int \frac{C\theta}{l} (2\pi r^3) dr$$

$$= \frac{C\theta}{l} \pi \frac{r^4}{2}$$

Properties of Material 2 ) Torsion of a cylinder

### TORSION OF A CYLINDER/TWISTING WIRE

Let,  $l$  = length of cylinder

$r$  = radius of cylinder

$\phi$  = angle of twist

$\theta$  = angle of shear

$\eta$  = modulus of rigidity

$\tau$  = restoring torque developed in the cylinder twisting

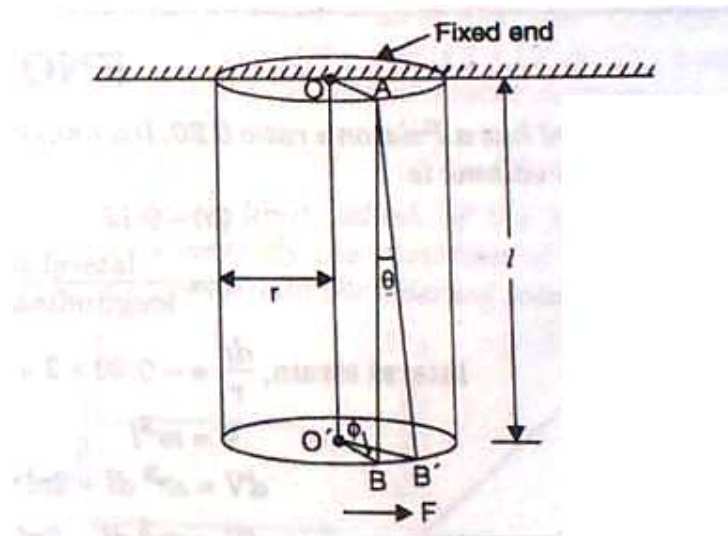
$c$  = restoring couple per unit twist

$F$  = tangential force applied at the free end.

(i) Relation between angles of shear and twist

$$BB' = l\theta = r\phi \text{ or}$$

$$\theta = \frac{r}{l} \phi$$



$$(ii) \tau = \frac{\pi \eta r^4}{2l} \phi$$

$$\tau = c\phi$$

(iii) Work done in twisting through angle  $\phi$  is

$$\frac{1}{2} c\phi^2, \text{ i.e., } \frac{\pi \eta r^4}{4l} \phi^2$$

Properties of Material 3 ) Coefficient of Resilience

3 kinds of Coefficient of Resilience

The amount of energy absorbed per unit volume of the body. This is affected by the class of deformation whether axial, bending, or torsional; hence there are three kinds of coefficients of resilience.

Some Authors refer Coefficient of Restitution as Coefficient of resilience.

If a ball falls from a height falling vertically, and just before hitting the ground, it has a speed of  $v_1$ . Then after hitting the ground it jumps upward with a vertical upward speed of  $v_2$ .

Then the coefficient of restitution  $e = \text{mod of } (v_2 / v_1)$

If a ball is moving at  $u_1$  and another is moving at  $u_2$ , they collide. After collision if these move at  $v_1$  and  $v_2$  then  $e = \text{mod of } (v_2 - v_1) / (u_2 - u_1)$



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$$e = \frac{\text{Velocity of Separation}}{\text{Velocity of approach}}$$

ie.  $e = \frac{V_2 - V_1}{U_2 - U_1} \dots(2)$

Properties of Material 4 ) Relations between various Elastic constants

## RELATIONS CONNECTING THE ELASTIC CONSTANTS

$$1. K = \frac{Y}{3(1 - 2\sigma)} \quad 3. \frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K}$$

$$2. \eta = \frac{Y}{2(1 + \sigma)} \quad 4. \sigma = \frac{3K - 2\eta}{2(3K + \eta)}$$

Write many times to memorize

### Relations between Elastic Constants $Y$ , $\eta$ , $K$ and $\sigma$

$$(i) \eta = \frac{Y}{2(1 + \sigma)} \quad (ii) K = \frac{Y}{3(1 - 2\sigma)}$$

$$(iii) \frac{3}{Y} = \frac{1}{3K} + \frac{1}{\eta} \quad (iv) \sigma = \frac{3K - 2\eta}{2\eta + 6K}$$

There is a mistake in the formula below.  $Y/\eta$  should be  $2(1 + \sigma)$

Note  $\beta = \frac{Y}{3(1 - 2\sigma)}$ ,  $\frac{Y}{\eta} = 2(1 + \sigma)$ ,  $Y = \frac{\sigma\eta\beta}{\eta + 3\beta}$ ,

Torsional rigidity  $C = \frac{k\eta r^4}{2l}$

Torsional couple (Torque)  $G = C\theta$ . If tangential stress is  $T$  then  $\frac{T}{r} = \eta$  where  $\phi$  is shear angle.  $\phi = \frac{x\theta}{l}$  where  $\theta$  is angle of twist.

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Poisson's ratio cannot exceed

- (a) 0.25 (b) 1.0  
(c) 0.75 (d) 0.5

(d)  $B = \frac{Y}{3(1-2\sigma)}$  if  $B = \infty$   $1 - 2\sigma \rightarrow 0$  or  
 $\sigma_{\max} = \frac{1}{2}$ .

A copper wire of cross-section  $A$  is under a tension  $T$ . Find the decrease in the cross-section area. Young's modulus is  $Y$  and Poisson's ratio is  $\sigma$ .

- (a)  $\frac{\sigma T}{2AY}$  (b)  $\frac{\sigma T}{AY}$   
(c)  $\frac{2\sigma T}{AY}$  (d)  $\frac{4\sigma T}{AY}$

$$\frac{\Delta r}{r} = \sigma \frac{\Delta l}{l} \text{ and } \frac{\Delta l}{l} = \frac{T}{AY}$$

$$\frac{\Delta A}{A} = \frac{2\Delta r}{r} = \frac{2\sigma T}{AY}$$

Properties of Material 5 ) Bending of the Beam

Depression of Beam at center

#### The Depression of a Beam at its Centre

The depression at the centre of a beam is given by

$$= \frac{MgL^3}{4bd^3Y}$$

$M$  = Suspended Mass,  $L$  = Length of the beam,  $b$  = Bread of the beam,  
 $Y$  = Young's modulus and  $d$  = Thickness of the beam

**\*SUPPORTED BEAM, CENTRALLY LOADED,**  
*[Assumption : Weight of the beam is ineffective.]*

(i) If the beam is of circular cross-section, then depression  $y$  is given by :

$$y = \frac{WL^3}{12Y\pi r^4}$$

where  $W$  is the load suspended at the middle of the beam,  $L$  is the length of the beam between two supported points,  $Y$  is Young's modulus of elasticity and  $r$  is the radius of the circular cross-section of the beam.

(ii) If the beam is of rectangular cross-section of breadth  $b$  and depth  $d$ , then depression at the middle is given by

$$y = \frac{WL^3}{4Ybd^3}$$

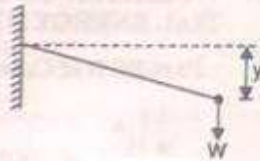
**THE CANTILEVER—DEPRESSION OF ITS LOADED END**

*[Assumption : Weight of cantilever is ineffective]*

$$Y = \frac{WL^3}{3YI}$$

For a beam of rectangular cross-section of breadth  $b$  and depth  $d$ ,  $I = \frac{bd^3}{12}$

$$\therefore y = \frac{WL^3 \times 12}{3Y \times bd^3} = \frac{4WL^3}{Ybd^3}$$



If the cross-section is square in shape, then  $b = d$ .

$$\therefore I = \frac{b^4}{12}$$

$$\therefore y = \frac{WL^3 \times 12}{3Yb^4} = \frac{4WL^3}{Yb^4}$$

For the beam of circular cross-section of radius  $r$ ,

$$I = \frac{\pi r^4}{4}$$

$$\therefore y = \frac{WL^3}{3Y \left[ \frac{\pi r^4}{4} \right]} = \frac{4WL^3}{3Y\pi r^4}$$

*For the same cross-sectional area and for given load, the ratio of depression for the beam of a square cross-section and circular cross-section is :*

- (a) 3 :  $\pi$   
(c) 1 :  $\pi$

- (b)  $\pi$  : 3  
(d)  $\pi$  : 1.

**Sol.**  $y_1 = \frac{4WL^3}{Yb^4}$ ,  $y_2 = \frac{4WL^3}{3Y\pi r^4}$

$$\frac{y_1}{y_2} = \frac{4WL^3}{Yb^4} \times \frac{3Y\pi r^4}{4WL^3} = \frac{3\pi r^4}{b^4} = \frac{3\pi r^4}{(\pi r^2)^2}$$

[ $\because b^2 = \pi r^2$ ]

$$= \frac{3}{\pi}$$

So, (a) is the right choice.

#### Properties of Material 6 ) Measurement of Radius of Curvature

*To measure the radius of curvature with a spherometer, we use the formula*

(a)  $R = \frac{h^2}{6} + \frac{l}{2}$

(b)  $R = \frac{l^2}{6h} + \frac{h}{2}$

(c)  $R = \frac{h^2}{2l} + \frac{l}{h}$

(d)  $R = \frac{2l^2}{h} + \frac{6}{l}$

A spherometer (Fig. 11) is used to determine the radius of curvature of a spherical surface. The theory of the method is briefly described below:

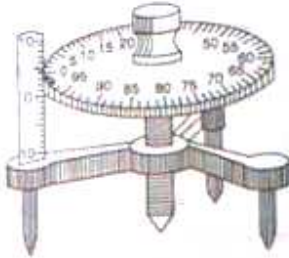


Fig. 11

In the Fig. 12,  $r^2 = h(2R - h)$

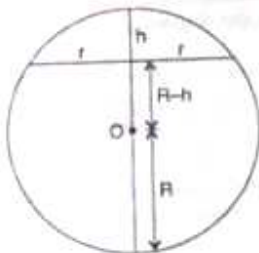


Fig. 12

On simplification,

$$R = \frac{r^2}{2h} + \frac{h}{2}$$

But

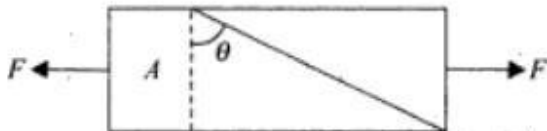
$$r = \frac{l}{\sqrt{3}}$$

[Think of an equilateral triangle of side  $l$ .]

$$\therefore R = \frac{l^2}{6h} + \frac{h}{2}$$

Properties of Material 7 ) Shear stress

A bar of cross-section  $A$  is subjected to equal and opposite tensile forces  $F$  at its ends. Consider a plane through the bar making an angle  $\theta$  with a plane at right angles to the bar. Then shearing stress will be maximum if  $\theta$



- (a)  $0^\circ$
- (b)  $30^\circ$
- (c)  $45^\circ$
- (d)  $60^\circ$
- (e)  $90^\circ$

(c) Shear stress =  $\frac{F \sin \theta}{A / \cos \theta} = \frac{F \sin 2\theta}{2A}$

Shear stress will be maximum if  $\sin 2\theta = 1$  or  $2\theta = 90^\circ$  i.e.  $\theta = 45^\circ$ .

Properties of Material 8 ) Thermal stress and force

Thermal Stress

(i) The thermal stress set up in the rod which is not free to expand or contract is given by,

$$\text{Stress in the rod} = \frac{F}{A} = Y \alpha (\theta_2 - \theta_1).$$

$Y$  = Young's modulus,  $\alpha$  = Linear coefficient of expansion and  $(\theta_2 - \theta_1)$  = Temperature difference.

(ii) Thermal force =  $F = YA \alpha (\theta_2 - \theta_1)$

(iii) Two different rods of different materials are joined end to end and the composite rod is fixed between the two supports. The temperature difference is  $(\theta_2 - \theta_1)$ . Then force is given by

$$F = \frac{L_1 \alpha_1 (\theta_2 - \theta_1) + L_2 \alpha_2 (\theta_2 - \theta_1)}{\frac{L_1}{A_1 Y_1} + \frac{L_2}{A_2 Y_2}}$$



### Properties of Material 9 ) Proof Resilience

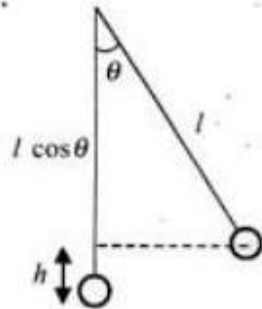
Proof resilience is related to

- (a) PE stored in an elastic body.
- (b) stiffness of a beam.
- (c) elastic fatigue.
- (d) elastic relaxation.

Ans : (a)

Properties of Material 10 ) Elongation in a Pendulum

A sphere of mass  $M$  kg is suspended by a metal wire of length  $L$  and diameter  $d$ . When in equilibrium, there is a gap of  $\Delta l$  between the sphere and the floor. The sphere is gently pushed aside so that it makes an angle  $\theta$  with the vertical. Find  $\theta_{\max}$  so that sphere fails to rub the Floor. Young's modulus of the wire is  $Y$ .



- (a)  $\sin^{-1} \left( 1 - \frac{Y\pi d^2 \Delta l}{8MgL} \right)$       (b)  $\tan^{-1} \left( 1 - \frac{Y\pi d^2 \Delta l}{8MgL} \right)$   
 (c)  $\cos^{-1} \left( 1 - \frac{Y\pi d^2 \Delta l}{8MgL} \right)$       (d) none

Ans :      (c)  $Y = \frac{Fl}{A\Delta l} = \frac{2Mg(1 - \cos\theta)L}{\pi \frac{d^2}{4} \Delta l}$

or  $1 - \cos\theta = \frac{Y\pi d^2 \Delta l}{8Mgl}$  or  $\cos\theta = 1 - \frac{Y\pi d^2 \Delta l}{8Mgl}$

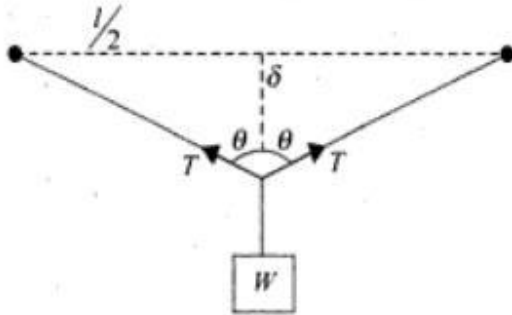
$$\frac{mv^2}{2} = mgl(1 - \cos\theta)$$

or  $\frac{mv^2}{l} = 2mg(1 - \cos\theta)$

$$\theta = \cos^{-1} \left( 1 - \frac{Y\pi d^2 \Delta l}{8MgL} \right)$$

Properties of Material 11 ) Depression at center of rod

A wire of length  $L$  is clamped at two ends so that it lies horizontally and without tension. A weight  $W$  is suspended from the middle point of the wire. The vertical depression is  $\delta$ . Young's modulus is  $Y$ .



- (a)  $\sqrt{\frac{2Tl^2}{4AY} + \frac{T^2l^2}{4A^2Y^2}}$       (b)  $\sqrt{\frac{2Tl^2}{4AY} - \frac{T^2l^2}{4A^2Y^2}}$   
 (c)  $\sqrt{\frac{2Tl^2}{4AY}}$       (d)  $\frac{Tl}{2AY}$

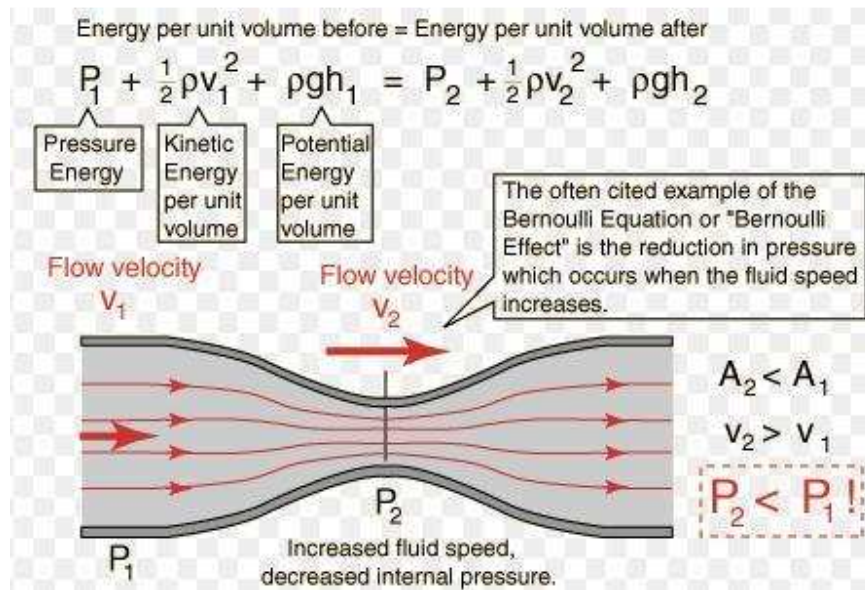
(a)  $2T \cos \theta = W$

or  $T = \frac{W}{2 \cos \theta}$

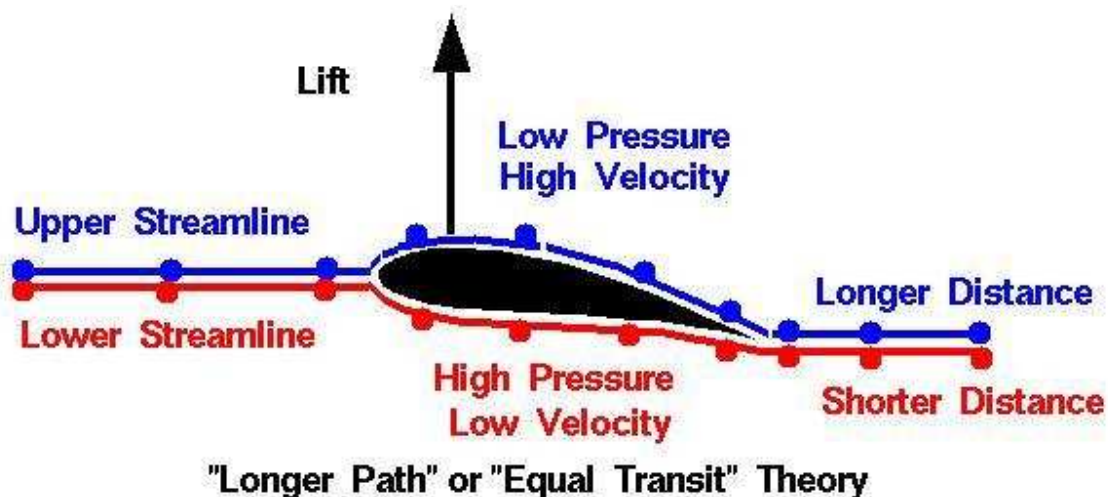
$$\Delta l = \frac{Tl}{2AY} \quad \delta = \sqrt{\left(\frac{l}{2} + \Delta l\right)^2 - \frac{l^2}{4}}$$

or  $\delta = \sqrt{\left(\frac{l}{2} + \frac{Tl}{2AY}\right)^2 - \frac{l^2}{4}} = \sqrt{\frac{2Tl^2}{4AY} + \frac{T^2l^2}{4A^2Y^2}}$

## Fluid 1 ) Bernoulli's Principle and Application



Differential velocity at top and bottom of an aircraft wing, for uplift



Dynamic lift in aircraft

Aeroplanes get the dynamic lift because of the shape of their wings. The upper surface of the wing is made more curved than the lower surface; air flows with greater speed above the wing; pressure above the wing is less. The wing gets dynamic lift upwards.

$$\text{Dynamic lift} = (P_2 - P_1)A = \frac{1}{2}\rho(v_1^2 - v_2^2)A$$

Where  $\rho$  is the density of air,  $A$  is the area of the wing,  $v_1$  and  $v_2$  are the speeds of air above and below the wing and  $P_1$  and  $P_2$  are pressures above and below the wing.

**Air is streaming past a horizontal air plane wing such that its speed is  $90 \text{ m s}^{-1}$  at the lower surface and  $120 \text{ m s}^{-1}$  over the upper surface. If the wing is 10 m long and has an average width of 2 m, the difference of pressure on the two sides and the gross lift on the wing are [Density of air =  $1.3 \text{ kg m}^{-3}$ ]**

(a) 5 Pa, 900 N

(b) 95 Pa, 900 N

(c) 4095 Pa, 900 N

(d) 4095 Pa, 81900 N.

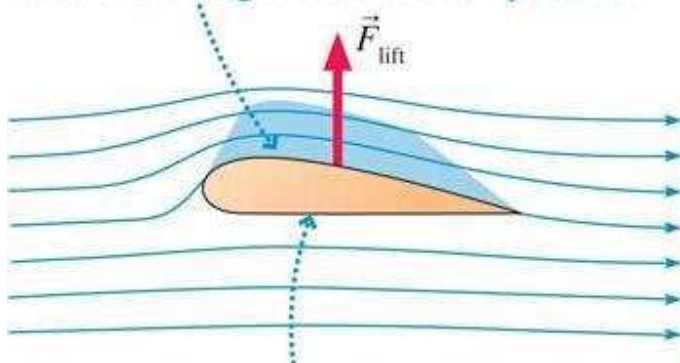
Ans :

Pressure Difference =  $\Delta P = \frac{1}{2} (\rho) v^2$

$$(d) P_2 - P_1 = \frac{1}{2} \times 1.3 [120^2 - 90^2] = 4095 \text{ Pa}$$

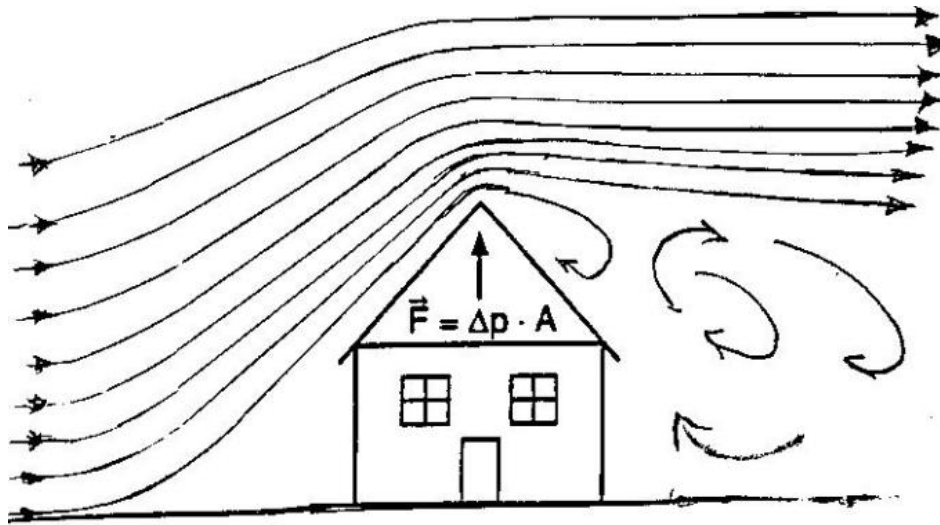
$$\text{Lift} = 4095 \times 2 \times 10 \text{ N} = 81900 \text{ N}$$

A pressure gradient is needed to accelerate the air around the curved upper surface of the wing. Thus the air just above the wing is a zone of low pressure.



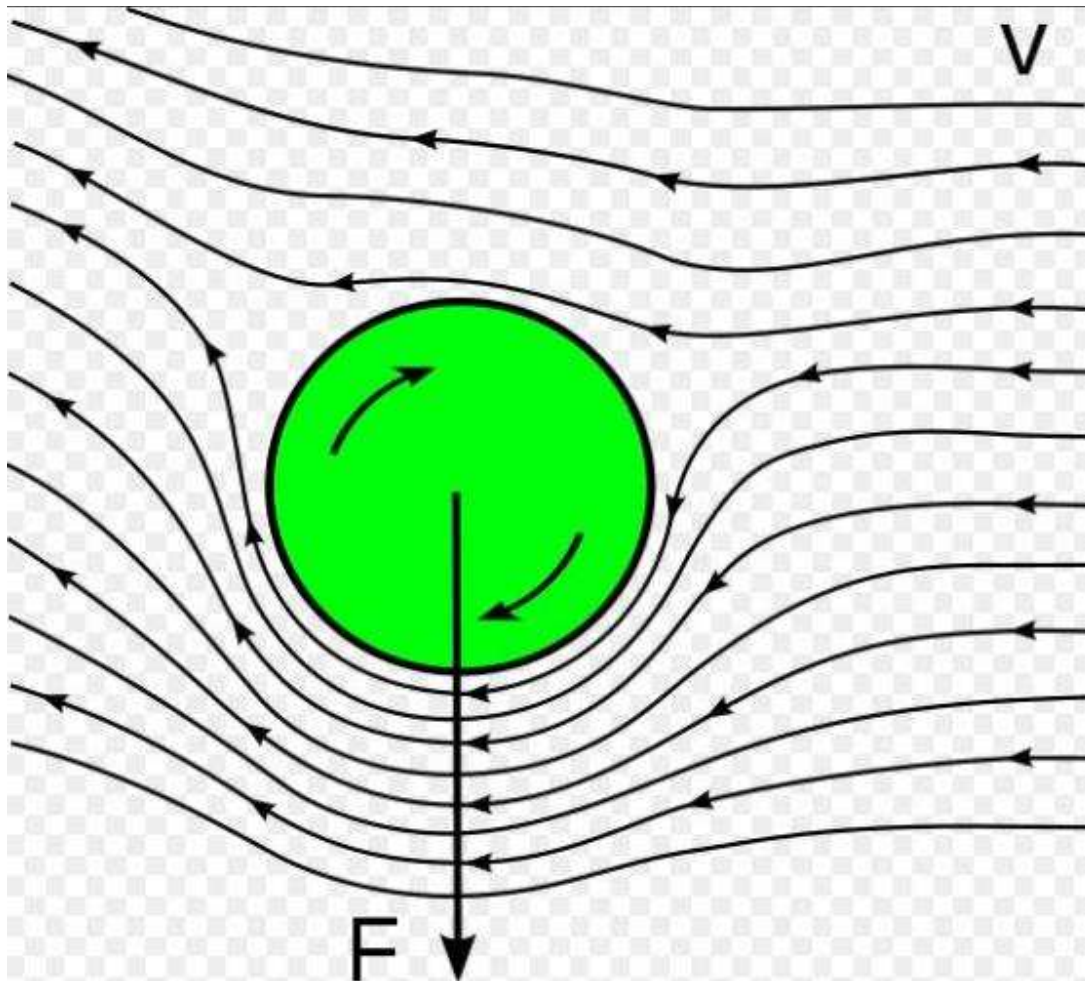
Because the pressure beneath the wing is higher than the pressure above, there's a net upward force on the wing. This is lift.

roof of hut being flown off due to strong wind

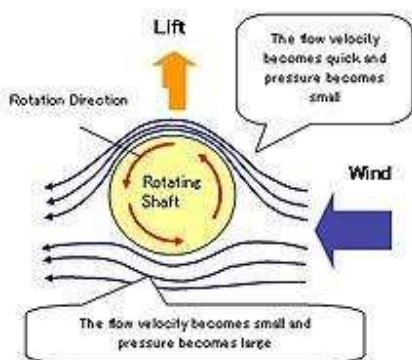




### Fluid 2 ) Magnus Effect Top Spin



### Magnus Effect lift



### Fluid 3 ) Reynold's Number

$$N_{Re} = \frac{D V C}{\eta}$$

**D = inside pipe diameter**

**V = average velocity**

**C = density**

**$\eta$  = absolute viscosity**

### Fluid 4 ) Surface Tension Formula

Work done = energy = Area  $\times$  Surface tension

Energy for film = 2(Area  $\times$  Surface tension)

Absorbed energy when drop of radius  $R$  splits into  $n$  identical drops of radius  $r$ , is

$$= 4\pi R^2(n^{1/3} - 1)T = 4\pi r^2 n^{2/3} (n^{1/3} - 1)T$$

Excess pressure inside the soap bubble =  $\frac{4T}{r}$

Excess pressure inside the liquid drop =  $\frac{2T}{r}$

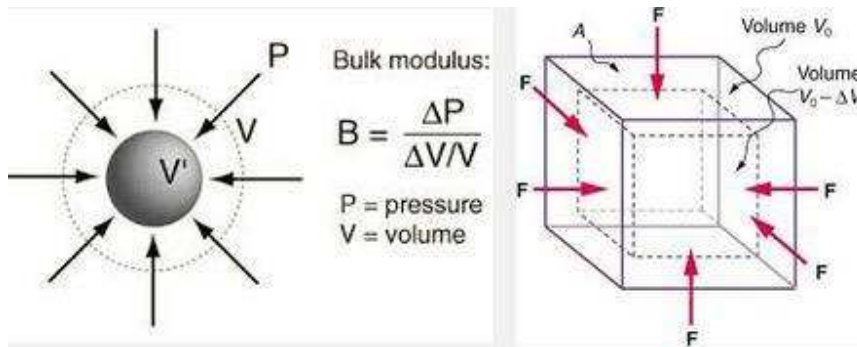
Difference between convex concave side is

$$p = T \left( \frac{1}{r_1} + \frac{1}{r_2} \right)$$

When two drops of radii  $r_1, r_2$  coalesce to form a new drop of radius  $R$  under isothermal condition, then  $R = \sqrt{r_1^2 + r_2^2}$

When a soap bubble of radius  $r_1$  and another of radius  $r_2$  are brought together the radius of the common interface is  $\frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2}$

Fluid 5 ) Bulk Modulus and Compression of liquid



$$\beta = -\frac{\delta V}{V \delta p} = \text{Compressibility}$$

$$-\frac{\delta V}{V} = \frac{\delta p}{K} \quad K = \rho \frac{dp}{d\rho}$$

$$V = \frac{1}{\rho} \quad K = \text{Bulk Modulus}$$

$\rho = \text{Density,}$

$V = \text{Volume, } p = \text{Pressure,}$

Find the density of water 2 km deep in a sea. Bulk modulus =  $2 \times 10^9$  Pa.

- (a)  $10^3 \text{ kgm}^{-3}$                       (b)  $1010 \text{ kgm}^{-3}$   
 (c)  $1100 \text{ kgm}^{-3}$                       (d)  $1040 \text{ kgm}^{-3}$

$$(b) \frac{\Delta V}{V} = \frac{P}{B} = \frac{2 \times 10^3 \times 10^3 \times 10}{2 \times 10^9} = .01$$

$$\frac{\Delta V}{V} = \frac{\Delta \delta}{\delta} \text{ or } \Delta \delta = 10 \text{ kg/m}^3.$$

density of water =  $1010 \text{ kg m}^{-3}$

The average depth of Indian Ocean is about 3000 m. Bulk modulus of water is  $2.2 \times 10^9 \text{ Nm}^{-2}$ ,  $g = 10 \text{ ms}^{-2}$ , then fractional compression  $\left(\frac{\Delta V}{V}\right)$  of water at the bottom of the Indian Ocean is

- (a) 1.36%      (b) 20.6%      (c) 13.9%      (d) 0.52%

**Interpret** (a) The pressure exerted by a 3000 m column of water on the bottom layer

$$p = h\rho g = 3000 \times 1000 \times 10 \\ = 3 \times 10^7 \text{ kg m}^{-1} \text{ s}^{-2} = 3 \times 10^7 \text{ Nm}^{-2}$$

Fractional compression  $\left(\frac{\Delta V}{V}\right)$

$$= \frac{\text{Stress}}{B} = \frac{(3 \times 10^7 \text{ Nm}^{-2})}{(2.2 \times 10^9)} = 1.36 \times 10^{-2}$$

$$\frac{\Delta V}{V} = 1.36\%$$

Find the volume density of elastic energy of fresh water at a depth of 1000 m

- (a)  $2.5 \text{ kJm}^{-3}$       (b)  $25 \text{ kJm}^{-3}$   
 (c)  $0.25 \text{ kJm}^{-3}$       (d) none

$$(b) \frac{dW}{V} = \frac{1}{2} P \frac{\Delta V}{V} = \frac{1}{2} P \left(\frac{P}{B}\right) \\ = \frac{(\rho gh)^2}{2 \times 2 \times 10^9} = \frac{(10^3 \times 10 \times 10^3)^2}{2 \times 2 \times 10^9} = 2.5 \times 10^4 \text{ J/m}^3.$$

A driver at a depth of 45 m exhales a bubble of air that is 1.0 cm in radius. Assuming ideal gas behaviour, what will be the radius of this bubble as it breaks the surface of water?

**Plan** Inside water  $P_{\text{Total}} = \text{atmospheric pressure} + \rho gh$

Using  $P_1V_1 = P_2V_2$ ,  $V_2$  at the surface of water is calculated ( $V_2$  is the volume of bubble at the surface), thus,  $r$  can be calculated.

**Solution** Atmospheric pressure = 1 atm.

Pressure due to depth of 45 m =  $\rho gh$

where  $\rho = \text{density of water} = 1 \text{ g cm}^{-3} = 1000 \text{ kg m}^{-3}$ ,  
 $g = 9.81 \text{ m s}^{-2}$ ,  $h = 45 \text{ m}$

$$\rho gh = 1000 \times 9.81 \times 45 \text{ N m}^{-2}$$

$$= \frac{1000 \times 9.81 \times 45}{101325} \text{ atm} = 4.36 \text{ atm}$$

$$(\because 1 \text{ atm} = 1.01325 \times 10^5 \text{ N m}^{-2})$$

$$\therefore P_1 = \text{atmospheric pressure} + \rho gh = 1 + 4.36 = 5.36 \text{ atm}$$

$$P_2 = 1 \text{ atm}$$

$$V_1 = \frac{4}{3} \pi r^3 = \frac{4}{3} \times \pi \times (1)^3 \text{ cm}^3$$

$$V_2 = \frac{4}{3} \pi r^3 = \text{volume of bubble at } P_2 \text{ (at the surface)}$$

using

$$P_1V_1 = P_2V_2$$

$$V_2 = \frac{P_1V_1}{P_2}$$

$$\frac{4}{3} \pi r^3 = \frac{5.36 \times \frac{4}{3} \pi (1)^3}{1}$$

$$r^3 = 5.36 \text{ cm}^3$$

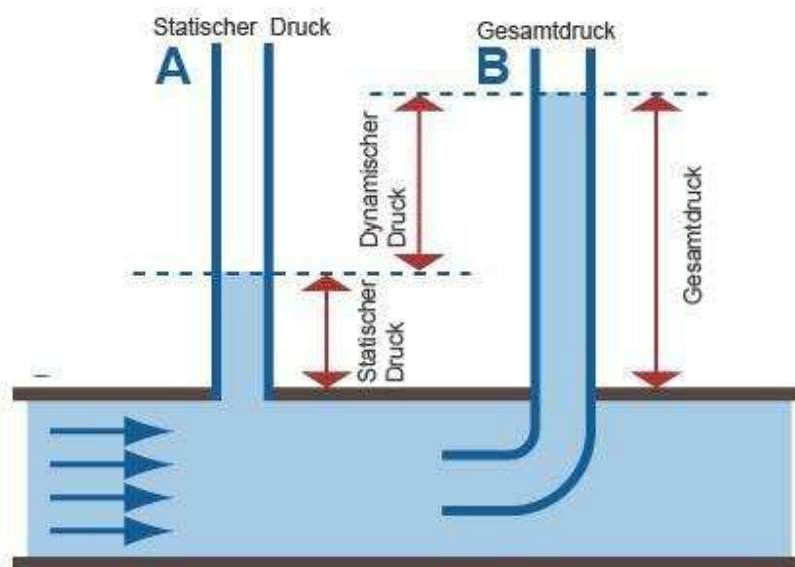
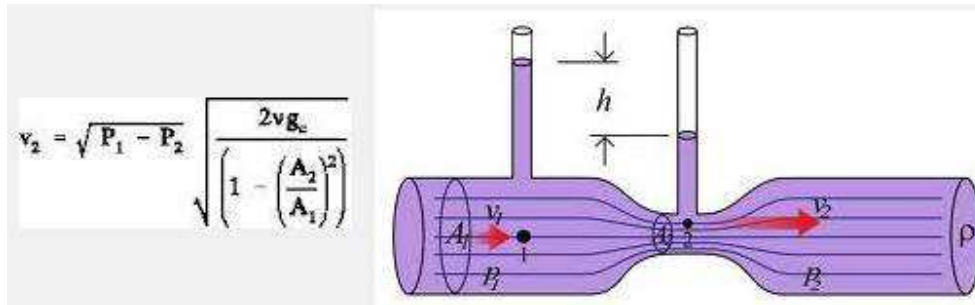
$$r = 1.75 \text{ cm}$$



Fluid 6 ) Time taken for water to go from  $h_1$  to  $h_2$

A cylindrical vessel of area of cross-section  $A$  has a hole of area of cross-section ' $a$ ' in its bottom. Time taken for the water level to decrease from  $h_1$  to  $h_2$  as water flows out from the hole is

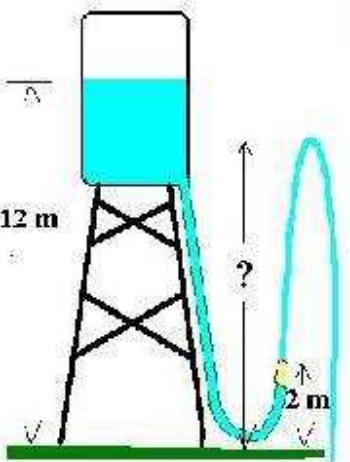
$$t = \frac{A}{a} \sqrt{\frac{2}{g}} (\sqrt{h_1} - \sqrt{h_2})$$



Application of Bernoulli's Equation in Siphon



## Application of Bernoulli's Equation



Water level is 12 m above ground. Hose nozzle is 2.0 m above ground.

