Spoon Feeding Relations & Functions

Simplified Knowledge Management Classes Bangalore

My name is Subhashish Chattopadhyay. I have been teaching for IIT-JEE, Various International Exams (such as IMO [International Mathematics Olympiad], IPhO [International Physics Olympiad], IChO [International Chemistry Olympiad]), IGCSE (IB), CBSE, I.Sc, Indian State Board exams such as WB-Board, Karnataka PU-II etc since 1989. As I write this book in 2016, it is my 25 th year of teaching. I was a Visiting Professor to BARC Mankhurd, Chembur, Mumbai, Homi Bhabha Centre for Science Education (HBCSE) Physics Olympics camp BARC Campus.

I am Life Member of …

- IAPT (Indian Association of Physics Teachers)

- IPA (Indian Physics Association)

- AMTI (Association of Mathematics Teachers of India)

- National Human Rights Association

- Men's Rights Movement (India and International)

- MGTOW Movement (India and International)

And also of

IACT (Indian Association of Chemistry Teachers)

The selection for National Camp (for Official Science Olympiads - Physics, Chemistry, Biology, Astronomy) happens in the following steps ….

1) **NSEP** (National Standard Exam in Physics) and **NSEC** (National Standard Exam in Chemistry) held around 24 rth November. Approx 35,000 students appear for these exams every year. The exam fees is Rs 100 each. Since 1998 the IIT JEE toppers have been topping these exams and they get to know their rank / performance ahead of others.

2) **INPhO** (Indian National Physics Olympiad) and **INChO** (Indian National Chemistry Olympiad). Around 300 students in each subject are allowed to take these exams. Students coming from outside cities are paid fair from the Govt of India.

3) The Top 35 students of each subject are invited at HBCSE (Homi Bhabha Center for Science Education) Mankhurd, near Chembur, BARC, Mumbai. After a 2-3 weeks camp the top 5 are selected to represent India. The flight tickets and many other expenses are taken care by Govt of India.

Since last 50 years there has been no dearth of "Good Books". Those who are interested in studies have been always doing well. This e-Book does not intend to replace any standard text book. These topics are very old and already standardized.

There are 3 kinds of Text Books

- The thin Books - Good students who want more details are not happy with these. Average students who need more examples are not happy with these. Most students who want to "Cram" quickly and pass somehow find the thin books "good" as they have to read less !!

- The Thick Books - Most students do not like these, as they want to read as less as possible. Average students are "busy" with many other things and have no time to read all these.

- The Average sized Books - Good students do not get all details in any one book. Most bad students do not want to read books of "this much thickness" also !!

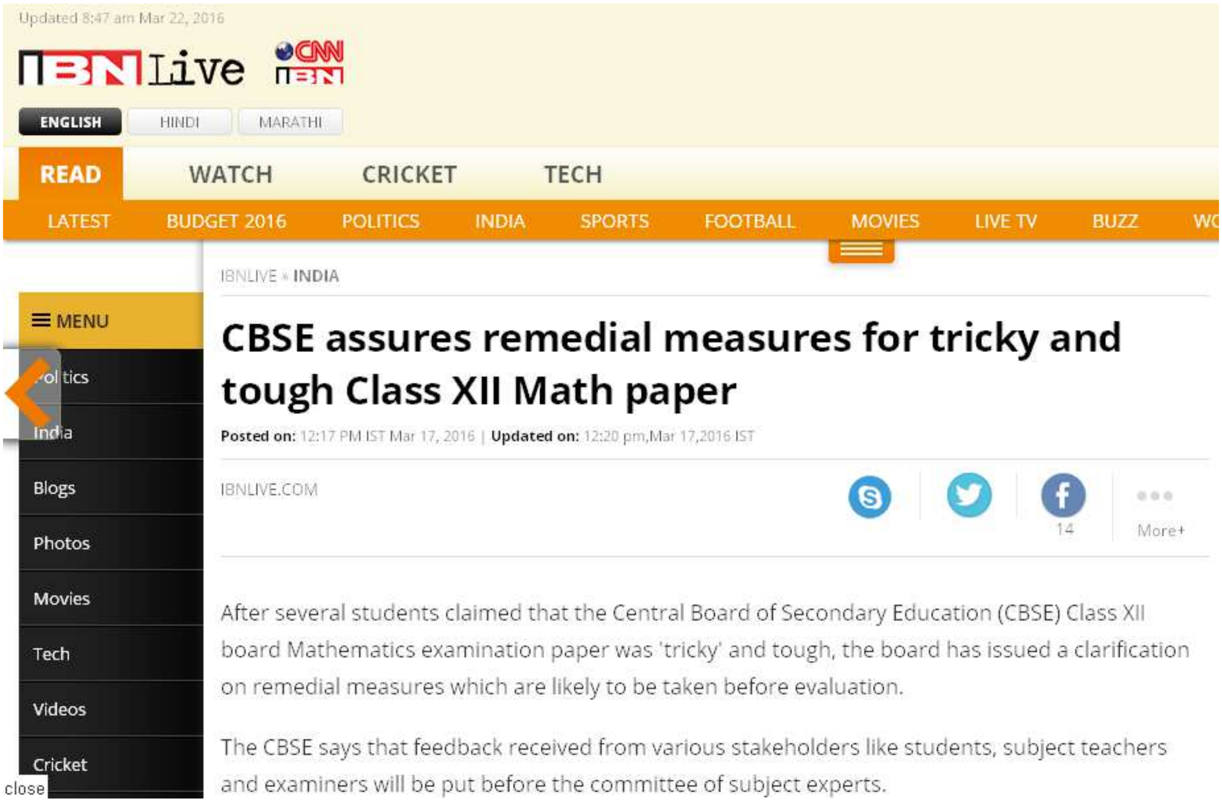
We know there can be no shoe that's fits in all.

Printed books are not e-Books! Can't be downloaded and kept in hard-disc for reading "later" ....

So if you read this book later, you will get all kinds of examples in a single place. This becomes a very good "Reference Material". I sincerely wish that all find this "very useful".

Students who do not practice lots of problems, do not do well. The rules of "doing well" had never changed .... Will never change !

