Spoon Feeding 3D Geometry

Simplified Knowledge Management Classes Bangalore

My name is Subhashish Chattopadhyay. I have been teaching for IIT-JEE, Various International Exams (such as IMO [International Mathematics Olympiad], IPhO [International Physics Olympiad], IChO [International Chemistry Olympiad]), IGCSE (IB), CBSE, I.Sc, Indian State Board exams such as WB-Board, Karnataka PU-II etc since 1989. As I write this book in 2016, it is my 25 th year of teaching. I was a Visiting Professor to BARC Mankhurd, Chembur, Mumbai, Homi Bhabha Centre for Science Education (HBCSE) Physics Olympics camp BARC Campus.

I am Life Member of …

- IAPT (Indian Association of Physics Teachers)

- IPA (Indian Physics Association)

- AMTI (Association of Mathematics Teachers of India)

- National Human Rights Association

- Men's Rights Movement (India and International)

- MGTOW Movement (India and International)

And also of

IACT (Indian Association of Chemistry Teachers)

The selection for National Camp (for Official Science Olympiads - Physics, Chemistry, Biology, Astronomy) happens in the following steps ….

1) **NSEP** (National Standard Exam in Physics) and **NSEC** (National Standard Exam in Chemistry) held around 24 rth November. Approx 35,000 students appear for these exams every year. The exam fees is Rs 100 each. Since 1998 the IIT JEE toppers have been topping these exams and they get to know their rank / performance ahead of others.

2) **INPhO** (Indian National Physics Olympiad) and **INChO** (Indian National Chemistry Olympiad). Around 300 students in each subject are allowed to take these exams. Students coming from outside cities are paid fair from the Govt of India.

3) The Top 35 students of each subject are invited at HBCSE (Homi Bhabha Center for Science Education) Mankhurd, near Chembur, BARC, Mumbai. After a 2-3 weeks camp the top 5 are selected to represent India. The flight tickets and many other expenses are taken care by Govt of India.

Since last 50 years there has been no dearth of “Good Books“. Those who are interested in studies have been always doing well. This e-Book does not intend to replace any standard text book. These topics are very old and already standardized.

There are 3 kinds of Text Books

- The thin Books - Good students who want more details are not happy with these. Average students who need more examples are not happy with these. Most students who want to “Cram” quickly and pass somehow find the thin books “good” as they have to read less !!

- The Thick Books - Most students do not like these, as they want to read as less as possible. Average students are “busy” with many other things and have no time to read all these.

- The Average sized Books - Good students do not get all details in any one book. Most bad students do not want to read books of “this much thickness“ also !!

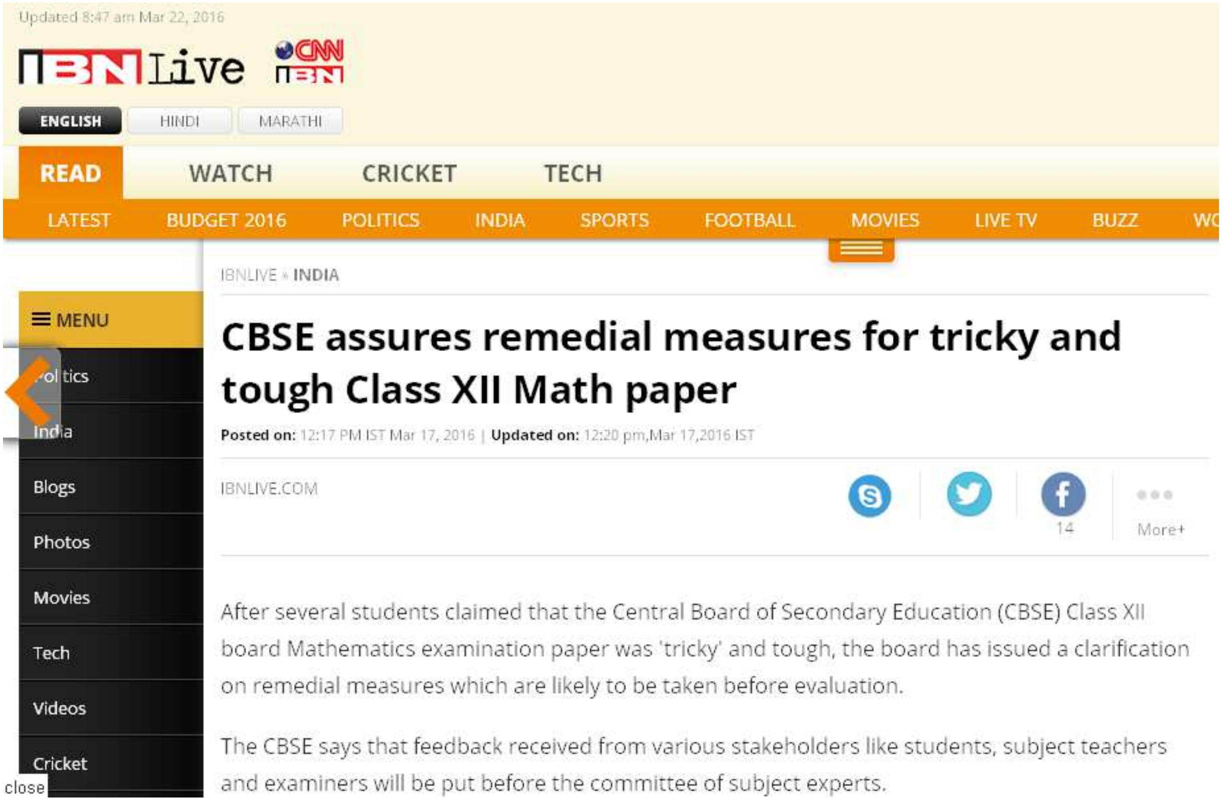
We know there can be no shoe that’s fits in all.

Printed books are not e-Books! Can’t be downloaded and kept in hard-disc for reading “later” ………

So if you read this book later, you will get all kinds of examples in a single place. This becomes a very good “Reference Material”. I sincerely wish that all find this “very useful”.

Students who do not practice lots of problems, do not do well. The rules of “doing well” had never changed …. Will never change !

After 2016 CBSE Mathematics exam, lots of students complained that the paper was tough!



Updated 8:47 am Mar 22, 2016

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CBSE assures remedial measures for tricky and tough Class XII Math paper

Posted on: 12:17 PM IST Mar 17, 2016 | **Updated on:** 12:20 pm,Mar 17,2016 IST

IBNLIVE.COM

After several students claimed that the Central Board of Secondary Education (CBSE) Class XII board Mathematics examination paper was 'tricky' and tough, the board has issued a clarification on remedial measures which are likely to be taken before evaluation.

The CBSE says that feedback received from various stakeholders like students, subject teachers and examiners will be put before the committee of subject experts.

On 21 st May 2016 the CBSE standard 12 result was declared. I loved the headline

INDIATODAY.IN NEW DELHI, MAY 21, 2016 | UPDATED 16:40 IST

CBSE Class 12 Results out: No leniency in Maths paper, high paper standard to be maintained in future

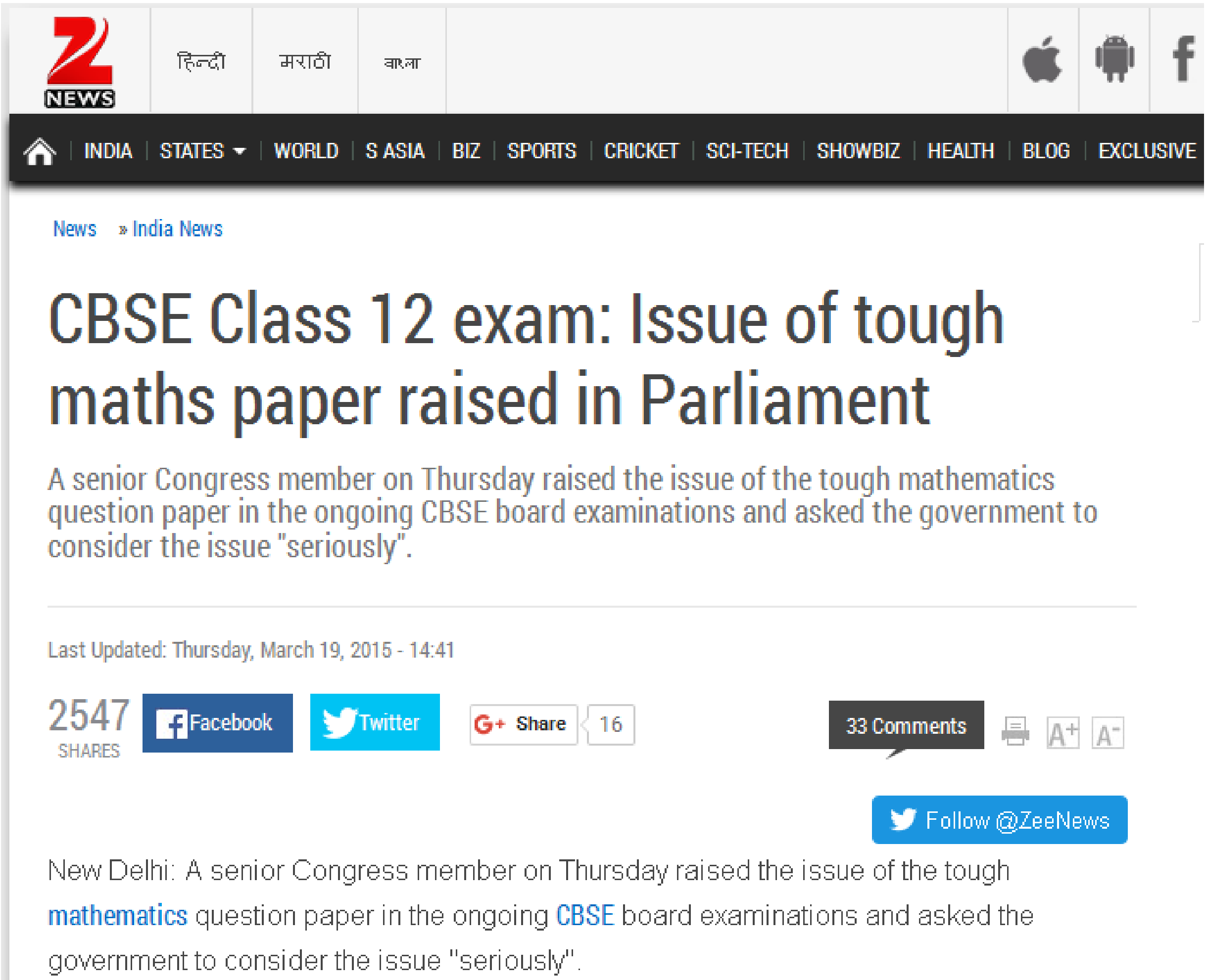
The CBSE Class 12 Mathematics board exam on March 14 reduced many students to tears as they found the paper quite lengthy and tough and many couldn't finish it on time. The results show an overall lowering of marks received in the Maths paper.

RELATED STORIES

- CBSE Board result 2016 declared! Thiruvanathpuram obtains the highest part percentage, check how your region scored
- Meet CBSE topper Sukriti Gupta: Check her percentage here!
- CBSE Class 12 Boards 2016: Results announced ahead of time!
- CBSE results declared at www.cbse.nic.in: Steps to check online
- Exclusive! CBSE declares Class 12 Results at www.cbseresults.nic.in and cbse.nic.in

The CBSE (Central Board of Secondary Education) Class 12 Board exam results have been announced today, i.e on May 21, around 10:30 am ahead of time. Students may check their scores at the official website, *www.cbseresults.nic.in*. (**Read: CBSE Class 12 Boards 2016: Results announced ahead of time! Check your score at cbseresults.nic.in**)

In 2015 also the same complain was there by many students



CBSE Class 12 exam: Issue of tough maths paper raised in Parliament

A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".

Last Updated: Thursday, March 19, 2015 - 14:41

New Delhi: A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".

So we see that by raising frivolous requests, even upto parliament, actually does not help. Many times requests from several quarters have been put to CBSE, or Parliament etc for easy Math Paper. These kinds of requests actually can-not be entertained, never will be.

In March 2016, students of Karnataka PU-II also complained the same, regarding standard 12 (PU-II Mathematics Exam). Even though the Math Paper was identical to previous year, most students had not even solved the 2015 Question Paper.

Friday, March 25, 2016 - 13:28

The NEWS Minute

HOME NEWS ANDHRA KARNATAKA KERALA TAMIL NADU TELANGANA CULTURE MEDIA BLOG

Exams

Online petition for lenient evaluation of K'taka II PU math paper gets over 8000 supporters

The campaign, which was launched on Monday, has garnered over 8000 supporters

TNM Staff | Wednesday, March 16, 2016 - 09:32

Follow @thenewsminute

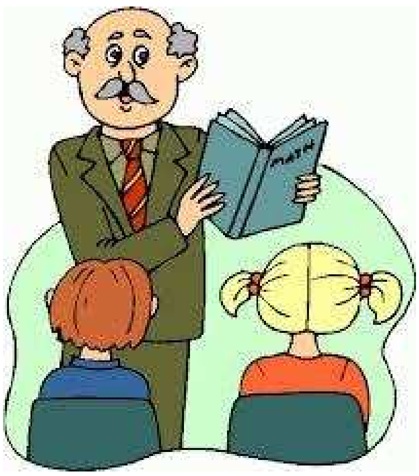
Share Tweet Email Share Reddit

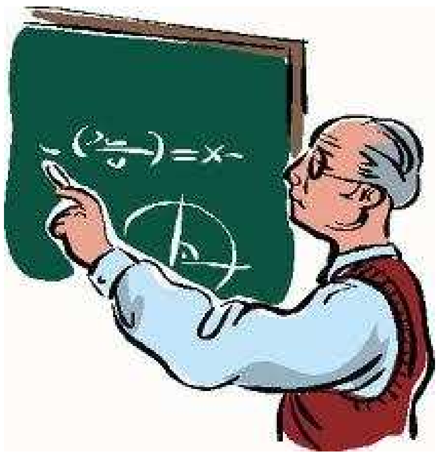
Following a "very tough" math paper that left many II PU students in tears, Saket Ravindran a student launched an online campaign demanding lenient evaluation.

These complains are not new. In fact since last 40 years, (since my childhood), I always see this; every year the same setback, same complain!

In this e-Book I am trying to solve this problem. Those students who practice can learn.

No one can help those who are not studying, or practicing.





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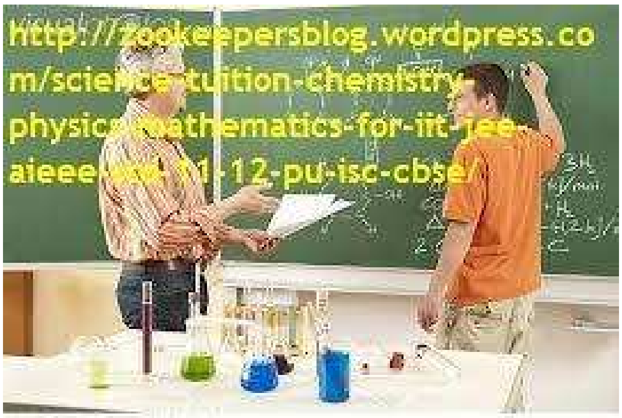


A very polite request :

I wish these e-Books are read only by Boys and Men. Girls and Women, better read something else; learn from somewhere else.

Preface

We all know that in the species "Homo Sapiens ", males are bigger than females. The reasons are explained in standard 10, or 11 (high school) Biology texts. **This shapes or size, influences all of our culture**. Before we recall / understand the reasons once again, let us see some random examples of the influence

Random - 1

If there is a Road rage, then who all fight ? (generally ?). Imagine two cars driven by adult drivers. Each car has a woman of similar age as that of the Man. The cars " touch " or " some issue happens". Who all comes out and fights ? Who all are most probable to drive the cars ?

(Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win)

Random - 2

Heavy metal music artists are all Men. Metallica, Black Sabbath, Motley Crue, Megadeth, Motorhead, AC/DC, Deep Purple, Slayer, Guns & Roses, Led Zeppelin, Aerosmith ….. the list can be in thousands. All these are grown-up Boys, known as Men.

(Men strive for perfection. Men are eager to excel. Men work hard. Men want to win.)

Random - 3

Apart from Marie Curie, only one more woman got Nobel Prize in Physics. (Maria Goeppert Mayer - 1963). So, … almost all are men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 4

The best Tabla Players are all Men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 5

History is all about, which Kings ruled. Kings, their men, and Soldiers went for wars. History is all about wars, fights, and killings by men.

Boys start fighting from school days. Girls do not fight like this

(Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.)

Random - 6

The highest award in Mathematics, the “ Fields Medal “ is around since decades. Till date only one woman could get that. (Maryam Mirzakhani - 2014). So, … almost all are men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 7

Actor is a gender neutral word. Could the movie like “ Top Gun “ be made with Female actors ? The best pilots, astronauts, Fighters are all Men.

Random - 8

In my childhood had seen a movie named " The Tower in Inferno ". In the movie when the tall tower is in fire, women were being saved first, as only one lift was working….

Many decades later another movie is made. A box office hit. " The Titanic ". In this also …. As the ship is sinking women are being saved. **Men are disposable**. Men may get their turn later…

Movies are not training programs. Movies do not teach people what to do, or not to do. Movies only reflect the prevalent culture. Men are disposable, is the culture in the society. Knowingly, unknowingly, the culture is depicted in Movies, Theaters, Stories, Poems, Rituals, etc. I or you can't write a story, or make a movie in which after a minor car accident the Male passengers keep seating in the back seat, while the both the women drivers come out of the car and start fighting very bitterly on the road. There has been no story in this world, or no movie made, where after an accident or calamity, Men are being helped for safety first, and women are told to wait.

Random - 9

Artists generally follow the prevalent culture of the Society. In paintings, sculptures, stories, poems, movies, cartoon, Caricatures, knowingly / unknowingly, “ the prevalent Reality “ is depicted. The opposite will not go well with people. If deliberately “ the opposite “ is shown then it may only become a special art, considered as a special mockery.

Random - 10

Men go to “girl / woman’s house” to marry / win, and bring her to his home. That is a sort of winning her. When a boy gets a “ Girl-Friend “, generally he and his friends consider that as an achievement. The boy who “ got / won “ a girl-friend feels proud. His male friends feel, jealous, competitive and envious. Millions of stories have been written on these themes. Lakhs of movies show this. Boys / Men go for “ bike race “, or say “ Car Race “, where the winner “ gets “ the most beautiful girl of the college.




(Men want to excel. Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.)

Prithviraj Chauhan ‘ went ` to “ pickup “ or “ abduct “ or “ win “ or “ bring “ his love. There was a Hindi movie (hit) song … “ Pasand ho jaye, to ghar se utha laye “. It is not other way round. Girls do not go to Boy’s house or man’s house to marry. Nor the girls go in a gang to “ pick-up “ the boy / man and bring him to their home / place / den.

Random - 11

Rich people; often are very hard working. Successful business men, establish their business (empire), amass lot of wealth, with lot of difficulty. Lots of sacrifice, lots of hard work, gets into this. Rich people's wives had no contribution in this wealth creation. Women are smart, and successful upto the extent to choose the right/rich man to marry. So generally what happens in case of Divorces ? Search the net on " most costly divorces " and you will know. The women;(who had no contribution at all, in setting up the business / empire), often gets in Billions, or several Millions in divorce settlements.

Number 1

Rupert & Anna Murdoch -- $1.7 billion

One of the richest men in the world, **Rupert Murdoch** developed his worldwide media empire when he inherited his father's Australian newspaper in 1952. He married Anna Murdoch in the '60s and they remained together for 32 years, springing off three children.

They split amicably in 1998 but soon Rupert forced Anna off the board of News Corp and the gloves came off. The divorce was finalized in June 1999 when Rupert agreed to let his ex-wife leave with $1.7 billion worth of his assets, $110 million of it in cash. Seventeen days later, Rupert married Wendi Deng, one of his employees.

Ted Danson & Casey Coates -- $30 million

Ted Danson's claim to fame is undoubtedly his decade-long stint as Sam Malone on NBC's celebrated sitcom Cheers . While he did other TV shows and movies, he will always be known as the bartender of that place where everybody knows your name. He met his future first bride Casey, a designer, in 1976 while doing Erhard Seminars Training.

Ten years his senior, she suffered a paralyzing stroke while giving birth to their first child in 1979. In order to nurse her back to health, Danson took a break from acting for six months. But after two children and 15 years of marriage, the infatuation fell to pieces. Danson had started seeing Whoopi Goldberg while filming the comedy, Made in America and this precipitated the 1992 divorce. Casey got $30 million for her trouble.

See https://zookeepersblog.wordpress.com/misandry-and-men-issues-a-short-summary-at-single-place/

See http://skmclasses.kinja.com/save-the-male-1761788732

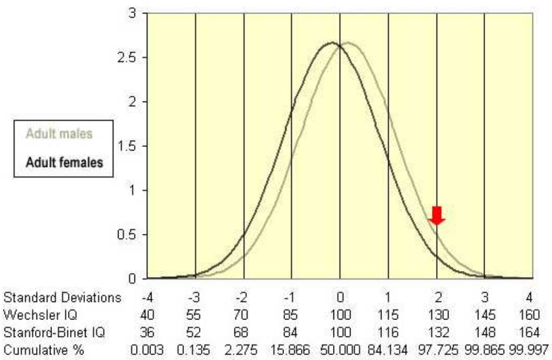
It was Boys and Men, who brought the girls / women home. The Laws are biased, completely favoring women. The men are paying for their own mistakes.

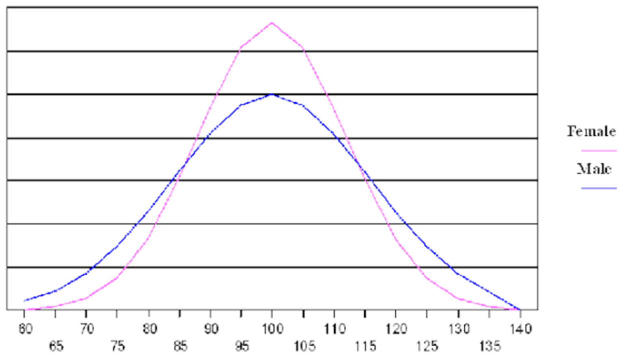
See https://zookeepersblog.wordpress.com/biased-laws/

(Man brings the Woman home. When she leaves, takes away her share of big fortune!)

Random - 12

A standardized test of Intelligence will never be possible. It never happened before, nor ever will happen in future; where the IQ test results will be acceptable by all. In the net there are thousands of charts which show that the intelligence scores of girls / women are lesser. Debates of Trillion words, does not improve performance of Girls.





I am not wasting a single second debating or discussing with anyone, on this. I am simply accepting ALL the results. IQ is only one of the variables which is required for success in life. Thousands of books have been written on “ Networking Skills “, EQ (Emotional Quotient), Drive, Dedication, Focus, “ Tenacity towards the end goal “ … etc. In each criteria, and in all together, women (in general) do far worse than men. Bangalore is known as “ ….. capital of India “. [Fill in the blanks]. The blanks are generally filled as “ Software Capital “, “ IT Capital “, “ Startup Capital “, etc. I am member in several startup eco-systems / groups. I have attended hundreds of meetings, regarding “ technology startups “, or “ idea startups “. These meetings have very few women. Starting up new companies are all “ Men’s Game “ / “ Men’s business “. Only in Divorce settlements women will take their goodies, due to Biased laws. There is no dedication, towards wealth creation, by women.

Random - 13

Many men, as fathers, very unfortunately treat their daughters as “ Princess “. Every “ non-performing “ woman / wife was “ princess daughter “ of some loving father. Pampering the girls, in name of “ equal opportunity “, or “ women empowerment “, have led to nothing.

See http://skmclasses.kinja.com/progressively-daughters-become-monsters-1764484338

See http://skmclasses.kinja.com/vivacious-vixens-1764483974

There can be thousands of more such random examples, where " Bigger Shape / size " of males have influenced our culture, our Society. **Let us recall the reasons**, that we already learned in standard 10 - 11, Biology text Books. In humans, women have a long gestation period, and also spends many years (almost a decade) to grow, nourish, and stabilize the child. (Million years of habit) Due to survival instinct Males want to inseminate. Boys and Men fight for the " facility (of womb + care) " the girl / woman may provide. Bigger size for males, has a winning advantage. Whoever wins, gets the " woman / facility ". The male who is of " Bigger Size ", has an advantage to win…. Leading to Natural selection over millions of years. In general " Bigger Males "; the " fighting instinct " in men; have led to wars, and solving tough problems (Mathematics, Physics, Technology, startups of new businesses, Wealth creation, Unreasonable attempts to make things [such as planes], Hard work ….)

So let us see the IIT-JEE results of girls. Statistics of several years show that there are around 17, (or less than 20) girls in top 1000 ranks, at all India level. Some people will yet not understand the performance, till it is said that … year after year we have around 980 boys in top 1000 ranks. Generally we see only 4 to 5 girls in top 500. In last 50 years not once any girl topped in IIT-JEE advanced. Forget about Single digit ranks, double digit ranks by girls have been extremely rare. It is all about " good boys ", " hard working ", " focused ", "**Bel-esprit**" **boys**.

In 2015, Only 2.6% of total candidates who qualified are girls (upto around 12,000 rank). while 20% of the Boys, amongst all candidates qualified. The Total number of students who appeared for the exam were around 1.4 million for IIT-JEE main. Subsequently 1.2 lakh (around 120 thousands) appeared for IIT-JEE advanced.

IIT-JEE results and analysis, of many years is given at https://zookeepersblog.wordpress.com/iit-jee-iseet-main-and-advanced-results/

In Bangalore it is rare to see a girl with rank better than 1000 in IIT-JEE advanced. We hardly see 6-7 boys with rank better than 1000. Hardly 2-3 boys get a rank better than 500.

See http://skmclasses.weebly.com/everybody-knows-so-you-should-also-know.html

Thousands of people are exposing the heinous crimes that Motherly Women are doing, or Female Teachers are committing. See https://www.facebook.com/WomenCriminals/

Some Random Examples must be known by all

Mother Admits On Facebook to Sleeping with 15 Yr Old Son, They Have a Baby Together - Alwayzturntup

Sometimes it hard to believe w From Alwayzturntup

ALWAYZTURNTUP.ME

It is extremely unfortunate that the " woman empowerment " has created. This is the kind of society and women we have now. I and many other sensible Men hate such women. Be away from such women, be aware of reality.

