Spoon Feeding Pair of Straight Lines

Simplified Knowledge Management Classes Bangalore

My name is Subhashish Chattopadhyay. I have been teaching for IIT-JEE, Various International Exams (such as IMO [International Mathematics Olympiad], IPhO [International Physics Olympiad], IChO [International Chemistry Olympiad]), IGCSE (IB), CBSE, I.Sc, Indian State Board exams such as WB-Board, Karnataka PU-II etc since 1989. As I write this book in 2016, it is my 25 th year of teaching. I was a Visiting Professor to BARC Mankhurd, Chembur, Mumbai, Homi Bhabha Centre for Science Education (HBCSE) Physics Olympics camp BARC Campus.

I am Life Member of …

- **IAPT (Indian Association of Physics Teachers)**
- **IPA (Indian Physics Association)**
- **AMTI (Association of Mathematics Teachers of India)**
- **National Human Rights Association**
- **Men's Rights Movement (India and International)**
- **MGTOW Movement (India and International)**

And also of

IACT (Indian Association of Chemistry Teachers)

The selection for National Camp (for Official Science Olympiads - Physics, Chemistry, Biology, Astronomy) happens in the following steps ….

1) **NSEP** (National Standard Exam in Physics) and **NSEC** (National Standard Exam in Chemistry) held around 24 rth November. Approx 35,000 students appear for these exams every year. The exam fees is Rs 100 each. Since 1998 the IIT JEE toppers have been topping these exams and they get to know their rank / performance ahead of others.

2) **INPhO** (Indian National Physics Olympiad) and **INChO** (Indian National Chemistry Olympiad). Around 300 students in each subject are allowed to take these exams. Students coming from outside cities are paid fair from the Govt of India.

3) The Top 35 students of each subject are invited at HBCSE (Homi Bhabha Center for Science Education) Mankhurd, near Chembur, BARC, Mumbai. After a 2-3 weeks camp the top 5 are selected to represent India. The flight tickets and many other expenses are taken care by Govt of India.

Since last 50 years there has been no dearth of "Good Books". Those who are interested in studies have been always doing well. This e-Book does not intend to replace any standard text book. These topics are very old and already standardized.

There are 3 kinds of Text Books

- The thin Books - Good students who want more details are not happy with these. Average students who need more examples are not happy with these. Most students who want to "Cram" quickly and pass somehow find the thin books "good" as they have to read less !!

- The Thick Books - Most students do not like these, as they want to read as less as possible. Average students are "busy" with many other things and have no time to read all these.

- The Average sized Books - Good students do not get all details in any one book. Most bad students do not want to read books of "this much thickness" also !!

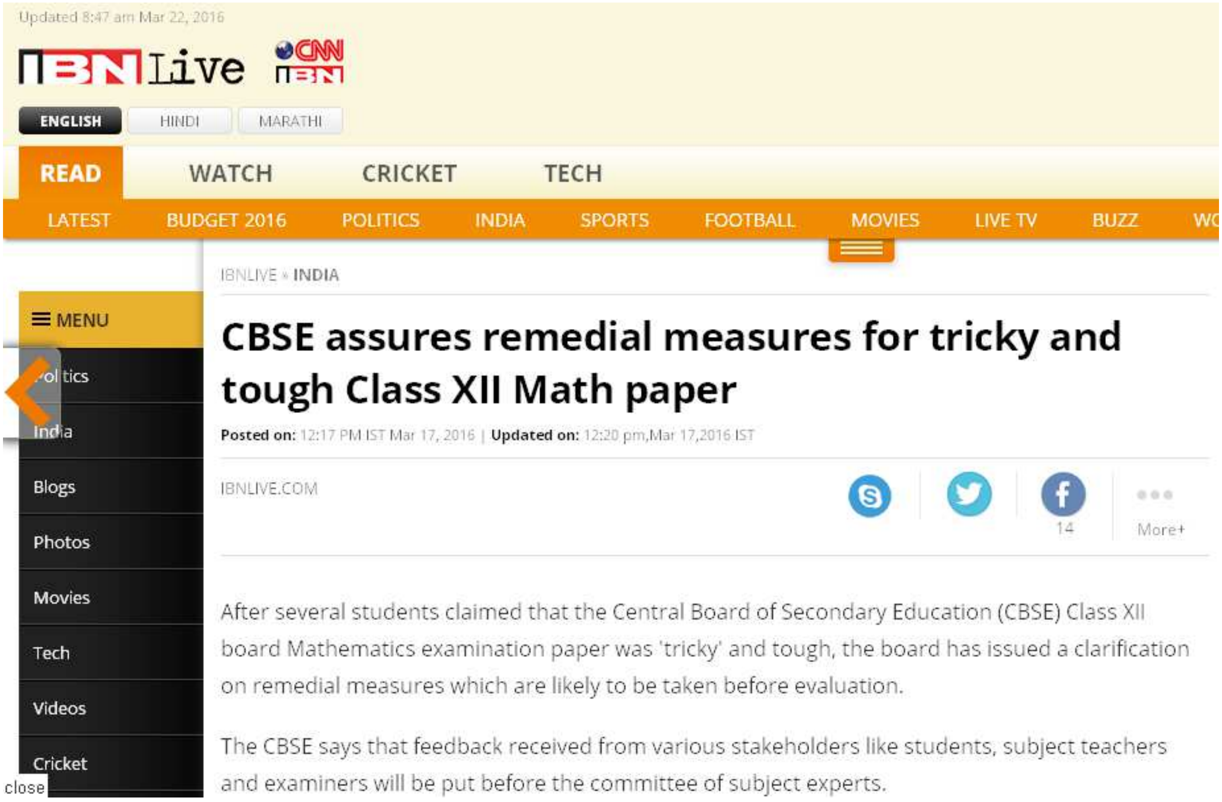
We know there can be no shoe that's fits in all.

Printed books are not e-Books! Can't be downloaded and kept in hard-disc for reading "later" ………

So if you read this book later, you will get all kinds of examples in a single place. This becomes a very good "Reference Material". I sincerely wish that all find this "very useful".

Students who do not practice lots of problems, do not do well. The rules of "doing well" had never changed …. Will never change !

After 2016 CBSE Mathematics exam, lots of students complained that the paper was tough!



Updated 8:47 am Mar 22, 2016

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IBNLIVE » INDIA

CBSE assures remedial measures for tricky and tough Class XII Math paper

Posted on: 12:17 PM IST Mar 17, 2016 | **Updated on:** 12:20 pm,Mar 17,2016 IST

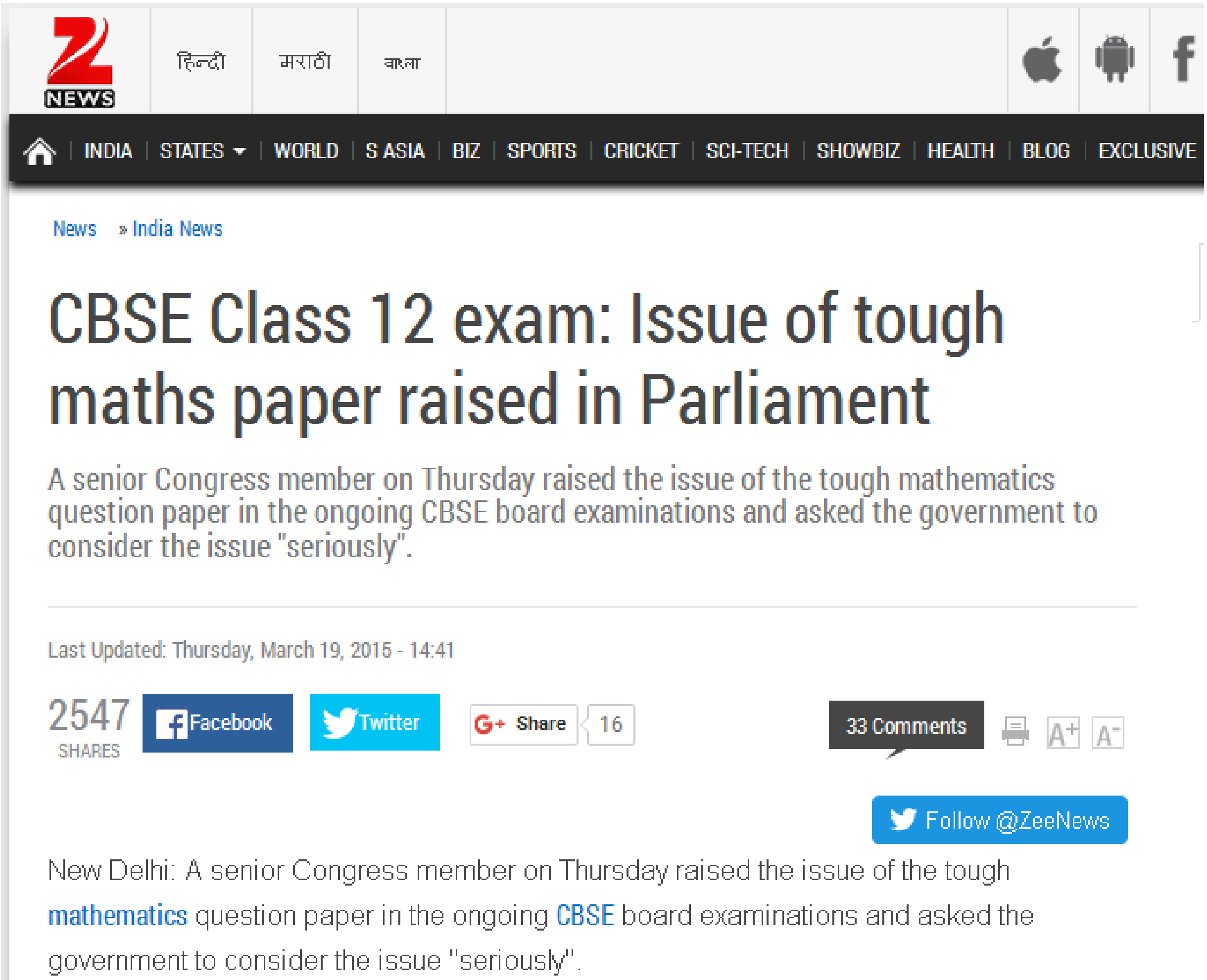
IBNLIVE.COM

14 More+

After several students claimed that the Central Board of Secondary Education (CBSE) Class XII board Mathematics examination paper was 'tricky' and tough, the board has issued a clarification on remedial measures which are likely to be taken before evaluation.

The CBSE says that feedback received from various stakeholders like students, subject teachers and examiners will be put before the committee of subject experts.

In 2015 also the same complain was there by many students



CBSE Class 12 exam: Issue of tough maths paper raised in Parliament

A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".

Last Updated: Thursday, March 19, 2015 - 14:41

New Delhi: A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".

In March 2016, students of Karnataka PU-II also complained the same, regarding standard 12 (PU-II Mathematics Exam). Even though the Math Paper was identical to previous year, most students had not even solved the 2015 Question Paper.

Friday, March 25, 2016 - 13:28

The NEWS Minute

HOME NEWS ANDHRA KARNATAKA KERALA TAMIL NADU TELANGANA CULTURE MEDIA BLOG

Exams

Online petition for lenient evaluation of K'taka II PU math paper gets over 8000 supporters

The campaign, which was launched on Monday, has garnered over 8000 supporters

TNM Staff | Wednesday, March 16, 2016 - 09:32

Follow @thenewsminute

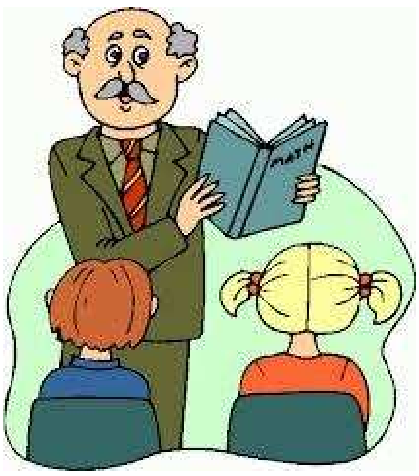
Share Tweet Email Share Reddit

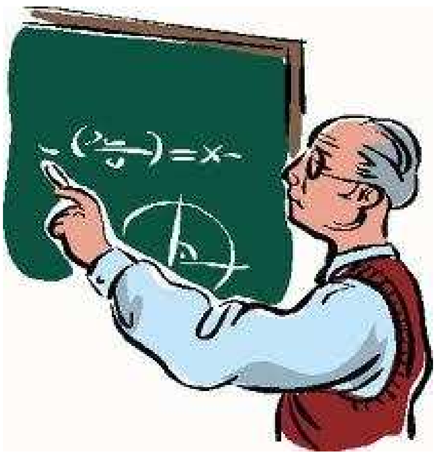
Following a "very tough" math paper that left many II PU students in tears, Saket Ravindran a student launched an online campaign demanding lenient evaluation.

These complains are not new. In fact since last 40 years, (since my childhood), I always see this; every year the same setback, same complain!

In this e-Book I am trying to solve this problem. Those students who practice can learn.

No one can help those who are not studying, or practicing.



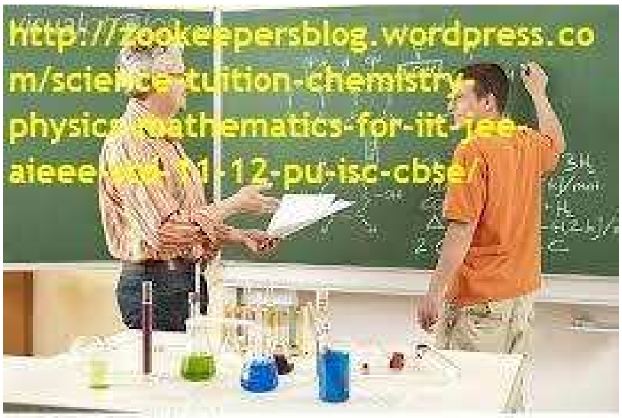


Learn more at http://skmclasses.weebly.com/iit-jee-home-tuitions-bangalore.html

Twitter - https://twitter.com/ZookeeperPhy

Facebook - https://www.facebook.com/IIT.JEE.by.Prof.Subhashish/

Blog - http://skmclasses.kinja.com



A very polite request :

I wish these e-Books are read only by Boys and Men. Girls and Women, better read something else; learn from somewhere else.

Preface

We all know that in the species "Homo Sapiens ", males are bigger than females. The reasons are explained in standard 10, or 11 (high school) Biology texts. **This shapes or size, influences all of our culture**. Before we recall / understand the reasons once again, let us see some random examples of the influence

Random - 1

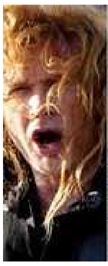
If there is a Road rage, then who all fight ? (generally ?). Imagine two cars driven by adult drivers. Each car has a woman of similar age as that of the Man. The cars " touch " or " some issue happens". Who all comes out and fights ? Who all are most probable to drive the cars ?




(Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win)

Random - 2

Heavy metal music artists are all Men. Metallica, Black Sabbath, Motley Crue, Megadeth, Motorhead, AC/DC, Deep Purple, Slayer, Guns & Roses, Led Zeppelin, Aerosmith ….. the list can be in thousands. All these are grown-up Boys, known as Men.






(Men strive for perfection. Men are eager to excel. Men work hard. Men want to win.)






Random - 3

Apart from Marie Curie, only one more woman got Nobel Prize in Physics. (Maria Goeppert Mayer - 1963). So, … almost all are men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 4

The best Tabla Players are all Men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 5

History is all about, which Kings ruled. Kings, their men, and Soldiers went for wars. History is all about wars, fights, and killings by men.

Boys start fighting from school days. Girls do not fight like this

(Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.)

Random - 6

The highest award in Mathematics, the “ Fields Medal “ is around since decades. Till date only one woman could get that. (Maryam Mirzakhani - 2014). So, … almost all are men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 7

Actor is a gender neutral word. Could the movie like “ Top Gun “ be made with Female actors ? The best pilots, astronauts, Fighters are all Men.

Random - 8

In my childhood had seen a movie named “ The Tower in Inferno “. In the movie when the tall tower is in fire, women were being saved first, as only one lift was working….

Many decades later another movie is made. A box office hit. “ The Titanic “. In this also …. As the ship is sinking women are being saved. **Men are disposable**. Men may get their turn later…

Movies are not training programs. Movies do not teach people what to do, or not to do. Movies only reflect the prevalent culture. Men are disposable, is the culture in the society. Knowingly, unknowingly, the culture is depicted in Movies, Theaters, Stories, Poems, Rituals, etc. I or you can’t write a story, or make a movie in which after a minor car accident the Male passengers keep seating in the back seat, while the both the women drivers come out of the car and start fighting very bitterly on the road. There has been no story in this world, or no movie made, where after an accident or calamity, Men are being helped for safety first, and women are told to wait.

Random - 9

Artists generally follow the prevalent culture of the Society. In paintings, sculptures, stories, poems, movies, cartoon, Caricatures, knowingly / unknowingly, “ the prevalent Reality “ is depicted. The opposite will not go well with people. If deliberately “ the opposite “ is shown then it may only become a special art, considered as a special mockery.

Random - 10

Men go to “girl / woman’s house” to marry / win, and bring her to his home. That is a sort of winning her. When a boy gets a “ Girl-Friend “, generally he and his friends consider that as an achievement. The boy who “ got / won “ a girl-friend feels proud. His male friends feel, jealous, competitive and envious. Millions of stories have been written on these themes. Lakhs of movies show this. Boys / Men go for “ bike race “, or say “ Car Race “, where the winner “ gets “ the most beautiful girl of the college.




(Men want to excel. Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.)

Prithviraj Chauhan ‘ went ` to “ pickup “ or “ abduct “ or “ win “ or “ bring “ his love. There was a Hindi movie (hit) song … “ Pasand ho jaye, to ghar se utha laye “. It is not other way round. Girls do not go to Boy’s house or man’s house to marry. Nor the girls go in a gang to “ pick-up “ the boy / man and bring him to their home / place / den.

Random - 11

Rich people; often are very hard working. Successful business men, establish their business (empire), amass lot of wealth, with lot of difficulty. Lots of sacrifice, lots of hard work, gets into this. Rich people's wives had no contribution in this wealth creation. Women are smart, and successful upto the extent to choose the right/rich man to marry. So generally what happens in case of Divorces ? Search the net on " most costly divorces " and you will know. The women;(who had no contribution at all, in setting up the business / empire), often gets in Billions, or several Millions in divorce settlements.

Number 1

Rupert & Anna Murdoch -- $1.7 billion

One of the richest men in the world, **Rupert Murdoch** developed his worldwide media empire when he inherited his father's Australian newspaper in 1952. He married Anna Murdoch in the '60s and they remained together for 32 years, springing off three children.

They split amicably in 1998 but soon Rupert forced Anna off the board of News Corp and the gloves came off. The divorce was finalized in June 1999 when Rupert agreed to let his ex-wife leave with $1.7 billion worth of his assets, $110 million of it in cash. Seventeen days later, Rupert married Wendi Deng, one of his employees.

Ted Danson & Casey Coates -- $30 million

Ted Danson's claim to fame is undoubtedly his decade-long stint as Sam Malone on NBC's celebrated sitcom Cheers . While he did other TV shows and movies, he will always be known as the bartender of that place where everybody knows your name. He met his future first bride Casey, a designer, in 1976 while doing Erhard Seminars Training.

Ten years his senior, she suffered a paralyzing stroke while giving birth to their first child in 1979. In order to nurse her back to health, Danson took a break from acting for six months. But after two children and 15 years of marriage, the infatuation fell to pieces. Danson had started seeing Whoopi Goldberg while filming the comedy, Made in America and this precipitated the 1992 divorce. Casey got $30 million for her trouble.

See https://zookeepersblog.wordpress.com/misandry-and-men-issues-a-short-summary-at-single-place/

See http://skmclasses.kinja.com/save-the-male-1761788732

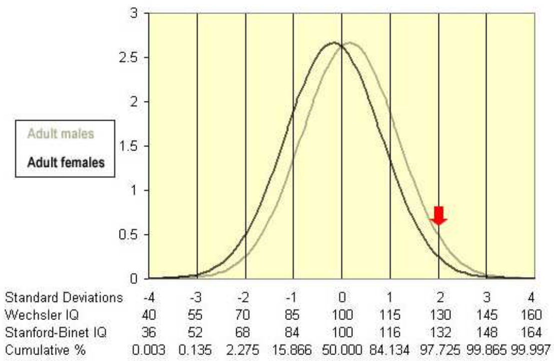
It was Boys and Men, who brought the girls / women home. The Laws are biased, completely favoring women. The men are paying for their own mistakes.

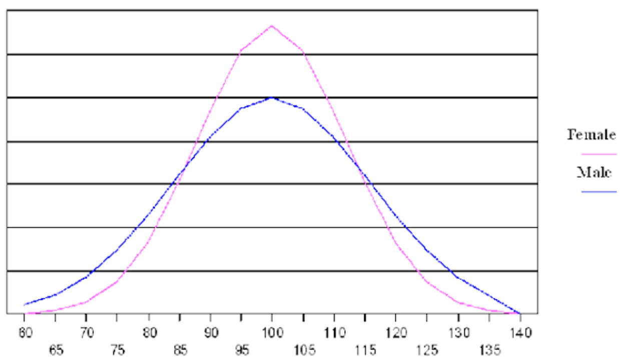
See https://zookeepersblog.wordpress.com/biased-laws/

(Man brings the Woman home. When she leaves, takes away her share of big fortune!)

Random - 12

A standardized test of Intelligence will never be possible. It never happened before, nor ever will happen in future; where the IQ test results will be acceptable by all. In the net there are thousands of charts which show that the intelligence scores of girls / women are lesser. Debates of Trillion words, does not improve performance of Girls.

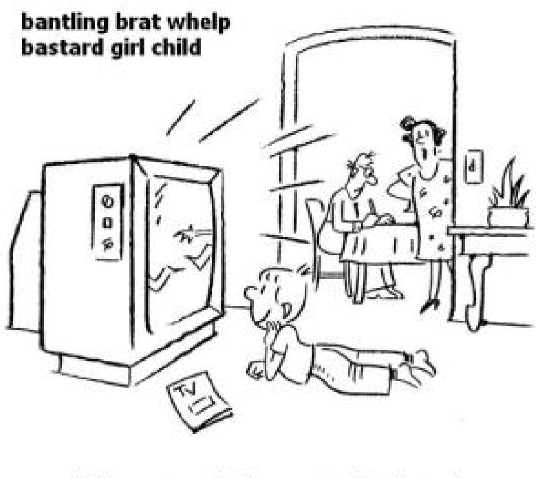




I am not wasting a single second debating or discussing with anyone, on this. I am simply accepting ALL the results. IQ is only one of the variables which is required for success in life. Thousands of books have been written on “ Networking Skills “, EQ (Emotional Quotient), Drive, Dedication, Focus, “ Tenacity towards the end goal “ … etc. In each criteria, and in all together, women (in general) do far worse than men. Bangalore is known as “ ….. capital of India “. [Fill in the blanks]. The blanks are generally filled as “ Software Capital “, “ IT Capital “, “ Startup Capital “, etc. I am member in several startup eco-systems / groups. I have attended hundreds of meetings, regarding “ technology startups “, or “ idea startups “. These meetings have very few women. Starting up new companies are all “ Men’s Game “ / “ Men’s business “. Only in Divorce settlements women will take their goodies, due to Biased laws. There is no dedication, towards wealth creation, by women.

Random - 13

Many men, as fathers, very unfortunately treat their daughters as “ Princess “. Every “ non-performing “ woman / wife was “ princess daughter “ of some loving father. Pampering the girls, in name of “ equal opportunity “, or “ women empowerment “, have led to nothing.



"Please turn it down - Daddy is trying to do your homework."

See http://skmclasses.kinja.com/progressively-daughters-become-monsters-1764484338

See http://skmclasses.kinja.com/vivacious-vixens-1764483974

There can be thousands of more such random examples, where " Bigger Shape / size " of males have influenced our culture, our Society. **Let us recall the reasons**, that we already learned in standard 10 - 11, Biology text Books. In humans, women have a long gestation period, and also spends many years (almost a decade) to grow, nourish, and stabilize the child. (Million years of habit) Due to survival instinct Males want to inseminate. Boys and Men fight for the " facility (of womb + care) " the girl / woman may provide. Bigger size for males, has a winning advantage. Whoever wins, gets the " woman / facility ". The male who is of " Bigger Size ", has an advantage to win…. Leading to Natural selection over millions of years. In general " Bigger Males "; the " fighting instinct " in men; have led to wars, and solving tough problems (Mathematics, Physics, Technology, startups of new businesses, Wealth creation, Unreasonable attempts to make things [such as planes], Hard work ….)

So let us see the IIT-JEE results of girls. Statistics of several years show that there are around 17, (or less than 20) girls in top 1000 ranks, at all India level. Some people will yet not understand the performance, till it is said that … year after year we have around 980 boys in top 1000 ranks. Generally we see only 4 to 5 girls in top 500. In last 50 years not once any girl topped in IIT-JEE advanced. Forget about Single digit ranks, double digit ranks by girls have been extremely rare. It is all about " good boys ", " hard working ", " focused ", "**Bel-esprit** " boys.

In 2015, Only 2.6% of total candidates who qualified are girls (upto around 12,000 rank). while 20% of the Boys, amongst all candidates qualified. The Total number of students who appeared for the exam were around 1.4 million for IIT-JEE main. Subsequently 1.2 lakh (around 120 thousands) appeared for IIT-JEE advanced.

IIT-JEE results and analysis, of many years is given at https://zookeepersblog.wordpress.com/iit-jee-iseet-main-and-advanced-results/

In Bangalore it is rare to see a girl with rank better than 1000 in IIT-JEE advanced. We hardly see 6-7 boys with rank better than 1000. Hardly 2-3 boys get a rank better than 500.

See http://skmclasses.weebly.com/everybody-knows-so-you-should-also-know.html

Thousands of people are exposing the heinous crimes that Motherly Women are doing, or Female Teachers are committing. See https://www.facebook.com/WomenCriminals/

Some Random Examples must be known by all

Mother Admits On Facebook to Sleeping with 15 Yr Old Son, They Have a Baby Together - Alwayzturntup

Sometimes it hard to believe w From Alwayzturntup

ALWAYZTURNTUP.ME

It is extremely unfortunate that the " woman empowerment " has created. This is the kind of society and women we have now. I and many other sensible Men hate such women. Be away from such women, be aware of reality.

'Sex with my son is incredible - we're in love and we want a baby'

Ben Ford, who ditched his wife when he met his mother Kim West after 30 years, claims what the couple are doing 'isn't incest'

MIRROR.CO.UK

Former Shelbyville ISD teacher who had sex with underage student gets 3 years in prison

After a two day break over the weekend, A Shelby County jury was back in the courtroom looking to conclude the trial of a former Shelbyville ISD teacher who had...

KLTV.COM | BY CALEB BEAMES

Woman sent to jail for the rest of her life after raping her four grandchildren is described as the 'most evil person' the judge has ever seen

Edwina Louis rape...

See More

Woman sent to jail for raping her four grandchildren

A Ohio grandmother has been sentenced to four consecutive life terms after being found guilty of the rape of her own grandchildren. Edwina Louis, 53, will spend the rest of her life behind bars.

DAILYMAIL.CO.UK

http://www.thenativecanadian.com/.../eastern-ontario-teacher-...

The N.C. Chronicles.: Eastern Ontario teacher charged with 36 sexual offences

anti feminism, Child abuse, children's rights, Feminist hypocrisy,

THENATIVECANADIAN.COM | BY BLACKWOLF

Hyd woman kills newborn boy as she wanted daughter - Times of India

Having failed to bear a daughter for the third time, a shopkeeper's wife slit the throat of her 24day-old son with a shaving blade and left him to die in a street on Tuesday night.Purnima's first child was a stillborn boy, followed by another boy born five years ago.

TIMESOFINDIA.INDIATIMES.COM

Montgomery's son, Alan Vonn Webb, took the stand and was a key witness in her conviction.

"I want to see her placed somewhere she can never do that to children ...

See More

Woman sentenced to 40 years in prison for raping her children

A Murfreesboro mother found guilty of raping her own children learned her fate on Wednesday.

WAFF.COM | BY DENNIS FERRIER

gentler sex? Violence against men.'s photo.

Women, the gentler sex? Violence against men.

April 8 at 1:38am

Like Page

In fact, the past decade has seen a dramatic increase in the number of incidents of women raping and sexually assaulting boys and men. On May 2014, Jezebel repo...

End violence against women . . . .

North Carolina Grandma Eats Her Daughter's New Born Baby After Smoking Bath Salts

Henderson, North Carolina– A North Carolina grandmother of 4 and recovering drug addict, is now in custody after she allegedly ate her daughter's newborn baby....

AZ-365.TOP

28-Year-Old Texas Teacher Accused of Sending Nude Picture to 14-Year-Old Former Student

BREITBART.COM

http://latest.com/.../attractive-girl-gang-lured-men-alleywa.../

Attractive Girl Gang Lured Men Into Alleyways Where Female Body Builder Would Attack Them

A Mexican street gang made up entirely of women has been accused of using their feminine wiles to lure men into alleyways and then beating them up and...

LATEST.COM

http://www.wfmj.com/.../youngstown-woman-convicted-of-raping-...

Youngstown woman convicted of raping a 1 year old is back in jail

A Youngstown woman who went to prison for raping a 1-year-old boy fifteen years ago is in trouble with the law again.

WFMJ.COM

End violence against women . . . .

Women are raping boys and young men

Rape advocacy has been maligned and twisted into a political agenda controlled by radicalized activists. Tim Patten takes a razor keen and well supported look into the manufactured rape culture and...

AVOICEFORMEN.COM | BY TIM PATTEN

Bronx Woman Convicted of Poisoning and Drowning Her Children

Lisette Bamenga researched methods on the Internet before she killed her son and daughter in 2012.

NYTIMES.COM | BY MARC SANTORA

A Russian-born newlywed slowly butchered her German husband — feeding strips of his flesh to their dog until he took his last breath. Svetlana Batukova, 46, was...

See More

She killed her husband and then fed him to her dog: police

A Russian-born newlywed butchered her German hubby — and fed strips of his flesh to her pooch, authorities said. Svetlana Batukova offed Horst Hans Henkels at their...

NYPOST.COM

Daily Mail
January 15, 2015

Mother charged with rape and sodomy of her son's 12-year-old friend

Mom, 30, 'raped and had oral sex with her son's 12-year-old friend'

Nicole Marie Smith, 30, (pictured) of St Charles County, Missouri, has been jailed after she allegedly targeted the 12-year-old boy at her home.