After 2016 CBSE Mathematics exam, lots of students complained that the paper was tough!



Updated 8:47 am Mar 22, 2016

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CBSE assures remedial measures for tricky and tough Class XII Math paper

Posted on: 12:17 PM IST Mar 17, 2016 | **Updated on:** 12:20 pm,Mar 17,2016 IST

IBNLIVE.COM

14

More+

After several students claimed that the Central Board of Secondary Education (CBSE) Class XII board Mathematics examination paper was 'tricky' and tough, the board has issued a clarification on remedial measures which are likely to be taken before evaluation.

The CBSE says that feedback received from various stakeholders like students, subject teachers and examiners will be put before the committee of subject experts.

In 2015 also the same complain was there by many students

हिन्दी मराठी বাংলা

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News » India News

CBSE Class 12 exam: Issue of tough maths paper raised in Parliament

A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".

Last Updated: Thursday, March 19, 2015 - 14:41

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New Delhi: A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".

In March 2016, students of Karnataka PU-II also complained the same, regarding standard 12 (PU-II Mathematics Exam). Even though the Math Paper was identical to previous year, most students had not even solved the 2015 Question Paper.

Friday, March 25, 2016 - 13:28

The NEWS Minute

HOME NEWS ANDHRA KARNATAKA KERALA TAMIL NADU TELANGANA CULTURE MEDIA BLOG

Exams

Online petition for lenient evaluation of K'taka II PU math paper gets over 8000 supporters

The campaign, which was launched on Monday, has garnered over 8000 supporters

TNM Staff | Wednesday, March 16, 2016 - 09:32

Follow @thenewsminute

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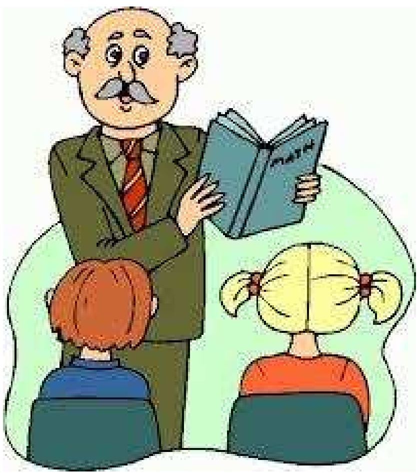
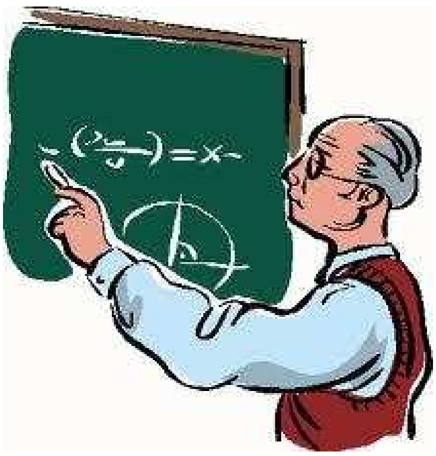
Following a "very tough" math paper that left many II PU students in tears, Saket Ravindran a student launched an online campaign demanding lenient evaluation.

These complains are not new. In fact since last 40 years, (since my childhood), I always see this; every year the same setback, same complain!

In this e-Book I am trying to solve this problem. Those students who practice can learn.

No one can help those who are not studying, or practicing.



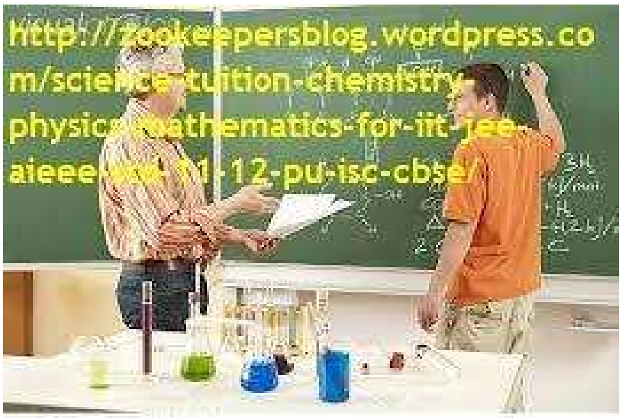


Learn more at http://skmclasses.weebly.com/iit-jee-home-tuitions-bangalore.html

Twitter - https://twitter.com/ZookeeperPhy

Facebook - https://www.facebook.com/IIT.JEE.by.Prof.Subhashish/

Blog - http://skmclasses.kinja.com



A very polite request :

I wish these e-Books are read only by Boys and Men. Girls and Women, better read something else; learn from somewhere else.

Preface

We all know that in the species “Homo Sapiens “, males are bigger than females. The reasons are explained in standard 10, or 11 (high school) Biology texts. **This shapes or size, influences all of our culture**. Before we recall / understand the reasons once again, let us see some random examples of the influence

Random - 1

If there is a Road rage, then who all fight ? (generally ?). Imagine two cars driven by adult drivers. Each car has a woman of similar age as that of the Man. The cars “ touch “ or “ some issue happens”. Who all comes out and fights ? Who all are most probable to drive the cars ?




(Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win)

Random - 2

Heavy metal music artists are all Men. Metallica, Black Sabbath, Motley Crue, Megadeth, Motorhead, AC/DC, Deep Purple, Slayer, Guns & Roses, Led Zeppelin, Aerosmith ….. the list can be in thousands. All these are grown-up Boys, known as Men.




(Men strive for perfection. Men are eager to excel. Men work hard. Men want to win.)

Random - 3

Apart from Marie Curie, only one more woman got Nobel Prize in Physics. (Maria Goeppert Mayer - 1963). So, … almost all are men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 4

The best Tabla Players are all Men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 5

History is all about, which Kings ruled. Kings, their men, and Soldiers went for wars. History is all about wars, fights, and killings by men.

Boys start fighting from school days. Girls do not fight like this

(Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.)

Random - 6

The highest award in Mathematics, the “ Fields Medal “ is around since decades. Till date only one woman could get that. (Maryam Mirzakhani - 2014). So, … almost all are men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 7

Actor is a gender neutral word. Could the movie like “ Top Gun “ be made with Female actors ? The best pilots, astronauts, Fighters are all Men.