'Sex with my son is incredible - we're in love and we want a baby'

Ben Ford, who ditched his wife when he met his mother Kim West after 30 years, claims what the couple are doing 'isn't incest'

MIRROR.CO.UK

Former Shelbyville ISD teacher who had sex with underage student gets 3 years in prison

After a two day break over the weekend, A Shelby County jury was back in the courtroom looking to conclude the trial of a former Shelbyville ISD teacher who had...

KLTV.COM | BY CALEB BEAMES

Woman sent to jail for the rest of her life after raping her four grandchildren is described as the 'most evil person' the judge has ever seen

Edwina Louis rape...

See More

Woman sent to jail for raping her four grandchildren

A Ohio grandmother has been sentenced to four consecutive life terms after being found guilty of the rape of her own grandchildren. Edwina Louis, 53, will spend the rest of her life behind bars.

DAILYMAIL.CO.UK

http://www.thenativecanadian.com/.../eastern-ontario-teacher-...

The N.C. Chronicles.: Eastern Ontario teacher charged with 36 sexual offences

anti feminism, Child abuse, children's rights, Feminist hypocrisy,

THENATIVECANADIAN.COM | BY BLACKWOLF

Hyd woman kills newborn boy as she wanted daughter - Times of India

Having failed to bear a daughter for the third time, a shopkeeper's wife slit the throat of her 24day-old son with a shaving blade and left him to die in a street on Tuesday night.Purnima's first child was a stillborn boy, followed by another boy born five years ago.

TIMESOFINDIA.INDIATIMES.COM

Montgomery's son, Alan Vonn Webb, took the stand and was a key witness in her conviction.

"I want to see her placed somewhere she can never do that to children ...

See More

Woman sentenced to 40 years in prison for raping her children

A Murfreesboro mother found guilty of raping her own children learned her fate on Wednesday.

WAFF.COM | BY DENNIS FERRIER

gentler sex? Violence against men.'s photo.

Women, the gentler sex? Violence against men.
April 8 at 1:38am ·

Like Page

In fact, the past decade has seen a dramatic increase in the number of incidents of women raping and sexually assaulting boys and men. On May 2014, Jezebel repo...

End violence against women . . . .

North Carolina Grandma Eats Her Daughter's New Born Baby After Smoking Bath Salts

Henderson, North Carolina– A North Carolina grandmother of 4 and recovering drug addict, is now in custody after she allegedly ate her daughter's newborn baby....

AZ-365.TOP

28-Year-Old Texas Teacher Accused of Sending Nude Picture to 14-Year-Old Former Student

BREITBART.COM

http://latest.com/.../attractive-girl-gang-lured-men-alleywa.../

Attractive Girl Gang Lured Men Into Alleyways Where Female Body Builder Would Attack Them

A Mexican street gang made up entirely of women has been accused of using their feminine wiles to lure men into alleyways and then beating them up and...

LATEST.COM

http://www.wfmj.com/.../youngstown-woman-convicted-of-raping-...

Youngstown woman convicted of raping a 1 year old is back in jail

A Youngstown woman who went to prison for raping a 1-year-old boy fifteen years ago is in trouble with the law again.

WFMJ.COM

End violence against women . . . .

Women are raping boys and young men

Rape advocacy has been maligned and twisted into a political agenda controlled by radicalized activists. Tim Patten takes a razor keen and well supported look into the manufactured rape culture and...

AVOICEFORMEN.COM | BY TIM PATTEN

Bronx Woman Convicted of Poisoning and Drowning Her Children

Lisette Bamenga researched methods on the Internet before she killed her son and daughter in 2012.

NYTIMES.COM | BY MARC SANTORA

A Russian-born newlywed slowly butchered her German husband — feeding strips of his flesh to their dog until he took his last breath. Svetlana Batukova, 46, was...

See More

She killed her husband and then fed him to her dog: police

A Russian-born newlywed butchered her German hubby — and fed strips of his flesh to her pooch, authorities said. Svetlana Batukova offed Horst Hans Henkels at their...

NYPOST.COM

Daily Mail
January 15, 2015

Mother charged with rape and sodomy of her son's 12-year-old friend

Mom, 30, 'raped and had oral sex with her son's 12-year-old friend'

Nicole Marie Smith, 30, (pictured) of St Charles County, Missouri, has been jailed after she allegedly targeted the 12-year-old boy at her home.

DAILYM.AI

April 4 at 4:48am

Female prison officers commit 90pc of sex assaults on male teens in US juvenile detention centres

Lawsuit in Idaho highlights the prevalence of sexual victimization of juvenile offenders.

IBTIMES.CO.UK | BY NICOLE ROJAS

This mother filmed herself raping her own son and then sold it to a man for $300. The courts just decide her fate. When you see what she got, you're going to be outraged.

Mother Who Filmed Herself Raping Her 1-Year-Old Son Receives Shocking Sentence

"...then used the money to buy herself a laptop..."

AMERICANEWS.COM

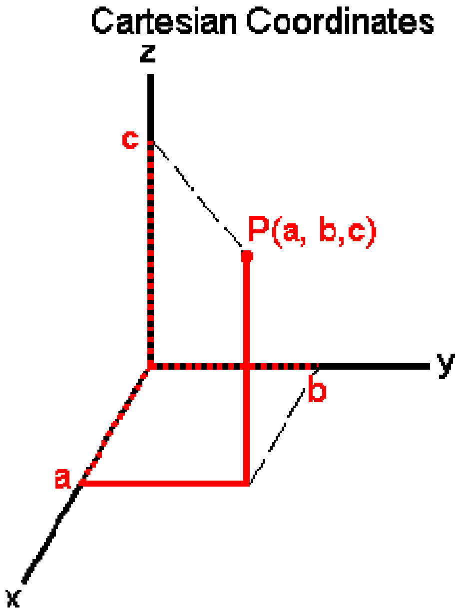
In several countries or rather in several regions of the world, family system has collapsed, due to bad nature and naughty acts of women. Particularly in Britain, and America, almost 50% people are alone, lonely, separated, divorced or failed marriages. In 2013, 48% children were born out of wedlock. It was projected that by 2016, more than 51% children will be born, to unmarried mothers. In these developed countries " paternity fraud " by women, are close to 20%. You can see several articles in the net, and in wikipedia etc. This means 1 out of 5 children are calling a wrong man as dad. The lonely, alone “ mothers " are frustrated. They see the children as burden. Love in the Society in general is lost, long time ago. The types of “ Mothers “ and “ Women “ we have now …………

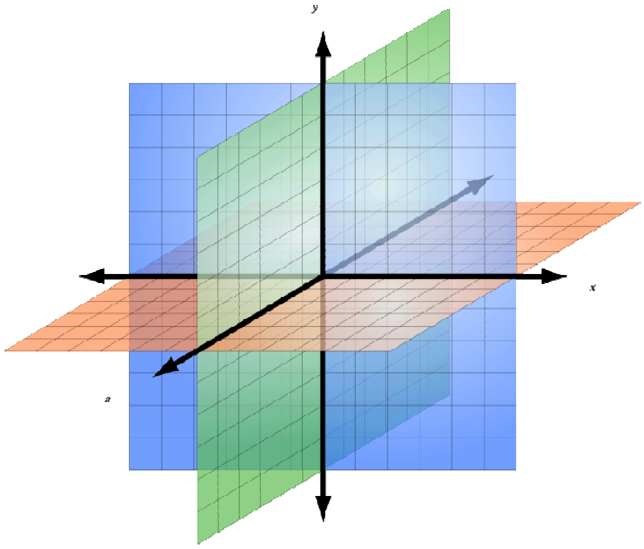
By now if you have assumed that Indian women are not doing any crime then please become friends with MRA Guri https://www.facebook.com/profile.php?id=100004138754180

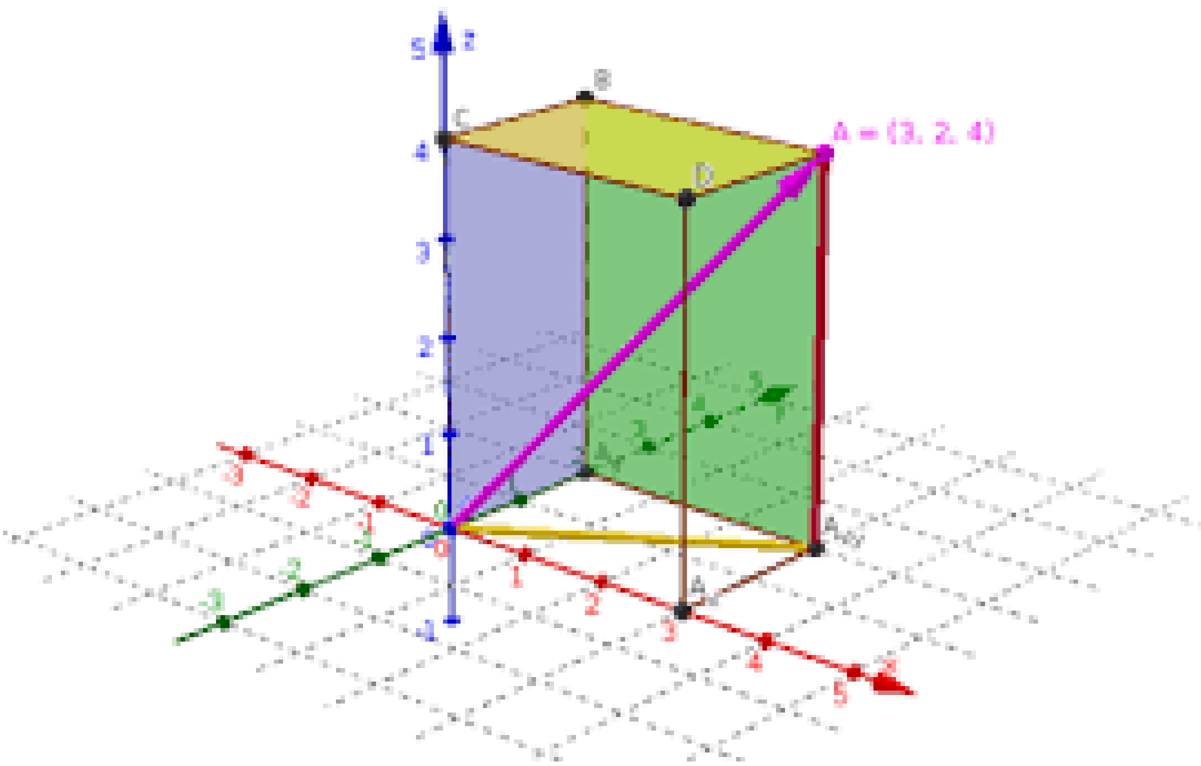
He has dedicated his life to expose Indian Criminals

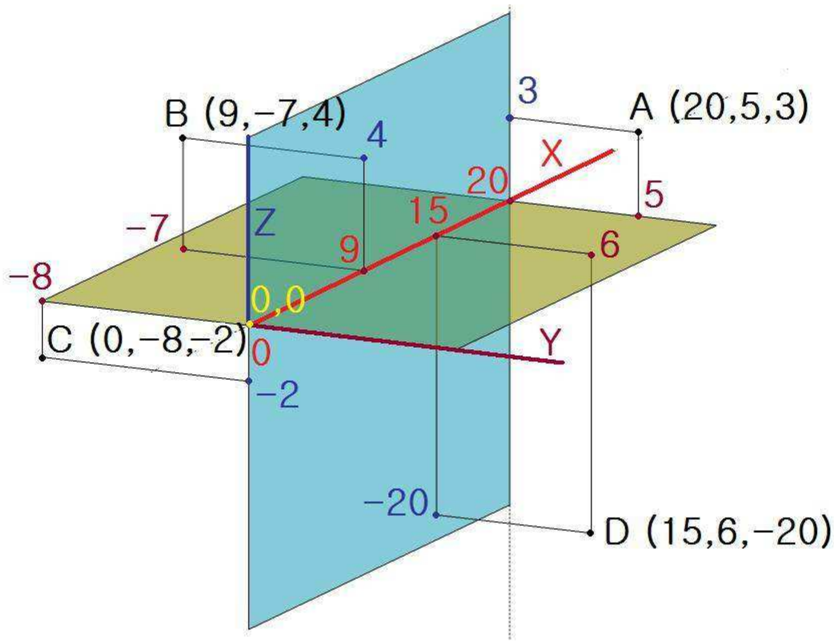
Professor Subhashish Chattopadhyay

Spoon Feeding Series - 3D Geometry









Distance Formula

Distance between $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Section Formulae

The coordinates of the point P which divides the join of $A\ (x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in the ratio $m : n$ are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

By taking $m = n$, we find the coordinates of the mid-point of AB as

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

Coordinates of any point on the join of two points $A\ (x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ are $\left(\frac{\lambda x_2 + x_1}{\lambda+1}, \frac{\lambda y_2 + y_1}{\lambda+1}, \frac{\lambda z_2 + z_1}{\lambda+1}\right)$

where $\lambda \neq -1$. If λ takes only positive real values, we get the coordinates of the points on the segment PQ (except P and Q) .

1. The coordinates of the point dividing the line joining $P\ (x_1, y_1, z_1)$ and $Q\ (x_2, y_2, z_2)$ in the ratio $m_1 : m_2$ internally are

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \frac{m_1z_2 + m_2z_1}{m_1 + m_2}\right).$$

2. The coordinates of the point dividing the line joining $P\ (x_1, y_1, z_1)$ and $Q\ (x_2, y_2, z_2)$ in the ratio $m_1 : m_2$ externally are

$$\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}, \frac{m_1z_2 - m_2z_1}{m_1 - m_2}\right).$$

3. The coordinates of the mid-point of the join of (x_1, y_1, z_1) and (x_2, y_2, z_2) are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right).$$

Vector forms

Internal Division: If the point R divides the join of PQ internally in the ratio of $m : n$, then position vector of $R(\vec{r})$ is

$$\vec{r} = \frac{m\vec{r_2} + n\vec{r_1}}{m+n}$$

External Division: If the point R divides the join of PQ externally in the ratio of $m : n$ i.e., internally in the ratio $m : (-n)$, then the position vector $R(\vec{r})$ is

$$\vec{r} = \frac{m\vec{r_2} - n\vec{r_1}}{m-n}$$

If is the point R is the mid point of the line joining PQ, then $m : n = 1 : 1$, therefore the position vector $R(\vec{r})$ is

$$\vec{r} = \frac{\vec{r_1} + \vec{r_2}}{2}$$

Coordinates of a general point

The co-ordinates of any point lying on the line joining points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2\ z_2)$ may be taken as $\left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}, \frac{kz_2 + z_1}{k+1}\right)$, which divides PQ in the ratio $k : 1$. This is called *general point* on the line PQ.

Centroid of Triangle

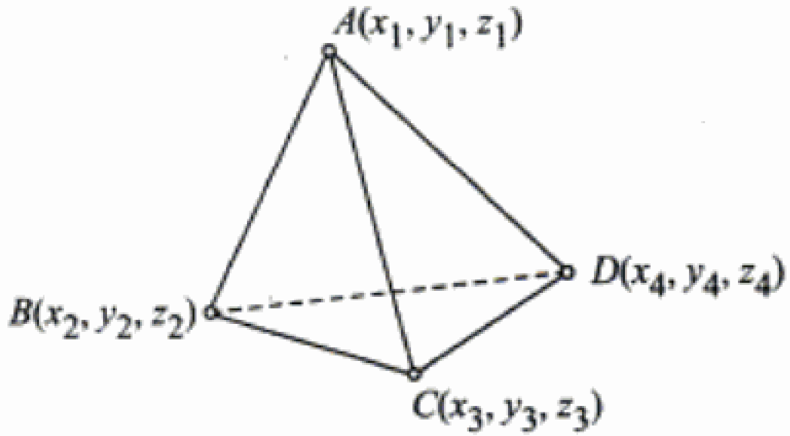
The coordinates of the centroid of the triangle ABC, whose vertices are $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$, are

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right).$$

Centroid of a Tetrahedron

If (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) and (x_4, y_4, z_4) are the vertices of a tetrahedron, then its centroid G is given by

$$\left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right)$$



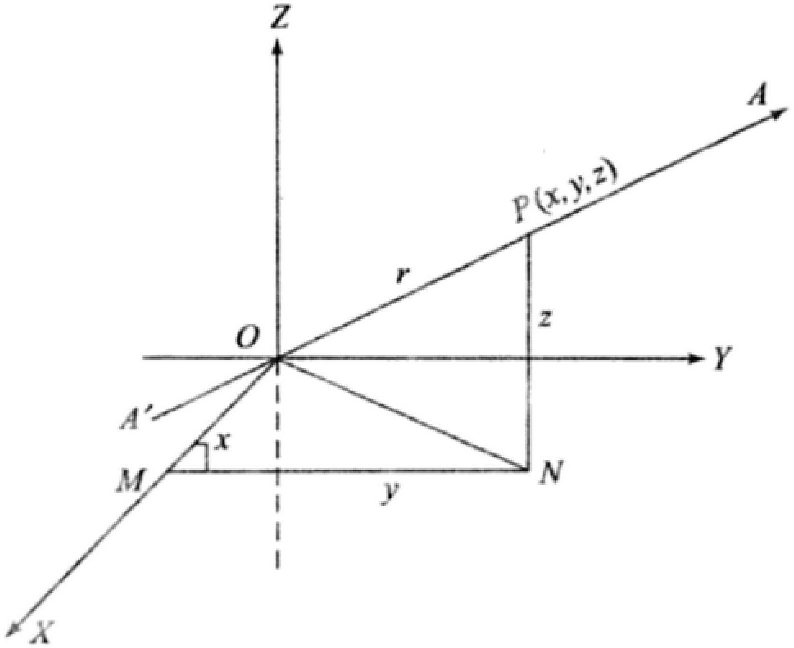
Volume of a tetrahedron

If $A_i(x_i, y_i, z_i)$, $i = 1,2,3,4$ are the veatices of a tetrahedron, its volume is equal to

$$\frac{1}{6}\left|\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}\right|$$

Direction-Cosines

If α, β, γ are the angles that a given directed line makes with the positive directions $X'OX$, $Y'OY$, $Z'OZ$ of the coordinate axes, then $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are called the *direction-cosines* of the line.



Let $P(x, y, z)$ be any point on $A'OA$, and let the measure of OP be r. Let PN be the perpendicular from P on the plane XOY, and NM be the perpendicular from N on OX. Then

$$x = r\cos\alpha,\ y = r\cos\beta,\ z = r\cos\gamma.$$

and $$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$$

Direction-Ratios

If l, m, n be the direction-cosines of a line OP and a, b, c be numbers proportional to l, m, n respectively, then a, b, c are called the direction ratios of OP. If l, m, n be the direction-cosines of a line and $k\,(\neq 0)$ be any number, then kl, km, kn are the direction-ratios of OP.

If a, b, c are the direction-ratios of a line, the actual direction-cosines $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are obtained from the relations

$$\frac{\cos\alpha}{a}=\frac{\cos\beta}{b}=\frac{\cos\gamma}{c}=\pm\frac{\sqrt{\cos^2\alpha+\cos^2\beta+\cos^2\gamma}}{\sqrt{a^2+b^2+c^2}}=\frac{\pm 1}{\sqrt{a^2+b^2+c^2}}$$

If P be the point (a, b, c), and $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are the direction cosines of the directed line OP, then

$$\cos\alpha=\frac{a}{\sqrt{(a^2+b^2+c^2)}},\ \cos\beta=\frac{b}{\sqrt{(a^2+b^2+c^2)}},$$

$$\cos\gamma=\frac{c}{\sqrt{(a^2+b^2+c^2)}}$$

The direction-cosines of PO are

$$-\frac{a}{\sqrt{(a^2+b^2+c^2)}},\ -\frac{b}{\sqrt{(a^2+b^2+c^2)}},\ -\frac{c}{\sqrt{(a^2+b^2+c^2)}}$$

Useful Results on Direction Cosines and Direction Ratios

If $P(x, y, z)$ is a point in space such that $\mathbf{r} = OP$ has direction cosines l, m, n, then

(a) $x = l\,|\mathbf{r}|,\ y = m\,|\mathbf{r}|,\ z = n\,|\mathbf{r}|$

(b) $l\,|\mathbf{r}|,\ m\,|\mathbf{r}|,\ n\,|\mathbf{r}|$ are projections of $\mathbf{r}$ on OX, OY, OZ respectively.

(c) $\mathbf{r} = |\mathbf{r}|\,(l\hat{i} + m\hat{j} + n\hat{k})$ and $\hat{r} = l\hat{i} + m\hat{j} + n\hat{k}$.

(d) $l^2 + m^2 + n^2 = 1$.

(e) If $\mathbf{r} = a\hat{i} + b\hat{j} + c\hat{k}$, then

(a) a, b, c are the direction ratios of $\mathbf{r}$.

(b) Direction cosines of $\mathbf{r}$ are given by

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}, m = \frac{b}{\sqrt{a^2 + b^2 + c^2}},$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

(f) Direction ratios of the line joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $x_2 - x_1, y_2 - y_1, z_2 - z_1$, and its direction cosines are

$$\frac{x_2 - x_1}{|\mathbf{PQ}|}, \frac{y_2 - y_1}{|\mathbf{PQ}|}, \frac{z_2 - z_1}{|\mathbf{PQ}|}$$

(g) The direction cosines of

(a) $\overrightarrow{OX}$ are (1, 0, 0)

(b) $\overrightarrow{OY}$ are (0, 1, 0)

(c) $\overrightarrow{OZ}$ are (0, 0, 1)

Angle Between Two Lines

(a) If θ is an angle between two lines whose direction-cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2) then

$$\cos\theta = l_1l_2 + m_1m_2 + n_1n_2.$$

The expressions for $\sin\theta$, $\tan\theta$ are given below:

$$\sin^2\theta = \begin{vmatrix} m_1 & n_1 \\ m_2 & n_2 \end{vmatrix}^2 + \begin{vmatrix} n_1 & l_1 \\ n_2 & l_2 \end{vmatrix}^2 + \begin{vmatrix} l_1 & m_1 \\ l_2 & m_2 \end{vmatrix}^2$$

and $$\tan\theta = \pm [\Sigma (l_1 m_2 - l_2 m_1)^2]^{1/2} / (l_1l_2 + m_1m_2 + n_1n_2)$$

The lines are parallel to each other if and only if

$$l_1/l_2 = m_1/m_2 = n_1/n_2.$$

Also the lines are perpendicular to each other provided

$$l_1l_2 + m_1m_2 + n_1n_2 = 0.$$

(b) Angle θ between two lines whose direction-ratios are a_1, b_1, c_1 and a_2, b_2, c_2 is given by

$$\cos\theta = \pm\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\sin\theta = \pm\frac{[(a_1b_2 - a_2b_1)^2 + (b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2]^{1/2}}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\tan\theta = \pm\frac{[(a_1b_2 - a_2b_1)^2 + (b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2]^{1/2}}{a_1a_2 + b_1b_2 + c_1c_2}$$

The lines are parallel to each other if and only if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Also, the lines are perpendicular to each other if and only if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0.$$

1. If $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$, then two vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ having direction cosines l_1, m_1, n_1 and l_2, m_2, n_2 are orthogonal.

2. If $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$ then two vectors are parallel.

3. Any vector equally inclined to all the three axes have direction cosines as

$$\left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right)$$

4. If any line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube, then

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$$

5. If l_1, m_1, n_1 and l_2, m_2, n_2 are the *d.c.*'s of two concurrent lines, then the *d.c.'s* of the lines bisecting the angles between them are proportional to $l_1 \pm l_2, m_1 \pm m_2, n_1 \pm n_2$.

6. The angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

7. The angle between a diagonal of a cube and the diagonal of a face of the cube is $\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$.

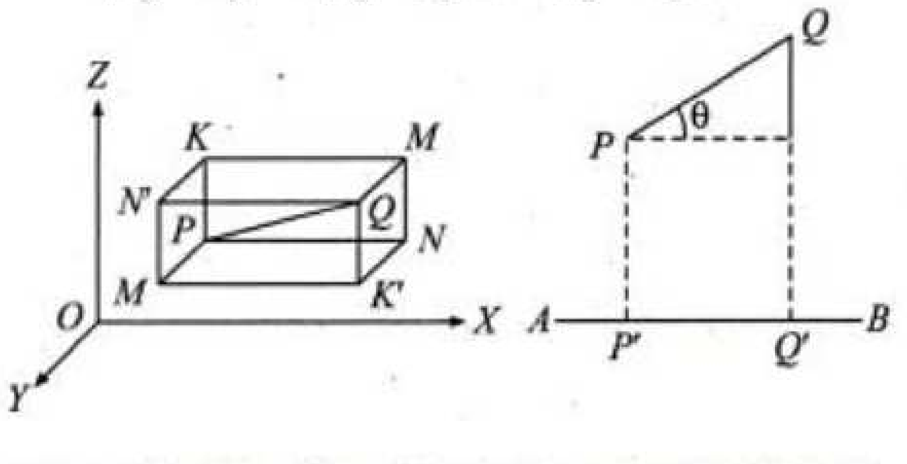
8. If the edges of a rectangular parallelopiped be a, b, c, then the angles between the two diagonals are $\cos^{-1}\left[\frac{\pm a^2 \pm b^2 \pm c^2}{a^2+b^2+c^2}\right]$.