DAILYM.AI

April 4 at 4:48am

Female prison officers commit 90pc of sex assaults on male teens in US juvenile detention centres

Lawsuit in Idaho highlights the prevalence of sexual victimization of juvenile offenders.

IBTIMES.CO.UK | BY NICOLE ROJAS

This mother filmed herself raping her own son and then sold it to a man for $300. The courts just decide her fate. When you see what she got, you're going to be outraged.

Mother Who Filmed Herself Raping Her 1-Year-Old Son Receives Shocking Sentence

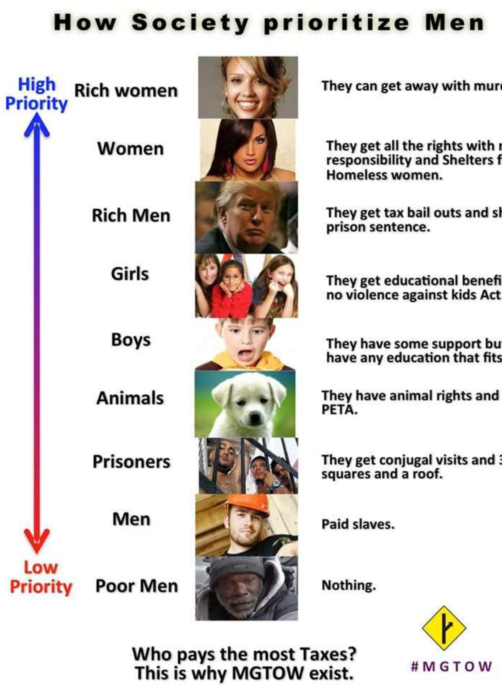
"...then used the money to buy herself a laptop..."

AMERICANEWS.COM

In several countries or rather in several regions of the world, family system has collapsed, due to bad nature and naughty acts of women. Particularly in Britain, and America, almost 50% people are alone, lonely, separated, divorced or failed marriages. In 2013, 48% children were born out of wedlock. It was projected that by 2016, more than 51% children will be born, to unmarried mothers. In these developed countries " paternity fraud " by women, are close to 20%. You can see several articles in the net, and in wikipedia etc. This means 1 out of 5 children are calling a wrong man as dad. The lonely, alone “ mothers " are frustrated. They see the children as burden. Love in the Society in general is lost, long time ago. The types of “ Mothers “ and “ Women “ we have now …………

By now if you have assumed that Indian women are not doing any crime then please become friends with MRA Guri https://www.facebook.com/profile.php?id=100004138754180

He has dedicated his life to expose Indian Criminals



Professor Subhashish Chattopadhyay

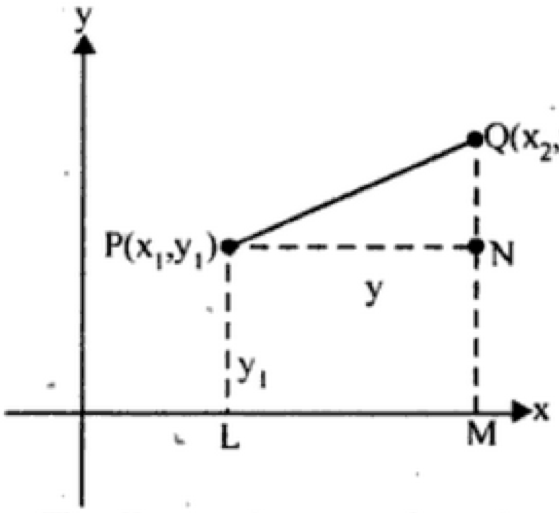
Spoon Feeding Series - Pair of Straight Lines

Before we discuss examples and problems let us see the formulae

Distance between two points

Here $QN = QM - NM = y_2 - y_1$.

$PN = OM - OL = x_2 - x_1$



$\therefore$ The distance between the points $P(x_1, y_1)$ and $Q(x_2\ y_2)$ is given by

$$PQ^2 = PN^2 + QN^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

i.e., $$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Question

The point (4, 1) undergoes the following successive transformations:

(i) reflection about the line $y = x$

(ii) translation through a distance 2 units along the positive x-axis.

then, the final coordinates of the point are

(a) (4, 3) (b) (3, 4) (c) (1, 4) (d) (4, 4)

Ans. (b)

Solution Let $Q(x, y)$ be the reflection of $P(4, 1)$ about the line $y = x$, then mid-point of PQ lies on this line and PQ is perpendicular to it. So we have

$$\frac{y+1}{2} = \frac{x+4}{2} \text{ and } \frac{y-1}{x-4} = -1.$$

$$\Rightarrow \quad x - y = -3 \text{ and } x + y = 5$$

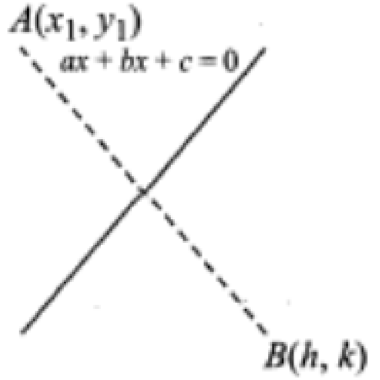
$$\Rightarrow \quad x = 1, y = 4$$

Therefore reflection of (4, 1) about $y = x$ is (1, 4). Next, this point is shifted. 2 units along the positive x-axis, the new coordinates are $(1 + 2, 4 + 0) = (3, 4)$

Image of a point

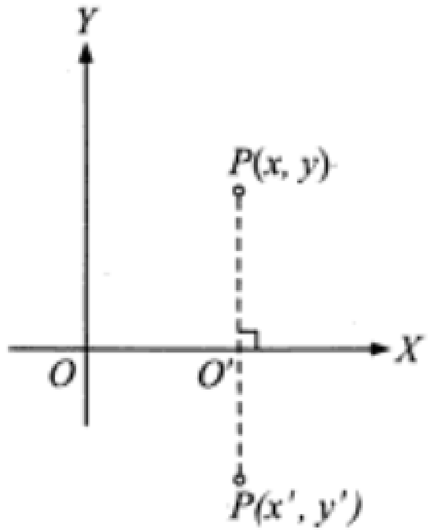
The image of a point with respect to the line mirror. The image of $A(x_1, y_1)$ with respect to the line mirror $ax + by + c = 0$ be $B(h, k)$ given by,

$$\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$$



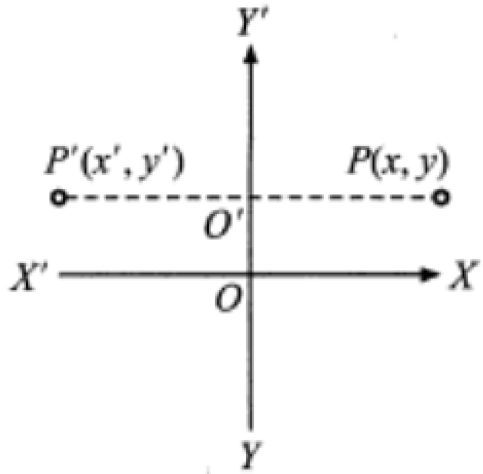
The image of a point with respect to x-axis: Let $P(x, y)$ be any point and $P'(x', y')$ its image after reflection in the x-axis, then

$x' = x$ and $y' = -y$, ($\because$ O' is the mid point of PP')



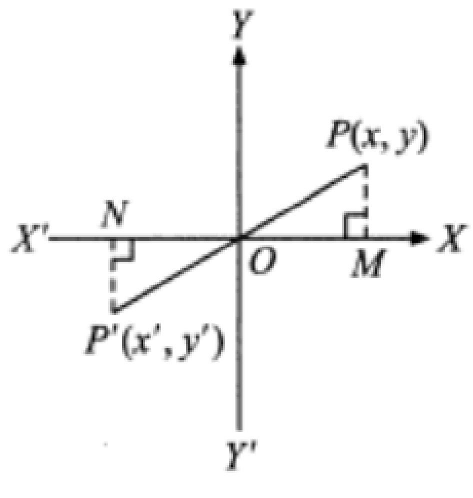
The image of a point with respect to y-axis: $P(x, y)$ be any point and $P'(x', y')$ its image after reflection in the y-axis, then

$x' = -x$ and $y' = y$ ($\because$ O' is the mid point of PP')



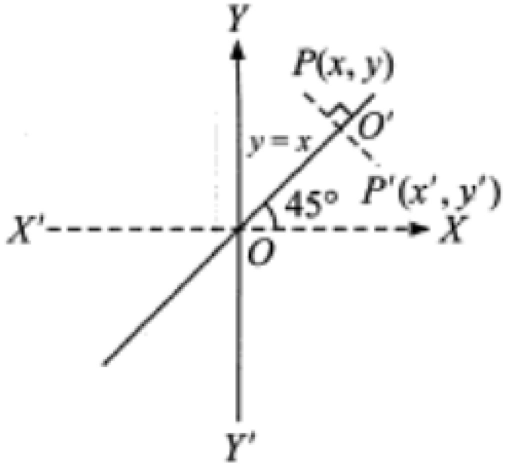
The image of a point with respect to the origin: Let $P(x, y)$ be any point and $P'(x', y')$ be its image after reflection through the origin, then

$x' = -x$ and $y' = -y$ ($\because$ O is the mid-point of PP')

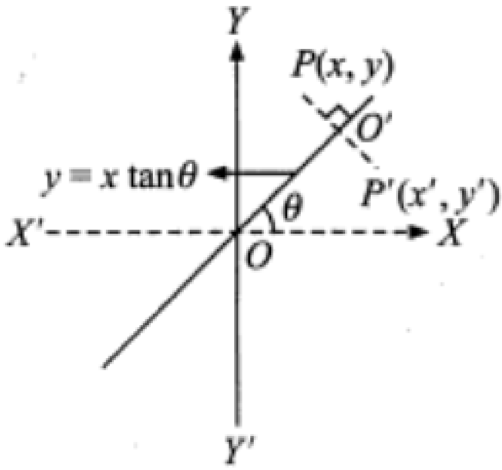


The image of a point with respect to the line $y = x$: Let $P(x, y)$ be any point and $P'(x', y')$ be its image after reflection in the line $y = x$, then,

$x' = y$ and $y' = x$ ($\because$ O' is the mid-point of PP')



The image of a point with respect to the line $y = x \tan\theta$: Let $P(x, y)$ be any point and $P'(x', y')$ be its image after reflection in the line $y = x \tan\theta$, then,

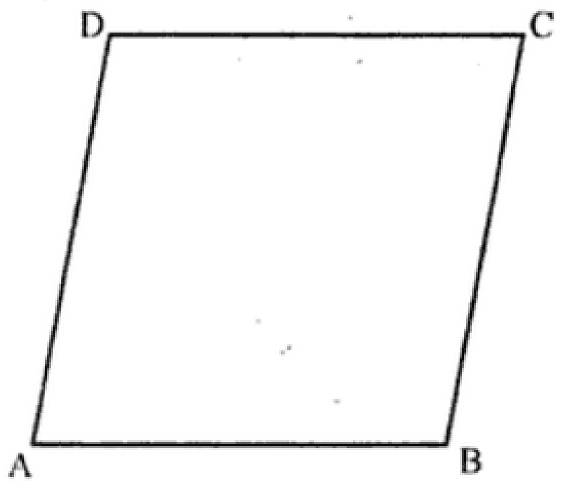


$x' = x \cos 2\theta + y \sin 2\theta$

$y' = x \sin 2\theta - y \cos 2\theta,$

($\because$ O' is the mid-point of PP')

A Rhombus is made by distorting a square



All four sides are equal. So AB = BC = CD = DA

Question

The diagonals of the parallelogram whose sides are $lx + my + n = 0$, $lx + my + n' = 0$, $mx + ly + n = 0$, $mx + ly + n' = 0$ include an angle

(a) $\frac{\pi}{3}$

(b) $\frac{\pi}{2}$

(c) $\tan^{-1}\left(\frac{l^2 - m^2}{l^2 + m^2}\right)$

(d) $\tan^{-1}\left(\frac{2lm}{l^2 + m^2}\right)$

Solution

(b). Since the distance between the parallel lines $lx + my + n = 0$ and $lx + my + n' = 0$ is same as the distance between the parallel lines $mx + ly + n = 0$ and $mx + ly + n' = 0$. Therefore, the parallelogram is a rhombus. Since the diagonals of a rhombus are at right angles, therefore the required angle is $\frac{\pi}{2}$.

Question

A rectangle has two opposite vertices at the points (1, 2) and (5, 5). If the other vertices lie on the line $x = 3$, then the coordinates of the other vertices are

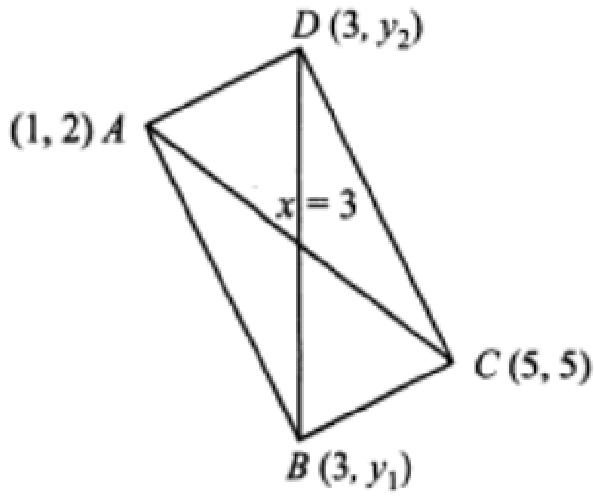
(a) (3, – 1), (3, – 6) (b) (3, 1), (3, 5)

(c) (3, 2), (3, 6) (d) (3, 1), (3, 6)

Solution

(d). Let $A \equiv (1, 2)$ and $C \equiv (5, 5)$. Since the vertices B and D lie on the line $x = 3$, therefore, let $B \equiv (3, y_1)$ and $D \equiv (3, y_2)$.

Since AC and BD bisect each other, so they have same middle point



i.e., $\frac{y_1 + y_2}{2} = \frac{2+5}{2}$

or $y_1 + y_2 = 7$ ...(1)

Also, $BD^2 = AC^2$

$\Rightarrow$ $(y_1 - y_2)^2 = (1-5)^2 + (2-5)^2 = 25$

or $y_1 - y_2 = \pm 5$ ...(2)

Solving (1) and (2), we get $y_1 = 6, y_2 = 1$

or $y_1 = 1, y_2 = 6$.

Thus, the other vertices of the rectangle are **(3, 1)** and **(3, 6).**

Question

Two points $(a, 3)$ and $(5, b)$ are the opposite vertices of a rectangle. If the other two vertices lie on the line $y = 2x + c$ which passes through the point (a, b) then the value of c is

(a) -7 (b) -4 (c) 0 (d) 7

Ans. (a)

Solution Mid point of the line joining the given points lie on the given line

$$\frac{3+b}{2} = 2\left(\frac{a+5}{2}\right) + c$$

$\Rightarrow$ $2a + 2c - b + 7 = 0$ (i)

Also since the given line passes through (a, b)

$$b = 2a + c \quad \text{(ii)}$$

Solving (i) and (ii) we get $c = -7$

Question

Two adjacent sides of a parallelogram are $4x + 5y = 0$ and $7x + 2y = 0$. If an equation to one of the diagonals is $11x + 7y - 9 = 0$, then an equation of the other diagonal is

(a) $x + y = 0$ (b) $7x - 11y = 0$

(c) $x - y = 0$ (d) none of these

Ans. (*c*)

Solution Since the given lines intersect at the origin O, one of the vertex is $O(0, 0)$. Let A and B be the points of intersection of the sides $4x + 5y = 0$ and $7x + 2y = 0$ respectively with the diagonal. $11x + 7y - 9 = 0$, then the coordinates of A and B are respectively $\left(\frac{5}{3}, \frac{-4}{3}\right)$ and $\left(-\frac{2}{3}, \frac{7}{3}\right)$. The coordinates of the mid point of AB are $\left(\frac{1}{2}, \frac{1}{2}\right)$. Since the other diagonal passes through the vertex (0, 0) and the mid-point $\left(\frac{1}{2}, \frac{1}{2}\right)$ of AB, its equation is $y = x$.

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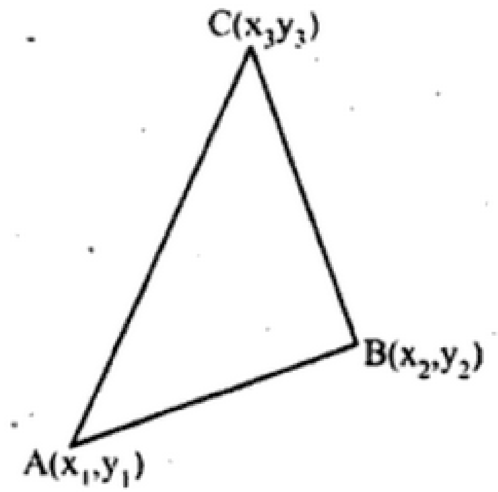
Area of a Triangle



The area of a triangle, the coordinates of whose vertices are (x_1, y_1) (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

or

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Question on Area

Let P (2, – 4) and Q (3, 1) be two given points. Let $R(x, y)$ be a point such that $(x - 2)(x - 3) + (y - 1)(y + 4) = 0$. If area of ΔPQR is $\frac{13}{2}$, then the number of possible positions of R are

(a) 2 (b) 3

(c) 4 (d) none of these

Solution

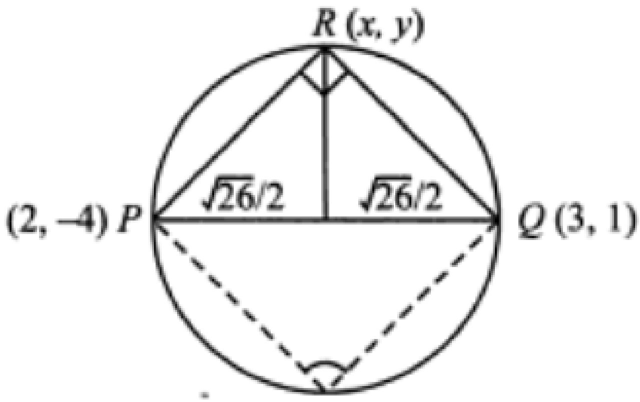
(a). We have

$$(x-2)(x-3)+(y-1)(y+4)=0$$

$\Rightarrow \quad \left(\frac{y+4}{x-2}\right)\times\left(\frac{y-1}{x-3}\right)=-1$

$\Rightarrow \quad RP \perp RQ$ or $\angle PRQ = \frac{\pi}{2}$.

$\therefore$ The point R lies on the circle whose diameter is PQ.



Now, area of $\Delta PQR = \frac{13}{2}$

$\Rightarrow \quad \frac{1}{2}\times\sqrt{26}\times(\text{altitude})=\frac{13}{2}$

$\Rightarrow \quad \text{altitude} = \frac{\sqrt{26}}{2} = \text{radius}$

$\Rightarrow$ there are **two** possible positions of R.

Condition of colinearity of 3 points

Three points $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are collinear if

i) Area of triangle ABC = 0 i.e.,

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 \text{ or } \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

or

ii) AB+BC=AC (or) AC+BC=AB (or) AC+AB=BC

In some cases a problem can be solved just by observation. Meaning the above determinant need not be evaluated.

The points $(a, b + c)$, $(b, c + a)$ and $(c, a + b)$ are

(a) vertices of an equilateral triangle
(b) concyclic
(c) vertices of a right angled triangle
(d) none of these

Ans. (d)

Solution As the given points lie on the line $x + y = a + b + c$, they are collinear.

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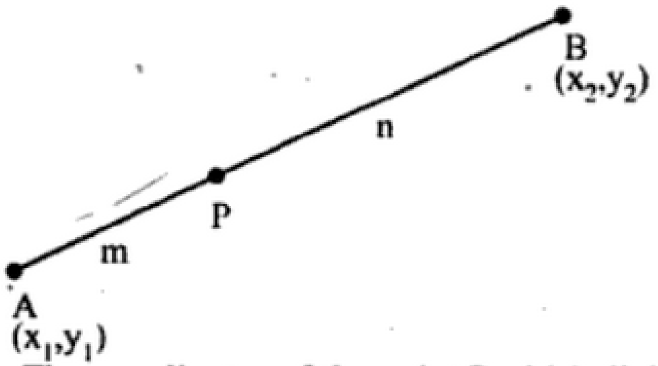
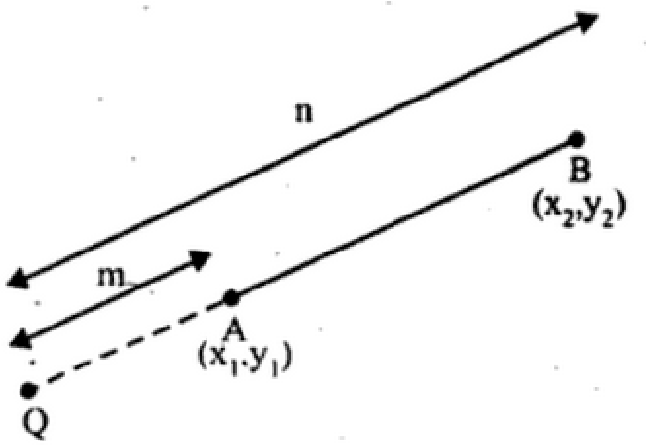
Section formula Internal Division



The coordinates of the point P which divides the line segment joining the points $A(x_1\ y_1)$ and $B(x_2\ y_2)$ internally in the ratio m:n are given by

$$P=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$$

Section formula External Division can have Two formulae. Depending on from which external side the division is being done



Here the external point Q is on the side of A

If m is the distance from A then m gets multiplied to coordinates of opposite point i.e.

$B(x_2 , y_2)$

The coordinates of the point Q which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2\ y_2)$ externally in the ratio m:n are given by

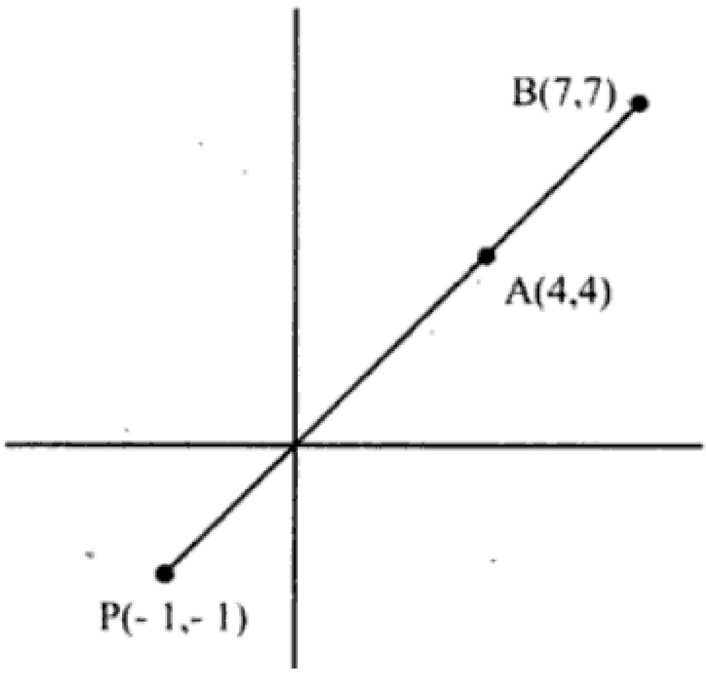
$$Q = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$

Note:

i) If P is the mid point of AB, then the coordinate of P is given by $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

ii) The co-ordinate of any point on AB can be written as $\left(\frac{\lambda x_2 + x_1}{\lambda + 1}, \frac{\lambda y_2 + y_1}{\lambda + 1}\right)$

The ratio in which the line joining the points A (4, 4) and B(7, 7) is divided by (-1, -1)



a) 7:4 externally **b)** 8:5 externally
c) 5:8 externally **d)** 4:7 externally

Ans (c)

$$PA = \sqrt{5^2 + 5^2} = 5\sqrt{2}$$

$$PB = \sqrt{8^2 + 8^2} = 8\sqrt{2}$$

$$\therefore \quad PA : PB = 5 : 8$$

thus P (-1, -1) divides AB externally in the ratio 5:8.

Question

If two vertices of a triangle are (–2, 3) and (5, –1), orthocentre lies at the origin and centroid on the line $x + y = 7$, then the third vertex lies at

(a) (7, 4) (b) (8, 14)
(c) (12, 21) (d) none of these

Ans. (d)

Solution Let $O(0, 0)$ be the orthocentre; $A(h, k)$ the third vertex; and $B(-2, 3)$ and $C(5, -1)$ the other two vertices. Then the slope of the line through A and O is k/h, while the line through B and C has the slope $(-1-3)/(5+2) = -4/7$. By the property of the orthocentre, these two lines must be perpendicular, so we have

$$\left(\frac{k}{h}\right)\left(-\frac{4}{7}\right) = -1 \Rightarrow \frac{k}{h} = \frac{7}{4} \qquad \text{(i)}$$

Also $$\frac{5-2+h}{3} + \frac{-1+3+k}{3} = 7$$

$\Rightarrow$ $$h + k = 16 \qquad \text{(ii)}$$

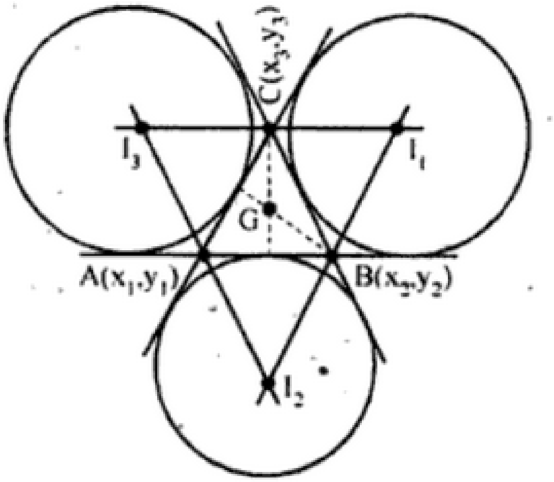
Which is not satisfied by the points given in (a), (b), or (c)

-

Coordinates of the centroid, in-centre and ex-centres of a triangle

Let $A(x_1, y_1)$ $B(x_2\ y_2)$ and $C(x_3\ y_3)$ be the three vertices of a triangle ABC.

i) Centroid of a triangle



Centroid is the point of intersection of medians, whose coordinates are given by

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

ii) In-centre of a triangle

In-centre is the point of intersection of internal angular bisectors, whose coordinates are given by

$$I = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

where a, b, c are the lengths of the sides BC, CA, AB respectively.

iii) Ex-centres of a triangle

The point of intersection I_1 of the external angular bisectors of $\angle B$ and $\angle C$ is one of the excentres of the triangle ABC and is given by

$$I_1 = \left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c} \right)$$

similarly the other ex-centres are given by

$$I_2 = \left(\frac{ax_1 - bx_2 + cx_3}{a-b+c}, \frac{ay_1 - by_2 + cy_3}{a-b+c} \right) \text{ and}$$

$$I_3 = \left(\frac{ax_1 + bx_2 - cx_3}{a+b-c}, \frac{ay_1 + by_2 - cy_3}{a+b-c} \right)$$

where a, b, c are the lengths of the sides BC, CA, AB respectively.

Question

If the vertices P, Q, R of a ΔPQR are rational points, which of the following points of the ΔPQR is (are) always rational point(s)?

(a) centroid (b) incentre

(c) circumcentre (d) orthocentre

(A rational point is a point both of whose coordinates are rational numbers)

Solution

(a). Let $P \equiv (x_1, y_1)$, $Q \equiv (x_2, y_2)$; $R \equiv (x_3, y_3)$, where x_i, y_i $(i = 1, 2, 3)$ are rational numbers.

Now, the centroid of ΔPQR is

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

which is rational point. Incentre, circumcentre and orthocentre depend on sides of the triangle which may not be rational even if vertices are so. For example, for P (0, 1) and Q (1, 0); $PQ = \sqrt{2}$.

Question

The points A (2, 3); B (3, 5), C (7, 7) and D (4, 5) are such that

(a) $ABCD$ is a parallelogram
(b) A,B,C and D are collinear
(c) D lies inside the triangle ABC
(d) D lies on the boundary of the triangle ABC

Ans. (c)

Solution Since $\frac{2+3+7}{3} = 4$, $\frac{3+5+7}{3} = 5$

$D = (4, 5)$ is the centroid of the triangle ABC and hence lies inside the ΔABC.

Question

Let A (-1, 5) B (3,1) C(5, a) be the vertices of a triangle ABC. If D, E, F are the middle points of BC, CA and AB respectively and area of triangle ABC is equal to four times the area of triangle DEF, then

a) $a = 3$ **b)** $a \neq 5$

c) for any real value of a

d) any real value except -1.

Ans (d)

Since A, B, C from a triangle

$$\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} \neq 0 \text{ i.e., } \begin{vmatrix} 1 & -1 & 5 \\ 1 & 3 & 1 \\ 1 & 5 & a \end{vmatrix} \neq 0$$

$4a + 4 \neq 0 \Rightarrow a \neq -1$

but since area of any triangle is always four times the area of a triangle formed by the mid points a, can be any real value except -1.

In some problems we find the Area pretty differently

The area of the triangle formed by the tangent to the curve $y = \dfrac{8}{4+x^2}$ at $x = 2$ and the coordinate axes is

(*a*) 2 sq. units (*b*) $\frac{7}{2}$ sq. units

(*c*) 4 sq. units (*d*) 8 sq. units.

Solution

(c) From $y = \frac{8}{4 + x^2}$,

when $x = 2, y = \frac{8}{4+4} = 1$

Also, $\frac{dy}{dx} = -\frac{8}{(4+x^2)^2}(2x) \Rightarrow \left[\frac{dy}{dx}\right]_{(2,1)} = -\frac{1}{2}$

$\therefore$ equation of tangent is

$$y - 1 = -\frac{1}{2}(x - 2) \quad \text{or} \quad x + 2y - 4 = 0 \qquad \text{...(1)}$$

Its intercepts on axes are (by putting $y = 0$ and $x = 0$ respectively) $a = 4, b = 2$

$\therefore$ Area $= \frac{1}{2}ab = \frac{1}{2} \times 4 \times 2 = 4$ sq. units.

-

Perpendicular Lines

If there is a line whose slope is m (assuming this line NOT parallel to x-axis) then the slope of the line which is perpendicular to this will be -1 / m

Meaning, product of the slopes of lines that are perpendicular is -1

If one of the lines is parallel to x-axis its slope is 0 while the line perpendicular will have a slope of infinity (∞) This line is parallel to y-axis. Product of 0 X ∞ is undefined. In this case we do not apply the -1 as product rule.