Random - 8

In my childhood had seen a movie named “ The Tower in Inferno “. In the movie when the tall tower is in fire, women were being saved first, as only one lift was working….

Many decades later another movie is made. A box office hit. “ The Titanic “. In this also …. As the ship is sinking women are being saved. **Men are disposable**. Men may get their turn later…

Movies are not training programs. Movies do not teach people what to do, or not to do. Movies only reflect the prevalent culture. Men are disposable, is the culture in the society. Knowingly, unknowingly, the culture is depicted in Movies, Theaters, Stories, Poems, Rituals, etc. I or you can’t write a story, or make a movie in which after a minor car accident the Male passengers keep seating in the back seat, while the both the women drivers come out of the car and start fighting very bitterly on the road. There has been no story in this world, or no movie made, where after an accident or calamity, Men are being helped for safety first, and women are told to wait.

Random - 9

Artists generally follow the prevalent culture of the Society. In paintings, sculptures, stories, poems, movies, cartoon, Caricatures, knowingly / unknowingly, “ the prevalent Reality “ is depicted. The opposite will not go well with people. If deliberately “ the opposite “ is shown then it may only become a special art, considered as a special mockery.

Random - 10

Men go to “girl / woman’s house” to marry / win, and bring her to his home. That is a sort of winning her. When a boy gets a “ Girl-Friend “, generally he and his friends consider that as an achievement. The boy who “ got / won “ a girl-friend feels proud. His male friends feel, jealous, competitive and envious. Millions of stories have been written on these themes. Lakhs of movies show this. Boys / Men go for “ bike race “, or say “ Car Race “, where the winner “ gets “ the most beautiful girl of the college.

(Men want to excel. Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.)

Prithviraj Chauhan ‘ went ` to “ pickup “ or “ abduct “ or “ win “ or “ bring “ his love. There was a Hindi movie (hit) song … “ Pasand ho jaye, to ghar se utha laye “. It is not other way round. Girls do not go to Boy’s house or man’s house to marry. Nor the girls go in a gang to “ pick-up “ the boy / man and bring him to their home / place / den.

Random - 11

Rich people; often are very hard working. Successful business men, establish their business (empire), amass lot of wealth, with lot of difficulty. Lots of sacrifice, lots of hard work, gets into this. Rich people's wives had no contribution in this wealth creation. Women are smart, and successful upto the extent to choose the right/rich man to marry. So generally what happens in case of Divorces ? Search the net on " most costly divorces " and you will know. The women;(who had no contribution at all, in setting up the business / empire), often gets in Billions, or several Millions in divorce settlements.

Number 1

Rupert & Anna Murdoch -- $1.7 billion

One of the richest men in the world, **Rupert Murdoch** developed his worldwide media empire when he inherited his father's Australian newspaper in 1952. He married Anna Murdoch in the '60s and they remained together for 32 years, springing off three children.

They split amicably in 1998 but soon Rupert forced Anna off the board of News Corp and the gloves came off. The divorce was finalized in June 1999 when Rupert agreed to let his ex-wife leave with $1.7 billion worth of his assets, $110 million of it in cash. Seventeen days later, Rupert married Wendi Deng, one of his employees.

Ted Danson & Casey Coates -- $30 million

Ted Danson's claim to fame is undoubtedly his decade-long stint as Sam Malone on NBC's celebrated sitcom Cheers . While he did other TV shows and movies, he will always be known as the bartender of that place where everybody knows your name. He met his future first bride Casey, a designer, in 1976 while doing Erhard Seminars Training.

Ten years his senior, she suffered a paralyzing stroke while giving birth to their first child in 1979. In order to nurse her back to health, Danson took a break from acting for six months. But after two children and 15 years of marriage, the infatuation fell to pieces. Danson had started seeing Whoopi Goldberg while filming the comedy, Made in America and this precipitated the 1992 divorce. Casey got $30 million for her trouble.

See https://zookeepersblog.wordpress.com/misandry-and-men-issues-a-short-summary-at-single-place/

See http://skmclasses.kinja.com/save-the-male-1761788732

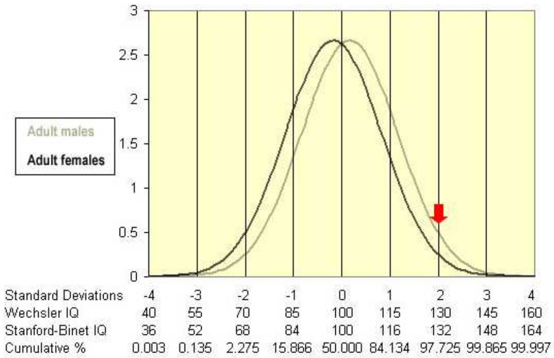
It was Boys and Men, who brought the girls / women home. The Laws are biased, completely favoring women. The men are paying for their own mistakes.

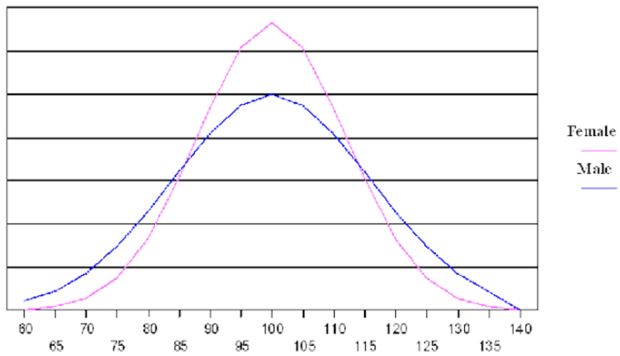
See https://zookeepersblog.wordpress.com/biased-laws/

(Man brings the Woman home. When she leaves, takes away her share of big fortune!)

Random - 12

A standardized test of Intelligence will never be possible. It never happened before, nor ever will happen in future; where the IQ test results will be acceptable by all. In the net there are thousands of charts which show that the intelligence scores of girls / women are lesser. Debates of Trillion words, does not improve performance of Girls.