Projection

Projection of a line joining the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ on another line whose direction cosines are l, m and n : Let PQ be a line segment where $P \equiv (x_1, y_1, z_1)$ and $Q \equiv (x_2, y_2, z_2)$ and AB be a given line with *d.c.*'s as l, m, n. If the line segment PQ makes angle θ with the line AB, then

Projection of PQ is $P'Q' = PQ\cos\theta$

$$= (x_2 - x_1)\cos\alpha + (y_2 - y_1)\cos\beta + (z_2 - z_1)\cos\gamma$$
$$= (x_2 - x_1)l + (y_2 - y_1)m + (z_2 - z_1)n.$$



To get the projection of vector $\vec{a}$ along the direction of $\vec{b}$ then take the dot product of $\vec{a}$ with the unit vector along $\vec{b}$

$\therefore$ Projection of $\vec{a}$ on $\vec{b}$ = $\vec{a}.\hat{b}$

If $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$ is any vector, then the projection of $\vec{r}$ on a line whose direction cosines are (l, m, n) is

$$|\vec{r}| \cos q = \vec{r}.(l\hat{i} + m\hat{j} + n\hat{k}) = al + bm + cn$$

where, $l\hat{i} + m\hat{j} + n\hat{k}$ is the unique unit vector along the line whose direction cosines are given

Straight Line

The vector equation of a straight line passing through a given point with position vector **a** and parallel to a given vector **b** is

$$\mathbf{r} = \mathbf{a} + \lambda\, \mathbf{b}$$

where λ is a scalar.

Cartesian Form The equation of a straight line with direction ratios a, b, c and passing through a fixed point (x_1, y_1, z_1) is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}.$$

The equation of a line whose direction cosines are l, m, n and which passes through the point (x_1, y_1, z_1) is

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{c}$$

The coordinates of any point on the line

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \text{ are given by}$$

$(x_1 + a\lambda, y_1 + b\lambda, z_1 + c\lambda)$, where λ is a real number.

Equation of x-axis:

$$\frac{x - 0}{1} = \frac{y - 0}{0} = \frac{z - 0}{0} \text{ or } y = 0, z = 0$$

Equation of y-axis:

$$\frac{x - 0}{0} = \frac{y - 0}{1} = \frac{z - 0}{0} \text{ or } x = 0, z = 0$$

Equation of z-axis:

$$\frac{x - 0}{0} = \frac{y - 0}{0} = \frac{z - 0}{1} \text{ or } x = 0, y = 0.$$

Vector Equation of a line passing through Two points

The vector equation of a line passing through two points with position vectors **a** and **b** is

$$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$$

Cartesian Form The equation of a line passing through two given points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

CHANGING UNSYMMETRICAL FORM TO SYMMETRICAL FORM

The unsymmetrical form of a line $ax + by + cz + d = 0$, $a'x + b'y + c'z + d' = 0$ can be changed to symmetrical form as follows:

$$\frac{x-\dfrac{bd'-b'd}{ab'-a'b}}{bc'-b'c}=\frac{y-\dfrac{da'-d'a}{ab'-a'b}}{ca'-c'a}=\frac{z}{ab'-a'b}$$

Angle between Two lines

Vector Form The angle between the two lines

$$\mathbf{r} = \mathbf{a}_1 + \lambda\,\mathbf{b}_1 \text{ and } \mathbf{r} = \mathbf{a}_2 + \mu\mathbf{b}_2$$

is given by $\cos\theta = \dfrac{\mathbf{b}_1\cdot\mathbf{b}_2}{|\mathbf{b}_1||\mathbf{b}_2|}$

Cartesian Form The angle between the two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$$

and $$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by $\cos\theta = \dfrac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\,\sqrt{a_2^2+b_2^2+c_2^2}}$.

Intersection of Two lines

Let the two lines be $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ ...(i)

and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ ...(ii)

Step I: Write the coordinates of general points on (i) and (ii). The coordinates of general points on (i) and (ii) are given by

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} = \lambda$$

and $$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{z_2} = \mu$$

respectively. i.e., $(a_1\lambda + x_1, b_1\lambda + y_1 + c_1\lambda + z_1)$ and $(a_2\mu + x_2, b_2\mu + y_2, c_2\mu + z_2)$.

Step II: If the lines (i) and (ii) intersect, then they have a common point.

$$a_1\lambda + x_1 = a_2\mu + x_2, b_1\lambda + y_1 = b_2\mu + y_2$$

and $c_1\lambda + z_1 = c_2\mu + z_2$.

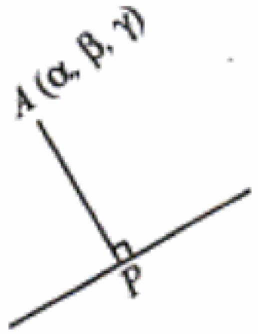
Step III: Solve any two of the equations in λ and μ obtained in step II. If the values of λ and μ satisfy the third equation, then the lines (i) and (ii) intersect, otherwise they do not intersect.

Step IV: To obtain the coordinates of the point of inter-section, substitute the value of λ (or μ) in the coordinates of general point (s) obtained in step 1.

Perpendicular from a point to a line

Let the equation of the line be

$$\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n} = r \text{ (say)}$$



and $A(\alpha, \beta, \gamma)$ be the given point. Then,

1. the coordinates of the foot of the perpendicular from A on the given line are

$$P(lr + a, mr + b, nr + c)$$

2. length of perpendicular (AP) is

$$\sqrt{(lr + a - \alpha)^2 + (mr + b - \beta)^2 + (nr + c - \gamma)^2}$$

3. equation of the perpendicular is given by

$$\frac{x-\alpha}{lr+a-\alpha} = \frac{y-\beta}{mr+b-\beta} = \frac{z-\gamma}{nr+c-\gamma}$$

where $r = (\alpha - a)\,l + (\beta - b)\,m + (\gamma - c)\,n$.

Vector Form

Length of the perpendicular from a point $A(\mathbf{r}_1)$ upon the line $\mathbf{r} = \mathbf{a} + \lambda\,\mathbf{b}$ is given by

$$= \frac{|(\mathbf{a} - \mathbf{r}_1) \times \mathbf{b}|}{|\mathbf{b}|}.$$

Alternate method

Find the foot of the perpendicular from the point (1, 6, 3) to line.

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}.$$

Also, find the length of the perpendicular and the equation of the perpendicular.

Any point on the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ can be taken as $(\lambda, 1 + 2\lambda, 2 + 3\lambda)$.

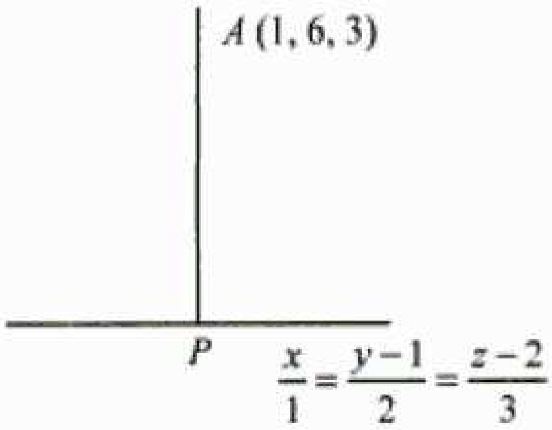
Let this point be P, the foot of perpendicular from $A(1, 6, 3)$ to the line is

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}.$$

Direction ratios of the given line are 1, 2, 3. Direction ratios of AP are

$$\lambda - 1,\ 1 + 2\lambda - 6,\ 2 + 3\lambda - 3$$

i.e., $\lambda - 1, 2\lambda - 5, 3\lambda - 1$



Since, AD is perpendicular to the given line

$\therefore \quad 1(\lambda - 1) + 2(2\lambda - 5) + 3(3\lambda - 1) = 0$

$\Rightarrow \quad \lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0$

$\Rightarrow \quad 14\lambda - 14 = 0 \Rightarrow \lambda = 1$

Thus, coordinates of P are $(1, 1 + 2, 2 + 3)$, i.e., $(1, 3, 5)$

$\therefore$ Foot of perpendicular is $(1, 3, 5)$.

Length of perpendicular is

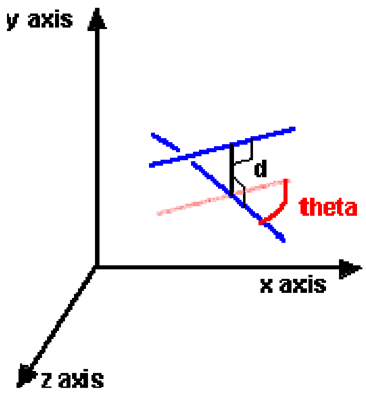
$$AP = \sqrt{(1-1)^2 + (3-6)^2 + (5-3)^2}$$
$$= \sqrt{0 + 9 + 4} = \sqrt{13}$$

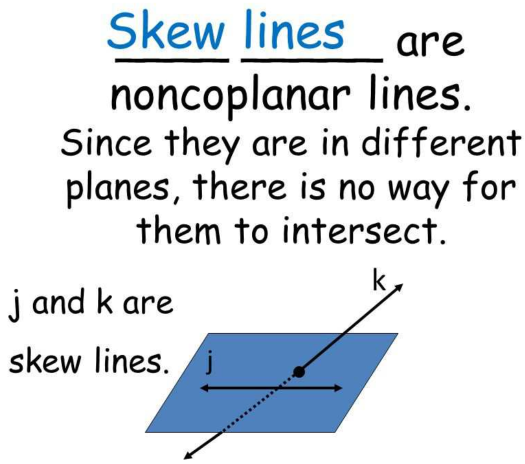
Equations of perpendicular, i.e., equations of AP are

$$\frac{x-1}{1-1} = \frac{y-6}{3-6} = \frac{z-3}{5-3}$$

i.e., $$\frac{x-1}{0} = \frac{y-6}{-3} = \frac{z-3}{2}$$

Skew Lines





If l_1 and l_2 are two skew lines, then the straight line which is perpendicular to each of these two non-intersecting lines is called the "*LIne of shortest distance.*"

The Plane

Equation of a Plane

A plane is represented by an equation of the first degree i.e. by $ax + by + cz + d = 0$. Conversely, every equation of the first degree in x, y, z represents a plane.

(a) Equation of the yz-plane is $x = 0$.

(b) Equation of the zx-plane is $y = 0$.

(c) Equation of the xy-plane is $z = 0$.

(d) If l, m, n be the direction cosines of the normal to a plane and p be the length of the perpendicular from the origin on the plane, then an equation of the plane is $lx + my + nz = p$.

(e) If a plane makes intercepts a, b, c on the axes of coordinates, its equation is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

Systems of Planes

An equation of a plane contains three independent constants, as such a plane is uniquely determined by three independent conditions. If we consider a plane satisfying just two given conditions, its equation will contain one arbitrary constant. If we consider a plane satisfying one given condition, its equation will contain two arbitrary constants.

We give below the equations of some well-known systems of planes:

(i) The equation $ax + by + cz + k = 0$ represents a system of planes parallel to the plane $ax + by + cz + d = 0$, k being a parameter.

(ii) The equation $ax + by + cz + k = 0$,

represents a system of planes perpendicular to the line $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$.

(iii) The equation $(a_1x + b_1y + c_1z + d_1) + k(a_2x + b_2y + c_2z + d_2) = 0$ represents a system of planes passing through the intersection of the planes

$$a_1x + b_1y + c_1z + d_1 = 0 \text{ and } a_2x + b_2y + c_2z + d_2 = 0,$$

k being a parameter.

(iv) The equation $A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$.
represents a system of planes passing through the point (x_1, y_1, z_1), the ratios of A, B, C being the two parameters.

It may be noted that (i)–(iii) above are examples of one-parameter family of planes and (iv) is an example of a two-parameter family of planes.

Equation of Line through two Given Points

If $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ be two given points, an equation of the line AB is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

The coordinates of a variable point on AB can be expressed in terms of a parameter λ in the form

$$x = \frac{\lambda x_2 + x_1}{\lambda + 1}, y = \frac{\lambda y_2 + y_1}{\lambda + 1}, z = \frac{\lambda z_2 + z_1}{\lambda + 1},$$

λ being any real number different from -1. In fact, (x, y, z) are the coordinates of the point which divides the join of A and B in the ratio $\lambda : 1$.

Changing unsymmetrical form to symmetrical form

The unsymmetrical form of a line

$$ax + by + cz + d = 0,\ a'x + b'y + c'z + d' = 0.$$

can be changed to symmetrical form as follows:

$$\frac{x - \dfrac{bd' - b'd}{ab' - a'b}}{bc' - b'c} = \frac{y - \dfrac{da' - d'a}{ab' - a'b}}{ca' - c'a} = \frac{z}{ab' - a'b}$$

Number of Constants in the Equation of a Line

The equation $\dfrac{x - a}{l} = \dfrac{y - b}{m} = \dfrac{z - c}{n}$,

of a line can be written as $x = (l/n)\,z + a - (lc/n),\ y = (m/n)\,z + b - (mc/n)$,
which are of the form $x = Az + B,\ y = Cz + D$

Therefore the *equation of a line contains four arbitrary constants.*

A Plane and a Straight Line

Let the equations

$$ax + by + cz + d = 0,\ \frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n},$$

represent a given plane and a straight line respectively.

(i) The line is perpendicular to the plane if and only if

$$\frac{a}{l} = \frac{b}{m} = \frac{c}{n},$$

(ii) The line is parallel to the plane if and only if $al + bm + cn = 0$.

(iii) The line lies in the plane if and only if

$$al + bm + cn = 0 \text{ and } a\alpha + b\beta + c\gamma + d = 0.$$

Angle between a Line and a Plane

The angle θ between the line

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n},$$

and the plane $ax + by + cz + d = 0$, is given by

$$\sin\theta = \frac{al + bm + cn}{\sqrt{(a^2+b^2+c^2)}\ \sqrt{(l^2+m^2+n^2)}}$$

Coplanar Lines

The lines $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$, and $\frac{x-\alpha'}{l'} = \frac{y-\beta'}{m'} = \frac{z-\gamma'}{n'}$ are coplanar if and only if

$$\begin{vmatrix} \alpha-\alpha' & \beta-\beta' & \gamma-\gamma' \\ l & m & n \\ l' & m' & n' \end{vmatrix} = 0$$

In case the above condition is satisfied, the equation of the plane containing the lines is

$$\begin{vmatrix} x-\alpha & y-\beta & z-\gamma \\ l & m & n \\ l' & m' & n' \end{vmatrix} = 0.$$

General Equation of a Plane Containing a Line

The general equation of the plane containing the line

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n},$$

is $$A(x-\alpha) + B(y-\beta) + C(z-\gamma) = 0,$$

where $$Al + Bm + Cn = 0.$$

Length of the Perpendicular from a Point to a Line

The length p of the perpendicular from a given point $P(x_1, y_1, z_1)$ to a given line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ (l, m, n are direction cosines of the line), is given by

$$p^2 = (x_1-\alpha)^2 + (y_1-\beta)^2 + (z_1-\gamma)^2 - [l(x_1-\alpha) + m(y_1-\beta) + n(z_1-\gamma)]^2.$$

Vectorial Equations

1. Parametric vectorial equation of the line through a point with position vector **a** and parallel to the vector **b** is

$$\mathbf{r} = \mathbf{a} + t\mathbf{b}$$

where t, is a scalar parameter.

2. Parametric vectorial equation of the line through two points with position vectors **a** and **b** is

$$\mathbf{r} = (1 - t)\mathbf{a} + t\mathbf{b}$$

where t, is a scalar parameter.

3. Parametric vectorial equation of a plane which passes through the point with position vector **a** and which is parallel to the vectors **b** and **c** is $\mathbf{r} = \mathbf{a} + t\mathbf{b} + p\mathbf{c}$

where, t and p are scalar parameters.

4. Parametric vectorial equation of a plane passing through two given points with position vectors **a** and **b** and parallel to the vectors **c** is

$$\mathbf{r} = \mathbf{a} + t(\mathbf{b} - \mathbf{a}) + p\mathbf{c} = (1 - t)\,\mathbf{a} + t\mathbf{b} + p\mathbf{c}$$

where t and p are scalar parameters.

5. Equation of the plane passing through three points with position vectors **a, b, c** is $\mathbf{r}.(\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}+\mathbf{a}\times\mathbf{b})=[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$.
6. *Normal form of the vector equation of a plane* $\mathbf{r}\cdot\mathbf{n}=p$ is vector equation of a plane, such that **n** is the unit vector normal to the plane and p is the length of the perpendicular from the origin to the plane.
7. $(\mathbf{r}-\mathbf{a})\cdot\mathbf{n}=0$ is vector equation of a plane normal to the vector **n** and passing through a point with position vector **a**.
8. *Angle between two planes* $\mathbf{r}\cdot\mathbf{n_1}=p_1$ and $\mathbf{r}\cdot\mathbf{n_2}=p_2$ is $\cos^{-1}\dfrac{\mathbf{n_1}\cdot\mathbf{n_2}}{|\mathbf{n_1}||\mathbf{n_2}|}$
9. *Angle between a line and a plane* whose vectorial equations are $\mathbf{r}=\mathbf{a}+\mathbf{b}t$ and $\mathbf{r}\cdot\mathbf{n}=q$ respectively is $\sin^{-1}\dfrac{\mathbf{n}\cdot\mathbf{b}}{|\mathbf{n}|\,|\mathbf{b}|}$.
10. *The perpendicular distance* of a point with position vector **a** from the plane $\mathbf{r}\cdot\mathbf{n}=q$ is $\dfrac{|q-\mathbf{a}\cdot\mathbf{n}|}{|\mathbf{n}|}$.
11. Equation of the plane containing two coplanar lines $\mathbf{r}=\mathbf{a}+t\mathbf{b}$ and $\mathbf{r}=\mathbf{c}+p\mathbf{d}$ is $(\mathbf{r}-\mathbf{a})\cdot\mathbf{b}\times\mathbf{d}=0\Rightarrow\quad\mathbf{r}\cdot\mathbf{b}\times\mathbf{d}=\mathbf{a}\cdot\mathbf{b}\times\mathbf{d}$.
12. Condition for the lines $\mathbf{r}=\mathbf{a}+t\mathbf{b}$ and $\mathbf{r}=\mathbf{c}+p\mathbf{d}$ to be *coplanar* is $[\mathbf{cbd}]=[\mathbf{abd}]$.
13. The line *LM* of shortest distance between the lines $\mathbf{r}=\mathbf{a}+t\mathbf{b}$ and $\mathbf{r}=\mathbf{c}+p\mathbf{d}$ is parallel to the vector $\mathbf{b}\times\mathbf{d}$.

Equation of a Plane through Two Given Points and Parallel to a Given Vector

Vector Form The equation of a plane through two given points having position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ and parallel to a given vector $\mathbf{m}$ is

$$(\mathbf{r} - \mathbf{r}_1) \cdot [(\mathbf{r}_2 - \mathbf{r}_1) \times \mathbf{m}] = 0$$

Cartesian Form The equation of a plane passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) and parallel to a line having direction ratios a, b, c is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x - x_2 & y - y_2 & z - z_2 \\ a & b & c \end{vmatrix} = 0.$$

Equation of a Plane Passing through a Given Point and Parallel to Two Given Vectors

Vectors Form The equation of a plane passing through a point having position vector $\mathbf{a}$ and parallel to two given vectors $\mathbf{b}$ and $\mathbf{c}$ is

$\mathbf{r} = \mathbf{a} + \lambda \mathbf{b} + \mu \mathbf{c}$, where λ and μ are scalars

or $(\mathbf{r} - \mathbf{a}) \cdot (\mathbf{b} \times \mathbf{c}) = 0$ or $\mathbf{r} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$.

Cartesian Form The equation of a plane passing through a point (x_1, y_1, z_1) and parallel to two lines having direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0.$$

Planes Parallel to a Given Plane

Cartesian Form Equation of a plane parallel to the plane $ax + by + cz + d = 0$ is $ax + by + cz + k = 0$, where k is a constant to be detemined by the given condition.

Vector Form The equation of a plane parallel to the plane $\mathbf{r} . \mathbf{n} = d_1$ is $\mathbf{r} . \mathbf{n} = d_2$, where d_2 is a constant to be determined by the given condition.

Angle between Two Planes

Angle between two planes is the angle between their normals.

Vector Form The angle between the planes $\mathbf{r} . \mathbf{n}_1 = d_1$ and $\mathbf{r} \cdot \mathbf{n}_2 = d_2$ is given by

$$\cos \theta = \frac{\mathbf{n}_1 \cdot \mathbf{n}_1}{|\mathbf{n}_1||\mathbf{n}_2|}.$$

Cartesian Form The angle between the planes

$$a_1x + b_1y + c_1z + d_1 = 0$$

and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos \theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

If $a_1a_2 + b_1b_2 + c_1c_2 = 0$, then the planes are perpendicular to each other.

- If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the planes are parallel to each other.

Angle between a Line and a Plane

The angle between a line and a plane is the angle between the line and the normal to the plane.

Vector Form If θ is the angle between the line $\mathbf{r} = \mathbf{a} + \lambda\,\mathbf{b}$ and the plane $\mathbf{r}\,.\,\mathbf{n} = d$, then

$$\sin\theta = \frac{\mathbf{b}\cdot\mathbf{n}}{|\mathbf{b}|\,|\mathbf{n}|}.$$

Cartesian Form If θ is the angle between the line

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$$ and the plane

$a_2x + b_2y + c_2z + d = 0$, then

$$\sin\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\,\sqrt{a_2^2 + b_2^2 + c_2^2}}.$$

If the line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ is parallel to the plane $a_2x + b_2y + c_2z + d = 0$, then $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

POINT OF INTERSECTION OF A LINE AND A PLANE

Working rule for finding the point of intersection of a line and a plane:

Step I: Write the coordinates of any point on the line in terms of some parameter r (say).

Step II: Substitute these coordinates in the equation of the plane to obtain the value of r.

Step III: Put the value of r in the coordinates of the point in step I.

The ratio in which the line segment PQ, joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$. is divided by plane

$$ax + by + cz + d = 0 \text{ is, } -\left(\frac{ax_1 + by_1 + cz_1 + d}{ax_2 + by_2 + cz_2 + d}\right)$$

Planes Bisecting the Angles between Two Planes

Cartesian Form The equations of the planes bisecting the angles between the planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are

$$\frac{(a_1x + b_1y + c_1z + d_1)}{\sqrt{a_1^2 + b_1^2 + c_1^2}} = \frac{(a_2x + b_2y + c_2z + d_2)}{\sqrt{a_2^2 + b_2^2 + c_2^2}} \quad ...(1)$$

Vector Form The equations of the planes bisecting the angles between the planes $\mathbf{r} \cdot \mathbf{n}_1 = d_1$ and $\mathbf{r} \cdot \mathbf{n}_2 = d_2$ are

$$\frac{|\mathbf{r}.\mathbf{n}_1 - \mathbf{d}_1|}{|\mathbf{n}_1|} = \frac{|\mathbf{r}.\mathbf{n}_2 - \mathbf{d}_2|}{|\mathbf{n}_2|}$$

or $$\mathbf{r}\cdot(\hat{\mathbf{n}}_1 \pm \hat{\mathbf{n}}_2) = \frac{d_1}{|\mathbf{n}_1|} \pm \frac{d_2}{|\mathbf{n}_2|}$$

Bisector of the Angle Containing the Origin After making the constant term in both the equations positive, the positive sign in (1) gives the bisector of the angle which contains the origin.

Bisector of Acute/Obtuse Angle

(a) Write the equations of the given planes such that their constant terms are positive.

(b) If $a_1a_2 + b_1b_2 + c_1c_2 > 0$, then origin lies in obtuse angle and hence positive sign in (1) gives the bisector of the obtuse angle.

(c) If $a_1a_2 + b_1b_2 + c_1c_2 < 0$, then origin lies in acute angle and hence positive sign in (1) gives the bisector of the acute angle.

Distance of a Point from a Plane

Vector Form The length of the perpendicular from a point having position vector **a** to the plane $\mathbf{r}\,.\,\mathbf{n} = d$ is given by

$$p = \frac{|\mathbf{a}.\mathbf{n} - d|}{|\mathbf{n}|}.$$

Cartesian Form The length of the perpendicular from a point $P(x_1, y_1, z_1)$ to the plane $ax + by + cz + d = 0$ is given by

$$p = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}.$$

Distance between Two Parallel Planes

Vector Form The distance between two parallel planes $\mathbf{r} \cdot \mathbf{n} = d_1$ and $\mathbf{r} \cdot \mathbf{n} = d_2$ is given by

$$p = \frac{|d_1 - d_2|}{|\mathbf{n}|}.$$

Cartesian Form The distance between two parallel planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_1x + b_1y + c_1z + d_2 = 0$ is, given by

$$p = \frac{|d_1 - d_2|}{\sqrt{a_1^2 + b_1^1 + c_1^2}}.$$

Planes Passing through the Intersection of Two Planes

Vector Form The equation of a plane passing through the intersection of the planes $\mathbf{r} \cdot \mathbf{n}_1 = d_1$ and $\mathbf{r} \cdot \mathbf{n}_2 = d_2$ is

$$(\mathbf{r} \cdot \mathbf{n}_1 - d_1) + k\,(\mathbf{r} \cdot \mathbf{n}_2 - d_2) = 0$$

or $\quad \mathbf{r} \cdot (\mathbf{n}_1 + k\mathbf{n}_2) = d_1 + kd_2,$

where k is an arbitrary constant.

Cartesian Form The equation of a plane passing through the intersection of planes

$a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is $(a_1x + b_1y + c_1z + d_1) + k\,(a_2x + b_2y + c_2z + d_2) = 0$, where k is an arbitrary constant.

Two Sides of a Plane

The two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ lie on the same side or the opposite sides of the plane $ax + by + cz + d = 0$ according as $ax_1 + by_1 + cz_1 + d$ and $ax_2 + by_2 + cz_2 + d$ have the same sign or the opposite signs.

Condition for a Line to Lie in a Plane

Vector Form If the line $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$ lines in the plane $\mathbf{r} \cdot \mathbf{n} = d$, then

$$\mathbf{b} \cdot \mathbf{n} = 0 \quad \text{and} \quad \mathbf{a} \cdot \mathbf{n} = d.$$

Cartesian Form If the line $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ lies in the plane $ax + by + cz + d = 0$, then

(a) $ax_1 + by_1 + cz_1 + d = 0$ and

(b) $al + bm + cn = 0$.