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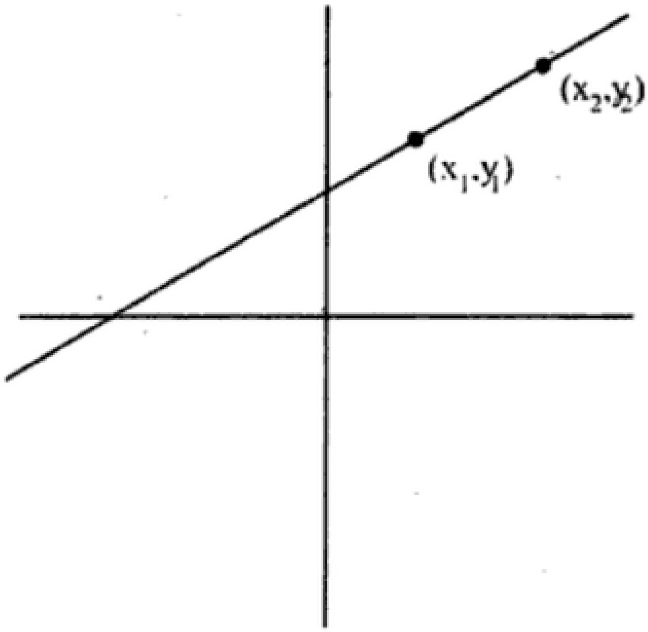
Equation of the line passing through two points



The equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is

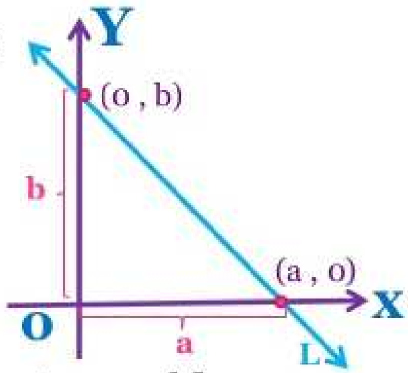
$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

-

The intercept form of a line

- Suppose a line **L** makes x-intercept a and y-intercept b on the axes. Obviously **L** meets x-axis at the point (a, o) and y-axis at the point (o, b).

By two-point form of the equation of the line, we have



$$y - 0 = \frac{b - 0}{0 - a}(x - a)$$

Or $$ay = -bx + ab$$

i.e., $$\frac{x}{a} + \frac{y}{b} = 1$$

Thus, equation of the line making intercepts **a** and **b** on x- and y-axis, respectively, is

$$\frac{x}{a} + \frac{y}{b} = 1$$

Question

Through the point $P(\alpha, \beta)$, where $\alpha\beta > 0$ the straight line $\frac{x}{a} + \frac{y}{b} = 1$ is drawn so as to form with coordinate axes a triangle of area S. If $ab > 0$, then the least value of S is

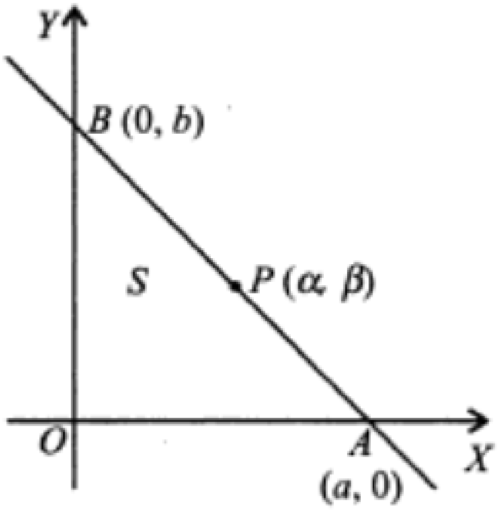
(a) $\alpha\beta$ (b) $2\alpha\beta$

(c) $4\alpha\beta$ (d) none of these

Solution

(b). The equation of the given line is

$$\frac{x}{a}+\frac{y}{b}=1 \quad \ldots(1)$$



This line cuts x-axis and y-axis at A $(a, 0)$ and B $(0, b)$ respectively.

Since area of $\Delta OAB = S$ (Given)

$\therefore \quad \left|\frac{1}{2}ab\right| = S$ or $ab = 2S \quad (\because\ ab > 0) \quad \ldots(2)$

Since the line (1) passes through the point P (α, β)

$\therefore \quad \frac{\alpha}{a}+\frac{\beta}{b}=1$ or $\frac{\alpha}{a}+\frac{a\beta}{2S}=1$ [Using (2)]

or $\quad a^2\beta - 2aS + 2\alpha S = 0.$

Since a is real, $\therefore\ 4S^2 - 8\alpha\beta S \geq 0$

or $\quad 4S^2 \geq 8\alpha\beta S$ or $S \geq 2\alpha\beta \quad \left(\because\ S=\frac{1}{2}ab>0 \text{ as } ab>0\right)$

Hence the least value of $S = 2\alpha\beta$.

Question

The number of integer values of m, for which the x-coordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer is

(a) 2 (b) 0 (c) 4 (d) 1

Ans. (a)

Solution x-coordinates of the points of intersection is given by $3x + 4(mx + 1) = 9$

$\Rightarrow \quad (3 + 4m)x = 5 \quad \Rightarrow \quad x = \frac{5}{3+4m}$

As x is an integer, $3 + 4m$ must be a divisors of 5

$\Rightarrow \quad 3 + 4m = \pm 1$ or ± 5

$\Rightarrow \quad m = -1$ or -2. (Considering the integer value only)

-

i) The equation of a line parallel to a given line $ax+by+c=0$ is $ax+by+\lambda=0$, where λ is constant.

ii) The equation of a line perpendicular to a given line $ax+by+c=0$ is $bx-ay+\lambda=0$, where λ is constant.

iii) The slope of the line $ax+by+c=0$ is given by

$$m = \frac{-a}{b}.$$

iv) For intercept on x-axis, put $y=0$. For intercept on y-axis, put $x=0$.

v) Angle θ between the lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$ is given by

$$\tan\theta = \left|\frac{a_2b_1 - a_1b_2}{a_1a_2 + b_1b_2}\right|$$

vi) The lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$ are

a) Coincident if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

b) Parallel if $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

c) intersecting if $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

d) Perpendicular if $a_1a_2 + b_1b_2=0$

Question on Angle

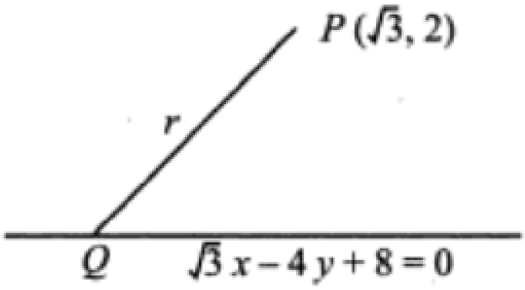
If the straight line drawn through the point $P(\sqrt{3}, 2)$ and making an angle $\frac{\pi}{6}$ with the x-axis meets the line $\sqrt{3}x - 4y + 8 = 0$ at Q, then the length of PQ is

(a) 4 (b) 5

(c) 6 (d) none of these

Solution

(c). The given line is



$\sqrt{3}\ x - 4y + 8 = 0$ ...(1)

Let $PQ = r$.

Then, the coordinates of Q are

$$\left(\sqrt{3} + r\cos\frac{\pi}{6},\ 2 + r\sin\frac{\pi}{6}\right) \text{ or } \left(\sqrt{3} + \frac{\sqrt{3}}{2}r,\ 2 + \frac{r}{2}\right).$$

Since the point Q lies on the given line,

$$\therefore \quad \sqrt{3}\left(\sqrt{3} + \frac{\sqrt{3}}{2}r\right) - 4\left(2 + \frac{r}{2}\right) + 8 = 0$$

$\Rightarrow \quad 6 + 3r - 16 - 4r + 16 = 0$ or $r = 6$

Hence, $PQ = \mathbf{6}$

Question

A line is drawn from the point $P(\alpha, \beta)$, making an angle θ with the positive direction of x-axis, to meet the line $ax + by + c = 0$ at Q. The length of PQ is

(a) $-\dfrac{a\alpha + b\beta + c}{a\cos\theta + b\sin\theta}$ (b) $\left|\dfrac{a\alpha + b\beta + c}{\sqrt{a^2 + b^2}}\right|$

(c) $\dfrac{a\alpha + b\beta + c}{a\cos\theta + b\sin\theta}$ (d) none of these

Solution

(a). Equation of a straight line passing through the point $P(\alpha, \beta)$ and making an angle θ with positive direction of x-axis is

$$\frac{x-\alpha}{\cos\theta} = \frac{y-\beta}{\sin\theta} = r \quad \text{(say)}$$

Coordinates of any point on this line are

$$(\alpha + r\cos\theta, \beta + r\sin\theta)$$

If it lies on the line $ax + by + c = 0$, then

$$a(\alpha + r\cos\theta) + b(\beta + r\sin\theta) + c = 0$$

$$\Rightarrow \quad r = -\frac{a\alpha + b\beta + c}{a\cos\theta + b\sin\theta}$$

Thus, $PQ = r = -\dfrac{a\alpha + b\beta + c}{a\cos\theta + b\sin\theta}$.

Question on Angle between lines

The two curves $x^3 - 3xy^2 + 5 = 0$ and $3x^2y - y^3 - 7 = 0$

(a) cut at right angles (b) touch each other

(c) cut at an angle $\frac{\pi}{4}$ (d) cut at an angle $\frac{\pi}{3}$.

Solution

(a) Differentiating $x^3 - 3xy^2 + 5 = 0$, we get

$$3x^2 - 3y^2 - 6xy\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{x^2 - y^2}{2xy}$$

Differentiating $3x^2y - y^3 - 7 = 0$, we get

$$6xy + 3x^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{2xy}{y^2 - x^2}$$

Since, product of slopes is $\frac{x^2 - y^2}{2xy} \times \frac{2xy}{y^2 - x^2} = -1$

$\therefore$ The two curves cut at right angle.

So we see in some problems we apply calculus rather than simple coordinate geometry formulae

Question

A line L has intercepts a and b on the coordinate axes. When the axes are rotated through an angle, keeping the origin fixed, the same line L has intercepts p and q. Then,

(a) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$

(b) $\frac{1}{a^2}-\frac{1}{b^2}=\frac{1}{p^2}-\frac{1}{q^2}$

(c) $\frac{1}{a^2}+\frac{1}{b^2}=2\left(\frac{1}{p^2}+\frac{1}{q^2}\right)$

(d) none of these

Solution

(a). Since the line L has intercepts a and b on the coordinate axes, therefore its equation is

$$\frac{x}{a}+\frac{y}{b}=1 \qquad ...(1)$$

When the axes are rotated, its equation with respect to the new axes and same origin will become

$$\frac{x}{p}+\frac{y}{q}=1 \qquad ...(2)$$

In both the cases, the length of the perpendicular from the origin to the line will be same.

$$\therefore \quad \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}=\frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}}$$

or $$\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$$

which is the required relation.

Question on Locus of midpoints of intercepts

Locus of the mid-points of the intercepts between the coordinate axes by the lines passing through $(a, 0)$ does not intersect

(a) x-axis (b) y-axis (c) $y = x$ (d) $y = a$

Ans. (b)

Solution Equation of any line through $(a, 0)$ be $\frac{x}{a}+\frac{y}{b}=1$, where b is a parameter. This line meets y-axis at $(0, b)$ and if (h, k) denotes the mid-point of the intercept of the line between the coordinate axes, then $h = a/2$, $k = b/2$ and then the locus of (h, k) is $x = a/2$. This clearly does not intersect y-axis.

Question

The line L has intercepts a and b on the coordinate axes. The coordinate axes are rotated through a fixed angle, keeping the origin fixed. If p and q are the intercepts of the line L on the new axes, then $\frac{1}{a^2} - \frac{1}{p^2} + \frac{1}{b^2} - \frac{1}{q^2}$ is equal to

(a) -1 (b) 0

(c) 1 (d) none of these

Ans. (b)

Solution Equation of the line L in the two coordinate systems is

$$\frac{x}{a} + \frac{y}{b} = 1, \quad \frac{X}{p} + \frac{Y}{q} = 1$$

where (X, Y) are the new coordinates of a point (x, y) when the axes are rotated through a fixed angle, keeping the origin fixed. As the length of the perpendicular from the origin has not changed.

$$\frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} = \frac{1}{\sqrt{\frac{1}{p^2} + \frac{1}{q^2}}} \Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$$

or

$$\frac{1}{a^2} - \frac{1}{p^2} + \frac{1}{b^2} - \frac{1}{q^2} = 0.$$

Question

If a straight line cuts intercepts from the axes of coordinates the sum of the reciprocals of which is a constant k, then the line passes through the fixed point

(a) (k, k) (b) $\left(\frac{1}{k}, \frac{1}{k}\right)$

(c) $(k, -k)$ (d) $(-k, k)$

Solution

(b). Let the equation of the line be

$$\frac{x}{a}+\frac{y}{b}=1 \qquad ...(1)$$

Its intercepts on x-axis and y-axis are a and b respectively.

Given: $\frac{1}{a}+\frac{1}{b}=k$

$$\Rightarrow \quad \frac{1}{ak}+\frac{1}{bk}=1 \quad \text{or} \quad \frac{1/k}{a}+\frac{1/k}{b}=1 \qquad ...(2)$$

From (2) it follows that the line (1) passes through the fixed point $\left(\frac{1}{k}, \frac{1}{k}\right)$.

Question

A line has intercepts a and b on the coordinate axes. When the axes are rotated through an angle α, keeping the origin fixed, the line makes equal intercepts on the coordinate axes, then $\tan\alpha =$

(a) $\frac{a+b}{a-b}$ (b) $\frac{a-b}{a+b}$

(c) a^2-b^2 (d) none of these

Ans. (b)

Solution Let the equation of the line in the original coordinate system be $\frac{x}{a}+\frac{y}{b}=1$. If (X, Y) denote the coordinates of any point $P(x, y)$ in the new coordinate system obtained by rotation of the axes through an angle, then

$$x = X\cos\alpha - Y\sin a,\ y = X\sin\alpha + Y\cos\alpha$$

So that the equation of the line with reference to new system of coordinates is

$$\frac{X\cos\alpha - Y\sin\alpha}{a} + \frac{X\sin\alpha + Y\cos\alpha}{b} = 1$$

or $$X\left(\frac{\cos\alpha}{a} + \frac{\sin\alpha}{b}\right) + Y\left(\frac{\cos\alpha}{b} - \frac{\sin\alpha}{a}\right) = 1$$

Since it makes equal intercepts on the coordinates axes.

$$b\cos\alpha + a\sin\alpha = a\cos\alpha - b\sin\alpha$$

$$\Rightarrow \quad (a-b)\cos\alpha = (a+b)\sin\alpha$$

$$\Rightarrow \quad \tan\alpha = \frac{a-b}{a+b}$$

Question

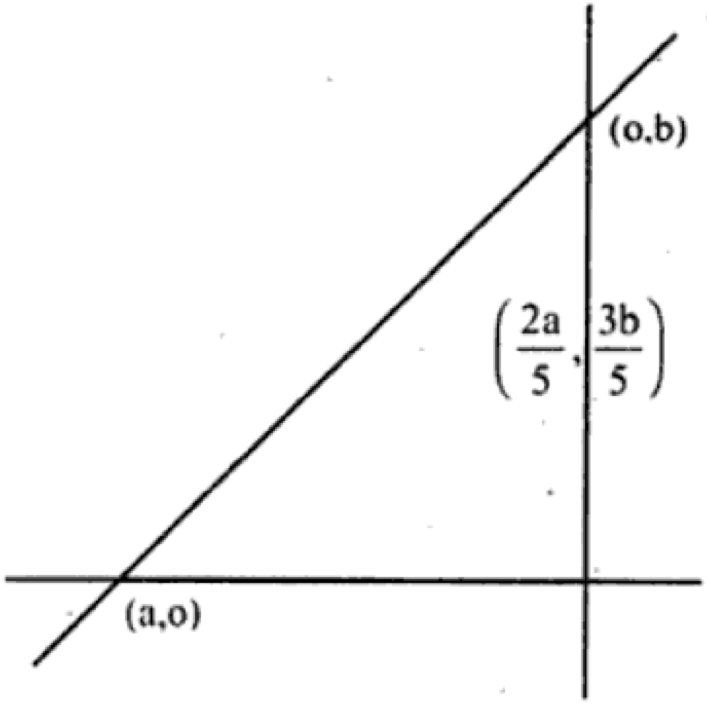
A straight line passes through the point (-2, 6) and the portion of the line intercepted between the axes is divided at this point in the ratio 3:2. The equation of the line is

a) 3x + y = 0 **b)** 2x -y + 10 = 0

c) x + 2y =10 **d)** 3x + 2y = 6

Ans (b)

Let the line be $\frac{x}{a} + \frac{y}{b} = 1$ ...(1)



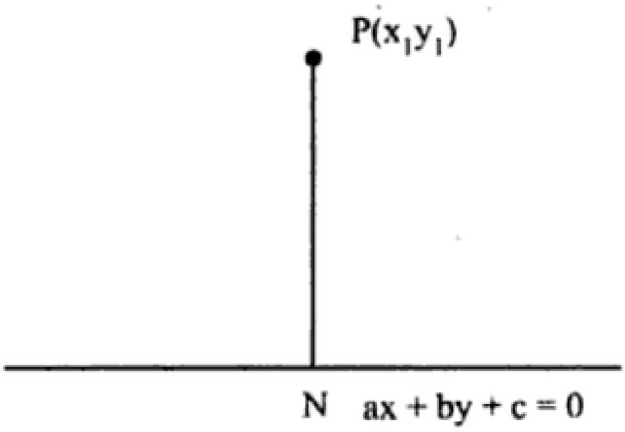
Here $\left(\frac{2a}{5}, \frac{3b}{5}\right) = (-2, 6)$

$\Rightarrow a = -5$ and $b = 10$

thus (1) becomes $2x - y + 10 = 0$

Distance of a point from a line

The length of the perpendicular from a point (x_1, y_1) to a line $ax+by+c=0$ is given by



$$PN = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

Note:

The length of the perpendicular from the origin to the line $ax+by+c=0$ is $\dfrac{|c|}{\sqrt{a^2+b^2}}$

Question on Length of Perpendiculars

If p_1, p_2 denote the lengths of the perpendiculars from the point (2, 3) on the lines given by $15x^2 + 31xy + 14y^2 = 0$, then if $p_1 > p_2$, $p_1^2 + \frac{1}{74} - p_2^2 + \frac{1}{13}$ is equal to

(a) -2 (b) 0

(c) 2 (d) none of these

Ans. (c)

Solution The lines given by $15x^2 + 31xy + 14y^2 = 0$ are

$$5x + 7y = 0 \text{ and } 3x + 2y = 0$$

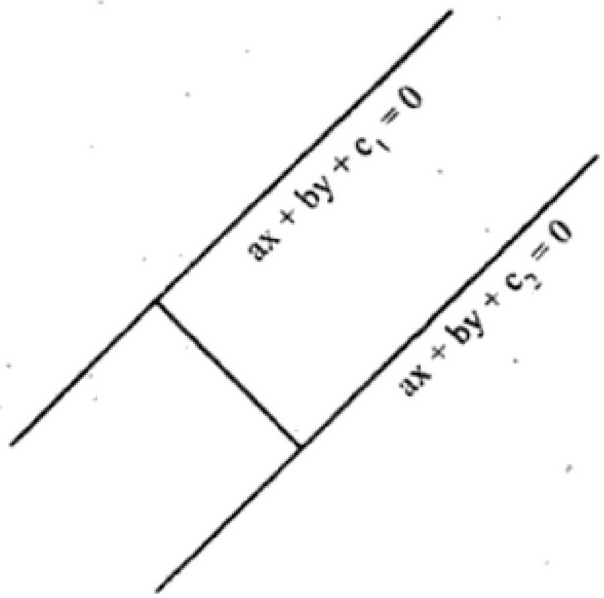
Length of the perpendiculars from (2, 3) on these lines are

$$p_1 = \frac{31}{\sqrt{74}} \text{ and } p_2 = \frac{12}{\sqrt{13}} \qquad [\because p_1 > p_2]$$

So that $p_1^2 + \frac{1}{74} - p_2^2 + \frac{1}{13} = \frac{961}{74} + \frac{1}{74} - \left(\frac{144}{13} - \frac{1}{13}\right) = 2.$

The distance between the parallel lines $ax+by+c_1=0$ and $ax+by+c_2=0$ is given by

$$\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}.$$



The two points (x_1, y_1) and (x_2, y_2) are on the same (or opposite) sides of the straight line $ax+by+c=0$ according to the quantities ax_1+by_1+c and ax_2+by_2+c have same (or opposite) signs.

Question

The distance between the parallel lines given by $(x+7y)^2 + 4\sqrt{2}\,(x+7y) - 42 = 0$ is

(a) 4/5 (b) $4\sqrt{2}$ (c) 2 (d) $10\sqrt{2}$

Ans. (c)

Solution The lines given by the equation are

$$(x + 7y - 3\sqrt{2})\,(x + 7y + 7\sqrt{2}) = 0$$

$$\Rightarrow \quad x + 7y - 3\sqrt{2} = 0 \quad \text{and} \quad x + 7y + 7\sqrt{2} = 0$$

$$\text{distance between these lines} = \left|\frac{7\sqrt{2} - (-3\sqrt{2})}{\sqrt{1^2 + 7^2}}\right| = 2.$$

Question

The equation of the straight line such that the length of the perpendicular from the origin is of length 4 and the inclination of this perpendicular to the x-axis is 135° is

a) $x - y + \sqrt{2} = 0$ **b)** $x - y + 4\sqrt{2} = 0$

c) $x - y + \frac{4}{\sqrt{2}} = 0$ **d)** None

Ans (b)

The equation of the line is $x \cos \alpha + y \sin \alpha = P$.

Here $P = 4$ and $\alpha = 135°$

$\Rightarrow x \cos 135° + y \cos 135° = 4$

$\Rightarrow x - y + 4\sqrt{2} = 0$

is the required equation.

-

Two lines given by the equations $ax + by + c = 0$ and $a'x + b'y + c' = 0$ are

(i) *parallel* (i.e., the slopes are equal), if $ab' = a'b$

(ii) *perpendicular* (i.e., the product of their slopes is -1), if $aa' + bb' = 0$

(iii) identical if $ab'c' = a'b'c = a'c'b$.

(iv) not parallel, then

(a) angle θ between them at their point of intersection is given by

$$\tan\theta = \pm\frac{m - m'}{1 + mm'} = \pm\frac{a'b - ab'}{aa' + bb'}$$

m, m' being the slopes of the two lines.

(b) the coordinates of their point of intersection are

$$\left(\frac{bc' - b'c}{ab' - a'b}, \frac{ca' - c'a}{ab' - a'b}\right)$$

(c) An equation of any line through their point of intersection is

$$(ax + by + c) + \lambda(a'x + b'y + c') = 0$$

where λ is a real number.

Question

If the equation $x+\sqrt{3}y+4=0$ is expressed in the normal form x cos α + y sin α = P, then

a) $P = 2$ **b)** $P = 4$

c) $\alpha = 120°$ **d)** $\alpha = 300°$

Ans (a)

The equation is $-x-\sqrt{3}y=4$

dividing by $\sqrt{1+\left(\sqrt{3}\right)^2}=2$,

we get

$$\frac{-1}{2}x-\frac{\sqrt{3}}{2}y=2$$

which is of the form x cos α + y sin α = P,

where $\cos\alpha=\frac{-1}{2}, \sin\alpha=\frac{-\sqrt{3}}{2}, P=2$.

Question

The line $(p+2q)x+(p-3q)y=p-q$ for different values of p and q passes through the fixed point

(a) $\left(\frac{3}{2}, \frac{5}{2}\right)$ (b) $\left(\frac{2}{5}, \frac{2}{5}\right)$

(c) $\left(\frac{3}{5}, \frac{3}{5}\right)$ (d) $\left(\frac{2}{5}, \frac{3}{5}\right)$

Solution

(d). The equation of the given line can be re-written as

$$p(x + y - 1) + q(2x - 3y + 1) = 0$$

which, clearly, passes through the point of intersection of the lines

$$x + y - 1 = 0 \quad ...(1)$$

and

$$2x - 3y + 1 = 0 \quad ...(2)$$

for different values of p and q.

Solving (1) and (2), we get the coordinates of the point of intersection as $\left(\frac{2}{5}, \frac{3}{5}\right)$.

-

The three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ are *concurrent* (intersect at a point) if and only if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Question

Lines $ax + by + c = 0$ where $3a + 2b + 4c = 0$, $a, b, c \in \mathbf{R}$ are concurrent at the point.

(a) (3, 2) (b) (2, 4) (c) (3, 4) (d) (3/4, 1/2)

Ans. (d)

Solution $3a + 2b + 4c = 0$

$$\Rightarrow \quad \frac{3}{4}a + \frac{1}{2}b + c = 0$$

$\Rightarrow$ $ax + by + c$ passes through (3/4, 1/2) for all values of a, b, c.

Question

If the lines $x + 2ay + a = 0$, $x + 3by + b = 0$ and $x + 4cy + c = 0$ are concurrent, then a, b, c are in

(a) A.P. (b) G.P.

(c) H.P. (d) none of these

Ans. (c)

Solution Since the given lines are concurrent

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0 \quad \Rightarrow \quad -bc + 2ac - ab = 0$$

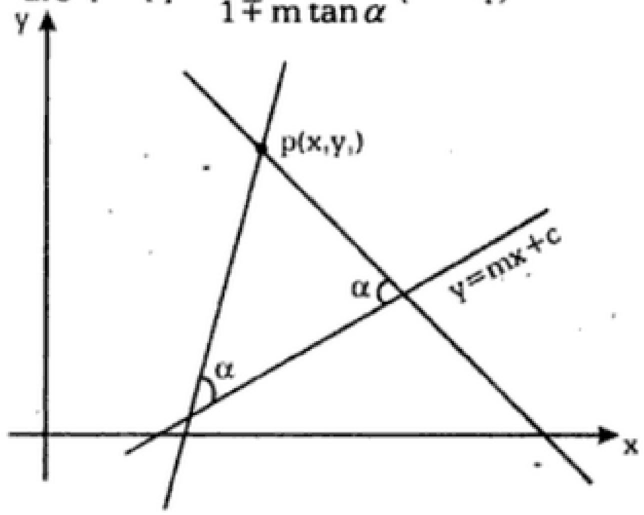
$$\Rightarrow \quad b = \frac{2ac}{a+c}$$

$\Rightarrow$ a, b, c are in H.P.

-

The equations of the straight lines which pass through a given point (x_1, y_1) and make a given angle α with the given straight line $y=mx+c$

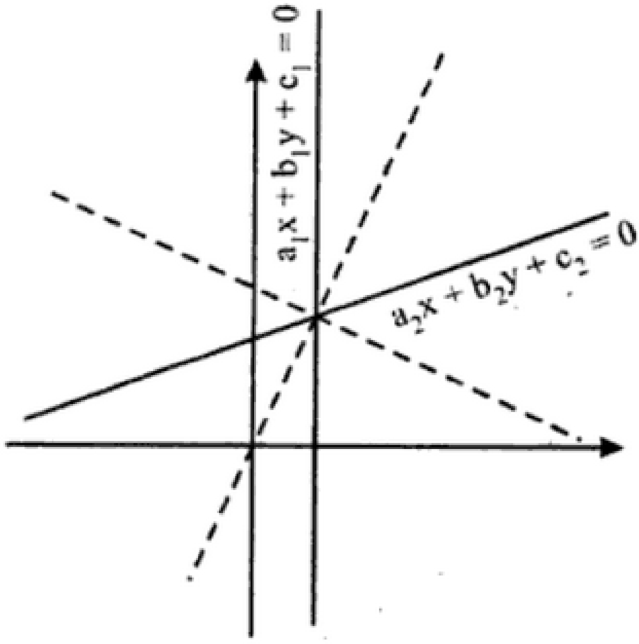
are $y - y_1 = \dfrac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$



The angle between the lines $x\cos\alpha_1 + y\sin\alpha_1 = P_1$ and $x\cos\alpha_2 + y\sin\alpha_2 = P_2$ is $\alpha_1 - \alpha_2$.

Equation of Internal and External bisectors of 2 Lines

The equation of the bisectors of the angles between the lines $a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2=0$ is given by



$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

Bisector of the angle containing the origin

If c_1, c_2 are positive, then the equation of the bisector of the angle containing the origin is

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = + \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

Bisector of Acute and Obtuse angle between lines

i) If c_1, c_2 are positive and if $a_1a_2+b_1b_2>0$, then

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}=+\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$$ is the obtuse angle bisector and

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}=-\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$$ is the acute angle bisector.

ii) If c_1,c_2 are positive and if $a_1a_2+b_1b_2<0$, then

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}=+\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$$

is the acute angle bisector and

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}=-\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$$

is the obtuse angle bisector.

If c_1, c_2 are positive and $a_1a_2+b_1b_2>0$, then the origin lies in the obtuse angle and the '+' sign gives the bisector of the obtuse angle. If $a_1a_2+b_1b_2<0$, then the origin lies in the acute angle and '+' sign gives the bisector of acute angle.

Question

If a line joining two points A (2, 0) and B (3, 1) is rotated about A in anticlockwise direction through an angle 15°, then equation of the line in the new position is

(a) $\sqrt{3}\ x + y = 2\sqrt{3}$ (b) $\sqrt{3}\ x - y = 2\sqrt{3}$

(c) $x + \sqrt{3}y = 2\sqrt{3}$ (d) $x - \sqrt{3}y = 2\sqrt{3}$

Ans. (b)

Solution Slope of $AB = \frac{1-0}{3-2} = 1$

$\Rightarrow \quad \angle BAX = 45°$ (Ref. Fig. 15.2)

If AC is the new position of the line AB then $\angle CAX = 45° + 15° = 60°$. and thus its equation is

$$y = \tan 60° (x - 2)$$

$$\Rightarrow \quad y = \sqrt{3}\ (x - 2) \quad \Rightarrow \quad \sqrt{3}x - y = 2\sqrt{3}$$

Question

The line $y = 3x$ bisects the angle between the lines $ax^2 + 2axy + y^2 = 0$ if $a =$

(a) 3 (b) 11 (c) 3/11 (d) 11/3

Ans. (c)

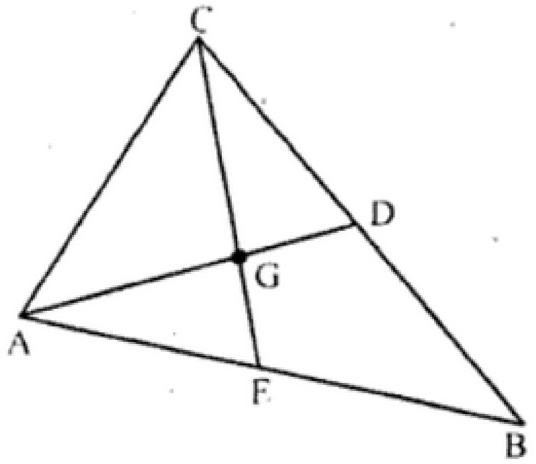
Solution Equation of the bisectors of the angles between the lines $ax^2 + 2axy + y^2 = 0$ is

$\frac{x^2 - y^2}{a-1} = \frac{xy}{a}$ which is satisfied by $y = 3x$ if $\frac{1-9}{a-1} = \frac{3}{a} \Rightarrow a = 3/11$

Question

Coordinates of Centroid, Orthocenter, Circumcenter of a Triangle

Centroid: The point of intersection of the medians of a triangle is called its centroid. It divides the median in the ratio 2:1.