- 1 What is (gauge) pressure in hose at nozzle?
- 2 What is water speed at nozzle?
- 3 How high does water squirt?

## Magnetic Properties of Materials 1 ) Diamagnetic, Paramagnetic, Ferrimagnetic, Antiferromagnetic

**Magnetic Properties :** Solids can be classified into different types depending upon their behaviour towards applied magnetic field.

**a. Diamagnetic Substances :** Which are weakly repelled by magnetic field. They have paired electrons.

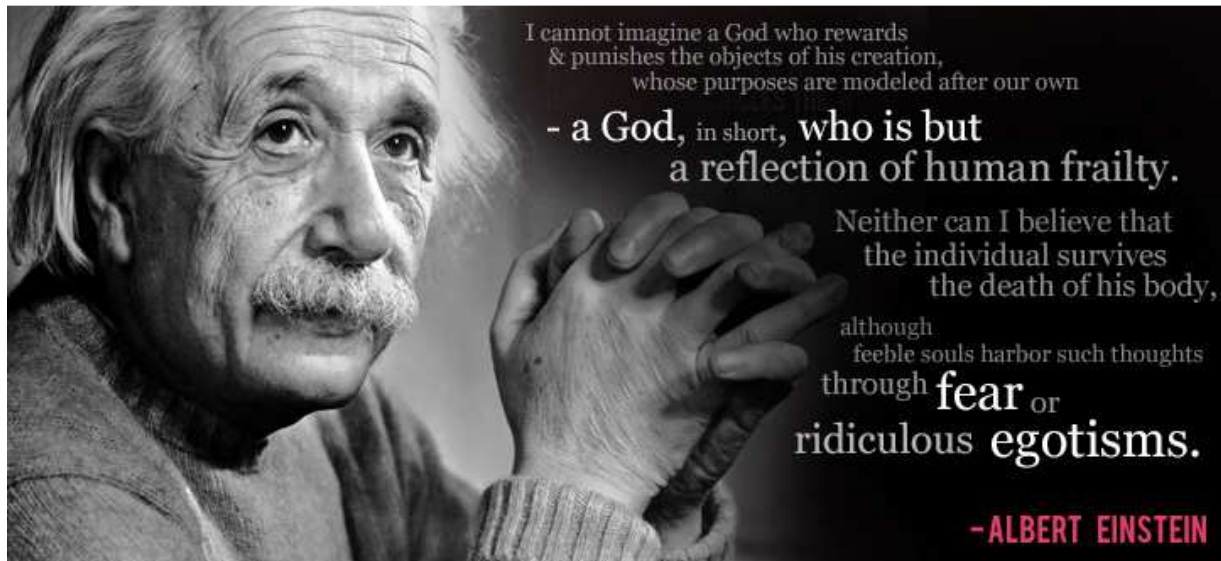
NaCl,  $V_2O_5$ ,  $TiO_2$ .

**b. Paramagnetic Substances :** Which are weakly attracted by magnetic field. They have permanent dipoles due to presence of unpaired electrons. They lose their magnetism on removal of magnetic field.  $TiO$ ,  $Ti_2O_3$ ,  $VO$ ,  $VO_2$ ,  $CuO$ .

**c. Ferromagnetic Substances :** Spontaneous alignment of magnetic dipoles of ions or atoms in same direction. It changes into paramagnetic substances at higher temperature.  $Fe$ ,  $Co$ ,  $Ni$ ,  $CrO_2$ .

**d. Ferrimagnetic Substances :** Alignment of magnetic dipoles of ions or atoms in such a way so that there is some net magnetic moment due to unequal number of parallel and anti-parallel magnetic dipoles. It also changes into paramagnetic substances at higher temperature.  $Fe_3O_4$ .

**e. Anti Ferromagnetic Substances :** Alignment of magnetic dipoles of ions or atoms in such a way so that there is no net magnetic moment (i.e. zero magnetic moment) due to equal number of parallel and anti-parallel magnetic dipoles.  $V_2O_3$ ,  $Cr_2O_3$ ,  $MnO$ ,  $Mn_2O_3$ ,  $MnO_2$ ,  $FeO$ ,  $Fe_2O_3$ ,  $CoO$ ,  $NiO$ .

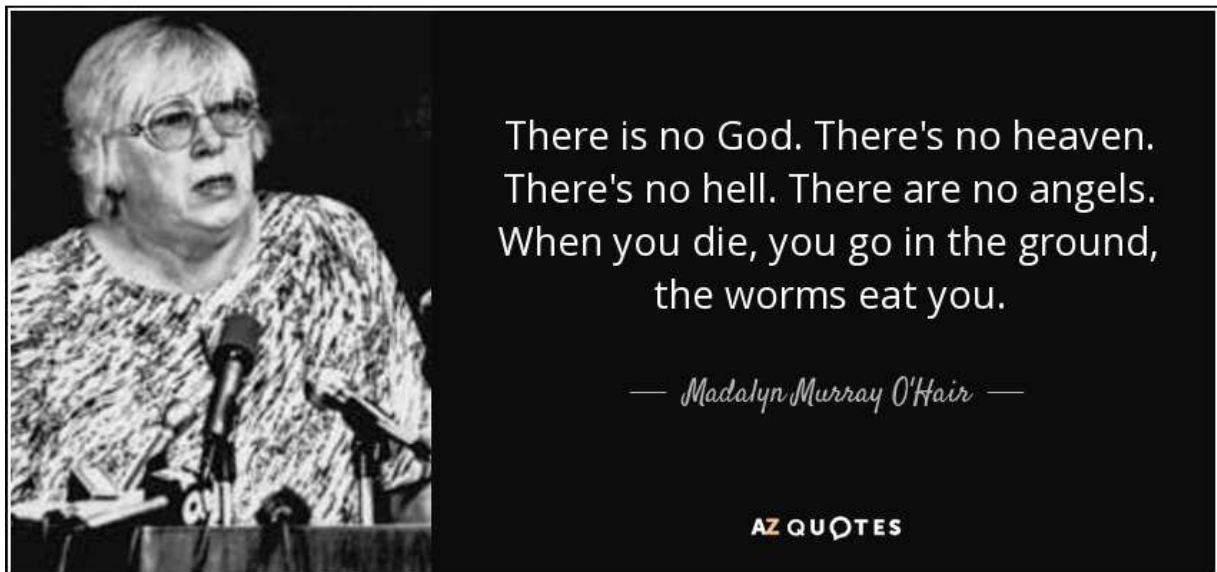


( Apart from Millions of smart people ) Several Nobel Laureates were Atheists.

Some famous examples have been [Albert Einstein](#) ( 1921 ), [Richard Feynman](#) ( 1965 ), [Erwin Schrödinger](#) ( 1933 ), [Paul Dirac](#) ( 1933 ), [Lawrence M. Krauss](#) ( 2011 ), [Niels Bohr](#) ( 1922 ), [Peter Higgs](#) ( 2013 ), [John Bardeen](#) ( **The only person receiving the Physics Nobel prize twice. 1956, 1972** ), [Frederick Sanger](#) ( **The only person receiving the Chemistry prize twice. 1958, 1980** ), [Marie Curie](#) ( 1903, 1911 ), [Frédéric Joliot-Curie](#) and [Irène Joliot-Curie](#) ( 1935 ), [Milton Friedman](#) ( 1976 ), [John Harsanyi](#) ( 1994 ), [Friedrich Hayek](#) ( 1974 ), [John Forbes Nash, Jr.](#) ( 1994 ), [Amartya Sen](#) ( 1998 ), [Subrahmanyan Chandrasekhar](#) ( 1983 ), [Enrico Fermi](#) ( 1938 ), [C. V. Raman](#) ( 1930 ), [Eugene Wigner](#) ( 1963 ), [Steven Weinberg](#) ( 1979 ), [Chen-Ning Yang](#) ( 1957 ) etc

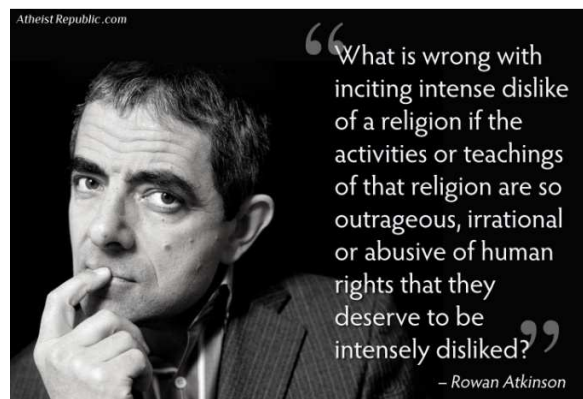
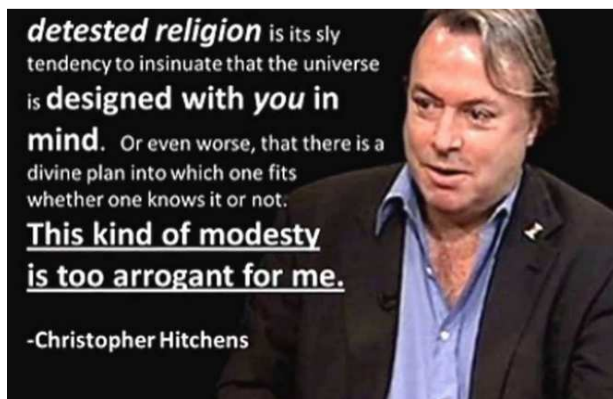
A bigger ( incomplete ) list can be seen at

[https://en.wikipedia.org/wiki/List\\_of\\_nonreligious\\_Nobel\\_laureates](https://en.wikipedia.org/wiki/List_of_nonreligious_Nobel_laureates)



( When the body is burnt, oxides are the ash. The gases and water vapor spread in the air )

My **personal favorites** ( among these Atheists ) are **Richard Feynman**, **Peter Higgs**, **Lawrence Krauss**.



Richard Feynman openly laughed ( Publicly and in class ) about Gods, Fairies etc. see <https://www.youtube.com/watch?v=j3mhkYbznBk>

and [https://www.youtube.com/results?search\\_query=Richard+Feynman](https://www.youtube.com/results?search_query=Richard+Feynman)

Peter Higgs was very unhappy about " Higgs Boson " being called "G..( I don't want to name this ) Particle". **Stupid Journalists**, Media, and dumb people kept repeating that word, and Peter requested to refrain from using this word.

**Lawrence Krauss openly laughs and ridicules the Theists or any non-Atheists. The crap of Agnosticism does not work with me or Krauss.**

We are in Modern Times. I am lucky to learn the correct things quite early in my life, in a so " peaceful " society. When I was in standard 9, ( in early 1980s ), I was writing a book on

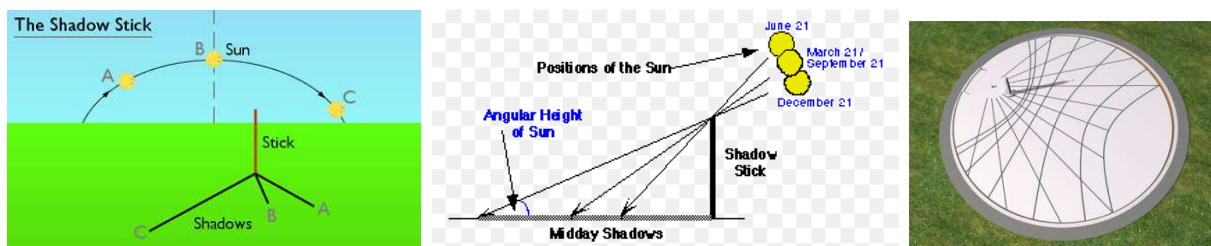
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Atheism. I was convinced to understand, learn, and imbibe the correct approach and knowledge.

But that was not the case previously. Copernicus used to discuss and explain people widely and randomly, that Earth is rotating around the Sun, and it is not a Geocentric" universe. Nicolaus Copernicus had to waste lot of time arguing, fighting and convincing the stupid.

Measuring something, which is very slow; is very difficult. I have asked lot of "educated / engineer / Software or IT ( senior position ) Parents" that " How do we know that Earth is moving around the Sun in 365 days or say 365.242196 days " ? **Believe me I never got an answer.** The Modern iPad / smartphone **community in general does not know how 365.24 days was measured almost thousand years ago !**

A metal triangle was set at top of buildings ( Mosques or churches ) and the position of the shadow was marked at a particular time. Say 8 AM everyday. The position of the shadow varied everyday. It was seen that after 365 days the shadow matched the position but after sometime, not exactly at 8 AM but after a few hours ( approx 6 hours ) so at around 2 PM or slightly before.



See details of this at <http://blog.world-mysteries.com/science/ancient-timekeepers-part-2-observing-the-sky/>

<http://blog.world-mysteries.com/science/ancient-timekeepers-part4-calendars/>

See the video [https://www.youtube.com/watch?v=IhqzW97\\_47w](https://www.youtube.com/watch?v=IhqzW97_47w)

<https://thecuriousastronomer.wordpress.com/2012/10/>

Much tougher questions are “ How many different kind of years do we have ? “

Or “ What is the difference between ‘ **Sidereal year** ‘ and ‘ **Tropical year** ‘ “

Meteors were coming from sky. These were called ‘ shooting stars ‘. Meteors often had Iron in them. Sidero is a combining form meaning “star,” “constellation,” used in the formation of compound words. Greeks used the word siderolite for Iron. Next the source of meteors; the sky itself was named the same. As year was measured using objects from sky; Sun and shadows; the year was named a “ **Sidereal Year** “

To avoid embarrassing people; I don't ask ....

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See the answers in <https://www.youtube.com/watch?v=cGjP3vAZGa4>

<https://www.youtube.com/watch?v=qgsrVyW53DY>

It took many centuries to introduce the leap year corrections. A century is a leap year only if divisible by 400 and not the rule of divisible by 4. Year 1900 was not a Leap year. But year 2000 was. I have met computer Science guys who are aware that Microsoft Database SQL-server do not accept some old dates, while Oracle database does not accept some specific dates of the past. But none whom I met knew the detailed or actual reasons.

See <https://zookeepersblog.wordpress.com/everyone-must-know-about-the-calendar/>

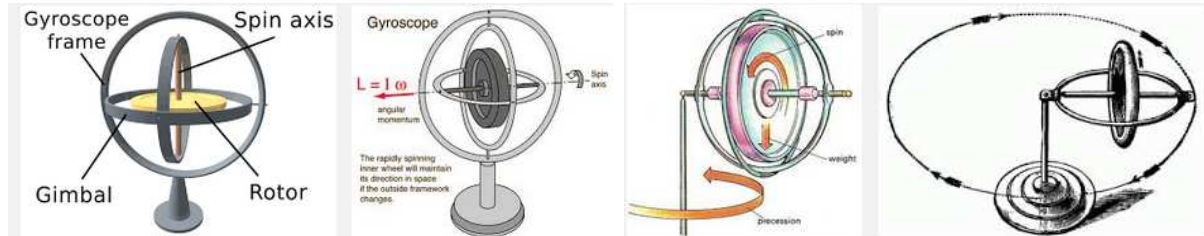
**“ How do you prove that day and night is happening due to rotation of Earth around its own axis in contrast to Sun is rotating around Earth “ ?**

No student from Bangalore, whom I met, answered this. Though conservation of Angular Momentum is in course. ( I am being polite ) Hardly met any parent who knew the explanation. See [https://www.youtube.com/watch?v=iqpV1236\\_Q0](https://www.youtube.com/watch?v=iqpV1236_Q0)

And <https://www.youtube.com/results?q=Foucault%27s+pendulum>

What about Gyroscopes ?

Approx 300 year back around 1750 the gyroscopes were made.



History of Gyroscope <http://www.gyroscopes.org/history.asp>

See about Gyroscopes in [https://www.youtube.com/watch?v=cquvA\\_lpEsA](https://www.youtube.com/watch?v=cquvA_lpEsA)

<https://www.youtube.com/watch?v=awXTZt86gz0>

<https://www.youtube.com/watch?v=zbdrrpXb-fY>

<https://www.youtube.com/watch?v=N92FYHHT1qM>

[https://en.wikipedia.org/wiki/Earth%27s\\_orbit](https://en.wikipedia.org/wiki/Earth%27s_orbit)

<https://www.youtube.com/watch?v=ZcWsjlGPPFQ>

Must see

<https://www.youtube.com/watch?v=SnMmBmzoVQc&list=PL68IJE2PG4AnVVM57WvOYbJDmqf4umHG1>

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Must know ...

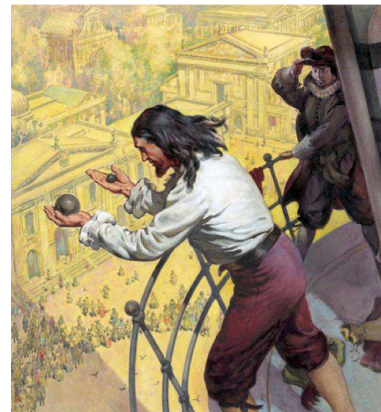
[https://www.youtube.com/watch?v=zjV3PQ4f6IM&list=PLTve54sz-eh\\_P29Sbbv\\_j3bC97OFaArOd](https://www.youtube.com/watch?v=zjV3PQ4f6IM&list=PLTve54sz-eh_P29Sbbv_j3bC97OFaArOd)

Tyco Brahe took the boldest step to create the " Foundation of Science ". Experiments or "Double blind experimental observations" are the supreme. The **Theory follows the experimental verification.**

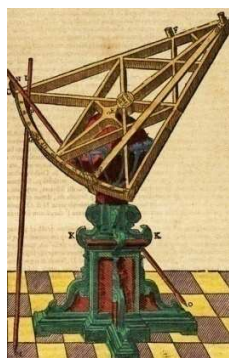
[ There are some universities who award M.Sc in Psychology. A psychologist may **guess** something .... **But that is not reality or truth.** Till something is experimentally verified it remains as a Perception. Truth is known only after experiments. Because the subject Psychology; completely stands of experimental verification; so the Master in Science degree. ]



Galileo Galilei  
(1564-1642)



Galileo was the first person who wanted to experimentally verify the speed of light.

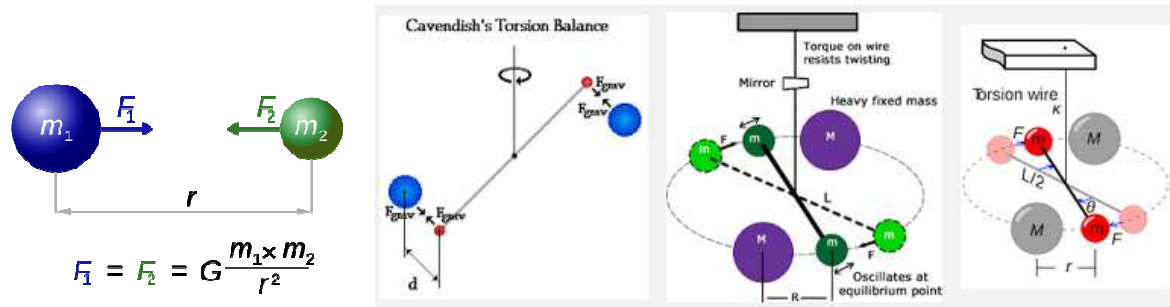


Tycho decided to observe the skies ( around 1573 ). In those days sky was synonymous to God. He had the courage to **go to the King** to ask for donations to make an observatory. He said to the king that "he wants to observe the Gods and take conclusions ". **Salute to Tycho's paradigm that even Gods can be observed and conclusions can be drawn.**

**Amazing leap to start Science.**

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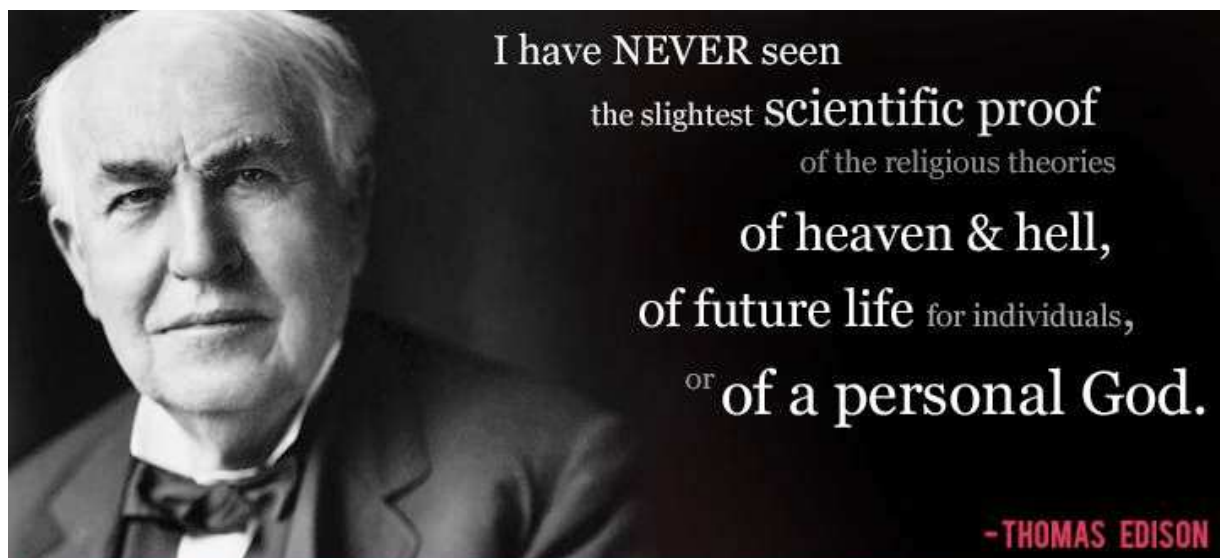




Since those days till now we observed and concluded about Kepler's Laws, Gravitation Laws, We concluded that there was no **Phlogiston** or Flogiston, Cavendish measuring value of  $G$ , measuring speed of light, X-Ray, **Electromagnetism / Maxwell's equations**, Radioactivity, No Aether was "observed" in Michelson Morley's experiments, Protons, Neutrons, General Theory of Relativity, Slowing of clocks at high speed, Bending of space, **Bending of light and gravitational lens**, YDSE, Quantum Mechanics, Ernst Ruska designed and built the first **electron microscope**, **Casimir Forces**, Virtual particles and more than 400 kinds of particles, Quarks, Unruh effect ( an accelerating thermometer shows higher temperature ), Negative Kelvin Temperature, Bose-Einstein condensates, Superconductivity, **Solution to EPR paradox by John Stewart Bell**, Violation of Parity in certain situations - Madam Wu, Yang and Lee, Quantum entanglement in Alain Aspect's Experiments, Black holes, mass of Neutrinos, Caesium Atomic Clocks, Dark Matter, Dark energy, Magnetic Monopole, Gravitational Waves, Nano Materials, Meta Materials, Quantum Computers .....

**No God was observed**, or **no role of God was observed**. There is no conspiracy theory going around in Science. Those who want to verify God have to die waiting

... Nothing ever will be reported regarding this illusion.



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[ **Stupids had proposed the phlogiston theory.** This was a superseded scientific theory that postulated that a fire-like element called phlogiston is contained within combustible bodies and released during combustion. The name comes from the Ancient Greek φλογιστόν phlogistón (burning up), from φλόξ phlóx (flame).]

In contrast see <http://www.americanscientist.org/issues/pub/burn-magnet-burn>

**Some examples of stupidity to show / explain by contrasts; will be the right approach.**

## Aristotle used goat urine and Hippocrates recommended pigeon droppings. For what?



As a treatment for baldness. Men have never found baldness an appealing trait, in spite of stories that bald men are sexier. (Stories usually spread by bald men.) Virtually anything that can be done to a bald pate has been tried to stimulate hair growth. The ancient Egyptians were fond of rancid crocodile or hippo fat. If it smelled bad, surely it must do some good. It didn't. Cleopatra experimented with a goo made of ground horse teeth and deer marrow to spur Julius Caesar's dormant

hair follicles into action. When this didn't work she traded him in for Mark Antony. During the Victorian era cold tea was brushed on the scalp, followed by citrus juice. In farming areas chickens were persuaded to leave deposits on a bald head and cows to lick it. Electric combs, suction caps and paint thinner have been tried. At a secluded farmhouse in Pennsylvania, Marcella Ferens rakes a glass instrument filled with a purple gas across the head to "sterilize the scalp." Then the subject holds a wire attached to some electrical machine while the operator holds a second wire as she massages the bald area with a secret formula. This forces the formula into the scalp. Some infomercials push shampoos with special emulsifiers to clean follicles as if baldness were due to plugged follicles. Others use jumbled language to promote spray paint to cover bald spots. The truth is that only Rogaine (minoxidil) rubbed on the scalp or Propecia (finasteride) taken orally have shown any effect in growing hair. Even with these the results are not impressive. The Bald Headed Men of America, headquartered appropriately in Morehead, North Carolina, was started when the founder was refused a job because he was bald. They take a different tack. If you want to waste your hormones growing hair....go ahead" Actually this is a wrong statement because it is high levels of dihydrotestosterone that can cause baldness. They are on firmer footing with their slogan. No rugs or drugs.

Aristotle used Goat Urine and Hippocrates recommended Pigeon droppings to cure baldness.

<http://dazeinfo.com/2010/06/22/superstitions-across-different-countries-an-overview/>

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## **Australians bathed inside rotting whales to 'cure' rheumatism**

**The Australian National Maritime Museum has revealed that sufferers of rheumatism were once advised to sit inside the festering carcasses of whales in order to relieve their symptoms.**

The museum has recently opened a new exhibit in Sydney, which seeks to uncover the diversity, origins and adaptation of whales, charting their development from land mammals to aquatic giants. The exhibition, entitled "Amazing Whales" also looks at the different relationships humans have had with the cetaceans, which includes their apparent medicinal qualities.

Those afflicted with rheumatism were advised to sit inside the belly of a dead whale for approximately 30 hours. If the patient could stay the course and withstand this bizarre practice, they were promised at least 12 months of relief from pain.

<http://www.wired.co.uk/article/whale-bath>

## **Weird Bizarre superstitions to cure disease**

<http://www.historyextra.com/feature/animals/10-historical-superstitions-we-carry-today>

<http://listverse.com/2013/01/21/10-crazy-cures-for-the-black-death/>

## **Millions of People are making money out of superstitions of Fools**

*Rebirthing Therapy, Reiki, Energy-Deflecting Golfer Pendant, Maggot Debridement Therapy, Leech Therapy, Beer spas, Ozone Anti-Aging ..... the list is very big.*

<http://webcoist.momtastic.com/2010/07/05/12-most-bizarre-modern-alternative-medical-treatments/>

<http://oddrandomthoughts.com/strange-and-bizarre-medicine-and-cures/>

<http://www.stylist.co.uk/life/13-strange-superstitions>



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So in simple words instead of taking opinions of Stupid Fools, or wasting any time arguing with them ..... Let study science correctly, without bias !

Most important physics experiments can be seen at

See <http://www.explainthatstuff.com/great-physics-experiments.html>

<http://physics-animations.com/Physics/English/top10.htm>

[https://en.wikipedia.org/wiki/List\\_of\\_experiments](https://en.wikipedia.org/wiki/List_of_experiments)

<https://www.quora.com/What-are-some-of-the-most-important-experiments-in-physics>

**Though my list will be as follows -**

Michelson-Morley experiment proving there was no Aether, Moseley 's experiment with X-Rays to discover Protons, Jagadish chandra Bose demonstrating controlled emission / transmission and receiving of Radio waves, Casimir experiments to show Casimir forces of virtual particles, Edington measuring bending of light, Flying atomic clocks in planes and confirming slowing down of time at high speeds, Victor Hess measured Radiation level variation at ground and high up in the atmosphere, Soviet physicist Sergey Vernov was the first to use radiosondes to perform cosmic ray readings with an instrument carried to high altitude by a balloon at heights up to 13.6 km, The proof of time dilation by Muon decay <https://debunkingrelativity.com/muons-time-dilation/> , Measurement of Space-time curvature near Earth and thereby the stress-energy tensor (which is related to the distribution and the motion of matter in space) in and near Earth [https://en.wikipedia.org/wiki/Gravity\\_Probe\\_B](https://en.wikipedia.org/wiki/Gravity_Probe_B) , Detecting Gravitational Waves.

[ In 1909 Theodor Wulf developed an electrometer, a device to measure the rate of ion production inside a hermetically sealed container, and used it to show higher levels of radiation at the top of the Eiffel Tower than at its base. However, his paper published in *Physikalische Zeitschrift* was not widely accepted. In 1911 Domenico Pacini observed simultaneous variations of the rate of ionization over a lake, over the sea, and at a depth of 3 meters from the surface. Pacini concluded from the decrease of radioactivity underwater that a certain part of the ionization must be due to sources other than the radioactivity of the Earth. In 1912, Victor Hess carried three enhanced-accuracy Wulf electrometers to an altitude of 5300 meters in a free balloon flight. He found the ionization rate increased approximately fourfold over the rate at ground level. Hess ruled out the Sun as the radiation's source by making a balloon ascent during a near-total eclipse. With the moon blocking much of the Sun's visible radiation, Hess still measured rising radiation at rising altitudes. He concluded "The results of my observation are best explained by the assumption that a radiation of very great penetrating power enters our atmosphere from above." In 1913-1914, Werner Kolhörster confirmed Victor Hess' earlier results by measuring the increased ionization rate at an altitude of 9 km. Hess received the Nobel Prize in Physics in 1936 for his discovery. Homi J. Bhabha derived an expression for the probability of scattering positrons by electrons, a process now known as Bhabha scattering. His classic paper, jointly with Walter Heitler, published in 1937

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described how primary cosmic rays from space interact with the upper atmosphere to produce particles observed at the ground level. Bhabha and Heitler explained the cosmic ray shower formation by the cascade production of gamma rays and positive and negative electron pairs. Soviet physicist Sergey Vernov was the first to use radiosondes to perform cosmic ray readings with an instrument carried to high altitude by a balloon. On 1 April 1935, he took measurements at heights up to 13.6 kilometers using a pair of Geiger counters in an anti-coincidence circuit to avoid counting secondary ray showers. ]

See [https://en.wikipedia.org/wiki/Cosmic\\_ray](https://en.wikipedia.org/wiki/Cosmic_ray)

<http://web.mit.edu/8.13/www/JLEperiments/JLExp14.pdf>

<http://web.mit.edu/lululiu/Public/pixx/not-pixx/muons.pdf>

[https://en.wikipedia.org/wiki/Time\\_dilation](https://en.wikipedia.org/wiki/Time_dilation)

<http://www.physics.rutgers.edu/ugrad/389/muon/muonphysics.pdf>

<http://www2.fisica.unlp.edu.ar/~veiga/experiments.html>

## Detecting Neutrons

Rutherford predicted the existence of the neutron in 1920. Twelve years later, his assistant James Chadwick found it. At Cambridge, Chadwick searched for the neutron. He tried in 1923, but did not find it. He tried again in 1928, with no success. In 1930, the German physicists Walther Bothe and Herbert Becker noticed something odd. When they shot alpha rays at beryllium (atomic number 4) the beryllium emitted a neutral radiation that could penetrate 200 millimeters of lead. In contrast, it takes less than one millimeter of lead to stop a proton. Bothe and Becker assumed the neutral radiation was high-energy gamma rays.

Marie Curie's daughter, Irene Joliot-Curie, and Irene's husband, Frederic, put a block of paraffin wax in front of the beryllium rays. They observed high-speed protons coming from the paraffin. They knew that gamma rays could eject electrons from metals. They thought the same thing was happening to the protons in the paraffin. Chadwick said the radiation could not be gamma rays. To eject protons at such a high velocity, the rays must have an energy of 50 million electron volts. An electron volt is a tiny amount of energy, only enough to keep a 75-watt light bulb burning for a tenth of a trillionth of a second. The alpha particles colliding with beryllium nuclei could produce only 14 million electron volts.

The law of conservation of energy states that energy can neither be created nor destroyed. It certainly looked as if energy was being created along with the neutral radiation. Chadwick had another explanation for the beryllium rays. He thought they were neutrons. He set up an experiment to test his hypothesis.

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**Chadwick put a piece of beryllium in a vacuum chamber with some polonium.** The polonium emitted alpha rays, which struck the beryllium. When struck, the beryllium emitted the mysterious neutral rays.

In the path of the rays, Chadwick put a target. When the rays hit the target, **they knocked atoms out of it.** The atoms, which became electrically charged in the collision, flew into a detector. Chadwick's detector was a chamber filled with gas. When a charged particle passed through the chamber, it ionized the gas molecules. The ions drifted toward an electrode. **Chadwick measured the current flowing through the electrode. Knowing the current, he could count the atoms and estimate their speed.** Chadwick used targets of different elements, measuring the energy needed to eject the atoms of each. Gamma rays could not explain the speed of the atoms. The only good explanation for his result was a neutral particle. **To prove that the particle was indeed the neutron, Chadwick measured its mass.** He could not weigh it directly. Instead he measured everything else in the collision and used that information to calculate the mass.

For his mass measurement, Chadwick bombarded boron with alpha particles. Like beryllium, boron emitted neutral rays. Chadwick placed a hydrogen target in the path of the rays. When the rays struck the target, protons flew out. Chadwick measured the velocity of the protons.

Using the laws of conservation of momentum and energy, Chadwick calculated the mass of the neutral particle. It was 1.0067 times the mass of the proton. The neutral radiation was indeed the long-sought neutron.