Condition for the Two Lines to be Intersecting (Coplanar) and the Equation of the Plane Containing Them

Vector Form If the lines $\mathbf{r} = \mathbf{a}_1 + \lambda\mathbf{b}_1$ and $\mathbf{r} = \mathbf{a}_2 + \mu\mathbf{b}_2$ are intersecting (coplanar), then

$$[\mathbf{a}_1\ \mathbf{b}_1\ \mathbf{b}_2] = [\mathbf{a}_2\ \mathbf{b}_1\ \mathbf{b}_2]$$

and the equation of the plane containing the two lines is

$$[\mathbf{r}\ \mathbf{b}_1\ \mathbf{b}_2] = [\mathbf{a}_1\ \mathbf{b}_1\ \mathbf{b}_2]$$

or $$[\mathbf{r}\ \mathbf{b}_1\ \mathbf{b}_2] = [\mathbf{a}_2\ \mathbf{b}_1\ \mathbf{b}_2]$$

Cartesian Form If the lines $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$ are intersecting (coplanar)

then $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$

and the equation of the plane containing the two lines is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

or $\begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$

IMAGE OF A POINT IN A PLANE

Let P and Q be two points and let π be a plane such that

(i) Line PQ is perpendicular to the plane π, and

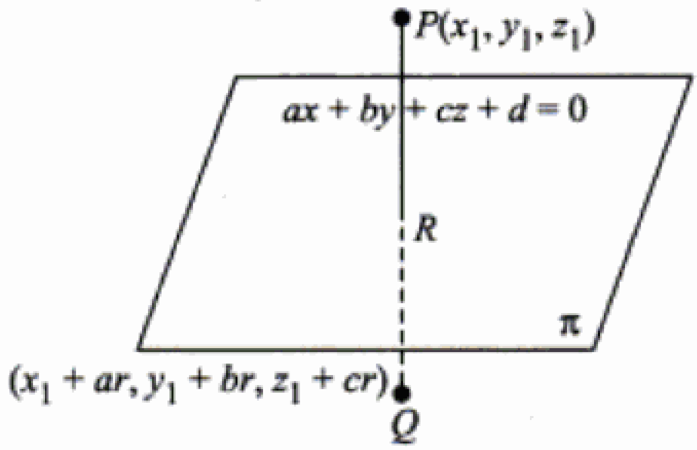
(ii) Mid-point of PQ lies on the plane π.

Then either of the point is the image of the other in the plane π.

To find the image of a point in a given plane, we proceed as follows

(i) Write the equations of the line passing through P and normal to the given plane as

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}.$$



(ii) Write the co-ofdinates of image Q as

$$(x_1 + ar, y_1 + br, z_1 + cr).$$

(iii) Find the co-ordinates of the mid-point R of PQ.

(iv) Obtain the value of r by putting the coordinates of R in the equation of the plane.

(v) Put the value of r in the coordinates of Q.

IMAGE OF A LINE ABOUT A PLANE

Let the line be $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and the plane be $a_2 x + b_2 y + c_2 z + d = 0$

Find point of intersection (say P) of the line and the plane. Find image (say Q) of point (x_1, y_1, z_1) about the plane. Line PQ is the reflected line.

Sphere

A sphere is the locus of a point which remains at a constant distance from a fixed point. The constant distance is called the **radius** and the fixed point is called the **centre** of the sphere.

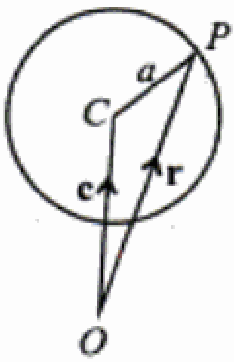
EQUATION OF A SPHERE

Vector Equation The vector equation of a sphere of radius a and centre having position vector $\mathbf{c}$ is $|\mathbf{r} - \mathbf{c}| = a$

The vector equation of a sphere of radius a with centre at the origin, is $|\mathbf{r}| = a$.

Cartesian Equation The equation of a sphere with centre (a, b, c) and radius k is given by

$$(x - a)^2 + (y - b)^2 + (z - c)^2 = k^2$$



The equation of a sphere with centre at origin and radius k is

$$x^2 + y^2 + z^2 = k^2.$$

General Equation of a Sphere

The general equation

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$

represents a sphere with centre $(-u, -v, -w)$ and radius equal to $\sqrt{u^2 + v^2 + w^2 - d}$.

Equation of a Sphere through Four Points

Equation of a sphere passing through four non-coplanar points (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) and (x_4, y_4, z_4) is

$$\begin{vmatrix} x^2+y^2+z^2 & x & y & z & 1 \\ x_1^2+y_1^2+z_1^2 & x_1 & y_1 & z_1 & 1 \\ x_2^2+y_2^2+z_2^2 & x_2 & y_2 & z_2 & 1 \\ x_3^2+y_3^2+z_3^2 & x_3 & y_3 & z_3 & 1 \\ x_4^2+y_4^2+z_4^2 & x_4 & y_4 & z_4 & 1 \end{vmatrix} = 0$$

or

(a) Assume the equation of the sphere to be

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0 \quad ...(1)$$

(b) Put the coordinates of four given points in Eqn. (1) to obtain four equations in u, v, w and d.

(c) Solve the four equations obtained in Step (b) to get the values of u, v, w, and d.

(d) Put the values of u, v, w and d obtained in Step (c) in Eqn. (1) to obtain the required equation of sphere.

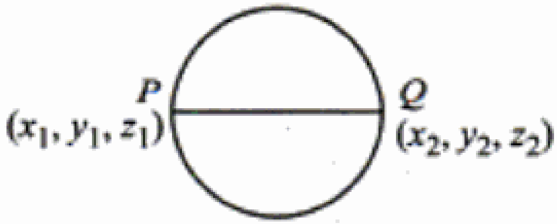
Equation of a Sphere, the Extremities of Diameter Being given

Cartesian Form The equation of a sphere described on the join of two points

$$P(x_1, y_1, z_1) \text{ and } Q(x_2, y_2, z_2)$$

as diameter is given by

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + (z - z_1)(z - z_2) = 0.$$



Vector Form The vector equation of a sphere, described on the join of two points P and Q, having position vectors **a** and **b**, as diameter, is given by

$$(\mathbf{r} - \mathbf{a}) . (\mathbf{r} - \mathbf{b}) = 0.$$

or $\quad |\mathbf{r}^2| - \mathbf{r}. (\mathbf{a} - \mathbf{b}) + \mathbf{a} . \mathbf{b} = 0$

or $\quad |\mathbf{r} - \mathbf{a}|^2 + |\mathbf{r} - \mathbf{b}|^2 = |\mathbf{a} - \mathbf{b}|^2$

Condition of Tangency

Vector Form Condition for the plane $\mathbf{r} . \mathbf{n} = d$ to touch the sphere $|\mathbf{r} - \mathbf{c}| = a$ is

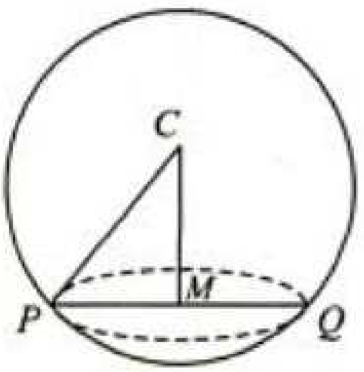
$$\frac{|\mathbf{c}.\mathbf{n} - d|}{|\mathbf{n}|} = a.$$

Cartesian Form Condition for the plane $lx + my + nz = p$ to touch the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ is $(ul + vm + wn + p)^2 = (l^2 + m^2 + n^2)(u^2 + v^2 + w^2 - d)$.

SECTION OF A SPHERE BY A PLANE

Consider a sphere intersected by a plane. The set of points common to both sphere and plane is called a *plane section* of a sphere. The plane section of a sphere is always a circle. The equations of the sphere and the plane taken together represent the plane section.

Let C be the centre of the sphere and M be the foot of the perpendicular from C on the plane. Then M is the centre of the circle and radius of the circle is given by $PM = \sqrt{CP^2 - CM^2}$.



The centre M of the circle is the point of intersection of the plane and line CM which passes through C and is perpendicular to the given plane.

Centre: The foot of the perpendicular from the centre of the sphere to the plane is the centre of the circle.

(Radius of circle)^2 = (Radius of sphere)^2 – (Perpendicular from centre of sphere on the plane)^2

Great Circle: The section of a sphere by a plane through the centre of the sphere is a great circle. Its centre and radius are the same as those of the given sphere.

Question

The direction cosines of the line joining the points (1, 2, – 3) and (– 2, 3, 1) are.

(a) – 3, 1, 4 (b) – 1, 5, – 2

(c) $\frac{-3}{\sqrt{26}}, \frac{1}{\sqrt{26}}, \frac{4}{\sqrt{26}}$ (d) $\frac{-1}{\sqrt{30}}, \frac{5}{\sqrt{30}}, \frac{-2}{\sqrt{30}}$

Ans. (c)

Solution The direction cosines of the given line are proportional to – 2 – 1, 3 – 2, 1 – (– 3) i.e. – 3, 1, 4.

Therefore, the actual direction cosines are

$$\frac{-3}{\sqrt{9+1+16}}, \frac{1}{\sqrt{9+1+16}}, \frac{4}{\sqrt{9+1+16}} \text{ i.e } \frac{-3}{\sqrt{26}}, \frac{1}{\sqrt{26}}, \frac{4}{\sqrt{26}}.$$

Question

A point is on the x-axis. What are its y-coordinates and z-coordinates?

Answer

If a point is on the x-axis, then its y-coordinates and z-coordinates are zero.

Question

There are three points A, B, C on axes at distances *a*, *b*, *c* respectively, then the coordinates of point which is equidistance from A, B, C and O, is

(a) (a, b, c) (b) $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$

(c) $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ (d) $\left(\frac{a}{4}, \frac{b}{4}, \frac{c}{4}\right)$.

Solution

Ans. (b)

Let point $P(x, y, z)$, be equidistance from A, B, C, O

$\therefore \quad PA = PO \Rightarrow PA^2 = PO^2$

$\Rightarrow \quad (x - a)^2 + y^2 + z^2 = x^2 + y^2 + z^2$

$\Rightarrow \quad -2ax + a^2 = 0 \Rightarrow x = a/2$

similarly $\quad y = b/2$, $z = c/2$

$\therefore$ Reqd. point is $P\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$.

Question

An equation of z-axis is

(a) $z = 0, x = 0$ (b) $z = 0, y = 0$
(c) $x = 0, y = 0$ (d) $x = k, y = -k.$, $(k \neq 0)$

Ans. (c)

Question

The ratio in which yz plane cuts the line joining the point $(-2, 4, 7)$ and $(3, -5, 8)$ is

(a) 2 : 3 (b) 3 : 2
(c) –2 : 3 (d) 4 : – 3.

Solution

Ans. (a)

On yz plane x coordinate of every point is 0. let this ratio be $\lambda : 1$

$$0 = \frac{3\lambda + 1 \times -2}{\lambda + 1}$$

$$\Rightarrow \quad 3\lambda - 2 = 0 \quad \Rightarrow \lambda = \frac{2}{3}$$

$\therefore$ It cuts in the ratio 2 : 3.

Question

A point is in the XZ-plane. What can you say about its y-coordinate?

Answer

If a point is in the XZ plane, then its y-coordinate is zero.

Question

The distance of the point (1, 2, 3) from x-axis is

(a) $\sqrt{13}$ (b) $\sqrt{5}$

(c) $\sqrt{10}$ (d) $\sqrt{14}$.

Solution

Ans. (a)

The distance of any point from x-axis

$$= \sqrt{y^2 + z^2} = \sqrt{4 + 9} = \sqrt{13}.$$

Question

The ratio in which the yz plane divides the segment joining the points $(-2, 4, 7)$ and $(3, -5, 8)$ is

(a) $2:3$ (b) $3:2$ (c) $4:5$ (d) $-7:8$

Ans. (a)

Solution Let yz plane divide the segment joining $(-2, 4, 7)$ and $(3, -5, 8)$ in the ratio $\lambda : 1$. Then

$\Rightarrow \quad \frac{3\lambda - 2}{\lambda + 1} = 0 \quad \Rightarrow \quad \lambda = \frac{2}{3}$ and the required ratio is $2:3$.

Question

If angles α, β, γ made by a line with positive axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$

(a) 2 (b) 3

(c) 4 (d) 1.

Solution

Ans. (a)

we know that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \quad 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$

$\Rightarrow \quad \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2.$

Question

Name the octants in which the following points lie:

(1, 2, 3), (4, −2, 3), (4, −2, −5), (4, 2, −5), (−4, 2, −5), (−4, 2, 5), (−3, −1, 6), (2, −4, −7)

Answer

The x-coordinate, y-coordinate, and z-coordinate of point (1, 2, 3) are all positive. Therefore, this point lies in octant I.

The x-coordinate, y-coordinate, and z-coordinate of point (4, −2, 3) are positive, negative, and positive respectively. Therefore, this point lies in octant **IV**.

The x-coordinate, y-coordinate, and z-coordinate of point (4, −2, −5) are positive, negative, and negative respectively. Therefore, this point lies in octant **VIII**.

The x-coordinate, y-coordinate, and z-coordinate of point (4, 2, −5) are positive, positive, and negative respectively. Therefore, this point lies in octant **V**.

The x-coordinate, y-coordinate, and z-coordinate of point (−4, 2, −5) are negative, positive, and negative respectively. Therefore, this point lies in octant **VI**.

The x-coordinate, y-coordinate, and z-coordinate of point (−4, 2, 5) are negative, positive, and positive respectively. Therefore, this point lies in octant II.

The x-coordinate, y-coordinate, and z-coordinate of point (−3, −1, 6) are negative, negative, and positive respectively. Therefore, this point lies in octant III.

The x-coordinate, y-coordinate, and z-coordinate of point (2, −4, −7) are positive, negative, and negative respectively. Therefore, this point lies in octant **VIII**.

Question

D.C.'s of the line equally inclined with axes are

(a) 1, 1, 1 (b) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

(c) $-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$ (d) $-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$.

Solution

Ans. (b), (c)

Let the line make angle α with coordinate axes

$\therefore \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$$\Rightarrow \quad \cos^2\alpha = \frac{1}{3} \Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{3}}$$

$\therefore$ D.C's of the lines are

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) \text{ or } \left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right).$$

Question

The coordinates of a point equidistant from the points $(a, 0, 0)$, $(0, a, 0)$, $(0, 0, a)$ and $(0, 0, 0)$ are.

(a) $(a/3, a/3, a/3)$ (b) $(a/2, a/2, a/2)$

(c) (a, a, a) (d) $(2a, 2a, 2a)$

Ans. (b)

Solution Let the coordinates of the required point be (x, y, z) then $x^2 + y^2 + z^2 = (x - a)^2 + y^2 + z^2 = x^2 + (y - a)^2 + z^2 = x^2 + y^2 + (z - a)^2$

$\Rightarrow \quad x = a/2 = y = z$. Hence the required point is $(a/2, a/2, a/2)$.

Question

A line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube then, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta =$

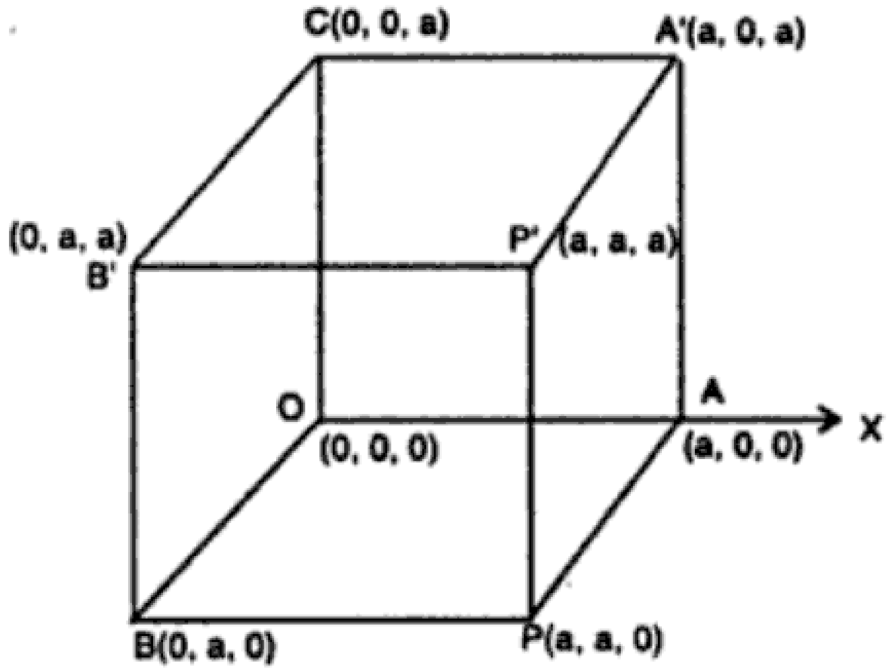
(a) $\frac{2}{3}$ (b) $\frac{4}{3}$

(c) $\frac{8}{3}$ (d) $\frac{1}{3}$. **(MNR 1998)**

Solution

Ans. (b)

Let a be the length of side of a cube and its diagonals are OP', PC, AB', A'B



D.C's of OP' $= \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

D.C's of PC $= \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}$

D.C's of AB' $= -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

D.C's of BA' $= \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Let D.C's of line be l, m, n, then

$$\cos\alpha = \frac{l}{\sqrt{3}} + \frac{m}{\sqrt{3}} + \frac{n}{\sqrt{3}}$$

$$\cos\beta = \frac{l}{\sqrt{3}} + \frac{m}{\sqrt{3}} - \frac{n}{\sqrt{3}}$$

$$\cos\gamma = \frac{-l}{\sqrt{3}} + \frac{m}{\sqrt{3}} + \frac{n}{\sqrt{3}}$$

$$\cos\delta = \frac{l}{\sqrt{3}} - \frac{m}{\sqrt{3}} + \frac{n}{\sqrt{3}}$$

$$\therefore \cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{5}(l^2 + m^2 + n^2) = \frac{4}{3}.$$

since $l^2 + m^2 + n^2 = 1$.

Question

Fill in the blanks:

Answer

(i) The x-axis and y-axis taken together determine a plane known as $\underline{\text{XY} - \text{plane}}$.

(ii) The coordinates of points in the XY-plane are of the form $\underline{(x, y, 0)}$.

(iii) Coordinate planes divide the space into $\underline{\text{eight}}$ octants.

Question

Which of the Triplets are D.C.'s of a line ?

(a) 1, 1, 1 **(b)** 1, – 1, 1

(c) 1, 1, – 1 **(d)** $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

Solution

Ans. (d)

Since $\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 = 1$

Question

If O is the origin and the line OP of length r makes an angle α with x-axis and lies in the xz plane, then the coordinate of P are

(a) $(r\cos\alpha, 0, r\sin\alpha)$ (b) $(0, 0, r\sin\alpha)$

(c) $(0, 0, r\cos\alpha)$ (d) $(r\cos\alpha, 0, 0)$

Ans. (a)

Solution Let the coordinate of P be (x, y, z).

Since OP lies in xz plane and makes an angle α with the x-axis, it makes angle $\pi/2 - \alpha$ with z-axis and $\pi/2$ with Y-axis. so, $x = r\cos\alpha$, $y = r\cos\pi/2$, $z = r\cos(\pi/2 - \alpha)$ are the required coordinates and therefore are $(r\cos\alpha, 0, r\sin\alpha)$.

Question

A line is such that it is inclined with y axis and z-axis at 60°, then at what angle is it inclined with x-axis ?

(a) 45° **(b) 30°**

(c) 75° **(d) 60°**

Solution

Ans. (a)

Let the line make angle α with x-axis, then

$$\cos^2\alpha + \cos^2 60 + \cos^2 60^\circ = 1$$

$$\Rightarrow \cos^2\alpha + \frac{1}{4} + \frac{1}{4} = 1$$

$$\Rightarrow \cos^2\alpha = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow \qquad \cos\alpha = \pm\frac{1}{\sqrt{2}}$$

$$\Rightarrow \qquad \alpha = \pm 45^\circ.$$

Question

If a line is equally inclined with the coordinate axes, then the angle of inclination is

(a) $\cos^{-1}(1/2)$ (b) $\cos^{-1}(1/\sqrt{2})$

(c) $\cos^{-1}(1/\sqrt{3})$ (d) $\cos^{-1}(\sqrt{3}/2)$

Ans. (c)

Solution Let the line be inclined at an angle α with each of the three coordinates axes, then the direction cosines of the line are $\cos\alpha$, $\cos\alpha$, $\cos\alpha$ and $3\cos^2\alpha = 1 \Rightarrow \cos\alpha = 1/\sqrt{3} \Rightarrow \alpha = \cos^{-1}(1/\sqrt{3})$

Question

The projection of line segment on axes are 12, 4, 3 respectively, then length of segment and its D.C.'s are

(a) 13, $\left(\frac{12}{13}, \frac{4}{13}, \frac{3}{13}\right)$ (b) 19, $\left(\frac{12}{19}, \frac{4}{19}, \frac{3}{19}\right)$

(c) 11, $\left(\frac{12}{11}, \frac{14}{11}, \frac{3}{11}\right)$ (d) none of these.

Solution

Ans. (a)

Let OP = r

Projection of OP on x-axis = 12

$\Rightarrow \quad r\cos\alpha = 12$

Similarly $r\cos\beta = 4$

$r\cos\gamma = 3$

$\Rightarrow \quad r^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$= 144 + 16 + 9 = 169$

$\Rightarrow \quad r = 13$

and $\quad \cos\alpha = \frac{12}{13}, \cos\beta = \frac{4}{\sqrt{13}}, \cos\gamma = \frac{3}{13}.$

Question

Find the distance between the following pairs of points:

(i) (2, 3, 5) and (4, 3, 1) (ii) (−3, 7, 2) and (2, 4, −1)

(iii) (−1, 3, −4) and (1, −3, 4) (iv) (2, −1, 3) and (−2, 1, 3)

Answer

The distance between points P(x_1, y_1, z_1) and P(x_2, y_2, z_2) is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

(i) Distance between points (2, 3, 5) and (4, 3, 1)

$$= \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2}$$

$$= \sqrt{(2)^2 + (0)^2 + (-4)^2}$$

$$= \sqrt{4+16}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5}$$

(ii) Distance between points (−3, 7, 2) and (2, 4, −1)

$$= \sqrt{(2+3)^2 + (4-7)^2 + (-1-2)^2}$$

$$= \sqrt{(5)^2 + (-3)^2 + (-3)^2}$$

$$= \sqrt{25+9+9}$$

$$= \sqrt{43}$$

(iii) Distance between points (−1, 3, −4) and (1, −3, 4)

$$= \sqrt{(1+1)^2 + (-3-3)^2 + (4+4)^2}$$

$$= \sqrt{(2)^2 + (-6)^3 + (8)^2}$$

$$= \sqrt{4+36+64} = \sqrt{104} = 2\sqrt{26}$$

(iv) Distance between points (2, −1, 3) and (−2, 1, 3)

$$= \sqrt{(-2-2)^2 + (1+1)^2 + (3-3)^2}$$
$$= \sqrt{(-4)^2 + (2)^2 + (0)^2}$$
$$= \sqrt{16+4}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5}$$

Question

D.C.'s of the line joining points (4, 3, – 5) and (– 2, 1, – 8) are

(a) 2, 4, – 13 **(b) 6, 2, 3**

(c) $\frac{6}{7}, \frac{2}{7}, \frac{3}{7}$ **(d) none of these.**

Solution

Ans. (c)

D.R's of the line are = 4 – (– 2), 3 – 1, – 5 – (– 8)

= 6, 2, 3

$\therefore$ **D.C's are**

$$= \frac{6}{\sqrt{36+4+9}}, \frac{2}{\sqrt{36+4+9}}, \frac{3}{\sqrt{36+4+9}}$$

$$= \frac{6}{7}, \frac{2}{7}, \frac{3}{7}.$$

Question

A line makes an angle of 60° with each of x and y axis, the angle which it makes with z axis is

(a) 30° (b) 45°
(c) 60° (d) none of these

Ans. (b)

Solution Let α be the angle which the line makes with z-axis, thus the direction cosines of the line are cos 60°, cos 60°, cos α.

$\Rightarrow \quad \cos^2 60° + \cos^2 60° + \cos^2 \alpha = 1 \Rightarrow \cos^2 \alpha = 1 - 1/4 - 1/4 = 1/2$

$\Rightarrow \quad \cos \alpha = \pm 1/\sqrt{2}$ so $\alpha = 45°$.

Question

If a, b, c and a', b', c' are D.R's of two mutually perpendicular lines, then

(a) $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$ (b) $aa' + bb' + cc' = 0$
(c) $aa' + bb' + cc' = 1$ (d) none of these.

Answer is (b)

Question

Show that the points (−2, 3, 5), (1, 2, 3) and (7, 0, −1) are collinear.

Answer

Let points (−2, 3, 5), (1, 2, 3), and (7, 0, −1) be denoted by P, Q, and R respectively.

Points P, Q, and R are collinear if they lie on a line.

$$PQ = \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2}$$
$$= \sqrt{(3)^2 + (-1)^2 + (-2)^2}$$
$$= \sqrt{9+1+4}$$
$$= \sqrt{14}$$

$$QR = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2}$$
$$= \sqrt{(6)^2 + (-2)^2 + (-4)^2}$$
$$= \sqrt{36+4+16}$$
$$= \sqrt{56}$$
$$= 2\sqrt{14}$$

$$PR = \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2}$$
$$= \sqrt{(9)^2 + (-3)^2 + (-6)^2}$$
$$= \sqrt{81+9+36}$$
$$= \sqrt{126}$$
$$= 3\sqrt{14}$$

Here, PQ + QR $= \sqrt{14} + 2\sqrt{14} = 3\sqrt{14}$ = PR

Hence, points P(−2, 3, 5), Q(1, 2, 3), and R(7, 0, −1) are collinear.

Question

If A(6, 3, 2), B(5, 1, 4), C(3, – 4, 7), D(0, 2, 5) are four points, then projection of CD on AB is

(a) $\frac{-13}{3}$ (b) $\frac{-13}{7}$

(c) $\frac{-3}{13}$ (d) $\frac{-7}{13}$.