I am not wasting a single second debating or discussing with anyone, on this. I am simply accepting ALL the results. IQ is only one of the variables which is required for success in life. Thousands of books have been written on " Networking Skills ", EQ (Emotional Quotient), Drive, Dedication, Focus, " Tenacity towards the end goal " … etc. In each criteria, and in all together, women (in general) do far worse than men. Bangalore is known as " ….. capital of India ". [Fill in the blanks]. The blanks are generally filled as " Software Capital ", " IT Capital ", " Startup Capital ", etc. I am member in several startup eco-systems / groups. I have attended hundreds of meetings, regarding " technology startups ", or " idea startups ". These meetings have very few women. Starting up new companies are all " Men's Game " / " Men's business ". Only in Divorce settlements women will take their goodies, due to Biased laws. There is no dedication, towards wealth creation, by women.

Random - 13

Many men, as fathers, very unfortunately treat their daughters as " Princess ". Every " non-performing " woman / wife was " princess daughter " of some loving father. Pampering the girls, in name of " equal opportunity ", or " women empowerment ", have led to nothing.

See http://skmclasses.kinja.com/progressively-daughters-become-monsters-1764484338

See http://skmclasses.kinja.com/vivacious-vixens-1764483974

There can be thousands of more such random examples, where " Bigger Shape / size " of males have influenced our culture, our Society. **Let us recall the reasons**, that we already learned in standard 10 - 11, Biology text Books. In humans, women have a long gestation period, and also spends many years (almost a decade) to grow, nourish, and stabilize the child. (Million years of habit) Due to survival instinct Males want to inseminate. Boys and Men fight for the " facility (of womb + care) " the girl / woman may provide. Bigger size for males, has a winning advantage. Whoever wins, gets the " woman / facility ". The male who is of " Bigger Size ", has an advantage to win.... Leading to Natural selection over millions of years. In general " Bigger Males "; the " fighting instinct " in men; have led to wars, and solving tough problems (Mathematics, Physics, Technology, startups of new businesses, Wealth creation, Unreasonable attempts to make things [such as planes], Hard work ....)

So let us see the IIT-JEE results of girls. Statistics of several years show that there are around 17, (or less than 20) girls in top 1000 ranks, at all India level. Some people will yet not understand the performance, till it is said that ... year after year we have around 980 boys in top 1000 ranks. Generally we see only 4 to 5 girls in top 500. In last 50 years not once any girl topped in IIT-JEE advanced. Forget about Single digit ranks, double digit ranks by girls have been extremely rare. It is all about " good boys ", " hard working ", " focused ", "**Bel-esprit** " **boys**.

In 2015, Only 2.6% of total candidates who qualified are girls (upto around 12,000 rank). while 20% of the Boys, amongst all candidates qualified. The Total number of students who appeared for the exam were around 1.4 million for IIT-JEE main. Subsequently 1.2 lakh (around 120 thousands) appeared for IIT-JEE advanced.

IIT-JEE results and analysis, of many years is given at https://zookeepersblog.wordpress.com/iit-jee-iseet-main-and-advanced-results/

In Bangalore it is rare to see a girl with rank better than 1000 in IIT-JEE advanced. We hardly see 6-7 boys with rank better than 1000. Hardly 2-3 boys get a rank better than 500.

See http://skmclasses.weebly.com/everybody-knows-so-you-should-also-know.html

Thousands of people are exposing the heinous crimes that Motherly Women are doing, or Female Teachers are committing. See https://www.facebook.com/WomenCriminals/

Some Random Examples must be known by all

It is extremely unfortunate that the " woman empowerment " has created. This is the kind of society and women we have now. I and many other sensible Men hate such women. Be away from such women, be aware of reality.

Mother Admits On Facebook to Sleeping with 15 Yr Old Son, They Have a Baby Together - Alwayzturntup

Sometimes it hard to believe w From Alwayzturntup

ALWAYZTURNTUP.ME

'Sex with my son is incredible - we're in love and we want a baby'

Ben Ford, who ditched his wife when he met his mother Kim West after 30 years, claims what the couple are doing 'isn't incest'

MIRROR.CO.UK

Woman sent to jail for the rest of her life after raping her four grandchildren is described as the 'most evil person' the judge has ever seen

Edwina Louis rape...

See More

Former Shelbyville ISD teacher who had sex with underage student gets 3 years in prison

After a two day break over the weekend, A Shelby County jury was back in the courtroom looking to conclude the trial of a former Shelbyville ISD teacher who had...

KLTV.COM | BY CALEB BEAMES

Woman sent to jail for raping her four grandchildren

A Ohio grandmother has been sentenced to four consecutive life terms after being found guilty of the rape of her own grandchildren. Edwina Louis, 53, will spend the rest of her life behind bars.

DAILYMAIL.CO.UK

http://www.thenativecanadian.com/.../eastern-ontario-teacher-...

The N.C. Chronicles.: Eastern Ontario teacher charged with 36 sexual offences

anti feminism, Child abuse, children's rights, Feminist hypocrisy,

THENATIVECANADIAN.COM | BY BLACKWOLF

Hyd woman kills newborn boy as she wanted daughter - Times of India

Having failed to bear a daughter for the third time, a shopkeeper's wife slit the throat of her 24day-old son with a shaving blade and left him to die in a street on Tuesday night.Purnima's first child was a stillborn boy, followed by another boy born five years ago.

TIMESOFINDIA.INDIATIMES.COM

Montgomery's son, Alan Vonn Webb, took the stand and was a key witness in her conviction.

"I want to see her placed somewhere she can never do that to children ...

See More

Woman sentenced to 40 years in prison for raping her children

A Murfreesboro mother found guilty of raping her own children learned her fate on Wednesday.

WAFF.COM | BY DENNIS FERRIER

gentler sex? Violence against men.'s photo.

Women, the gentler sex? Violence against men.

Like Page

April 8 at 1:38am

In fact, the past decade has seen a dramatic increase in the number of incidents of women raping and sexually assaulting boys and men. On May 2014, Jezebel repo...

End violence against women . . . .