<http://ansnuclearcafe.org/2011/10/19/pioneers102011/>

( **As I write these words { 2016 } GUT [ General Unified Theory ] is being modified to introduce a 5th fundamental force, because some heavy particles have been observed at CERN and various other experiments and Producing Gravitational waves at will, without mass )**



Learn Science from <https://www.youtube.com/user/cassiopeiaproject/videos>

Some easy Physics ( much easier than IIT-JEE )

[https://www.youtube.com/channel/UClISRiiRVQuDfgxl\\_QN\\_Fmw/videos](https://www.youtube.com/channel/UClISRiiRVQuDfgxl_QN_Fmw/videos)

( Pradeep Kshetrapal Sir's Videos are at -

<https://www.youtube.com/user/PradeepKshetrapal/videos> )

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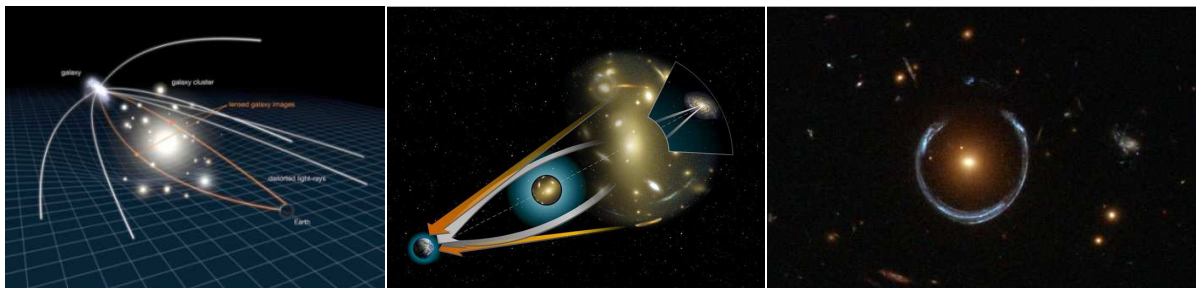
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IIT-JEE is extremely tough for most humans. A productive PhD in Physics, or actually contributing to growth of the subject is much more tougher ( than IIT JEE ). { I personally know quite a few IIT-JEE single or double digit rankers, joining for PhD and then dropped out due to performance }. **Most people have an illusion that they can argue with Scientists and imagine to ask some " smart " questions which the Scientists will not able to answer, so the argument is won, and existence of God is proved.** As if Scientist are eagerly sitting or waiting to answer every crap asked. I can only say; that most scientists ( since more than 100 years ) have stopped wasting their time arguing or convincing fools. I am not a Scientist. Even being a simple teacher, I do not try to teach fools, or argue with anyone.

[ **For History of Physics I recommend**

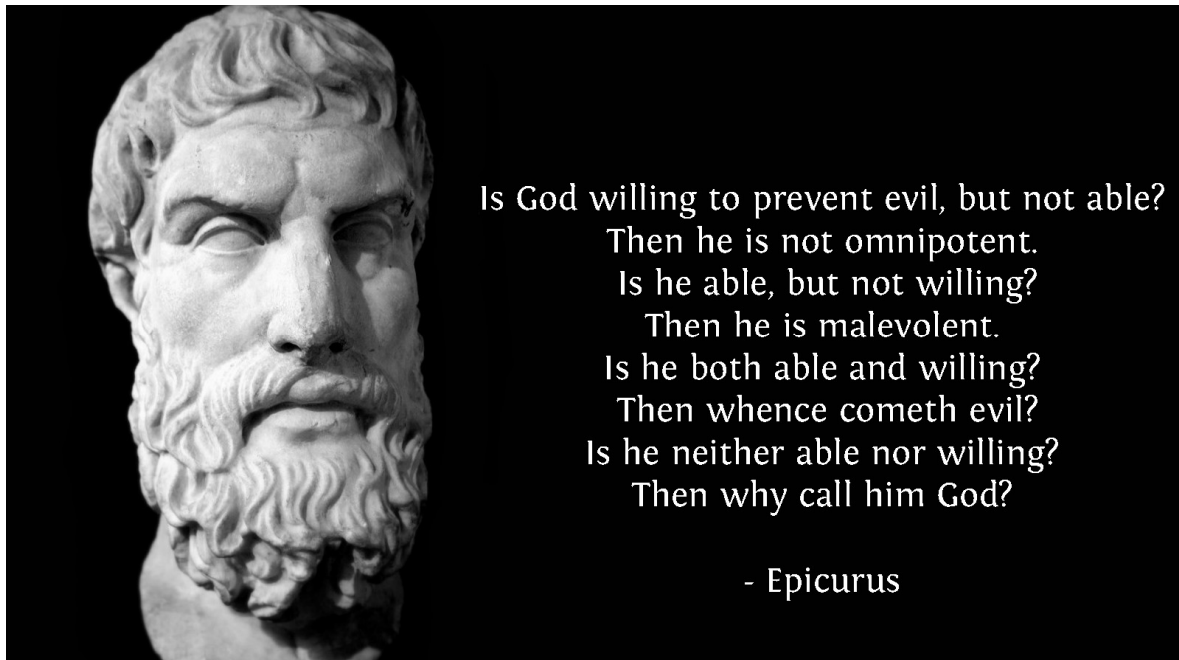
<http://www.historyworld.net/wrldhis/PlainTextHistories.asp?ParagraphID=kqq> ]

[ Gravitational lens and Einstein ring due to bending of light by mass ]



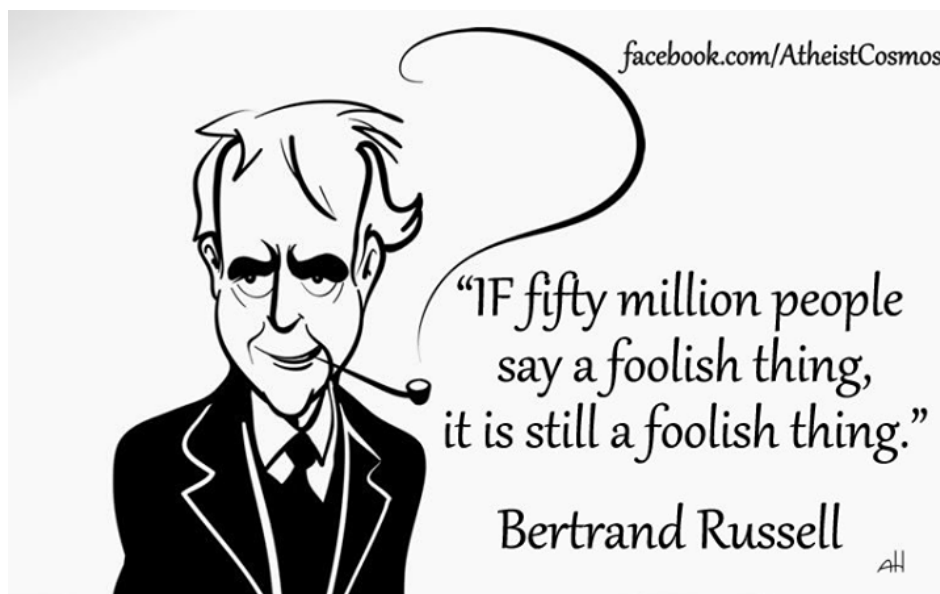
Recall what I said at the beginning of the book .... " **Someone will learn only by his hard work, his desire to learn.** " No arguments or no 'time wasting' with fools. There is too much of good material ( data, books, videos etc ) out and free in this world. If someone wants to learn, can learn; instead of wasting time arguing. Since centuries stupids and/or fools are being eliminated in various exams. Entrance exam, is a misnomer. These are elimination tests. The society has systems of Interviews, Peer reviews, appraisals, Thesis evaluation etc... to eliminate crap, foolish things, and nonsense.

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Religion and/or " war between religions " mostly to decide whose God is better; have killed millions. Instead of fighting and killing; to decide which custom to follow; how to dress; what rituals to do on a daily basis; better to spend time experimenting and developing new things, new technologies, new ideas. Scientists ( **the men** ) are busy; and **always will be busy!** **Rather, in war; with new frontiers of knowledge;** not in arguments, verbal wars, or physical wars. **Atheism is the most peaceful Doctrine.**

“**Bertrand Arthur William Russell**” the famous Philosopher, Mathematician, Logician, received 1950 Nobel Prize for Literature.



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So those who want to learn can continue learning ...

See [https://www.youtube.com/results?search\\_query=History+of+science](https://www.youtube.com/results?search_query=History+of+science)

See

[https://www.youtube.com/results?search\\_query=history+of+science+the+complete+full+documentary+](https://www.youtube.com/results?search_query=history+of+science+the+complete+full+documentary+)

I will choose only two extreme examples of what Human beings have “ **seen** “ by now ...

**For far and big** ) Very powerful cameras ready with video recording facilities were scanning the sky. Coincidentally the “place or region “ a camera was looking had an event ( many million years back though ) of a black hole devouring a star.

<https://www.youtube.com/watch?v=O3Z5AS3TTS4>

<https://www.youtube.com/watch?v=x7ZX10UbMus>

**For small** ) Photographs of molecules and subsequently atoms

<https://www.youtube.com/watch?v=yqLlgIaz1L0>

<https://www.youtube.com/watch?v=ofp-OHlq6Wo>

<https://www.youtube.com/watch?v=oSCX78-8-q0>

<https://www.youtube.com/watch?v=RTLeWlqynW4>

<https://www.youtube.com/watch?v=J3xLuZnKhLY>

<https://www.youtube.com/watch?v=SMgi2j9Ks9k>

[https://www.youtube.com/watch?v=V0KjXsGRvoA&list=PLC3E0tG-9im\\_kuMwYIM7-NZR62VyWZ6rl](https://www.youtube.com/watch?v=V0KjXsGRvoA&list=PLC3E0tG-9im_kuMwYIM7-NZR62VyWZ6rl)

Enjoy

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### Spoon Feeding Series - Mirrors Prisms Lens Slabs - Optics

History of Optics can be seen at

<http://www.angelfire.com/ga/astronomyclubaugusta/History/hist.htm>

And <http://www.benr.net/technology/historyofoptics/>

History of Wave Optics

[http://lightandmatter.com/html\\_books/lm/ch32/ch32.html](http://lightandmatter.com/html_books/lm/ch32/ch32.html)

<https://www.boundless.com/physics/textbooks/boundless-physics-textbook/wave-optics-26/>

<http://www.ece.umd.edu/~taylor/optics.htm>

<http://light.ece.illinois.edu/ECE460/PDF/Brief%20history%20of%20optics.pdf>

[http://ocw.mit.edu/courses/mechanical-engineering/2-71-optics-spring-2009/video-lectures/lecture-1-course-organization-introduction-to-optics/MIT2\\_71S09\\_lec01.pdf](http://ocw.mit.edu/courses/mechanical-engineering/2-71-optics-spring-2009/video-lectures/lecture-1-course-organization-introduction-to-optics/MIT2_71S09_lec01.pdf)

**Two plane mirrors are inclined to each other such that a ray of light incident on the first mirror and parallel to the second is reflected from the second mirror parallel to the first mirror. Determine the angle between the two mirrors. Also determine the total deviation produced in the incident ray due to the two reflections.**

**Solution.** From fig.

$$3\theta = 180^\circ$$

$$\therefore \theta = 60^\circ$$

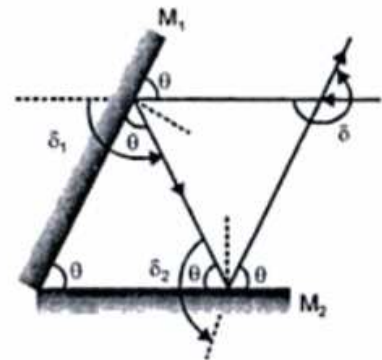
$$\delta_1 = 180^\circ - 2 \times 60^\circ = 60^\circ$$

$$\delta_2 = 180^\circ - 2 \times 60^\circ = 60^\circ$$

$$\therefore \text{Total deviation} = \delta_1 + \delta_2 = 120^\circ \text{ or } 240^\circ$$

**Alternatively :**

From fig.  $\delta = 180^\circ + \theta = 240^\circ$  anticlockwise or  $120^\circ$  clockwise.



**Two plane mirrors are inclined to each other at an angle  $\theta$ . A ray of light is reflected first at one mirror and then at the other. Find the total deviation of the ray.**

## Solution

**Solution.** Let  $\alpha$  = Angle of incidence for  $M_1$   
 $\beta$  = Angle of incidence for  $M_2$   
 $\delta_1$  = Deviation due to  $M_1$   
 $\delta_2$  = Deviation due to  $M_2$

From fig. 1.23,

$$\delta_1 = \pi - 2\alpha, \quad \delta_2 = \pi - 2\beta$$

Also ray is rotated in same sense *i.e.*, anticlockwise

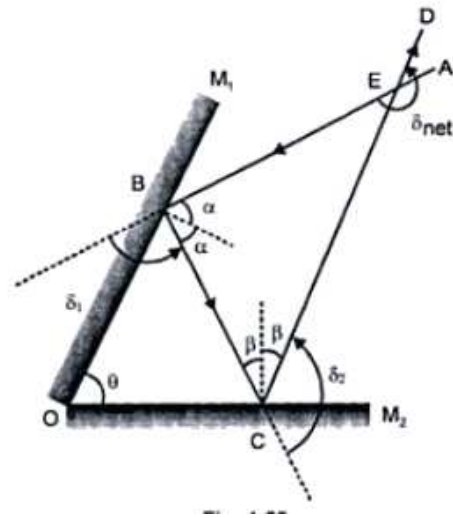
$$\therefore \delta_{net} = \text{Total deviation} = \delta_1 + \delta_2 \\ = 2\pi - 2(\alpha + \beta)$$

Now in  $\triangle OBC$ ,  $\angle OBC + \angle BCO + \angle COB = 180^\circ$

$$\text{or } (90^\circ - \alpha) + (90^\circ - \beta) + \theta = 180^\circ$$

$$\text{or } \alpha + \beta = \theta$$

$$\text{Hence, } \delta_{net} = 2\pi - 2\theta$$



**Alternative method :**

$$\alpha = \angle BEC + \angle CEA + \angle AED$$

$$\angle BEC = \angle AED \quad (\text{Vertically opposite angle})$$

$$= 180^\circ - 2(\alpha + \beta)$$

$$= 180^\circ - 2\theta$$

$$[\because \theta = \alpha + \beta]$$

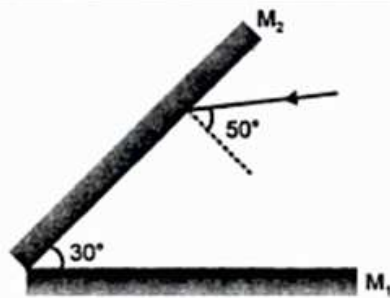
$$\angle CEA = 2\alpha + 2\beta$$

$$= 2(\alpha + \beta) = 2\theta$$

$$\therefore \delta = (180^\circ - 2\theta) + 2\theta + (180^\circ - 2\theta)$$

$$= 360^\circ - 2\theta$$

Calculate deviation suffered by incident ray in situation as shown in fig. after three successive reflections.



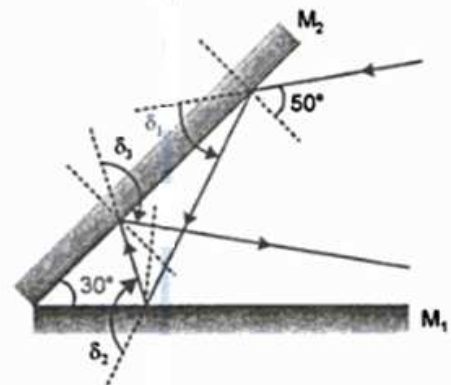
**Solution.** From fig. 1.25

$$\delta_1 = 180^\circ - 2 \times 50^\circ = 100^\circ$$

$$\delta_2 = 180^\circ - 2 \times 20^\circ = 140^\circ$$

$$\delta_3 = 180^\circ - 2 \times 10^\circ = 160^\circ$$

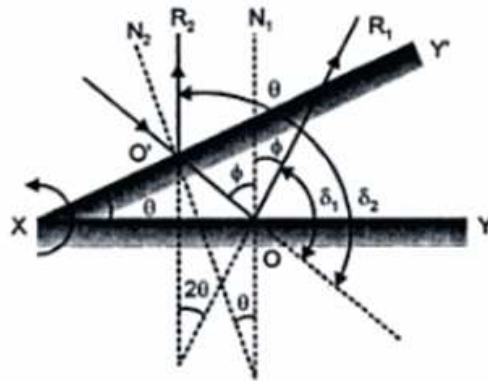
$$\therefore \delta = \text{Total deviation } (100^\circ + 140^\circ + 160^\circ) = 100^\circ \text{ or } 260^\circ$$



A ray of light is incident on a plane mirror at a certain point. The mirror is capable of rotation about an axis passing through X as shown in fig. 1.28. Prove that if the mirror turns through a certain angle, the reflected ray turns through double the angle.

**Solution.** Let XY be the position of mirror initially and mirror be rotated to a new position XY' through an angle  $\theta$  about an axis passing through X.





$$\delta_1 = \text{Deviation in first position of mirror} \\ = \pi - 2\phi$$

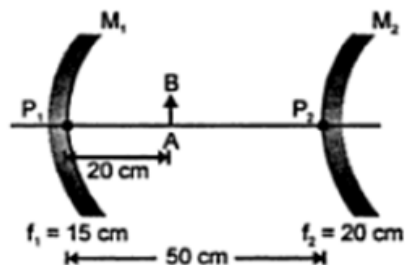
$$\delta_2 = \text{Deviation in second position of mirror} \\ = \pi - 2(\phi - \theta)$$

$$[\because \text{Angle of incidence in second position} = \phi - \theta]$$

$$\therefore \delta_1 - \delta_2 = \pi - 2(\phi - \theta) - (\pi - 2\phi) = 2\theta$$

Hence, reflected ray rotates twice the angle turned by the mirror.

**Find out position, size and nature of image of an object of height 2 mm kept between two mirrors in situation as shown in fig after two successive reflection considering first reflection at concave mirror and then at convex mirror.**



**Solution.** Consider reflection at concave mirror ( $M_1$ )

$$u = -20 \text{ cm}$$

$$f = -15 \text{ cm}$$

Using mirror equation we get,

$$v = \frac{u \cdot f}{u - f} = \frac{(-20)(-15)}{-20 - (-15)} = -60 \text{ cm}$$

$$m_1 = \frac{-v}{u} = \frac{-(-60)}{-20} = -3 \text{ (Inverted)}$$

$$\therefore A'B' = m_1(AB) = 6 \text{ mm}$$

The image ( $A'B'$ ) formed by concave mirror acts as object for convex mirror. Now, consider reflection at convex mirror ( $M_2$ ).

$$u = +10 \text{ cm}$$

$$f = +20 \text{ cm}$$

$$\therefore v = \frac{v \cdot f}{u - f} = \frac{(10)(20)}{10 - 20} = -20 \text{ cm}$$

$$m_2 = v = \frac{-v}{u} = \frac{-(-20)}{10} = 2 \text{ (Erect)}$$

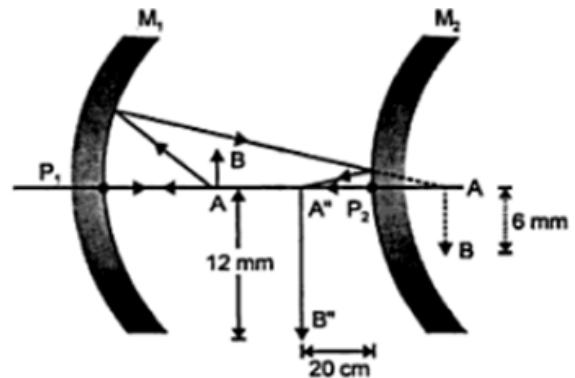
$$\therefore A^*B^* = m_2(AB) = 12 \text{ mm}$$

Hence, for final image  $A^*B^*$

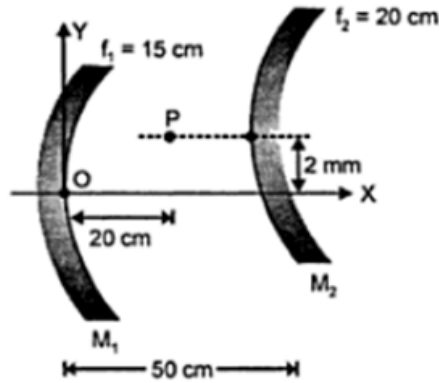
**Position :** 20 cm in front of convex mirror ( $M_2$ )

**Size :** 12 mm

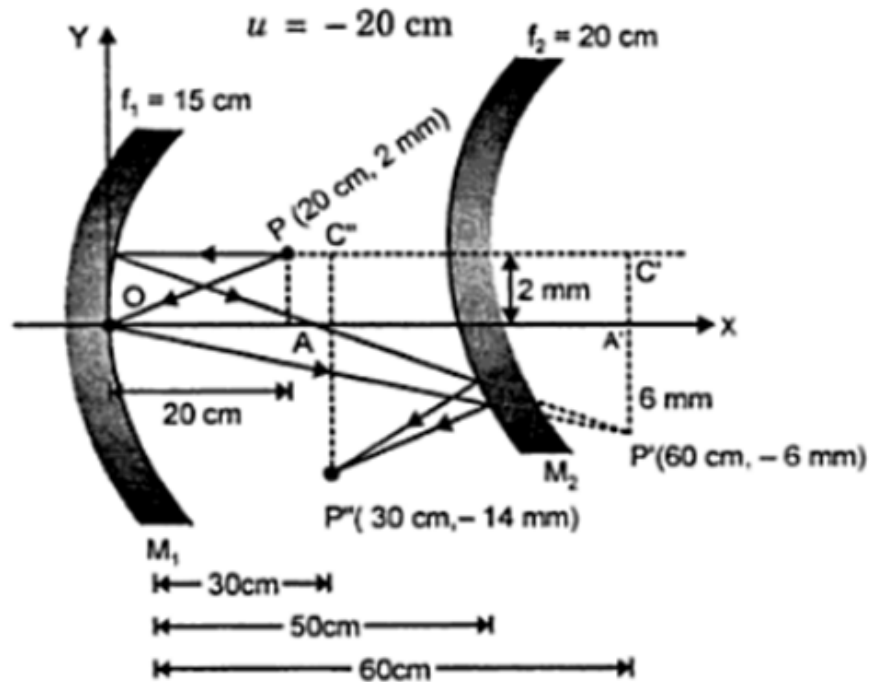
**Nature :** Real and inverted



**Find the co-ordinates of image of point object P formed after two successive reflection in situation as shown in fig. considering first reflection at concave mirror and then at convex mirror.**



**Solution.** For reflection at concave mirror  $M_1$



$$f_1 = -15 \text{ cm}$$

$$v_1 = \frac{u \cdot f_1}{u - f_1} = \frac{(-20)(-15)}{-20 + 15}$$

or

$$v_1 = -60 \text{ cm}$$

$$\text{Magnification } (m_1) = -\frac{v_1}{u} = -\frac{-60}{-20} = -3 \quad (\text{Inverted})$$

$\therefore$

$$A'P' = m_1 (AP) = 3 \times 2 = 6 \text{ mm}$$

For reflection at convex mirror  $M_2$

$$u = +10 \text{ cm}$$

$$f_2 = +20 \text{ cm}$$

$$v_2 = \frac{u \cdot f_2}{u - f_2} = \frac{(10)(20)}{10 - 20} = -20 \text{ cm}$$

$$\text{Magnification } m_2 = -\frac{v_2}{u} \Rightarrow -\frac{-20}{10} \Rightarrow 2$$

$\therefore$

$$C'P' = m_2 (C'P) = 2 \times 8 = 16 \text{ mm}$$

So, the co-ordinate of image of point object P (30 cm, -14 mm).

**A concave mirror forms on a screen a real image of thrice the linear dimensions of the object. Object and screen are moved until the image is twice the size of the object. If the shift of the object is 6 cm, find the shift of the screen and the focal length of the mirror.**

**Solution.** Initial magnification = 3

i.e.,  $\frac{v_1}{u_1} = 3$  or  $v_1 = 3u_1$

$\therefore f = \frac{u_1 v_1}{u_1 + v_1} = \frac{u_1(3u_1)}{u_1 + 3u_1} = \frac{3}{4} u_1$  ... (1)

In the second case,

i.e.,  $\frac{v_2}{u_2} = 2$  or  $v_2 = 2u_2$

$\therefore f = \frac{u_2 v_2}{u_2 + v_2} = \frac{u_2(2u_2)}{u_2 + 2u_2} = \frac{2}{3} u_2$  ... (2)

From (1) and (2),  $\frac{3}{4} u_1 = \frac{2}{3} u_2$

or  $u_2 = \frac{9}{8} u_1$

It is given that the shift of the object = 6 cm

$\therefore \frac{9}{8} u_1 - u_1 = 6$

or  $u_1 = 6 \times 8 = 48$  cm

and  $v_1 = 3 \times 48 = 144$  cm

$\therefore f = \frac{3}{4} u_1 = \frac{3}{4} \times 48 = 36$  cm

$\therefore u_2 = \frac{9}{8} u_1 = \frac{9}{8} \times 48 = 54$  cm

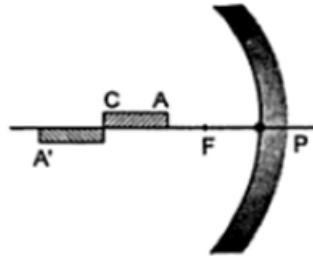
$v_2 = 2 u_2 = 2 \times 54 = 108$  cm

Thus, Shift of the screen =  $v_1 - v_2 = 144 - 108 = 36$  cm.

A thin rod of length  $\frac{f}{3}$  is placed along the principal axis of a concave mirror of focal length  $f$  such that its image just touches the rod. Calculate magnification.

**Solution.** Since image touches the rod, the rod must be placed with one end at centre of curvature.

**Case (I) :**



For A :

$$u = -\left(2f - \frac{f}{3}\right) = -\frac{5f}{3}$$

$$f = -f$$

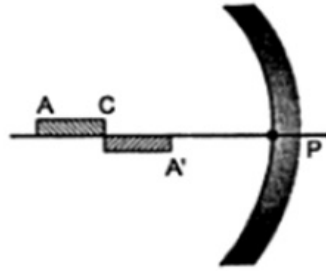
$$v = \frac{u \cdot f}{u - f} = \frac{\left(-\frac{5f}{3}\right)(-f)}{-\frac{5f}{3} - (-f)} = \frac{5f}{2}$$

$$m = \frac{\text{Length of Image}}{\text{Length of Object}}$$

$$= \frac{v_A - v_C}{u_A - u_C} = \frac{-\frac{5f}{2} - (-2f)}{-\frac{5f}{3} - (-2f)} = -\frac{3}{2}$$



**Case (II) :**



For A:

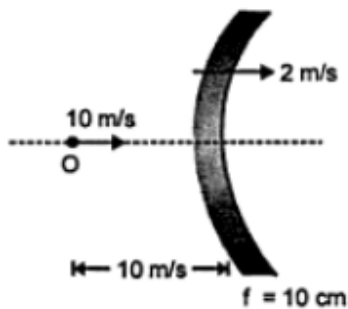
$$u = -\left(2f + \frac{f}{3}\right) = -\frac{7f}{3}$$

$$f = -f$$

$$v = \frac{u \cdot f}{u - f} = \frac{\left(-\frac{7f}{3}\right)(-f)}{-\frac{7f}{3} - (-f)} = -\frac{7f}{4}$$

$$m = \frac{v_A - v_C}{u_A - u_C} = \frac{-\frac{7f}{4} - (-2f)}{-\frac{7f}{3} - (-2f)} = -\frac{3}{4}$$

**Find the velocity of image in situation as shown**



**Solution.**

$$m = \frac{f}{f - u} = \frac{10}{10 - (-10)} = \frac{1}{2}$$

We know that,

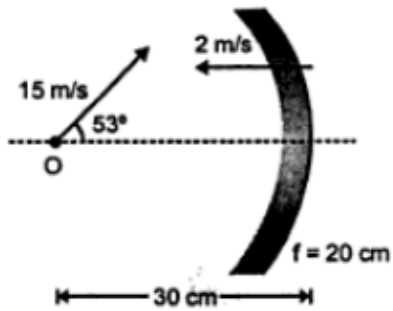
$$\vec{V}_{I/m} = -m^2 \vec{V}_{O/m}$$

$\therefore$

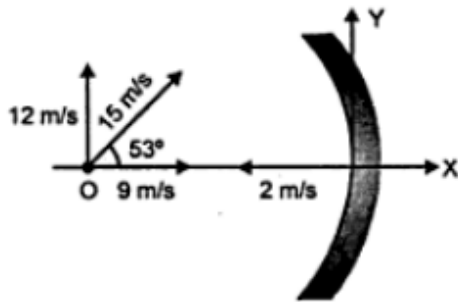
$$V_{I/m} = -\left(\frac{1}{2}\right)^2 \times 8 = -2 \text{ m/s}$$

Hence, image will appear to be moving with a speed of 2 m/s towards mirror.

**Find the velocity of image in situation as shown**



**Solution**



$$\vec{V}_O = \text{Velocity of object} = (9 \hat{i} + 12 \hat{j}) \text{ m/s}$$

$$\vec{V}_m = \text{Velocity of mirror} = -2 \hat{i} \text{ m/s}$$

$$m = \frac{f}{f-u} = \frac{-20}{-20 - (-30)} = -2$$

For velocity component parallel to optical axis

$$(\vec{V}_{I/m})_{\parallel} = -m^2 (\vec{V}_{O/m})_{\parallel}$$

$$(\vec{V}_{I/m})_{\parallel} = -(-2)^2 11 \hat{i} = -44 \hat{i} \text{ m/s}$$

For velocity component perpendicular to optical axis

$$\begin{aligned} (\vec{V}_{I/m})_{\perp} &= (\vec{V}_{O/m})_{\perp} \\ &= -(-2) 12 \hat{j} = -24 \hat{j} \text{ m/s} \end{aligned}$$

$\therefore$

$$\begin{aligned} \vec{V}_{I/m} &= \text{Velocity of image w. r. t. mirror} \\ &= (\vec{V}_{I/m})_{\parallel} + (\vec{V}_{I/m})_{\perp} \\ &= (-44 \hat{i} - 24 \hat{j}) \text{ m/s} \end{aligned}$$

Also,

$$\vec{V}_{I/m} = \vec{V}_I - \vec{V}_m$$

or

$$\begin{aligned}\vec{V}_I &= \vec{V}_{I/m} + \vec{V}_m \\ &= (-44 \hat{i} - 24 \hat{j}) - 2 \hat{i} \\ &= (-46 \hat{i} - 24 \hat{j}) \text{ m/s}\end{aligned}$$

A small block of mass  $m$  and a concave mirror of radius  $R$  fitted with a stand, lie on a smooth horizontal table with a separation  $d$  between them. The mirror together with its stand has a mass  $m$ . The block is pushed at  $t = 0$  towards the mirror so that it starts moving towards the mirror at a constant speed  $V$  and collides with it. The collision is perfectly elastic. Find the velocity of the image (a) at a time  $t < d/V$ , (b) at a time  $t > d/V$ .

Solution. (a)

$$t < \frac{d}{V}$$

$$u = -(d - Vt)$$

$$f = \frac{-R}{2}$$

We know that

$$\vec{V}_{I/m} = -m^2 \vec{V}_{O/m}$$

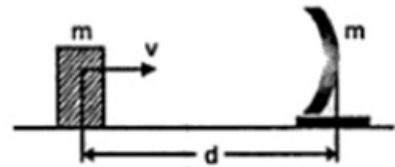
Here,

$$m = \frac{f}{f - u}$$

$$= \frac{-R/2}{-R/2 + (d - Vt)} = \frac{-R}{2(d - Vt) - R}$$

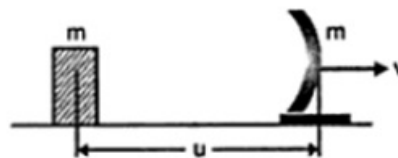
$\therefore$

$$\begin{aligned}v_1 = \text{velocity of image} &= - \left[ \frac{-R}{2(d - Vt) - R} \right]^2 v \\ &= \frac{R^2 V}{[2(d - Vt) - R]^2}\end{aligned}$$



$$(b) \quad t > \frac{d}{V}$$

Block will collide with mirror assembly after time  $T = \frac{d}{V}$ . From conservation of linear momentum, block and mirror assembly will exchange their momentum *i.e.*, block will stop and mirror starts moving with velocity  $V$ .



$$u = -V \left( t - \frac{d}{V} \right)$$

$$\text{Also,} \quad m = \frac{f}{f+u} = \frac{-R/2}{-R/2 + V \left( t - \frac{d}{V} \right)}$$

$$\text{We know that} \quad \vec{V}_{I/m} = -m^2 \vec{V}_{O/m}$$

$$\text{or} \quad \vec{V}_1 - \vec{V}_m = -m^2 (\vec{V}_{O/m})$$

Let us assume rightward direction as positive

$$v_1 - V = -m^2 (-V)$$

$$\text{or} \quad v_1 = (1 + m^2)V$$

$$\therefore v_1 = V \left[ 1 + \frac{R^2}{[2(Vt - d) - R^2]} \right]$$

**A gun of mass  $M$  fires a bullet of mass  $m$  with a horizontal speed  $V$ . The gun is fitted with a concave mirror of focal length  $f$  facing towards the receding bullet. Find the speed of separation of the bullet and the image just after the gun was fired.**

**Solution.** From conservation of linear momentum, we get

$$MV' = mV$$

$$\therefore V' = \frac{mV}{M}$$

Just after the bullet is fired,

$$u \rightarrow 0$$

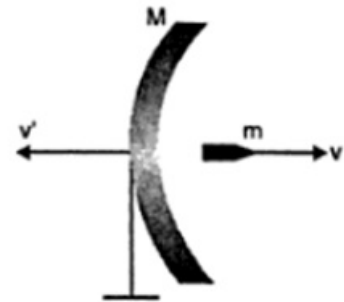
$$\therefore m = \text{magnification} = \frac{f}{f-u} = 1$$

We know that  $\vec{V}_{I/m} = -m^2 \vec{V}_{O/m}$

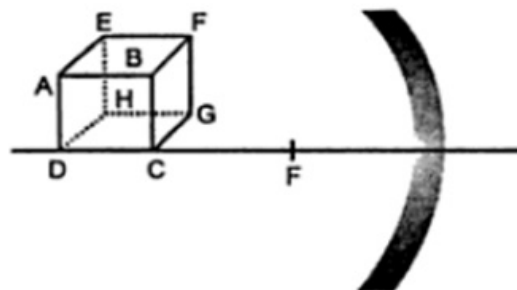
$$\begin{aligned} \vec{V}_{O/m} &= \vec{V}_O - \vec{V}_m \\ &= V - (-V') = V \left(1 + \frac{m}{M}\right) \end{aligned}$$

Now,  $\vec{V}_{I/m} = -\vec{V}_{O/m}$

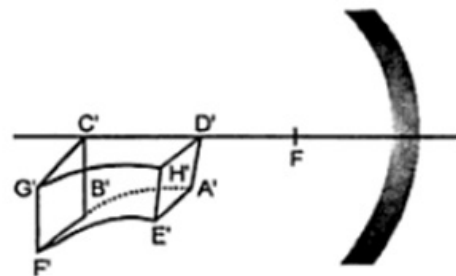
Hence, speed of separation between bullet and its image will be  $2V \left(1 + \frac{m}{M}\right)$



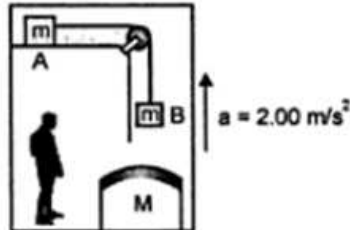
**A cube is placed in front of concave mirror in situation as shown. Draw the approximate image of cube.**



**Solution.** The cube can be imagined to be a structure made of 8 sticks. Sticks parallel to principal axis will have curve image except stick DC which is lying along the principal axis. This is because magnification is not a linear function of object distance  $\left(m = \frac{f}{f-u}\right)$ . Further, images of all sticks perpendicular to principal axis will be straight. With these consideration, the image of cube will look like as shown in fig.