If (x_1, y_1) (x_2, y_2) and (x_3, y_3) are the vertices of a triangle, then the coordinates of its centroid are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Question on Centroid

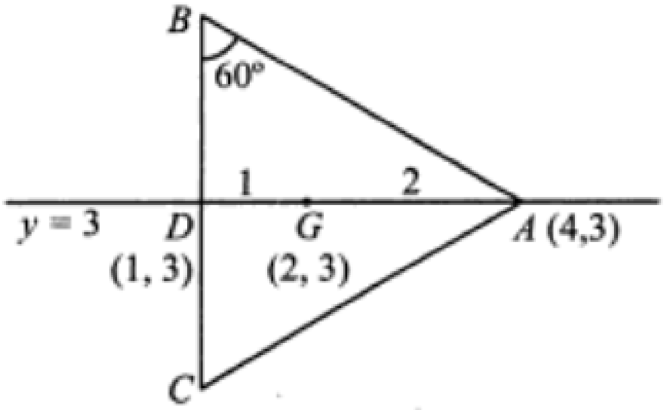
If the centroid and a vertex of an equilateral triangle are (2, 3) and (4, 3) respectively, then the other two vertices of the triangle are

(a) $(1, 3 \pm \sqrt{3})$ (b) $(2, 3 \pm \sqrt{3})$

(c) $(1, 2 \pm \sqrt{3})$ (d) $(2, 2 \pm \sqrt{3})$

Solution

(a). G being the centroid, divides AD in the ratio 2 : 1.



Since $AG = 2$, $\quad \therefore \quad GD = 1$,

$\therefore$ Coordinates of D, using section formula, are $D(1, 3)$.

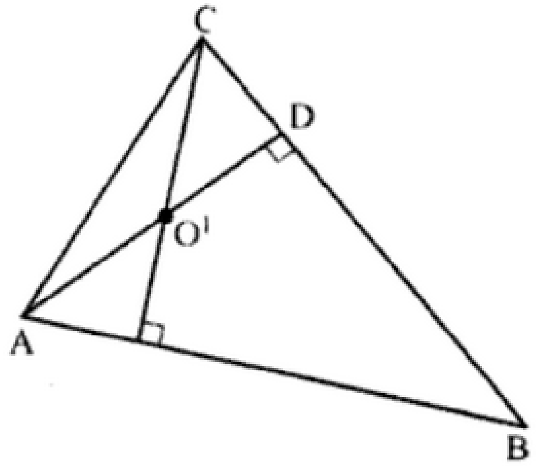
Now $AD = 1 + 2 = 3$, $\therefore \tan 60° = \dfrac{3}{BD} \Rightarrow BD = \sqrt{3}$.

$\therefore \quad B \equiv (1, 3 + \sqrt{3})$ and $C \equiv (1, 3 - \sqrt{3})$.

Orthocentre

The point of intersection of the altitudes of a triangle is called its orthocentre

To determine the orthocentre, first we find equations of line passing through vertices and perpendicular to the opposite sides. Solving two of these three equations we get the co-ordinates of orthocentre.



If angles A, B and C and vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ of a ΔABC are given, then orthocentre of ΔABC is given by

$$\left(\frac{x_1 \tan A + x_2 \tan B + x_3 \tan C}{\tan A + \tan B + \tan C}, \frac{y_1 \tan A + y_2 \tan B + y_3 \tan C}{\tan A + \tan B + \tan C}\right)$$

If any two lines out of three lines, i.e., AB, BC and CA are perpendicular, then orthocentre is the point of intersection of two perpendicular lines.

The orthocentre of the triangle with vertices (0, 0), (x_1, y_1) and (x_2, y_2) is

$$\left\{(y_1 - y_2)\left[\frac{x_1 x_2 - y_1 y_2}{x_2 y_1 - x_1 y_2}\right],\right.$$

$$\left.(x_1 - x_2)\left[\frac{x_1 x_2 + y_1 y_2}{x_1 y_2 - x_2 y_1}\right]\right\}$$

Question on Orthocenter

The orthocentre of the triangle formed by the lines $xy = 0$ and $2x + 3y - 5 = 0$ is

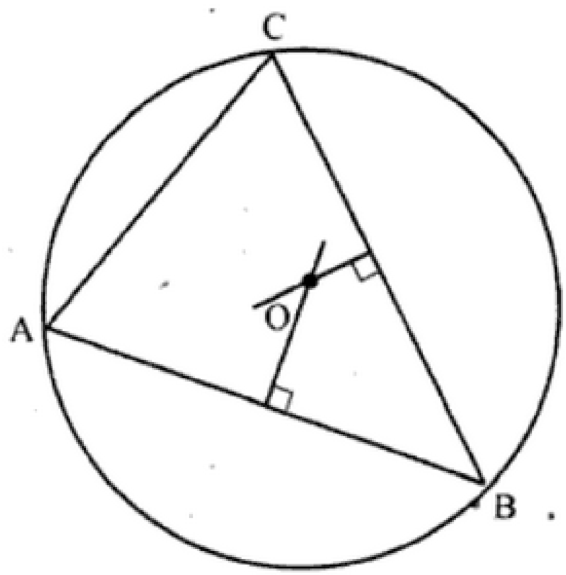
(a) (2, 3) (b) (3, 2) (c) (0, 0) (d) (5, – 5)

Ans. (c)

Solution The given triangle is right angled at (0, 0) which is therefore the orthocentre of the triangle.

Circumcentre

The point of intersection of the perpendicular bisectors of the sides of a triangle is called its circum-centre. It is equidistant from the vertices of a triangle.



Note:

The circumcentre O, centroid G and orthocentre O′ of a triangle ABC are collinear such that G divides O′O in the ratio 2:1
i.e., O′G:OG=2:1

Question

If the circumcentre of a triangle lies at the origin and the centroid is the middle point of the line joining the points $(a^2 + 1, a^2 + 1)$ and $(2a, -2a)$; then the orthocentre lies on the line

(a) $y = (a^2 + 1)x$ (b) $y = 2ax$

(c) $x + y = 0$ (d) $(a-1)^2 x - (a+1)^2 y = 0$

Ans. (d)

Solution We know from geometry that the circumcentre, centroid and orthocentre of a triangle lie on a line. So the orthocentre of the triangle lies on the line joining the circumcentre (0, 0) and the centroid $\left(\frac{(a+1)^2}{2}, \frac{(a-1)^2}{2}\right)$

i.e. $$\frac{(a+1)^2}{2} y = \frac{(a-1)^2}{2} x$$

or $$(a-1)^2 x - (a+1)^2 y = 0.$$

Question

If the equations of the sides of a triangle are $x + y = 2$, $y = x$ and $\sqrt{3}y + x = 0$, then which of the following is an exterior point of the triangle?

(a) orthocentre (b) incentre

(c) centroid (d) none of these

Solution

(a). The lines $y = x$ and $\sqrt{3}y + x = 0$ are inclined at 45° and 150°, respectively, with the positive direction of x-axis. So, the angle between the two lines is an obtuse angle. Therefore, orthocentre lies outside the given triangle, whereas incentre and centroid lie within the triangle (In any triangle, the centroid and the incentre lie within the triangle).

Question

The equations to the sides of a triangle are $x - 3y = 0$, $4x + 3y = 5$ and $3x + y = 0$. The line $3x - 4y = 0$ passes through the

(a) incentre (b) centroid

(c) circumcentre (d) orthocentre of the triangle

Ans. (d)

Solution Two sides $x - 3y = 0$ and $3x + y = 0$ of the triangle being perpendicular to each other, the triangle is right angled at the origin, the point of intersection of these sides. So that origin is the orthocentre of the triangle and the line $3x - 4y = 0$ passes through this orthocentre.

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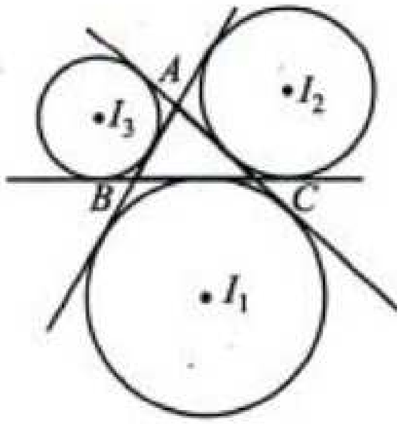
Ex-Centres of a Triangle A circle touches one side outside the triangle and the other two extended sides then circle is known as excircle.



Let ABC be a triangle then there are three excircles, with three excentres I_1, I_2, I_3 opposite to vertices A, B and C respectively. If the vertices of triangle are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ then

$$I_1 = \left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c}\right)$$

$$I_2 = \left(\frac{ax_1 - bx_2 + cx_3}{a-b+c}, \frac{ay_1 - by_2 + cy_3}{a-b+c}\right)$$

$$I_3 = \left(\frac{ax_1 + bx_2 - cx_3}{a+b-c}, \frac{ay_1 + by_2 - cy_3}{a+b-c}\right).$$

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Family of lines through the intersection of two given lines

The equation of family of lines passing through the intersection of the lines

$L_1 = a_1x+b_1y+c_1=0$ and $L_2=a_2x+b_2y+c_2=0$ is

$(a_1x+b_1y+c_1) + \lambda(a_2x+b_2y+c_2) = 0$, where λ is a parameter i.e., $L_1+\lambda L_2=0$.

Formulae specific to Pair of Straight Lines

Homogeneous equation of second degree in x and y

A general homogenous equation of degree 2 always represent two straight lines, real or imaginary, through the origin. Conversely, the equal of a pair of lines through origin is a second degree homogeneous equation in x and y.

The equation of the form $ax^2+2hxy+by^2=0$ is called a homogeneous equation of degree 2 in x and y, where a, b, h are constants.

let $ax^2 + 2hxy+by^2=0$ ...(1)

$$b\left(\frac{y}{x}\right)^2 + 2h\left(\frac{y}{x}\right) + a = 0$$

The general equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of Straight lines only if

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0 \quad \text{i.e., iff} \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

For easy remembering note that the first row of the Determinant is coeffs of x terms

$(a)x^2 + 2(h)xy$ …. + 2 (g) x …..

Similarly the second row is made of coeffs of y terms. i.e.

2 (h) xy + $(b)y^2$ + 2 (f) y ….

The last row of the determinant is the last 3 constants of last 3 terms. i.e. g, f, and c

Equation of the lines joining the origin to the points of intersection of a line and a conic.

Let $L \equiv lx + my + n = 0$

and $S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

be the equations of a line and a *conic*, respectively. Writing the equation of the line as $\frac{lx+my}{-n} = 1$ and making $S = 0$ homogeneous with its help, we get

$$S = ax^2 + 2hxy + by^2 + 2(gx + fy)\left(\frac{lx+my}{-n}\right) + c\left(\frac{lx+my}{-n}\right)^2 = 0$$

which being a homogeneous equation of second degree, represents a pair of straight lines through the origin and passing through the points common to $S = 0$ and $L = 0$.

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Equation of the pair of lines through the origin perpendicular to the pair of lines $ax^2 + 2hxy + by^2 = 0$ is $bx^2 - 2hxy + ay^2 = 0$.

Question

If the slope of one of the lines represented by $ax^2 - 6xy + y^2 = 0$ is square of the other, then

(a) $a = 1$ (b) $a = 2$ (c) $a = 4$ (d) $a = 8$

Ans. (d)

Solution Let the lines represented by the given equation be $y = mx$ and $y = m^2x$, then

$$m + m^2 = 6 \text{ and } m^3 = a$$

$\Rightarrow$ $m = 2$ or -3

and so $a = 8$ or -27

Question

If the pairs of lines $x^2 + 2xy + ay^2 = 0$ and $ax^2 + 2xy + y^2 = 0$ have exactly one line in common then the joint equation of the other two lines is given by

(a) $3x^2 + 8xy - 3y^2 = 0$
(b) $3x^2 + 10xy + 3y^2 = 0$
(c) $y^2 + 2xy - 3x^2 = 0$
(d) $x^2 + 2xy - 3y^2 = 0$

Ans. (b)

Solution Let $y = mx$ be a line common to the given pairs of lines, then

$am^2 + 2am + 1 = 0$ and $m^2 + 2m + a = 0 \Rightarrow \dfrac{m^2}{2(1-a)} = \dfrac{m}{a^2 - 1} = \dfrac{1}{2(1-a)}$

$\Rightarrow m^2 = 1$ and $m = -\dfrac{a+1}{2} \Rightarrow (a + 1)^2 = 4 \Rightarrow a = 1$ or -3

But for $a = 1$, the two pairs have both the lines common, so $a = -3$ and the slope m of the line common to both the pairs is 1.

Now $x^2 + 2xy + ay^2 = x^2 + 2xy - 3y^2 = (x - y)(x + 3y)$

and $ax^2 + 2xy + y^2 = -3x^2 + 2xy + y^2 = -(x - y)(3x + y)$

So the equation of the required lines is

$$(x + 3y)(3x + y) = 0 \Rightarrow 3x^2 + 10xy + 3y^2 = 0.$$

Question

If one of the lines given by the equation $2x^2 + axy + 3y^2 = 0$ coincide with one of those given by $2x^2 + bxy - 3y^2 = 0$ and the other lines represented by them be perpendicular, then

(a) $a = -5, b = 1$ (b) $a = 5, b = -1$

(c) $a = 5, b = 1$ (d) none of these

Ans. (c)

Solution Let $\frac{2}{3}x^2 + \frac{a}{3}xy + y^2 = (y - mx)(y - m'x)$

and $\frac{2}{-3}x^2 + \frac{b}{-3}xy + y^2 = \left(y + \frac{1}{m}x\right)(y - m'x)$

then $m + m' = -\frac{a}{3}, mm' = \frac{2}{3}$ (i)

$\frac{1}{m} - m' = \frac{-b}{3}, -\frac{m'}{m} = -\frac{2}{3}$ (ii)

$\Rightarrow$ $m^2 = 1 \Rightarrow m = \pm 1$

If $m = 1, m' = \frac{2}{3} \Rightarrow a = -5, b = -1$

If $m = -1, m' = -\frac{2}{3} \Rightarrow a = 5, b = 1.$

Question

If pairs of lines $3x^2 - 2pxy - 3y^2 = 0$ and $5x^2 - 2q\,xy - 5y^2 = 0$ are such that each pair bisects the angle between the other pair, then pq is equal to

(a) -1 (b) -3 (c) -5 (d) -15

Ans. (d)

Solution Equation of the bisectors of angles between the lines $3x^2 - 2pxy - 3y^2 = 0$ is

$$\frac{x^2 - y^2}{3+3} = \frac{xy}{-p} \text{ or } x^2 + \frac{6}{p}xy - y^2 = 0 \qquad \text{(i)}$$

If (i) represents the lines $5x^2 - 2qxy - 5y^2 = 0$

or $x^2 - (2q/5)\,xy - y^2 = 0$

then $\frac{6}{p} = \frac{-2q}{5} \Rightarrow pq = -15$

Question

If the area of the triangle formed by the pair of lines $8x^2 - 6xy + y^2 = 0$ and the line $2x + 3y = a$ is 7, then a is equal to

(a) 14 (b) $14\sqrt{2}$
(c) 28 (d) none of these

Ans. (c)

Solution Equations of the sides of the given triangle are $y = 2x$, $y = 4x$ and $2x + 3y = a$

$\therefore$ vertices of the triangle formed by these lines are $(a/8, a/4)$, $(a/14, 2a/7)$ and $(0, 0)$

$$\text{Area of the triangle} = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ \frac{a}{8} & \frac{a}{4} & 1 \\ \frac{a}{14} & \frac{2a}{7} & 1 \end{vmatrix} = \frac{a^2}{112} = 7 \Rightarrow a = 28.$$

Question

The lines joining the origin to the points of intersection of $x^2 + y^2 + 2gx + c = 0$ and $x^2 + y^2 + 2fy - c = 0$ are at right angles, if

(a) $g^2 + f^2 = c$ (b) $g^2 - f^2 = c$

(c) $g^2 - f^2 = 2c$ (d) $g^2 + f^2 = c^2$

Ans. (c)

Solution Subtracting the given equations we get

$$2gx - 2fy + 2c = 0 \quad \Rightarrow \quad gx - fy + c = 0$$

$$\Rightarrow \quad \frac{gx - fy}{c} = -1 \qquad \text{(i)}$$

Now to obtain the lines joining the origin to the points of intersection of the given circles we make the equation of the first circle homogeneous with the help of (i), which gives

$$x^2 + y^2 + 2gx\left(\frac{gx - fy}{-c}\right) + c\left(\frac{gx - fy}{c}\right)^2 = 0$$

$$\Rightarrow \quad c(x^2 + y^2) - 2gx(gx - fy) + (gx - fy)^2 = 0$$

These lines are at right angles if

$$c - 2g^2 + g^2 + c + f^2 = 0$$

$$g^2 - f^2 = 2c.$$

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Question on Locus

If $P(1, 0)$, $Q(-1, 0)$ and $R(2, 0)$ are three given points. The point S satisfies the relation $SQ^2 + SR^2 = 2SP^2$. The locus of S meets PQ at the point

(a) $(0, 0)$ (b) $(2/3, 0)$

(c) $(-3/2, 0)$ (d) $(0, -2/3)$

Ans. (c)

Solution Let S be the point (x, y)

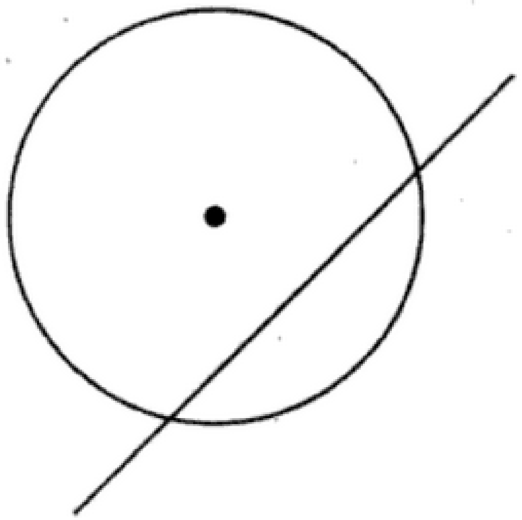
then $(x+1)^2 + y^2 + (x-2)^2 + y^2 = 2[(x-1)^2 + y^2]$

$\Rightarrow 2x + 3 = 0$, the locus of S and equation of PQ is $y = 0$.

So the required points is $(-3/2, 0)$.

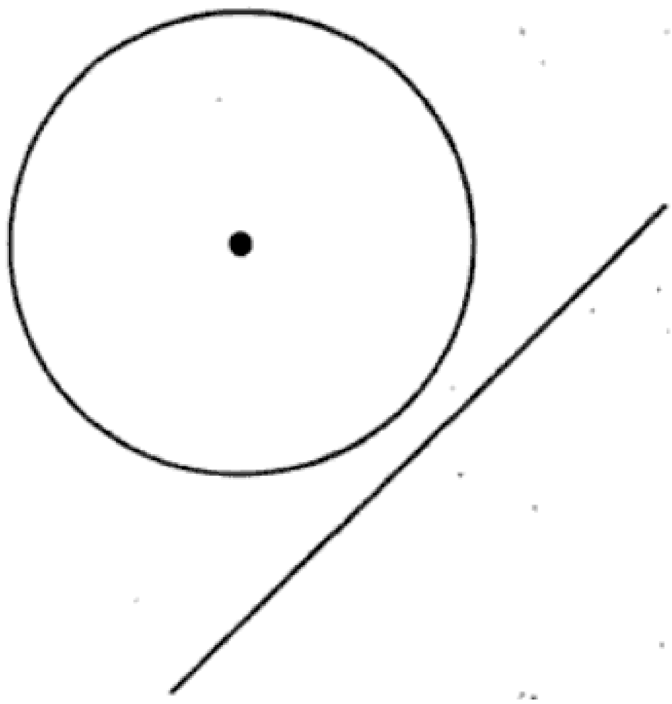
Formulae related to circles

The line y = mx + c intersects the circle $x^2 + y^2 = a^2$ at two distinct points if the length of the perpendicular from the centre is less than the radius of the circle.



ie., $\left|\dfrac{c}{\sqrt{1+m^2}}\right| < a$

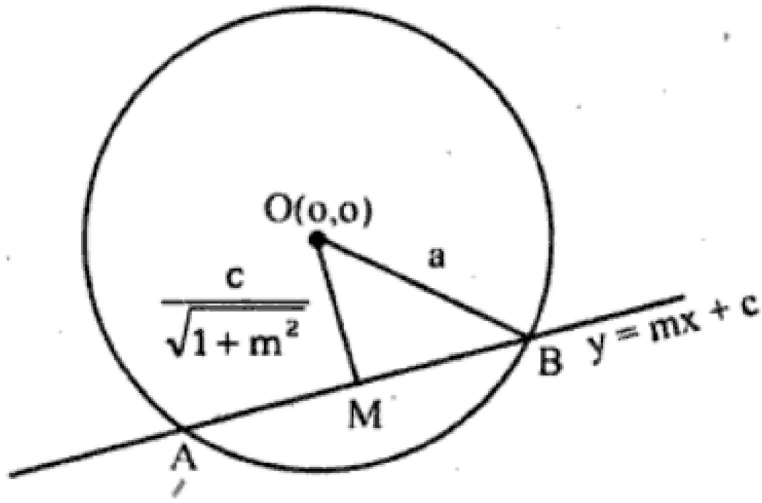
The line does not intersect the circle $x^2 + y^2 = a^2$ if the length of the perpendicular, from the centre is greater than the radius of the circle



ie., $\left|\frac{c}{\sqrt{1+m^2}}\right| > a$

iii) The length of the intercept cut off from a line $y = mx + c$ by a circle $x^2 + y^2 = a^2$ is

$$2MB = 2\sqrt{\frac{a^2(1+m^2)-c^2}{(1+m^2)}}$$



Question on Tangent

The point on the curve $y = 6x - x^2$ where the tangent is parallel to x-axis is

(*a*) (0, 0) (*b*) (2, 8)

(*c*) (6, 0) (*d*) (3, 9).

Solution

(*d*) $\frac{dy}{dx} = 6 - 2x$

$\therefore \quad \frac{dy}{dx} = 0 \Rightarrow x = 3.$

$\therefore \quad y = 18 - 9 = 9 \quad \therefore$ Point is (3, 9).

Question

For the curve $x = t^2 - 1, y = t^2 - t$, the tangent line is perpendicular to x- axis, where

(a) $t = 0$ (b) $t \to \infty$

(c) $t = \frac{1}{\sqrt{3}}$ (d) $t = -\frac{1}{\sqrt{3}}$.

Solution

(a) $\frac{dx}{dt} = 2t,$

Tangent is perpendicular to x-axis if $\frac{dx}{dt} = 0 \Rightarrow t = 0$.

Question

The point on the curve $y^2 = x$, the tangent at which makes an angle of 45° with x-axis will be given by

(a) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (b) $\left(\frac{1}{2}, \frac{1}{2}\right)$

(c) $(2, 4)$ (d) $\left(\frac{1}{4}, \frac{1}{2}\right)$.

Solution

(d) $y^2 = x \Rightarrow 2y\frac{dy}{dx} = 1$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2y} = \tan 45° = 1$ (given)

$\Rightarrow y = \frac{1}{2}$. $\therefore x = \frac{1}{4}$

$\therefore$ Point is $\left(\frac{1}{4}, \frac{1}{2}\right)$.

Question

If tangent to the curve $x = at^2, y = 2at$ is perpendicular to x-axis then its point of contact is

(a) (a, a) (b) $(0, a)$

(c) $(a, 0)$ (d) $(0, 0)$.

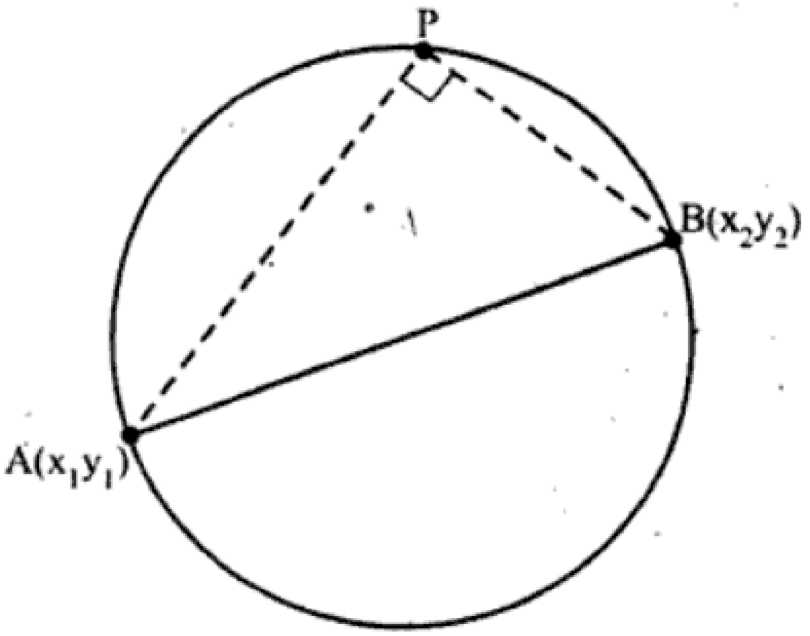
Solution

(d) $\frac{dx}{dt} = 2at, \frac{dy}{dt} = 2a \Rightarrow \frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$

$\Rightarrow \frac{1}{t} = \infty \Rightarrow t = 0 \Rightarrow$ Point is $(0, 0)$.

Equation of the circle when the end points of a diameter are given

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be the end points of a diameter of circle and let P be any point on circle.



Now, since the angle subtended at the point P in the semicircle APB is a right angle.

$m_1m_2 = -1$ (m_1 = slope of AP, m_2 = slope of BP)

$$\frac{y-y_1}{x-x_1} \times \frac{y-y_2}{x-x_2} = -1$$

ie., $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Condition for two intersecting circles to be orthogonal

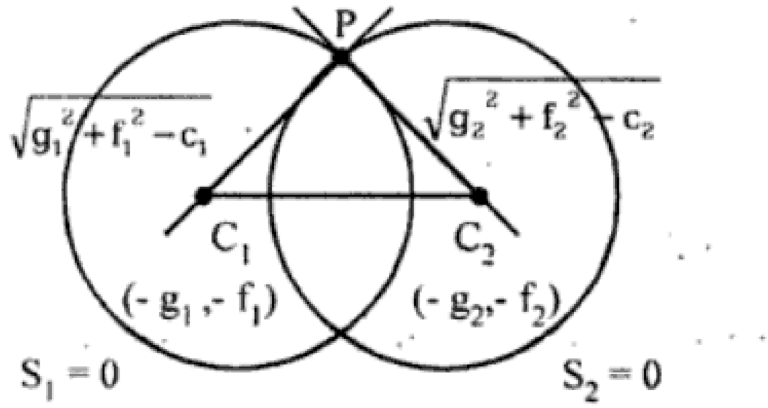
Definition

Two intersecting circles are said to cut each other orthogonally when the tangents at the point of intersection of the two circles are at right angles.

Let the circles

$S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + C_1 = 0$ and

$S_2 \equiv x^2 + y^2 + 2g_2x + 2f_2y + C_2 = 0$



intersect orthogonally, then $\angle C_1PC_2 = 90°$

ie., ΔC_1PC_2 is right angled

$\therefore C_1C_2^2 = C_1P^2 + C_2P^2$

$(g_1 - g_2)^2 + (f_1 - f_2)^2 = (g_1^2 + f_1^2 - c_1) + (g_2^2 + f_2^2 - c_2)$

$\Rightarrow 2g_1g_2 + 2f_1f_2 = c_1 + c_2$ is the required condition that S_1 and S_2 intersect orthogonally.

Some important results

i) The equation of chord joining two points θ_1 and θ_2 on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is

$$(x + g)\cos\frac{\theta_1+\theta_2}{2} + (y + f)\sin\frac{\theta_1+\theta_2}{2} = r\cos\left(\frac{\theta_1-\theta_2}{2}\right),$$ where r is the radius of the circle.

ii) The equation of the tangent at $P(\theta)$ on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $(x+ g)\cos\theta + (y + f)\sin\theta = \sqrt{g^2+f^2-c}$

iii) The locus of the point of intersection of two tangents drawn to the circle $x^2 + y^2 = a^2$ which makes an constant angle α to each other is $x^2 + y^2 - 2a^2 = 4a^2(x^2 + y^2 - a^2)\cot^2\alpha$.

Question

The equation of tangent to the circle $x^2 + y^2 + 6x + 4y - 12 = 0$ at (6,2) is

a) $4x - 9y - 6 = 0$ **b)** $9x + 4y + 12 = 0$

b) $3x - 9y = 0$ **d)** $2x - 3y = 6$

Ans (b)

Note:

The equation of tangent at (x_1, y_1) is

$xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$

thus the equation of tangent at (6,2) is

$6x + 2y + 3(x+6) + 2(y+2) - 12 = 0$

i.e., $9x + 4y + 12 = 0$.

Question

The line $y = m(x-a) + a\sqrt{1+m^2}$ touches the circle $x^2 + y^2 = 2ax$

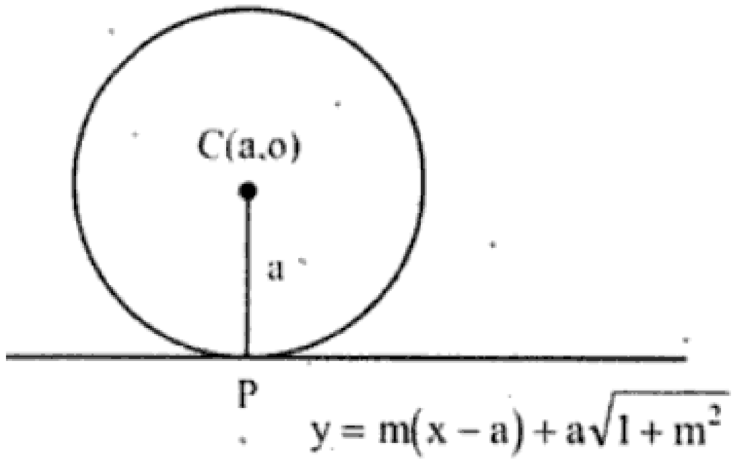
a) for only two real values of m

b) for only one real value of m

c) for no real value of m

d) for all real values of m

Ans (d)



The centre and radius of the circle $x^2 + y^2 - 2ax$ are (a, 0) and a respectively.