North Carolina Grandma Eats Her Daughter's New Born Baby After Smoking Bath Salts

Henderson, North Carolina– A North Carolina grandmother of 4 and recovering drug addict, is now in custody after she allegedly ate her daughter's newborn baby....

AZ-365.TOP

28-Year-Old Texas Teacher Accused of Sending Nude Picture to 14-Year-Old Former Student

BREITBART.COM

http://latest.com/.../attractive-girl-gang-lured-men-alleywa.../

Attractive Girl Gang Lured Men Into Alleyways Where Female Body Builder Would Attack Them

A Mexican street gang made up entirely of women has been accused of using their feminine wiles to lure men into alleyways and then beating them up and...

LATEST.COM

http://www.wfmj.com/.../youngstown-woman-convicted-of-raping-...

Youngstown woman convicted of raping a 1 year old is back in jail

A Youngstown woman who went to prison for raping a 1-year-old boy fifteen years ago is in trouble with the law again.

WFMJ.COM

End violence against women . . . .

Women are raping boys and young men

Rape advocacy has been maligned and twisted into a political agenda controlled by radicalized activists. Tim Patten takes a razor keen and well supported look into the manufactured rape culture and...

AVOICEFORMEN.COM | BY TIM PATTEN

Bronx Woman Convicted of Poisoning and Drowning Her Children

Lisette Bamenga researched methods on the Internet before she killed her son and daughter in 2012.

NYTIMES.COM | BY MARC SANTORA

A Russian-born newlywed slowly butchered her German husband — feeding strips of his flesh to their dog until he took his last breath. Svetlana Batukova, 46, was...

See More

She killed her husband and then fed him to her dog: police

A Russian-born newlywed butchered her German hubby — and fed strips of his flesh to her pooch, authorities said. Svetlana Batukova offed Horst Hans Henkels at their...

NYPOST.COM

Daily Mail
January 15, 2015 ·

Mother charged with rape and sodomy of her son's 12-year-old friend

Mom, 30, 'raped and had oral sex with her son's 12-year-old friend'

Nicole Marie Smith, 30, (pictured) of St Charles County, Missouri, has been jailed after she allegedly targeted the 12-year-old boy at her home.

DAILYM.AI

April 4 at 4:48am ·

Female prison officers commit 90pc of sex assaults on male teens in US juvenile detention centres

Lawsuit in Idaho highlights the prevalence of sexual victimization of juvenile offenders.

IBTIMES.CO.UK | BY NICOLE ROJAS

This mother filmed herself raping her own son and then sold it to a man for $300. The courts just decide her fate. When you see what she got, you're going to be outraged.

Mother Who Filmed Herself Raping Her 1-Year-Old Son Receives Shocking Sentence

"...then used the money to buy herself a laptop..."

AMERICANEWS.COM

In several countries or rather in several regions of the world, family system has collapsed, due to bad nature and naughty acts of women. Particularly in Britain, and America, almost 50% people are alone, lonely, separated, divorced or failed marriages. In 2013, 48% children were born out of wedlock. It was projected that by 2016, more than 51% children will be born, to unmarried mothers. In these developed countries " paternity fraud " by women, are close to 20%. You can see several articles in the net, and in wikipedia etc. This means 1 out of 5 children are calling a wrong man as dad. The lonely, alone " mothers " are frustrated. They see the children as burden. Love in the Society in general is lost, long time ago. The types of " Mothers " and " Women " we have now …………

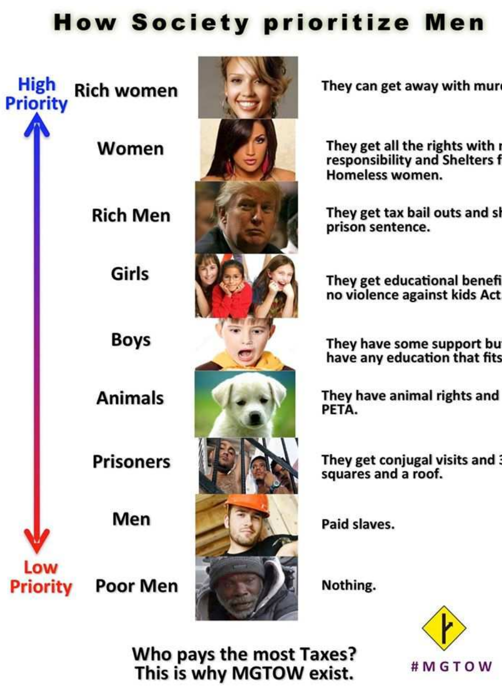

This is the type of women we have in this world. These kind of women were also someones daughter
Mother Stabs Her Baby 90 Times With Scissors After He Bit Her While Breastfeeding Him!
Eight-month-old Xiao Bao was discovered by his uncle in a pool of blood Needed 100 stitches after the incident; he is now recovering in hospital Reports say his...
MOMMABUZZ.COM


MY PUSSY MY RULES
NO GOD IN MY VAGINA


NATIONAL GO TOPLESS DAY
GOING TOPLESS IS LEGAL IN NY !!!


NUDITY ISN'T SEXUAL


TOPLESS : ÉGALITÉ POUR TOUS OU POUR PERSONNE !
SORRY TO BOTHER YOU, BUT WE ARE CHANGING THE WORLD!
LES SEINS DES FEMMES NE SONT PAS INDÉCENTS



Professor Subhashish Chattopadhyay

-

CBSE Standard 12 Math Survival Guide - Relations & Functions by Prof. Subhashish Chattopadhyay SKMClasses Bangalore Useful for IIT-JEE, I.Sc. PU-II, Boards, IGCSE IB AP-Mathematics and other exams

Spoon Feeding Series - Relations & Functions

Each set is a collection, but each collection need not be a set.
For example, a collection of beautiful women in Delhi is just a collection and not a set, for the term beautiful is not well defined. Only well defined collection of objects forms a set.

(i) The collection of all natural numbers less than 50 forms a set as it is well defined.

(ii) It is not a set as the term 'good' is not well defined.

(iii) It forms a set as it is well defined.