Consider the situation shown in fig The elevator is going up with an acceleration of  $2.00 \text{ m/s}^2$  and the focal length of the mirror is  $12.0 \text{ cm}$ . All the surfaces are smooth and the pulley is light. The mass-pulley system is released from rest (with respect to the elevator) at  $t = 0$  when the distance of B from the mirror is  $42.0 \text{ cm}$ . Find the distance between the image of the block B and the mirror at  $t = 0.200 \text{ s}$ . Take  $g = 10 \text{ m/s}^2$ .



**Solution.** Let us assume that the acceleration of blocks A and B is 'a' with respect to lift and  $a_L$  is acceleration of lift.

Consider block B  
System : Block (B)

Frame of reference : Lift

$$mg + ma_L - T = ma \quad \dots(1)$$

Now, consider block A

System : Block (A)

Frame of reference : Lift

$$N = mg + ma_L \quad \dots(2)$$

$$T = ma \quad \dots(3)$$

On adding eq. (1) and eq. (3), we get

$$a = \frac{g + a_L}{2} = \frac{10 + 2}{2} = 6 \text{ m/s}^2$$

$$\therefore \text{Distance fallen by block (B)} = \frac{1}{2}at^2$$

$$= \frac{1}{2} \times 6 \times (0.2)^2$$

$$= 0.12 \text{ m or } 12 \text{ cm}$$

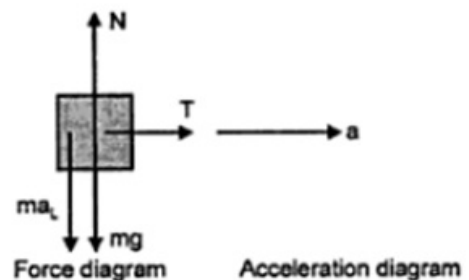
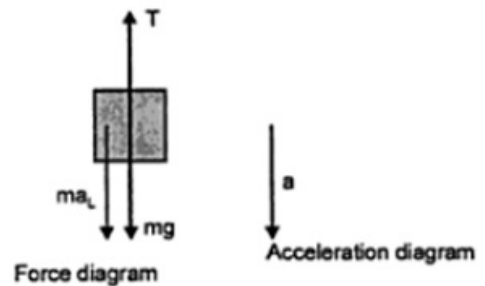
Now, consider reflection at convex mirror

$$u = -(42 - 12) = -40 \text{ cm}$$

$$f = +12 \text{ cm}$$

$$\therefore v = \frac{uf}{u-f} = \frac{(-40)(12)}{-40-12} = 8.75 \text{ cm}$$

Therefore, the distance between the image of block (B) and mirror is  $8.57 \text{ cm}$ .



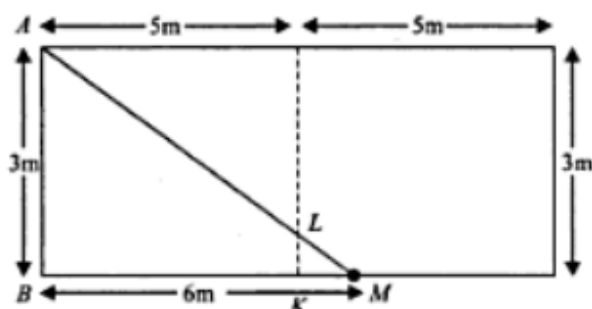


A room is 3m high and 5m long. A man is standing in front of one of the walls 1m from the wall. A mirror is to be installed on the wall. Find the height (minimum) of the mirror so that complete image of the wall behind him is seen.

- (a) 1.5 m                      (b) 1 m  
 (c) 2 m                         (d) 0.5 m

**Solution** (d)  $\triangle ABM$  and  $\triangle KLM$  are similar

$$\therefore \frac{KL}{AB} = \frac{KM}{MB} \Rightarrow KL = \frac{3 \times 1}{6} = \frac{1}{2} \text{ m}$$



A beam is incident parallel on the prism shown in Fig. Find the angle between emerging rays.

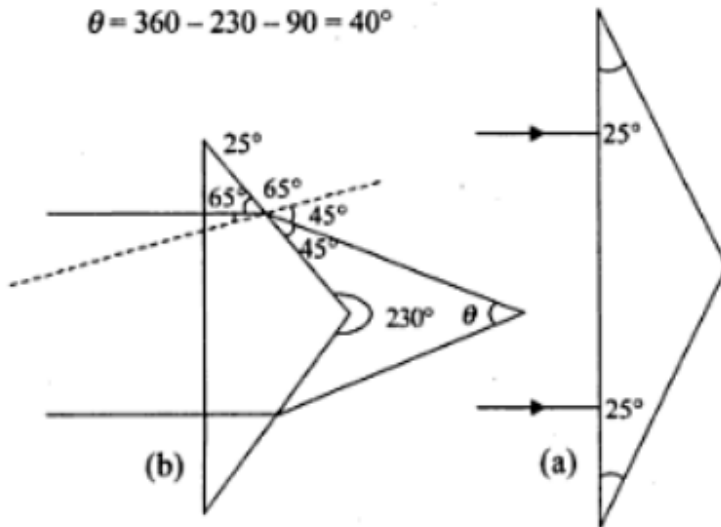
$$\mu_{\text{prism}} = 1.66.$$

- (a)  $180^\circ$                       (b)  $120^\circ$   
 (c)  $135^\circ$                       (d)  $40^\circ$

**Solution** (d)  $\sin C = \frac{1}{1.66}$  or  $C = 37^\circ$

$$\begin{aligned} \therefore \text{ray is refracted out} \\ \sin r &= \sin 25 (1.66) \\ &= 0.4226 (1.66) = .706 \\ \therefore r &= 45^\circ \end{aligned}$$

$$\theta = 360 - 230 - 90 = 40^\circ$$



A slide projector is to project a (35 mm × 23 mm) slide on a 2m × 2m screen. Find the focal length of the lens used if screen is 10 m away from the lens.

- (a) 15.1 cm                      (b) 17.2 cm  
(c) 16.1 cm                      (d) 18.2 cm

**Solution** (b)  $M = \frac{200}{3.5} = \frac{10}{u}$  or  $u = \frac{7}{40}$  m; using

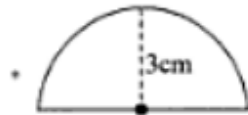
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \quad \text{i.e.,}$$

$$\frac{1}{f} = \frac{1}{10} + \frac{40}{7} \text{ or } f = 17.2 \text{ cm}$$

A paper weight is hemispherical with radius 3 cm. It is kept on a printed page, the printed letter will appear at a height \_\_\_\_\_ cm from the centre of the hemisphere when viewed vertically.

- (a) 0                                      (b) 1 cm  
(c) 2 cm                                (d) 1.21 cm

**Solution** (a)  $\frac{1.5}{v} - \frac{1}{-3} = \frac{1.5-1}{-3}$   $v = -3$  cm.



A 1 m long rod is half dipped in a swimming pool. The sunlight is incident at  $45^\circ$  on the rod. Find the length of the shadow on the bed of swimming pool.

- (a) 73 cm                      (b) 78.25 cm  
 (c) 74.17 cm                  (d) 81.5 cm

**Solution** (d) Length of shadow =  $x + 0.5$  m

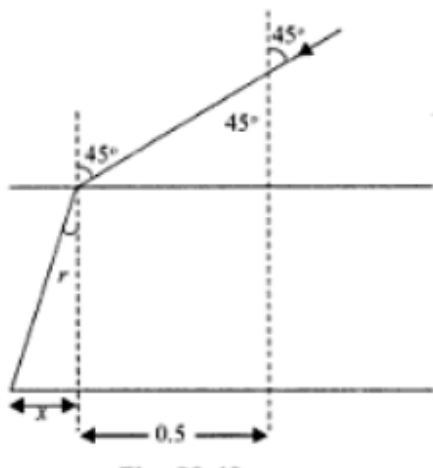
$$x = 0.5 \tan r$$

$$\sin r = .707 \times \frac{3}{4} \text{ or}$$

$$r = 32^\circ 12'$$

$$l = 50 (\tan 32^\circ 12') + 50 \text{ cm}$$

$$= 50 (.6297) + 50 = 81.5 \text{ cm}$$



A diverging lens of  $f = 20$  cm and a converging mirror  $f = 10$  cm are placed 5 cm apart coaxially. Where shall an object be placed so that object and its real image coincide?

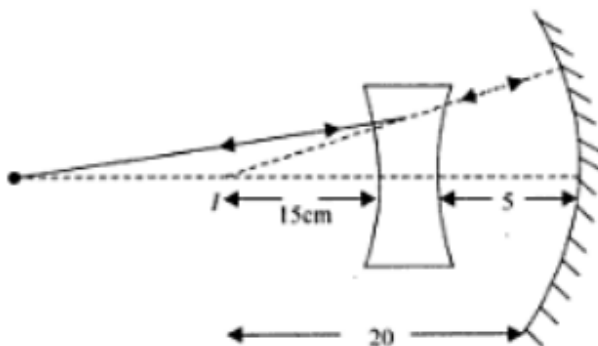
- (a) 60 cm away from lens (b) 15 cm away from lens  
(c) 20 cm away from lens (d) 45 cm away from lens

**Solution** (a) If the rays are to retrace the path, light ray must fall normal on the mirror. Hence  $f$  should be 20 cm from mirror and 15 cm from lens.

$$\frac{1}{-15} - \frac{1}{x} = \frac{1}{-20}$$

or  $\frac{1}{x} = \frac{1}{-15} + \frac{1}{20} = \frac{1}{-60}$

$x = -60$  cm i.e. 60 cm away from lens.



A fish looking up through the water sees outside world contained in a circular horizon. The refractive index of

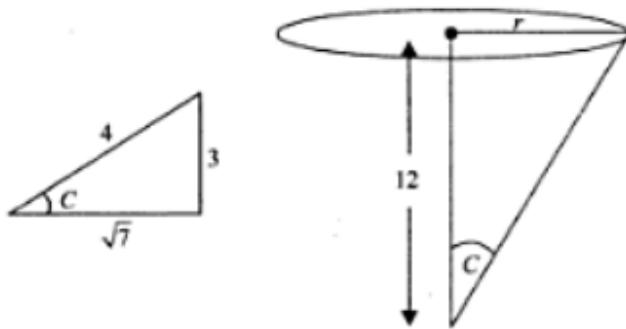
water is  $\frac{4}{3}$  and the fish is 12 cm below the surface. The radius of this circle in cm is

[AIEEE 2005]

- (a)  $36\sqrt{7}$                       (b)  $\frac{36}{\sqrt{7}}$   
 (c)  $36\sqrt{5}$                       (d)  $4\sqrt{5}$

**Solution** (b)  $\sin C = \frac{3}{4}$  and  $\tan C = \frac{3}{\sqrt{7}} = \frac{r}{12}$

or  $r = \frac{36}{\sqrt{7}}$  cm.



Two point white dots are 1 mm apart on a black paper. They are viewed by eye of pupil of diameter 3 mm. Approximately what is the maximum distance upto which these dots can be resolved by the eye.

[AIEEE 2005]

- (a) 5 m                              (b) 6 m  
 (c) 1 m                              (d) 4 m

**Solution** (a)  $\frac{1.22\lambda}{3\text{mm}} = \frac{1\text{mm}}{d}$

or  $d = \frac{3 \times 10^{-6}}{1.22 \times 5 \times 10^{-7}} = 5 \text{ m}$

A thin glass ( $\mu = 1.5$ ) lens has power  $-5D$  in air. Its power in a medium of refractive index 1.6 will be

- (a)  $\frac{5}{8} D$                       (b)  $\frac{25}{8} D$   
(c)  $-\frac{5}{8} D$                       (d)  $-\frac{25}{8} D$

**Solution** (a)  $f_m = \frac{(\mu_l - 1)}{\left(\frac{\mu_l - 1}{\mu_m}\right)} f_a = \frac{\frac{1}{2} \times (-20)}{\frac{3}{2} - 1}$   
 $= \frac{\frac{1}{2} \times 20 \times 3.2}{.2} = 160 \text{ cm}$

The angular resolution of a telescope of 10 cm diameter at a wavelength of  $5000 \text{ \AA}$  is of the order of

[CBSE 2005]

- (a)  $10^6 \text{ rad}$                       (b)  $10^{-2} \text{ rad}$   
(c)  $10^{-4} \text{ rad}$                       (d)  $10^{-6} \text{ rad}$

**Solution** (d)  $\frac{\lambda}{d} = \frac{5 \times 10^{-7}}{10^{-1}} = 10^{-6}$



A tank of height 33.25 cm is completely filled with liquid ( $\mu = 1.33$ ). An object is placed at the bottom of the tank on the axis of concave mirror as shown in Fig.

Image of the object is formed at 25 cm below the surface of the liquid. Focal length of the mirror is

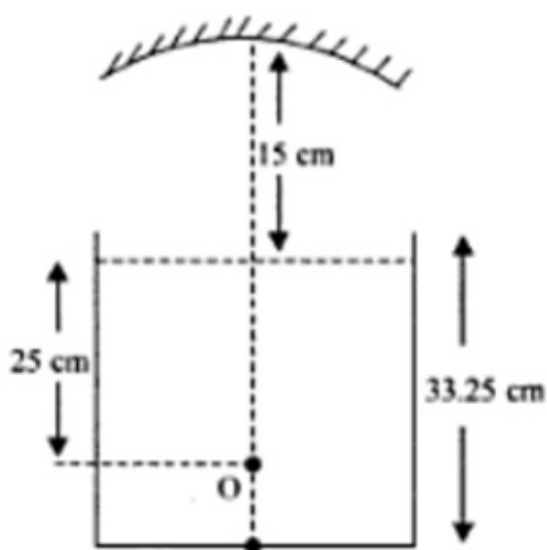
[IIT 2005]

- (a) 10 cm                      (b) 15 cm  
(c) 20 cm                      (d) 25 cm

**Solution** (c) Apparent depth =  $\frac{33.25}{1.33} = 25$  cm

when object is at  $2f$  image is formed at  $2f$ .

$\therefore 2f = 15 + 25 = 40$  cm and  $f = 20$  cm



A telescope has an objective lens of focal length 200 cm and an eyepiece with focal length 2 cm. It is used to see a 50 m tall building at a distance of 2 km. What is the height of the image of the building formed by the objective lens ?

[AIIMS 2005]

- (a) 5 cm                      (b) 10 cm  
(c) 1 cm                      (d) 2 cm

**Solution** (a)  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

or  $\frac{1}{v} = \frac{1}{200} - \frac{1}{2000 \times 100}$

or  $v = 200$  cm.

Using  $\frac{v}{u} = \frac{I}{O}$

$$I = \frac{2}{2000} \times 50 = \frac{1}{20} \text{ m or 5 cm.}$$

*A.* Resolving power of a telescope is more if the diameter of the objective lens is more.

*R.* Objective lens of large diameter collects more light.

[AIIMS 2005]

- (a) both *A* and *R* are correct and *R* is correct explanation of *A*  
(b) *A* and *R* both are correct but *R* is not correct explanation of *A*  
(c) *A* is true but *R* is false  
(d) both *A* and *R* are false

**Solution** (b)  $RP = \frac{D}{1.22\lambda} \therefore A$  is correct. Though *R* is

correct but for larger resolution objects making small angle be distinguished or very close objects should be distinguished.

A convex lens of focal length 80 cm and a concave lens of focal length 50 cm are combined together. What will be their resultant power?

[BHU 2005]

- (a) 0.65 D                      (b) -0.65 D  
(c) 0.75 D                      (d) -0.75 D

**Solution** (d)  $P_1 = 1.25 D$  and  $P_2 = -2 D$

$$P_{\text{net}} = P_1 + P_2 = -0.75 D$$

A plane slab is kept over various colour letters. The letter which appears least raised is

[BHU 2005]

- (a) Red                              (b) Green  
(c) Violet                              (d) Blue

**Solution** (a)  $\mu = A + \frac{B}{\lambda^2}$

$$\therefore \mu \text{ is minimum for Red and App. depth} = \frac{\text{Real depth}}{\mu}$$

A convex lens forms the full image of the object on a screen. If half of the lens is covered with an opaque object then

[BHU 2005]

- (a) the image disappears  
(b) half the image is seen  
(c) full image of same intensity is seen  
(d) full image of decreased intensity is seen.

**Solution** (d)

Time taken by light to pass through 4 mm thick glass slab of refractive index 1.5 will be

- (a)  $8 \times 10^{-11}$  s                      (b)  $2 \times 10^{-11}$  s  
(c)  $8 \times 10^{-8}$  s                      (d)  $2 \times 10^{-8}$  s

[BHU 2005]

**Solution** (b)  $t = \frac{4 \times 10^{-3} \times 1.5}{3 \times 10^8} = 2 \times 10^{-11}$  s

A lens acts as a converging lens in air and diverging lens in water. The refractive index of the lens is

[BHU 2005]

- (a) = 1                                      (b) < 1.33  
(c) > 1.33                                (d) < 1

**Solution** (b)

A light passing through air has wavelength  $6000 \text{ \AA}$ . Wavelength when same ray passes through a glass slab of refractive index 1.5 is

[BHU 2005]

- (a)  $4000 \text{ \AA}$                               (b)  $2000 \text{ \AA}$   
(c)  $8000 \text{ \AA}$                               (d)  $1200 \text{ \AA}$

**Solution** (a)  $\lambda' = \frac{\lambda}{\mu} = \frac{6000}{1.5} = 4000 \text{ \AA}$

A professor reads a greeting card on his 50th birthday with + 2.5 D glasses keeping the card 25 cm away. 10 years later he reads the greeting card with same glass keeping the card 50 cm away. What power glasses should he wear now?

- (a) 2D (b) 0.5 D  
(c) 2.25 D (d) 4.5 D

**Solution** (d)  $\frac{1}{f'} = \frac{1}{25} - \frac{1}{50} = \frac{1}{50}$  or

$$P = 2 \text{ D}$$

$$P_{\text{net}} = 2.5 + 2 = 4.5 \text{ D}$$

A simple microscope is rated  $5\times$  for a normal relaxed eye. What will be its magnifying power for a farsighted man whose near point is 40 cm ?

- (a)  $5\times$  (b)  $3\times$   
(c)  $8\times$  (d)  $13\times$

**Solution** (c) For a relaxed eye  $M = \frac{D}{f} \therefore f = 5 \text{ cm}$

$$\text{In case II } M = \frac{40}{5} = 8$$

A particle is moving at a constant speed  $v$  from a large distance towards a concave mirror of radius  $R$  along the principal axis. Find the speed of the image as a function of the distance  $x$  of the particle from the mirror.

**Solution** Let  $y$  represent the image distance and  $x$  the object distance from the mirror. Then

$$\frac{1}{y} + \frac{1}{-x} = -\frac{2}{R}$$

or  $\frac{1}{y} = \frac{-2}{R} + \frac{1}{x} = \frac{-2x + R}{Rx}$

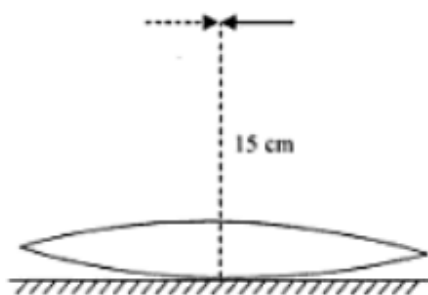
or  $y = \frac{Rx}{R-2x}$  ..... (1)

Differentiating equation. (1)

$$\frac{dy}{dt} = \frac{R \frac{dx}{dt}}{(R-2x)} + \frac{2Rx \frac{dx}{dt}}{(R-2x)^2}$$

or  $\frac{dy}{dt} = \frac{[R(R-2x) + 2Rx] \frac{dx}{dt}}{(R-2x)^2} = \frac{R^2 v}{(R-2x)^2}$

When an equiconvex lens ( $\mu_{\text{lens}} = 1.5$ ) is placed over a plane mirror as shown, then object needle and its image coincide at 15 cm. When a liquid of refractive index  $\mu$  is filled in the gap between mirror and lens, the object needle and its image coincide at 40 cm. Find the ref. index  $\mu$  of the liquid.



(a)  $\frac{8}{5}$

(b)  $\frac{5}{8}$

(c)  $\frac{13}{8}$

(d)  $\frac{5}{13}$

(e)  $\frac{13}{5}$



**Solution** (c) From case (i) we get  $f_{\text{lens}} = 15 \text{ cm}$

since  $\mu_{\text{lens}} = 1.5 \quad \therefore f_{\text{lens}} = R_{\text{lens}} = 15 \text{ cm}$

In case (ii) focal length of the combination = 40 cm  
[combination of lens + combination of (planoconcave) liquid lens].

$$\frac{1}{40} = \frac{1}{15} + \frac{1}{f_{\text{liquid lens}}}$$

$$\text{or } \frac{1}{f_{\text{liq. lens}}} = -\frac{1}{15} + \frac{1}{40}$$

$$f_{\text{liq. lens}} = -24 \text{ cm}$$

$$\frac{1}{f_{\text{liq. lens}}} = (\mu - 1) \left[ \frac{1}{R} - \frac{1}{\infty} \right] \text{ or } \frac{1}{-24}$$

$$= (\mu - 1) \left[ \frac{1}{-15} \right] \text{ or}$$

$$\mu = \frac{13}{8}$$

A particle executes SHM of amplitude 1 cm along principal axis of a convex lens of focal length 12 cm. The mean position of oscillation is at 20 cm from the lens. Find the amplitude of oscillation of the image of the particle.

- (a) 2 cm                      (b) 2.6 cm  
(c) 1 cm                      (d) 2.3 cm

**Solution** (d)  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v_1} = \frac{1}{12} - \frac{1}{21} = \frac{9}{21 \times 12}$$

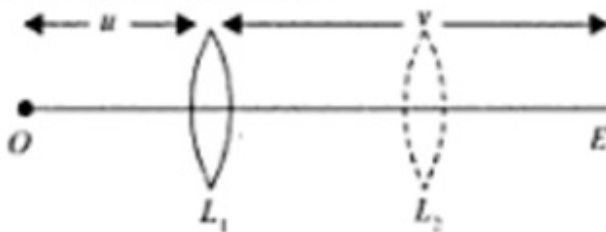
or  $v_1 = 28 \text{ cm}$

$$\frac{1}{v_2} = \frac{1}{12} - \frac{1}{19} = \frac{7}{19 \times 12}$$

or  $v_2 = \frac{19 \times 12}{7} = \frac{228}{7} = 32.6 \text{ cm}$

$$\Delta x = 32.6 - 28 = 4.6 \text{ cm amplitude} = \frac{\Delta x}{2} = 2.3 \text{ cm}$$

Object and screen are fixed 90 cm apart. The lens is displaced. Two sharp images are obtained when lens is at  $L_1$  and  $L_2$  respectively such that  $I_1 = 4I_2$ . Find the focal length of the lens.



(a) 18 cm

(b) 15 cm

(c) 16 cm

(d) 20 cm

**Solution** (d)  $O = \sqrt{I_1 I_2} = 2 I_2 = \frac{I_1}{2}$

$$\frac{v}{u} = 2$$

$$v = 2u$$

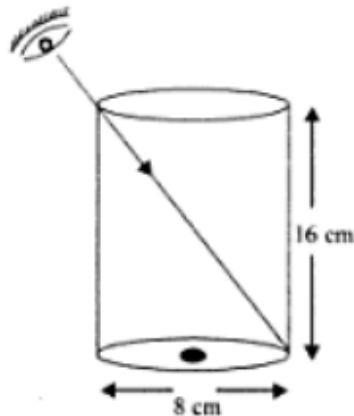
$$v + u = 90 \text{ cm}$$

$$3u = 90 \text{ cm or } u = 30 \text{ cm}$$

$$\frac{1}{f} + \frac{1}{v} + \frac{1}{u} = \frac{1}{60} + \frac{1}{30} = \frac{3}{60}$$

or  $f = 20 \text{ cm}$

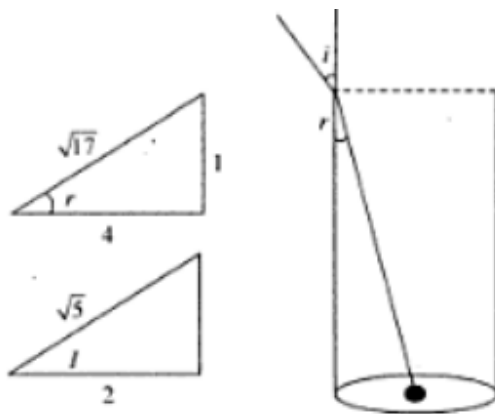
You are looking at the rim from the vertical side to see the opposite edge at the bottom of a 16 cm high and 8 cm diameter vessel. A friend fills it with a liquid of refractive index  $\mu$  so that a coin placed at the centre becomes visible. What is the value of  $\mu$ ?



- (a) 1.34                      (b) 1.6  
(c) 1.73                      (d) 1.84

**Solution** (d)  $\tan i = \frac{1}{2} \mu = \frac{\sin i}{\sin r} = \frac{1/\sqrt{5}}{1/\sqrt{17}}$

$$= \sqrt{\frac{17}{5}} = \sqrt{3.4} = 1.84$$



A very large array radio telescope of NASA has a special arrangement of reflectors which gives an effective aperture of 36 km diameter. What is the limit of resolution? ( $\lambda = 10^{-2}$  m)

**Solution**

$$\begin{aligned} \sin \theta \approx \theta &= \frac{1.22 \times 10^{-2}}{36 \times 10^3} \\ &= 3.4 \times 10^{-7} \text{ rad.} \end{aligned}$$

A person cannot see beyond 5 m. How can his vision be corrected?

**Solution**

His far point is now 5 m. It should be made  $\infty$ . Object at infinity should give a virtual image at 5 m. i.e., for him the

object should appear to be at 5 m. Therefore,  $\frac{-1}{5} - \frac{1}{-\infty} = \frac{1}{f}$ .

A concave lens is required and it should have a focal length = -5 m.

$$\Rightarrow \text{i.e., } P_{\text{lens}} = -0.2 \text{ D}$$

The near point and far point of a person are 40 cm and 400 cm, respectively. He wants to read a book kept at

25 cm. Find the focal length of the lens required and the distance upto which objects can be viewed.

### **Solution**

Put  $u = -25$ ,

$v = -40$

$$\frac{1}{f_{\text{lens}}} = \frac{1}{-40} - \frac{1}{-25} \Rightarrow f_{\text{lens}} = +\frac{200}{3} \text{ cm}$$

With this lens at what distance an object will give virtual image at his far point of 400 cm?

$$\frac{1}{-400} - \frac{1}{u} = \frac{1}{+200/3}$$

$$\Rightarrow u = -\frac{400}{7} \text{ cm}$$

What focal length should the reading spectacles have for a person whose D value is 40 cm?

### **Solution**

D value > 25 cm

$\Rightarrow$  hypermetropia

$\therefore$  at  $u = -40$  cm, his  $v = +2.5$  cm

$\Rightarrow$  his  $F_{\text{min}}$  is given by

$$\frac{1}{2.5} - \frac{1}{-40} = \frac{1}{f_{\text{min}}} \Rightarrow f_{\text{min}} = \frac{40}{17} \text{ cm}$$




= 2.35 cm as against  $\frac{25}{11} = 2.27$  cm for a normal person.  $\Rightarrow$  This requires a convex lens so that

$$\frac{1}{40/17} + \frac{1}{f_{\text{lens}}} = \frac{1}{25/11} \Rightarrow f_{\text{lens}} = +\frac{200}{3} \text{ cm.}$$

The same question can be alternatively posed by asking what kind of lens should give a virtual image at 40 cm, for an object at 25 cm?

$$\frac{1}{-40} - \frac{1}{-25} = \frac{1}{f_{\text{lens}}} \Rightarrow f_{\text{lens}} = +\frac{200}{3} \text{ cm}$$


### Modified lenses

(i) A lens  of focal length  $f$  is cut  into two halves and kept in contact as shown: 

The focal length now has the same value,  $f$ .

(ii) The focal length of each half is not  $\frac{f}{2}$  but  $2f$  because each part has a focal length  $f'$  given by

$$\frac{1}{f'} = (n-1) \left( \frac{1}{R} - \frac{1}{\infty} \right) = \frac{(n-1)}{R} = \frac{1}{2f}$$


(iii) If kept in contact, but the other way  the focal


length is  $f$  because  $\frac{1}{f'} = \frac{1}{2f} + \frac{1}{2f} = \frac{1}{f}$


(iv) If kept as shown  the focal length is again  $f$ .



(v) If cut as shown and kept , the focal length  $f$  is that of

the uncut lens; the focal length of each half  is  $f$  and the intensity of image due to only one part will be half of the intensity of the image of original lens, because the area exposed to incident rays is one half of that of the full lens.

(vi) If the two halves are kept with a small gap  the focal length is  $f$ , but the contrast of the image will be reduced.

(vii) If the two halves are kept as shown,  the resultant focal length will be  $\left(\frac{1}{f'} = \frac{1}{f} + \frac{1}{f}\right) \Rightarrow f' = \frac{f}{2}$ .

A convex lens ( $n = 1.5$ ) has radii 40 cm and 60 cm. The 40 cm surface is silvered. What is the focal length of this lens?

**Solution**

$$\frac{1}{f_{\text{lens}}} = (1.5 - 1) \left( \frac{1}{+60} - \frac{1}{-40} \right) \Rightarrow \frac{1}{f_{\text{lens}}} = +\frac{5}{240}$$

$$\frac{1}{f_{\text{mir}}} = \frac{1}{-40/2} = -\frac{1}{20}$$

$$\therefore \frac{1}{f_{\text{silv.lens}}} = \frac{1}{f_{\text{mir}}} - \frac{2}{f_{\text{lens}}} = -\frac{1}{20} - \frac{10}{240} = -\frac{22}{240}$$

$$\Rightarrow f_{\text{silv.lens}} = -\frac{120}{11} \text{ cm}$$

$$\Rightarrow \text{behaves like a concave mirror of } f = -\frac{120}{11} \text{ cm}$$

The convex side of a plano-convex lens ( $n = 1.5$ ) with  $R = 60$  cm is silvered. An object 2 cm tall is located 25 cm in front of this. Describe image details.

### **Solution**

Obviously, if the silvered side faces this object, it does not produce an image.

$$\frac{1}{f_{\text{lens}}} = (1.5 - 1) \left( \frac{1}{\infty} - \frac{1}{-60} \right) = \frac{1}{120}$$

$$\frac{1}{f_{\text{mirr}}} = \frac{1}{-60/2} = -\frac{1}{30}$$

$$\therefore \frac{1}{f_{\text{sys}}} = -\frac{1}{30} - \frac{2}{120} = -\frac{1}{20}$$

$$\Rightarrow f_{\text{sys}} = -20 \text{ cm}$$

$\therefore$  The system behaves like a concave mirror with  $f = -20 \text{ cm}$

$$\therefore \frac{1}{v} + \frac{1}{-25} = \frac{1}{-20}$$

$$v = -100 \text{ cm}$$

negative sign for a mirror indicates real image.

$$m = -\frac{v}{u} = \frac{+100}{-25} = -4$$

The image is inverted, with height = 8 cm

A convex lens of focal length 40 cm is placed 20 cm away from another convex lens of focal length 50 cm. An object 2 cm tall is 60 cm in front of the first lens. Determine its image characteristics.

### ***Solution***

Refraction by the first lens:

$$\frac{1}{v'} - \frac{1}{-60} = \frac{1}{+40} \Rightarrow v' = +120 \text{ cm}$$

$$m_1 = \frac{+120}{-60} = -2$$

Refraction by the second lens:

$$u = 120 - 20 = +100$$

$$\frac{1}{v} - \frac{1}{+100} = \frac{1}{+50} \Rightarrow v = +\frac{100}{3} \text{ cm}$$

$$m_2 = \frac{+100 / 3}{+100} = +\frac{1}{3},$$

Total magnification is  $m_1 m_2 = -\frac{2}{3}$

Final image is  $\frac{100}{3}$  cm to the right of the second lens,  
is real and is inverted,  $\frac{4}{3}$  cm tall.

A convex lens of focal length 40 cm and a concave lens of focal length 30 cm are separated by a gap of 20 cm; object is 1 cm tall at 20 cm in front of the first (convex) lens. Describe image details.

### **Solution**

At lens 1:

$$\frac{1}{v'} - \frac{1}{-20} = \frac{1}{+40}$$

$$\Rightarrow v' = -40 \text{ cm}$$

$$m_1 = \frac{-40}{-20} = +2$$

At lens 2:

$$u = -40 - 20 = -60$$

$$\frac{1}{v} - \frac{1}{-60} = \frac{1}{-30}$$

$$\Rightarrow v = -20 \text{ cm}$$

$$m_2 = \frac{-20}{-60} = +\frac{1}{3}$$

$$m_1 m_2 = +\frac{2}{3}$$

Image: 20 cm to left of the second lens  $\Rightarrow$  at the optic centre of the first lens, virtual, erect and  $\frac{2}{3}$  cm tall.

Two lenses one of which is crown glass with  $\omega_1 = \frac{1}{30}$  and the other flint glass with  $\omega_2 = \frac{1}{20}$ . How can an achromatic combination, (No dispersion), of focal length 40 cm, be formed?

### **Solution**

Let  $n_1, R_{11}, R_{12}, \omega_1$  and  $n_2, R_{21}, R_{22}, \omega_2$  represent the parameters of three lenses.

$$\frac{1}{f_1} = (n_1 - 1) \left( \frac{1}{R_{11}} - \frac{1}{R_{12}} \right) = (n_1 - 1)K_1$$

$$\frac{1}{f_2} = (n_2 - 1)K_2$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = (n_1 - 1)K_1 + (n_2 - 1)K_2$$

For no dispersion,  $\frac{df}{d\lambda} = 0$

$$\Rightarrow -\frac{1}{f^2} \cdot \frac{df}{d\lambda} = 0 = K_1 \frac{dn_1}{d\lambda} + K_2 \frac{dn_2}{d\lambda}$$

$$0 = \frac{1}{f_1(n_1 - 1)} \cdot \frac{dn_1}{d\lambda} + \frac{1}{f_2(n_2 - 1)} \frac{dn_2}{d\lambda}$$

$$\Rightarrow \frac{dn_1}{f_1(n_1 - 1)} + \frac{dn_2}{f_2(n_2 - 1)} = 0$$

But  $\omega_1 = \frac{dn_1}{n_1 - 1}$

and  $\omega_2 = \frac{dn_2}{(n_2 - 1)}$

$$\therefore \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

$$\Rightarrow \frac{1}{30f_1} + \frac{1}{20f_2} = 0 \quad \text{eqn ( 1 )}$$



If the lenses are placed in juxtaposition,  $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

$$\Rightarrow \frac{1}{40} = \frac{1}{f_1} + \frac{1}{f_2} \quad \text{--- (2)}$$

Solving equations (1) and (2)

$$f_1 = +\frac{40}{3} \text{ cm (convex)}$$

$$f_2 = -20 \text{ cm (concave)}$$

A thin converging lens of focal length 25 cm forms an image on a screen at 5 m from the lens. The screen is brought closer to the lens by 18 cm. By what distance should the object be shifted away from/towards the lens so that its image on the screen is sharp again?

### **Solution**

The object should be moved away. The object is always between  $f$  and  $2f$  ( $\because$  image is between  $2f$  and  $\infty$  in both cases). Let us apply the formula and verify.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow u = \frac{fv}{f - v} = \frac{(+25)(+500)}{(25 - 500)}$$

$$\Rightarrow u = \frac{-500}{19} = -26.32 \text{ cm}$$

$$u' = \frac{(+25)(+482)}{(25 - 482)} = -26.37 \text{ cm}$$

$\therefore \Delta u = u' - u \approx -0.5 \text{ mm}$  (should be moved away by  $\approx 0.5 \text{ mm}$ )

$\Delta u$  being small, we could have applied approximation as below.