Solution

Ans. (a)

$$\text{D.R's of AB} = 6-5, 3-1, 2-4$$
$$= 1, 2, -2$$
$$\text{D.C's of AB} = \frac{1}{3}, \frac{2}{3}, \frac{-2}{3}$$

$$\text{Projection of CD on AB} = (3-0).\frac{1}{3}$$
$$+ (-4-2).\frac{2}{3} + (7-5)\left(\frac{-2}{3}\right)$$
$$= 1 - 4 - \frac{4}{3}$$
$$= \frac{-13}{3}.$$

Question

If a plane meets the co-ordinate axes in A, B, C such that the centroid of the triangle ABC is the point $(1, r, r^2)$, then equation of the plane is

(a) $x + ry + r^2z = 3r^2$ (b) $r^2x + ry + z = 3r^2$

(c) $x + ry + r^2z = 3$ (d) $r^2x + ry + z = 3$

Ans. (b)

Solution Let an equation of the required plane be

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

This meets the coordinates axes in

$$A\ (a, 0, 0),\ B\ (0, b, 0) \text{ and } C\ (0, 0, c).$$

So that the coordinates of the centroid of the triangle ABC are $(a/3, b/3, c/3) = (1, r, r^2)$ (given) $\Rightarrow$ $a = 3, b = 3r, c = 3r^2$
and the required equation of the plane is

$$\frac{x}{3} + \frac{y}{3r} + \frac{z}{3r^2} = 1 \quad \text{or} \quad r^2x + ry + z = 3r^2.$$

Question

Angle between two lines whose D. R's are $1, 1, 2$; and $\sqrt{3} - 1, -\sqrt{3} - 1, 4$ respectivley, is

(a) $\cos^{-1}\left(\frac{1}{65}\right)$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$.

Solution

Ans. (c)

$$\cos\theta = \frac{6}{\sqrt{6}\,\sqrt{24}} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

by formula

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\,\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Question

Verify the following:

(i) (0, 7, −10), (1, 6, −6) and (4, 9, −6) are the vertices of an isosceles triangle.

(ii) (0, 7, 10), (−1, 6, 6) and (−4, 9, 6) are the vertices of a right angled triangle.

(iii) (−1, 2, 1), (1, −2, 5), (4, −7, 8) and (2, −3, 4) are the vertices of a parallelogram.

Answer

(i) Let points (0, 7, −10), (1, 6, −6), and (4, 9, −6) be denoted by A, B, and C respectively.

$$\begin{aligned} AB &= \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2} \\ &= \sqrt{(1)^2 + (-1)^2 + (4)^2} \\ &= \sqrt{1+1+16} \\ &= \sqrt{18} \\ &= 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2} \\ &= \sqrt{(3)^2 + (3)^2} \\ &= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \end{aligned}$$

$$CA = \sqrt{(0-4)^2 + (7-9)^2 + (-10+6)^2}$$
$$= \sqrt{(-4)^2 + (-2)^2 + (-4)^2}$$
$$= \sqrt{16+4+16} = \sqrt{36} = 6$$

Here, AB = BC ≠ CA

Thus, the given points are the vertices of an isosceles triangle.

(ii) Let (0, 7, 10), (−1, 6, 6), and (−4, 9, 6) be denoted by A, B, and C respectively.

$$AB = \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2}$$
$$= \sqrt{(-1)^2 + (-1)^2 + (-4)^2}$$
$$= \sqrt{1+1+16} = \sqrt{18}$$
$$= 3\sqrt{2}$$

$$BC = \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2}$$
$$= \sqrt{(-3)^2 + (3)^2 + (0)^2}$$
$$= \sqrt{9+9} = \sqrt{18}$$
$$= 3\sqrt{2}$$

$$CA = \sqrt{(0+4)^2 + (7-9)^2 + (10-6)^2}$$
$$= \sqrt{(4)^2 + (-2)^2 + (4)^2}$$
$$= \sqrt{16+4+16}$$
$$= \sqrt{36}$$
$$= 6$$

$$\text{Now, } AB^2 + BC^2 = \left(3\sqrt{2}\right)^2 + \left(3\sqrt{2}\right)^2 = 18 + 18 = 36 = AC^2$$

Therefore, by Pythagoras theorem, ABC is a right triangle.

Hence, the given points are the vertices of a right-angled triangle.

(iii) Let (−1, 2, 1), (1, −2, 5), (4, −7, 8), and (2, −3, 4) be denoted by A, B, C, and D respectively.

$$\begin{aligned} AB &= \sqrt{(1+1)^2 + (-2-2)^2 + (5-1)^2} \\ &= \sqrt{4+16+16} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(4-1)^2 + (-7+2)^2 + (8-5)^2} \\ &= \sqrt{9+25+9} = \sqrt{43} \end{aligned}$$

$$\begin{aligned} CD &= \sqrt{(2-4)^2 + (-3+7)^2 + (4-8)^2} \\ &= \sqrt{4+16+16} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

$$\begin{aligned} DA &= \sqrt{(-1-2)^2 + (2+3)^2 + (1-4)^2} \\ &= \sqrt{9+25+9} = \sqrt{43} \end{aligned}$$

Here, AB = CD = 6, BC = AD = $\sqrt{43}$

Hence, the opposite sides of quadrilateral ABCD, whose vertices are taken in order, are equal.

Therefore, ABCD is a parallelogram.

Hence, the given points are the vertices of a parallelogram.

Question

If the line joining (1, 2, – 1) and (– 1, 0, 1) is $\frac{x-1}{l} = \frac{y-2}{m} = \frac{z+1}{n}$, the values of ($l$, n, n) are

(a) (– 1, 0, 1) (b) (1, 1 – 1)

(c) (1, 2, – 1) (d) (0, 1, 0).

(MP PET 1992)

Solution

Ans. (b)

D.R's of the line joining points (1, 2, – 1) and (– 1, 0, 1) are

$$[1-(-1), (2-0), (-1, -1)]$$

or 2, 2, – 2

$\therefore$ $l, m, n = 1, 1, -1.$

Question

Algebraic sum of the intercepts made by the plane $x + 3y - 4z + 6 = 0$ on the axes is

(a) $-13/2$ (b) $19/2$ (c) $-22/3$ (d) $26/3$

Ans. (a)

Solution Equation of the plane can be written as

$$\frac{x}{-6} + \frac{y}{-2} + \frac{z}{3/2} = 1$$

So the intercepts on the coordinates axes are $-6, -2, 3/2$ and the required sum is $-6 - 2 + 3/2 = -13/2$.

Question

The distance of the point (1, – 2, 3) from the planes $x - y + 2 = 5$ measured along the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is

(a) 1 **(b) $\frac{6}{7}$**

(c) $\frac{7}{6}$ **(d) none of these.**

(AICBSE 1984)

Solution

Ans. (a)

The line passing through the point (1, – 2, 3) and parallel to the line is

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = r$$

$$\Rightarrow \quad \frac{x-1}{\frac{2}{7}} = \frac{y+2}{\frac{3}{2}} = \frac{z-3}{\frac{-6}{7}} = r$$

$$\Rightarrow \quad x = 1 + \frac{2r}{7},\ y = -2 + \frac{3r}{7},$$

$z = 3 - \frac{6r}{7}$ which lies on the plane $x - y + z = 5$

$$\therefore \ 1 + \frac{2r}{7} + 2 \frac{-3r}{7} + 3 \frac{-6r}{7} = 5 \Rightarrow r = 1.$$

Question

Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, −1).

Answer

Let P (x, y, z) be the point that is equidistant from points A(1, 2, 3) and B(3, 2, −1).

Accordingly, PA = PB

$$\Rightarrow PA^2 = PB^2$$

$$\Rightarrow (x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$$

$$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$$

$\Rightarrow -2x - 4y - 6z + 14 = -6x - 4y + 2z + 14$

$\Rightarrow -2x - 6z + 6x - 2z = 0$

$\Rightarrow 4x - 8z = 0$

$\Rightarrow x - 2z = 0$

Thus, the required equation is $x - 2z = 0$.

Question

The angle between the line $\frac{x+1}{3} = \frac{y-1}{2} = \frac{z-2}{4}$ and the plane $2x + y - 3z + 4 = 0$ is

(a) $\sin^{-1}\left[\frac{4}{\sqrt{406}}\right]$ (b) $\sin^{-1}\left[\frac{-4}{\sqrt{406}}\right]$

(c) $\sin^{-1}\left[\frac{4}{14\sqrt{29}}\right]$ (d) none of these.

(CBSE 1981)

Solution

Ans. (b)

$$\cos(90° - \theta) = \frac{3 \cdot 2 + 2 \cdot 1 + 4(-3)}{\sqrt{9+4+16}\,\sqrt{4+1+9}}$$

$$\Rightarrow \qquad \sin\theta = \frac{-4}{\sqrt{29}\,\sqrt{14}} = \frac{-4}{\sqrt{406}}.$$

Question

Equations of a line passing through (2, –1, 1) and parallel to the line whose equations are

$$\frac{x-3}{2}=\frac{y+1}{7}=\frac{z-2}{-3} \text{ are}$$

(a) $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-1}{2}$ (b) $\frac{x-2}{2}=\frac{y+1}{7}=\frac{z-1}{-3}$

(c) $\frac{x-2}{2}=\frac{y-7}{-1}=\frac{z+3}{1}$ (d) $\frac{x-3}{2}=\frac{y+1}{-1}=\frac{z-2}{1}$

Ans. (b)

Solution The required line passes through (2, – 1, 1) and its direction cosines are proportional to 2, 7, –3 so its equation is

$$\frac{x-2}{2}=\frac{y+1}{7}=\frac{z-1}{-3}$$

Question

Lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ are mutually perpendicular, if

(a) $aa' + cc' = 1$ (b) $aa' + cc' = -1$

(c) $ac + a'c' = 1$ (d) $ac + a'c' = -1$.

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Solution

Ans. (b)

$$\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c} \text{ and } \frac{x-b'}{a'} = \frac{y}{1} = \frac{z-d'}{c'}$$

are perpendicular to each other, then $aa' + 1 + cc'$ $= 0$.

Question

Find the equation of the set of points P, the sum of whose distances from A (4, 0, 0) and B (−4, 0, 0) is equal to 10.

Answer

Let the coordinates of P be (x, y, z).

The coordinates of points A and B are (4, 0, 0) and (−4, 0, 0) respectively.

It is given that PA + PB = 10.

$$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} + \sqrt{(x+4)^2 + y^2 + z^2} = 10$$

$$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} = 10 - \sqrt{(x+4)^2 + y^2 + z^2}$$

On squaring both sides, we obtain

$$\Rightarrow (x-4)^2 + y^2 + z^2 = 100 - 20\sqrt{(x+4)^2 + y^2 + z^2} + (x+4)^2 + y^2 + z^2$$

$$\Rightarrow x^2 - 8x + 16 + y^2 + z^2 = 100 - 20\sqrt{x^2 + 8x + 16 + y^2 + z^2} + x^2 + 8x + 16 + y^2 + z^2$$

$$\Rightarrow 20\sqrt{x^2 + 8x + 16 + y^2 + z^2} = 100 + 16x$$

$$\Rightarrow 5\sqrt{x^2 + 8x + 16 + y^2 + z^2} = (25 + 4x)$$

On squaring both sides again, we obtain

$25(x^2 + 8x + 16 + y^2 + z^2) = 625 + 16x^2 + 200x$

$\Rightarrow 25x^2 + 200x + 400 + 25y^2 + 25z^2 = 625 + 16x^2 + 200x$

$\Rightarrow 9x^2 + 25y^2 + 25z^2 - 225 = 0$

Thus, the required equation is $9x^2 + 25y^2 + 25z^2 - 225 = 0$.

Question

The equation of the line passing through the points (a, b, c) and $(a - b, b - c, c - a)$ is

(a) $\frac{x-a}{a-b} = \frac{y-b}{b-c} = \frac{z-c}{c-a}$

(b) $\frac{x-a}{b} = \frac{y-b}{c} = \frac{z-c}{a}$

(c) $\frac{x-a}{a} = \frac{y-b}{b} = \frac{z-c}{c}$

(d) $\frac{x-a}{2a-b} = \frac{y-b}{2b-c} = \frac{z-c}{2c-a}$.

Solution

Ans. (b)

the equation of Reqd. line is

$$\frac{x-a}{a-b-a} = \frac{x-b}{b-c-b} = \frac{x-c}{c-a-c}$$

$$\Rightarrow \quad \frac{x-a}{-b} = \frac{x-b}{-c} = \frac{x-c}{-a}$$

$$\Rightarrow \quad \frac{x-a}{b} = \frac{y-b}{c} = \frac{z-c}{a}.$$

Question

If M denotes the mid-point of the line joining A $(4\mathbf{i} + 5\mathbf{j} - 10\mathbf{k})$ and B $(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$, then equation of the plane through M and perpendicular to AB is

(a) $\mathbf{r}\cdot(-5\mathbf{i} - 3\mathbf{j} + 11\mathbf{k}) + 135/2 = 0$

(b) $\mathbf{r}\cdot\left(\frac{3}{2}\mathbf{i}+\frac{7}{2}\mathbf{j}-\frac{9}{2}\mathbf{k}\right)+\frac{135}{2} = 0$

(c) $\mathbf{r}\cdot(4\mathbf{i} + 5\mathbf{j} - 10\mathbf{k}) + 4 = 0$

(d) $\mathbf{r}\cdot(-\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + 4 = 0$

Ans. (a)

Solution Middle point M of AB is

$$M\left(\frac{1}{2}(4\mathbf{i}+5\mathbf{j}-10\mathbf{k}-\mathbf{i}+2\mathbf{j}+\mathbf{k})\right) = M\left(\frac{3}{2}\mathbf{i}+\frac{7}{2}\mathbf{j}-\frac{9}{2}\mathbf{k}\right)$$

Also $\mathbf{AB} = -\mathbf{i}+2\mathbf{j}+\mathbf{k}-(4\mathbf{i}+5\mathbf{j}-10\mathbf{k}) = -5\mathbf{i}-3\mathbf{j}+11\mathbf{k}$

So the plane passing through M and perpendicular to the direction $\mathbf{AB}$ is

$$\left[\mathbf{r}-\left(\frac{3}{2}\mathbf{i}+\frac{7}{2}\mathbf{j}-\frac{9}{2}\mathbf{k}\right)\right]\cdot(-5\mathbf{i}-3\mathbf{j}+11\mathbf{k}) = 0$$

$$\mathbf{r}\cdot(-5\mathbf{i}-3\mathbf{j}+11\mathbf{k})+135/2=0$$

Question

. The distance between parallel planes $2x - 2y + z + 3 = 0$ and $4x - 4y + 2z + 5 = 0$ is

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) 2.

Solution

Ans. (c)

$$d = \frac{\left|3 - \frac{5}{2}\right|}{\sqrt{4 + 4 + 1}} = \frac{1}{2 \times 3} = \frac{1}{6}$$

Question

Find the coordinates of the point which divides the line segment joining the points (−2, 3, 5) and (1, −4, 6) in the ratio (i) 2:3 internally, (ii) 2:3 externally.

Answer

(i) The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio $m:n$ are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right).$$

Let R (x, y, z) be the point that divides the line segment joining points(−2, 3, 5) and (1, −4, 6) internally in the ratio 2:3

$$x=\frac{2(1)+3(-2)}{2+3}, y=\frac{2(-4)+3(3)}{2+3}, \text{ and } z=\frac{2(6)+3(5)}{2+3}$$

$$\text{i.e., } x=\frac{-4}{5}, y=\frac{1}{5}, \text{ and } z=\frac{27}{5}$$

Thus, the coordinates of the required point are $\left(-\frac{4}{5}, \frac{1}{5}, \frac{27}{5}\right)$.

(ii) The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) externally in the ratio $m:n$ are

$$\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right).$$

Let R (x, y, z) be the point that divides the line segment joining points(−2, 3, 5) and (1, −4, 6) externally in the ratio 2:3

$$x=\frac{2(1)-3(-2)}{2-3}, y=\frac{2(-4)-3(3)}{2-3}, \text{ and } z=\frac{2(6)-3(5)}{2-3}$$

$$\text{i.e., } x=-8, y=17, \text{ and } z=3$$

Thus, the coordinates of the required point are (−8, 17, 3).

Question

The number of lines which are equally inclined with coordinate axes, is

(a) 2 (b) 4

(c) 6 (d) 8.

Answer (a)

Question

Equation of the plane through (3, 4, –1) which is parallel to the plane $\mathbf{r}\cdot(2\mathbf{i}-3\mathbf{j}+5\mathbf{k})+7=0$ is

(a) $\mathbf{r}\cdot(2\mathbf{i}-3\mathbf{j}+5\mathbf{k})+11=0$ (b) $\mathbf{r}\cdot(3\mathbf{i}+4\mathbf{j}-\mathbf{k})+11=0$

(c) $\mathbf{r}\cdot(3\mathbf{i}+4\mathbf{j}-\mathbf{k})+7=0$ (d) $\mathbf{r}\cdot(2\mathbf{i}-3\mathbf{j}+5\mathbf{k})-7=0$

Ans. (a)

Solution Equation of any plane parallel to the given plane is

$$\mathbf{r}\cdot(2\mathbf{i}-3\mathbf{j}+5\mathbf{k})+\lambda=0.$$

If $\mathbf{r}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$, we get

$$2x-3y+5k+\lambda=0$$

This plane passes through the point (3, 4, –1)

if $2\times3-3\times4+5(-1)+\lambda=0$ or if $\lambda=11$

and hence the equation of the required plane is

$$\mathbf{r}\cdot(2\mathbf{i}-3\mathbf{j}+5\mathbf{k})+11=0$$

Question

If planes $ax + by + cz + d = 0$ and $a'x + b'y + c'z + d' = 0$ are mutualiy perpendicular then

(a) $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$

(b) $\frac{a}{a'} + \frac{b}{b'} + \frac{c}{c'} = 0$

(c) $aa' + bb' + cc' + dd' = 0$

(d) $aa' + bb' + cc' = 0.$

Answer (d)

Question

Given that P (3, 2, −4), Q (5, 4, −6) and R (9, 8, −10) are collinear. Find the ratio in which Q divides PR.

Answer

Let point Q (5, 4, −6) divide the line segment joining points P (3, 2, −4) and R (9, 8, −10) in the ratio k:1.

Therefore, by section formula,

$$(5,4,-6) = \left(\frac{k(9)+3}{k+1}, \frac{k(8)+2}{k+1}, \frac{k(-10)-4}{k+1} \right)$$

$$\Rightarrow \frac{9k+3}{k+1} = 5$$

$$\Rightarrow 9k+3 = 5k+5$$

$$\Rightarrow 4k = 2$$

$$\Rightarrow k = \frac{2}{4} = \frac{1}{2}$$

Thus, point Q divides PR in the ratio 1:2.

Question

The angle between lines $2x = 3y = -z$ and $6x = -y = -4z$ is

(a) $0°$ (b) $30°$

(c) $45°$ (d) $90°$.

Solution

Ans. (d)

$$2x = 3y = -z \quad \text{and} \quad 6x = -y = -4z$$

$$\Rightarrow \quad \frac{x}{3} = \frac{y}{2} = \frac{\pm z}{\pm 6} \quad \text{and} \quad \frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$$

$$\cos\theta = \frac{3 \times 2 + 2 \times -12 + (-6) \times (-3)}{\sqrt{9+4+36}\,\sqrt{4+144+9}}$$

$$= \frac{0}{7\sqrt{151}} = 0 \quad \Rightarrow \quad \theta = \frac{\pi}{2}.$$

Question

The ratio in which the plane $2x - 1 = 0$ divides the line joining $(-2, 4, 7)$ and $(3, -5, 8)$ is

(a) 2 : 3 (b) 4 : 5 (c) 7 : 8 (d) 1 : 1

Ans. (d)

Solution Let the required ratio be $k : 1$, then the coordinates of the point which divides the join of the points $(-2, 4, 7)$ and $(3, -5, 8)$ in this ratio are given by

$$\left(\frac{3k-2}{k+1}, \frac{-5k+4}{k+1}, \frac{8k+7}{k+1}\right)$$

As this point lies on the plane $2x - 1 = 0$.

$\Rightarrow \quad \frac{3k-2}{k+1} = \frac{1}{2} \Rightarrow k = 1$ and thus the required ratio is 1 : 1.

Question

A line passes through the points $(6, -7, -1)$ and $(2, -3, 1)$. If the angle α which the line makes with the positive direction of x-axis is acute, the direction cosines of the line are

(a) 2/3, – 2/3, – 1/3 (b) 2/3, 2/3, – 1/3

(c) 2/3, – 2/3, 1/3 (d) 2/3, 2/3, 1/3

Ans. (a)

Solution The direction cosines of the given line are proportional to

$$2-6, -3+7, 1+1, \text{ i.e. } -2, 2, 1$$

the direction cosines are therefore $= \frac{\mp 2}{3}, \frac{\pm 2}{3}, \frac{\pm 1}{3}$.

Since the angle α which the line makes with positive direction of x-axis is acute, $\cos\alpha > 0 \Rightarrow \cos\alpha = 2/3$,

Thus, required direction cosines are 2/3, –2/3, –1/3.

Question

Find the ratio in which the YZ-plane divides the line segment formed by joining the points (−2, 4, 7) and (3, −5, 8).

Answer

Let the YZ planedivide the line segment joining points (−2, 4, 7) and (3, −5, 8) in the ratio $k:1$.

Hence, by section formula, the coordinates of point of intersection are given by

$$\left(\frac{k(3)-2}{k+1}, \frac{k(-5)+4}{k+1}, \frac{k(8)+7}{k+1}\right)$$

On the YZ plane, the x-coordinate of any point is zero.

$$\frac{3k-2}{k+1}=0$$

$$\Rightarrow 3k-2=0$$

$$\Rightarrow k=\frac{2}{3}$$

Thus, the YZ plane divides the line segment formed by joining the given points in the ratio 2:3.

Question

The d.r. of normal to the plane through (1, 0, 0), (0, 1, 0) which makes an angle $\pi/4$ with the plane $x + y = 3$ are

(a) $1, \sqrt{2}, 1$ (b) $1, 1, \sqrt{2}$ (c) $1, 1, 2$ (d) $\sqrt{2}, 1, 1$

Ans. (b)

Solution Equation of a plane through (1, 0, 0) is $A(x - 1) + By + Cz = 0$. Since it passes through the point (0, 1, 0), $-A + B = 0 \Rightarrow A = B$ and as it makes an angle $\pi/4$ with $x + y = 3$

$$\frac{A+B}{\sqrt{1+1}\sqrt{A^2+B^2+C^2}} = \pm\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad (A + B)^2 = A^2 + B^2 + C^2$$

$$\Rightarrow \quad 2A^2 = C^2$$

So $$\frac{A}{1} = \frac{B}{1} = \frac{C}{\pm\sqrt{2}}$$

and the required d.r. are $1, 1, \sqrt{2}$.

Question

Using section formula, show that the points A (2, −3, 4), B (−1, 2, 1) and $C\left(0,\frac{1}{3},2\right)$ are collinear.

Answer

The given points are A (2, −3, 4), B (−1, 2, 1), and $C\left(0,\frac{1}{3},2\right)$.

Let P be a point that divides AB in the ratio $k:1$.

Hence, by section formula, the coordinates of P are given by

$$\left(\frac{k(-1)+2}{k+1},\frac{k(2)-3}{k+1},\frac{k(1)+4}{k+1}\right)$$

Now, we find the value of k at which point P coincides with point C.

By taking $\frac{-k+2}{k+1}=0$, we obtain k = 2.

For k = 2, the coordinates of point P are $\left(0,\frac{1}{3},2\right)$.

i.e., $C\left(0,\frac{1}{3},2\right)$ is a point that divides AB externally in the ratio 2:1 and is the same as point P.

Hence points A, B, and C are collinear

Question

A plane which passes through the point (3, 2, 0) and the line $\frac{x-4}{1}=\frac{y-7}{5}=\frac{z-4}{4}$ is

(a) $x-y+z=1$ (b) $x+y+z=5$

(c) $x+2y-z=1$ (d) $2x-y+z=5$

Ans. (a)

Solution Let the equation of the plane be

$$a(x-3)+b(y-2)+cz=0$$

Since it contains the given line.

$$a(4-3)+b(7-2)+4c=0$$

$$\Rightarrow \quad a+5b+4c=0$$

which is satisfied by the d.r 1, – 1, 1 of the plane in (a).

Question

Find the coordinates of the points which trisect the line segment joining the points P (4, 2, −6) and Q (10, −16, 6).

Answer

Let A and B be the points that trisect the line segment joining points P (4, 2, −6) and Q (10, −16, 6)

Question

A line makes the same angle θ with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals

(a) 3/5 (b) 1/5 (c) 2/3 (d) 2/5

Ans. (a)

Solution We have $\cos^2\theta + \cos^2\theta + \cos^2\beta = 1$

$\Rightarrow$ $2\cos^2\theta = \sin^2\beta = 3\sin^2\theta = 3(1 - \cos^2\theta)$

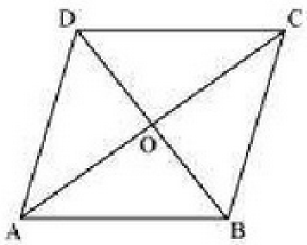
$\Rightarrow$ $5\cos^2\theta = 3 \Rightarrow \cos^2\theta = 3/5.$

Question

Three vertices of a parallelogram ABCD are A (3, −1, 2), B (1, 2, −4) andC (−1, 1, 2). Find the coordinates of the fourth vertex.

Answer

The three vertices of a parallelogram ABCD are given as A (3, −1, 2), B (1, 2, −4), and C (−1, 1, 2). Let the coordinates of the fourth vertex be D (x, y, z).



We know that the diagonals of a parallelogram bisect each other.

Therefore, in parallelogram ABCD, AC and BD bisect each other.

$\therefore$Mid-point of AC = Mid-point of BD

$$\Rightarrow \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$$

$$\Rightarrow (1,0,2) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$$

$$\Rightarrow \frac{x+1}{2} = 1, \frac{y+2}{2} = 0, \text{ and } \frac{z-4}{2} = 2$$

$\Rightarrow x = 1$, $y = -2$, and $z = 8$

Thus, the coordinates of the fourth vertex are (1, −2, 8).