(iv) It is not a set as the term 'most' is not well defined. A writer may be talented in the eye of one person, but he may not be talented in the eye of some other person.

(v) It is not a set as the term 'difficult' is not well defined.
A topic may be difficult for one person but may not be difficult for another person, so the term 'difficult' is vague.

(vi) It forms a set as it is well defined.

(vii) It forms a set as it is well defined.

(viii) It forms a set as it is well defined.

(ix) It is not a set as the term 'most dangerous' is not well defined.
The notion of dangerous animals differs from person to person.

(x) It forms a set as it is well defined.

By the definition of equality of ordered pairs

$$\left(\frac{a}{3}+1,\ b-\frac{2}{3}\right)=\left(\frac{5}{3},\ \frac{1}{3}\right)$$

$\Rightarrow \quad \frac{a}{3}+1=\frac{5}{3}$ and $b-\frac{2}{3}=\frac{1}{3}$

$\Rightarrow \quad \frac{a}{3}=\frac{5}{3}-1$ and $b=\frac{1}{3}+\frac{2}{3}$

$\Rightarrow \quad \frac{a}{3}=\frac{5-3}{3}$ and $b=\frac{1+2}{3}$

$\Rightarrow \quad \frac{a}{3}=\frac{2}{3}$ and $b=\frac{3}{3}$

$\Rightarrow \quad a=2$ and $b=1$

By the definition of equality of ordered pairs

$$(x+1,1)=(3,y-2)$$

$\Rightarrow \quad x+1=3$ and $1=y-2$

$\Rightarrow \quad x=3-1$ and $1+2=y$

$\Rightarrow \quad x=2$ and $3=y$

$\Rightarrow \quad x=2$ and $y=3$

We have,

$$(x,-1)\in\{(a,b):b=2a-3\}$$

and, $(5,y)\in\{(a,b):b=2a-3\}$

$\Rightarrow \quad -1=2\times x-3$ and $y=2\times 5-3$

$\Rightarrow \quad -1=2x-3$ and $y=10-3$

$\Rightarrow \quad 3-1=2x$ and $y=7$

$\Rightarrow \quad 2=2x$ and $y=7$

$\Rightarrow \quad x=1$ and $y=7$

We have,

$$a + b = 5$$
$$\Rightarrow \quad a = 5 - b$$
$$\therefore \quad b = 0 \Rightarrow a = 5 - 0 = 5,$$
$$b = 3 \Rightarrow a = 5 - 3 = 2,$$
$$b = 6 \Rightarrow a = 5 - 6 = -1,$$

Hence, the required set of ordered pairs (a, b) is $\{(-1,6), (2,3), (5,0)\}$

We have,

$$a \in \{2,4,6,9\}$$
and, $\quad b \in \{4,6,18,27\}$

Now, a/b stands for 'a divides b'. For the elements of the given sets, we find that $2/4$, $2/6$, $2/18$, $6/18$, $9/18$ and $9/27$

$\therefore \quad \{(2,4),\ (2,6),\ (2,18),\ (6,18),\ (9,18),\ (9,27)\}$ are the required set of ordered pairs (a, b)

We have,

$$A = \{1,2\} \text{ and } B = \{1,3\}$$
Now, $\quad A \times B = \{1,2\} \times \{1,3\}$
$$= \{(1,1),\ (1,3),\ (2,1),\ (2,3)\}$$
and, $\quad B \times A = \{1,3\} \times \{1,2\}$
$$= \{(1,1),\ (1,2),\ (3,1),\ (3,2)\}$$

We have,

$$A = \{1, 2, 3\} \text{ and } B = \{3, 4\}$$

$$\therefore \quad A \times B = \{1, 2, 3\} \times \{3, 4\}$$

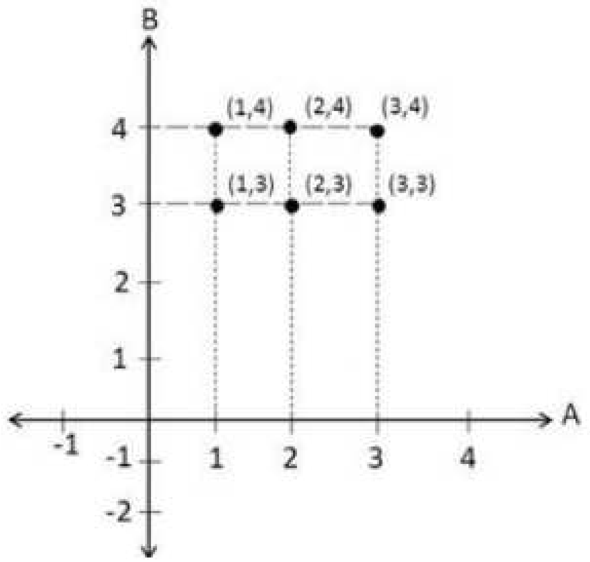
$$= \{(1,3),\ (1,4),\ (2,3),\ (2,4),\ (3,3),\ (3,4)\}$$

In order to represent $A \times B$ graphically, we follow the following steps:

(a) Draw two mutually perpendicular line one horizontal and other vertical.

(b) On the horizontal line represent the element of set A and on the vertical line represent the elements of set B.

(c) Draw vertical dotted lines through points representing elements of A on horizontal line and horizontal lines through points representing elements of B on the vertical line points of intersection of these lines will represent $A \times B$ graphically.