The length of perpendicular from (a, 0) to the line $y - mx + am - a\sqrt{1+m^2} = 0$ is

$$CP = \left| \frac{0 - ma + am - a\sqrt{1+m^2}}{\sqrt{1+m^2}} \right| = a$$

since the $\perp^r$ distance from centre to the line is equal to the radius the line touches the circle for all real values of m.

Question on Angle of intersection

The angle of intersection of the curves $y = x^2$ and $6y = 7 - x^3$ at (1, 1) is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) None of these.

Solution

(c) $y = x^2 \Rightarrow \frac{dy}{dx} = 2x \Rightarrow m_1 = 2$

$$6y = 7 - x^3 \Rightarrow \frac{dy}{dx} = -\frac{1}{2} \Rightarrow m_2 = -\frac{1}{2}$$

$\therefore \quad m_1 m_2 = -1$ at (1, 1)

$\Rightarrow \quad \theta = \frac{\pi}{2}$.

Question

If a, x_1, x_2 are in G.P. with common ratio r, and b, y_1, y_2 are in G.P. with common ratio s where $s - r = 2$, then the area of the triangle with vertices (a, b), (x_1, y_1) and (x_2, y_2) is

(a) $|ab(r^2 - 1)|$ (b) $ab(r^2 - s^2)$

(c) $ab(s^2 - 1)$ (d) $abrs$

Ans. (a)

Solution Area of the triangle

$$= \frac{1}{2}\left|\begin{vmatrix} a & b & 1 \\ ar & bs & 1 \\ ar^2 & bs^2 & 1 \end{vmatrix}\right| = \frac{1}{2}|ab(r-1)(s-1)(s-r)|$$

$$= |ab(r-1)(r+1)| = |ab(r^2 - 1)|$$

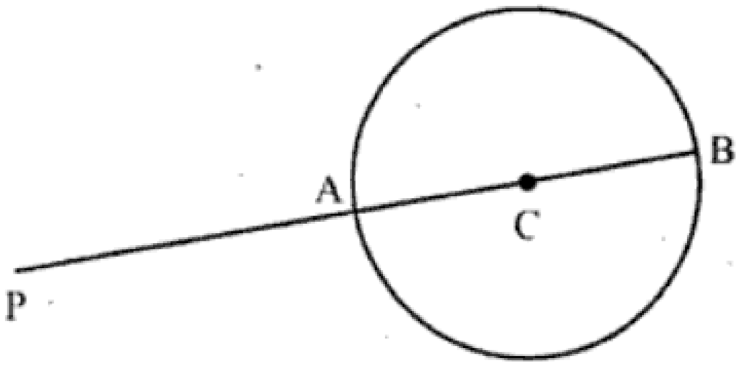
Question

Let $S \equiv x^2 + y^2 - 4x + 6y - 12 = 0$ and $P = (-13, 17)$ and consider the statements

A: The nearest point on S from P is (-1,1)
B: The farthest point on S from P is (5,-7), then

a) only statement A is true
b) only statement B is true
c) both the statements A and B are true
d) neither statement A nor statement B is true

Ans (c)



Here centre, C= (2, -3)

radius

$$= \sqrt{4+9+12} = 5$$

$$CP = \sqrt{(2+13)^2 + (-3-17)^2} = \sqrt{625} = 25 > r$$

$\Rightarrow$ P lies outside the circle.

let A, B be the nearest and farthest points on the circle from P

$\therefore$ PA+AC = CP $\Rightarrow$ PA+5 = 25 $\Rightarrow$ PA = 20

Also

PB=PC+CB $\Rightarrow$ PB = 25+5 25 $\Rightarrow$ PB = 30

Now A divides PC in the ratio

PA:AC=20:5=4:1

$$\Rightarrow A = \left(\frac{4(2)+1(-13)}{4+1}, \frac{4(-13)+1(17)}{4+1}\right)$$

$$= (-1,1)$$

Now B divides PC in the ratio PB : BC =

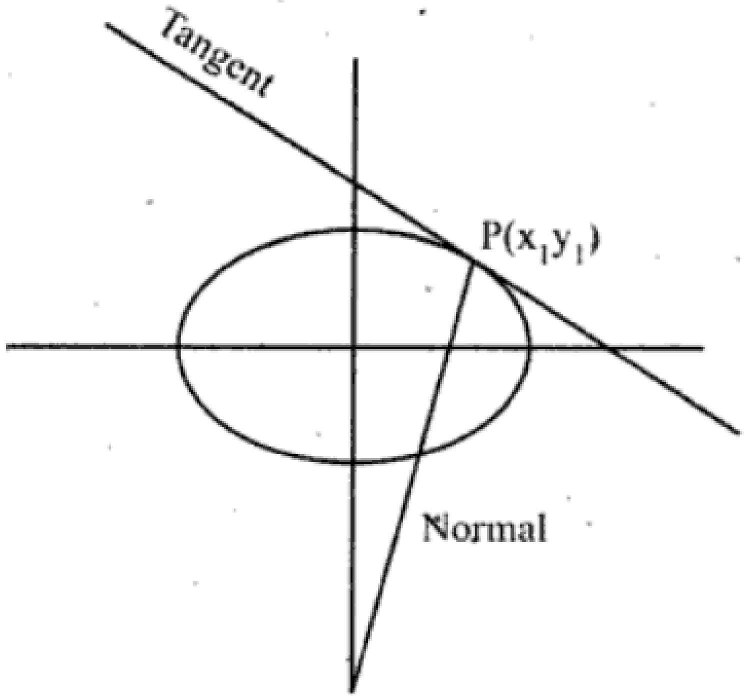
30:5=6:1 externally

$$\therefore \quad B = \left(\frac{6(2)-1(-13)}{6-1}, \frac{6(-3)-1(17)}{6-1}\right)$$

$$= (5,-7)$$

Formulae related to ellipse

The equation of tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $P(x_1 y_1)$ is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$



The equation of normal to the ellipse

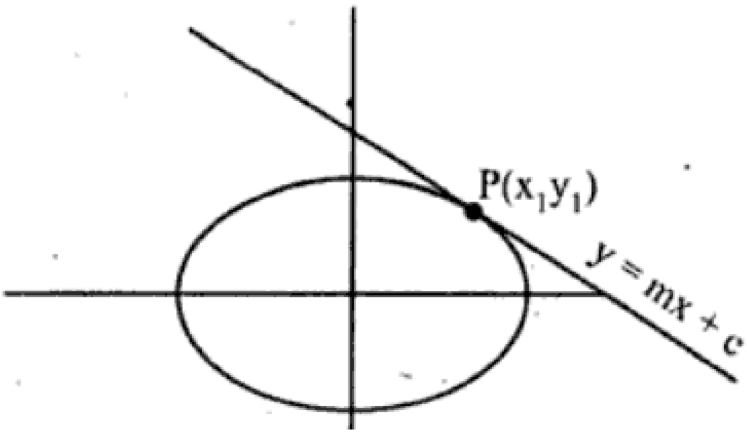
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $P(x_1 y_1)$ is $\frac{a^2 x}{x_1}-\frac{b^2 y}{y_1}=a^2-b^2$

Note:

Four normals can be drawn from any point to the ellipse.

Condition for y = mx + c to be a tangent to the ellipse and points of tangency

The equation of tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $P(x_1 y_1)$ is



$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1 \quad ...(1)$$

Given $-mx + y = c$ ..(2)

(1) and (2) represent the same line

thus $\dfrac{\frac{x_1}{a^2}}{-m} = \dfrac{\frac{y_1}{b^2}}{1} = \dfrac{1}{c}$

$$\Rightarrow x_1 = \frac{-a^2m}{c}, \; y_1 = \frac{b^2}{c}$$

Since $P(x_1, y_1)$ lies on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

we get, $\dfrac{x_1^2}{a^2} + \dfrac{y_1^2}{b^2} = 1$

$$\Rightarrow \frac{a^4m^2}{c^2a^2} + \frac{b^4}{c^2b^2} = 1$$

Formulae related to Hyperbola

Parametric equations of the hyperbola

A point (x, y) on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ can be represented as $x = a\sec\theta$, $y = b\tan\theta$ in a single parameter θ. These equations are called parametric equations of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$. The point $(a\sec\theta, b\tan\theta)$ is simply denoted by θ.

Some important results

i) The equation of the chord joining the points $(a \sec \alpha, b \tan \alpha)$ and $(a \sec \beta, b \tan \beta)$ is

$$\frac{x}{a}\cos\frac{\alpha-\beta}{2}-\frac{y}{b}\sin\frac{\alpha+\beta}{2}=\cos\frac{\alpha+\beta}{2}.$$

ii) The equation of the tangent at $P(\theta)$ on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

$$\frac{x\sec\theta}{a}-\frac{y\tan\theta}{b}=1$$

iii) The equation of the normal at $P(\theta)$ on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

$$\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2$$

iv) The condition that the line $lx + my + n = 0$ may be a normal to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ is } \frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2 + b^2)^2}{n^2}$$

v) If P is a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with foci S and S^1, then $S^1P - SP = 2a$.

vi) The locus of point of intersection of perpendicular tangents to an hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is a circle $x^2 + y^2 = a^2 - b^2$ called director circle of the hyperbola.

vii) The locus of the feet of perpendiculars drawn the foci to any tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is a circle $x^2 + y^2 = a^2$, called auxiliary circle of the hyperbola.

Question

The distance between the points (a cos α, a sin α) and (a cos β, a sin β) is

a) $a\sin\frac{\alpha-\beta}{2}$ **b)** $2a\cos\frac{\alpha-\beta}{2}$

c) $a\cos\frac{\alpha-\beta}{2}$ **d)** $2a\sin\frac{\alpha-\beta}{2}$

Ans (d)

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{(a\cos\beta-a\cos\alpha)^2+(a\sin\beta-a\sin\alpha)^2}$$

$$=\sqrt{a^2\left(2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}\right)^2+a^2\left(2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}\right)^2}$$

$$=2a\sin\frac{\alpha-\beta}{2}\sqrt{\sin^2\frac{\alpha+\beta}{2}+\cos^2\frac{\alpha+\beta}{2}}$$

$$=2a\sin\frac{\alpha-\beta}{2}$$

Question

The points (3,3) (9,0) and (12,21) are the

a) Collinear

b) Vertices of an equilateral triangle

c) Vertices of isosceles triangle

d) Vertices of right-angled triangle.

Ans (d)

Let A(3,3) B(9,0) and C(12,21)

$$AB = \sqrt{6^2 + (-3)^2} = \sqrt{36+9} = \sqrt{45}$$

$$BC = \sqrt{3^2 + 21^2} = \sqrt{9+441} = \sqrt{450}$$

$$CA = \sqrt{9^2 + 18^2} = \sqrt{81+324} = \sqrt{405}$$

Since $AB^2 + AC^2 + BC^2$, ABC is a right angled triangle with $\angle A = 90°$.

Question

The distance between the mid point and the point which divides externally in the ratio 2:5 the line joining the points (7,6) and (-3, -4).

a) $\frac{35}{3}$ **b)** $\frac{70}{3}$

c) $\frac{35}{3\sqrt{2}}$ **d)** $\frac{35\sqrt{2}}{3}$

Ans (d)

Let P be the mid point and θ be the point which divides A(7,6) and B(-3,-4) externally in the ratio 2:5

$$P = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{7-3}{2}, \frac{6-4}{2}\right) = (2,1)$$

$$Q = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$

$$PQ = \sqrt{\left(\frac{41}{3} - 2\right)^2 + \left(\frac{38}{3} - 1\right)^2}$$

$$= \sqrt{\left(\frac{35}{3}\right)^2 + \left(\frac{35}{3}\right)^2} = \frac{35}{3}\sqrt{2}.$$

Question

The co-ordinate of the incentre of a triangle whose vertices are (0, 0) (-3, 4) and (8,6)

a) $\left(\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right)$

b) $\left(\frac{7(3+\sqrt{5})}{2}, \frac{3-\sqrt{5}}{2}\right)$

c) $\left(\frac{(3-\sqrt{5})}{2}, \frac{7(3-\sqrt{5})}{2}\right)$

d) $\left(\frac{(3+\sqrt{5})}{2}, \frac{7(3+\sqrt{5})}{2}\right)$

Ans (c)

Let A(0,0) B(-3, 4) and C (8,6)
BC = a, CA = b, AB = C
then

$$a = \sqrt{(8+3)^2 + (6-4)^2} = 5\sqrt{5}$$

$$b = \sqrt{(8-0)^2 + (6-0)^2} = 10$$

$$c = \sqrt{(-3-0)^2 + (4-0)^2} = 5$$

Incentre

$$= \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$$

$$= \left(\frac{5\sqrt{5} \times (0) + 10 \times (-3) + 5 \times (8)}{5\sqrt{5} + 5 + 10}, \frac{5\sqrt{5} \times (0) + 10 \times (4) + 5 \times (6)}{5\sqrt{5} + 5 + 10} \right)$$

$$= \left(\frac{2}{3+\sqrt{5}}, \frac{14}{(3+\sqrt{5})} \right)$$

$$= \left(\frac{3-\sqrt{5}}{2}, \left(\frac{7(3-\sqrt{5})}{2} \right) \right)$$

Question

Let P (2, 0) and Q (0, 2) be two points and O be the origin. If A (x, y) is a point such that $xy > 0$ and $x + y < 2$, then

(a) A cannot be inside the ΔOPQ

(b) A lies outside the ΔOPQ

(c) A lies either inside ΔOPQ or in the third quadrant

(d) none of these

Solution

(c). Since $xy > 0$, therefore the point A lies either in the first quadrant or in the third quadrant. Since $x + y < 2$, therefore the point A lies either inside the ΔOPQ or in the third quadrant.

Question

The eccentricity of the conic $9x^2 + 25y^2 - 18x - 100y - 116 = 0$ is

a) $\frac{5}{4}$ **b)** $\frac{4}{5}$

c) $\frac{3}{5}$ **d)** None

Ans (b)

The equation can be written as

$9x^2 - 18x - 25y^2 - 100y = 116$

$9(x^2 - 2x) + 25(y^2 - 4y) = 116$

$9(x^2 - 2x + 1) + 25(y^2 - 4y + 4) = 116 + 9 + 100$

$9(x-1)^2 + 25(y-2)^2 = 225$

$$\Rightarrow \frac{(x-1)^2}{25} + \frac{(y-2)^2}{9} = 1$$

which is the ellipse with centre at (1, 2)

$a^2 = 25,\ b^2 = 9$

thus

$b^2 = a^2(1 - e^2)$

$\Rightarrow 9 = 25(1 - e^2)$

$$\Rightarrow e = \frac{4}{5}$$

Question

The vertices of a triangle are A(1, 1) B(4, 5) and C(6, 13), and A equals

a) $\sin^{-1}\left(\frac{63}{65}\right)$ **b)** $\cos^{-1}\left(\frac{63}{65}\right)$

c) $\tan^{-1}\left(\frac{63}{65}\right)$ **d)** $\sin^{-1}\left(\frac{\sqrt{251}}{65}\right)$

Ans (b)

Here $a = BC = \sqrt{(4-6)^2 + (5-13)^2} = \sqrt{68}$

$b = CA = \sqrt{(6-1)^2 + (13-1)^2} = \sqrt{169} = 13$

$c = AB = \sqrt{(4-1)^2 + (5-1)^2} = 5$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{169 + 25 - 68}{2 \times 13 \times 5} = \frac{63}{65}$$

$$\Rightarrow A = \cos^{-1}\left(\frac{63}{65}\right)$$

Question

If the points (x_1, y_1) (x_2, y_2) (x_3, y_3) lie on a straight line which of the following need not be true.

a) $\begin{vmatrix} x_1 & 1 & y_1 \\ x_2 & 1 & y_2 \\ x_3 & 1 & y_3 \end{vmatrix} = 0$

b) $\frac{1}{2}\begin{bmatrix} x_1(y_2 - y_3) - x_2(y_3 - y_1) \\ + x_3(y_1 - y_2) \end{bmatrix} = 0$

c) $\frac{y_2 - y_3}{x_2 x_3} + \frac{y_3 - y_1}{x_3 x_1} + \frac{y_1 - y_2}{x_1 x_2} = 0$

d) $\frac{y_2 - y_1}{y_3 - y_2} = \frac{x_2 - x_1}{x_3 - x_2}$

Ans (b)

If (x_1, y_1) (x_2, y_2) and (x_3, y_3) are collinear, then $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$.

Question

Number of equilateral triangles with $y = \sqrt{3}(x - 1) + 2$ and $y = \sqrt{3}x$ as two of its sides, is

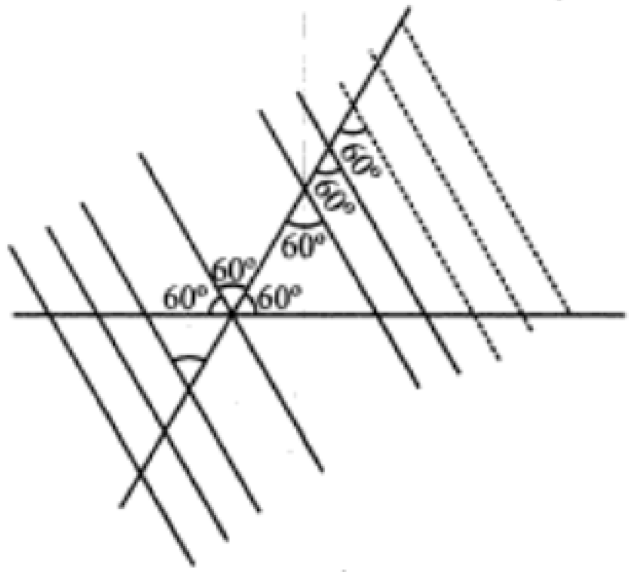
(a) 0 (b) 1

(c) 2 (d) none of these

Solution

(d). The sides are,

$$y = \sqrt{3}(x-1)+2 \text{ and } y = -\sqrt{3}\,x$$



The two lines are at an angle of 60° to each other. Now any line parallel to obtuse angle bisector will make equilateral triangle with these lines as its two sides.

Question

The parabola whose focus is (- 3, 2) and the directrix is x + y = 4 is

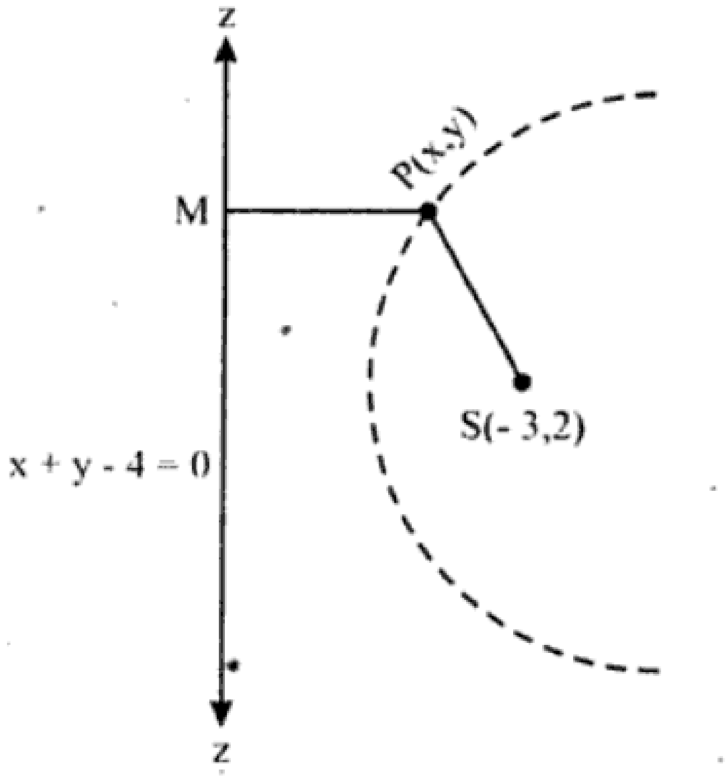
a) $y^2 = 8x$

b) $y^2 = 8x + 2 + 2y$

c) $x^2 + y^2 - 2xy + 20x + 10 = 0$

d) $x^2 + 2x = 8y$

Ans (c)



Let P(x, y) be the any point on the parabola

We have SP = PM

$\Rightarrow SP^2 = PM^2$

$$\Rightarrow (x+3)^2 + (y-2)^2 = \left(\frac{x+y-4}{\sqrt{1+1}}\right)^2$$

$$\Rightarrow 2\left[x^2 + y^2 + 6x - 4y + 13\right]$$

$$= \left[x^2 + y^2 + 16 + 2xy - 8y - 8x\right]$$

$$\Rightarrow x^2 + y^2 - 2xy + 20x + 10 = 0$$

Question

The equation of directrix of the parabola $4y^2 + 12x - 12y + 39 = 0$ is

a) $x+\frac{5}{2}=0$ **b)** $x+\frac{7}{2}=0$

c) $y-\frac{3}{2}=0$ **d)** None

Ans (d)

The equation of the parabola can be written

as $4\left(y^2-3y+\frac{9}{4}\right)=-12x-39+9$

$\Rightarrow 4\left(y-\frac{3}{2}\right)^2=-12\left(x+\frac{5}{2}\right)$

$\Rightarrow y^2=-4ax,$

Where $x=x+\frac{5}{2}$, $y=y-\frac{3}{2}$ and $a=\frac{3}{4}$

thus the vertex is $\left(-\frac{5}{2},\frac{3}{2}\right)$

thus equation of directrix is $x = a$

$x+\frac{5}{2}=\frac{3}{4}\Rightarrow x=\frac{-7}{4}$

Question

If 2, 5, 9 are the ordinates of vertices of the triangle inscribed in a parabola $2y^2 = x$, then the area of triangle is

a) 42 **b)** 8

c) 84 **d)** 72

Ans (c)

Note:

If y_1, y_2, y_3 are the ordinates of vertices of the triangle inscribed in a parabola $y^2 = 4ax$, the area of the triangle is

$$\frac{1}{8a}\left|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)\right|$$

Here $a = \frac{1}{8}, y_1 = 2, y_2 = 5, y_3 = 9$

$$\therefore \text{Area} = \frac{1}{8\left(\frac{1}{8}\right)}\left|(2-5)(5-9)(9-2)\right|$$

$= 3.4.7$

$= 84$ sq.units

Question

If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in G.P. with the same common ratio, then the points (x_1, y_1), (x_2, y_2) (x_3, y_3)

(a) lie on a straight line (b) lie on an ellipse

(c) lie on a circle (d) are vertices of a triangle

Solution

(a). Let $\frac{x_2}{x_1} = \frac{x_3}{x_2} = r$ and $\frac{y_2}{y_1} = \frac{y_3}{y_2} = r$

$\Rightarrow \quad x_2 = x_1 r, x_3 = x_1 r^2, y_2 = y_1 r$ and $y_3 = y_1 r^2$.

We have,

$$\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} x_1 & y_1 & 1 \\ x_1 r & y_1 r & 1 \\ x_1 r^2 & y_1 r^2 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} x_1 & y_1 & 1 \\ 0 & 0 & 1-r \\ 0 & 0 & 1-r \end{vmatrix}$$

[Applying $R_3 \to R_3 - rR_2$ and $R_2 \to R_2 - rR_1$]

$= 0 \qquad (\because R_2$ and R_3 are identical)

Thus, $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ lie on a **straight line.**

Question

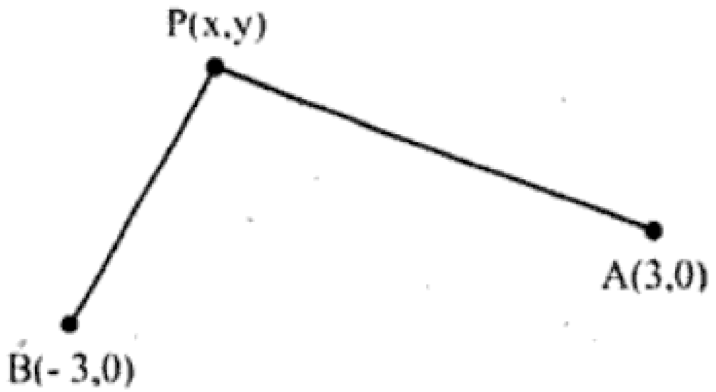
A point moves so that its distance from (3, 0) is twice the distance from (- 3, 0), then the locus of the point

a) is a circle with centre (- 5, 1)
b) is a straight line
c) is an ellipse
d) None of the above

Solution

Ans (d)

Let the moving point be P(x, y)



Given PA = 2PB

thus $PA^2 = 4PB^2$

$(x-3)^2 + y^2 = 4((x+3)^2 + y^2)$

$x^2+y^2 -6x+9=4x^2 + 4y^2 + 24x + 36$

$3x^2 + 3y^2 + 30x + 27 = 0$

$x^2 + y^2 + 10x + 9=0$

Question

If the equation of a line in the intercept form $\frac{x}{a} + \frac{y}{b} = 1$ is transformed to the normal form $x \cos \alpha + y \sin \alpha = P$, then

a) $P^2 = a^2 + b^2$ **b)** $a^2 + b^2 = \frac{1}{P^2}$

c) $P = (a + b)^2$ **d)** $\frac{1}{P^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Solution

Ans (d)

Here P is the distance of the line from the origin

$\therefore$ P = distance of $\frac{x}{a}+\frac{y}{b}=1$ from (0,0)

$$=\left|\frac{0+0-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right|$$

$$\therefore \quad \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{P^2}$$

Question

Let $L_1 \equiv 3x + 4y + 7 = 0$ and $L_2 = 28x - 21y + 50 = 0$ and $L_3 = 12x + 16y + 7 = 0$ then which of the following is true

a) L_1, L_2 are not perpendicular to each other

b) L_2, L_3 intersect at only one point

c) L_1, L_3 intersect at atleast one point.

d) L_1, L_2, L_3 are concurrent lines

Solution

Ans (b)

Here $L_1 \perp^r L_2$, $L_1 \|^l L_3$ and $L_2 \perp^r L_3$

thus L_2, L_3 intersect at only one point.

Question

The coordinate of the orthocentre of the triangle formed by the line $y + x - 6 = 0$, $y - x + 2 = 0$ and $5x - 3y + 2 = 0$

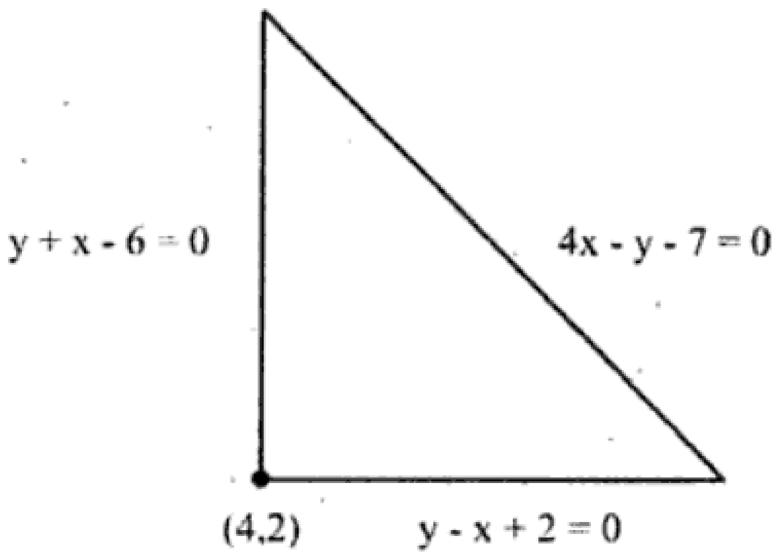
a) $\left(\frac{5}{2}, \frac{5}{2}\right)$ **b)** $\left(\frac{5}{2}, \frac{3}{2}\right)$

c) $\left(\frac{3}{2}, \frac{3}{2}\right)$ **d)** None

Solution

Ans (d)

Since the lines $y + x - 6 = 0$, $y - x + 2 = 0$ are perpendicular, the point of intersection of these two lines gives the orthocentre.



thus (4, 2) is the orthocentre.

Question

Let $ax + by + c = 0$ be a variable straight line, where a, b and c are first, third and seventh terms of an increasing A.P. Then, the variable straight line always passes through a fixed point which lies on

(a) $x^2 + y^2 = 4$
(b) $x^2 + y^2 = 13$
(c) $y^2 = 2x$
(d) $2x + 3y = 9$

Solution

(b). Let d be the common difference of A.P., then

$b = a + 2d$ and $c = a + 6d$. Clearly, $(b - a) \times 3 = c - a$

$\Rightarrow \quad 2a - 3b + c = 0$

Thus, the straight line $ax + by + c = 0$ passes through the point $(2, -3)$ which also satisfies $x^2 + y^2 = 13$

Question

The lines $2x - y - 5 = 0$, $3x - y - 6 = 0$ and $4x - y - 7 = 0$

a) forms a right angle triangle
b) forms an equilateral triangle
c) are concurrent
d) are neither concurrent nor forms a triangle

Solution

Ans (c)

Here

$$\begin{vmatrix} 2 & -1 & -5 \\ 3 & -1 & -6 \\ 4 & -1 & -7 \end{vmatrix} = 2(7-6)+1(3)-5(1)$$

$$= 0$$

thus the lines are concurrent.