Considering  $\Delta u$  to be small, differentiate  $u = \frac{fv}{f-v}$

$$\begin{aligned} \Rightarrow du &= \frac{f^2}{(f-v)^2} dv \\ &= \frac{25^2}{(-475)^2} (-18) \approx -0.5 \text{ mm} \end{aligned}$$

A source and a screen are fixed at a distance  $\ell$  from each other. A thin lens is placed between them such that the image is focussed on the screen. Determine

- (i) the focal length of the lens
- (ii) the position of the lens.

**Solution**

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Writing  $v = |\ell - |u|| \Rightarrow \frac{1}{+\ell - |u|} - \frac{1}{-|u|} = \frac{1}{f}$

$$\Rightarrow \frac{1}{\ell - |u|} + \frac{1}{|u|} = \frac{1}{f}$$

$$\Rightarrow |u|^2 - \ell|u| + f\ell = 0$$

$$\Rightarrow |u| = \frac{\ell}{2} \left[ 1 \pm \sqrt{1 - \frac{4f}{\ell}} \right]$$

Case (i)

$$\text{If } 1 - \frac{4f}{\ell} < 0 \Rightarrow f > \frac{\ell}{4},$$

then  $|u|$  is not real which is not possible  $\therefore f \leq \frac{\ell}{4}$

Case (ii)

$$\text{If } f = \frac{\ell}{4}, \text{ then } |u| = \frac{\ell}{2}$$

Case (iii)

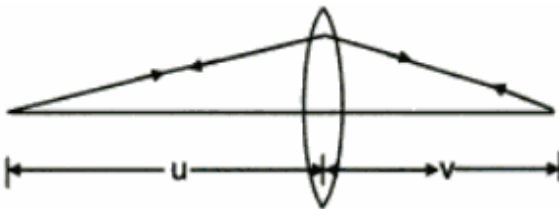
If  $f < \frac{\ell}{4}$ , then  $|u|$  has two values

$$\frac{\ell}{2} \left[ 1 + \sqrt{1 - \frac{4f}{\ell}} \right]$$

and

$$\frac{\ell}{2} \left[ 1 - \sqrt{1 - \frac{4f}{\ell}} \right]$$

you will find that if  $|u|$  is one of them,  $v$  is the other value i.e., the values of  $|u|$  and  $v$  are interchangeable.



$$\text{If } u = \frac{\ell}{2}, v = \frac{\ell}{2},$$

the equation  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$  gives the position of the focus,

$$f = \frac{\ell}{4}$$

An object and a screen are fixed and a lens is being moved from one point along the axis to another resulting in sharp images of heights  $h_1$  and  $h_2$ . What is the object height?

### **Solution**

Let height of object =  $h$

$$h_1 = h \frac{v}{|u|}$$

$$h_2 = h \frac{|u|}{v}$$

$$\Rightarrow h^2 = h_1 h_2 \Rightarrow h = \sqrt{h_1 h_2}$$

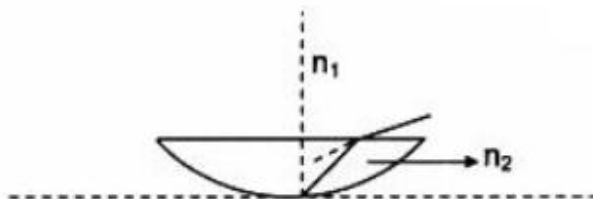
A plano convex lens of thickness 5 cm is kept on a flat table with its curved surface in contact with the table. The apparent depth of the bottom point is 4 cm. When the lens is reversed, the depth is  $\frac{100}{21}$  cm.

What is its focal length?

### **Solution**

Let the refractive index of lens be  $n_2$ , that of medium  $n_1$  (need not be air). We cannot use lens formula because thickness of lens is very large.

Case (i)



You can use either apparent depth formula or spherical surface formula.

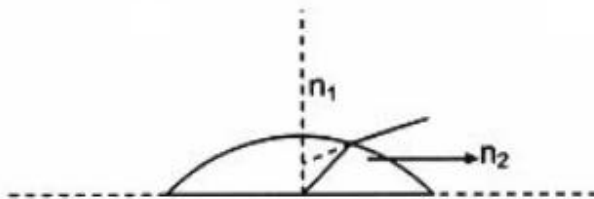
Application of depth formula:

$$4 = \frac{5}{n_{21}} \Rightarrow n_{21} = \frac{5}{4}$$

(or) spherical surface formula: ( $R = \infty$ , for plane)

$$\frac{1}{-4} - \frac{n_{21}}{-5} = \frac{1 - n_{21}}{\infty} \Rightarrow n_{21} = \frac{5}{4}$$

Case (ii)



(Here we cannot use apparent depth formula because interface is not plane)

$$\frac{1}{-100/21} - \frac{n_{21}}{-5} = \frac{1 - n_{21}}{R}$$

$$\Rightarrow \frac{-21}{100} + \frac{1}{4} = \frac{-1}{4R} \Rightarrow R = \frac{-100}{16} \text{ cm}$$

$$\frac{1}{f} = (n_{21} - 1) \left( \frac{1}{\infty} - \frac{1}{R} \right)$$

$$= \left( \frac{5}{4} - 1 \right) \left( \frac{16}{100} \right) \Rightarrow f = 25 \text{ cm}$$

Given a convex lens of focal length  $f$ , find the minimum distance between object and screen, below which a real image on the screen cannot be obtained, wherever the lens is kept?

### **Solution**

If the distance between object and screen is  $\ell$ , we saw that formula  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$  leads to the equation

$$|u|^2 - \ell|u| + \ell f = 0$$

$\Rightarrow$  Discriminant  $\ell^2 - 4\ell f$  should be  $\geq 0 \Rightarrow \ell \geq 4f$

$\Rightarrow \ell_{\min} = 4f.$

The power of a lens ( $n = 1.5$ ) is  $+5.0$  D. Find its power when immersed in water ( $n = \frac{4}{3}$ ).

### **Solution**

$$P = \frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\Rightarrow +5 = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \quad \text{--- (1)}$$

$$\left[ f = \frac{1}{+5} = +0.20 \text{ metre} \right]$$

$$P_w = \frac{n_w}{f_w} = (n - n_w) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \quad \text{--- (2)}$$



$$\frac{(2)}{(1)} \Rightarrow \frac{P_w}{+5} = \frac{1.5 - \frac{4}{3}}{1.5 - 1} \Rightarrow P_w = \frac{+5}{3} \text{ D}$$

$$\left[ f_w = \frac{n_w}{P_w} = \frac{4/3}{5/3} = 0.80 \text{ m} \right]$$

In the above problem, if the two surfaces of the lens are of equal radius (i) what is the radius of curvature? (ii) what is its power with the lens kept with air on one side and water on the other side?

### ***Solution***

The formula is

$$\frac{n_3}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}$$

By putting  $u = -\infty$  and  $v = f$ ,

$$\frac{n_3}{f} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}$$

and if  $n_3 = n_1 = 1$ ,

$$\frac{1}{f} = (n_2 - 1) \left( \frac{1}{R} - \frac{1}{-R} \right) = (n_2 - 1) \frac{2}{R}$$

(i) Now take the data for  $n_3 = n_1 = \text{air}$

$$+5 = \frac{1}{f} = \frac{2(1.5 - 1)}{R} = \frac{1}{R} \Rightarrow R = f = 0.2 \text{ m}$$

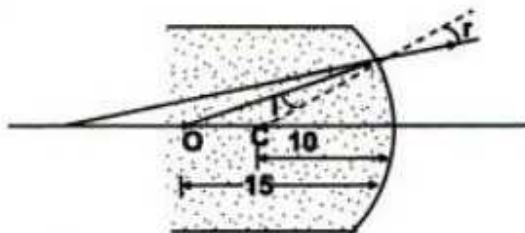
(ii) Using data for one side air and other side water:

$$P' = \frac{4/3}{f'} = \frac{1.5 - 1}{0.2} + \frac{\frac{4}{3} - 1.5}{-0.2}$$

$$\Rightarrow P' = +\frac{10}{3}D \text{ focal length } f' = 0.4 \text{ m}$$

An object is immersed in a medium of refractive index  $n = 2.0$ . Object is 15 cm from the concave surface whose radius of curvature is 10 cm. Locate the image.

**Solution**



$$u = -15, n_1 = 2, n_2 = 1, R = -10$$

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{1}{v} - \frac{2}{-15} = \frac{1 - 2}{-10} \Rightarrow v = -30 \text{ cm (virtual)}$$

What is the relationship between  $u$  and  $v$  if the refracting surface is plane?

**Solution**

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

For plane surface,  $R = \infty \Rightarrow \frac{n_2}{v} = \frac{n_1}{u} \Rightarrow v = \frac{n_2}{n_1}u$

[we already know this from the apparent depth formula with object real; this gives always virtual image]

To obtain a real image at infinity in a medium 1 (with refractive index  $n_1$ ) out of a real object in a medium 2 (with refractive index  $n_2$ ) and at a distance  $d$  from the interface, what should be the geometry of the interface?

**Solution**

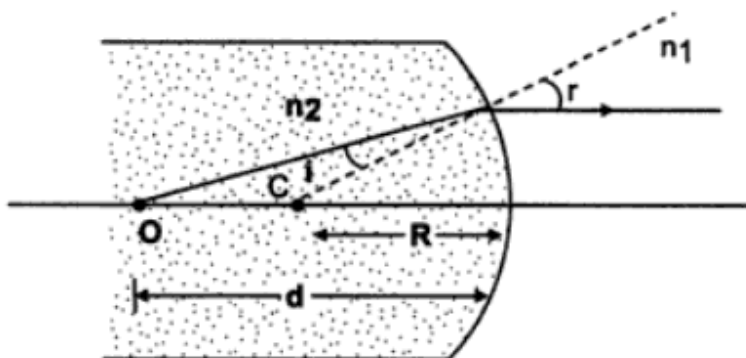
$$\frac{n_1}{\infty} - \frac{n_2}{-d} = \frac{n_1 - n_2}{R}$$

Clearly it cannot be a plane (whose  $R = \infty$ )

$\therefore$  surface can be only spherical, whose  $R = \left(\frac{n_1 - n_2}{n_2}\right)d$ ,  
 $d$ , being modulus value, is always positive

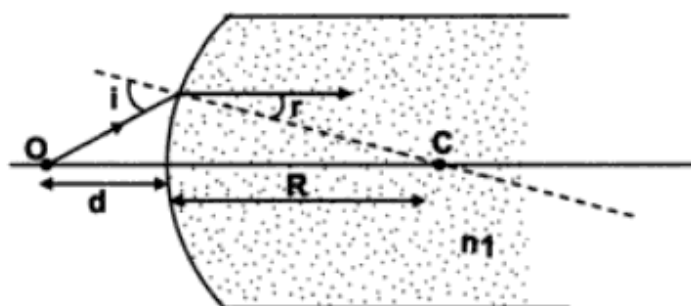
$\therefore$   $R$  can be negative (concave) with  $n_2 > n_1$

$$|R| = \left(\frac{n_2 - n_1}{n_2}\right)d$$



Or  $R$  can be positive  $\Rightarrow$  (convex) when  $n_1 > n_2$

$$R = \left( \frac{n_1 - n_2}{n_2} \right) d$$



A crown glass prism of  $11^\circ$  is combined with a flint glass prism to form a combination giving dispersion without deviation. Given the refractive indices as

	Crown	Flint
Red	1.51	1.62
Blue	1.52	1.64

- Find
- (1) The angle of the flint glass prism
  - (2) The net angular dispersion produced

### **Solution**

Refractive index of mean ray (crown)

$$n = \frac{1.51 + 1.52}{2} = 1.515$$

**Refractive index of mean ray(flint)**

$$n' = \frac{1.62 + 1.64}{2} = 1.63$$

Condition for no deviation is  $\frac{A}{A'} = -\frac{n'-1}{n-1}$

$$\begin{aligned} \Rightarrow A' &= -\frac{(n-1)A}{n'-1} \\ &= -\frac{(1.515-1)11}{(1.63-1)} = -8.99^\circ \end{aligned}$$

Angle of flint glass prism =  $8.99^\circ$

Net angular dispersion =  $(n_b - n_r)A + (n_b' - n_r') A' = (1.52 - 1.51)11 + (1.64 - 1.62)(-8.99) = -0.07^\circ$

$\therefore$  Net deviation  $d = (n-1)A - \frac{(n'-1)}{(n_b' - n_r')} (n_b - n_r)$

$$A = (n-1)A \left[ 1 - \frac{\omega}{\omega'} \right], \text{ where}$$

$n$  = refractive index of first prism for mean light

$\omega$  = dispersive power of material of first prism and

$\omega'$  = dispersive power of material of second prism

Calculate the dispersive power of the material of a prism with respect to the given pair of colours of light.  $n_v = 1.538$  and  $n_r = 1.527$

**Solution**

$$\text{Mean refractive index } n = \frac{n_v + n_r}{2}$$

$$= \frac{1.538 + 1.527}{2}$$
$$= 1.5325$$

$$\text{Dispersive power } \omega = \frac{n_v - n_r}{n - 1}$$
$$= \frac{1.538 - 1.527}{1.5325 - 1} = 0.021$$

In one dispersion experiment

$$d_v = 3.25^\circ, d_b = 3^\circ \text{ and}$$

$d_r = 2.75^\circ$ . Calculate angular dispersion, mean deviation and dispersive power for

(i) violet – red pair and (ii) blue – red pair

### ***Solution***

Case I (violet – red pair)

$$\text{Angular dispersion} = d_v - d_r = 3.25^\circ - 2.75^\circ = 0.5^\circ$$

$$\text{Mean deviation} = d = \frac{d_v + d_r}{2} = \frac{3.25^\circ + 2.75^\circ}{2} = 3^\circ$$



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$$\begin{aligned}\text{Dispersive power} &= \frac{d_v - d_r}{d} \\ &= \frac{0.5^\circ}{3^\circ} = 0.167\end{aligned}$$

Case II ( blue – red pair)

$$\begin{aligned}\text{Angular dispersion} &= d_b - d_r = 3^\circ - 2.75^\circ \\ &= 0.25^\circ\end{aligned}$$

$$\text{Mean deviation} = d = \frac{d_b + d_r}{2} = \frac{3^\circ + 2.75^\circ}{2} = 2.875^\circ$$

$$\text{Dispersive power} = \frac{d_b - d_r}{d} = \frac{0.25^\circ}{2.875^\circ} = 0.087$$

A prism of angle  $60^\circ$  is of a material of refractive index  $\sqrt{2}$ . Calculate angles of minimum deviation and maximum deviation.

**Solution**

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(i)  $A = 60^\circ, n = \sqrt{2}$

At  $\delta_{\min}, r = \frac{A}{2} \Rightarrow r = 30^\circ$

$$n = \frac{\sin i}{\sin r} \Rightarrow \sqrt{2} = \frac{\sin i}{\sin 30^\circ} \Rightarrow \sin i = \frac{1}{\sqrt{2}} \Rightarrow i = 45^\circ$$

$$\delta_{\min} = 2i - A = 90^\circ - 60^\circ = 30^\circ$$

$$\theta_c = \sin^{-1} \frac{1}{\sqrt{2}} = 45^\circ \Rightarrow A > \theta_c$$

(ii) Now, since  $r_1 = \theta_c$  and  $A = r_1 + r_2$ ,

We have  $r_2 = A - r_1 = A - \theta_c$

$$\therefore i_2 = n \sin r_2 = n \sin (A - \theta_c)$$

$\therefore$  Applicable formula is

$$\delta_{\max} = 90^\circ + \sin^{-1} [n \sin(A - \theta_c)] - A$$

$$= 90^\circ + \sin^{-1} [\sqrt{2} \cdot$$

$$\sin(60^\circ - 45^\circ)] - 60^\circ = 90^\circ + 22^\circ - 60^\circ = 52^\circ$$

(We may also calculate  $\delta_{\max}$  without the use of this formula:

For  $\delta_{\max}, i_1 = 90^\circ$

$$\therefore r_1 = \theta_c = 45^\circ$$

$$\Rightarrow r_2 = A - r_1 = 60^\circ - 45^\circ = 15^\circ$$

$$\therefore \frac{\sin i_2}{\sin 15^\circ} = \sqrt{2}$$

$$\Rightarrow \sin i_2 = \sqrt{2} \sin 15^\circ = 0.37 \Rightarrow i_2 = 22^\circ$$

$$\therefore \delta_{\max} = i_1 + i_2 - A = 90^\circ + 22^\circ - 60^\circ = 52^\circ$$

If a prism ( $n = \sqrt{2}$ ) has  $D = 2A$ , what is  $A$ ?

**Solution**

$$\sqrt{2} = \frac{\sin \frac{A+D}{2}}{\sin \frac{A}{2}} = \frac{\sin \frac{3A}{2}}{\sin \frac{A}{2}}$$

let  $\theta = \frac{A}{2}$

$$\sqrt{2} = \frac{\sin 3\theta}{\sin \theta} = 3 - 4 \sin^2 \theta$$

$$\Rightarrow \theta = 39^\circ 4' \Rightarrow A = 2\theta = 78^\circ 8'$$

**To print a photograph from a negative, the time of exposure to light from a lamp placed 0.5 m away is 2.5 second. How much exposure time is required if the lamp is placed 1 m away ?**

**Solution. Let**

**$E$  = Amount of light falling on the photographic film**

**$x$  = Distance of the lamp**

**and**

**$t$  = Time of exposure**

**then,**

$$E \propto \frac{t}{x^2}$$

**$\therefore$**

$$\frac{E_1}{E_2} = \frac{t_1 x_2^2}{t_2 x_1^2}$$

**According to problem,**

$$E_1 = E_2$$

**$\therefore$**

$$x_1^2 t_2 = x_2^2 t_1$$

**or**

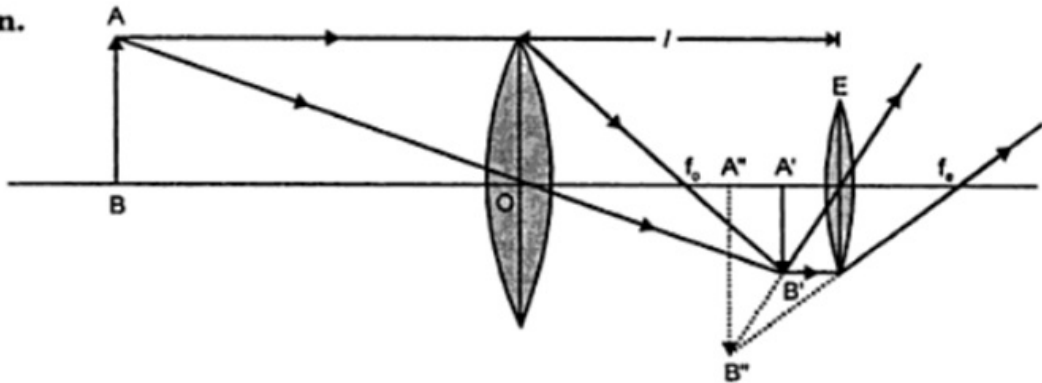
$$t_2 = \frac{x_2^2}{x_1^2} t_1 = 10 \text{ sec Ans.}$$

**A telescope has an objective of focal length 50 cm and an eye piece of focal length 5 cm. The least distance of distinct vision is 25 cm.**

**The telescope is focussed for distinct vision on a scale 200 cm away from the objective. Calculate :**

- (i) the separation between the objective and the eyepiece**
- (ii) the magnification produced.**

**Solution.**



Let AB be the position of the object

$$u = 200 \text{ cm}, \quad f = 50 \text{ cm}$$

$\therefore$  The image  $A'B'$  formed by the objective is a distance  $v$  from it

$$\therefore \quad v = \frac{uf}{u-f} = \frac{200 \times 50}{200-50} = \frac{1000}{15} = \frac{200}{3} \text{ cm}$$

This serves as an object for the eyepiece

The distance between  $A'B'$  and the eyepiece is  $u' = l - \frac{200}{3}$ , where  $l$  is the separation between the lenses. The images (final) distance is  $v' = 25$  cm from the eyepiece and  $f' = 5$  cm.

$$\therefore \quad u' = \frac{v'f'}{v'-f'} = \frac{-25 \times 5}{-25-5} = \frac{25}{6} \text{ cm}$$

$$\therefore \quad l = \frac{25}{6} + \frac{200}{3} = \frac{425}{6} = 70.83 \text{ cm}$$

$$\text{Total magnification } m_o \times m_e = \frac{v}{u} \times \frac{v'}{u'} = \frac{200}{3} \times \frac{1}{200} \times 25 \times \frac{6}{25} = 2$$

**For a normal eye, the far point is at infinity and the near point of distinct vision is 25 cm in front of eye. The cornea of the eye provides a converging power of about 40 D and the least converging power of the eye-lens behind the cornea is about 20 D. From this rough data, estimate the range of accommodation of a normal eye.**

**Solution.** To see objects at infinity, the eye uses its least converging power *i.e.*, (40 + 20) D or 60 D

Using lens equation, we get

$$\frac{1}{v} - \frac{1}{-\infty} = \frac{60}{100} \Rightarrow v = \frac{5}{3} \text{ cm}$$

To focus an object at the near point, if the focal length is  $f$ , then

$$\frac{1}{f} = \frac{1}{5/3} - \frac{1}{-25} = \frac{25}{16} \text{ cm}$$

$$\therefore P = \frac{100 \times 16}{25} = 64 \text{ D}$$

Hence, Power of eye-lens = (64 – 40) D = 24 D

Therefore, the range of accommodation of the eye-lens is approximately 20 D to 24 D.

I read quite a few books on “ **Quantum Mechanics** “. Almost all started discussing YDSE, Young’s Double Slit Experiment, in the first page. YDSE was one of the most important inputs for “ Quantum Mechanics “. [ The others being **black body radiation**, wien’s Law, Ultra Violet Catastrophe, Photoelectric effect, Unexplained assumptions of Bohr, Zeeman effect, Stark effect, Fine Structure constant, Flame test of various elements, emission / absorption / Molecular spectra, Molecular shape / structure giving inputs for Orbital shapes / orientation, Uncertainty principle, De Broglie’s wavelength of Particles etc ]

Read History of Quantum Mechanics at [http://www-history.mcs.stand.ac.uk/HistTopics/The\\_Quantum\\_age\\_begins.html](http://www-history.mcs.stand.ac.uk/HistTopics/The_Quantum_age_begins.html)

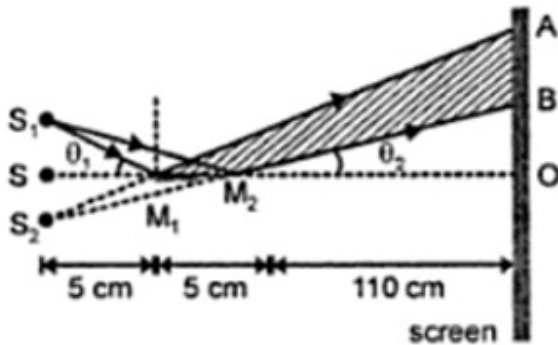
[https://en.wikipedia.org/wiki/History\\_of\\_quantum\\_mechanics](https://en.wikipedia.org/wiki/History_of_quantum_mechanics)

[https://www.youtube.com/watch?v=xkA\\_0QJerY8](https://www.youtube.com/watch?v=xkA_0QJerY8)

[https://www.youtube.com/watch?v=\\_sdoGjhQOtM](https://www.youtube.com/watch?v=_sdoGjhQOtM)

Professor H C Verma has written a book on “ Quantum Mechanics “. This also starts with YDSE discussions.

**Example** A Lloyd's mirror of length 5 cm is illuminated with monochromatic light of wavelength  $\lambda = 6000 \text{ \AA}$  from a narrow slit 1 mm from its plane and 5 cm in its plane from its near edge. Find the fringe width on a screen 120 cm from the slit and width of interference on the screen.



**Solution.** In plane mirror, image is formed as behind it, as the object is in front of it.

So,  $d = 2 \text{ mm} = 0.2 \text{ cm}$  ;  $\lambda = 6000 \text{ \AA} = 6000 \times 10^{-8} \text{ cm}$  ;  $D = 120 \text{ cm}$

$$\therefore \text{Fringe width } \beta = \frac{\lambda D}{d} = \frac{6 \times 10^{-5} \times 120}{0.20} = 0.036 \text{ cm}$$

The width of the fringe pattern is AB. From the fig. 4.46

$$\tan \theta_1 = \frac{0.1}{5} \quad \text{and} \quad \tan \theta_2 = \frac{0.1}{10}$$

In right angled triangle  $AM_1O$  and  $BM_2O$

$$\tan \theta_1 = \frac{0.1}{5} = \frac{OA}{M_1O}$$

$$\text{i.e.,} \quad OA = 115 \times \frac{0.1}{5} \text{ cm}$$

$$\text{and} \quad \tan \theta_2 = \frac{0.1}{10} = \frac{OB}{OM_2}$$

$$\text{i.e.,} \quad OB = 110 \times \frac{0.1}{10} \text{ cm}$$

$$\therefore \text{Width of fringe pattern} = OA - OB = \frac{115 \times 0.1}{5} - \frac{110 \times 0.1}{10} = 1.2 \text{ cm}$$



**In the usual layout for interference fringes, two identical slits, each of width  $a$  are kept apart by  $d$  from centre of centre. Find :**

- (a) the difference of path differences between rays from the bottom and top of slits i.e.,  $\delta\Delta = \Delta_b - \Delta_t$ ,  
 (b) the maximum value of  $a$  at which interference fringes continue to be sharp. Take  $D$  = distance between the screen and the slits.

**Solution.** (a) The rays from the top of the slits may be assumed to come from ideal sources with their pole (equidistant point on the screen) at O. Then

$$\Delta_t = \frac{xd}{D}$$

Similarly,

$$\Delta_b = \frac{x'd}{D}$$

$\therefore$

$$\Delta_b - \Delta_t = \frac{(x' - x)d}{D}$$

But

$$x' - x = a$$

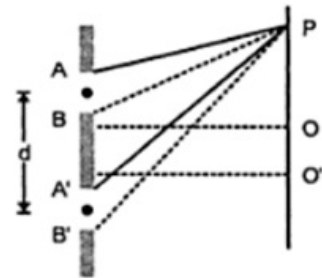
$\therefore$

$$\delta\Delta = \Delta_b - \Delta_t = \frac{ad}{D}$$

(b) If  $\delta\Delta = \frac{\lambda}{2}$ , the maximum from top edges will be superimposed on to the minimum from the bottom edges, owing to which the interference pattern will disappear completely

$\therefore$

$$\frac{\lambda}{2} = \frac{ad}{D} \Rightarrow a = \frac{\lambda D}{2d}$$



**In Young's experiment, the source is red light of wavelength  $7 \times 10^{-7}$  m. When a thin glass plate of refractive index 1.5 at this wavelength is put in the path of one of the interfering beams, the central bright fringe shifts by  $10^{-3}$  m in the position previously occupied by the 5<sup>th</sup> bright fringe. Find the thickness of the plate. When the source is now changed to green light of wavelength  $5 \times 10^{-7}$  m, the central fringe shifts to a position initially occupied by the 6<sup>th</sup> bright fringe due to red light. Find the refractive index of glass for the green light. Also estimate the change in fringe width due to the change in wavelength.**

**Solution.** As due to presence of glass plate path difference changes by  $(\mu - 1)t$ , so according to given problem,

$$(\mu_R - 1)t = 5\lambda_R$$

*i.e.,* 
$$t = \frac{5 \times 7 \times 10^{-6}}{(1.5 - 1)} = 7 \mu\text{m}$$

Now, when red light is replaced by green light

$$(\mu_G - 1)t = 6\lambda_R$$

So 
$$\frac{\mu_R - 1}{\mu_G - 1} = \frac{5}{6}$$

*i.e.,* 
$$\mu_G - 1 = \frac{6}{5}(1.5 - 1) \quad \text{or} \quad \mu_G = 1.6$$

Further as 
$$5\beta_R = 10^{-3}$$

*i.e.,* 
$$\beta_R = 2 \times 10^{-4}$$

So, 
$$\frac{\beta_G}{\beta_R} = \frac{\lambda_G}{\lambda_R} = \frac{5}{7}$$

*i.e.,* 
$$\frac{\beta_G - \beta_R}{\beta_R} = \frac{5}{7} - 1 = -\frac{2}{7}$$

So, 
$$\Delta\beta = -\frac{2}{7} \times 2 \times 10^{-4} = 0.57 \times 10^{-4} \text{ m}$$

*i.e.,* Fringe will decrease by  $0.57 \times 10^{-4} \text{ m}$  when red light replaced by green light.

**In Young's double slit experiment set-up with light of wavelength  $\lambda = 6000 \text{ \AA}$ , distance between the two slits is 2 mm and distance between the plane of slits and the screen is 2 m. The slits are of equal intensity. When a sheet of glass of refractive index 1.5 (which permits only a fraction  $\eta$  of the incident light to pass through) and thickness  $8000 \text{ \AA}$  is placed in front of the lower slit, it is observed that the intensity at a point P, 0.15 mm above the central maxima does not change. Find the value of  $\eta$ .**

**Solution.** In absence of glass sheet, path difference at P,

$$\Delta x = \frac{yd}{D} = \frac{0.15 \times 10^{-3} \times 2 \times 10^{-3}}{2} = 1.5 \times 10^{-7} \text{ m}$$

Corresponding phase difference at P,

$$\phi = \left( \frac{2\pi}{\lambda} \right) (\Delta x) = \left( \frac{2\pi}{6000 \times 10^{-10}} \right) (1.5 \times 10^{-7}) = \frac{\pi}{2}$$

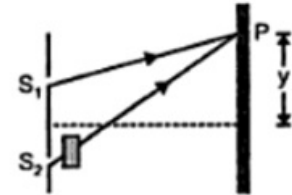
$$\therefore \text{Intensity at P, } I = 4I_0 \cos^2 \frac{\phi}{2} = 2I_0$$

Phase difference when glass sheet is introduced,

$$\begin{aligned} \phi' &= \phi + \frac{2\pi}{\lambda} (\mu - 1) t \\ &= \frac{\pi}{2} + \frac{2\pi}{6000 \times 10^{-10}} (1.5 - 1) (8000 \times 10^{-10}) = \frac{11\pi}{6} \end{aligned}$$

$$\text{The intensity at P is now, } I' = I_0 + \eta I_0 + 2\sqrt{\eta I_0^2} \cos \frac{11\pi}{6} = 2I_0$$

$$\therefore \eta = 0.21$$

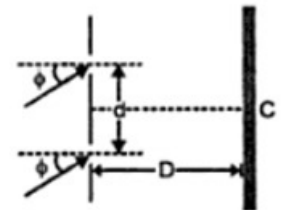


Light of wavelength  $\lambda = 500 \text{ nm}$  falls on two narrow slits placed a distance  $d = 50 \times 10^{-4} \text{ cm}$  apart, at an angle  $\phi = 30^\circ$  relative to the slits shown in fig.

On the lower slit a transparent slab of thickness  $0.1 \text{ mm}$  and refractive index

$\frac{3}{2}$  is placed. The interference pattern is observed on a screen at a distance  $D = 2 \text{ m}$  from the slits. Then calculate :

- Position of the central maxima ?
- The order of minima closest to centre C of screen ?
- How many fringes will pass over C, if we remove the transparent slab from the lower slit ?



**Solution.** (a) Path difference,  $\Delta x = d \sin \phi + d \sin \theta - (\mu - 1) t$

For central maxima,  $\Delta x = 0$

$$\begin{aligned} \therefore \sin \theta &= \frac{(\mu - 1) t}{d} - \sin \phi \\ &= \frac{(3/2 - 1)(0.1)}{50 \times 10^{-3}} - \sin 30^\circ = \frac{1}{2} \end{aligned}$$

$$\therefore \theta = 30^\circ$$

(b) At C,  $\theta = 0^\circ$ ,

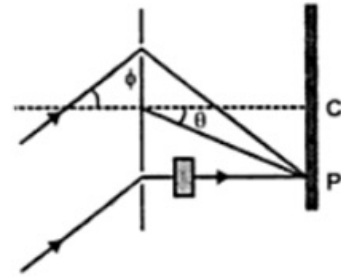
$$\begin{aligned} \text{Therefore, } \Delta x &= d \sin \phi - (\mu - 1) t = (50 \times 10^{-3}) \left( \frac{1}{2} \right) - (3/2 - 1)(0.1) \\ &= 0.025 - 0.05 = -0.025 \text{ mm} \end{aligned}$$

Substituting  $\Delta x = n\lambda$ ,

$$\text{We get } n = \frac{\Delta x}{\lambda} = \frac{-0.025}{500 \times 10^{-6}} = -50$$

Hence, at C, there will be maxima. Therefore, closest to C order of minima is 49.

$$(c) \text{ Number of fringes shifted upwards } = \frac{(\mu - 1) t}{\lambda} = \frac{(3/2 - 1)(0.1)}{500 \times 10^{-6}} = 100$$



**In a Young experiment the light source is at distance  $l_1 = 20 \mu\text{m}$  and  $l_2 = 40 \mu\text{m}$  from the slits. The light of wavelength  $\lambda = 500 \text{ nm}$  is incident on slits separated at a distance  $10 \mu\text{m}$ . A screen is placed at a distance  $D = 2 \text{ m}$  away from the slits as shown in fig. Find :**

- the values of  $\theta$  relative to the central line where maxima appear on the screen ?
- how many maxima will appear on the screen ?
- what should be minimum thickness of a slab of refractive index 1.5 be placed on the path of one of the ray so that minima occurs at C ?

**Solution.** (a) The optical path difference between the beams arriving at P,

$$\Delta x = (l_1 - l_2) + d \sin \theta$$

The condition for maximum intensity is,

$$\Delta x = n\lambda$$

Thus, 
$$\sin \theta = \frac{1}{d} [\Delta x - (l_1 - l_2)]$$

$$= \frac{1}{d} [n\lambda - (l_1 - l_2)]$$

$$= \frac{1}{10 \times 10^{-6}} [n \times 500 \times 10^{-9} - 20 \times 10^{-6}] = 2 \left[ \frac{n}{40} - 1 \right]$$

Hence, 
$$\theta = \sin^{-1} \left[ 2 \left( \frac{n}{40} - 1 \right) \right]$$

(b) 
$$|\sin \theta| \leq 1$$

$$\therefore -1 \leq 2 \left[ \frac{n}{40} - 1 \right] \leq 1$$

or 
$$-20 \leq (n - 40) \leq 20$$

or 
$$20 \leq n \leq 60$$

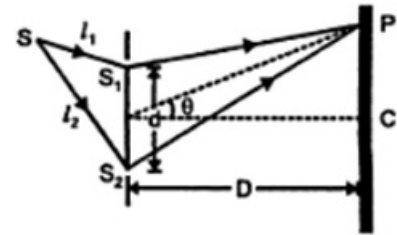
Hence, Number of maxima =  $60 - 20 = 40$

(c) At C, phase difference, 
$$\phi = \left( \frac{2\pi}{\lambda} \right) (l_2 - l_1) = \left( \frac{2\pi}{500 \times 10^{-9}} \right) (20 \times 10^{-6}) = 80 \pi$$

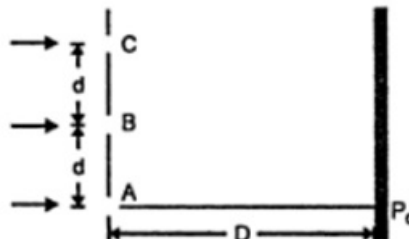
Hence, maximum intensity will appear at C. For minimum intensity at C,

$$(\mu - 1)t = \frac{\lambda}{2}$$

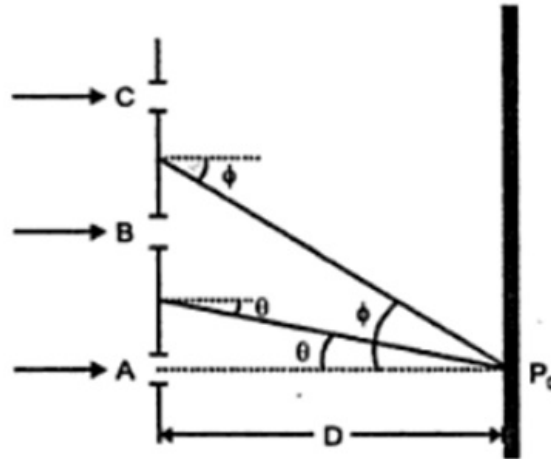
or 
$$t = \frac{\lambda}{2(\mu - 1)} = \frac{500 \times 10^{-9}}{2 \times 0.5} = 500 \text{ nm}$$



shows three equidistant slits being illuminated by a monochromatic parallel beam of light. Let  $BP_0 - AP_0 = \lambda/3$  and  $D \gg \lambda$ . (a) Show that in this case  $d = \sqrt{2\lambda D/3}$ . (b) Show that the intensity at  $P_0$  is three times the intensity due to any of the three slits individually.