Question

A line with directional cosines proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The coordinates of each of the points of intersection are given by

(a) $(3a, 2a, 3a), (a, a, 2a)$ (b) $(3a, 2a, 3a), (a, a, a)$

(c) $(3a, 3a, 3a), (a, a, a)$ (d) $(2a, 3a, 3a), (2a, a, a)$

Ans. (b)

Solution Let the points be $P(r, r-a, r)$ on the first line and $Q(2r'-a, r', r')$ on the second line

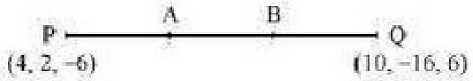
then $$\frac{r-2r'+a}{2} = \frac{r-a-r'}{1} = \frac{r-r'}{2}$$

$\Rightarrow$ $r = 3a$, $r' = a$, so the required points are

$P(3a, 2a, 3a)$ and $Q(a, a, a)$

Question

Find the lengths of the medians of the triangle with vertices A (0, 0, 6), B (0, 4, 0) and (6, 0, 0).



Point A divides PQ in the ratio 1:2. Therefore, by section formula, the coordinates of point A are given by

$$\left(\frac{1(10)+2(4)}{1+2}, \frac{1(-16)+2(2)}{1+2}, \frac{1(6)+2(-6)}{1+2}\right)=(6,-4,-2)$$

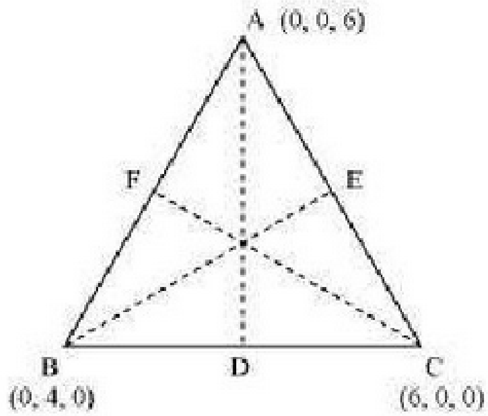
Point B divides PQ in the ratio 2:1. Therefore, by section formula, the coordinates of point B are given by

$$\left(\frac{2(10)+1(4)}{2+1}, \frac{2(-16)+1(2)}{2+1}, \frac{2(6)-1(6)}{2+1}\right)=(8,-10,2)$$

Thus, (6, −4, −2) and (8, −10, 2) are the points that trisect the line segment joining points P (4, 2, −6) and Q (10, −16, 6).

Answer

Let AD, BE, and CF be the medians of the given triangle ABC.



Since AD is the median, D is the mid-point of BC.

$\therefore$Coordinates of point D = $\left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2}\right)$ = (3, 2, 0)

$$AD = \sqrt{(0-3)^2 + (0-2)^2 + (6-0)^2} = \sqrt{9+4+36} = \sqrt{49} = 7$$

Since BE is the median, E is the mid-point of AC.

$\therefore$ Coordinates of point E = $\left(\frac{0+6}{2}, \frac{0+0}{2}, \frac{6+0}{2}\right) = (3, 0, 3)$

$$BE = \sqrt{(3-0)^2 + (0-4)^2 + (3-0)^2} = \sqrt{9+16+9} = \sqrt{34}$$

Since CF is the median, F is the mid-point of AB.

$\therefore$ Coordinates of point F = $\left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2}\right) = (0, 2, 3)$

$$\text{Length of } CF = \sqrt{(6-0)^2 + (0-2)^2 + (0-3)^2} = \sqrt{36+4+9} = \sqrt{49} = 7$$

Thus, the lengths of the medians of ΔABC are $7, \sqrt{34}$, and 7.

Question

If the straight lines.

$x = 1 + s$, $y = -3 - \lambda s$, $z = 1 + \lambda s$ and $x = t/2$, $y = 1 + t$, $z = 2 - t$, with parameters s and t are coplanar, then λ equals

(a) $-1/2$ (b) -1 (c) -2 (d) 0

Ans. (c)

Solution Equation of the lines can be written as $\frac{x-1}{1} = \frac{y+3}{-\lambda} = \frac{z-1}{\lambda}$ and $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{-2}$. Since these are coplanar

$$\begin{vmatrix} 1 & -4 & -1 \\ 1 & 2 & -2 \\ 1 & -\lambda & \lambda \end{vmatrix} = 0$$

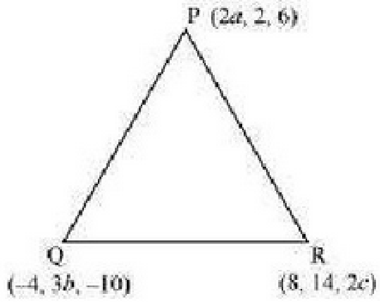
$$\Rightarrow \quad \begin{vmatrix} 1 & -4 & - \\ 0 & 6 & -1 \\ 0 & -\lambda+4 & \lambda+1 \end{vmatrix} = 0$$

$$\Rightarrow \quad 6(\lambda + 1) + (4 - \lambda) = 0 \quad \Rightarrow \quad \lambda = -2$$

Question

If the origin is the centroid of the triangle PQR with vertices P (2*a*, 2, 6), Q (−4, 3*b*, −10) and R (8, 14, 2*c*), then find the values of *a*, *b* and *c*.

Answer



It is known that the coordinates of the centroid of the triangle, whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3), are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$.

Therefore, coordinates of the centroid of ΔPQR

$$=\left(\frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3}\right)=\left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$$

It is given that origin is the centroid of ΔPQR.

$$\therefore (0,0,0)=\left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$$

$$\Rightarrow \frac{2a+4}{3}=0, \frac{3b+16}{3}=0 \text{ and } \frac{2c-4}{3}=0$$

$$\Rightarrow a=-2, b=-\frac{16}{3} \text{ and } c=2$$

Thus, the respective values of *a*, *b*, and *c* are $-2, -\frac{16}{3}$, and 2.

Question

The angles between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

(a) 45° (b) 30° (c) 0° (d) 90°

Ans. (d)

Solution Lines can be written as

$$\frac{x}{3} = \frac{y}{2} = \frac{z}{-6} \text{ and } \frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$$

Since $3 \times 2 + 2 \times (-12) + (-6)(-3) = 0$

The lines are perpendicular.

Question

Find the coordinates of a point on y-axis which are at a distance of $5\sqrt{2}$ from the point P (3, −2, 5).

Answer

If a point is on the y-axis, then x-coordinate and the z-coordinate of the point are zero.

Let A (0, b, 0) be the point on the y-axis at a distance of $5\sqrt{2}$ from point P (3, −2, 5).

Accordingly, $AP = 5\sqrt{2}$

$$\therefore AP^2 = 50$$
$$\Rightarrow (3-0)^2 + (-2-b)^2 + (5-0)^2 = 50$$
$$\Rightarrow 9 + 4 + b^2 + 4b + 25 = 50$$
$$\Rightarrow b^2 + 4b - 12 = 0$$
$$\Rightarrow b^2 + 6b - 2b - 12 = 0$$
$$\Rightarrow (b+6)(b-2) = 0$$
$$\Rightarrow b = -6 \text{ or } 2$$

Thus, the coordinates of the required points are (0, 2, 0) and (0, −6, 0).

Question

If the angle θ between the lines $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is such that $\sin\theta = 1/3$, then the value of λ is

(a) 3/4 (b) $-4/3$ (c) 5/3 (d) $-3/5$.

Ans. (c)

Solution Since the line makes an angle θ with the plane, it makes an angle $\pi/2 - \theta$ with the normal to the plane.

$$\therefore \quad \cos\left(\frac{\pi}{2} - \theta\right) = \frac{2(1) + (-1)(2) + (\sqrt{\lambda})(2)}{\sqrt{1+4+4} \times \sqrt{4+1+\lambda}}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{2\sqrt{\lambda}}{3\sqrt{\lambda+5}} \quad \Rightarrow \quad \lambda + 5 = 4\lambda$$

$$\Rightarrow \quad \lambda = 5/3.$$

Question

A point R with x-coordinate 4 lies on the line segment joining the pointsP (2, −3, 4) and Q (8, 0, 10). Find the coordinates of the point R.

[**Hint** suppose R divides PQ in the ratio k: 1. The coordinates of the point R are given by $\left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$]

Answer

The coordinates of points P and Q are given as P (2, −3, 4) and Q (8, 0, 10).

Let R divide line segment PQ in the ratio k:1.

Hence, by section formula, the coordinates of point R are given by

$$\left(\frac{k(8)+2}{k+1}, \frac{k(0)-3}{k+1}, \frac{k(10)+4}{k+1}\right) = \left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$$

It is given that the x-coordinate of point R is 4.

$$\therefore \frac{8k+2}{k+1} = 4$$

$$\Rightarrow 8k+2 = 4k+4$$

$$\Rightarrow 4k = 2$$

$$\Rightarrow k = \frac{1}{2}$$

Therefore, the coordinates of point R are $\left(4, \frac{-3}{\frac{1}{2}+1}, \frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1}\right) = (4, -2, 6)$

Question

If a line makes an angle $\pi/4$ with the positive direction of each of x-axis and y-axis, then the angle which the line makes with the positive direction of z-axis is

(a) $\pi/6$ (b) $\pi/3$ (c) $\pi/4$ (d) $\pi/2$

Ans. (d)

Solution If θ is the required angle then $\cos^2(\pi/4) + \cos^2(\pi/4) + \cos^2\theta = 1$

$$\Rightarrow \quad \cos^2\theta = 0 \quad \Rightarrow \quad \cos\theta = 0$$

$$\Rightarrow \quad \theta = \pi/2$$

Question

If A and B be the points (3, 4, 5) and (−1, 3, −7), respectively, find the equation of the set of points P such that $PA^2 + PB^2 = k^2$, where k is a constant.

Answer

The coordinates of points A and B are given as (3, 4, 5) and (−1, 3, −7) respectively.

Let the coordinates of point P be (x, y, z).

On using distance formula, we obtain

$$PA^2 = (x-3)^2 + (y-4)^2 + (z-5)^2$$
$$= x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z$$
$$= x^2 - 6x + y^2 - 8y + z^2 - 10z + 50$$
$$PB^2 = (x+1)^2 + (y-3)^2 + (z+7)^2$$
$$= x^2 + 2x + y^2 - 6y + z^2 + 14z + 59$$

Now, if $PA^2 + PB^2 = k^2$, then

$$\left(x^2 - 6x + y^2 - 8y + z^2 - 10z + 50\right) + \left(x^2 + 2x + y^2 - 6y + z^2 + 14z + 59\right) = k^2$$
$$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2$$
$$\Rightarrow 2\left(x^2 + y^2 + z^2 - 2x - 7y + 2z\right) = k^2 - 109$$

$$\Rightarrow x^2+y^2+z^2-2x-7y+2z=\frac{k^2-109}{2}$$

Thus, the required equation is $x^2+y^2+z^2-2x-7y+2z=\frac{k^2-109}{2}$.

Question

Let $\mathbf{a}=\mathbf{i}+\mathbf{j}+\mathbf{k}$, $\mathbf{b}=\mathbf{i}-\mathbf{j}+2\mathbf{k}$ and $\mathbf{c}=x\mathbf{i}+(x-2)\ \mathbf{j}-\mathbf{k}$.

If the vector $\mathbf{c}$ lies in the plane of $\mathbf{a}$ and $\mathbf{b}$, then x equals

(a) 0 (b) 1 (c) -4 (d) -2

Ans. (d)

Solution Since the three vectors are coplanar

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1 & -2 & 1 \\ x & -2 & -1-x \end{vmatrix}=0$$

$$\Rightarrow -2(-1-x)+2=0$$

$$\Rightarrow x=-2$$

Question

The point in which the line $\frac{x+1}{-1}=\frac{y-12}{5}=\frac{z-7}{-2}$ cuts the surface $11x^2-5y^2-z^2=0$ is

(a) $(2,-3,1)$ (b) $(2,3,-1)$

(c) $(1,2,3)$ (d) $(1,2,-3)$

Solution

(a, c). Let $\frac{x+1}{-1} = \frac{y-12}{5} = \frac{z-7}{2} = r.$

Any point on the line is $(-r-1, 5r+12, 2r+7)$ for every value of r.

If this point lies on the surface $11x^2 - 5y^2 + z^2 = 0$, then

$$11(-r-1)^2 - 5(5r+12)^2 + (2r+7)^2 = 0.$$

i.e., $110r^2 + 550r + 660 = 0,$

i.e., $r^2 + 5r + 6 = 0$

i.e., $(r+3)(r+2) = 0, \quad$ i.e., $r = -3, -2$

For these two values of r, the two points in which the given line cuts the surface are $(2, -3, 1)$ and $(1, 2, 3)$.

Question

The shortest distance from the plane $12x + 4y + 3z = 327$ to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is

(a) $11\frac{3}{4}$ (b) 13 (c) 39 (d) 26

Ans. (b)

Solution The centre of the sphere is $(-2, 1, 3)$ and its radius is

$$\sqrt{4+1+9+155} = 13$$

Length of the perpendicular from the centre of the sphere on the plane is

$$\left|\frac{-24+4+9-327}{\sqrt{144+16+9}}\right| = \frac{338}{13} = 26$$

So the plane is outside the sphere and the required distance is equal to $26 - 13 = 13$.

Question

The radius of the circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is cut by the plane $x + 2y + 2z + 7 = 0$ is

(a) 2 (b) 3 (c) 4 (d) 1

Ans. (b)

Solution Centre of the sphere is (– 1, 1, 2) and its radius is

$$\sqrt{1+1+4+19} = 5.$$

Length of the perpendicular from the centre on the plane is

$$\left|\frac{-1+2+4+7}{\sqrt{1+4+4}}\right| = 4$$

Radius of the required circle is $\sqrt{5^2 - 4^2} = 3$.

Question

The edge of a cube is of length '*a*' then the shortest distance between the diagonal of a cube and an edge skew to it is

(a) $a\sqrt{2}$ (b) a

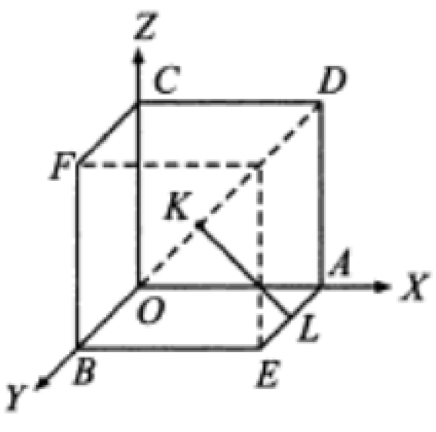
(c) $\sqrt{2}/a$ (d) $a/\sqrt{2}$

Solution

(d). Requaried distance = KL

$$= \sqrt{\left(a - \frac{a}{2}\right)^2 + 0^2 + \left(0 - \frac{a}{2}\right)^2}$$

$$= \frac{a}{\sqrt{2}}.$$



Question

Statement-1: The radius of the circle in which the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z - 2 = 0$ is cut by the plane $x + 2y + 3z - 6 = 0$ is 4.
Statement-2: A plane passing through the centre of a sphere cuts the sphere in a circle of maximum radius.

Ans. (a)

Solution Statement-2 is true as in this case the radius of the circle is equal to the radius of the sphere, the distance between the centre of the sphere and the plane is zero, minimum possible. Using this in statement-1, the centre (1, – 2, 3) of the sphere lies on the plane so the radius of the circle = $\sqrt{(+1)^2 + (-2)^2 + (3)^2 + 2} = 4$, the radius of the sphere and the statement-1 is also true.

Question

The equation of the plane through the line $x + y + z + 3 = 0 = 2x - y + 3z + 1$ and parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ is

(a) $x - 5y + 3z = 7$ (b) $x - 5y + 3z = -7$
(c) $x + 5y + 3z = 7$ (d) $x + 5y + 3z = -7$

Solution

(a). Any plane through the given line

$$2x - y + 3z + 1 + \lambda(x + y + z + 3) = 0$$

(From $S + \lambda S' = 0$)

If this plane is parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$, then the normal to the plane is also perpendicular to the above line or

$$(2 + \lambda)1 + (\lambda - 1)2 + (3 + \lambda)3 = 0.$$

(From $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$)

This gives $\lambda = -\frac{3}{2}$ and the required plane is

$$x - 5y + 3z - 7 = 0.$$

Question

Statement-1: If the straight lines

$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ and

$\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k} + \mu(3\mathbf{i} + a\mathbf{j} + 2\mathbf{k})$ intersect at a point, then the integer a is equal to -5

Statement-2: The plane $x + y - z = K$ touches the sphere $x^2 + y^2 + z^2 - 6x + 8y + 2z + 1 = 0$, then $K^2 = 75$

Ans. (b)

Solution In statement-1, Since the lines intersect, the shortest distance between the lines is zero.

$$\Rightarrow \quad [(2-1)\mathbf{i} + (3-2)\mathbf{j} + (1-3)\mathbf{k}].(a\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \times (3\mathbf{i} + a\mathbf{j} + 2\mathbf{k}) = 0$$

$$\Rightarrow \quad \begin{vmatrix} 1 & 1 & -2 \\ a & 2 & 3 \\ 3 & a & 2 \end{vmatrix} = 0 \Rightarrow 2a^2 + 5a - 25 = 0$$

$$\Rightarrow \quad a = 5/2, \text{ or } -5.$$

as the integral value of a is – 5, the statement-1 is true.
In statement-2, centre of the sphere is (3, – 4, – 1) and the radius is $\sqrt{9+16+1-1} = 5$.

Distance of the centre from the plane is $\left|\frac{3-4+1-K}{\sqrt{3}}\right| = 5$, the radius of the sphere

$\Rightarrow$ $K^2 = 3 \times 25 = 75$ so the statement-2 is also true but does not imply statement-1.

Question

The equation of the plane containing the line

$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and the point (0, 7, – 7) is

(a) $x + y + z = 2$ (b) $x + y + z = 3$
(c) $x + y + z = 0$ (d) none of these

Solution

(c). Any plane containing $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ is

$$a(x + 1) + b(y - 3) + c(z + 2) = 0, \quad ...(1)$$

where $$-3a + 2b + c = 0. \quad ...(2)$$

If the plane through (0, 7, – 7), then

$$a + 4b - 5c = 0 \quad ...(3)$$

From (2) and (3), $\frac{a}{-10-4} = \frac{b}{1-15} = \frac{c}{-12-2}$,

i.e., $\frac{a}{1} = \frac{b}{1} = \frac{c}{1}$.

Hence the plane (1) becomes

$$(x + 1) + (y - 3) + (z + 2) = 0, \text{ i.e., } x + y + z = 0.$$

Question

The reflection of the point A (1, 0, 0) in the line

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} \text{ is}$$

(a) (3, – 4, – 2) (b) (5, – 8, – 4) (c) (1, –1, – 10) (d) (2, – 3, 8)

Ans. (b)

Solution Any point P on the given line is $(2r + 1, -3r - 1, 8r - 10)$

So the direction ratios of AP are $2r, -3r - 1, 8r - 10$.

Now AP is perpendicular to the given line if

$$2(2r) - 3(-3r - 1) + 8(8r - 10) = 0$$

$$\Rightarrow \quad 77r - 77 = 0 \quad \Rightarrow \quad r = 1$$

and thus the coordinates of P, the foot of the perpendicular from A on the line are (3, – 4, – 2).

Let $B(a, b, c)$ be the reflection of A in the given line. Then P is the mid-point of AB

$$\frac{a+1}{2} = 3, \frac{b}{2} = -4, \frac{c}{2} = -2 \Rightarrow a = 5, b = -8, c = -4$$

Thus the coordinates of required point are (5, – 8, – 4).

Question

The equation of the plane passing through the straight line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{4}$ and perpendicular to the plane $x + 2y + z = 12$ is

(a) $9x + 2y - 5z + 4 = 0$ (b) $9x - 2y - 5z + 4 = 0$

(c) $9x + 2y + 5z + 4 = 0$ (d) none of these

Solution

(b). Any plane through the given line is

$$a(x-1)+b(y+1)+c(z-3)=0, \quad ...(1)$$

where $2a-b+4c=0 \quad ...(2)$

If this plane is perpendicular to $x+2y+z=12$, then their normals are also perpendicular to each other.

$\therefore \quad a+2b+c=0 \quad ...(3)$

From (2) and (3), $\frac{a}{-1-8}=\frac{b}{4-2}=\frac{c}{4+1}$,

i.e., $\frac{a}{-9}=\frac{b}{2}=\frac{c}{5}$.

$\therefore$ plane (1) becomes

$$-9(x-1)+2(y+1)+5(z-3)=0.$$

i.e., $9x-2y-5z+4=0$.

Question

An equation of the plane passing through the line of intersection of the planes $x+y+z=6$ and $2x+3y+4z+5=0$ and passing through (1, 1, 1) is

(a) $2x+3y+4z=9$ (b) $x+y+z=3$
(c) $x+2y+3z=6$ (d) $20x+23y+26z=69$.

Ans. (d)

Solution Equation of any plane through the line of intersection of the given planes is $2x+3y+4z+5+\lambda(x+y+z-6)=0$
It passes through (1, 1, 1) if $(2+3+4+5)+\lambda(1+1+1-6)=0$
$\Rightarrow \lambda=14/3$ and the required equation is therefore, $20x+23y+26z=69$.

Question

The position vectors of points A and B are $\hat{i} - \hat{j} + 3\hat{k}$ and $3\hat{i} + 3\hat{j} + 3\hat{k}$ respectively. The equation of a plane is $\mathbf{r} . (5\hat{i} + 2\hat{j} - 7\hat{k}) + 9 = 0$. The points A and B

(a) lie on the plane
(b) are on the same side of the plane
(c) are on the opposite side of the plane
(d) none of these

Solution

(c). The position vectors of two given points are $\mathbf{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\mathbf{b} = 3\hat{i} + 3\hat{j} + 3\hat{k}$ and the equation of the given plane is $\mathbf{r} \cdot (5\hat{i} + 2\hat{j} - 7\hat{k}) + 9 = 0$ or $\mathbf{r} \cdot \mathbf{n} + \mathbf{d} = 0$.

We have, $\mathbf{a} \cdot \mathbf{n} + \mathbf{d} = (\hat{i} - \hat{j} - 3\hat{k}) \cdot (5\hat{i} + 2\hat{j} - 7\hat{k}) + 9$
$= 5 - 2 - 21 + 9 < 0$

and $\mathbf{b} \cdot \mathbf{n} + d = (3\hat{i} + 3\hat{j} - 3\hat{k}) \cdot (5\hat{i} + 2\hat{j} - 7\hat{k}) + 9$
$= 15 + 6 - 21 + 9 > 0$

So, the points **a** and **b** are on the opposite sides of the plane.

Question

Equation of the plane through three points A, B, C with position vectors $-6\mathbf{i}+3\mathbf{j}+2\mathbf{k}, 3\mathbf{i}-2\mathbf{j}+4\mathbf{k}, 5\mathbf{i}+7\mathbf{j}+3\mathbf{k}$ is

(a) $\mathbf{r}.(\mathbf{i}-\mathbf{j}+7\mathbf{k})+23=0$ (b) $\mathbf{r}.(\mathbf{i}+\mathbf{j}+7\mathbf{k})=23$

(c) $\mathbf{r}.(\mathbf{i}+\mathbf{j}-7\mathbf{k})+23=0$ (d) $\mathbf{r}.(\mathbf{i}-\mathbf{j}-7\mathbf{k})=23$

Ans. (a)

Solution Equation of the plane passing through three points A, B, C with position vectors **a**, **b**, **c** is

$$\mathbf{r}.(\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a})=\mathbf{a}\,.\,\mathbf{b}\times\mathbf{c}$$

So that if **a**, **b**, **c** represent the given vectors, then

$$(\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a})=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -6 & 3 & 2\\ 3 & -2 & 4\end{vmatrix}+\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 3 & -2 & 4\\ 5 & 7 & 3\end{vmatrix}+\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 5 & 7 & 3\\ -6 & 3 & 2\end{vmatrix}$$

$$=\mathbf{i}\,(12+4-6-28+14-9)-\mathbf{j}\,(-24-6+9-20+10+18)+\mathbf{k}\,(12-9+21+10+15+42)=-13\mathbf{i}+13\mathbf{j}-91\mathbf{k}$$

and

$$\mathbf{a}.\mathbf{b}\times\mathbf{c}=\begin{vmatrix}-6 & 3 & 2\\ 3 & -2 & 4\\ 5 & 7 & 3\end{vmatrix}=299$$

So the required equation of the plane is

$$\mathbf{r}.(-13\mathbf{i}+13\mathbf{j}-91\mathbf{k})=299$$

or $$\mathbf{r}.(\mathbf{i}-\mathbf{j}+7\mathbf{k})+23=0$$

Question

Lines $\mathbf{r}=\mathbf{a}_1+t\mathbf{b}_1$ and $\mathbf{r}=\mathbf{a}_2+s\mathbf{b}_2$ lie on a plane if

(a) $\mathbf{a}_1\times\mathbf{a}_2=\mathbf{O}$ (b) $\mathbf{b}_1\times\mathbf{b}_2=\mathbf{O}$

(c) $(\mathbf{a}_2-\mathbf{a}_1).(\mathbf{b}_1\times\mathbf{b}_2)=\mathbf{O}$ (d) none of these

Solution

(c). Lines lie in a plane if

$$(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2) = 0$$

$\because$ $\mathbf{b}_1 \times \mathbf{b}_2$ is a vector $\perp$ to $\mathbf{b}_1, \mathbf{b}_2$.

Question

The lines whose vector equations are

$$\mathbf{r} = \mathbf{a} + t\mathbf{b}, \mathbf{r} = \mathbf{c} + t'\mathbf{d}$$

are coplanar if

(a) $(\mathbf{a} - \mathbf{b}) \cdot \mathbf{c} \times \mathbf{d}) = 0$ (b) $(\mathbf{a} - \mathbf{c}) \cdot \mathbf{b} \times \mathbf{d} = 0$

(c) $(\mathbf{b} - \mathbf{c}) \cdot \mathbf{a} \times \mathbf{d} = 0$ (d) $(\mathbf{b} - \mathbf{d}) \cdot \mathbf{a} \times \mathbf{c} = 0$

Ans. (b)

Solution The given lines are coplanar, if the normal to the plane containing these lines is perpendicular to both of them. Since the given lines are parallel to the vectors **b** and **d**, the normal to the plane is parallel to $\mathbf{b} \times \mathbf{d}$, which is perpendicular to the line joining the points on the plane with position vectors **a** and **c** $\Rightarrow (\mathbf{a} - \mathbf{c}) \cdot \mathbf{b} \times \mathbf{d} = 0$, which is the required condition for the given lines to be coplanar.