Question

The equation of the line passing through (x_1, y_1) and parallel to the line $ax + by + c = 0$ is

a) $a(x - x_1) + b(y - y_1) = 0$

b) $ax + by + c = x_1 + y_1$

c) $ax + by + ax_1 + by_1 + c = 0$

d) None

Solution

Ans (a)

Any line parallel to $ax + by + c = 0$ is given by $ax + by + k = 0$ ...(1)

Since (1) passes through (x_1, y_1), we have

$ax_1 + by_1 + k = 0 \Rightarrow k = -ax_1 - by_1$

thus (1) becomes

$ax + by - ax_1 - by_1 = 0$

ie., $a(x - x_1) + b(y - y_1) = 0$

Question

The incentre of the triangle by the lines $x = 0$, $y = 0$ and $x \cos \alpha + y \sin \alpha = P$ is

a) $\left(\dfrac{P}{\sin\alpha}, \dfrac{P}{\cos\alpha}\right)$

b) $\left(\dfrac{P}{1+\cos\alpha}, \dfrac{P}{1+\sin\alpha}\right)$

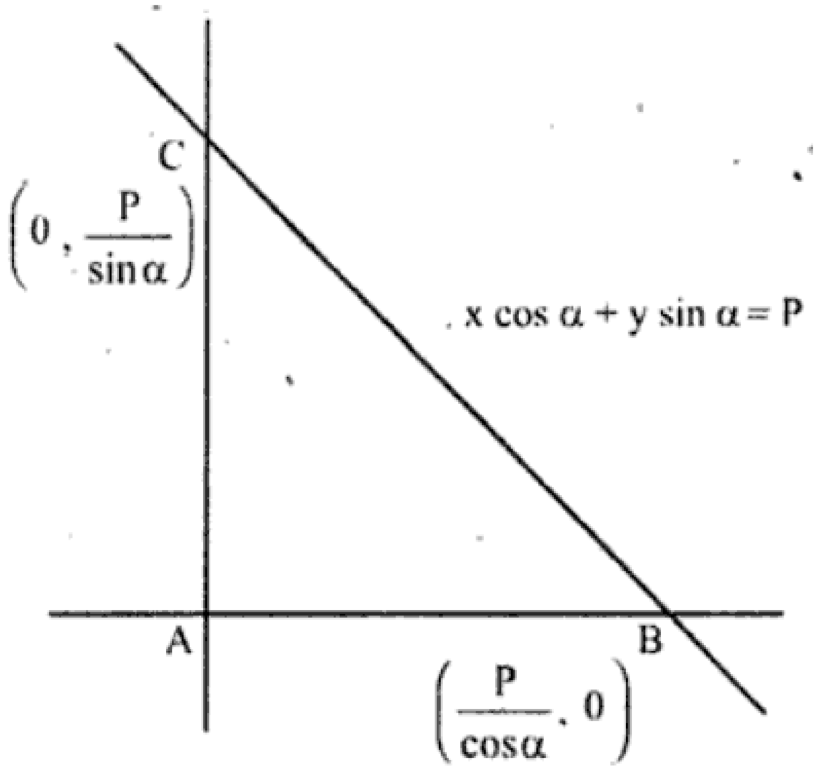
c) $\left(\dfrac{P}{1+\sin\alpha+\cos\alpha}, \dfrac{P}{1+\sin\alpha+\cos\alpha}\right)$

d) None

Solution

Ans (c)

Let A(0, 0) and $B\left(\dfrac{P}{\cos\alpha}, 0\right)$ $C\left(0, \dfrac{P}{\sin\alpha}\right)$ be the vertices of triangle OAB



$$a = BC = \sqrt{\frac{P^2}{\cos^2\alpha} + \frac{P^2}{\sin^2\alpha}} = \frac{P}{\cos\alpha \sin\alpha}$$

$$b = AC = \frac{P}{\sin\alpha}$$

$$c = AB = \frac{P}{\cos\alpha}$$

∴ Incentre of the triangle ABC

$$= \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

$$= \left(\frac{0 + \frac{P^2}{\sin\alpha\cos\alpha} + 0}{\frac{P}{\sin\alpha\cos\alpha} + \frac{P}{\sin\alpha} + \frac{P}{\cos\alpha}}, \frac{0 + 0 + \frac{P^2}{\sin\alpha\cos\alpha}}{\frac{P}{\sin\alpha\cos\alpha} + \frac{P}{\sin\alpha} + \frac{P}{\cos\alpha}} \right)$$

$$= \left(\frac{P}{1 + \sin\alpha + \cos\alpha}, \frac{P}{1 + \sin\alpha + \cos\alpha} \right)$$

Question

The value of 'm' so that the line $3x + 3y - m = 0$, $3x + 3y + 6 = 0$ and $6x + 5y - 9 = 0$ are concurrent is

a) -6 **b)** 6

c) 3 **d)** for no value of m

Solution

Ans (a)

Since the lines are concurrent, we have

$$\begin{vmatrix} 3 & 3 & -m \\ 3 & 3 & 6 \\ 6 & 5 & -9 \end{vmatrix} = 0$$

$3(-27 - 30) - 3(-27 - 36) - m(15 - 18) = 0$

$-171 + 189 + 3m = 0$

$\Rightarrow m = -6$

Question

If P and P_1 be the perpendiculars from the origin upon the lines x sec α + y cosec α = c and x cos α - y sin α = c cos 2α then

a) $P^2 + 4P_1^2 = c^2$ **b)** $P^2 + P_1^2 = 4c^2$

c) $4P^2 + P_1^2 = c^2$ **d)** $4(P^2 + P_1^2) = c^2$

Solution

Ans (c)

$$P = \left|\frac{-c}{\sqrt{\sec^2\alpha + \operatorname{cosec}^2\alpha}}\right|,$$

$$P_1 = \left|\frac{-c\cos 2\alpha}{\sqrt{\cos^2\alpha + \sin^2\alpha}}\right|$$

$$4P^2 + P_1^2 = \frac{4c^2}{\sec^2\alpha + \operatorname{cosec}^2\alpha} + c^2\cos^2 2\alpha$$

$= 4c^2 \cos^2\alpha \sin^2\alpha + c^2 \cos^2 2\alpha$

$= c^2(\sin^2 2\alpha + \cos^2 2\alpha)$

$= c^2.$

Question

The angle between the lines 3x + y - 7 = 0 and x + 2y + 9 = 0 is

a) 60° **b)** 45°

c) 15° **d)** None

Solution

Ans (b)

The lines are $y = -3x + 7,\ y = \frac{-x}{2} - \frac{9}{2}$ whose slopes are $m_1 = -3,\ m_2 = \frac{-1}{2}$

$$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{-3 + \frac{1}{2}}{1 + (-3)\left(\frac{-1}{2}\right)}\right|$$

$$= \left|\frac{\frac{-5}{2}}{\frac{5}{2}}\right| = 1 \Rightarrow \theta = 45°$$

Question

The line joining A ($b \cos \alpha$, $b \sin \alpha$) and B ($a \cos \beta$, $a \sin \beta$) is produced to the point M (x, y) so that $AM : MB = b : a$, then $x \cos \frac{\alpha + \beta}{2} + y \sin \frac{\alpha + \beta}{2} =$

(a) -1 (b) 0 (c) 1 (d) $a^2 + b^2$

Ans. (b)

Solution As M divides AB externally in the ratio $b : a$

$$x = \frac{b(a \cos \beta) - a\,(b \cos \alpha)}{b - a} \text{ and } y = \frac{b\,(a \sin \beta) - a\,(b \sin \alpha)}{b - a}$$

$$\Rightarrow \quad \frac{x}{y} = \frac{\cos \beta - \cos \alpha}{\sin \beta - \sin \alpha} = \frac{2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}}{2 \cos \frac{\alpha + \beta}{2} \sin \frac{\beta - \alpha}{2}}$$

$$\Rightarrow \quad x\cos\frac{\alpha+\beta}{2} + y\sin\frac{\alpha+\beta}{2} = 0.$$

Question

The equation of the circle which has the centres of the circles whose equations are $x^2 + y^2 - 16x - 18y + 20 = 0$ and $x^2 + y^2 - 3x + y - 4 = 0$ as the end point of its diameter is

a) $x^2 + y^2 - 17x - 16y = 0$

b) $x^2 + y^2 - 16x + 17y + 15 = 0$

c) $x^2 + y^2 + 16x - 17y + 15 = 0$

d) None

Solution

Ans (a)

The centres of the given circles are $C_1(8,9)$ and $C_2\left(\frac{3}{2}, -\frac{-1}{2}\right)$ C_1, C_2 as the end points of diameter is

$$\left(x-\frac{3}{2}\right)(x-8)+\left(y+\frac{1}{2}\right)(y-9)=0$$

$$(2x-3)(x-8)+(2y+1)(y-9)=0$$

Question

The length of the intercept made by the line $y = 2x + 1$ on the circle $x^2 + y^2 = 2$ is

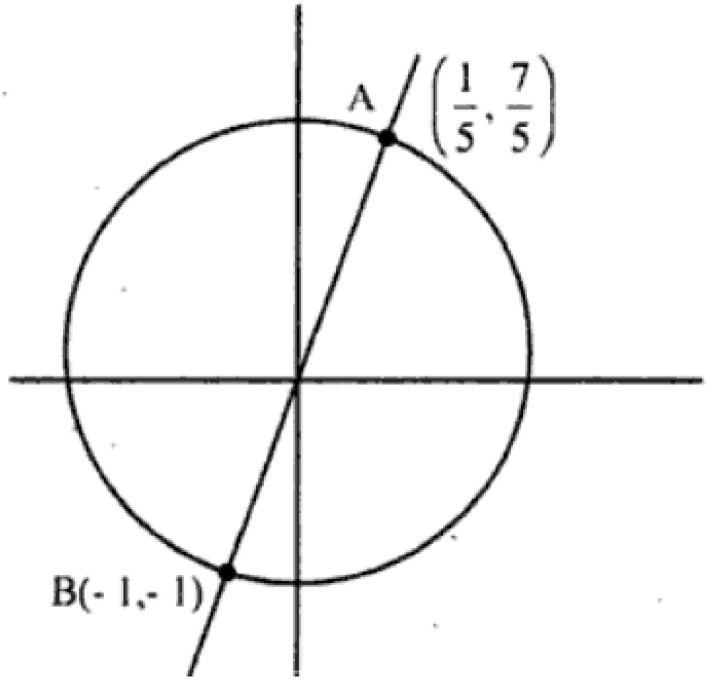
a) $\frac{6}{\sqrt{5}}$ b) $6\sqrt{5}$

c) $6\sqrt{2}$ d) None

Solution

Ans (a)

Solving the equations $y=2x+1$ and $x^2+y^2=2$, we get $x^2+(2x+1)^2=2$



$$\Rightarrow 5x^2 + 4x - 1 = 0$$

$$\Rightarrow x = \frac{1}{5}, -1$$

$$\Rightarrow y = \frac{7}{5}, -1$$

Hence the coordinates of the points of intersection are

$$A\left(\frac{1}{5}, \frac{7}{5}\right) \text{ and } B(-1, -1)$$

$$AB = \sqrt{\left(\frac{1}{5}+1\right)^2 + \left(\frac{7}{5}+1\right)^2} = \sqrt{\frac{36}{25} + \frac{144}{25}}$$

$$= \frac{\sqrt{180}}{5} = \frac{6}{\sqrt{5}}$$

Question

The angle formed by the abscissa and the tangent to the parabola $y = x^2 + 4x - 17$ at the point $\left(\frac{5}{2}, -\frac{3}{4}\right)$ is

(a) $\tan^{-1} 2$ **(b)** $\tan^{-1} 5$

(c) $\tan^{-1} 7$ **(d)** None of these.

Solution

(*d*) Slope of x-axis is 0.

$y = x^2 + 4x - 17 \Rightarrow \frac{dy}{dx} = 2x + 4$

$\therefore$ slope of tangent to parabola at $P\left(\frac{5}{2}, -\frac{3}{4}\right)$

$$= 2\left(\frac{5}{2}\right) + 4 = 9$$

If θ is the angle between x-axis and the tangent at P then $\tan\theta = 9 \Rightarrow \theta = \tan^{-1} 9$.

Question

The parametric equations of the circle $x^2 + y^2 + 8x - 6y = 0$ are

a) $x = 4 + 5\cos\theta,\ y = 3 + 5\sin\theta$

b) $x = -4 + 5\cos\theta,\ y = 3 + 5\sin\theta$

c) $x = 4 + 5\cos\theta,\ y = -3 + 5\sin\theta$

d) $x = -4 + 5\cos\theta,\ y = -3 + 5\sin\theta$

Solution

Ans (b)

The circle is $(x + 4)^2 + (y - 3)^2 = 25$

thus the parametric equation is

$x + 4 = 5\cos\theta,\ y - 3 = 5\sin\theta$

ie., $x = -4 + 5\cos\theta,\ y = 3 + 5\sin\theta$.

Question

The angle between the tangents drawn from (0, 0) to the circle $x^2 + y^2 + 4x - 6y + 4 = 0$ is

a) $\sin^{-1}\left(\frac{5}{13}\right)$ **b)** $\sin^{-1}\left(\frac{5}{12}\right)$

c) $\sin^{-1}\left(\frac{12}{13}\right)$ **d)** $\frac{\pi}{2}$

Solution

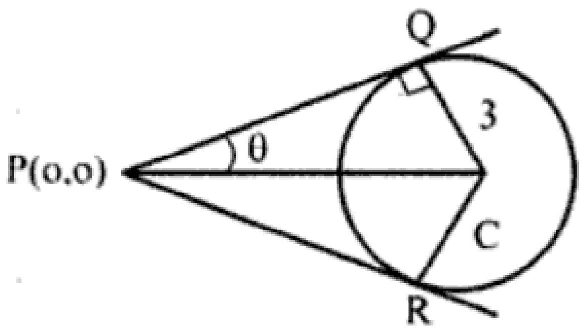
Ans (c)

The centre of the circle, c = (- 2, 3)

radius of the circle, $r = \sqrt{4+9-4} = 3$

PQ = length of the tangent from P(0, 0)to the circle

$= \sqrt{4} = 2.$



From ΔPQC, we have $\tan\theta = \frac{QC}{PQ} = \frac{3}{2}$

$$\therefore \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\left(\frac{3}{2}\right)}{1+\frac{9}{4}} = \frac{12}{13}$$

thus $2\theta = \sin^{-1}\left(\frac{12}{13}\right)$.

Question

The distance between the point (1, 1) and the tangent to the curve $y = e^{2x} + x^2$ drawn from the point $x = 0$ is

(a) $\frac{1}{\sqrt{5}}$ (b) $\frac{-1}{\sqrt{5}}$

(c) $\frac{2}{\sqrt{5}}$ (d) $\frac{-2}{\sqrt{5}}$.

Solution

(c) Putting $x = 0$ in $y = e^{2x} + x^2$ ...(1)

we get $y = 1$

$\therefore$ the given point is P(0, 1)

From (1), $\frac{dy}{dx} = 2e^{2x} + 2x$

$\Rightarrow \left[\frac{dy}{dx}\right]_P = 2$

$\therefore$ equation of tangent at P to (1) is

$$y - 1 = 2(x - 0) \Rightarrow 2x - y + 1 = 0 \quad ...(2)$$

$\therefore$ Required distance

= Length of $\perp$ from (1, 1) to (2)

$$= \frac{2 - 1 + 1}{\sqrt{4+1}} = \frac{2}{\sqrt{5}}.$$

Question

The tangent to the curve $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$ at the points $\theta = (2n + 1)\pi$, $n \in \mathbb{Z}$ are parallel to

(a) x-axis (b) y-axis

(c) $y = x$ (d) $x + y = 0$.

Solution

(a) $$\frac{dy}{dx} = \frac{-\sin\theta}{1 - \cos\theta}$$

$\therefore$ $$\frac{dy}{dx} = 0 \text{ for } \theta = (2n + 1)\pi$$

$\therefore$ The tangent is parallel to x-axis.

Question

The circles $x^2 + y^2 - 8x + 6y + 21 = 0$, $x^2 + y^2 + 4x - 10y - 115 = 0$

a) touch externally

b) touch internally

c) intersect at two points

d) None

Solution

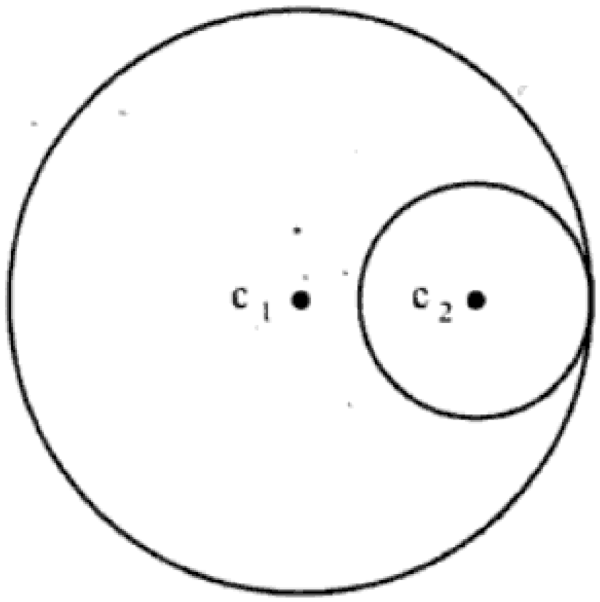
Ans (b)

the centres of the circles are $C_1 = (4, -3)$, $C_2 = (-2, 5)$ the radii are

$r_1 = \sqrt{16+9-21} = 2$, $r_2 = \sqrt{4+25+115} = 12$

Here $C_1C_2 = \sqrt{36+64} = 10$

Since $C_1C_2 = |r_1 - r_2|$, the circles touch each other internally.



Question

At (0, 0), the curve $y^2 = x^3 + x^2$

(a) touches x-axis

(b) bisects the angle between the axes

(c) makes an angle of 60° with ox

(d) None of these.

Solution

(b) $y^2 = x^3 + x^2 \Rightarrow 2y\frac{dy}{dx} = 3x^2 + 2x$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2 + 2x}{2y} = \frac{3x^2 + 2x}{2\sqrt{x^3 + x^2}} = \frac{3x + 2}{2\sqrt{1 + x}}$$

$$\therefore \left.\frac{dy}{dx}\right|_{(0,\,0)} = \frac{2}{2} = 1 \Rightarrow \theta = 45°$$

$\therefore$ the curve bisects the angle between the axes.

Question

The tangent to the curve $y = 2x^2 - x + 1$ is parallel to the line $y = 3x + 9$ at the point

(a) (2, 3) (b) (2, – 1)

(c) (2, 1) (d) (1, 2).

Solution

(d) $y = 2x^2 - x + 1$

$\Rightarrow \quad \frac{dy}{dx} = 4x - 1$

Also, slope of $y = 3x + 9$ is 3.

$\therefore \quad 4x - 1 = 3 \Rightarrow x = 1$

$\therefore$ From (1), $y = 2(1)^2 - 1 + 1 = 2$

$\therefore$ Point is (1, 2).

Question

The number of common tangents to the circles $x^2 + y^2 - 2x + 4y + 4 = 0$, $x^2 + y^2 + 4x - 2y + 1 = 0$ are

a) 0 b) 1

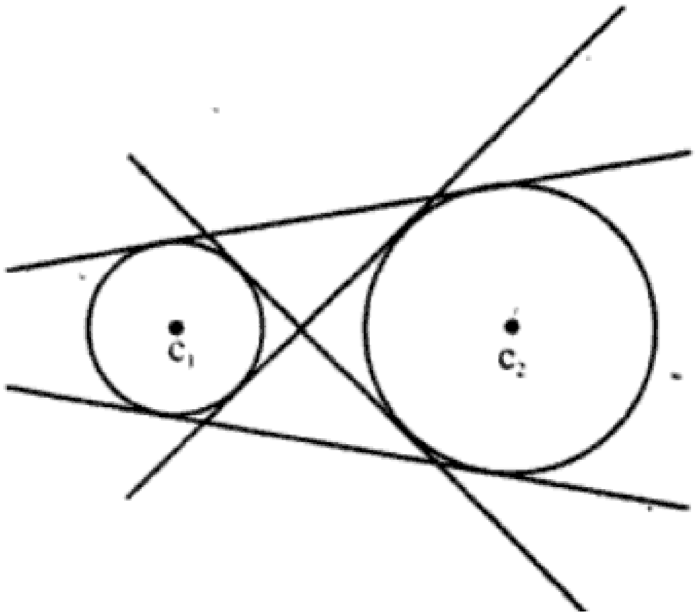
c) 2 d) 4

Solution

Ans (d)

The centres of the circles are $c_1 = (1, -2)$, $c_2 = (-2, 1)$ the radii are

$r_1 = \sqrt{1+4-4} = 1$, $r_2 = \sqrt{4+1-1} = 2$.



Here $C_1C_2 = \sqrt{9+9} = 3\sqrt{2}$.

Since $C_1C_2 > r_1 + r_2$, the circles are non-overlapping circles thus 4 common tangents.

Question

The radius of the director circle of the ellipse

$\frac{x^2}{6} + \frac{y^2}{4} = 1$ is

a) $\sqrt{10}$ b) 10

c) 5 d) $\sqrt{5}$

Solution

Ans (a)

Note:

The locus of point of intersection of perpendicular tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $x^2 + y^2 = a^2 + b^2$ called director circle of the ellipse.

$\therefore x^2 + y^2 = 6 + 4$

ie., $x^2 + y^2 = 10$, is the equation of the director circle whose radius is $\sqrt{10}$.

Question

The locus of the point of intersection of feet of perpendicular from focus on the tangent drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (a > b) is $x^2 + y^2 = 7$, then

a) $a = 7$ **b)** $b = 7$

c) $a^2 = 7$ **d)** $b^2 = 7$

Solution

Ans (c)

Note:

The locus of the point of intersection of feet of perpendicular from focus on the tangent drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $x^2 + y^2 = a^2$ called auxiliary circle.

$\therefore a^2 = 7$.

Question

The equation of the normal to the ellipse

$\frac{x^2}{10}+\frac{y^2}{5}=1$ **at** $(\sqrt{8}, 1)$ **is**

a) $10x + 5y = 1$ **b)** $y=\sqrt{2}(x+1)$

c) $x=\sqrt{2}(y+1)$ **d)** $y=\sqrt{8}(x+1)$

Solution

Ans (c)

The equation of normal is

$$\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$$

ie., $\frac{10x}{\sqrt{8}}-\frac{5y}{1}=10-5$

$$\frac{2x}{\sqrt{8}}-y=1 \qquad \frac{x}{\sqrt{2}}=1+y$$

$$\Rightarrow x=\sqrt{2}(1+y)$$

Question

If the lines joining the origin to the intersection of the line $y = mx + 2$ and the curve $x^2 + y^2 = 1$ are at right angles, then

(a) $m^2 = 1$ (b) $m^2 = 3$

(c) $m^2 = 7$ (d) $2m^2 = 1$

Ans. (c)

Solution Joint equation of the lines joining the origin and the point of intersection of the line $y = mx + 2$ and the curve $x^2 + y^2 = 1$ is

$$x^2 + y^2 = \left(\frac{y - mx}{2}\right)^2$$

$\Rightarrow$ $$x^2 (4 - m^2) + 2mxy + 3y^2 = 0$$

Since these lines are at right angles

$$4 - m^2 + 3 = 0 \Rightarrow m^2 = 7.$$

Question

The equations of the tangents to the ellipse $\frac{x^2}{28} + \frac{y^2}{16} = 1$ which makes an angle $60°$ with the major axis are

a) $y = \sqrt{3}x \pm 10$ **b)** $y = \sqrt{3}x \pm \sqrt{65}$

c) $x = \sqrt{3}y \pm 28$ **d)** $x = \sqrt{3}y \pm 7$

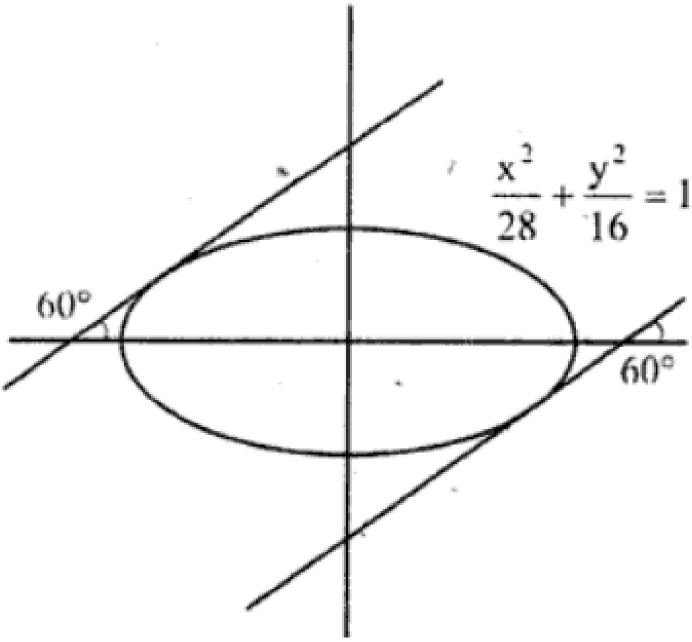
Solution

Ans (a)

Here slope of tangent = tan 60°

$m = \sqrt{3}$

∴ The equation of tangent is



$$y = mx \pm \sqrt{a^2m^2 + b^2}$$

$$y = \sqrt{3}x \pm \sqrt{28 \times 3 + 16}$$

$$y = \sqrt{3}x \pm 10.$$

Question

The number of tangents to $\frac{x^2}{25} + \frac{y^2}{16} = 1$ through (5, 0) is

a) 0 **b)** 1

c) 2 **d)** 3

Solution

Ans (b)

Since the points (5, 0) lies on the ellipse

$\frac{x^2}{25} + \frac{y^2}{16} = 1$ there is only one tangent (5, 0)

Question

The tangents at any point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the tangents at the vertices A and A^1 in L and M respectively. then AL. A^{1}M =

a) a^2 **b)** b^2

c) ab **d)** a^2b^2

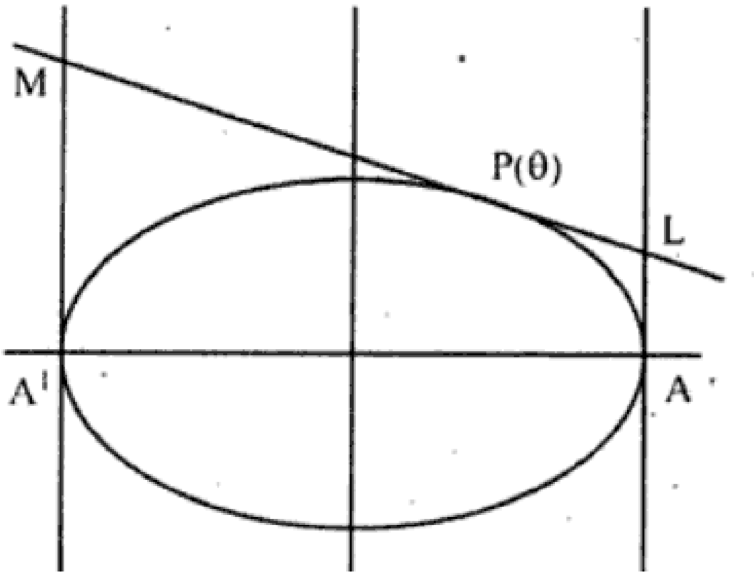
Solution

Ans (b)

The equation of tangent at $P(\theta)$ to the ellipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

$$\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1 \qquad ...(1)$$



at L, x = a $\therefore \frac{a\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$

$$\Rightarrow y = \frac{b}{\sin\theta}(1 - \cos\theta)$$

$$\Rightarrow AL = \frac{b}{\sin\theta}(1 - \cos\theta)$$

at , x = - a $\Rightarrow y = \frac{b}{\sin\theta}(1 + \cos\theta)$

$$\Rightarrow A'M = \frac{b}{\sin\theta}(1 + \cos\theta)$$

thus AL. A'M $= \frac{b^2}{\sin^2\theta}(1 - \cos^2\theta) = b^2$.

Question

If the equal sides AB and AC (each equal to a) of a right angled isosceles triangle ABC be produced to P and Q so that $BP \cdot CQ = AB^2$, then the line PQ always passes through the fixed point

(a) $(a, 0)$

(b) $(0, a)$

(c) (a, a)

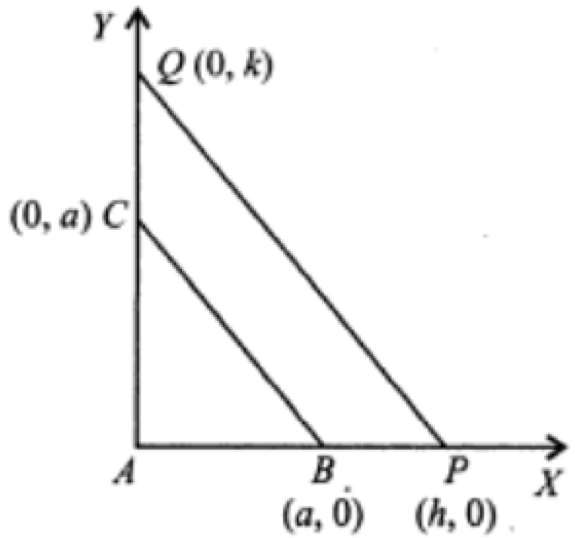
(d) none of these

Solution

(c). We take A as the origin and AB and AC as x-axis and y-axis respectively.

Let $AP = h$, $AQ = k$.

Equation of the line PQ is



$$\frac{x}{h} + \frac{y}{k} = 1 \qquad ...(1)$$

Given, $BP \cdot CQ = AB^2$

$\Rightarrow \quad (h - a)(k - a) = a^2$

$\Rightarrow \quad hk - ak - ah + a^2 = a^2 \quad$ or $\quad ak + ha = hk$

or $$\frac{a}{h} + \frac{a}{k} = 1 \qquad ...(2)$$

From (2), it follows that line (1) i.e. PQ passes through the fixed point (a, a).

Question

The lines $\frac{x}{a}+\frac{y}{b}=1$, $\frac{x}{a}+\frac{y}{b}=2$, $\frac{x}{b}+\frac{y}{a}=1$

$\frac{x}{b}+\frac{y}{a}=2$

a) forms square of side 1 unit

b) forms square of side $\frac{ab}{\sqrt{a^2+b^2}}$

c) forms rhombus of side $\frac{ab}{\sqrt{a^2+b^2}}$

d) forms rhombus of side $\frac{1}{\sqrt{a^2+b^2}}$

Solution

Ans (c)

Let $\frac{x}{a}+\frac{y}{b}=1$ ...(1)

$\frac{x}{a}+\frac{y}{b}=2$ ...(2)

and $\frac{x}{b}+\frac{y}{a}=1$ ...(3)

$\frac{x}{b}+\frac{y}{a}=2$ ...(4)

d_1 = distance between the parallel lines (1)

and (2) $= \dfrac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}} = \dfrac{ab}{\sqrt{a^2+b^2}}$

d_2 = distance between the parallel lines (3)

and (4) $= \dfrac{1}{\sqrt{\frac{1}{b^2}+\frac{1}{a^2}}} = \dfrac{ab}{\sqrt{a^2+b^2}}$

Since $d_1 = d_2$, ie., the distances between the pairs of parallel lines are equal, Hence ABCD is a rhombus.