We have,

$$A = \{1,2,3\} \text{ and } B = \{2,4\}$$

$$\therefore \quad A \times B = \{1,2,3\} \times \{2,4\}$$
$$= \{(1,2),\ (1,4),\ (2,2),\ (2,4),\ (3,2),\ (3,4)\},$$

$$B \times A = \{2,4\} \times \{1,2,3\}$$
$$= \{(2,1),\ (2,2),\ (2,3),\ (4,1),\ (4,2),\ (4,3)\},$$

$$A \times A = \{1,2,3\} \times \{1,2,3\}$$
$$= \{(1,1),\ (1,2),\ (1,3),\ (2,1),\ (2,2),\ (2,3),\ (3,1),\ (3,2),\ (3,3)\},$$

$$B \times B = \{2,4\} \times \{2,4\}$$
$$= \{(2,2),\ (2,4),\ (4,2),\ (4,4)\},$$

and, $(A \times B) \cap (B \times A)$
$$= \{(1,2),\ (1,4),\ (2,2),\ (2,4),\ (3,2),\ (3,4)\} \cap \{(2,1),\ (2,2),\ (2,3),\ (4,1),\ (4,2),\ (4,3)\}$$
$$= \{(2,2)\}$$

$$\Rightarrow \quad (A \times B) \cap (B \times A) = \{(2,2)\}.$$

Let (a,b) be an arbitrary element of $(A \times B) \cap (B \times A)$. Then,

$$(a,b) \in (A \times B) \cap (B \times A)$$
$$\Leftrightarrow \quad (a,b) \in A \times B \quad \text{and} \quad (a,b) \in B \times A$$
$$\Leftrightarrow \quad (a \in A \text{ and } b \in B) \quad \text{and} \quad (a \in B \text{ and } b \in A)$$
$$\Leftrightarrow \quad (a \in A \text{ and } a \in B) \quad \text{and} \quad (b \in A \text{ and } b \in B)$$
$$\Leftrightarrow \quad a \in A \cap B \quad \text{and} \quad b \in A \cap B$$

Hence, the sets $A \times B$ and $B \times A$ have an element in comon iff the sets A and B have an element in common.

Since $(x,1)$, $(y,2)$, $(z,1)$ are elements of $A \times B$. Therefore, $x,\ y,\ z \in A$ and $1,2 \in B$

It is given that $n(A) = 3$ and $n(B) = 2$

$\therefore \quad x,\ y,\ z \in A$ and $n(A) = 3$

$\Rightarrow \quad A = \{x, y, z\}$

$1,2 \in B$ and $n(B) = 2$

$\Rightarrow \quad B = \{1, 2\}$.

We have,

$A = \{1, 2, 3, 4\}$

and, $R = \{(a,b) = a \in A,\ b \in A,\ a \text{ divides } b\}$

Now,

a/b stands for 'a divides b'. For the elements of the given sets, we find that $1/1$, $1/2$, $1/3$, $1/4$, $2/2$, $3/3$ and $4/4$

$\therefore \ R = \{(1,1),\ (1,2),\ (1,3),\ (1,4),\ (2,2),\ (2,4),\ (3,3),\ (4,4)\}$

We have,

$A = \{-1, 1\}$

$$\begin{aligned} \therefore \quad A \times A &= \{-1,1\} \times \{-1,1\} \\ &= \{(-1,-1),\ (-1,1),\ (1,-1),\ (1,1)\} \end{aligned}$$

$$\begin{aligned} \therefore \quad A \times A \times A &= \{-1,1\} \times \{(-1,-1),\ (-1,1),\ (1,-1),\ (1,1)\} \\ &= \{(-1,-1,-1),\ (-1,-1,1),\ (-1,1,-1),\ (-1,1,1),\ (1,-1,-1),\ (1,-1,1),\ (1,1,-1),\ (1,1,1)\} \end{aligned}$$

If $P = \{m, n\}$ and $Q = \{n, m\}$,

Then,

$P \times Q = \{(m,n),\ (m,m),\ (n,n),\ (n,m)\}$

We have,

$$A = \{1, 2\}$$

$$\therefore \quad A \times A = \{1, 2\} \times \{1, 2\}$$
$$= \{(1,1),\ (1,2),\ (2,1),\ (2,2)\}$$

$$\therefore \quad A \times A \times A = \{1,2\} \times \{(1,1),\ (1,2),\ (2,1),\ (2,2)\}$$
$$= \{(1,1,1),\ (1,1,2),\ (1,2,1),\ (1,2,2),\ (2,1,1),\ (2,1,2),\ (2,2,1),\ (2,2,2)\}$$

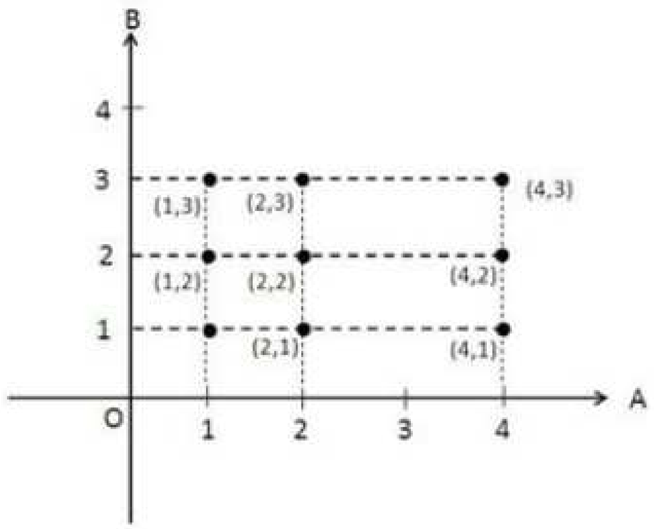
We have,

$$A = \{1, 2, 4\} \text{ and } B = \{1, 2, 3\}$$

$$\therefore \quad A \times B = \{1, 2, 4\} \times \{1, 2, 3\}$$
$$= \{(1,1),\ (1,2),\ (1,3),\ (2,1),\ (2,2),\ (2,3),\ (4,1),\ (4,2),\ (4,3)\}$$

Hence, we represent A on the horizontal line and B on vertical line.