Question

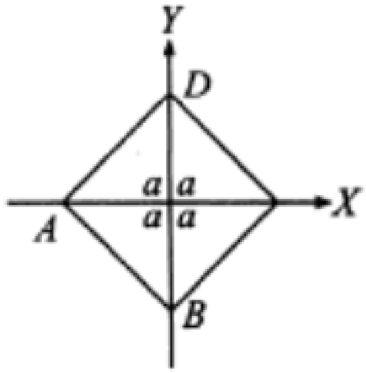
A square $ABCD$ of diagonal $2a$ is folded along the diagonal AC so that the planes DAC and BAC are at right angle. The shortest distance between DC and AB is

(a) $\sqrt{2}a$ (b) $2a/\sqrt{3}$

(c) $2a/\sqrt{5}$ (d) $(\sqrt{3}/2)a$

Solution

(b). When folded coordinates will be $D(0, 0, a)$; $C(a, 0, 0)$; $A(-a, 0, 0)$; $B(0, -a, 0)$



Equation of DC is,

$$\frac{x}{a} = \frac{y}{0} = \frac{z-a}{-a}$$

Equation of AB is,

$$\frac{x+a}{a} = \frac{y}{-a} = \frac{z}{0}$$

$\therefore$ Shortest distance $= \frac{2a}{\sqrt{3}}$.

Question

The shortest distance between the skew lines $l_1 : \mathbf{r} = \mathbf{a}_1 + \lambda\,\mathbf{b}_1$ and $l_2 : \mathbf{r} = \mathbf{a}_2 + \mu\,\mathbf{b}_2$ is

(a) $\frac{|(\mathbf{a}_2 - \mathbf{a}_1)\cdot \mathbf{b}_1 \times \mathbf{b}_2|}{|\mathbf{b}_1 \times \mathbf{b}_2|}$

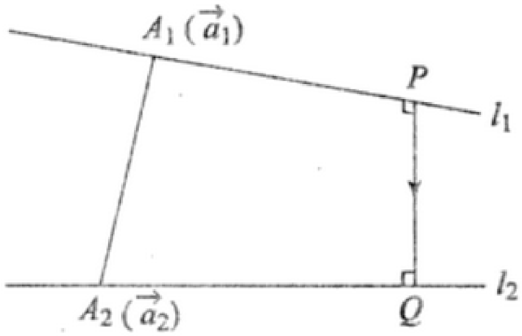
(b) $\frac{|(\mathbf{a}_1 - \mathbf{b}_1)\cdot \mathbf{a}_2 \times \mathbf{b}_2|}{|\mathbf{b}_1 \times \mathbf{b}_2|}$

(c) $\frac{|(\mathbf{a}_2 - \mathbf{b}_2)\cdot \mathbf{a}_1 \times \mathbf{b}_1|}{|\mathbf{b}_1 \times \mathbf{b}_2|}$

(d) $\frac{|(\mathbf{a}_1 - \mathbf{b}_2)\cdot \mathbf{b}_1 \times \mathbf{a}_2|}{|\mathbf{b}_1 \times \mathbf{a}_2|}$

Ans. (a)

Solution Let PQ be the shortest distance vector between l_1 and l_2. Now l_1 passes through A_1 $(\mathbf{a}_1)$ and is parallel to $\mathbf{b}_1$ and l_2 passes through A_2 $(\mathbf{a}_2)$ and is parallel to $\mathbf{b}_2$. Since **PQ** is perpendicular to both l_1 and l_2 it is parallel to $\mathbf{b}_1 \times \mathbf{b}_2$.



Let $\mathbf{n}$ be the unit vector along **PQ**.

Then $\mathbf{n} = \dfrac{\mathbf{b}_1 \times \mathbf{b}_2}{|\mathbf{b}_1 \times \mathbf{b}_2|}$.

Let d be the shortest distance between the given lines l_1 and l_2

$|\mathbf{PQ}| = d$ and $\mathbf{PQ} = d\,\hat{n}$.

Next PQ being the line of shortest distance between l_1 and l_2 is the projection of the line joining the points A_1 $(\mathbf{a}_1)$ and A_2 $(\mathbf{a}_2)$ on $\hat{n}$.

$$|\mathbf{PQ}| = |\mathbf{A}_1\mathbf{A}_2 \cdot \mathbf{n}| \quad \Rightarrow \quad d = \left| \frac{(\mathbf{a}_2 - \mathbf{a}_1).\, \mathbf{b}_1 \times \mathbf{b}_2}{|\mathbf{b}_1 \times \mathbf{b}_2|} \right|$$

Question

The equation of the plane containing the line $\mathbf{r} = \hat{i} + \hat{j} + t(2\hat{i} + \hat{j} + 4\hat{k})$, is

(a) $\mathbf{r} \cdot (\hat{i} + 2\hat{j} - \hat{k}) = 3$

(b) $\mathbf{r} \cdot (\hat{i} + 2\hat{j} - \hat{k}) = 6$

(c) $\mathbf{r} \cdot (-\hat{i} - 2\hat{j} + \hat{k}) = 3$

(d) none of these

Solution

(a). The position vector of any point on the given line is

$\hat{i} + \hat{j} + t\ (2\hat{i} + \hat{j} + 4\hat{k}) = (1 + 2t)\ \hat{i}\ + (1 + t)\ \hat{j}\ + 4t\hat{k}$

This lies on $\mathbf{r} \cdot (\hat{i} + 2\hat{j} - \hat{k}) = 3$

if $\quad (1 + 2t) \cdot 1 + (1 + t) \cdot 2 + 4t\,(-1) = 3$

i.e., if $\quad 1 + 2t + 2 + 2t - 4t = 3$. i.e., if $3 = 3$ which is true.

Hence the plane $\mathbf{r} \cdot (\hat{i} + 2\hat{j} - \hat{k}) = 3$ contains the given line.

Question

The foot of the perpendicular from the origin to the join of $A\,(-9, 4, 5)$ and $B\,(11, 0, -1)$ divides AB in the ratio

(a) 2 : 3 (b) 3 : 2
(c) 1 : 1 (d) none of these

Ans: (c)

Solution Let D be the foot of the perpendicular from the origin to the join of A and B and divide AB in the ratio $k : 1$, then the coordinates of D are

$$\left(\frac{11k-9}{k+1}, \frac{0k+4}{k+1}, \frac{-k+5}{k+1}\right)$$

So that the direction cosines of OD are proportional to

$$11k-9, 4, 5-k$$

and direction cosines of AB are proportional to

$$11+9,\ 0-4,\ -1-5 \text{ i.e. } 20, -4, -6. \text{ or } 10, -2, -3.$$

Since OD is perpendicular to AB.

$$10\,(11k-9) - 2(4) - 3(5-k) = 0$$

$$\Rightarrow \quad 110k - 90 - 8 - 15 + 3k = 0 \Rightarrow \quad 113k = 113 \quad \Rightarrow \quad k = 1$$

Question

The line of intersection of the planes $\mathbf{r}\cdot(3\hat{i}-\hat{j}+\hat{k})=1$ and $\mathbf{r}\cdot(\hat{i}+4\hat{j}-2\hat{k})=2$ is parallel to the vector

(a) $-2\hat{i}+7\hat{j}+13\hat{k}$ (b) $2\hat{i}+7\hat{j}-13\hat{k}$

(c) $-2\hat{i}-7\hat{j}+13\hat{k}$ (d) $2\hat{i}+7\hat{j}+13\hat{k}$

Solution

(a). The line of intersection of the planes

$\mathbf{r}\cdot(3\hat{i}-\hat{j}+\hat{k})=1$ and

$\mathbf{r}\cdot(\hat{i}+4\hat{j}-2\hat{k})=2$ is $\perp$ to each of the normal vectors

$$\mathbf{n}_1 = 3\hat{i}-\hat{j}+\hat{k} \text{ and } \mathbf{n}_2 = \hat{i}+4\hat{j}-2\hat{k}$$

$\therefore$ It is parallel to the vector

$$\mathbf{n}_1\times\mathbf{n}_2 = (3\hat{i}-\hat{j}+\hat{k})\times(\hat{i}+4\hat{j}-2\hat{k})$$

$$= -2\hat{i}+7\hat{j}+13\hat{k}.$$

Question

Cosine of the angle between the lines whose vector equations are $\mathbf{r} = 3\mathbf{i}+2\mathbf{j}-4\mathbf{k}+\lambda(\mathbf{i}+2\mathbf{j}+2\mathbf{k})$ and $\mathbf{r} = 5\mathbf{i}-2\mathbf{k}+\mu(3\mathbf{i}+2\mathbf{j}+6\mathbf{k})$; λ, μ being parameters, is

(a) $-1/3\sqrt{29}$ (b) $3/7\sqrt{29}$ (c) $23/29$ (d) $19/21$

Ans. (d)

Solution Direction vectors of the given lines are

$$\mathbf{i}+2\mathbf{j}+2\mathbf{k} \text{ and } 3\mathbf{i}+2\mathbf{j}+6\mathbf{k}.$$

The angle θ between the given lines is equal to the angle between these vectors.

Hence $\cos\theta = \frac{(\mathbf{i}+2\mathbf{j}+2\mathbf{k})}{\sqrt{1+4+4}} \cdot \frac{(3\mathbf{i}+2\mathbf{j}+6\mathbf{k})}{\sqrt{9+4+36}} = \frac{3+4+12}{3\times 7} = \frac{19}{21}$

Question

Gives the line L : $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-3}{-1}$ and the plane $\pi : x - 2y = 0$. Of the following assertions, the only one that is always true is

(a) L is $\perp$ to π (b) L lies in π

(c) L is parallel to π (d) none of these

Solution

(b). Since $3(1) + 2(-2) + (-1)(-1) = 3 - 4 + 1 = 0$,

$\therefore$ given line is $\perp$ to the normal to the plane i.e., given line is parallel to the given plane.

Also $(1, -1, 3)$ lies on the plane $x - 2y - z = 0$ if

$$1 - 2(-1) - 3 = 0 \text{ i.e., } 1 + 2 - 3 = 0$$

which is true $\therefore$ L lies in plane π.

Question

The cartesian equation of the plane passing through the line of intersection of the planes $\mathbf{r} \cdot (2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) = 1$ and $\mathbf{r} \cdot (\mathbf{i} - \mathbf{j}) + 4 = 0$ and perpendicular to the plane $\mathbf{r} \cdot (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + 8 = 0$ is

(a) $3x - 4y + 4z = 5$ (b) $x - 2y + 4z = 3$
(c) $5x - 2y - 12z + 47 = 0$ (d) $2x + 3y + 4 = 0$

Ans. (c)

Solution Equation of any plane passing through the intersection of the planes $\mathbf{r} \cdot (2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) = 1$ and $\mathbf{r} \cdot (\mathbf{i} - \mathbf{j}) + 4 = 0$ is

$$2x - 3y + 4z - 1 + \lambda (x - y + 4) = 0$$

or $$(2 + \lambda)x - (3 + \lambda)y + 4z + 4\lambda - 1 = 0$$

This plane is perpendicular to the plane $\mathbf{r} \cdot (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + 8 = 0$ if

$\Rightarrow \quad 2(2 + \lambda) + (3 + \lambda) + 4 = 0.$

$\Rightarrow \quad 11 + 3\lambda = 0 \quad \Rightarrow \quad \lambda = -11/3.$

and the required equation of the plane is

$$3(2x - 3y + 4z - 1) - 11(x - y + 4) = 0 \Rightarrow 5x - 2y - 12z + 47 = 0$$

Question

Radius of the circle $\mathbf{r}^2 + \mathbf{r} \cdot (2\hat{i} - 2\hat{j} - 4\hat{k}) - 19 = 0$

$\mathbf{r} \cdot (\hat{i} - 2\hat{j} + 4\hat{k}) + 8 = 0$

(a) 5 (b) 4
(c) 3 (d) 2

Solution

(b). Given circle is intersection of sphere

$$x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0 \quad \text{...(i)}$$

and plane

$$x - 2y + 2z + 8 = 0 \quad \text{...(ii)}$$

Centre of sphere is (– 1, 1, 2).

p = Length of the ⊥ from, (– 1, 1, 2) upon (ii)

$$= \frac{-1-2+4+8}{\sqrt{1+4+4}} = \frac{9}{3} = 3$$

R = Radius of the sphere = $\sqrt{1+1+4+19} = 5$

Radius of the circle = $\sqrt{R^2 - p^2} = \sqrt{25-9} = \sqrt{16} = 4.$

Question

The shortest distances between the lines

$\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$ and $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$ is

(a) $4\sqrt{5}$ (b) $4\sqrt{17}$ (c) $4\sqrt{3}$ (d) $8\sqrt{2}$

Ans. (c)

Solution Let l, m, n be the direction cosines of the line of shortest distance, then as it is perpendicular to the given lines $2l - 7m + 5n = 0$; $2l + m - 3n = 0$

$$\Rightarrow \quad \frac{l}{16} = \frac{m}{16} = \frac{n}{16} \Rightarrow \frac{l}{1} = \frac{m}{1} = \frac{n}{1} = \frac{\sqrt{l^2 + m^2 + n^2}}{\sqrt{1+1+1}}$$

$$\Rightarrow \quad l = m = n = 1/\sqrt{3}.$$

Now the shortest distance between the given lines is the projection of the join of the points (3, –15, 9) and (– 1, 1, 9) on the line of shortest distance. Hence the required distance = $\frac{1}{\sqrt{3}} |(3+1) + (-15-1) + 9 - 9| = 4\sqrt{3}$.

Question

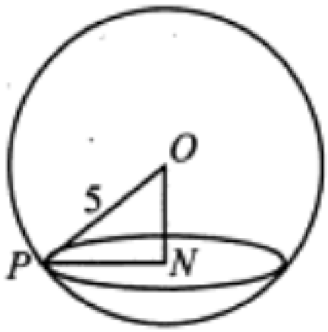
The position vector of the centre of the circle $|\mathbf{r}| = 5$, $\mathbf{r}\cdot(\hat{i}+\hat{j}+\hat{k}) = 3\sqrt{3}$ is

(a) $\sqrt{3}(\hat{i}+\hat{j}+\hat{k})$ (b) $\hat{i}+\hat{j}+\hat{k}$

(c) $3(\hat{i}+\hat{j}+\hat{k})$ (d) none of the above

Solution

(a). The equation of ON is $\mathbf{r} = \lambda\,(\mathbf{i}+\mathbf{j}+\mathbf{k})$ ...(i)



Since it passes through the origin and is parallel to the vector $(\mathbf{i}+\mathbf{j}+\mathbf{k})$, any pt. on it is $\lambda\,(\mathbf{i}+\mathbf{j}+\mathbf{k})$. If this pt. lies on the plane $\mathbf{r}\cdot(\mathbf{i}+\mathbf{j}+\mathbf{k}) = 3\sqrt{3}$

then $\lambda\,(\mathbf{i}+\mathbf{j}+\mathbf{k})\cdot(\mathbf{i}+\mathbf{j}+\mathbf{k}) = 3\sqrt{3}$

or $\lambda\,(1+1+1) = 3\sqrt{3}$

$\therefore\quad \lambda = \sqrt{3}$

Putting the value of λ in (*i*), we get the position vector N i.e., centre of the circle as $\sqrt{3}(\mathbf{i}+\mathbf{j}+\mathbf{k})$.

Question

The centre of the sphere

$(x + 1)(x - 1) + (y - 2)(y + 2) + (z - 3)(z + 3) = 0$ is

(a) $(-1, 2, 3)$ (b) $(1, -2, -3)$ (c) $(0, 0, 0)$ (d) $(1, 2, 3)$

Ans. (c)

Solution The given equation is the equation of a sphere on the line joining $(-1, 2, 3)$ and $(1, -2, -3)$ as a diameter and hence the mid-point $(0, 0, 0)$ of this line segment is the centre of the sphere.

Question

Equation of the line passing through (1, 1, 1) and parallel to the plane $2x + 3y + z + 5 = 0$ is

(a) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{1}$

(b) $\frac{x-1}{-1} = \frac{y-1}{1} = \frac{z-1}{-1}$

(c) $\frac{x-1}{3} = \frac{y-1}{2} = \frac{z-1}{1}$

(d) $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{1}$

Solution

(b). If the direction ratios of the line are l, m, n then it is perpendicular to the normal to the plane.

$\therefore \quad 2l + 3m + n = 0.$

And the only values of l, m, n that satisfy this equation are $-1, 1, -1$.

$\therefore$ (b) is the correct answer.

Question

If (u, v, w) be the centre of the sphere which passes through the points (0, 0, 0), (0, 2, 0), (1, 0, 0) and (0, 0, 4) then $u + v + w$ is equal to

(a) 3/2 (b) 5/2 (c) 7/2 (d) 9/2

Ans. (c)

Solution Let the equation of the sphere be

$x^2 + y^2 + z^2 - 2ux - 2vy - 2wz + d = 0.$

Since the sphere passes through the given points

$$d = 0,\ 4 - 4v = 0,\ 1 - 2u = 0,\ 16 - 8w = 0$$

$$\Rightarrow \quad u = 1/2,\ v = 1,\ w = 2 \quad \Rightarrow \quad u + v + w = 1/2 + 1 + 2 = 7/2.$$

Question

The distance between the line $\mathbf{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ and the plane $\mathbf{r}\cdot(\hat{i} + 5\hat{j} + \hat{k}) = 5$ is

(a) $\frac{10}{9}$ (b) $\frac{10}{3\sqrt{3}}$

(c) $\frac{10}{3}$ (d) none of these

Solution

(b). The given line is $\mathbf{r} = \mathbf{a} + t\mathbf{b}$
where $\mathbf{a} = 2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{b} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and given plane is $\mathbf{r} \cdot \mathbf{n} = \mathbf{p}$,
where $\mathbf{n} = \hat{i} + 5\hat{j} + \hat{k}$, $p = 5$.

Since $\mathbf{b} \cdot \mathbf{n} = 1 - 5 + 4 = 0$,

$\therefore$ given line is parallel to the given plane $\therefore$ the distance between the line and the plane is equal to length of the perpendicular from the point $\mathbf{a} = 2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ on the line in the given plane.

$$\therefore \text{ Reqd. distance} = \left| \frac{(2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})\cdot(\mathbf{i} + 5\mathbf{j} + \mathbf{k}) - 5}{\sqrt{1 + 25 + 1}} \right|$$

$$= \left| \frac{2 - 10 + 3 - 5}{\sqrt{27}} \right| = \frac{10}{3\sqrt{3}}.$$

Question

The plane $2x + 2y - z = k$ touches the sphere $x^2 + y^2 + z^2 - 4x + 2y - 6z + 5 = 0$ and makes a positive intercept on the axis of z, then $k =$

(a) -10 (b) -8 (c) 8 (d) 10

Ans. (a)

Solution Centre of the given sphere is $(2, -1, 3)$ and is radius is $\sqrt{4 + 1 + 9 - 5} = 3$. As the given plane touches the given sphere

$$\frac{2 \times 2 + 2(-1) - (3) - k}{\sqrt{2^2 + 2^2 + (-1)^2}} = \pm 3 \quad \Rightarrow \quad k = 8 \quad \text{or} \quad k = -10$$

Since the plane makes a positive intercept on z-axis $k < 0$ and the required value of $k = -10$.

Question

The radius of the sphere passing through the point (α, β, γ) and the circle $x^2 + y^2 = a^2, z = 0$ is

(a) k (b) $\sqrt{\frac{k^2}{4}+a^2}$ (c) $\sqrt{\frac{k^2}{4}-a^2}$ (d) $\sqrt{k^2+a^2}$

where $k = (a^2 - \alpha^2 - \beta^2 - \gamma^2)/\gamma$

Ans. (b)

Solution Equation of any sphere passing through the given circle is

$$x^2 + y^2 + z^2 - a^2 + \lambda z = 0 \qquad (1)$$

If it passes through the point (α, β, γ), then $\alpha^2 + \beta^2 + \gamma^2 - a^2 + \lambda\gamma = 0$

$$\Rightarrow \qquad \lambda = \left(a^2 - \alpha^2 - \beta^2 - \gamma^2\right)/\gamma = k$$

Centre of the sphere (1) is $(0, 0, -\lambda/2)$ and the radius = $\sqrt{\frac{k^2}{4}+a^2}$.

Question

The locus of $x^2 + y^2 + z^2 = 0$ is

(a) a circle (b) a sphere

(c) (0, 0, 0) (d) none of these

Solution

(c). $x^2 + y^2 + z^2 = 0 \Rightarrow x = 0, y = 0, z = 0.$

Question

$L: x - 1 = y + 1 = z, S: x^2 + y^2 + z^2 = 14.$

Statement-1: L is at a distance $\sqrt{2}$ from the centre of the sphere S.

Statement-2: Intercept made by the line L on the sphere S is of length $4\sqrt{3}$.

Solution

L meets the spheres S at the points $P(3, 1, 2)$ and $Q(-1, -3, -2)$

$\Rightarrow \quad PQ = \sqrt{(3+1)^2 + (1+3)^2 + (2+2)^2} = 4\sqrt{3}$ so the statement-2 is True.

Distance of L from the centre of the sphere.

$$= \sqrt{14 - \left(2\sqrt{3}\right)^2} = \sqrt{2}$$

Showing the statement-1 is true using statement-2.

Question

The locus of a point, such that the sum of the squares of its distances from the planes $x + y + z = 0$, $x - z = 0$ and $x - 2y + z = 0$ is 9, is

(a) $x^2 + y^2 + z^2 = 3$ (b) $x^2 + y^2 + z^2 = 6$

(c) $x^2 + y^2 + z^2 = 9$ (d) $x^2 + y^2 + z^2 = 12$

Solution

(c). Let the variable point be (α, β, γ), then according to the question

$$\left(\frac{|\alpha+\beta+r|}{\sqrt{3}}\right)^2 + \left(\frac{|\alpha-\gamma|}{\sqrt{2}}\right)^2 + \left(\frac{|\alpha-2\beta+\gamma|}{\sqrt{6}}\right)^2 = 9$$

$\Rightarrow \quad \alpha^2 + \beta^2 + \gamma^2 = 9.$

So, the locus of the point is $x^2 + y^2 + z^2 = 9$.

Question

The sphere on the join of $(2, 3, 5)$ and $(4, 9, -3)$ as the extremities of a diameter meets the line $x = y = z$ at the point (k, k, k), then k satisfies

(a) $3k^2 + 20k - 20 = 0$ (b) $3k^2 - 20k + 20 = 0$

(c) $3k^2 - 10k + 10 = 0$ (d) $3k^2 + 10k - 10 = 0$

Solution

Equation of the sphere is

$$(x-2)(x-4)+(y-3)(y-9)+(z-5)(z+3)=0$$

meets $x=y=z=k$ at $3k^2-20k+20=0$.

Question

The coordinates of a point which is equidistant from the points (o, o, o), (a, o, o), (o, b, o) and (o, o, c) are given by

(a) $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$ (b) $\left(\frac{-a}{2}, \frac{-b}{2}, \frac{c}{2}\right)$

(c) $\left(\frac{a}{2}, \frac{-b}{2}, \frac{-c}{2}\right)$ (d) $\left(\frac{-a}{2}, \frac{b}{2}, \frac{-c}{2}\right)$

Solution

(a). Sphere passing through $(a, 0, 0)$ $(0, b, 0)$ $(0, 0, c)$ and $(0, 0, 0)$ is $x^2+y^2+z^2-ax-by-cz=0$. Its centre $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$ is equidistant from given points.

Question

Equation of the sphere through the circle $x^2+y^2+z^2=16$, $3x+4y+5z+1=0$ and the point $(2, 3, 4)$ is

(a) $x^2+y^2+z^2-3x-4y-5z=17$

(b) $3x^2+3y^2+3z^2-3x-4y-5z-49=0$

(c) $x^2+y^2+z^2+3x+4y+5z=15$ (d) none of these

Solution

Equation of the required sphere is $x^2 + y^2 + z^2 - 16 + \lambda(3x + 4y + 5z + 1) = 0$. Since it passes through the point (2, 3, 4). $2^2 + 3^2 + 4^2 - 16 + \lambda(6 + 12 + 20 + 1) = 0$ which gives $\lambda = -1/3$.

Question

Perpendicular distance of the point (3, 4, 5) from the y-axis, is

(a) $\sqrt{34}$
(b) $\sqrt{41}$
(c) 4
(d) 5

Solution

(a). Distance of (α, β, γ) from y-axis is given by

$$d = \sqrt{\alpha^2 + \gamma^2}$$

$\therefore$ Distance (d) of (3, 4, 5) from y-axis is

$$d = \sqrt{3^2 + 5^2} = \sqrt{9 + 25} = \sqrt{34}.$$

Question

The plane $x + y + z + k = 0$ touches the sphere $x^2 + y^2 + z^2 = k^2$ for

(a) all values of k
(b) only one non zero value of k
(c) finite non zero values of k
(d) no non zero value of k

Solution

Centre of the sphere is (0, 0, 0) and the radius is k. Plane touches the sphere if $\frac{0+0+0+k}{\sqrt{1+1+1}} = \pm k$ which is not true for any non zero values of k.

Question

The number of straight lines that are equally inclined to three dimensional coordinate axes, is

(a) 2 (b) 4

(c) 6 (d) 8

Solution

(b). If α, β, γ are the angles made by the line with x, y, and z-axis respectively, then

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Given $\alpha = \beta = \gamma$, $\therefore$ $3\cos^2\alpha = 1$

or $\cos\alpha = \pm\frac{1}{\sqrt{3}}$

Possible direction cosines are $\left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right)$.

Different sets of Dc's are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$,

$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right), \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

and $\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Thus, four lines are equally inclined to axes.

Question

The sphere $x^2 + y^2 + z^2 - ax - by - cz = 0$ passes through the centre of the sphere

(a) $x^2 + y^2 + z^2 - 2x = a^2$ (b) $x^2 + y^2 + z^2 - 2y = b^2$

(c) $x^2 + y^2 + z^2 - 2z = c^2$ (d) $x^2 + y^2 + z^2 = a^2 + b^2 + c^2$

Solution

The sphere passes through the origin which is centre of the sphere in (d).