Question

The number of points on $x + y = 4$ that lie at a unit distance from the line $4x + 3y - 10 = 0$ is / are

a) 0 **b)** 1
c) 2 **d)** None

Solution

Ans (c)

An arbitrary point P on x + y = 4 can be taken as P(t, 4 - t)

Given, perpendicular distance of P from 4x + 3y - 10 = 0 = 1

$$\left|\frac{4t+3(4-t)-10}{\sqrt{4^2+3^2}}\right| = 1$$

$\Rightarrow t = -7$ or $t = 3$

Hence the required points are (- 7, 11) and (3,1).

Question

If a, b, c form a G.P., then twice the sum of the ordinates of the points of intersection of the line $ax + by + c = 0$ and the curve $x + 2y^2 = 0$ is

(a) $\frac{b}{a}$ (b) $\frac{c}{a}$

(c) $\frac{a}{c}$ (d) $\frac{a}{b}$

Solution

(a). Let a, b, c be in G.P. with common ratio r.

Then, $b = ar$ and $c = ar^2$.

So, the equation of the line is $ax + by + c = 0$

$\Rightarrow \quad ax + ary + ar^2 = 0 \quad \Rightarrow \quad x + ry + r^2 = 0$

This line cuts the curve $x + 2y^2 = 0$

Eliminating x, we get $2y^2 - ry + r^2 = 0$

If the roots of the quadratic equation are y_1 and y_2, then

$$y_1 + y_2 = \frac{r}{2} \Rightarrow 2(y_1 + y_2) = r = \frac{b}{a} = \frac{c}{b}.$$

Question

If 3a = 4b, then the line 6x - 8y + 5 = 0 divides the line joining points (a, b) and (1, 2)

a) externally in the ration 1:2

b) internally

c) in the ratio 0:1

d) None

Solution

Ans (b)

Let L = 6x - 8y + 5

Substitute (a, b) in L,

L = 6a - 8b + 5 = 2(3a - 4b) + 5 = 5 > 0

Subtitute (1, 2) in L,

L = 6 - 16 + 5 = - 5 < 0

thus L divides the line joining (a, b) and (1, 2) internally.

Question

If $\alpha + \beta + \gamma = 0$, the line $3\alpha x + \beta y + 2\gamma = 0$ passes through the fixed point

(a) $\left(2, \frac{2}{3}\right)$ (b) $\left(\frac{2}{3}, 2\right)$

(c) $\left(-2, \frac{2}{3}\right)$ (d) none of these

Solution

(b). The given line is $3\alpha x + \beta y + 2\gamma = 0$

$\Rightarrow 3\alpha x + \beta y + 2(-\alpha - \beta) = 0$ ($\because \alpha + \beta + \gamma = 0$)

$\Rightarrow \alpha(3x - 2) + \beta(y - 2) = 0$

$\Rightarrow$ the given line passes through the point of intersection of the lines $3x - 2 = 0$ and $y - 2 = 0$ i.e., $\left(\frac{2}{3}, 2\right)$, for all values of α and β.

Question

Which of the following is not the bisector of the angles between 3x - 4y + 7 = 0 and 12x - 5y - 8 = 0

a) $99x - 77y + 51 = 0$

b) $21x + 27y - 131 = 0$

c) $8x - 10y + 15 = 0$

d) None

Solution

Ans (c)

The equations of the angular bisector of $3x - 4y + 7$ and $12x - 5y - 8 = 0$ are

$$\frac{3x-4y+7}{\sqrt{9+16}} = \pm\frac{12x-5y-8}{\sqrt{144+25}}$$

$$13(3x-4y+7) = \pm 5(12x-5y-8)$$

$\Rightarrow 39x - 52y + 91 = 60x - 25y - 40$ and

$39x - 52y + 91 = -60x + 25y + 40$

$\Rightarrow 21x + 27y - 131 = 0$ and $99x - 77y + 51 = 0$

Question

The equation of the straight line which passes through the points of intersection of the lines $L_1 = x + 2y - 5 = 0$ and $L_2 = 3x + 7y - 17 = 0$ and perpendicular to the line $3x + 4y = 10$ is

a) $37L_1 + 11L_2 = 0$

b) $37L_1 - 11L_2 = 0$

c) $11L_1 - 37L_2 = 0$

d) $11L_1 + 37L_2 = 0$

Solution

Ans (b)

Any straight line passing through the point of intersection of L_1 and L_2 is given by

$L_1 + \lambda L_2 = 0$ ...(1)

$(x + 2y - 5) + \lambda(3x + 7y - 17) = 0$

$(3\lambda + 1)x + (2 + 7\lambda)y - 5 - 17\lambda = 0$ ...(2)

Scince line (2) is perpendicular to $3x + 4y = 10$ we have, product of slopes = - 1

ie., $-\left(\frac{3\lambda+1}{7\lambda+2}\right)\left(\frac{-3}{4}\right) = -1$

$\Rightarrow \lambda = \frac{-11}{37}$

$\therefore$ From (1) we get $37L_1 - 11L_2 = 0$

Question

If a, b, c are in A.P., a, x, b are in G.P. and b, y, c are in G.P., the point (x, y) lies on

(a) a straight line (b) a circle

(c) an ellipse (d) a hyperbola

Ans. (b)

Solution We have $2b = a + c$, $x^2 = ab$, $y^2 = bc$ so that $x^2 + y^2 = b(a + c) = 2b^2$ which is a circle.

Question

The four lines $px \pm qy \pm r = 0$ enclose a rhombus whose area is

a) $\dfrac{2r^2}{pq}$ b) $\dfrac{2rp}{q}$

c) $\dfrac{2pq}{r^2}$ d) $2pqr$

Answer

(a)

Question

The straight lines given by the equation $(x^2 + y^2)\sin^2\alpha = (x\cos\alpha - y\sin\alpha)^2$ are inclined at an angle of

a $\tan^{-1}\alpha$ b) α

c) 2α d) $\tan^{-1}2\alpha$

Solution

Ans (c)

The equation is

$x^2(\sin^2\alpha - \cos^2\alpha) + 2xy\sin\alpha\cos\alpha = 0$

$\Rightarrow x^2\cos 2a - xy\sin 2a = 0$

have $a = \cos 2\alpha$, $2h = -\sin 2\alpha$, $b = 0$

$$\therefore \tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b} = \frac{2\sqrt{\left(\frac{-\sin 2\alpha}{2}\right)^2 - 0}}{\cos 2\alpha + 0}$$

$= \tan 2\alpha$

$\Rightarrow \theta = 2\alpha.$

Question

The straight lines represented by the equations $x^2(\tan^2\theta + \cos^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$ makes angles θ_1 and θ_2 with the x-axis, then $\tan\theta_1 - \tan\theta_2 =$

a) 1 b) 2

c) 3 d) 4

Solution

Ans (b)

$x^2(\tan^2\theta + \cos^2)$

$- 2xy\tan\theta + y^2\sin^2\theta = 0$ ...(1)

Let the lines represented by $y = m_1x$ and $y = m_2x$ where

$m_1 = \tan\theta_1$, $m_2 = \tan\theta_2$.

$$m_1 + m_2 = \frac{-2h}{b} = \frac{2\tan\theta}{\sin^2\theta} \text{ and}$$

$$m_1m_2 = \frac{a}{b} = \frac{\tan^2\theta + \cos^2\theta}{\sin^2\theta}$$

$$\therefore (\tan\theta_1 - \tan\theta_2)^2 = (m_1 - m_2)^2$$

$$= (m_1 + m_2)^2 - 4m_1m_2$$

$$= \frac{4\tan^2\theta}{\sin^4\theta} - \frac{4(\tan^2\theta + \cos^2\theta)}{\sin^2\theta}$$

$$= 4\left[\frac{\tan^2\theta - \tan^2\theta\sin^2\theta - \cos^2\theta\sin^2\theta}{\sin^4\theta}\right]$$

$$= 4\left[\frac{\tan^2\theta\left(1-\sin^2\theta\right)-\sin^2\theta\cos^2\theta}{\sin^4\theta}\right]$$

$$= 4\left[\frac{\sin^2\theta-\sin^2\theta\cos^2\theta}{\sin^4\theta}\right]$$

$=4.$

thus $\tan\theta_1 - \tan\theta_2 = 2.$

Question

The second degree equation $x^2 + 3xy + 2y^2 + 3x + 5y + 2 = 0$ represents

a) parabola

b) ellipse

c) hyperbola

d) pair of straight lines

Solution

Ans (d)

Here $a=1$, $h=\frac{3}{2}$, $b=2$, $g=\frac{3}{2}$, $f=\frac{5}{2}$, $c=2$

thus $abc + 2fgh - af^2 - bg^2 - ch^2$

$$= 1.(2)(2)+2\left(\frac{5}{2}\right)\left(\frac{3}{2}\right)\left(\frac{3}{2}\right)$$

$$-1\left(\frac{5}{2}\right)^2-2\left(\frac{3}{2}\right)^2-2\left(\frac{3}{2}\right)^2 = 0$$

thus the second dgree equation represents pair of straight lines.

Question

The area enclosed by $2|x| + 3|y| \leq 6$ is

(a) 3 sq. units (b) 4 sq. units

(c) 12 sq. units (d) 24 sq. units

Ans. (c)

Solution The given inequality is equivalent to the following system of inequalities.

$2x + 3y \leq 6$, when $x \geq 0, y \geq 0$

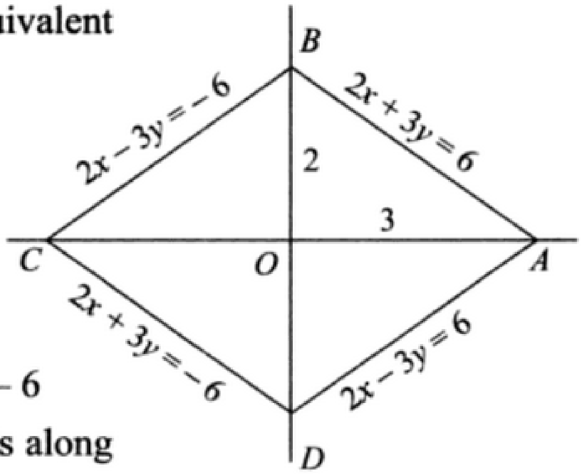
$2x - 3y \leq 6$, when $x \geq 0, y \leq 0$

$-2x + 3y \leq 6$, when $x \leq 0, y \geq 0$

$-2x - 3y \leq 6$, when $x \leq 0, y \leq 0$

which represents a rhombus with sides

$$2x \pm 3y = 6 \text{ and } 2x \pm 3y = -6$$



Length of the diagonals is 6 and 4 units along x-axis and y-axis.

∴ The required area

$$= \frac{1}{2} \times 6 \times 4 = 12 \text{ sq. units.}$$

Question

The lines $3x^2 + 8xy - 3y^2 = 0$ and $3x^2 + 8xy - 3y^2 + 2x - 4y - 1 = 0$ form

a) rhombus but not a square

b) rectangle but not a square

c) parallelogram but not a rectangle

d) None

Solution

Ans (d)

$3x^2 + 8xy - 3y_2 = 3x^2 + 9xy - xy - 3y^2$

$= 3x(x+3y) - y(x+3y)$

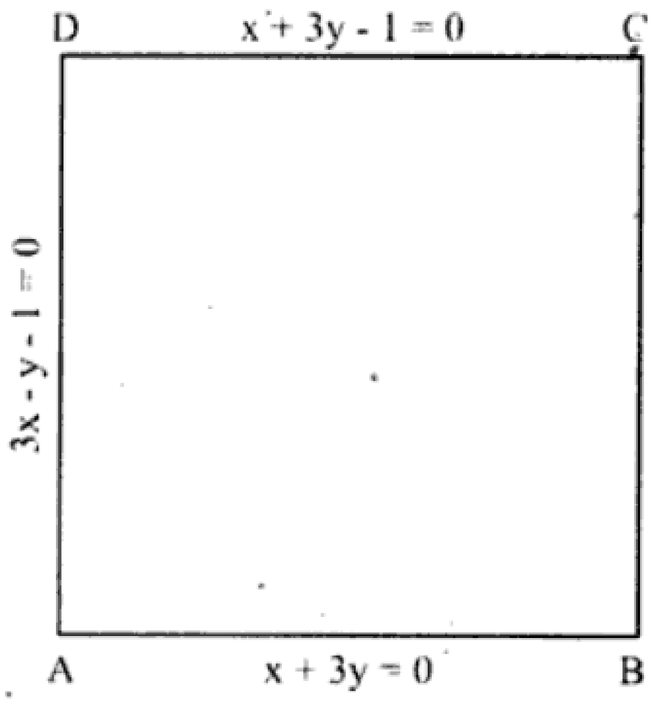
$= (x+3y)(3x-y).$

$\therefore 3x^2 + 8xy - 3y^2 = 0$ represents pair of lines $x+3y=0, 3x-y=0$

also $3x^2+8xy-3y^2+2x-4y-1=(x+3y+l)(3x-y+m)$ equating the coefficients of x and y, we get $3l+m=2$ and $-l+3m=-4$

on solving we get $l=+1$, $m=-1$

$\therefore 3x^2 + 8xy - 3y^2 + 2x - 4y - 1 = 0$, represents pair of line $x+3y+1=0$ and $3x-y-1=0$



Since the lines are perpendicular and the distance between the parallel lines are same, the figure forms an square.

Question

The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ will represent a pair of parallel straight lines if

a) $h^2 = ab$ **b)** $\frac{a}{h} = \frac{h}{b} \neq \frac{g}{f}$

c) $\frac{a}{h} = \frac{h}{b} = \frac{f}{g}$ **d)** $\frac{a}{h} = \frac{h}{b} = \frac{g}{f}$

Solution

Ans (d)

Since the lines are parallel we have

$$h^2 = ab \Rightarrow \frac{h}{b} = \frac{a}{h} \qquad ...(1)$$

since it represent pair of lines, we have

$abc+2fgh-af^2 - bg^2-ch^2=0$

$\Rightarrow (ab-h^2)c+2fgh - af^2-bg^2=0$

$\Rightarrow af^2-2fgh+bg^2=0$

$$\Rightarrow \frac{h^2}{b}f^2 - 2fgh + bg^2 = 0$$

$\Rightarrow h^2f^2-2fghb+b^2g^2=0$

$\Rightarrow (hf-bg)^2 = 0$

$$\Rightarrow hf - bg = 0 \Rightarrow \frac{h}{b} = \frac{g}{f} \qquad ...(2)$$

thus from (1) and (2), we get

$$\frac{a}{h} = \frac{h}{b} = \frac{g}{f}.$$

Question

If $x^2 + ky^2 + x - 3y - h = 0$ represents pair of perpendicular lines, then

a) $k = -1, h = 2$ **b)** $k = 1, h = -2$

c) $k = -1, h = -2$ **d)** $k = 1, h = 2$

Solution

Ans (a)

If $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents pair of $\perp^r$ lines then $a=b$ and $abc+2fgh-af^2-bg^2-ch^2=0$

$a=b \Rightarrow k=-1$, and

here $a=1, h=0, b=-1$,

$g=\frac{1}{2}, f=\frac{-3}{2}, c=h$

and $abc+2fgh-af^2-bg^2-ch^2=0 \Rightarrow h=2$

Question

The locus of the point of intersection of the lines $x \sin\theta + (1-\cos\theta)y = a\sin\theta$ and $x\sin\theta - (1+\cos\theta)y + a\sin\theta = 0$ is

(a) $x^2 - y^2 = a^2$ (b) $x^2 + y^2 = a^2$

(c) $y^2 = ax$ (d) none of these

Ans. (b)

Solution From the given equations we have

$$\frac{1-\cos\theta}{\sin\theta} = \frac{a-x}{y} \text{ and } \frac{1+\cos\theta}{\sin\theta} = \frac{a+x}{y}$$

Multiplying we get $\frac{1-\cos^2\theta}{\sin^2\theta} = \frac{a^2-x^2}{y^2} \Rightarrow x^2+y^2=a^2$

Question

If every point on the line $(a_1 - a_2)x + (b_1 - b_2)y = c$ is equidistant from the points (a_1, b_1) and (a_2, b_2) then $2c =$

(a) $a_1^2 - b_1^2 + a_2^2 - b_2^2$ (b) $a_1^2 + b_1^2 + a_2^2 + b_2^2$

(c) $a_1^2 + b_1^2 - a_2^2 - b_2^2$ (d) none of these

Ans. (c)

Solution Let (h, k) be any point on the given line then

$$(h - a_1)^2 + (k - b_1)^2 = (h - a_2)^2 + (k - b_2)^2$$

$\Rightarrow$ $2(a_1 - a_2)h + 2(b_1 - b_2)k = a_1^2 + b_1^2 - a_2^2 - b_2^2$

$\Rightarrow$ $(a_1 - a_2)h + (b_1 - b_2)k = (1/2)(a_1^2 + b_1^2 - a_2^2 - b_2^2)$ (i)

Since (h, k) lies on the given line

$$(a_1 - a_2)h + (b_1 - b_2)k = c \quad \text{(ii)}$$

Comparing (i) and (ii) we get $c = (1/2)(a_1^2 + b_1^2 - a_2^2 - b_2^2)$.

Question

Equations of the straight lines passing through the point (4, 3) and making intercepts on the coordinate axes whose sum is -1 are

(a) $x/2 + y/3 = 1$ and $x/2 + y/1 = 1$
(b) $x/2 - y/3 = -1$ and $x/(-2) + y/1 = -1$
(c) $x/2 + y/3 = -1$ and $x/(-2) + y/1 = -1$
(d) $x/2 - y/3 = 1$ and $x/(-2) + y/1 = 1$

Ans. (d)

Solution Let the equation of the line be $\frac{x}{a} + \frac{y}{-1-a} = 1$. Since it passes through (4, 3), $\frac{4}{a} + \frac{3}{-1-a} = 1$

$\Rightarrow$ $a^2 = 4 \Rightarrow a = \pm 2$

and the required equations are

$$\frac{x}{2}+\frac{y}{-3}=1 \quad \text{and} \quad \frac{x}{-2}+\frac{y}{1}=1.$$

Question

If non zero numbers a, b, c are in H.P, then the straightline $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point. That point is

(a) $(1, -2)$ (b) $(1, -1/2)$
(c) $(-1, 2)$ (d) $(-1, -2)$

Ans. (a)

Solution a, b, c are in H.P.

$$\Rightarrow \quad \frac{1}{b}-\frac{1}{a}=\frac{1}{c}-\frac{1}{b}$$

$$\Rightarrow \quad \frac{1}{a}-\frac{2}{b}+\frac{1}{c}=0$$

which shows that the given line passes through the point $(1, -2)$.

Question

The line parallel to x-axis passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$ where $(a, b) \neq (0, 0)$ is

(a) above x-axis at a distance 3/2 from it.
(b) above x-axis at a distance 2/3 from it.
(c) below x-axis at a distance 3/2 from it.
(d) below x-axis at a distance 2/3 from it.

Ans. (c)

Solution Eliminating x, we get the $(2b^2 + 2a^2)y + 3b^2 + 3a^2 = 0$
$\Rightarrow$ $y = -3/2$ which is the required line and hence below x-axis at a distance 3/2 from it.

Question

A straight line through the point $A(3, 4)$ is such that its intercept between the axes is bisected at A. Its equation is

(a) $3x + 4y = 25$ (b) $x + y = 7$
(c) $3x - 4y + 7 = 0$ (d) $4x + 3y = 24$

Ans. (d)

Solution Let the equation be $\frac{x}{a} + \frac{y}{b} = 1$ where $\frac{a}{2} = 3$, $\frac{b}{2} = 4$

$\Rightarrow$ $a = 6$, $b = 8$ and the required equation is $8x + 6y = 48$ or $4x + 3y = 24$

Question

Let $A(h, k)$, $B(1, 1)$ and $C(2, 1)$ be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1, then the set of values which k can take is given by

(a) $\{1, 3\}$ (b) $\{0, 2\}$
(c) $\{-1, 3\}$ (d) $\{-3, -2\}$

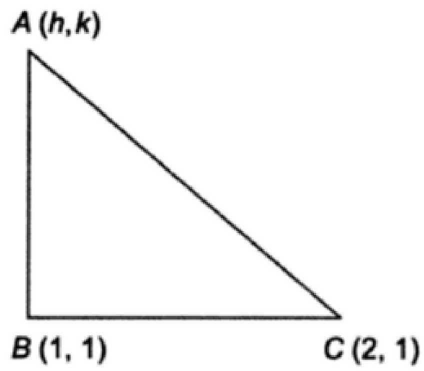
Ans. (c)

Solution Equation of BC is $y = 1$. AB is perpendicular to BC through $B(1, 1)$ so in equation is $x = 1 \Rightarrow h = 1$.

Area of the $\Delta ABC = \frac{1}{2} AB \times BC = 1$

$\Rightarrow$ $AB = 2 \Rightarrow |k - 1| = 2$

$\Rightarrow$ $k = -1$ or 3



Question

Let $A(2, -3)$ and $B(-2, 1)$ be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line

(a) $3x + 2y = 5$ (b) $2x - 3y = 7$
(c) $2x + 3y = 9$ (d) $3x - 2y = 3$

Ans. (c)

Solution Let $C(h, k)$ be the vertex, then the centroid is

$\left(\frac{h+2-2}{3}, \frac{k-3+1}{3}\right)$ i.e. $(h/3, (k-2)/3)$ which lies on $2x + 3y = 1$

$$\Rightarrow \quad 2\,\frac{h}{3} + \frac{3(k-2)}{3} = 1$$

$\Rightarrow$ $2h + 3k = 9$ and the locus of (h, k) is $2x + 3y = 9$.

Question

If the sum of the slopes of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times their product, then the value of c is

(a) 2 (b) -1
(c) 1 (d) -2

Ans. (a)

Solution If m_1, m_2 are the slopes, then $m_1 + m_2 = -2c/7$, $m_1 m_2 = -1/7$

$$m_1 + m_2 = 4\,m_1 m_2$$

$$\Rightarrow \quad c = 2$$

Question

Locus of mid point of the portion between the axes of $x \cos \alpha + y \sin \alpha = p$, where p is constant is

(a) $x^2 + y^2 = 4/p^2$ (b) $x^2 + y^2 = 4p^2$

(c) $1/x^2 + 1/y^2 = 2/p^2$ (d) $1/x^2 + 1/y^2 = 4/p^2$

Ans. (d)

Solution If (h, k) is the mid-point, then

$$h = p/2 \cos\alpha,\ k = p/2 \sin\alpha$$

so $(p/2h)^2 + (p/2k)^2 = \cos^2 \alpha + \sin^2 \alpha = 1$

$\Rightarrow$ $1/h^2 + 1/k^2 = 4/p^2$

Locus of (h, k) is $1/x^2 + 1/y^2 = 4/p^2$

Question

A triangle with vertices $(4, 0)$, $(-1, -1)$, $(3, 5)$ is

(a) isosceles and right angled
(b) isosceles but not right angled
(c) right angled but not isosceles
(d) neither right angled nor isosceles

Ans. (a)

Solution Length of the sides are $\sqrt{(4+1)^2 + 1} = \sqrt{26}$

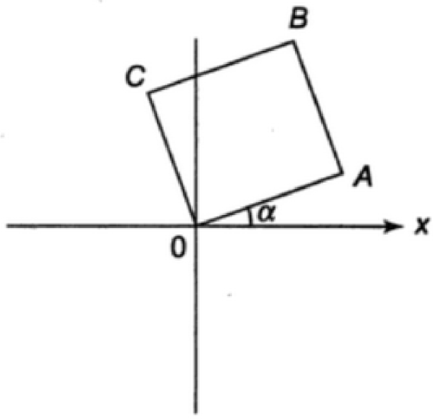
$$\sqrt{(4-3)^2 + 5^2} = \sqrt{26} \text{ and } \sqrt{(3-1)^2 + (5+1)^2} = \sqrt{52}$$

Showing that the triangle is isosceles and right angled.

Question

A square of side a lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle α $(0 < \alpha < \pi/4)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is

(a) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$
(b) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$
(c) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$
(d) $y(\cos\alpha - \sin\alpha) - x(\sin x - \cos\alpha) = a$



Ans. (c)

Solution Coordinates of A are $(a\cos\alpha, a\sin\alpha)$ and of C are $(a\cos(\alpha + \pi/2), a\sin(\alpha + \pi/2))$ i.e. $(-a\sin\alpha, a\cos\alpha)$

So the equation of the diagonal AC is

$$y - a\sin\alpha = \frac{a(\sin\alpha - \cos\alpha)}{a(\cos\alpha + \sin\alpha)}(x - a\cos\alpha)$$

Question

If algebraic sum of distances of a variable line from points (2, 0), (0, 2) and (– 2, – 2) is zero, then the line passes through the fixed point

(a) (– 1, – 1) (b) (0, 0)
(c) (1, 1) (d) (2, 2)

Ans. (b)

Solution Let the equation of the variable line be

$$ax + by + c = 0$$

then according to the given condition

$$\frac{2a+c}{\sqrt{a^2+b^2}} + \frac{2b+c}{\sqrt{a^2+b^2}} + \frac{-2a-2b+c}{\sqrt{a^2+b^2}} = 0$$

$$\Rightarrow \quad c = 0$$

which shows that the line passes through (0, 0) for all values of a and b.

An equation of a straight line passing through the intersection of the straight lines $3x - 4y + 1 = 0$ and $5x + y - 1 = 0$ and making non-zero, equal intercepts on the axes is

(a) $22x + 22y = 13$ (b) $23x + 23y = 11$

(c) $11x + 11y = 23$ (d) $8x - 3y = 0$

Ans. (b)

Solution Equation of any line through the point of intersection of the given lines is

$$(3x - 4y + 1) + k\,(5x + y - 1) = 0 \quad (1)$$

or $$(3 + 5k)x + (k - 4)y + 1 - k = 0$$

or $$\frac{x}{(k-1)/(3+5k)} + \frac{y}{(k-1)/(k-4)} = 1$$

Since x-intercept = y-intercept

$$\Rightarrow \quad \frac{k-1}{3+5k} = \frac{k-1}{k-4} \quad \Rightarrow \quad (k-1)(3 + 5k - k + 4) = 0$$

$$\Rightarrow \quad k = 1 \text{ or } k = -7/4$$

For $k = 1$, (1) becomes $8x - 3y = 0$ which makes zero intercepts on the axes.

A straight line through the origin O meets the parallel lines $4x + 2y = 9$ and $2x + y + 6 = 0$ at points P and Q respectively. The point O divides the segment PQ in the ratio

(a) 1 : 2 (b) 3 : 4 (c) 2 : 1 (d) 4 : 3

Ans. (b)

Solution It is clear that the lines lie on opposite side of the origin O. Let the equation of any line through O be $\frac{x}{\cos\theta} = \frac{y}{\sin\theta}$. If $OP = r_1$ and $OQ = r_2$ then the coordinates of P are $(r_1 \cos \theta, r_1 \sin \theta)$ and that of Q are $(-r_2 \cos \theta, -r_2 \sin \theta)$

Since P lies on $4x + 2y = 9$, $2r_1(2 \cos \theta + \sin \theta) = 9$

and Q lies on $2x + y + 6 = 0$, $-r_2(2 \cos \theta + \sin \theta) = -6$

so that $\frac{r_1}{r_2} = \frac{9}{12} = \frac{3}{4}$

and the required ratio is thus 3 : 4.

Alternately Let the equation of the line through O be $y = mx$ then coordinates of A and B are respectively $\left(\frac{9}{4+2m}, \frac{9m}{4+2m}\right)$ and $\left(\frac{-6}{2+m}, \frac{-6m}{2+m}\right)$ so that

$$\frac{OA}{OB} = \frac{9}{|4+2m|} \times \frac{|2+m|}{6} = \frac{3}{4}$$

Question

Let $P = (-1, 0)$, $Q = (0, 0)$ and $R = (3, 3\sqrt{3})$ be three points. Then the equation of the bisector of the angle PQR is

(a) $\frac{\sqrt{3}}{2} + y = 0$

(b) $x + \sqrt{3}y = 0$

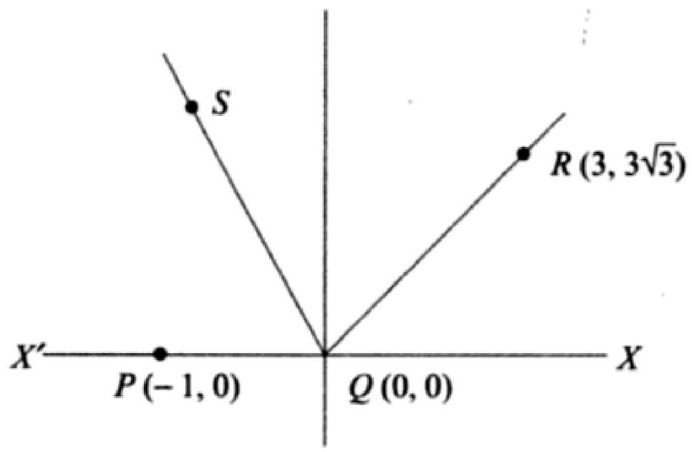
(c) $\sqrt{3}x + y = 0$

(d) $x + \frac{\sqrt{3}}{2}y = 0$

Ans. (c)

Solution Let the equation of QS, the bisector of angle PQR be $y = mx$.

Slope of $QR = \sqrt{3} = \tan 60^\circ$



$\Rightarrow \quad \angle PQR = 120^\circ \Rightarrow \angle PQS = 60^\circ$

$\Rightarrow \quad m = -\tan 60^\circ = -\sqrt{3}$ and thus the required equation of the bisector is $\sqrt{3}x + y = 0$.

Question

If the pair of lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie along the diameter of a circle and divide the circle into four sectors such that the area of one of the sector is thrice the area of another sector, then

(a) $3a^2 + 10ab + 3b^2 = 0$ (b) $3a^2 + 2ab + 3b^2 = 0$

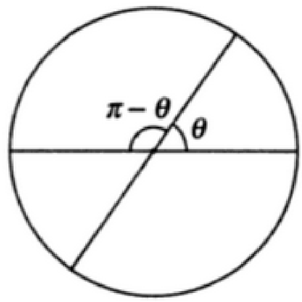
(c) $3a^2 - 10ab + 3b^2 = 0$ (d) $3a^2 - 2ab + 3b^2 = 0$

Ans. (b)

Solution As the area of one of the sectors is thrice that of the other, $\pi - \theta = 3\theta$

$\Rightarrow \quad \theta = \pi/4$

$\Rightarrow$ angle between the lines is $3\pi/4$ or $\pi/4$



i.e. $$\pm 1 = \frac{2\sqrt{(a+b)^2 - ab}}{a+b}$$

$\Rightarrow \quad (a + b)^2 = 4(a^2 + b^2 + ab)$

$\Rightarrow \quad 3a^2 + 2ab + 3b^2 = 0$

Question

If one of the lines $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines $xy = 0$, then m is

(a) $-1/2$ (b) -2

(c) 1 (d) 2

Ans. (c)

Solution Equation of the bisectors of $xy = 0$ is $y^2 - x^2 = 0$ which is satisfied by the given equation if $m = 1$.