**Solution. (a)**



$$BP_0 - AP_0 = \frac{\lambda}{3}$$

or

$$d \sin \theta = \frac{\lambda}{3}$$

or

$$d \tan \theta = \frac{\lambda}{3} \quad (\text{For small angle } \tan \theta \approx \sin \theta \approx \theta)$$

$\therefore$

$$d \left( \frac{d/2}{D} \right) = \frac{\lambda}{3}$$

or

$$d = \sqrt{\frac{2D\lambda}{3}}$$

(b)

$$\Delta x_{A/B} = \text{path difference between waves coming from A and B} = \frac{\lambda}{3}$$

$\therefore$

$$\begin{aligned} \phi_{A/B} &= \text{phase difference} \\ &= \frac{2\pi}{\lambda} \Delta x_{A/B} = \frac{2\pi}{3} \end{aligned}$$

Similarly,

$$\Delta x_{B/C} = d \sin \phi$$

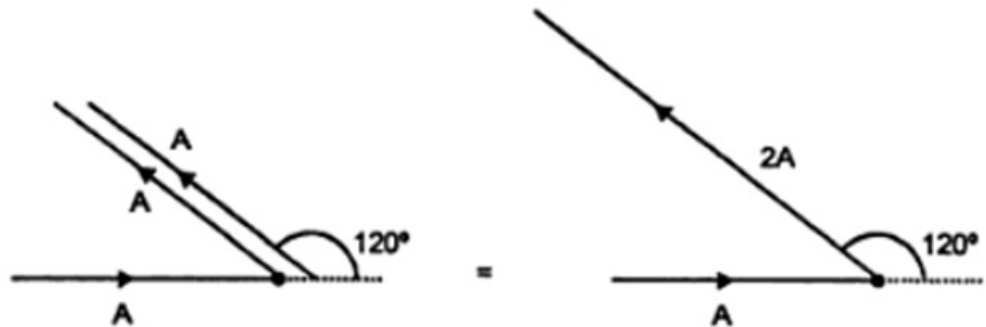
$$= d \left( \frac{3d/2}{D} \right) = \frac{3d^2}{2D} = \lambda$$

$\therefore$

$$\phi_{B/C} = 2\pi$$



Now, phase diagram of the waves arriving at  $P_0$  is as shown below :



$\therefore$  Amplitude of resultant wave is given by

$$A' = \sqrt{A^2 + (2A)^2 + 2(A)(2A) \cos 120^\circ}$$

$$= \sqrt{3}A$$

As intensity  $(I) \propto A^2$

$\therefore$  Intensity at  $P_0$  will be three times the intensity due to any of the three slits individually.

Two glass plates enclose a wedge shaped air film, touching at one edge and are separated by a wire of 0.05 mm diameter at a distance of 15 cm from the edge. Calculate the fringe width. Monochromatic light of  $\lambda = 6000 \text{ \AA}$  from a broad source falls normally on the film.

Solution.

$$x = 15 \text{ cm}$$

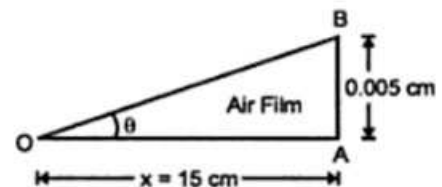
$$\lambda = 6000 \text{ \AA} = 6000 \times 10^{-8} \text{ cm}$$

$$AB = 0.005 \text{ cm}$$

$$\text{Fringe width, } \beta = \frac{\lambda}{2\theta}$$

$$\theta = \frac{AB}{OA} = \frac{0.005}{15}$$

$$\beta = \frac{\lambda}{2\theta} = \frac{6000 \times 10^{-8} \times 15}{2 \times 0.005} = 0.09 \text{ cm}$$



White light may be considered to have  $\lambda$  from  $4000 \text{ \AA}$  to  $7500 \text{ \AA}$ . If an oil film has thickness  $10^{-4} \text{ cm}$ , deduce the wavelength in the visible region for which the reflection along the normal direction will be (i) weak (ii) strong. Take  $\mu$  of oil as 1.4.

**Solution.** Here,

$$r = 0^\circ; \mu = 1.4; t = 10^{-4} \text{ cm}$$

$$\therefore 2 \mu t = 2 \times 1.4 \times 10^{-4} \text{ cm} = 2.8 \times 10^{-4} \times 10^8 \text{ \AA} = 28000 \text{ \AA}$$

(i) Condition for weak reflection (destructive interference) is given by

$$2 \mu t = n \lambda$$

$$\text{or } \lambda = \frac{2 \mu t}{n} = \frac{28000}{n}$$

The value of  $n$  should be selected such that  $\lambda$  lies between  $4000 \text{ \AA}$  and  $7500 \text{ \AA}$ . This will be possible if

$$\lambda = \frac{28000}{4} = 7000 \text{ \AA} \quad (\text{for } n = 4)$$

$$\lambda = \frac{28000}{5} = 5600 \text{ \AA} \quad (\text{for } n = 5)$$

$$\lambda = \frac{28000}{6} = 4667 \text{ \AA} \quad (\text{for } n = 6)$$

$$\lambda = \frac{28000}{7} = 4000 \text{ \AA} \quad (\text{for } n = 7)$$

The other values of  $n$  are not allowed as for those values of  $n$ ,  $\lambda$  does not lie within the given wavelength range of  $4000 \text{ \AA}$  to  $7500 \text{ \AA}$ . Hence, all above values of  $\lambda$  cause weak reflection.

(ii) For strong reflection (constructive interference), we have

$$2 \mu t = (2n + 1) \left( \frac{\lambda}{2} \right)$$

$$\therefore \lambda = \frac{2 \times 2 \mu t}{2n + 1} = \frac{2 \times 28000}{2n + 1} = \frac{56000}{2n + 1}$$

The possible values of  $\lambda$  in this case are given by

$$\lambda = \frac{56000}{9} = 6222 \text{ \AA} \quad (\text{for } n = 4)$$

$$\lambda = \frac{56000}{11} = 5091 \text{ \AA} \quad (\text{for } n = 5)$$

$$\lambda = \frac{56000}{13} = 4300 \text{ \AA} \quad (\text{for } n = 6)$$

Hence, only the above values of  $n$  will cause strong reflection because the range will not be within desired wavelengths, if  $n$  is different.

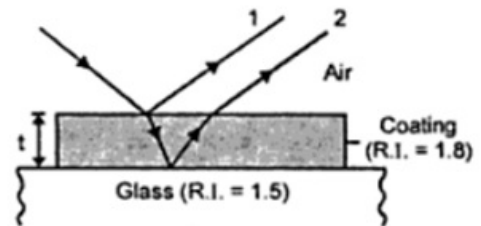
**A glass plate of refractive index 1.5 is coated with a thin layer of thickness  $t$  and refractive index 1.8. Light of wavelength  $\lambda$  travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surfaces of the layer and the two reflected rays interfere. Write the condition for their constructive interference. If  $\lambda = 648 \text{ nm}$ , obtain the least value of  $t$  for which the rays interfere constructively.**

**Solution.** The ray reflected from upper surface suffer a phase change of  $\pi$  due to reflection, at denser media, so the condition of constructive interference for normal incidence.

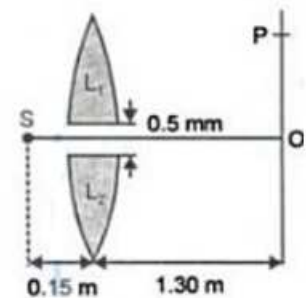
$$2\mu t + \frac{\lambda}{2} = n\lambda \quad \text{or} \quad 2\mu t = \frac{(2n-1)\lambda}{2}$$

For minimum value of  $t$ ,  $n = 1$

$$\therefore t_{\min} = \frac{\lambda}{4\mu} = 90 \text{ nm}$$



**In fig.** shown, **S** is a monochromatic point source emitting light of wavelength  $\lambda = 500 \text{ nm}$ . A thin lens of circular shape and focal length  $0.10 \text{ m}$  is cut into two identical halves  $L_1$  and  $L_2$  by a plane passing through a diameter. The two halves are placed symmetrically about the central axis  $SO$  with a gap of  $0.5 \text{ mm}$ . The distance along the axis from  $S$  to  $L_1$  and  $L_2$  is  $0.15 \text{ m}$ , while that from  $L_1$  and  $L_2$  to  $O$  is  $1.30 \text{ m}$ . The screen at  $O$  is normal to  $SO$ .



- (i) If the third intensity maximum occurs at the point **P** on the screen, find distance **OP**.
- (ii) If the gap between  $L_1$  and  $L_2$  is reduced from its original value of  $0.5 \text{ mm}$ , will the distance **OP** increase, decrease or remain the same ?

**Solution. (i)** As shown in fig. each part of the lens will form image of S which will act as coherent sources.

From lens equations, we can write

$$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$$

or  $v = 30 \text{ cm}$

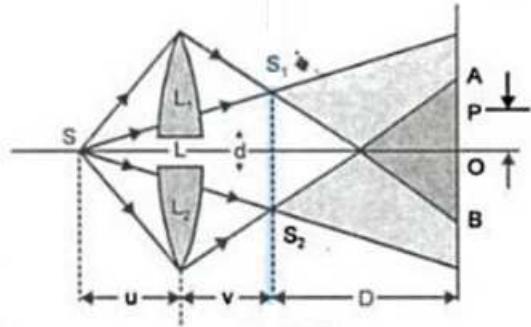
$$m = \frac{-v}{u} = -2$$

$$d = 3 \times 0.5 \text{ mm} = 1.5 \text{ mm}$$

And from fig.  $D = 1.30 - 0.30 = 1 \text{ m}$

Now, from the theory of interference the position  $y$  of a point P on the screen is given by

$$y = \frac{D}{d}(\Delta x)$$



and as point is third maximum

$$\Delta x = 3\lambda$$

So,

$$y = \frac{D}{d} (3\lambda)$$

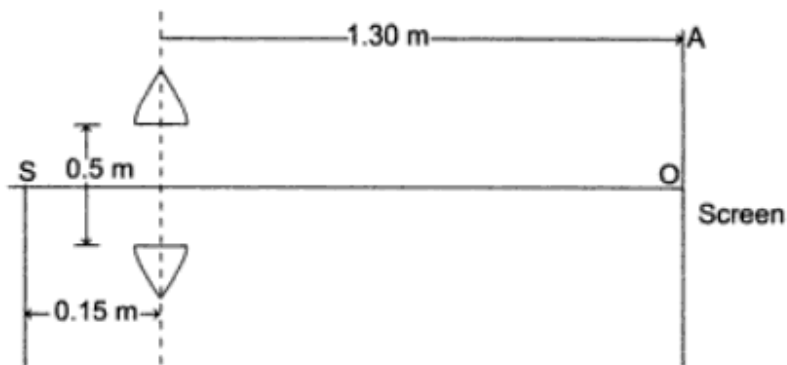
or

$$y = \frac{5 \times 10^{-7}}{0.5 \times 10^{-3}} \times 10^{-3} \text{ m} = 1 \text{ mm}$$

(ii) If gap between  $L_1$  and  $L_2$  is reduced then  $d$  will decrease. As  $\beta = \frac{D\lambda}{d}$  and  $OP = 3\beta$ , therefore OP will increase.

**In the figure shown,  $S$  is a monochromatic point source emitting light of wavelength  $\lambda = 500 \text{ nm}$ . A thin lens of circular shape and focal length  $0.10 \text{ m}$  is cut into two identical halves  $L_1$  and  $L_2$  by a plane passing through a diameter. The two are placed symmetrically about the central axis so with a gap of  $0.5 \text{ mm}$ . The distance along the axis from  $S$  to  $L_1$  and  $L_2$  is  $0.15 \text{ m}$ , while that from  $L_1$  and  $L_2$  to  $O$  is  $1.30 \text{ m}$ . The screen at  $O$  is normal to  $SO$ .**

- (i) **If the third intensity maximum occurs at the point  $A$  on the screen, find the distance  $OA$ .**
- (ii) **If the gap between  $L_1$  and  $L_2$  is reduced from its original value of  $0.5 \text{ mm}$ , will the distance  $OA$  increase, decrease or remain the same?**

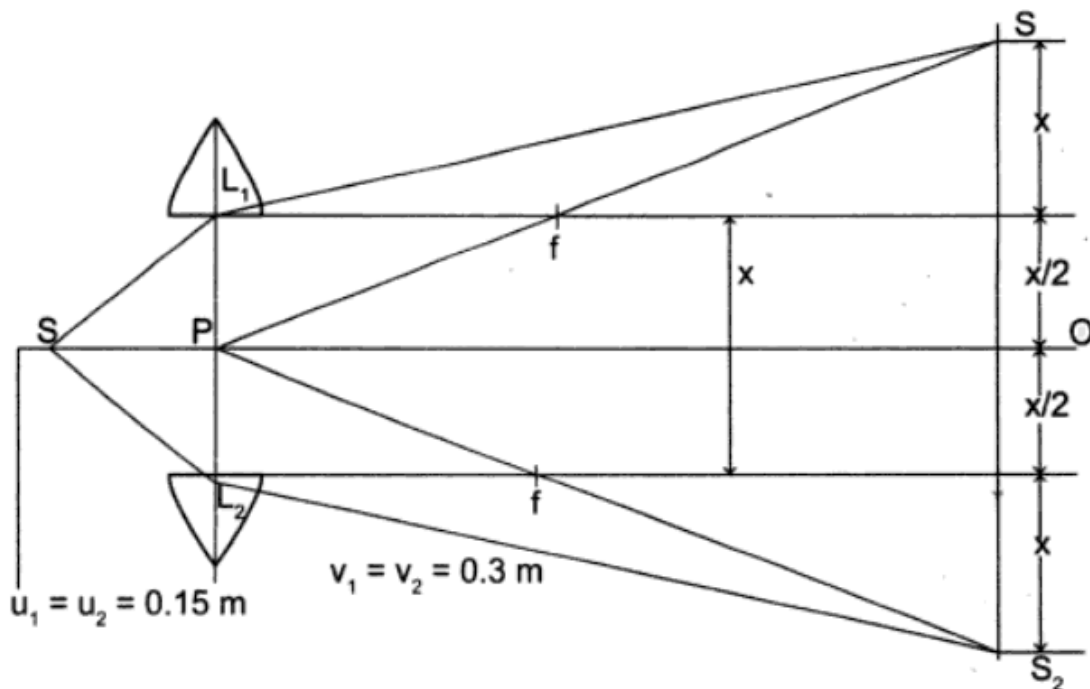


**Solution**

For  $L_1$  and  $L_2$ :  $u_1 = u_2 = 0.15$  m, and  $f = 0.1$  m

$$\therefore \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\text{or } \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{0.1} - \frac{1}{0.15}$$





or  $v = 0.3 \text{ m}$

$$\Rightarrow \frac{v_1}{u_1} = \frac{v_2}{u_2} = 2$$

Hence distance between two source  $S_1$  and  $S_2$   
 $= 3x = 1.5 \text{ mm}$

- (i) The maximum intensity on the screen is observed, when the distance of maximum from O,

$$x_n = n \frac{\lambda D}{d}$$

Given:  $n = 3,$

$$D = 1.3 - 0.3 = 1 \text{ m}$$

$$d = S_1 S_2 = 1.5 \text{ mm},$$

$$\lambda = 500 \text{ nm}$$

$$\therefore OA = x_3$$

$$= \frac{3 \times 500 \times 10^{-9} \times 1}{1.5 \times 10^{-3}} = 1 \times 10^{-3} \text{ m.}$$

- (ii) If gap between  $L_1$  and  $L_2$  is reduced from its original value of **0.5 mm**, then  $d$  will reduce and hence OA will increase.

**In a biprism experiment with sodium light, bands of width 0.0195 cm are observed at 100 cm from the slit. On introducing a convex lens 30 cm away from the slit, two images of the slit are seen 0.7 cm apart, at 100 cm distance from the slit. Calculate the wavelength of sodium light.**

**Solution.**  $\beta = \frac{\lambda D}{d}$  or  $\lambda = \frac{\beta d}{D}$   $\therefore$

Here,  $\beta = 0.0195 \text{ cm}$  ;  $D = 100 \text{ cm}$

For a convex lens  $\frac{1}{O} = \frac{v}{u}$ ,  $v + u = 100 \text{ cm}$

$$u = 30 \text{ cm} \quad \text{or} \quad \frac{0.7}{O} = \frac{70}{30} \text{ cm} \quad \text{or} \quad O = 0.30 \text{ cm}$$

*i.e.*, Distance between the two coherent sources

$$d = O = 0.30 \text{ cm}$$

$$\therefore \lambda = \frac{0.0195 \times 0.30}{100} = 5850 \times 10^{-8} \text{ cm} \quad \text{or} \quad \lambda = 5850 \text{ \AA}$$

**Interference bands are produced by a Fresnel's biprism in the focal plane of a reading microscope. The focal plane is 100 cm distant from the slit. A lens is inserted between the biprism and microscope and gives two images of the slit for two positions of lens. In one, separation between them is 4.05 mm and in other 2.90 mm. If sodium light is used, find the distance between interference bands. ' $\lambda$ ' for sodium light =  $5886 \times 10^{-8} \text{ cm}$ .**

**Solution.** Here  $\lambda = 5886 \times 10^{-8} \text{ cm}$  ;  $D = 100 \text{ cm}$ ;  $d_1 = 4.05 \text{ mm} = 0.405 \text{ cm}$

$$d_2 = 2.90 \text{ mm} = 0.290 \text{ cm}$$

$$d = \sqrt{d_1 d_2} = \sqrt{0.405 \times 0.290}$$

$$\beta = \frac{\lambda D}{d} = \frac{5886 \times 10^{-8} \times 100}{\sqrt{0.405 \times 0.290}} = 0.017 \text{ cm}$$

**A source of light of wavelength  $5000 \text{ \AA}$  is placed at one end of a table  $200 \text{ cm}$  long and  $5 \text{ mm}$  above its flat well polished top. Find the fringe-width of the interference bands located on a screen at the end of the table.**

**Solution.** Distance of source  $S$  from the table  
 $= 5 \text{ mm} = 0.5 \text{ cm}$

Distance of  $S'$  from table  $= 0.5 \text{ cm}$

If ' $d$ ' is the distance between  $S$  and  $S'$

$$d = 0.5 + 0.5 = 1 \text{ cm}$$

$$D = 200 \text{ cm}$$

$$\lambda = 5000 \text{ \AA} = 5000 \times 10^{-8} \text{ cm} = 5 \times 10^{-5} \text{ cm}$$

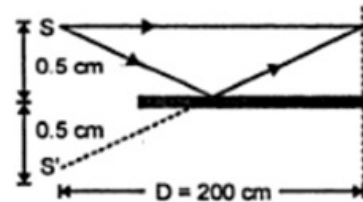
Since,

$$\beta = \frac{\lambda D}{d}$$

$\therefore$

$$\beta = \frac{5 \times 10^{-5} \times 200}{1} = 10^{-2} \text{ cm}$$

$$\beta = 0.01 \text{ cm}$$



**Two point sources of radiation:  $S_1$  and  $S_2$  radiate waves of same frequency and are excited by the same oscillator. They are also in phase with each other.  $S_1$  is placed at origin  $O$ , while  $S_2$  is placed at the point  $(0, 4)$  on  $y$ -axis. Find the points of maxima of received intensity if a detector is moved towards positive  $x$ -axis starting from origin. The wavelength of radiation is  $1 \text{ m}$ .**

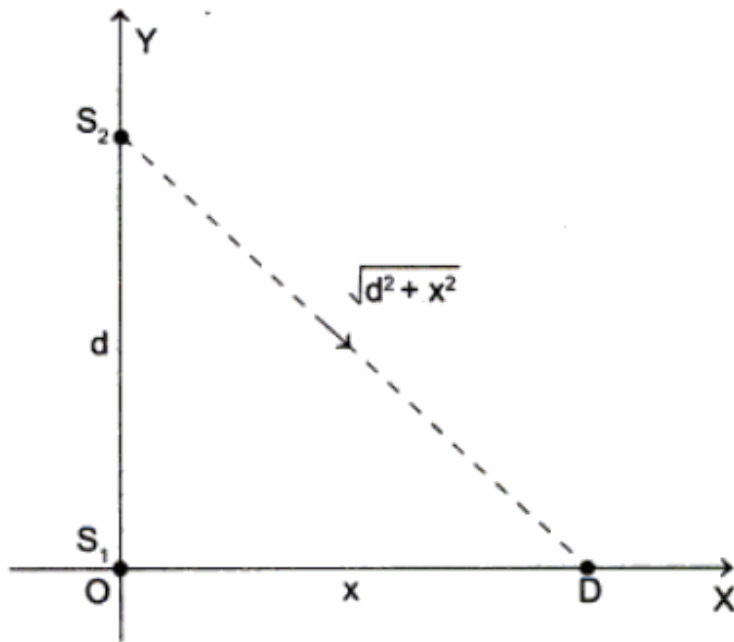
**Solution** Let the detector be at the point  
 $P(x, 0)$

$$\text{Let } S_1 S_2 = d = 4 \text{ m}$$

$\Rightarrow$  Path difference,

$$\Delta x = S_2 P - S_1 P$$

$$= \sqrt{x^2 + d^2} - x$$



For constructive interference

$$\begin{aligned} \Delta x &= n\lambda \\ &= \sqrt{x^2 + d^2} - x \end{aligned}$$

Using  $d = 4 \text{ m}$ ,  $\lambda = 1 \text{ m}$ , we have

$$\sqrt{x^2 + 16} - x = n$$

Solving, we get  $x = \frac{8}{n} - \frac{n}{2}$

It is easily seen that the path difference decreases from  $\Delta x \simeq 4 \text{ m}$  to  $(\Delta x \rightarrow 0 \text{ at } x \rightarrow \infty)$

Hence,  $n$  can have value  $n = 3, 2, 1$ .

Going away from O towards the  $x$ -axis,

first maxima occurs at  $x = \frac{8}{3} - \frac{3}{2} = \frac{7}{6} \text{ m}$  (for  $n = 3$ )

second maxima occurs at  $x = \frac{8}{2} - \frac{2}{2} = 3 \text{ m}$  (for  $n = 2$ )

third maxima occurs at  $x = \frac{8}{1} - \frac{1}{2} = 7.5 \text{ m}$  (for  $n = 1$ )

**The path difference of a certain point on screen in a double-slit experiment is one-eighth of the wavelength. Find the ratio of intensity at this point and the intensity at central maximum.**

**Solution** At central maximum, constructive interference occurs.

$$\therefore I_{\text{center}} = I_{\text{max}} (\Delta \phi = 0)$$

At the given point,  $\Delta \phi = \frac{2\pi}{\lambda}$  (Path difference)

$$= \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \frac{\pi}{4}$$

$$I = I_{\text{max}} \cos^2 \frac{\Delta \phi}{2} = I_{\text{max}} \cos^2 \left( \frac{\pi}{8} \right)$$

$$\therefore \frac{I}{I_{\text{max}}} = \frac{1 + (1/\sqrt{2})}{2} = \frac{\sqrt{2} + 1}{2\sqrt{2}}$$

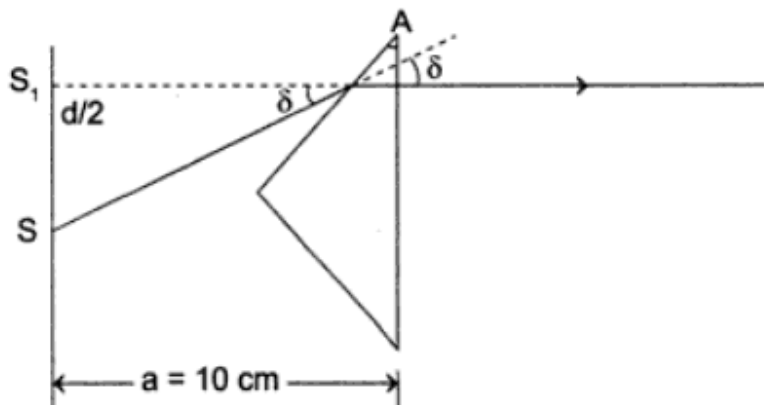
**A Fresnel's biprism of angle  $2^\circ$  is illuminated by light of wavelength  $\lambda = 5890 \text{ \AA}$  by a slit source 10 cm from it. Calculate the fringe width on a screen 90 cm from biprism, if the R.I. of biprism is 1.5.**

☞ **Solution** Here,  $\lambda = 5890 \times 10^{-8} \text{ cm}$   
 $D = 90 + 10 = 100 \text{ cm}$

$$\mu = 1.5$$

$$\text{Angle of biprism, } A = 2^\circ = \frac{\pi}{90} \text{ radians}$$

*The distance  $d$  is found as shown:*





The minimum deviation of the thin prism is given as

$$\delta = A(\mu - 1)$$

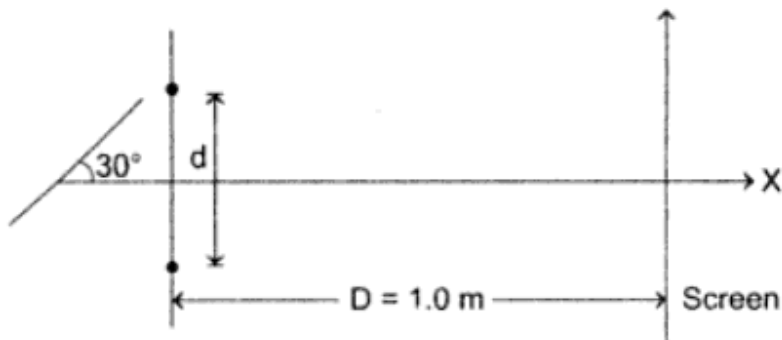
$$\therefore \tan \delta \cong \delta = \frac{d}{2a}$$

$$\begin{aligned} \therefore d &= 2a \times A(\mu - 1) \\ &= 2 \times 10 \times \frac{\pi}{90} (1.5 - 1) = \frac{\pi}{9} \text{ cm} \end{aligned}$$

$$\begin{aligned} \therefore w &= \frac{\lambda D}{d} \\ &= \frac{5890 \times 10^8 \times 100}{\frac{\pi}{9}} = \mathbf{0.01687 \text{ cm.}} \end{aligned}$$

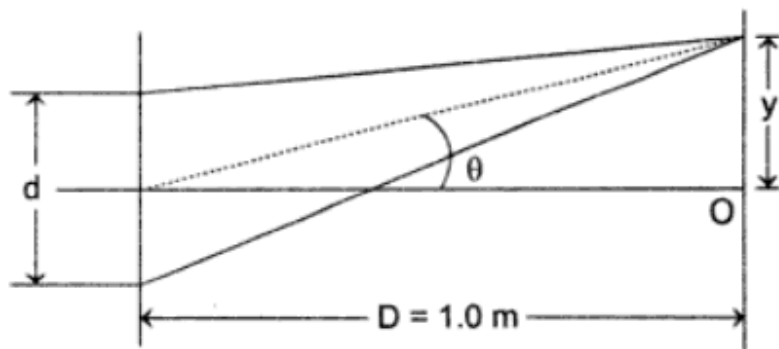
**A coherent parallel beam of microwave of wavelength  $\lambda = 0.5$  mm falls on a Young's double slit apparatus. The separation between the slits is 1.0 mm. The intensity of microwaves is measured on a screen placed parallel to the plane of the slits at a distance of 1.0 mm as shown in the figure.**

- (i) If the incident beam falls normally on the double slit apparatus, find the  $y$  co-ordinates of all the interference minima on the screen.**



(ii) If the incident beam makes an angle of  $30^\circ$  with the x-axis (as in the dotted arrow in the figure), find the y co-ordinates of the first minima on either side of the central maximum.

**Solution**



(i) The interference minima will occur on the screen provided

$$d \sin \theta = \left( m + \frac{1}{2} \right) \lambda; \quad m = 0, 1, 2, \dots$$

Only those values of  $m$  are allowed for which  $\sin \theta$  does not exceed  $\pm 1$ .

For the given data, we get

$$\sin \theta = \left(m + \frac{1}{2}\right) \frac{\lambda}{D} = \left(m + \frac{1}{2}\right)$$

$$\left(\frac{0.5 \text{ mm}}{1.0 \text{ mm}}\right) = \frac{1}{2} \left(m + \frac{1}{2}\right)$$

Thus, the allowed values of  $m$  are  $+1$ ,  $0$ ,  $-1$  and  $-2$ . Hence, four minima will be observed on the screen; distance on the screen is given by

$$y = D \tan \theta = 1.0$$

$$\left(\frac{\sin \theta}{\cos \theta}\right) = 1.0 \frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}}$$

$$\text{For } m = 1, \sin \theta = \frac{3}{4}, y = \left(\frac{3}{\sqrt{7}}\right) m$$

$$\text{For } m = 0, \sin \theta = \frac{1}{4}, y = \left(\frac{1}{\sqrt{15}}\right) m$$

$$\text{For } m = -1, \sin \theta = \frac{-1}{4}, y = -\frac{1}{\sqrt{15}} m$$

$$\text{For } m = -2, \sin \theta = \frac{-3}{4}, y = -\frac{3}{\sqrt{7}} m$$

(ii) For angle of incidence,  $\alpha = 30^\circ$ , the condition for minima is

$$d \sin \theta - d \sin \alpha = \left(m + \frac{1}{2}\right)\lambda,$$

$m = 0, 1, 2, \dots$

From given data, we get

$$\begin{aligned}\sin \theta &= \sin \alpha + \left(m + \frac{1}{2}\right)\frac{\lambda}{d} \\ &= \sin 30^\circ + \left(m + \frac{1}{2}\right)\left(\frac{0.5 \text{ mm}}{1.0 \text{ mm}}\right) \\ &= \frac{1}{2} + \left(m + \frac{1}{2}\right)\frac{1}{2} \\ &= \frac{1}{2}\left(m + \frac{3}{2}\right)\end{aligned}$$

---

For the central maximum, we have

$$d \sin \theta - d \sin \alpha = m\lambda, \text{ where, } m = 0$$

$$\text{This gives } \sin \theta = \frac{1}{2}, \quad y = \left(\frac{1}{\sqrt{3}}\right)m$$

First minimum above the central maximum occurs when

$$m = -1, \quad \sin \theta = \frac{1}{4}, \quad y = \left(\frac{1}{\sqrt{15}}\right)m$$

First minimum below the central maximum occurs when

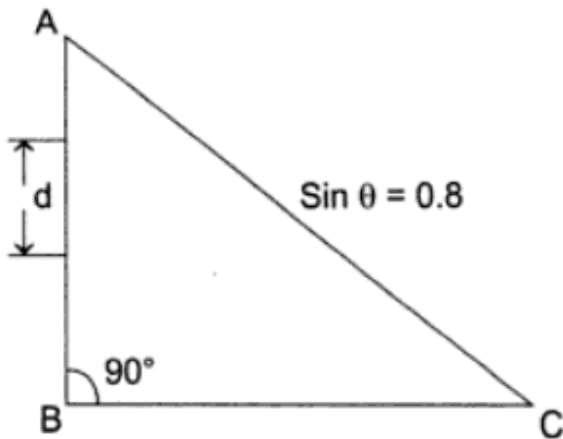
$$m = -2, \quad \sin \theta = \frac{-1}{4}, \quad y = -\left(\frac{1}{\sqrt{15}}\right)m$$

**Two parallel beams of light  $P$  and  $Q$  (separation  $d$ ) containing radiation of wavelengths  $4000 \text{ \AA}$  and  $5000 \text{ \AA}$  (which are mutually coherent in each wavelength separately) are incident normally on a prism as shown in the figure. The refractive index of the prism as a function of wavelength is given by the relation.**

$$\mu(\lambda) = 1.20 + \frac{b}{\lambda^2}$$

**where,  $\lambda$  is in  $\text{\AA}$  and  $b$  is a positive constant. The value of  $b$  is such that the condition for total reflection at the face  $AC$  is just satisfied for one wavelength and is not satisfied for the other.**

- (i) Find the value of  $b$ .**
- (ii) Find the deviation of the beams transmitted through the face  $AC$ .**
- (iii) A convergent lens is used to bring these transmitted beams into focus. If the intensities of the upper and the lower beams, immediately after transmission from the face  $AC$ , are  $4I$  and  $I$  respectively, find the resultant intensity at the focus.**



☞ **Solution** Greater the  $\mu$ , smaller the wavelength and smaller the critical angle.

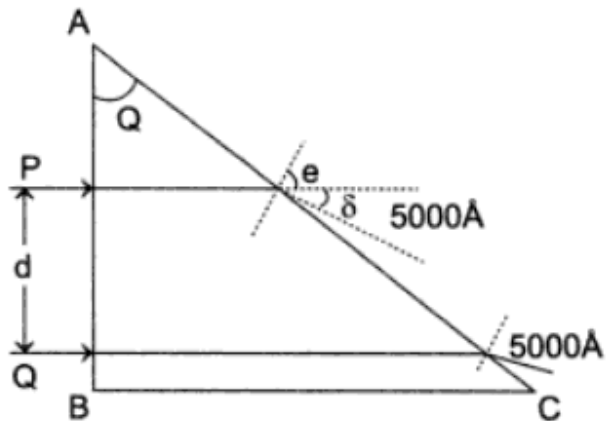
For just total internal reflection,

$$\theta = \text{critical angle.}$$

Critical angle for  $4000 \text{ \AA} = \theta$

$$\begin{aligned} \text{(i) } \mu_{4000} &= \frac{1}{\sin \theta} = \frac{1}{0.8} \\ &= 1.20 + \frac{b}{(4 \times 10^{-7})^2} \end{aligned}$$





$$\begin{aligned} \therefore b &= (1.25 - 1.20) (4 \times 10^{-7})^2 \\ &= 0.05 \times (4 \times 10^{-7})^2 \\ &= \mathbf{0.8 \times 10^{-14} \text{ m}^2} \end{aligned}$$

$$\begin{aligned} \text{(ii) } \mu_{5000} &= 1.20 + \frac{0.8 \times 10^{-14}}{25 \times 10^{-14}} \\ &= 1.20 + .032 = 1.232 \end{aligned}$$

Let,  $e$  = angle of emergence of the 5000 Å ray of light

Solve the remaining part

**In a Young's double slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes observed in the same segment of the screen is**

- (a) 12 (b) 18  
(c) 24 (d) 30

☞ **Explanation** Fringe width,  $w = \frac{D\lambda}{d}$  and

$$n_1 w_1 = n_2 w_2$$

$$\therefore n_2 = 12 \left( \frac{600}{400} \right) = 18$$

**The ratio of the intensity at the centre of a bright fringe to the intensity at a point one-quarter of the distance between two fringes from the centre is**

- (a) 1 (b) 1/2  
(c) 4 (d) 16

☞ **Explanation** Two waves of a single source having an amplitude  $A$  interfere. The resulting amplitude  $A_r$  is given by

$$A_r^2 = A_1^2 + A_2^2 + 2A_1A \cos \delta$$

where  $A_1 = A_2 = A$  and  $\delta =$  phase difference between the waves

$$\therefore I_r = I_1 + I_2 + 2\sqrt{I_1I_2} \cos \delta$$

When the maxima occurs at the centre,  $\delta = 0$

$$\therefore I_r = 4I \quad \dots(i)$$

Since the phase difference between two successive fringes is  $2\pi$ , the phase difference between two points separated by a distance equal to one quarter of the distance between

the two successive fringes,

$$\delta = (2\pi)\left(\frac{1}{4}\right) = \frac{\pi}{2} \text{ radians}$$

$$\Rightarrow I_{r_2} = 4I \cos^2\left(\frac{\pi/2}{2}\right) = 2I \quad \dots(ii)$$

From equations (i) and (ii), we get

$$\frac{I_{r_1}}{I_{r_2}} = \frac{4I}{2I} = 2$$



Clearly, the resolving power depends upon :

(i) the wavelength  $\lambda$  of the light (ii) the refractive index  $\mu$  of the medium between the object and the objective of the microscope (iii) the angle  $\theta$  subtended by the radius of the objective on one of the objects.