Question

The locus of the foot of the perpendicular from the origin on the variable plane through the fixed point $(2, -4, 6)$ is a sphere of radius

(a) $\sqrt{56}$ (b) $\sqrt{14}$

(c) $2\sqrt{12}$ (d) $3\sqrt{10}$

Solution

(b). Let $P(u, v, w)$ be the foot of the perpendicular from the origin to the plane, then OP is normal to the plane, so that direction ratios of the normal to the plane are u, v, w and as it passes through the point $(2, -4, 6)$, its equations is $u(x-2) + v(y+4) + w(z-6) = 0$.

Since (u, v, w) lies on it.

$$u(u-2) + v(v+4) + w(w-6) = 0$$

$$\Rightarrow \quad u^2 + v^2 + w^2 - 2u + 4v - 6w = 0$$

The locus of (u, v, w) is

$$x^2 + y^2 + z^2 - 2x + 4y - 6z = 0.$$

which is a sphere of radius $= \sqrt{1+4+9} = \sqrt{14}$

Question

The foot of the perpendicular from (a, b, c) on the line $x = y = z$ is the point (r, r, r) where

(a) $r = a + b + c$
(b) $r = 3(a + b + c)$
(c) $3r = a + b + c$
(d) none of these.

Solution

Direction ratios of the perpendicular are $r - a$, $r - b$, $r - c$ and those of the line are 1, 1, 1. So $1.(r - a) + 1.(r - b) + 1.(r - c) = 0$

$\Rightarrow \quad 3r = a + b + c.$

Question

The cartesian equation of the plane

$\vec{r} = (1 + \lambda - \mu)\hat{i} + (2 - \lambda)\hat{j} + (3 - 2\lambda + 2\mu)\hat{k}$ is

(a) $2x + y = 5$
(b) $2x - y = 5$
(c) $2x + z = 5$
(d) $2x - z = 5$

Solution

(c). We have,

$$\vec{r} = (1+\lambda-\mu)\hat{i} + (2-\lambda)\hat{j} + (3-2\lambda+2\mu)\hat{k}$$

$$\Rightarrow \quad \vec{r} = (\hat{i}+2\hat{j}+3\hat{k}) + \lambda(\hat{i}-\hat{j}-2\hat{k}) + \mu(-\hat{i}+2\hat{k})$$

which is a plane passing through

$\vec{a} = \hat{i}+2\hat{j}+3\hat{k}$ and parallel to the vectors

$\vec{b} = \hat{i}-\hat{j}-2\hat{k}$ and $\vec{c} = -\hat{i}+2\hat{k}$.

Therefore, it is $\perp$ to the vector

$$\vec{n} = \vec{b}\times\vec{c} = -2\hat{i}-\hat{k}.$$

Hence, its vector equation is $(\vec{r}-\vec{a}).\vec{n} = 0$

$$\Rightarrow \quad \vec{r}.\vec{n} = \vec{a}.\vec{n}$$

$$\Rightarrow \vec{r}.(-2\hat{i}-\hat{k}) = -2-3 \Rightarrow \vec{r}.(2\hat{i}+\hat{k}) = 5$$

So, the cartesian equation is

$$(x\hat{i}+y\hat{j}+z\hat{k}).(2\hat{i}+\hat{k}) = 5$$

or, $2x+z=5$.

Question

The points (1, – 2, 3), (2, 3, – 4) (0, – 7, 10) are the vertices of

(a) a right angled triangles
(b) isosceles triangle
(c) equilateral triangle
(d) none of these

Solution

Lengths of the sides are $\sqrt{(2-1)^2+(3+2)^2+(-4-3)^2} = \sqrt{75}, \sqrt{75}$ and $\sqrt{300}$. So the triangle is isosceles.

Question

The lines $\mathbf{r} = \mathbf{i} - \mathbf{j} + \lambda(2\mathbf{i} + \mathbf{k})$ and $\mathbf{r} = 2\hat{i} - \hat{j} + \mu(\hat{i} + \hat{j} - \hat{k})$

(a) intersect each others

(b) do not intersect

(c) intersect at $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + \mathbf{x}$

(d) are parallel.

Solution

Since $\begin{vmatrix} 2-1 & -1+1 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix} = -1 \neq 0$, the lines do not intersect.

Question

Equation of a line passing through the point whose position vector is $2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ and in the direction of the vector $3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$ is

(a) $4x + 3y = 17,\ 5y - 4z = 1$

(b) $4x - 3y = 17$
$5y + 4z = 1$

(c) $4x + 5y = 12,\ 3y + 4z = 1$

(d) $5y + 4z = 1$
$4x + 3z = 1$

Solution

Equation of the line is $\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k} + \lambda(3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k})$ so the line passes through $(2, -3, 4)$ and the direction ratios are $3, 4, -5$ and so its equation is $\dfrac{x-2}{3} = \dfrac{y+3}{4} = \dfrac{z-4}{-5}$

$\Rightarrow \quad 4x - 3y = 17,\ 5y + 4z = 1$

Question

The plane passing through the points (– 2, – 2, 2) and containing the line joining the points (1, 1, 1) and (1, – 1, 2) makes intersects on the coordinates axes, the sum of whose length is

(a) 3 (b) 4 (c) 6 (d) 12

Solution

Equation of the plane be $a(x+2)+b(y+2)+c(z-2)=0$. As it passes through (1, 1, 1) and (1, – 1, 2), $\frac{a}{1}=\frac{b}{-3}=\frac{c}{-6}$. Equation of the plane is $\frac{x}{-8}+\frac{y}{8/3}+\frac{z}{8/6}=1$ and the required sum = 12.

Question

If the foot of the perpendicular from the origin to a plane is (a, b, c), then the equation of the plane is

(a) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ (b) $ax+by+cz=1$

(c) $ax+by+cz=a^2+b^2+c^2$ (d) $ax+by+cz=0$

Solution

Let $P(a, b, c)$, d.r. of the normal to the plane are a, b, c and as it passes through (a, b, c) its equation is $a(x-a)+b(y-b)+c(z-c)=0$

Question

If $\boldsymbol{r}.\boldsymbol{n}=q$ is the equation of a plane normal to the vector $\boldsymbol{n}$, the length of the perpendicular from the origin on the plane is

(a) q (b) $|\mathbf{x}|$ (c) $q|\mathbf{x}|$ (d) $\frac{q}{|\mathbf{x}|}$

Solution

Equation of the plane is $\mathbf{r}.\frac{\mathbf{n}}{|\mathbf{x}|}=\frac{q}{|\mathbf{n}|}$ i.e., $\mathbf{r}.\mathbf{n}=\frac{q}{|\mathbf{n}|}$. So the required length $=q/|\mathbf{n}|$.

Question

The line of shortest distance between the lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$ intersects the first line at the point

(a) (1, 1, 1) (b) (– 1, – 1, – 1)
(c) (3, 3, 3) (d) (– 3, – 3, – 3)

Solution

Only the point (– 1, – 1, – 1) in (b) lies on the first line.

Question

Equation of the plane passing through the points $\mathbf{i}+\mathbf{j}-2\mathbf{k}$, $2\mathbf{i}-\mathbf{j}+\mathbf{k}$ and $\mathbf{i}+2\mathbf{j}+\mathbf{k}$ is

(a) $\mathbf{r}.(4\mathbf{i}+2\mathbf{j})=20$ (b) $\mathbf{r}.(9\mathbf{i}+3\mathbf{j}-\mathbf{k})=14$
(c) $\mathbf{r}.(9\mathbf{i}+3\mathbf{j}-\mathbf{k})=6$ (d) none of these

Solution

Let the plane be $\mathbf{r}.(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})=d$.

$x+y-2z=2x-y+z=x+2y+z=d.$

$\Rightarrow \quad \frac{x}{9}=\frac{y}{3}=\frac{z}{-1}$ and $d=14$.

Question

If the line $\frac{x-1}{2} = \frac{y-3}{a} = \frac{z+1}{3}$ lies in the plane $bx + 2y + 3z - 4 = 0$, then

(a) $a = 11/2, b = 1$ (b) $a = -5/2, b = -7$

(c) $a = -11/2, b = 1$ (d) $a = 1, b = -11/2$

Solution

$$b \times 1 + 2 \times 3 + 3 \times (-1) - 4 = 0 \quad \Rightarrow \quad b = 1$$
$$2b + 2a + 9 = 0 \quad \Rightarrow \quad a = -11/2.$$

Question

Equation of a plane bisecting an angle between the plane $\mathbf{r}.(\mathbf{i}+2\mathbf{j}+2\mathbf{k}) = 19$ and $\mathbf{r}.(4\mathbf{i}-3\mathbf{j}+12\mathbf{k})+3=0$, passing through the point with position vector $\mathbf{i}+7\mathbf{j}-\mathbf{k}$ is

(a) $\mathbf{r}.(\mathbf{i}+35\mathbf{j}-10\mathbf{k}) - 256 = 0$

(b) $\mathbf{r}.(25\mathbf{i}+17\mathbf{j}+62\mathbf{k})-238=0$

(c) $\mathbf{r}.(\mathbf{i}+2\mathbf{j}+2\mathbf{k})-13=0$ (d) $\mathbf{r}.(4\mathbf{i}-3\mathbf{j}+12\mathbf{k})+29=0$

Solution

Equation of the plane is $\frac{x+2y+2z-19}{\sqrt{1+4+4}} = \pm\frac{4x-3y+12z+3}{\sqrt{16+9+144}}$ since it passes through the vector $\mathbf{i}+7\mathbf{j}-\mathbf{k}$ or the point $(1, 7, -1)$,

$\frac{1+14-2-19}{3} = \pm\frac{4-21-12+3}{13}$ which is true for the + ve sign.

Hence the required plane is $x + 35y - 10z - 256 = 0$ or $\mathbf{r}.(\mathbf{i}+35\mathbf{j}-10\mathbf{k}) - 256 = 0$.

Question

The plane containing the lines

$\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}$ passes through

(a) (0, 0, 0) (b) (1, 0, 1)

(c) (1, – 1, 1) (d) (– 1, 1, 0)

Solution

Let the equation of the plane be $lx + my + mz + d = 0$, $3l + 5m + 7n = 0$ and $l + 4m + 7n = 0$.

$\Rightarrow$ $l/1 = m/-2 = n/1$ and the equation of the plane is $x - 2y + z + d = 0$ since it passes through (– 1, – 3, – 5) and (2, 4, 6), $d = 0$ and the plane passes through (0, 0, 0).

Question

If θ denotes the acute angle between the line $\mathbf{r} = (\mathbf{i} + 2\mathbf{j} - \mathbf{k}) + \lambda(\mathbf{i} - \mathbf{j} + \mathbf{k})$ and the plane $\mathbf{r}.(2\mathbf{i} - \mathbf{j} + \mathbf{k}) = 4$, then $\sin\theta + \sqrt{2}\cos\theta =$

(a) $1/\sqrt{2}$ (b) 1 (c) $\sqrt{2}$ (d) $1+\sqrt{2}$

Solution

Direction ratios of the line are 1, – 1, 1 and that of the normal to the plane are 2, – 1, 1,

so $\sin\theta = \frac{2+1+1}{\sqrt{3}\sqrt{6}} = \frac{4}{\sqrt{18}}$, $\cos\theta = \frac{\sqrt{2}}{\sqrt{18}}$.

$\sin\theta + \sqrt{2}\cos\theta = \frac{4}{\sqrt{18}} + \frac{2}{\sqrt{18}} = \frac{6}{\sqrt{18}} = \sqrt{2}$

Question

The lines $\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}$ and $\frac{x-4}{2}=\frac{y+0}{0}=\frac{z+1}{3}$

(a) do not intersect
(b) intersect
(c) intersect at (4, 0, 4)
(d) intersect at (1, 1, – 1)

Solution

Any point on the first line is $(3r + 1, - r + 1, - 1)$ and a point on the second line is $(2k + 4, 0, 3k - 1)$. The two points are same if $r = 1$ and $k = 0$ and the point of intersection of (4, 0, – 1).

Question

An equation of a line through A (3, 4, – 7) and B (1, – 1, 6) is

(a) $\mathbf{r} = 3\mathbf{i} - 4\mathbf{j} + 7\mathbf{k} + \lambda(2\mathbf{i} - 5\mathbf{j} - 13\mathbf{k})$

(b) $\mathbf{r} = \mathbf{i} - \mathbf{j} + 6\mathbf{k} - \lambda(2\mathbf{i} - 5\mathbf{j} - 13\mathbf{k})$

(c) $\frac{x-3}{2}=\frac{y-4}{-5}=\frac{z+7}{-13}$

(d) $\frac{x-1}{-2}=\frac{y+1}{-5}=\frac{z-6}{13}$

Solution

Equation of the line is $\frac{x-1}{3-1}=\frac{y+1}{4+1}=\frac{z-6}{-7-6}$

Question

If the vector $2\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$ is inclined at angles α, β, γ with the coordinate axes, then

(a) $3\cos\alpha = 2/\sqrt{62}$
(b) $2\cos\beta = -3/\sqrt{62}$
(c) $\cos\gamma = 7/\sqrt{62}$
(d) $2\cos\alpha = -3\cos\beta = 7\cos\gamma$

Solution

$\cos \alpha = 2/\sqrt{62}$, $\cos \beta = -3/\sqrt{62}$, $\cos \gamma = 7/\sqrt{62}$.

Question

The points (0, 7, 10), (– 1, 6, 6) and (– 4, 9, 6) are the vertices of
(a) a right angled isosceles triangle (b) equilateral triangle
(c) an isosceles triangle (b) an obtuse angled triangle

Solution

Length of the sides are 18, 18 and 36.

Question

P (1, 1, 1) and Q (λ, λ, λ) are two points in the space such that $PQ = \sqrt{27}$, the value of λ can be
(a) – 4 (b) – 3 (c) 2 (d) 4

Solution

$3(\lambda - 1)^2 = 27 \quad \Rightarrow \quad \lambda = -2 \text{ or } 4.$

Question

P (0, 5, 6), Q (1, 4, 7), R (2, 3, 7) and S (3, 4, 6) are four points in the space. The point farthest from the origin O (0, 0, 0) is
(a) P (b) Q (c) R (d) S

Solution

$OP^2 = 0 + 5^2 + 6^2 = 61$, $OQ^2 = 66$, $OR^2 = 62$, $OS^2 = 61$

Question

If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of two perpendicular lines, then

(a) $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$
(b) $l_1 l_2 + m_1 m_2 + n_1 n_2 = 1$
(c) $\frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2} = 1$
(d) $\frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2} = 0$

Solution

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2 = 0 \text{ if } \theta = \pi/2.$$

Question

$l = m = n = 1$ represents the direction ratios of

(a) x-axis
(b) y-axis
(c) z-axis
(d) none of these

Solution

Direction ratios of x, y, and z axis are 1,0,0; 0,1,0 and 0,0,1

Question

The coordinate of the foot of the perpendicular from the point (a, b, c) on z-axis is

(a) $(a, 0, 0)$ (b) $(0, b, 0)$ (c) $(0, 0, c)$ (d) $(a, b, 0)$

Solution

For a point on z axis x = 0 and y = 0

Question

An equation of the XOY plane is

(a) $x = 0$ (b) $y = 0$ (c) $z = 0$ (d) $z = c, c \neq 0$

Solution

In this plane z coordinates are 0

Question

The coordinate of the middle point of the line joining the points $(-1, -1, 1)$ and $(-1, 1, -1)$ are

(a) $(0, 0, 0)$ (b) $(-1, 0, 0)$ (c) $(0, -1, 1)$ (d) $(0, 1, -1)$.

Solution

$$\text{Coordinates are } \left(\frac{-1-1}{2}, \frac{-1+1}{2}, \frac{1-1}{2}\right) = (-1, 0, 0)$$

:-{D

To recall standard integrals

$f(x)$	$\int f(x)dx$	$f(x)$	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1}$ $(n \neq -1)$	$[g(x)]^n g'(x)$	$\frac{[g(x)]^{n+1}}{n+1}$ $(n \neq -1)$
$\frac{1}{x}$	$\ln\|x\|$	$\frac{g'(x)}{g(x)}$	$\ln\|g(x)\|$
e^x	e^x	a^x	$\frac{a^x}{\ln a}$ $(a > 0)$
$\sin x$	$-\cos x$	$\sinh x$	$\cosh x$
$\cos x$	$\sin x$	$\cosh x$	$\sinh x$
$\tan x$	$-\ln\|\cos x\|$	$\tanh x$	$\ln \cosh x$
$\operatorname{cosec} x$	$\ln\left\|\tan \frac{x}{2}\right\|$	$\operatorname{cosech} x$	$\ln\left\|\tanh \frac{x}{2}\right\|$
$\sec x$	$\ln\|\sec x + \tan x\|$	$\operatorname{sech} x$	$2\tan^{-1} e^x$
$\sec^2 x$	$\tan x$	$\operatorname{sech}^2 x$	$\tanh x$
$\cot x$	$\ln\|\sin x\|$	$\coth x$	$\ln\|\sinh x\|$
$\sin^2 x$	$\frac{x}{2} - \frac{\sin 2x}{4}$	$\sinh^2 x$	$\frac{\sinh 2x}{4} - \frac{x}{2}$
$\cos^2 x$	$\frac{x}{2} + \frac{\sin 2x}{4}$	$\cosh^2 x$	$\frac{\sinh 2x}{4} + \frac{x}{2}$

$f(x)$	$\int f(x)\,dx$	$f(x)$	$\int f(x)\,dx$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a}$	$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert \quad (0<\lvert x\rvert<a)$
	$(a>0)$	$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert \quad (\lvert x\rvert>a>0)$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\frac{x}{a}$	$\frac{1}{\sqrt{a^2+x^2}}$	$\ln\left\lvert\frac{x+\sqrt{a^2+x^2}}{a}\right\rvert \quad (a>0)$
	$(-a<x<a)$	$\frac{1}{\sqrt{x^2-a^2}}$	$\ln\left\lvert\frac{x+\sqrt{x^2-a^2}}{a}\right\rvert \quad (x>a>0)$
$\sqrt{a^2-x^2}$	$\frac{a^2}{2}\Big[\sin^{-1}\left(\frac{x}{a}\right)$	$\sqrt{a^2+x^2}$	$\frac{a^2}{2}\left[\sinh^{-1}\left(\frac{x}{a}\right)+\frac{x\sqrt{a^2+x^2}}{a^2}\right]$
	$+\frac{x\sqrt{a^2-x^2}}{a^2}\Big]$	$\sqrt{x^2-a^2}$	$\frac{a^2}{2}\left[-\cosh^{-1}\left(\frac{x}{a}\right)+\frac{x\sqrt{x^2-a^2}}{a^2}\right]$

Some series Expansions -

$$\frac{\pi}{2}=\left(\frac{2}{1}\frac{2}{3}\right)\left(\frac{4}{3}\frac{4}{5}\right)\left(\frac{6}{5}\frac{6}{7}\right)\left(\frac{8}{7}\frac{8}{9}\right)\ldots$$

$$\pi=\frac{4}{1}-\frac{4}{3}+\frac{4}{5}-\frac{4}{7}+\frac{4}{9}-\frac{4}{11}+\frac{4}{13}-\cdots$$

$$\frac{\pi}{4}=\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\cdots$$

$$\pi=\sqrt{12}\left(1-\frac{1}{3\cdot 3}+\frac{1}{5\cdot 3^2}-\frac{1}{7\cdot 3^3}+\cdots\right)$$

$$\frac{\pi^2}{6}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots=\sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\int_0^{\pi/2} \log \sin x \, dx = -\frac{\pi}{2} \log 2 = \frac{\pi}{2} \log \frac{1}{2}$$

Solve a series problem

If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$ upto $\infty = \frac{\pi^2}{6}$, then value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ up to ∞ is

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{6}$ (c) $\frac{\pi^2}{8}$ (d) $\frac{\pi^2}{12}$

Ans. (c)

Solution We have $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ upto ∞

$$= \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} \cdots \text{upto } \infty$$

$$- \frac{1}{2^2}\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right]$$

$$= \frac{\pi^2}{6} - \frac{1}{4}\left(\frac{\pi^2}{6}\right) = \frac{\pi^2}{8}$$

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \ldots \infty = \frac{\pi^2}{12}$$

$$\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \ldots \infty = \frac{\pi^2}{24}$$

$$\frac{\sin\sqrt{x}}{\sqrt{x}} = 1 - \frac{x}{3!} + \frac{x^2}{5!} - \frac{x^3}{7!} + \frac{x^4}{9!} - \frac{x^5}{11!} + \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = \sum_{k=0}^{n} \frac{(-1)^k x^{2k}}{(2k)!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots = \sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots = \sum_{k=0}^{n} \frac{x^{2k}}{(2k)!}$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots = \sum_{k=0}^{n} \frac{x^{2k+1}}{(2k+1)!}$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad (-1 \le x < 1)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} \cdots + \frac{2^{2n}\left(2^{2n}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \qquad |x| < \frac{\pi}{2}$$

$$\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \cdots + \frac{E_n x^{2n}}{(2n)!} + \cdots \quad |x| < \frac{\pi}{2}$$

$$\csc x = \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^5}{15120} + \cdots + \frac{2\left(2^{2n-1}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \qquad 0 < |x| < \pi$$

$$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - - \cdots - \frac{2^{2n} B_n x^{2n-1}}{(2n)!} - \cdots \quad 0 < |x| < \pi$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$$

$$\sec x = 1 + \frac{x^2}{\lfloor 2} + \frac{5x^4}{\lfloor 4} + \cdots$$

$$\log(\cos x) = -\frac{x^2}{\lfloor 2} - \frac{2x^4}{\lfloor 4} - \cdots$$

$$\log(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \cdots$$

$$\sin^{-1} x = x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \cdots \quad |x| < 1$$

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$= \frac{\pi}{2} - \left(x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \cdots \right) \quad |x| < 1$$

$$\tan^{-1} x = \begin{cases} x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots & |x| < 1 \\ \pm\frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \cdots & \begin{cases} + \text{ if } x \geq 1 \\ - \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$$

$$= \frac{\pi}{2} - \left(\frac{1}{x} + \frac{1}{2\cdot 3x^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5x^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \cdots \right) \quad |x| > 1$$

$$\csc^{-1} x = \sin^{-1}(1/x)$$

$$= \frac{1}{x} + \frac{1}{2\cdot 3x^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5x^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \cdots \quad |x| > 1$$

$$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$$

$$= \begin{cases} \frac{\pi}{2} - \left(x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \right) & |x| < 1 \\ p\pi + \frac{1}{x} - \frac{1}{3x^3} + \frac{1}{5x^5} + \cdots & \begin{cases} p = 0 \text{ if } x \geq 1 \\ p = 1 \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\ln x = 2\left[\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \ldots\right]$$

$$= 2\sum_{n=1}^{\infty} \frac{1}{2n-1}\left(\frac{x-1}{x+1}\right)^{2n-1} \quad (x > 0)$$

$$\ln x = \frac{x-1}{x} + \frac{1}{2}\left(\frac{x-1}{x}\right)^2 + \frac{1}{3}\left(\frac{x-1}{x}\right)^3 + \ldots$$

$$= \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{x-1}{x}\right)^n \quad (x > \frac{1}{2})$$

$$\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1}\frac{1}{n}(x-1)^n \quad (0 < x \le 2)$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1}\frac{1}{n}x^n \quad (|x| < 1)$$

$$\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots\infty \, (-1 \le x < 1)$$

$$\log_e(1+x) - \log_e(1-x) =$$

$$\log_e \frac{1+x}{1-x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\infty\right) (-1 < x < 1)$$

$$\log_e\left(1+\frac{1}{n}\right) = \log_e \frac{n+1}{n} = 2 \left[\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \ldots\infty\right]$$

$$\log_e (1+x) + \log_e (1-x) = \log_e(1-x^2) = -2\left(\frac{x^2}{2} + \frac{x^4}{4} + \ldots\infty\right) (-1 < x < 1)$$

$$\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots = \frac{1}{1 \cdot 2} + \frac{1}{3.4} + \frac{1}{5.6} + \ldots$$

Important Results

(i) (a) $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cos^n x}{\sin^n x + \cos^n x} dx$

(b) $\int_0^{\pi/2} \frac{\tan^n x}{1 + \tan^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{dx}{1 + \tan^n x}$

(c) $\int_0^{\pi/2} \frac{dx}{1 + \cot^n x} = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{1 + \cot^n x} dx$

(d) $\int_0^{\pi/2} \frac{\tan^n x}{\tan^n x + \cot^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{\tan^n x + \cot^n x} dx$

(e) $\int_0^{\pi/2} \frac{\sec^n x}{\sec^n x + \operatorname{cosec}^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\operatorname{cosec}^n x}{\sec^n x + \operatorname{cosec}^n x} dx$ where, $n \in R$

(ii) $\int_0^{\pi/2} \frac{a^{\sin^n x}}{a^{\sin^n x} + a^{\cos^n x}} dx = \int_0^{\pi/2} \frac{a^{\cos^n x}}{a^{\sin^n x} + a^{\cos^n x}} dx = \frac{\pi}{4}$

(iii) (a) $\int_0^{\pi/2} \log \sin x \, dx = \int_0^{\pi/2} \log \cos x \, dx = -\frac{\pi}{2} \log 2$

(b) $\int_0^{\pi/2} \log \tan x \, dx = \int_0^{\pi/2} \log \cot x \, dx = 0$

(c) $\int_0^{\pi/2} \log \sec x \, dx = \int_0^{\pi/2} \log \operatorname{cosec} x \, dx = \frac{\pi}{2} \log 2$

(iv) (a) $\int_0^{\infty} e^{-ax} \sin bx \, dx = \frac{b}{a^2 + b^2}$

(b) $\int_0^{\infty} e^{-ax} \cos bx \, dx = \frac{a}{a^2 + b^2}$

(c) $\int_0^{\infty} e^{-ax} x^n \, dx = \frac{n!}{a^n + 1}$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left(x + \sqrt{x^2 - a^2}\right) + C$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left(\frac{x - a}{x + a}\right) + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left(\frac{a + x}{a - x}\right) + C$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{a^2 + x^2}\, dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right) + C$$

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Good Luck to you for your Preparations, References, and Exams

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