Question

Equation of a line which is parallel to the line common to the pair of lines given by $6x^2 - xy - 12y^2 = 0$ and $15x^2 + 14xy - 8y^2 = 0$ and the sum of whose intercepts on the axes is 7, is

(a) $2x - 3y = 42$ (b) $3x + 4y = 12$

(c) $5x - 2y = 10$ (d) none of these

Ans. (b)

Solution $6x^2 - xy - 12y^2 = 0$

$$\Rightarrow \quad (2x - 3y)(3x + 4y) = 0 \quad \text{(i)}$$

and $15x^2 + 14xy - 8y^2 = 0$

$$\Rightarrow \quad (5x - 2y)(3x + 4y) = 0 \quad \text{(ii)}$$

Equation of the line common to (i) and (ii) is

$$3x + 4y = 0 \quad \text{(iii)}$$

Equation of any line parallel to (ii) is

$$3x + 4y = k \text{ or } \frac{x}{k/3} + \frac{y}{k/4} = 1$$

If $\frac{k}{3} + \frac{k}{4} = 7$, then $k = 12$ and the equation of the required line is $3x + 4y = 12$

Question

If $ab \neq 0$, the equation $ax^2 + 2xy + by^2 + 2ax + 2by = 0$ represents a pair of straight lines if

(a) $a + b = 2$ (b) $a - b = 2$

(c) $ab = 2$ (d) $ab^2 + a^2b = 2$

Ans. (a)

Solution We know that the equation

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

In the given equation $c = 0, g = a, f = b, h = 1$.

$\therefore$ The required condition is

$$2b \times a \times 1 - ab^2 - ba^2 = 0$$

$\Rightarrow$ $$ab(2 - a - b) = 0$$

which is satisfied by (a).

Question

If one of the lines given by $6x^2 + xy - y^2 = 0$ coincides with one of the lines given by $3x^2 - axy + y^2 = 0$ then the values of a are given by

(a) $a^2 - 3a + 28 = 0$ (b) $2a^2 - a - 28 = 0$

(c) $2a^2 - 15a + 28 = 0$ (d) none of these

Ans. (b)

Solution $6x^2 + xy - y^2 = 0 \Rightarrow (3x - y)(2x + y) = 0$

$\Rightarrow$ $y/x = -2$ or $y/x = 3$

If $y/x = -2$ coincides with $3x^2 - axy + y^2 = 0$

then $3 - a(-2) + 4 = 0 \Rightarrow a = -7/2$

If $y/x = 3$ coincides with $3x^2 - axy + y^2 = 0$

then $3 - 3a + 9 = 0 \Rightarrow a = 4$

So the values of a are given by $(a + 7/2)(a - 4) = 0$

or $$2a^2 - a - 28 = 0$$

Question

The sine of the angle between the pair of lines represented by the equation $x^2 - 7xy + 12y^2 = 0$ is

(a) 1/12 (b) 1/13

(c) $1/\sqrt{170}$ (d) none of these

Ans. (c)

Solution If θ is the angle between the given lines, then

$$\tan\theta = \frac{2\sqrt{(7/2)^2 - 12}}{1+12} = \pm\frac{1}{13} \quad \Rightarrow \quad \sin\theta = \pm\frac{1}{\sqrt{170}}$$

Question

The square of the differences of the slopes of the lines represented by the equation $x^2(\sec^2\theta - \sin^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$ is

(a) 1 (b) 2 (c) 4 (d) 8

Ans. (c)

Solution If m_1 and m_2 are the slopes of the given lines

then $$m_1 + m_2 = \frac{2\tan\theta}{\sin^2\theta} \text{ and } m_1 m_2 = \frac{\sec^2\theta - \sin^2\theta}{\sin^2\theta}$$

so that $$(m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1m_2$$

$$= \frac{4\tan^2\theta - 4(\sec^2\theta - \sin^2\theta)\sin^2\theta}{\sin^4\theta} = 4$$

Question

The lines joining the origin to the points of intersection of $3x^2 + \lambda xy - 4x + 1 = 0$ and $2x + y - 1 = 0$ are at right angles for

(a) $\lambda = -4$ (b) $\lambda = 4$
(c) $\lambda = 7$ (d) all values of λ

Ans. (d)

Solution Equation of the lines joining the origin to the points of intersection of the given lines is

$$3x^2 + \lambda xy - 4x(2x + y) + 1\cdot(2x + y)^2 = 0$$

(Making the equation of the pair of lines homogeneous with the help of the equation of the line)

$\Rightarrow \quad x^2 - \lambda xy - y^2 = 0$

which are perpendicular for all values of λ.

Question

If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents parallel straight lines, then

(a) $hf = bg$ (b) $h^2 = bc$
(c) $a^2f = b^2g$ (d) none of these

Ans. (a)

Solution Since the equation represents a pair of straight lines

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0 \quad (1)$$

Also as they are parallel $h^2 = ab$ (2)

From (1) and (2) we get

$$2fgh = af^2 + bg^2$$

or $(f\sqrt{a} - g\sqrt{b})^2 = 0 \Rightarrow f\sqrt{a} = g\sqrt{b}$

$\Rightarrow \quad af^2 = bg^2$

$\Rightarrow \quad abf^2 = b^2 g^2$

$\Rightarrow \quad h^2 f^2 = b^2 g^2 \Rightarrow hf = bg$

Question

Statement-1: The points $A(3, 4)$, $B(2, 7)$, $C(4, 4)$ and $D(3, 5)$ are such that one of them lies inside the triangle formed by the other points.

Statement-2: Centroid of a triangle lies inside the triangle.

Ans. (a)

Solution $\dfrac{3+2+4}{3} = 3, \dfrac{4+7+4}{3} = 5$

$\Rightarrow$ $D(3, 5)$ is the centroid of the triangle ABC.

Using statement-2 which is true, statement-1 is also true.

Question

Statement-1: If the circumcentre of a triangle lies at the origin and centroid is the mid point of the line joining the points (2, 3) and (4, 7), then its orthocentre lies on the line $5x - 3y = 0$

Statement-2: Circumcentre, centroid and orthocentre of a triangle lie on the same line

Ans. (a)

Solution From geometry, statement-2 is True. Using it in statement-1, orthocentre lies on the line joining (0, 0) and $\left(\dfrac{4+2}{2}, \dfrac{7+3}{2}\right)$ i.e. (3, 5) which is $5x - 3y = 0$ and so the statement-1 is also true.

Question

Statement-1: If the perpendicular bisector of the line segment joining $P(1, 4)$ and $Q(k, 3)$ has y-intercept equal to -4, then $k^2 - 16 = 0$

Statement-2: Centroid of an isoceles triangle ABC lies on the perpendicular bisector of the base BC where $B = C$.

Ans. (b)

Solution Any point $L(x, y)$ on the perpendicular bisector in statement-1 is equidistant from P and Q. Locus of L is $(x-1)^2 + (y-4)^2 = (x-k)^2 + (y-3)^2$

$$\Rightarrow \qquad 2(k+1)x - 2y = k^2 - 8$$

$$y\text{-intercept} = -\frac{k^2-8}{2} = -4 \quad \Rightarrow \quad k^2 - 16 = 0$$

So statement-1 is True but does not follow from statement-2 which is also true, as the perpendicular bisector of BC is also the median through A.

Question

Statement-1: Circumcentre of the triangle formed by the lines $x + y = 0$, $x - y = 0$ and $x - 7 = 0$ is $(7, 0)$

Statement-2: Circumcentre of a triangle lies inside the triangle.

Ans. (c)

Solution In statement-1, the triangle is right angled with hypotenuse $x - 7 = 0$ and the vertices of the hypotenuse are $(7, 7)$ and $(7, -7)$, circumcentre is the mid-point $(7, 0)$ of the hypotenuse. So statement-1 is True. Statement-2 is false as the circumcentre of an obtuse angled triangle lies outside the triangle and that of the right angled is on the hypotenuse.

Question

Statement-1: Equation of the pair of lines through the origin perpendicular to the pair of lines $7x^2 - 55xy - 8y^2 = 0$ is $8x^2 - 55xy - 7y^2 = 0$

Statement-2: Equation of the pair of lines through the origin perpendicular to the pair to lines $ax^2 + 2hxy + by^2 = 0$ is $bx^2 - 2hxy + ay^2 = 0$

Ans. (a)

Solution Statement-2 is True because if m_1 and m_2 are the slopes of $ax^2 + 2hxy + by^2 = 0$, then $m_1 + m_2 = -\frac{2h}{b}$, $m_1 m_2 = \frac{a}{b}$.

$\Rightarrow \quad \frac{1}{m_1} + \frac{1}{m_2} = -\frac{2h}{a}$ and $\frac{1}{m_1 m_2} = \frac{b}{a}$. Equation of the lines with slope $-\frac{1}{m_1}$ and $-\frac{1}{m_2}$ is $y^2 + \left(\frac{1}{m_1} + \frac{1}{m_2}\right)xy + \frac{1}{m_1 m_2}x^2 = 0$

$$\Rightarrow \qquad y^2 - \frac{2h}{a}xy + \frac{b}{a}x^2 = 0 \quad \Rightarrow \quad bx^2 - 2hxy + ay^2 = 0$$

Using it statement-1 is also true.

Question

Statement-1: Pair and straight liens represented by the equation $(3 + 2\lambda)x^2 + 5xy + (\lambda - 6)y^2 = 0$ are perpendicular if $\lambda = 1$

Statement-2: Pair of straight lines represented by the equation. $ax^2 + 2hxy + by^2 = 0$ are perpendicular if $a - b = 0$

Ans. (c)

Solution Statement-2 is false because the lines are perpendicular if $m_1 m_2 = -1$ (From Ex. 73) $\Rightarrow$ $a + b = 0$. Using this result, lines in statement-1 are perpendicular if $3 + 2\lambda + \lambda - 6 - 0$ $\Rightarrow$ $\lambda = 1$ and the statement-1 is True.

Question

Statement-1: If θ is an angle between the lines represented by $2x^2 \cos^2 \alpha + 2xy \cos 2\alpha + 2y^2 \sin^2 \alpha = 0$ then $\tan^2 \theta = \cos 4\alpha$.
Statement-2: $x^2 + 2xy + y^2 + 2x + 2y - 15 = 0$ represents a pair of parallel lines.

Ans. (b)

Solution In statement-1 $\tan \theta = \pm \dfrac{2\sqrt{\cos^2 2\alpha - 4\sin^2 \alpha \cos^2 \alpha}}{2\cos^2 \alpha + 2\sin^2 \alpha}$.

$\Rightarrow$ $\tan^2 \theta = \cos^2 2\alpha - \sin^2 2\alpha = \cos 4\alpha$ and statement-1 is true

In statement-2, $(x + y)^2 + 2(x + y) - 15 = 0$

$\Rightarrow$ $(x + y + 5)(x + y - 3) = 0 \Rightarrow x + y + 5 = 0, x + y - 3 = 0$ a pair of parallel straight lines and so statement-2 is also true but does not lead to statement-1.

Question

If a, b, c are unequal and different from 1 such that the points $\left(\dfrac{a^3}{a-1}, \dfrac{a^2-3}{a-1}\right), \left(\dfrac{b^3}{b-1}, \dfrac{b^2-3}{b-1}\right)$, and $\left(\dfrac{c^3}{c-1}, \dfrac{c^2-3}{c-1}\right)$ are collinear, then

(a) $bc + ca + ab + abc = 0$
(b) $a + b + c = abc$

(c) $bc + ca + ab = abc$

(d) $bc + ca + ab - abc = 3(a + b + c)$

Ans. (d)

Solution Suppose the given points lie on the line

$$lx + my + n = 0$$

then a, b, c are the roots of the equation.

$$lt^3 + m(t^2 - 3) + n(t - 1) = 0$$

or $$lt^3 + mt^2 + nt - (3m + n) = 0$$

$\Rightarrow$ $$a + b + c = -m/l$$

$$bc + ca + ab = n/l$$

$$abc = (3m + n)/l$$

Eliminating l, m, n, we get

$$abc = -3(a + b + c) + bc + ca + ab$$

$\Rightarrow \quad bc + ca + ab - abc = 3(a + b + c)$

Question

If two vertices of a triangle are (–2, 3) and (5, –1), orthocentre lies at the origin and centroid on the line $x + y = 7$, then the third vertex lies at

(a) (7, 4) (b) (8, 14)
(c) (12, 21) (d) none of these

Ans. (d)

Solution Let $O(0, 0)$ be the orthocentre; $A(h, k)$ the third vertex; and $B(-2, 3)$ and $C(5, -1)$ the other two vertices. Then the slope of the line through A and O is k/h, while the line through B and C has the slope $-4/7$. By the property of the orthocentre, these two lines must be perpendicular, so we have

$$\left(\frac{k}{h}\right)\left(-\frac{4}{7}\right) = -1 \Rightarrow \frac{k}{h} = \frac{7}{4} \quad \text{(i)}$$

Also
$$\frac{5-2+h}{3} + \frac{-1+3+k}{3} = 7$$

$\Rightarrow$
$$h + k = 16$$

Which is not satisfied by the points given in (a), (b) or (c).

Question

The points A (2, 3); B (3, 5), C (7, 7) and D (4, 5) are such that

(b) $ABCD$ is a parallelogram
(b) A,B,C and D are collinear
(c) D lies inside the triangle ABC
(d) D lies on the boundary of the triangle ABC

Ans. (c)

Solution Since $\frac{2+3+7}{3} = 4$, $\frac{3+5+7}{3} = 5$

$D = (4, 5)$ is the centroid of the triangle ABC and hence lies inside the ΔABC.

Question

If $p, x_1, x_2 \dots x_i, \dots$ and $q, y_1, y_2 \dots, y_i, \dots$ are in A.P., with common difference a and b respectively, then the centre of mean position of the points $A_i(x_i, y_i)$, $i = 1, 2 \dots n$ lies on the line

(a) $ax - by = aq - bp$ (b) $bx - ay = ap - bq$

(c) $bx - ay = bp - aq$ (d) $ax - by = bq - ap$

[**Note**. Centre of Mean Position $\left(\frac{\Sigma xi}{n}, \frac{\Sigma yi}{n}\right)$]

Ans. (c)

Solution Let the coordinates of the centre of mean position of the points A_i, $i = 1, 2, \dots n$ be (x, y), then

$$x = \frac{x_1 + x_2 + \cdots + x_n}{n}, \; y = \frac{y_1 + y_2 + \cdots + y_n}{n}$$

$$\Rightarrow \quad x = \frac{np + a(1 + 2 + \cdots + n)}{n}, \; y = \frac{nq + b(1 + 2 + \cdots + n)}{n}$$

$$\Rightarrow \quad x = p + \frac{n(n+1)}{2n} a, \; y = q + \frac{n(n+1)}{2n} b$$

$$\Rightarrow \quad x = p + \frac{n+1}{2} a, \; y = q + \frac{n+1}{2} b$$

$$\Rightarrow \quad 2\frac{(x-p)}{a} = 2\frac{(y-q)}{b} \quad \Rightarrow \quad bx - ay = bp - aq.$$

Question

The point (4, 1) undergoes the following successive transformations:

(i) reflection about the line $y = x$

(ii) translation through a distance 2 units along the positive x-axis.

then, the final coordinates of the point are

(a) (4, 3) (b) (3, 4) (c) (1, 4) (d) (4, 4)

Ans. (b)

Solution Let $Q(x, y)$ be the reflection of $P(4, 1)$ about the line $y = x$, then mid-point of PQ lies on this line and PQ is perpendicular to it. So we have

$$\frac{y+1}{2} = \frac{x+4}{2} \text{ and } \frac{y-1}{x-4} = -1.$$

$$\Rightarrow \quad x - y = -3 \text{ and } x + y = 5$$

$$\Rightarrow \quad x = 1, y = 4$$

Therefore reflection of (4, 1) about $y = x$ is (1, 4). Next, this point is shifted. 2 units along the positive x-axis, the new coordinates are (1 + 2, 4 + 0) = (3, 4)

Question

If the lines joining the origin to the intersection of the line $y = mx + 2$ and the curve $x^2 + y^2 = 1$ are at right angles, then

(a) $m^2 = 1$ (b) $m^2 = 3$

(c) $m^2 = 7$ (d) $2m^2 = 1$

Ans. (c)

Solution Joint equation of the lines joining the origin and the point of intersection of the line $y = mx + 2$ and the curve $x^2 + y^2 = 1$ is

$$x^2 + y^2 = \left(\frac{y - mx}{2}\right)^2$$

$$\Rightarrow \quad x^2 (4 - m^2) + 2mxy + 3y^2 = 0$$

Since these lines are at right angles

$$4 - m^2 + 3 = 0 \Rightarrow m^2 = 7.$$

Question

If one of the lines given by the equation $2x^2 + axy + 3y^2 = 0$ coincide with one of those given by $2x^2 + bxy - 3y^2 = 0$ and the other lines represented by them be perpendicular, then

(a) $a = -5, b = 1$ (b) $a = 5, b = -1$

(c) $a = 5, b = 1$ (d) none of these

Ans. (c)

Solution Let $\frac{2}{3}x^2 + \frac{a}{3}xy + y^2 = (y - mx)(y - m'x)$

and $$\frac{2}{-3}x^2 + \frac{b}{-3}xy + y^2 = \left(y + \frac{1}{m}x\right)(y - m'x)$$

then $$m + m' = -\frac{a}{3},\ mm' = \frac{2}{3} \quad \text{(i)}$$

$$\frac{1}{m} - m' = \frac{-b}{3},\ -\frac{m'}{m} = -\frac{2}{3} \quad \text{(ii)}$$

$$\Rightarrow \qquad m^2 = 1 \Rightarrow m = \pm 1$$

$$\text{If } m = 1, m' = \frac{2}{3} \Rightarrow a = -5, b = -1$$

$$\text{If } m = -1, m' = -\frac{2}{3} \Rightarrow a = 5, b = 1.$$

Question

The line $y = 3x$ bisects the angle between the lines $ax^2 + 2axy + y^2 = 0$ if $a =$

(a) 3 (b) 11 (c) 3/11 (d) 11/3

Ans. (c)

Solution Equation of the bisectors of the angles between the lines $ax^2 + 2axy + y^2 = 0$ is

$\frac{x^2 - y^2}{a-1} = \frac{xy}{a}$ which is satisfied by $y = 3x$ if $\frac{1-9}{a-1} = \frac{3}{a} \Rightarrow a = 3/11$

Question

A line passing through the point $P(2, 3)$ meets the lines represented by $x^2 - 2xy - y^2 = 0$ at the points A and B such that $PA \cdot PB = 17$, the equation of the line is

(a) $x = 2$ (b) $y = 3$

(c) $3x - 2y = 0$ (d) none of these

Ans. (b)

Solution Let the equation of the line through $P(2, 3)$ making an angle θ with the positive direction of x-axis be $\frac{x-2}{\cos\theta} = \frac{y-3}{\sin\theta}$.

Then the coordinates of any point on this line at a distance r from P are $(2 + r\cos\theta, 3 + r\sin\theta)$. If $PA = r_1$ and $PB = r_2$, then r_1, r_2 are the roots of the equation.

$(2 + r\cos\theta)^2 - 2(2 + r\cos\theta)(3 + r\sin\theta) - (3 + r\sin\theta)^2 = 0$

$\Rightarrow \quad r^2(\cos 2\theta - \sin 2\theta) - 2r(\cos\theta + 5\sin\theta) - 17 = 0$

$$\Rightarrow \quad 17 = \mathbf{PA} \cdot \mathbf{PB} = r_1 r_2 = \frac{17}{\cos 2\theta - \sin 2\theta}$$

$\Rightarrow \quad \cos 2\theta - \sin 2\theta = 1$ which is satisfied by $\theta = 0$ and thus the equation of the line is $y = 3$.

:-{D

To recall standard integrals

$f(x)$	$\int f(x)dx$	$f(x)$	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1} \quad (n \neq -1)$	$[g(x)]^n g'(x)$	$\frac{[g(x)]^{n+1}}{n+1} \quad (n \neq -1)$
$\frac{1}{x}$	$\ln \lvert x \rvert$	$\frac{g'(x)}{g(x)}$	$\ln \lvert g(x) \rvert$
e^x	e^x	a^x	$\frac{a^x}{\ln a} \quad (a > 0)$
$\sin x$	$-\cos x$	$\sinh x$	$\cosh x$
$\cos x$	$\sin x$	$\cosh x$	$\sinh x$
$\tan x$	$-\ln \lvert \cos x \rvert$	$\tanh x$	$\ln \cosh x$
$\operatorname{cosec} x$	$\ln \left\lvert \tan \frac{x}{2} \right\rvert$	$\operatorname{cosech} x$	$\ln \left\lvert \tanh \frac{x}{2} \right\rvert$
$\sec x$	$\ln \lvert \sec x + \tan x \rvert$	$\operatorname{sech} x$	$2 \tan^{-1} e^x$
$\sec^2 x$	$\tan x$	$\operatorname{sech}^2 x$	$\tanh x$
$\cot x$	$\ln \lvert \sin x \rvert$	$\coth x$	$\ln \lvert \sinh x \rvert$
$\sin^2 x$	$\frac{x}{2} - \frac{\sin 2x}{4}$	$\sinh^2 x$	$\frac{\sinh 2x}{4} - \frac{x}{2}$
$\cos^2 x$	$\frac{x}{2} + \frac{\sin 2x}{4}$	$\cosh^2 x$	$\frac{\sinh 2x}{4} + \frac{x}{2}$

$f(x)$	$\int f(x)\,dx$	$f(x)$	$\int f(x)\,dx$
$\frac{1}{a^2+x^2}$	$\frac{1}{a} \tan^{-1} \frac{x}{a}$ $(a > 0)$	$\frac{1}{a^2-x^2}$	$\frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert \quad (0 < \lvert x \rvert < a)$
		$\frac{1}{x^2-a^2}$	$\frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert \quad (\lvert x \rvert > a > 0)$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1} \frac{x}{a}$ $(-a < x < a)$	$\frac{1}{\sqrt{a^2+x^2}}$	$\ln \left\lvert \frac{x+\sqrt{a^2+x^2}}{a} \right\rvert \quad (a > 0)$
		$\frac{1}{\sqrt{x^2-a^2}}$	$\ln \left\lvert \frac{x+\sqrt{x^2-a^2}}{a} \right\rvert \quad (x > a > 0)$
$\sqrt{a^2-x^2}$	$\frac{a^2}{2}\left[\sin^{-1}\left(\frac{x}{a}\right) + \frac{x\sqrt{a^2-x^2}}{a^2}\right]$	$\sqrt{a^2+x^2}$	$\frac{a^2}{2}\left[\sinh^{-1}\left(\frac{x}{a}\right) + \frac{x\sqrt{a^2+x^2}}{a^2}\right]$
		$\sqrt{x^2-a^2}$	$\frac{a^2}{2}\left[-\cosh^{-1}\left(\frac{x}{a}\right) + \frac{x\sqrt{x^2-a^2}}{a^2}\right]$

Some series Expansions -

$$\frac{\pi}{2} = \left(\frac{2}{1}\frac{2}{3}\right)\left(\frac{4}{3}\frac{4}{5}\right)\left(\frac{6}{5}\frac{6}{7}\right)\left(\frac{8}{7}\frac{8}{9}\right)\ldots$$

$$\pi = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \frac{4}{13} - \cdots$$

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$$

$$\pi = \sqrt{12}\left(1 - \frac{1}{3\cdot 3} + \frac{1}{5\cdot 3^2} - \frac{1}{7\cdot 3^3} + \cdots\right)$$

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots = \sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\int_0^{\pi/2} \log \sin x \, dx = -\frac{\pi}{2}\log 2 = \frac{\pi}{2}\log\frac{1}{2}$$

Solve a series problem

If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$ upto $\infty = \frac{\pi^2}{6}$, then value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ up to ∞ is

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{6}$ (c) $\frac{\pi^2}{8}$ (d) $\frac{\pi^2}{12}$

Ans. (c)

Solution We have $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ upto ∞

$$= \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} \cdots \text{ upto } \infty$$

$$- \frac{1}{2^2}\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right]$$

$$= \frac{\pi^2}{6} - \frac{1}{4}\left(\frac{\pi^2}{6}\right) = \frac{\pi^2}{8}$$

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \ldots \infty = \frac{\pi^2}{12}$$

$$\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \ldots \infty = \frac{\pi^2}{24}$$

$$\frac{\sin\sqrt{x}}{\sqrt{x}} = 1 - \frac{x}{3!} + \frac{x^2}{5!} - \frac{x^3}{7!} + \frac{x^4}{9!} - \frac{x^5}{11!} + \ldots.$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = \sum_{k=0}^{n} \frac{(-1)^k x^{2k}}{(2k)!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots = \sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots = \sum_{k=0}^{n} \frac{x^{2k}}{(2k)!}$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots = \sum_{k=0}^{n} \frac{x^{2k+1}}{(2k+1)!}$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad (-1 \le x < 1)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} \cdots + \frac{2^{2n}\left(2^{2n}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \qquad |x| < \frac{\pi}{2}$$

$$\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \cdots + \frac{E_n x^{2n}}{(2n)!} + \cdots \quad |x| < \frac{\pi}{2}$$

$$\csc x = \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^5}{15120} + \cdots + \frac{2\left(2^{2n-1}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \qquad 0 < |x| < \pi$$

$$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - - \cdots - \frac{2^{2n} B_n x^{2n-1}}{(2n)!} - \cdots \quad 0 < |x| < \pi$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$$

$$\sec x = 1 + \frac{x^2}{\lfloor 2} + \frac{5x^4}{\lfloor 4} + \cdots$$

$$\log(\cos x) = -\frac{x^2}{\lfloor 2} - \frac{2x^4}{\lfloor 4} - \cdots$$

$$\log(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \cdots$$

$$\sin^{-1} x = x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \cdots \quad |x| < 1$$

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$= \frac{\pi}{2} - \left(x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \cdots\right) \quad |x| < 1$$

$$\tan^{-1} x = \begin{cases} x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots & |x| < 1 \\ \pm\frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \cdots & \begin{cases} + \text{ if } x \geq 1 \\ - \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$$

$$= \frac{\pi}{2} - \left(\frac{1}{x} + \frac{1}{2\cdot 3x^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5x^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \cdots\right) \quad |x| > 1$$

$$\csc^{-1} x = \sin^{-1}(1/x)$$

$$= \frac{1}{x} + \frac{1}{2\cdot 3x^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5x^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \cdots \quad |x| > 1$$

$$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$$

$$= \begin{cases} \frac{\pi}{2} - \left(x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots\right) & |x| < 1 \\ p\pi + \frac{1}{x} - \frac{1}{3x^3} + \frac{1}{5x^5} + \cdots & \begin{cases} p = 0 \text{ if } x \geq 1 \\ p = 1 \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\ln x = 2\left[\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \ldots\right]$$

$$= 2\sum_{n=1}^{\infty} \frac{1}{2n-1}\left(\frac{x-1}{x+1}\right)^{2n-1} \quad (x > 0)$$

$$\ln x = \frac{x-1}{x} + \frac{1}{2}\left(\frac{x-1}{x}\right)^2 + \frac{1}{3}\left(\frac{x-1}{x}\right)^3 + \ldots$$

$$= \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{x-1}{x}\right)^n \quad (x > \frac{1}{2})$$

$$\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}(x-1)^n \quad (0 < x \le 2)$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}x^n \quad (|x| < 1)$$

$$\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots\infty(-1 \le x < 1)$$

$$\log_e(1+x) - \log_e(1-x) =$$

$$\log_e \frac{1+x}{1-x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\infty\right)(-1 < x < 1)$$

$$\log_e\left(1+\frac{1}{n}\right) = \log_e \frac{n+1}{n} = 2 \quad \left[\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \ldots\infty\right]$$

$$\log_e (1 + x) + \log_e (1 - x) = \log_e(1-x^2) = -2\left(\frac{x^2}{2} + \frac{x^4}{4} + \ldots\infty\right)(-1 < x < 1)$$

$$\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots \quad = \frac{1}{1\cdot 2} + \frac{1}{3.4} + \frac{1}{5.6} + \ldots$$

Important Results

(i) (a) $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cos^n x}{\sin^n x + \cos^n x} dx$

(b) $\int_0^{\pi/2} \frac{\tan^n x}{1 + \tan^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{dx}{1 + \tan^n x}$

(c) $\int_0^{\pi/2} \frac{dx}{1 + \cot^n x} = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{1 + \cot^n x} dx$

(d) $\int_0^{\pi/2} \frac{\tan^n x}{\tan^n x + \cot^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{\tan^n x + \cot^n x} dx$

(e) $\int_0^{\pi/2} \frac{\sec^n x}{\sec^n x + \operatorname{cosec}^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\operatorname{cosec}^n x}{\sec^n x + \operatorname{cosec}^n x} dx$ where, $n \in R$

(ii) $\int_0^{\pi/2} \frac{a^{\sin^n x}}{a^{\sin^n x} + a^{\cos^n x}} dx = \int_0^{\pi/2} \frac{a^{\cos^n x}}{a^{\sin^n x} + a^{\cos^n x}} dx = \frac{\pi}{4}$

(iii) (a) $\int_0^{\pi/2} \log \sin x \, dx = \int_0^{\pi/2} \log \cos x \, dx = -\frac{\pi}{2} \log 2$

(b) $\int_0^{\pi/2} \log \tan x \, dx = \int_0^{\pi/2} \log \cot x \, dx = 0$

(c) $\int_0^{\pi/2} \log \sec x \, dx = \int_0^{\pi/2} \log \operatorname{cosec} x \, dx = \frac{\pi}{2} \log 2$

(iv) (a) $\int_0^{\infty} e^{-ax} \sin bx \, dx = \frac{b}{a^2 + b^2}$

(b) $\int_0^{\infty} e^{-ax} \cos bx \, dx = \frac{a}{a^2 + b^2}$

(c) $\int_0^{\infty} e^{-ax} x^n \, dx = \frac{n!}{a^n + 1}$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left(x + \sqrt{x^2 - a^2}\right) + C$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left(\frac{x - a}{x + a}\right) + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left(\frac{a + x}{a - x}\right) + C$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{a^2 + x^2}\, dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right) + C$$

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Good Luck to you for your Preparations, References, and Exams

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