To increase the resolving power of a microscope,  $\mu$  is increased by using a suitable oil between the object and the objective. Such objectives are called oil-immersion objectives.

If the object is to be photographed and not seen visually,  $\lambda$  is increased by using ultraviolet light of wavelength 2750 Å. In this case, the glass lenses are to be replaced by quartz. This is because glass is opaque to ultraviolet light. In an electron microscope, an electron beam behaves like a wave of wavelength 1000 times smaller than those of visible light. It is for this reason that the resolving power of microscope is very large.

(viii) **Resolving power of a telescope.** The resolving power of a telescope is the reciprocal of the smallest angular separation between two distant objects whose images are separated in the telescope. This is given by  $d\theta = \frac{1.22\lambda}{a}$  where  $d\theta$  is the angle subtended by the point object at the objective,  $\lambda$  is the wavelength of light used and  $a$  is the diameter of the telescope objective. Clearly, a telescope with a larger aperture objective gives a high resolving power.

**In Fresnel biprism** both the sources  $S_1$  and  $S_2$  are virtual

$$D = a + b$$

$$d = 2a\delta = 2a(\mu - 1)\alpha$$

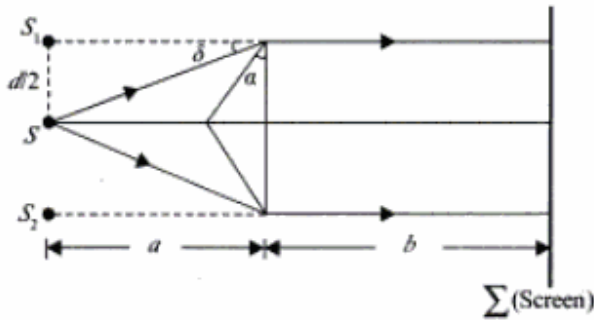
where  $\alpha$  is angle of biprism.

$$\beta = \frac{\lambda D}{d} = \frac{\lambda(a+b)}{2a(\mu-1)\alpha}$$

$$x_n = \frac{n\lambda D}{d} = \frac{n\lambda(a+b)}{2a(\mu-1)\alpha} \text{ for } n\text{th bright fringe.}$$

$$x_n = \frac{(2n-1)\lambda D}{2d} = \frac{(2n-1)\lambda(a+b)}{4\alpha(\mu-1)\alpha} \text{ for } n\text{th dark}$$

fringe.



**Fringe pattern in fresnel biprism**

If displacement method is used then  $d = \sqrt{d_1 d_2}$

If Fresnel biprism is immersed in a liquid of refractive index  $\mu'$ , then

$$\beta_{new} = \frac{\frac{\lambda}{\mu'}(a+b)}{2a\left(\frac{\mu}{\mu'}-1\right)\alpha} = \frac{\lambda(a+b)}{2a(\mu-\mu')\alpha}$$

**In Lloyd's Mirror:** Condition of  $n$ th bright and dark fringe obtained in Lloyd's mirror gets reversed to what was obtained in YDSE; because of reflection an additional phase shift of  $\pi$

or an additional path difference  $\frac{\lambda}{2}$  is achieved.

That is,  $x_n = \frac{n\lambda D}{d}$  for  $n$ th dark fringe

and  $x_n = \frac{(2n-1)\lambda D}{2d}$  for  $n$ th bright fringe.

In Lloyd's mirror one of the sources is real and other is virtual or image source.

Path difference  $= 2\mu t \cos r = (2n+1) \frac{\lambda}{2}$  for  $n$ th bright fringe and  $2\mu t \cos r = n\lambda$  for  $n$ th dark fringe. In reflected light

$$\left. \begin{array}{l} \text{Path difference } 2\mu t \cos r = n\lambda \\ 2\mu t \cos r = (2n+1) \frac{\lambda}{2} \end{array} \right\} \text{for refracted or transmitted light}$$

Interference	Diffraction
1. Fringes are formed due to superposition of wave trains emitted from two coherent sources.	Fringes are formed due to superposition of bent rays or due to superposition of secondary wavelets.
2. Intensity of each fringe is equal	Intensity falls as the fringe order increases.
3. Number of fringes is and quite large.	Number of fringes is finite (small).
4. Fringe width is equal for each fringe.	Fringe width of primary and secondary maxima are different.

**Brewster's Law** If light is incident on the interface of two media such that the angle between reflected and refracted radiations is  $90^\circ$  then reflected rays are completely polarised. Angle of incidence is called angle of polarisation ( $\theta_p$ ).

$$\text{Then } \mu = \tan \theta_p$$

**Malus Law** When the plane of polarisation is rotated by an angle  $\theta$  then intensity of emergent light is given by  $I = I_0 \cos^2 \theta$ .  $I_0$  is intensity of incident polarised light. In birefracting analysis there are two rays – ordinary and extraordinary. The extraordinary ray does not follow law of refraction. If the **velocity of extraordinary ray is greater** than that of **ordinary ray** such crystals are called **negative crystals**. Examples of negative crystal are Iceland spar, tourmaline, sapphire, ruby, emerald and apatite. If the **ordinary ray has higher velocity** than the **extraordinary ray** then such crystals are called **positive crystals**. Examples of positive crystals are quartz, iron oxide.

If the amplitude of two waves are unequal and angle between the two is  $\frac{\pi}{2}$  or path difference is  $\frac{\lambda}{4}$  then an elliptically polarised wave front results, it could be elliptically



polarised if amplitudes are equal but the angle between the two is  $0 < \theta < \pi/2$ .

In YDSE, an electron beam is used to obtain interference pattern. If speed of electrons is increased

- (a) no interference pattern will be observed
- (b) distance between the consecutive fringes will increase
- (c) distance between two consecutive fringes will decrease
- (d) distance between two consecutive fringes remains same

[IIT Screening 2005]

**Solution** (c)  $\lambda = \frac{h}{mv}$ ; if  $v$  increases,  $\lambda$  decreases.

Therefore  $\beta = \frac{\lambda D}{d}$  will decrease.

In YDSE the angular position of a point on the central maxima whose intensity is  $\frac{1}{4}$ th of the maximum intensity.

(a)  $\sin^{-1}\left(\frac{\lambda}{d}\right)$

(b)  $\sin^{-1}\left(\frac{\lambda}{2d}\right)$

(c)  $\sin^{-1}\left(\frac{\lambda}{3d}\right)$

(d)  $\sin^{-1}\left(\frac{\lambda}{4d}\right)$

[IIT Screening 2005]

**Solution** (c)  $2 \cos \theta/2 = 1$

$$\cos \theta/2 = \frac{1}{2} \text{ or } \phi = \frac{2\pi}{3}$$

$$\frac{2\pi}{\lambda} d \sin \theta = \frac{2\pi}{3} \text{ or } \phi = \sin^{-1}\left(\frac{\lambda}{3d}\right)$$

A YDSE uses a monochromatic source. The shape of the fringe formed on the screen, is

- (a) hyperbola (b) circle  
(c) straight line (d) parabola

**Solution** (c)

When an unpolarised light of intensity,  $I_o$  is incident on a polarising sheet, the intensity of the light which does not get transmitted is

- (a)  $\frac{I_o}{2}$  (b)  $I_o/4$   
(c) zero (d)  $I_o$

The intensity of principal maxima in the single slit diffraction pattern is  $I_o$ ? What will be its intensity when slit width is doubled?

- (a)  $2I_o$  (b)  $4I_o$   
(c)  $I_o$  (d)  $\frac{I_o}{2}$

**Solution** (c)

Two waves of intensity  $I$  undergo interference. The maximum intensity obtained is

- (a)  $I/2$  (b)  $2I$   
(c)  $I$  (d)  $4I$

**Solution** (d)  $I_{\max} = I + I + 2\sqrt{I} \sqrt{I} \cos \theta = 4I$  (for  $\theta = 0$ )

The wave theory in its original form was first postulated by

- (a) Issac Newton (b) Thomas Young  
(c) Christian Huygens (d) Augustin Jean Fresnel.

Solution ( c )



Select the right option.

- (a) Christian Huygens, a contemporary of Newton established the wave theory of light by assuming that light waves are transverse.
- (b) Maxwell provided the compelling theoretical evidence that light is transverse in nature.
- (c) Thomas Young experimentally proved the wave behaviour of light and Huygens assumption.
- (d) All the statements given above correctly answer the question, what is light.

Solution ( b )

In placing a thin sheet of mica of thickness  $12 \times 10^{-5} \text{ cm}$  in the path of one of the interfering beams in YDSE, the central fringe shifts equal to a fringe width. Find the refractive index of mica. Given  $\lambda = 600 \text{ nm}$ .

- (a) 1.5
- (b) 1.48
- (c) 1.61
- (d) 1.56

**Solution** (a)  $\frac{\lambda D}{d} = (\mu - 1) t \frac{D}{d}$  or  $\mu = \frac{\lambda}{t} + 1 = 1.5$

The waves emitted by a radio transmitter are

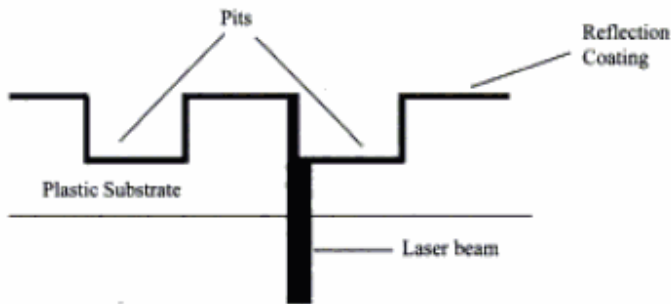
- (a) linearly polarised
- (b) unpolarised
- (c) monochromatic
- (d) elliptically polarised

**Solution** (a)

A CD (Compact disc) is read from the bottom by a semiconductor laser with wavelength  $790 \text{ nm}$  passing through a plastic substrate of refractive index 1.8. When the beam encounters a pit, part of the beam is reflected from the pit and part from the flat region. These two beams interfere with each other. What must be the minimum depth of the pit so that part of the beam reflected from the pit and part reflected from the flat surface cancel out? (This cancellation allows the player to recognise beginning and end of a pit).

- (a)  $0.197 \mu\text{m}$
- (b)  $0.395 \mu\text{m}$
- (c)  $0.22 \mu\text{m}$
- (d)  $0.11 \mu\text{m}$

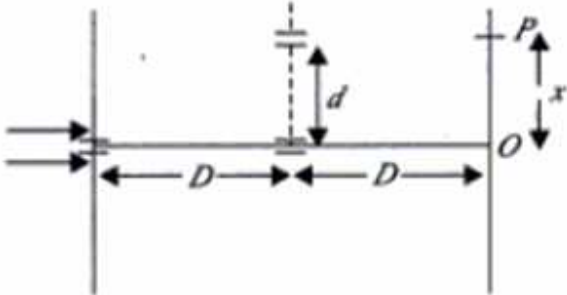
**Solution** (d)  $\lambda = \frac{\lambda_{air}}{\mu}$  and  $t = \frac{\lambda}{4} = \frac{\lambda_{air}}{4\mu} = \frac{790}{4 \times 1.8}$   
 $= 110 \text{ nm} = 0.11 \mu\text{m}.$

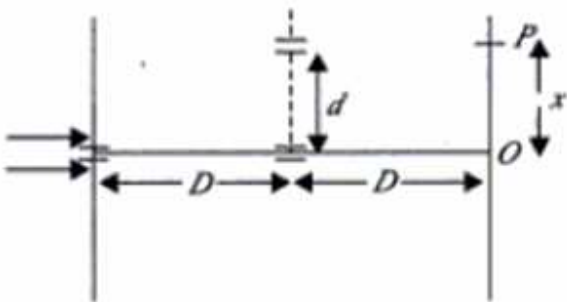


In a single slit diffraction pattern, (a) find the intensity at a point where the total phase difference between the wavelets from top to bottom of the slit is 66 rad. (b) If this point is  $7^\circ$  away from the central maxima. Find the width of slit. Given:  $\lambda = 600 \text{ nm}.$

**Solution** (a)  $I = I_0 \left[ \frac{\sin(33 \text{ rad})}{33 \text{ rad}} \right]^2 = 9.2 \times 10^{-4} I_0$

(b)  $a = \frac{B\lambda}{2\pi \sin \theta} = \frac{(66 \text{ rad}) 600 \times 10^{-9}}{2\pi \sin 7^\circ}$   
 $= 5.16 \times 10^{-5} \text{ m or } 0.052 \text{ mm (nearly).}$

Consider the arrangement shown in  The distance  $D$  is large compared to  $d$ . Find minimum value of  $d$  so that there is a dark fringe at  $O$ . For the same value of  $d$  find  $x$  at which next bright fringe is formed.



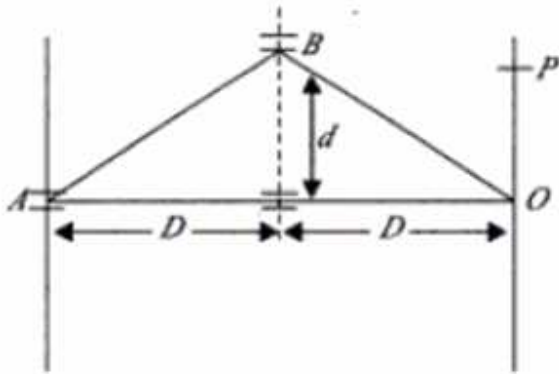
**Solution** Path difference =  $AB + BO - 2D$

$$2\sqrt{(D^2 + d^2)} - 2D = \frac{\lambda}{2}$$

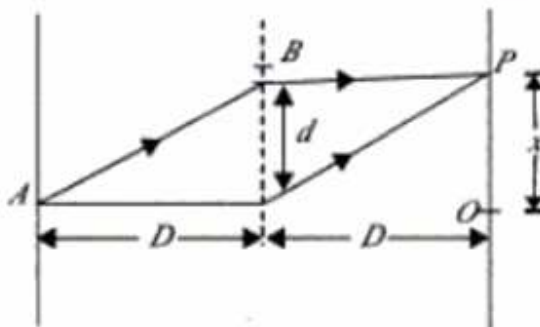
or  $2\sqrt{(D^2 + d^2)} = \frac{\lambda}{2} + 2D$

or  $4(D^2 + d^2) = \frac{\lambda^2}{4} + 4D^2 + 2\lambda D$

Eliminate  $\frac{\lambda^2}{4}$  as  $\lambda \ll D$ . or  $d = \sqrt{\frac{\lambda D}{2}}$



illustrates that if  $PO = x = d$ , path difference will be zero and we will observe first maxima.



A convex lens of diameter 8 cm is used to focus a parallel beam of light of wavelength 620 nm. Light is focussed at a distance 20 cm. from the lens. What would be the radius of central bright fringe?

**Solution** (a)  $R = \frac{1.22\lambda D}{r} = \frac{1.22 \times 620 \times 10^{-9} \times 0.2}{4 \times 10^{-2}}$   
 $= 3.8 \times 10^{-6} \text{ m.}$

A glass plate ( $n = 1.53$ ) that is  $485 \mu\text{m}$  thick and surrounded by air is illuminated by a beam of white light normal to the plate. (a) What wavelengths in the visible spectrum (400 to 700 nm) are intensified in the reflected beam? (b) What wavelengths are intensified in transmitted beam?

**Solution** (a) In reflected light  $2\mu t = (2n + 1) \frac{\lambda}{2}$

$$\lambda = \frac{4\mu t}{2n+1} = \frac{2970}{2n+1} \text{ nm} = 594 \text{ nm}, 424 \text{ nm}$$

(b) In transmitted light  $2\mu t = n\lambda$  or  $\lambda = \frac{2\mu t}{n} = \frac{1485}{n}$   
 $= 495 \text{ nm}$

In case of linearly polarised light the magnitude of electric field vector

- (a) varies periodically with time
- (b) increases and decreases linearly with time
- (c) does not change with time
- (d) is parallel to the direction of propagation

**Solution** (a)  $\because E = E_0 \sin(\omega t - kx)$ , it varies periodically with time.

**In a Young's Double Slit Experiment for interference of light, the slits are 0.2 cm apart and are illuminated by yellow light ( $\lambda = 600 \text{ nm}$ ). What would be the fringe width on a screen placed 1 m from the plane of slits if the whole system is immersed in water of index  $4/3$  ?**



**Sol.** Fringe width depends on medium  $D$  and  $d$ .  $\Rightarrow$  Fringe width  $= \frac{\lambda_m D}{d}$  ( $\lambda_m$  = wavelength in the medium)

By the definition of refractive index :  $\mu_m = \frac{\lambda_a}{\lambda_m} \Rightarrow \lambda_m = \frac{\lambda_a}{\mu_m} = \frac{600}{\frac{4}{3}} = \left(\frac{1800}{4}\right) \text{mm}$

Given that,  $D = 1 \text{ m}$  ;  $d = 0.2 \text{ cm}$

Then, Fringe width  $= \frac{\lambda_m D}{d} = \left(\frac{1800 \times 10^{-9}}{4 \times 0.2 \times 10^{-2}}\right) = \left(\frac{9}{4} \times 10^{-4}\right) \text{m} = 0.225 \text{ mm}$

[Ans. 0.225 mm]

**In Young's double slit experiment the slits are 0.5 mm apart and the interference is observed on a screen at a distance of 100 cm from the slit. It is found that the 9th bright fringe is at a distance of 7.5 mm from the second dark fringe from the center of the fringe pattern on same side. Find the wavelength of the light used.**

**Sol.** In Young's Double Slit Experiment, (YDSE), for bright pattern:  $Y_b = \pm \frac{n\lambda D}{d}$  ( $n = 0, 1, 2, \dots$ )

For dark pattern,  $y_d = \pm \frac{(2n-1)\lambda D}{2d}$  ( $n = 1, 2, 3, \dots$ )

For the second dark pattern above the point 'O',  $y_{2nd} = \frac{3\lambda D}{2d}$

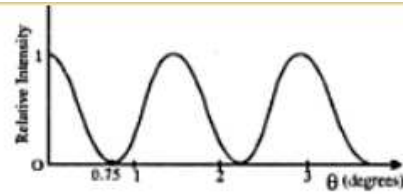
For the ninth bright pattern above the point 'O',  $Y_{9th} = \frac{9\lambda D}{d}$

Given that,  $Y_9 - Y_2 = 7.5 \text{ mm}$

$\Rightarrow \frac{9\lambda D}{d} - \frac{3\lambda D}{2d} = 7.5 \times 10^{-3} \Rightarrow \lambda = 5000 \text{ \AA}$

[Ans. 5000 Å]

**Light of wavelength 520 nm passing through a double slit, produces interference pattern of relative intensity versus deflection angle  $\theta$  as shown in the figure. Find the separation  $d$  between the slits.**



**Sol.** Formula of intensity,  $I = 4I_0 \cos^2 \frac{\Delta\phi}{2}$ , Where  $I_1 = I_2 = I_0$  then,  $\frac{I}{4I_0} = \cos^2 \frac{\Delta\phi}{2}$

$$\text{So, } I_r = \cos^2 \frac{\Delta\phi}{2}, \text{ here } I_r = \frac{I}{4I_0}$$

In the given question, when  $\theta = 0.75$ ,  $I_r = 0$ . By concept of YDSE, we know that,  $\Delta x = d \sin \theta = d\theta$

$$\text{then, } \Delta x = \frac{\lambda}{2} \text{ (For } I_r = 0) \Rightarrow \frac{\lambda}{2} = d\theta \Rightarrow d = \frac{\lambda}{2\theta} \quad (\text{where } \theta \text{ is in radian)}$$

Putting the values,  $\lambda = 520 \times 10^{-9} \text{m}$ ,  $\theta = \frac{0.75 \times \pi}{180} \text{rad}$ . In the above equation, we get  
 $d = 1.99 \times 10^{-5} \text{m}$

**[Ans.  $1.99 \times 10^{-2} \text{mm}$ ]**

**In a YDSE apparatus,  $d = 1 \text{mm}$ ,  $\lambda = 600 \text{nm}$  and  $D = 1 \text{m}$ . The slits produce same intensity on the screen. Find the minimum distance between two points on the screen having 75% intensity of the maximum intensity.**

**Sol.** For minimum distance, one point should be above the central maxima & the other point

should be below the central maxima  $I = 4I_0 \cos^2 \frac{\Delta\phi}{2}$  (Where  $I_1 = I_2 = I_0$ )

$$\text{Given that, } I = 75\% \text{ of maximum intensity} = \frac{75}{100} \times 4I_0 = 3I_0$$

We need the path difference at the point where intensity =  $3I_0$ . For this we need  $\Delta\phi$  at that

$$\text{point, } \Delta\phi = \frac{\pi}{3} \text{ (From the above equation)}$$

$$\text{So, } \frac{2\pi}{\lambda} (\Delta x) = \frac{\pi}{3} \Rightarrow \Delta x = \frac{\lambda}{6} \quad \& \quad \Delta x = \frac{\lambda}{6} = \frac{dy}{D} \Rightarrow y = \frac{yD}{6d}$$

Since one point is above & one below the central maxima, so, minimum distance,  $2y = 0.2 \text{mm}$

**[Ans.  $0.2 \text{mm}$ ]**

**The distance between two slits in a YDSE apparatus is 3mm. The distance of the screen from the slits is 1m. Microwaves of wavelength 1 mm are incident on the plane of the slits normally. Find the distance of the first maxima on the screen from the central maxima.**

### Solution

In YDSE, Path difference at a point P,  $\Delta x = S_2P - S_1P$   
 $= d \sin \theta$  For the maxima,  $\Delta x = \pm n\lambda \Rightarrow \sin \theta = \pm \left( \frac{n\lambda}{d} \right)$ .

Since  $\lambda$  and  $d$  are both comparable,  $\sin \theta \neq \theta$

( $\because \theta$  is not small)  $\Rightarrow \sin \theta = \pm \frac{n}{3}$

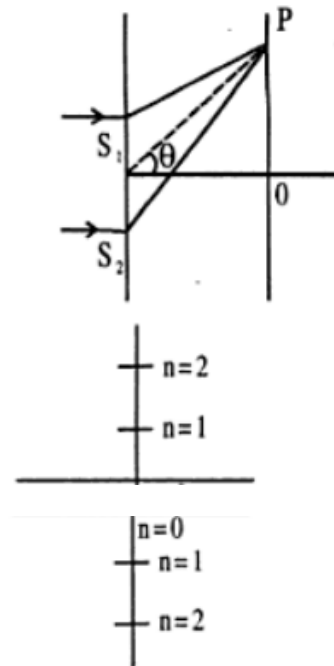
For 1<sup>st</sup> maxima,  $n = 1 \Rightarrow \sin \theta = \frac{1}{3}$

[For above the central maxima]

$\Rightarrow \tan \theta = \frac{1}{\sqrt{8}} = \frac{y}{D} \therefore y = \frac{D}{\sqrt{8}} = 35.35 \text{ cm}$

&  $n = 0, 1, 2$ , only for satisfying the equation  $\sin \theta = \pm \frac{n}{3}$

( $n = 3$  not possible as  $\theta = \frac{\pi}{2}$ )  $\Rightarrow \therefore 5$  maxima present.



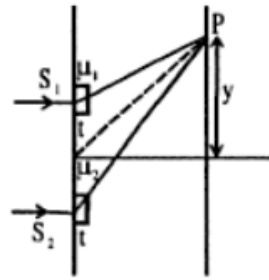
**[Ans. 35.35 cm app., 5]**

**One slit of a double slit experiment is covered by a thin glass plate of refractive index 1.4 and the other by a thin glass plate of refractive index 1.7. The point on the screen, where central bright fringe was formed before the introduction of the glass sheets, is now occupied by the 5<sup>th</sup> bright fringe. Assuming that both the glass plates have the same thickness and wavelength of light used is  $4800 \text{ \AA}$ , find their**

**Thickness**

### Solution

$$\begin{aligned} \text{Path difference at a point P, } \Delta x &= (S_2P - t + \mu_2 t) - (S_1P - t + \mu_1 t) \\ &= (S_2P - S_1P) + (\mu_2 - \mu_1)t \\ &= \frac{dy}{D} + (\mu_2 - \mu_1)t; (\because S_2P - S_1P = d \sin \theta \approx d \tan \theta = \frac{dy}{D}) \end{aligned}$$



$$\text{For 5th bright fringe, } \Delta x = 5\lambda; \therefore 5\lambda = \frac{dy}{D} + (\mu_2 - \mu_1)t$$

Given that 5th bright pattern is formed at the point where central bright fringe was formed before introducing the glass sheet.

$$\therefore 5\lambda = (\mu_2 - \mu_1)t (\because y = 0) \Rightarrow t = \frac{5\lambda}{\mu_2 - \mu_1} = \frac{5 \times 4800 \times 10^{-10}}{0.3} = 8 \times 10^{-6} \text{ m}$$

**8 μm]**

**A monochromatic light of  $\lambda = 5000 \text{ \AA}$  is incident on two slits separated by a distance of  $5 \times 10^{-4} \text{ m}$ . The interference pattern is seen on a screen placed at a distance of 1 m from the slits. A thin glass plate of thickness  $1.5 \times 10^{-6} \text{ m}$  & refractive index  $\mu = 1.5$  is placed between one of the slits & the screen. Find the intensity at the centre of the screen, if the intensity there is  $I_0$  in the absence of the plate. Also find the lateral shift of the central maximum.**

**Sol.** Path difference at a point P,  $\Delta x = (S_2P) - (S_1P - t + \mu t) = (S_2P - S_1P) + (t - \mu t)$   
 $= \frac{dy}{D} - t(\mu - 1)$

For finding intensity at the center of the screen, Put  $y = 0$ .

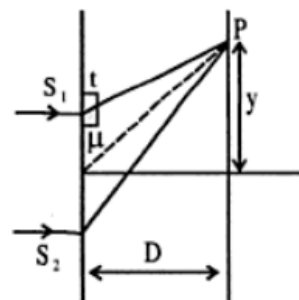
$$\therefore |\Delta x| = t(\mu - 1); \therefore \Delta \phi = \frac{2\pi}{\lambda} |\Delta x| = \frac{2\pi}{\lambda} \times t(\mu - 1)$$

$$I = I_0 \cos^2 \frac{D\phi}{2} \text{ Putting the values of } \Delta \phi$$

in the above equation we get,  $I = 0$

For lateral shift, Put  $n = 0$  (for the central maxima)

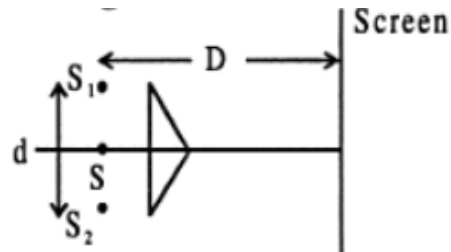
$$\therefore \Delta x = \pm n\lambda = 0 \Rightarrow \frac{dy}{D} = t(\mu - 1) \Rightarrow y = \frac{Dt(\mu - 1)}{d} \text{ Putting respective values, } y = 1.5 \text{ mm}$$



**[Ans. 0, 1.5 mm]**

**In a Biprism experiment with sodium light, bands of width 0.0195 cm are observed at 100 cm from slit. On introducing a convex lens 30 cm away from the slit between biprism and screen, two images of the slit are seen 0.7 cm apart at 100 cm distance from the slit. Calculate the wavelength of sodium light.**

In the Biprism experiment –



Here, S is the source of light and S<sub>1</sub> and S<sub>2</sub> are the images of S through the biprism.

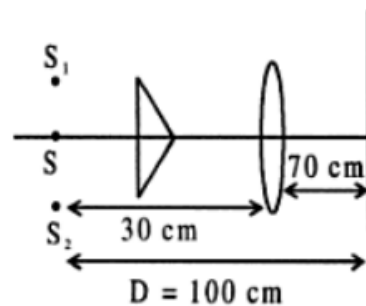
d = Distance between S<sub>1</sub> and S<sub>2</sub> ; D = Distance between slits and the screen.

$$\text{Then } \beta \text{ (fringe width)} = \frac{\lambda D}{d}$$

Introduce a convex lens 30 cm away from slit between biprism. From the given diagram, for convex lens, u = -30 cm ; v = 70 cm

$$\text{Then, } m = \frac{v}{u} = \frac{d'}{d} = \frac{h_1}{h_0} \Rightarrow \frac{70}{-30} = \frac{0.7}{d}, \text{ Given that, } d' = 0.7 \text{ cm} \Rightarrow d = -0.3 \text{ cm}$$

-ve sign shows that S<sub>1</sub> & S<sub>2</sub> positions have been interchanged. Also  $\beta$  = fringe width = 0.0195 cm



$$\therefore \beta = \frac{\lambda D}{d} \Rightarrow \lambda = \frac{d\beta}{D}$$

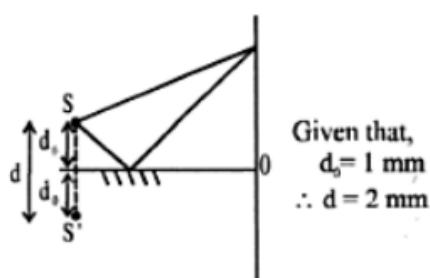
Putting values in the above equation we get,  $\lambda = 5.85 \times 10^{-7} \text{ m} = 5850 \text{ \AA}$   
 $\lambda = 5850 \text{ \AA}$

**A long narrow horizontal slit lies 1mm above a plane mirror. The interference pattern produced by the slit and its image is viewed on a screen distant 1m from the slit. The wavelength of light is 600nm. Find the distance of first maximum above the mirror.**

Solution

S' is the image of S. Now, S' & S will function as two slits as in YDSE.

$\therefore \Delta x = (S'P - SP) + \frac{\lambda}{2}$  (Where  $\frac{\lambda}{2}$  is the extra path changed due to reflection from mirror)



For 1st maxima,  $\Delta x = \lambda$  ;  $\therefore \lambda = \frac{d\lambda}{D} + \frac{\lambda}{2}$

$$\Rightarrow \frac{\lambda}{2} = \frac{dy}{D} ; \therefore y = \frac{\lambda D}{2d} = \frac{600 \times 10^{-9} \times 1}{2 \times 2 \times 10^{-3}} = 0.15 \text{ mm}$$

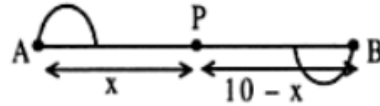
For bright pattern,  $\Delta x = \pm n\lambda$  putting  $n = 0$ , we get  $y < 0$  which means central bright is shifted below 0. [Ans. 0.15 mm]

**One radio transmitter A operating at 60.0 MHz is 10.0 m from another similar transmitter B that is  $180^\circ$  out of phase with transmitter A. How far must an observer move from transmitter A towards transmitter B along the line connecting A and B to reach the nearest point where the two beams are in phase ?**

Solution

Path difference at a point P,  $\Delta x = |AP - BP| = |x - (10-x)| = |2x - 10|$  So,  $\Delta\phi = \frac{2\pi}{\lambda}$ .

$\Delta x + \pi$  (Since sources are out of phase)



For waves to be in the same phase,  $\Delta\phi = 2n\pi$  where,  $n = 0, \pm 1, \pm 2$

$$\Rightarrow \frac{2\pi}{\lambda} \Delta x + \pi = 2n\pi \Rightarrow \frac{2\Delta x}{\lambda} = (2n - 1) \Rightarrow \frac{2|x - 10|}{\lambda} = (2n - 1)$$

$$\text{We know that } C = f\lambda \Rightarrow 3 \times 10^8 = 60 \times 10^5 \times \lambda \Rightarrow \lambda = 5 \text{ m}$$

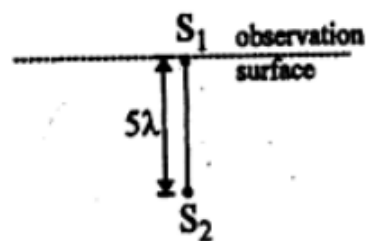
$$\therefore \frac{2}{5} |2x - 10| = (2n - 1) \Rightarrow \frac{2}{5} (2x - 10) = (2n - 1) \Rightarrow x = \frac{5(2n - 1)}{4} + 5$$

$$\text{Putting } n = 0, \quad x = \frac{15}{4} \text{ m} \Rightarrow \text{Putting } n = 1, \quad x = 5 + \frac{5}{4} = \frac{25}{4} \text{ m}$$

Putting  $n = -1$ ,  $x = \frac{5}{4} \text{ m}$ , Putting  $n = -2$ ,  $x < 0$  which is not possible. So minimum value

$$\text{of } x = \frac{5}{4} \text{ m} = 1.25 \text{ m} \quad [\text{Ans. } 1.25 \text{ m}]$$

**Two microwave coherent point sources emitting waves of wavelength  $\lambda$  are placed at  $5\lambda$  distance apart. The interference is being observed on a flat non-reflecting surface along a line passing through one source, in a direction perpendicular to the line joining the second source (refer figure) Considering  $\lambda$  as 4 mm, calculate the positions of maxima and draw shape of interference pattern. Take initial phase difference between the two sources to be zero.**





Path difference at a point i.e. P,  $\Delta x = S_2P - S_1P = \sqrt{(5\lambda)^2 + x^2} - x$

$$\therefore \Delta x = \pm n\lambda$$

For right of  $S_1$ , we take +ve values of n.

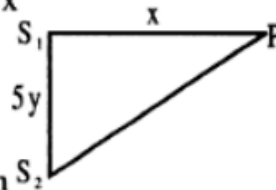
$$\therefore 25\lambda^2 + x^2 = (x + n\lambda)^2$$


Putting  $n = 1$ ,  $25\lambda^2 + x^2 = (x + \lambda)^2 \Rightarrow x = 12\lambda = 12 \times 4 = 48 \text{ mm}$

Putting  $n = 2$ ,  $25\lambda^2 + x^2 = (x + 2\lambda)^2 \Rightarrow x = \frac{21\lambda^2}{4\lambda} = \frac{21\lambda}{4} = 21 \text{ mm}$

Putting  $n = 3$ ,  $25\lambda^2 + x^2 = (x + 3\lambda)^2 \Rightarrow 16\lambda^2 - 6x\lambda = 0 \Rightarrow x = \frac{16\lambda}{6} = \frac{16 \times 4}{6} = \frac{32}{3} \text{ mm}$

Putting different values of n, we get  $x = 48 \text{ mm}, 21 \text{ mm}, \frac{32}{3} \text{ mm}, \frac{9}{2} \text{ mm}, 0 \text{ mm}$  and shape of fringe is circular in nature since line joining  $S_1$  &  $S_2$  is  $\perp$  to the screen



[Ans. 48, 21,  $\frac{32}{3}$ ,  $\frac{9}{2}$ , 0, m.m ; 

**A lens ( $\mu = 1.5$ ) is coated with a thin film of refractive index 1.2 in order to reduce the reflection from its surface at  $\lambda = 4800 \text{ \AA}$ . Find the minimum thickness of the film which will minimize the intensity of the reflected light.**

Solution

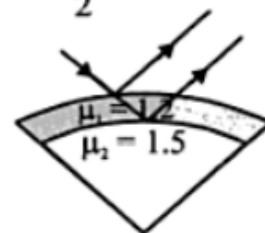
Path difference between two reflected ray in air,

$$\Delta X = 2\mu_1 t + \left( \frac{\lambda}{2} - \frac{\lambda}{2} \right) = 2\mu_1 t$$

For the minimum intensity,  $\Delta X = (2n - 1) \frac{\lambda}{2} \Rightarrow 2\mu_1 t = (2n - 1) \frac{\lambda}{2}$

$$t = \frac{(2n - 1)\lambda}{4\mu_1} = \frac{\lambda}{4\mu_1} \quad (\text{For minimum thickness, } n = 1)$$

$$t = \frac{\lambda}{4\mu_1} = \frac{4800 \times 10^{-10}}{4 \times 1.2} = 10^{-7} \text{ m [Ans. } 10^{-7} \text{ m]}$$



**A broad source of light of wavelength 680nm illuminates normally two glass plates 120mm long that meet at one end and are separated by a wire 0.048 mm in diameter at the other end. Find the number of bright fringes formed over the distance of 120 mm.**



### Solution

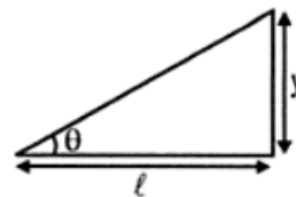
In the case of variable thickness (wedge shaped) film,

$$\text{Fringe width, } \beta = \frac{\lambda}{2\mu\theta} \Rightarrow \tan \theta = \theta = \frac{y}{\ell}$$

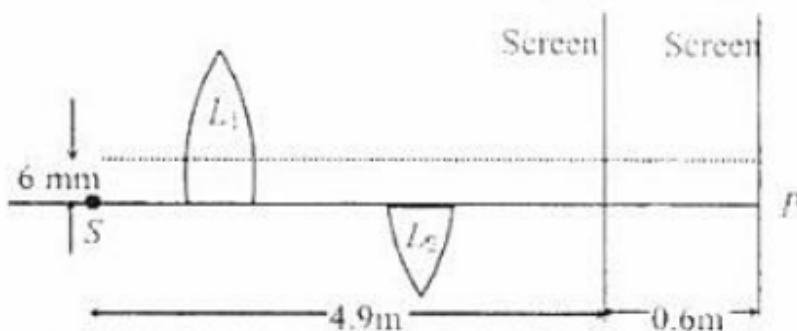
Let  $N$  be the no. of fringes formed over length ( $\ell$ )

$$\text{Then, } \ell = N\beta, \ell = N \left( \frac{\lambda}{2\mu\theta} \right) \Rightarrow N = \frac{2\mu\ell\theta}{\lambda}$$

$$\Rightarrow N = \frac{2\mu y}{\lambda} \quad (\because \theta = \frac{y}{\ell}) \Rightarrow N = \frac{2 \times 1 \times 0.048 \times 10^{-3}}{680 \times 10^{-9}} = (141) \quad (\because \mu = 1) \quad [\text{Ans. 141}]$$



**A thin convex lens of focal length  $f = 0.6 \text{ m}$  is cut into two unequal parts  $L_1$  and  $L_2$ . One part is shifted along the cutting plane axis as shown in the figure. A monochromatic line source  $S$ , perpendicular to the plane of paper, emitting light of wavelength  $\lambda = 600 \text{ nm}$ , is placed on the cutting plane axis. A screen with slits where the images of  $S$  is formed by these two pieces of the lens separately is placed perpendicular to the optical axis from the source at  $4.9 \text{ m}$ . There is an another screen placed at distance  $0.6 \text{ m}$  normal to optical axis where fringes are observed due to interference of the light passing through the holes. Find the position of central maximum from  $P$ . [Dotted line represents the principal axis of lens  $L_1$ ]**



Let  $x$  = distance between lenses,  $D$  is distance between the source and first screen. As images of the source due to both pieces of lens are observed on the screen, from displacement method,

$$x = \sqrt{D(D - 4f)} = \sqrt{4.9(4.9 - 2.4)} = 3.5 \text{ m}$$

Let  $m_1$  and  $m_2$  are the magnification by the lenses  $L_1$  and  $L_2$ ,  $m_1 = -\frac{D+x}{D-x} = -6$ ,  $m_2 = -\frac{1}{6}$

Let  $S_1$  is the image formed by  $L_1$  of  $S$  and  $h_1$  is the height of  $S_1$  form  $P' \Rightarrow h_1 = m_1 h = 36 \text{ mm}$

Let  $S_2$  is the image formed by  $L_2$  of  $S$  and  $h_2$  is the height of  $S_2$  form  $P' \Rightarrow h_2 = m_2 h = 1.0 \text{ mm}$

Distance between  $S_1$  and  $S_2$  ( $d$ ) =  $h_1 - h_2 = 36.0 - 1.0 = 35.0 \text{ mm}$

Position of central maximum  $O$  from  $P$  is  $6 + 1 + \frac{35}{2} = 24.5 \text{ mm}$

**In Young's experiment, the source is red light of wavelength  $7 \times 10^{-7} \text{ m}$ . When a thin glass plate of refractive index 1.5 at this wavelength is put in the path of one of the interfering beams, the central bright fringe shifts by  $10^{-3} \text{ m}$  to the position previously occupied by the 5th bright fringe. Find the thickness of the plate. When the source is now changed to green light of wavelength  $5 \times 10^{-7} \text{ m}$ , the central fringe shifts to a position initially occupied by the 6th bright fringe due to red light. Find the refractive index of glass for the green light. Also estimate the change in fringe width due to the change in wavelength. [JEE '97(I)]**

### Solution

(i) Path difference due to the glass slab,  $\Rightarrow \Delta x = (\mu - 1)t = (1.5 - 1)t = 0.5 t$

Due to this slab, 5 red fringes have been shifted upwards. Therefore

$$\Delta x = 5\lambda_{\text{red}} \text{ or } 0.5 t = (5) (7 \times 10^{-7} \text{ m})$$

$$\therefore t = \text{thickness of glass slab} = 7 \times 10^{-6} \text{ m}$$

(ii) Let  $\mu'$  be the refractive index for green light then,  $\Delta x' = (\mu' - 1) t$

Now the shifting is of 6 fringes of red light. Therefore,

$$\Delta x' = 6\lambda_{\text{red}} \quad \therefore t(\mu - 1) = 6\lambda_{\text{red}}$$

$$\therefore (\mu' - 1) = \frac{(6) \times (7 \times 10^{-7})}{7 \times 10^{-6}} = 0.6 \quad \Rightarrow \quad \mu' = 1.6$$

(iii) In part (i), shifting of 5 bright fringes was equal to  $10^{-3} \text{ m}$ , which implies that

$$5\omega_{\text{red}} = 10^{-3} \text{ m} \quad [\text{Here } \omega = \text{Fringe width}] \Rightarrow \omega_{\text{red}} = \frac{10^{-3}}{5} \text{ m} = 0.2 \times 10^{-3} \text{ m}$$

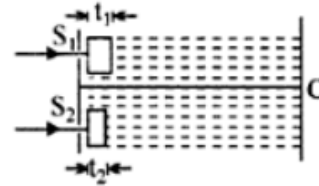
$$\text{Now since } \omega = \frac{\lambda D}{d} \text{ or } \omega \propto \lambda \quad \Rightarrow \quad \therefore \quad \frac{\omega_{\text{green}}}{\omega_{\text{red}}} = \frac{\lambda_{\text{green}}}{\lambda_{\text{red}}}$$

$$\therefore \omega_{\text{green}} = \omega_{\text{red}} \frac{\lambda_{\text{green}}}{\lambda_{\text{red}}} = (0.2 \times 10^{-3}) \left( \frac{5 \times 10^{-7}}{7 \times 10^{-7}} \right) \Rightarrow \omega_{\text{green}} = 0.143 \times 10^{-3} \text{ m}$$

$$\therefore \Delta\omega = \omega_{\text{green}} - \omega_{\text{red}} = (0.143 - 0.2) \times 10^{-3} \text{ m} \Rightarrow \Delta\omega = -5.71 \times 10^{-5} \text{ m}$$

**[Ans.  $7 \mu\text{m}$ , 1.6,  $400 / 7 \mu\text{m}$  (decrease)]**

A screen is at a distance  $D = 80$  cm from a diaphragm having two narrow slits  $S_1$  and  $S_2$  which are  $d = 2$  mm apart. Slit  $S_1$  is covered by a transparent sheet of thickness  $t_1 = 2.5 \mu\text{m}$  and  $S_2$  by another sheet of thickness  $t_2 = 1.25 \mu\text{m}$  as shown in the figure. Both sheets are made of same material having



refractive index  $\mu = 1.40$ . Water is filled in the space between diaphragm and screen. A monochromatic light beam of wavelength  $\lambda = 5000 \text{ \AA}$  is incident normally on the diaphragm. Assuming intensity of beam to be uniform and slits of equal width, calculate ratio of intensity at C to maximum intensity of interference pattern obtained on the screen, where C is foot of perpendicular bisector of  $S_1S_2$ . (Refractive index of water,  $\mu_w = 4/3$ )

Path difference at a point P in air,

$$\Delta x = (S_2P - t_2 + \mu t_2) - (S_1P - t_1 + \mu t_1) \Rightarrow \Delta x = (S_2P - S_1P) - (t_2 - t_1)\mu_1 + (t_2 - t_1)\mu_2$$

We know that,  $(S_2P - S_1P) = \frac{yd}{D}$  At point C,  $y = 0$  Hence,  $S_2P - S_1P = 0$  at point C,

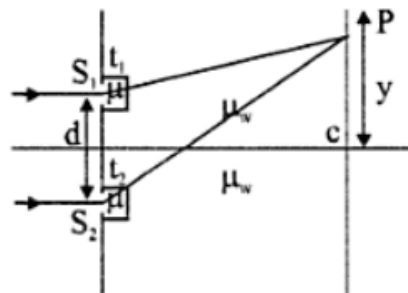
$$\text{Then, } \Delta x = (t_2 - t_1)\mu_2 - (t_2 - t_1)\mu_1$$

$$\Delta x = -\left(1.25 \times \frac{1}{15}\right) \mu\text{m} \quad ; \quad \Delta\phi = \frac{2\pi}{\lambda_{\text{air}}} \Delta x = \left(-\frac{\pi}{3}\right)$$

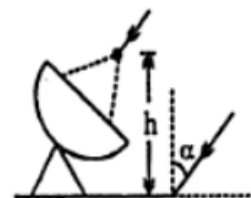
Intensity at a point C,

$$I_c = 4I_0 \cos^2 \frac{\Delta\phi}{2} \Rightarrow I_c = 3I_0 \text{ (since, } \Delta\phi = -\frac{\pi}{3}\text{)}$$

$$\text{Ratio} = \frac{I_c}{I_{\text{max}}} = \frac{3I_0}{4I_0} = \frac{3}{4} \quad [\text{Ans. } 3/4]$$



Radio waves coming vertically at  $\angle\alpha$  are received by a radar after reflection from a nearby water surface & directly. What should be the height of antenna from water surface so that it records a maximum intensity. (wavelength =  $\lambda$ ).

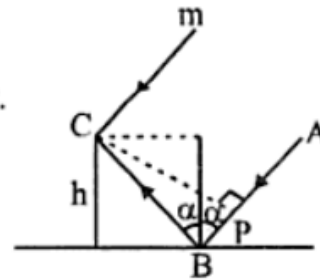


## Solution

For the maximum intensity path difference,  $\Delta x = n\lambda$

At the point C, two rays interfere, one is mc and other is ABC.

Then, path difference,  $\Delta x = (ABC + \frac{\lambda}{2}) - (mC)$



(since, in the case of reflection at point B, there is an extra path change equal to  $\frac{\lambda}{2}$ )

$$\Rightarrow \Delta x = (ABC - mC) + \frac{\lambda}{2} \Rightarrow \Delta x = (PB + BC) + \frac{\lambda}{2} \quad (\because AP = mC)$$

$$\Rightarrow \Delta x = \left[ \frac{h}{\cos \alpha} + \frac{h}{\cos \alpha} (\cos 2\alpha) \right] + \frac{\lambda}{2} \Rightarrow \Delta x = \frac{h}{\cos \alpha} (1 + \cos 2\alpha) + \frac{\lambda}{2}$$

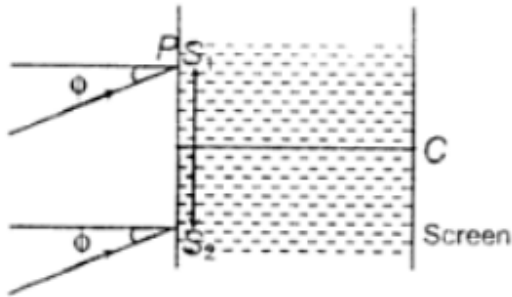
$$\Rightarrow \Delta x = \frac{2h \cos^2 \alpha}{\cos \alpha} + \frac{\lambda}{2} \Rightarrow \Delta x = 2h \cos \alpha + \frac{\lambda}{2}$$

$$\text{For the maximum intensity, } \Delta x = n\lambda \Rightarrow 2h \cos \alpha + \frac{\lambda}{2} = n\lambda \Rightarrow h = \frac{\lambda}{4 \cos \alpha} \quad (\text{for, } n = 1)$$

[Ans.  $\frac{\lambda}{4 \cos \alpha}$ ]

**In a young's double slit is experiment a parallel beam containing wavelengths  $\lambda = 4000 \text{ \AA}$  and  $\lambda_2 = 5600 \text{ \AA}$  is incident at an angle  $\phi = 30^\circ$  on a diaphragm having narrow slits at a separation  $d = 2 \text{ mm}$ . The screen is placed at a distance  $D = 40 \text{ cm}$  from the slits. A mica slab of thickness  $t = 5 \text{ mm}$  is placed in front of one of the slits and the whole apparatus is submerged in water. If the central bright fringe is observed at C, which is equidistant from both the slits. Calculate**

- the refractive index of the slab.
- the distance of the first black line from C.



**Solution**

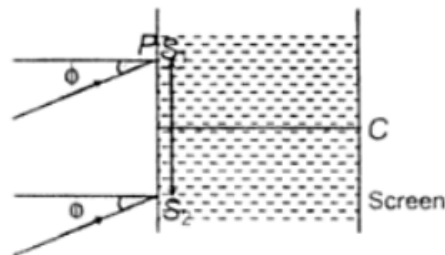
(a) Total phase difference at C,  $\Delta\phi = kd \sin \phi - Kt(\mu' - 1)$  for central maxima at

C,  $\Delta\phi = 0$   $\left[ \text{Here } \mu' = \frac{\mu_s}{\mu_w} \text{ \& } k = \frac{2\pi}{\lambda} \right] t = \frac{d \sin \phi}{(\mu' - 1)}$

$\Rightarrow \frac{2 \times 10^{-3} \times \sin 30^\circ}{(\mu' - 1)} = 5 \times 10^{-3} \quad \mu' = 1.2$

$\Rightarrow \mu = 1.2 \times (4/3) = 1.6$

Hence refractive index of mica slab = 1.6



(b) A black line is formed at the position where dark fringe is formed for both the wavelength.

The distance of the first black line from center bright line  $y = \frac{(2n - 1)\lambda D}{2d}$  .....(i)

For black line;  $\frac{(2n_1 - 1)\lambda_1 D}{2d} = \frac{(2n_2 - 1)\lambda_2 D}{2d}$

$\frac{(2n_2 - 1)\lambda_1 D}{2d} = \frac{(2n_2 - 1)\lambda_2 D}{2d} \quad \frac{(2n_1 - 1)}{(2n_2 - 1)} = \frac{\lambda_2}{\lambda_1}$ ; where  $\lambda_1 = \frac{\lambda_1}{\mu_w}$  and  $\lambda_2 = \frac{\lambda_2}{\mu_w}$



$$\frac{(2n_1 - 1)}{(2n_2 - 1)} = \frac{7}{5} \quad \text{For minimum value, } n_1 = 4 \text{ and } n_2 = 3.$$

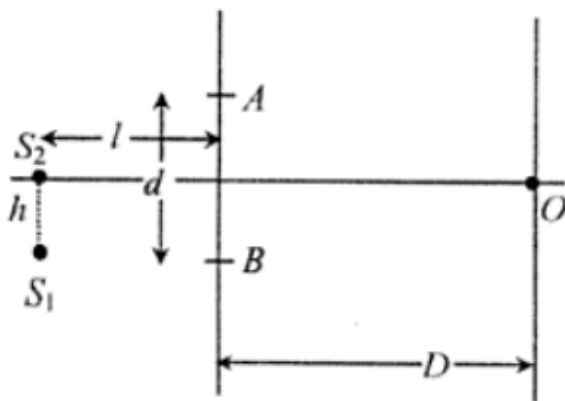
Hence distance of the first black line

$$y = \frac{(2 \times 4 - 1) 4000 \times 10^{-10} \times 40 \times 10^{-2} \times 3}{2 \times 2 \times 10^{-3} \times 4} = 2.1 \times 10^{-4} \text{ m} = 210 \mu\text{m}$$

**In a Young Double Slit Experiment, a source  $S_1$  of white light is kept at a distance  $h$  from the central line such that at the central point  $O$ , green light is missing. When another monochromatic green light source  $S_2$  is kept on central line it forms central maximum at  $O$ .**

- (a) Find the minimum distance,  $h_{\min}$  of source from central line.  
 (b) If  $h = 2 h_{\min}$ , at what minimum distance from point  $O$  maximum intensity of green light will appear. Assume intensity of the monochromatic source and that of green light in white light source to be same. ( $D = 1 \text{ m}$ ,  $d = 1 \text{ mm}$ ,  $l = 0.5 \text{ m}$ ,  $\lambda_{\text{green}} = 500 \text{ nm}$ )

Solution



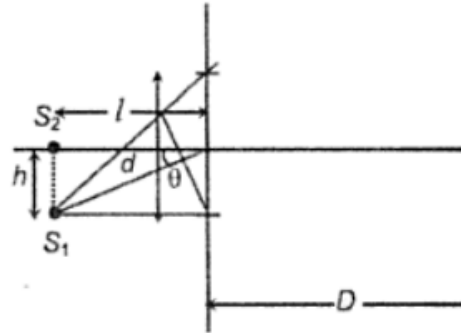
Here

(a)  $\Delta x = d \sin \theta = d \tan \theta = d \times \frac{h}{l}$  For green light to be missing  $\Delta$

$$x = \frac{n\lambda_g}{2} \Rightarrow h = \frac{n\lambda_g l}{2d}$$

For minimum  $h$ ,  $n$  should be equal to 1

$$\text{or } h_{\min} = \frac{\lambda_g l}{2d} = \frac{5 \times 10^{-7} \times 0.5}{2 \times 10^{-3}} = 1.25 \times 10^{-4} \text{ m}$$



Here fringe width  $\beta = \frac{\lambda_g D}{d} = \frac{5 \times 10^{-7} \times 1}{10^{-3}} = 5 \times 10^{-4} \text{ m}$ .

(b) If intensity due to  $S_2$  at any point on the screen is,  $I_2 = 4 I_0 \cos^2 \frac{\phi}{2}$ , then intensity due to

$S_1$  at the same point  $I_1 = 4 I_0 \cos^2 \left[ \frac{\phi + \phi_1}{2} \right]$  where,  $\phi_1 = \left( \frac{hd}{l} \right) \times \frac{2\pi}{\lambda} = 2 \times \frac{\lambda l}{2d} \times \frac{d}{l} \times \frac{2\pi}{\lambda} = 2\pi$

i.e.  $I_1 = 4 I_0 \cos^2 \left[ \frac{\phi + 2\pi}{2} \right] = 4 I_0 \cos^2 \frac{\phi}{2} = I_2 \quad \therefore \text{total intensity } I = I_1 + I_2 = 8 I_0 \cos^2 \frac{\phi}{2}$

$\therefore$  minimum distance of maximum from  $O = \frac{\beta}{2} = 2.5 \times 10^{-4} \text{ m}$

**In a Young Double Slit Experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes observed in the same segment of the screen is given by [JEE (Scr.) 2001]**

- (A) 12                      (B) 18                      (C) 24                      (D) 30

**Sol.(B)** Fringe width,  $\omega = \frac{\lambda D}{d} \propto \lambda$

When the wavelength is decreased from 600 nm to 400 nm, fringe width will also decrease by a factor of  $\frac{4}{6}$  or  $\frac{2}{3}$ . or the number of fringes in the same segment will increase by a factor of  $3/2$ . Therefore, number of fringes observed in the same segment =  $12 \times \frac{3}{2} = 18$

**Concept :**

Since  $\omega \propto \lambda$ , and if YDSE apparatus is immersed in a liquid of refractive index  $\mu$ , the wavelength  $\lambda$  and thus the fringe width will decrease  $\mu$  times.

$PR = d$

$\therefore PO = d \sec \theta$  and  $CO = PO \cot 2\theta = d \sec \theta \cos 2\theta$

path difference between the two rays is,  $\Delta x = CO + PO = (d \sec \theta + d \sec \theta \cos 2\theta)$

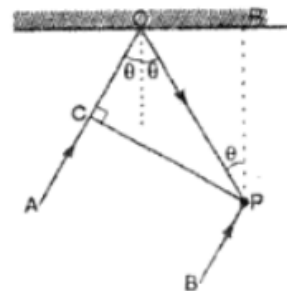
path difference between the two rays is,  $\Delta\phi = \pi$  (one is reflected, while another is direct)

Therefore, condition for constructive interference should be

$$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \dots$$

or  $d \sec \theta (1 + \cos 2\theta) = \frac{\lambda}{2}$

or  $\left(\frac{d}{\cos \theta}\right) (2 \cos^2 \theta) = \frac{\lambda}{2}$



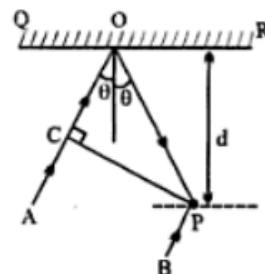
**In the adjacent diagram, CP represents a wavefront and AO and BP, the corresponding two rays. Find the condition on  $\theta$  for constructive interference at P between the ray BP and reflected ray OP. [JEE (Scr.) 2003]**

(A)  $\cos \theta = \frac{3\lambda}{2d}$

(B)  $\cos \theta = \frac{\lambda}{4d}$

(C)  $\sec \theta - \cos \theta = \frac{\lambda}{d}$

(D)  $\sec \theta - \cos \theta = \frac{4\lambda}{d}$



or  $\cos \theta = \frac{\lambda}{4d}$

**In a Young's Double Slit Experiment, two wavelengths of 500 nm and 700 nm were used. What is the minimum distance from the central maximum where their maximas coincide again? Take  $D/d = 10^3$ . Symbols have their usual meanings. [JEE 2004]**  
Let  $n_1$  bright fringe corresponding to wavelength  $\lambda_1 = 500$  nm coincides with  $n_2$  bright fringe corresponding to wavelength  $\lambda_2 = 700$  nm.

$$\therefore n_1 \frac{\lambda_1 D}{d} = n_2 \frac{\lambda_2 D}{d} \quad \text{or} \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{7}{5}$$

This implies that 7th maxima of  $\lambda_1$  coincides with 5th maxima of  $\lambda_2$ . Similarly 14th maxima of  $\lambda_1$  will coincide with 10th maxima of  $\lambda_2$  and so on.

$$\therefore \text{Minimum distance} = \frac{n_1 \lambda_1 D}{d} = 7 \times 5 \times 10^{-7} \times 10^{-3} = 3.5 \times 10^{-3} \text{ m} = 3.5 \text{ mm}$$

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<https://archive.org/details/PhysicsWaveOpticsPart7HuygensPrincipleCBSEClass12>

**14 solved examples in Wave Optics**

<https://archive.org/details/SolvedExampleOnNatureOfLightAndInterferenceWO18X3>

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**3 Videos in Polarization**

<https://archive.org/details/PolarizationByDichroismPOL08A>

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**8 Videos on Interference**

<https://archive.org/details/InterferenceDueToThinWedgeShapedFilmWO21A>

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### To recall standard integrals

$f(x)$	$\int f(x)dx$	$f(x)$	$\int f(x)dx$
$x^n$	$\frac{x^{n+1}}{n+1} \quad (n \neq -1)$	$[g(x)]^n g'(x)$	$\frac{[g(x)]^{n+1}}{n+1} \quad (n \neq -1)$
$\frac{1}{x}$	$\ln x $	$\frac{g'(x)}{g(x)}$	$\ln g(x) $
$e^x$	$e^x$	$a^x$	$\frac{a^x}{\ln a} \quad (a > 0)$
$\sin x$	$-\cos x$	$\sinh x$	$\cosh x$
$\cos x$	$\sin x$	$\cosh x$	$\sinh x$
$\tan x$	$-\ln \cos x $	$\tanh x$	$\ln \cosh x$
$\operatorname{cosec} x$	$\ln \tan \frac{x}{2} $	$\operatorname{cosech} x$	$\ln \tanh \frac{x}{2} $
$\sec x$	$\ln \sec x + \tan x $	$\operatorname{sech} x$	$2 \tan^{-1} e^x$
$\sec^2 x$	$\tan x$	$\operatorname{sech}^2 x$	$\tanh x$
$\cot x$	$\ln \sin x $	$\operatorname{coth} x$	$\ln \sinh x $
$\sin^2 x$	$\frac{x}{2} - \frac{\sin 2x}{4}$	$\sinh^2 x$	$\frac{\sinh 2x}{4} - \frac{x}{2}$
$\cos^2 x$	$\frac{x}{2} + \frac{\sin 2x}{4}$	$\cosh^2 x$	$\frac{\sinh 2x}{4} + \frac{x}{2}$

$f(x)$	$\int f(x) dx$	$f(x)$	$\int f(x) dx$
$\frac{1}{a^2+x^2}$	$\frac{1}{a} \tan^{-1} \frac{x}{a}$ $(a > 0)$	$\frac{1}{a^2-x^2}$	$\frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  \quad (0 <  x  < a)$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1} \frac{x}{a}$ $(-a < x < a)$	$\frac{1}{x^2-a^2}$	$\frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  \quad ( x  > a > 0)$
$\sqrt{a^2-x^2}$	$\frac{a^2}{2} \left[ \sin^{-1} \left( \frac{x}{a} \right) + \frac{x\sqrt{a^2-x^2}}{a^2} \right]$	$\frac{1}{\sqrt{a^2+x^2}}$	$\ln \left  \frac{x+\sqrt{a^2+x^2}}{a} \right  \quad (a > 0)$
		$\frac{1}{\sqrt{x^2-a^2}}$	$\ln \left  \frac{x+\sqrt{x^2-a^2}}{a} \right  \quad (x > a > 0)$
		$\sqrt{a^2+x^2}$	$\frac{a^2}{2} \left[ \sinh^{-1} \left( \frac{x}{a} \right) + \frac{x\sqrt{a^2+x^2}}{a^2} \right]$
		$\sqrt{x^2-a^2}$	$\frac{a^2}{2} \left[ -\cosh^{-1} \left( \frac{x}{a} \right) + \frac{x\sqrt{x^2-a^2}}{a^2} \right]$

### Some series Expansions -



$$\frac{\pi}{2} = \left(\frac{2}{1}\frac{2}{3}\right)\left(\frac{4}{3}\frac{4}{5}\right)\left(\frac{6}{5}\frac{6}{7}\right)\left(\frac{8}{7}\frac{8}{9}\right)\dots$$

$$\pi = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \frac{4}{13} - \dots$$

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots$$

$$\pi = \sqrt{12} \left(1 - \frac{1}{3 \cdot 3} + \frac{1}{5 \cdot 3^2} - \frac{1}{7 \cdot 3^3} + \dots\right)$$

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\int_0^{\pi/2} \log \sin x \, dx = -\frac{\pi}{2} \log 2 = \frac{\pi}{2} \log \frac{1}{2}$$

Solve a series problem

If  $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$  upto  $\infty = \frac{\pi^2}{6}$ , then value of

$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$  up to  $\infty$  is

- (a)  $\frac{\pi^2}{4}$       (b)  $\frac{\pi^2}{6}$       (c)  $\frac{\pi^2}{8}$       (d)  $\frac{\pi^2}{12}$

Ans. (c)

**Solution** We have  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$  upto  $\infty$

$$= \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \dots \text{ upto } \infty$$

$$- \frac{1}{2^2} \left[1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots\right]$$

$$= \frac{\pi^2}{6} - \frac{1}{4} \left(\frac{\pi^2}{6}\right) = \frac{\pi^2}{8}$$

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \dots \text{ upto } \infty = \frac{\pi^2}{12}$$

$$\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \dots \text{ upto } \infty = \frac{\pi^2}{24}$$

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$$\frac{\sin \sqrt{x}}{\sqrt{x}} = 1 - \frac{x}{3!} + \frac{x^2}{5!} - \frac{x^3}{7!} + \frac{x^4}{9!} - \frac{x^5}{11!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots = \sum_{k=0}^n \frac{(-1)^k x^{2k}}{(2k)!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots = \sum_{k=0}^n \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots = \sum_{k=0}^n \frac{x^{2k}}{(2k)!}$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots = \sum_{k=0}^n \frac{x^{2k+1}}{(2k+1)!}$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \quad (-1 \leq x < 1)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \dots + \frac{2^{2n} (2^{2n} - 1) B_n x^{2n-1}}{(2n)!} + \dots \quad |x| < \frac{\pi}{2}$$

$$\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \dots + \frac{E_n x^{2n}}{(2n)!} + \dots \quad |x| < \frac{\pi}{2}$$

$$\csc x = \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^5}{15120} + \dots + \frac{2(2^{2n-1} - 1) B_n x^{2n-1}}{(2n)!} + \dots \quad 0 < |x| < \pi$$

$$\cot x = \frac{1}{x} - \frac{x}{3} + \frac{x^3}{45} - \frac{2x^5}{945} - \dots - \frac{2^{2n} B_n x^{2n-1}}{(2n)!} - \dots \quad 0 < |x| < \pi$$

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$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \dots$$

$$\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{4} + \dots$$

$$\log(\cos x) = -\frac{x^2}{2} - \frac{2x^4}{4} - \dots$$

$$\log(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \dots$$

$$\sin^{-1} x = x + \frac{1}{2} \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \frac{x^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{x^7}{7} + \dots \quad |x| < 1$$

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$= \frac{\pi}{2} - \left( x + \frac{1}{2} \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \frac{x^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{x^7}{7} + \dots \right) \quad |x| < 1$$

$$\tan^{-1} x = \begin{cases} x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots & |x| < 1 \\ \pm \frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \dots & \begin{cases} + \text{if } x \geq 1 \\ - \text{if } x \leq -1 \end{cases} \end{cases}$$

$$\sec^{-1} x = \cos^{-1} \left( \frac{1}{x} \right)$$

$$= \frac{\pi}{2} - \left( \frac{1}{x} + \frac{1}{2 \cdot 3x^3} + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5x^5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7x^7} + \dots \right) \quad |x| > 1$$

$$\csc^{-1} x = \sin^{-1} (1/x)$$

$$= \frac{1}{x} + \frac{1}{2 \cdot 3x^3} + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5x^5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7x^7} + \dots \quad |x| > 1$$

$$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$$

$$= \begin{cases} \frac{\pi}{2} - \left( x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \right) & |x| < 1 \end{cases}$$

$$\begin{cases} p\pi + \frac{1}{x} - \frac{1}{3x^3} + \frac{1}{5x^5} + \dots & \begin{cases} p = 0 \text{ if } x \geq 1 \\ p = 1 \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\ln x = 2 \left[ \frac{x-1}{x+1} + \frac{1}{3} \left( \frac{x-1}{x+1} \right)^3 + \frac{1}{5} \left( \frac{x-1}{x+1} \right)^5 + \dots \right]$$

$$= 2 \sum_{n=1}^{\infty} \frac{1}{2n-1} \left( \frac{x-1}{x+1} \right)^{2n-1} \quad (x > 0)$$

$$\ln x = \frac{x-1}{x} + \frac{1}{2} \left( \frac{x-1}{x} \right)^2 + \frac{1}{3} \left( \frac{x-1}{x} \right)^3 + \dots$$

$$= \sum_{n=1}^{\infty} \frac{1}{n} \left( \frac{x-1}{x} \right)^n \quad \left( x > \frac{1}{2} \right)$$

$$\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \dots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} (x-1)^n \quad (0 < x \leq 2)$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \dots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} x^n \quad (|x| < 1)$$

$$\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots \infty \quad (-1 \leq x < 1)$$

$$\log_e(1+x) - \log_e(1-x) =$$

$$\log_e \frac{1+x}{1-x} = 2 \left( x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \infty \right) \quad (-1 < x < 1)$$

$$\log_e \left( 1 + \frac{1}{n} \right) = \log_e \frac{n+1}{n} = 2 \left[ \frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \dots \infty \right]$$

$$\log_e(1+x) + \log_e(1-x) = \log_e(1-x^2) = -2 \left( \frac{x^2}{2} + \frac{x^4}{4} + \dots \infty \right) \quad (-1 < x < 1)$$

$$\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots = \frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \dots$$

## Important Results

$$(i) (a) \int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cos^n x}{\sin^n x + \cos^n x} dx$$

$$(b) \int_0^{\pi/2} \frac{\tan^n x}{1 + \tan^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{dx}{1 + \tan^n x}$$

$$(c) \int_0^{\pi/2} \frac{dx}{1 + \cot^n x} = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{1 + \cot^n x} dx$$

$$(d) \int_0^{\pi/2} \frac{\tan^n x}{\tan^n x + \cot^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{\tan^n x + \cot^n x} dx$$

$$(e) \int_0^{\pi/2} \frac{\sec^n x}{\sec^n x + \operatorname{cosec}^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\operatorname{cosec}^n x}{\sec^n x + \operatorname{cosec}^n x} dx \text{ where, } n \in R$$

$$(ii) \int_0^{\pi/2} \frac{a^{\sin x}}{a^{\sin x} + a^{\cos x}} dx = \int_0^{\pi/2} \frac{a^{\cos x}}{a^{\sin x} + a^{\cos x}} dx = \frac{\pi}{4}$$

$$(iii) (a) \int_0^{\pi/2} \log \sin x dx = \int_0^{\pi/2} \log \cos x dx = -\frac{\pi}{2} \log 2$$

$$(b) \int_0^{\pi/2} \log \tan x dx = \int_0^{\pi/2} \log \cot x dx = 0$$

$$(c) \int_0^{\pi/2} \log \sec x dx = \int_0^{\pi/2} \log \operatorname{cosec} x dx = \frac{\pi}{2} \log 2$$

$$(iv) (a) \int_0^{\infty} e^{-ax} \sin bx dx = \frac{b}{a^2 + b^2}$$

$$(b) \int_0^{\infty} e^{-ax} \cos bx dx = \frac{a}{a^2 + b^2}$$

$$(c) \int_0^{\infty} e^{-ax} x^n dx = \frac{n!}{a^{n+1}}$$

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$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln(x + \sqrt{x^2 - a^2}) + C$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2}) + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left(\frac{x-a}{x+a}\right) + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left(\frac{a+x}{a-x}\right) + C$$

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{a^2 + x^2} dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \sinh^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \cosh^{-1}\left(\frac{x}{a}\right) + C$$

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