Graphical representation of $A \times B$ is as shown below:



We have,

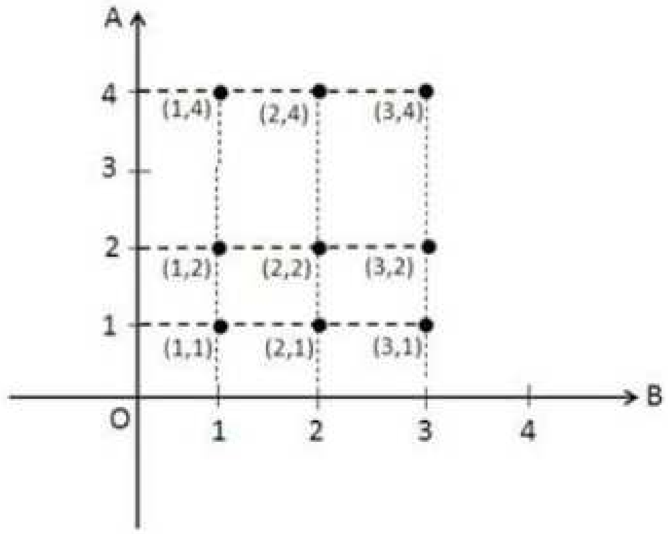
$A = \{1, 2, 4\}$ and $B = \{1, 2, 3\}$

$\therefore \quad B \times A = \{1, 2, 3\} \times \{1, 2, 4\}$

$= \{(1,1),\ (1,2),\ (1,4),\ (2,1),\ (2,2),\ (2,4),\ (3,1),\ (3,2),\ (3,4)\}$

Hence, we represent B on the horizontal line and A on vertical line.

Graphical representation of $B \times A$ is as shown below:



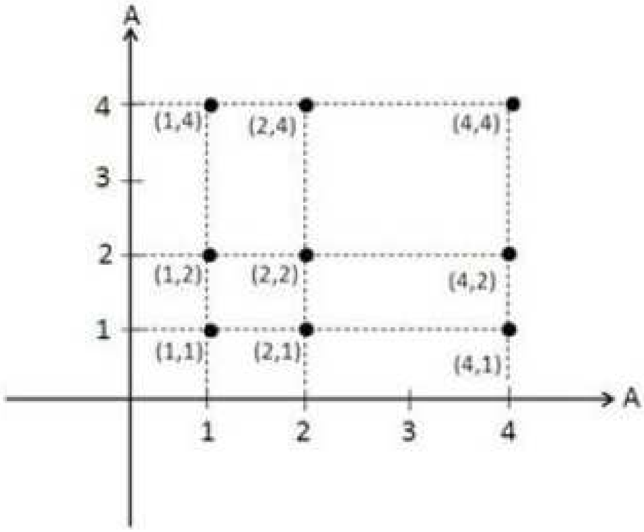
We have,

$$A = \{1, 2, 4\}$$

$$\therefore \quad A \times A = \{1, 2, 4\} \times \{1, 2, 4\}$$

$$= \{(1,1),\ (1,2),\ (1,4),\ (2,1),\ (2,2),\ (2,4),\ (4,1),\ (4,2),\ (4,4)\}$$

Graphical representation of $A \times A$ is shown below:



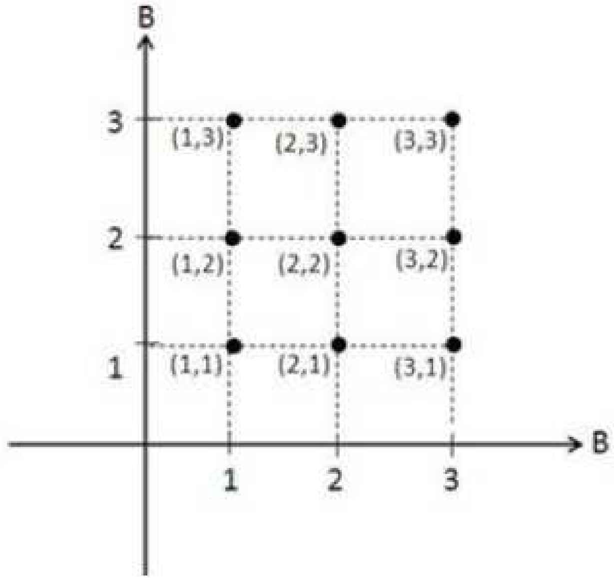
We have,

$B = \{1, 2, 3\}$

$\therefore \quad B \times B = \{1, 2, 3\} \times \{1, 2, 3\}$

$= \{(1,1),\ (1,2),\ (1,3),\ (2,1),\ (2,2),\ (2,3),\ (3,1),\ (3,2),\ (3,3)\}$

Graphical representation of $B \times B$ is shown below:



Function = Let A and B be two non-empty sets. A relation f from A to B, i.e., a sub-set of $A \times B$, is called a function (or a mapping or a map) from A to B, if

(i) for each $a \in A$ there exists $b \in B$ such that $(a, b) \in f$

(ii) $(a, b) \in f$ and $(a, c) \in f \Rightarrow b = c$

If $(a, b) \in f$, then 'b' is called the image of 'a' under f

If a function f is expresed as the set of ordered pairs, the domain f is the set of all first components of members of f and the range of f is the set of second components of members of f.

Function = Let A and B be two non-empty sets. Then a function 'f' from set A to set B is a rule or method or correspondence which associates elements of set A to elements of set b such that:

(i) all elements of set A are associated to element in set B.

(ii) an element of se A is associated to a unique element in set B.

In other words, a function 'f' from a set A to set B associates each element of set A to a unique element of set b.

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In other words, a function 'f' from a set A to set B associates each element of set A to a unique element of set b.

Function is a type of relation. But in a function no two ordered pairs have the same first element. For eg: R_1 and R_2 are two relations.

Clearly, R_1 is a function, but R_2 is not a function because two ordered pairs $(1,2)$ and $(1,4)$ have the same first element.
This means every function is a relation but every relation is not a function.

We have,

$$f(x) = x^2 - 2x - 3$$

Now,

$$\begin{aligned} f(-2) &= (-2)^2 - 2(-2) - 3 \\ &= 4 + 4 - 3 \\ &= 5 \end{aligned}$$

$$\begin{aligned} f(-1) &= (-1)^2 - 2(-1) - 3 \\ &= 1 + 2 - 3 \\ &= 0 \end{aligned}$$

$$\begin{aligned} f(-0) &= (-0)^2 - 2 \times 0 - 3 \\ &= -3 \end{aligned}$$

$$\begin{aligned} f(1) &= (1)^2 - 2 \times 1 - 3 \\ &= 1 - 2 - 3 \\ &= -4 \end{aligned}$$

$$\begin{aligned} f(2) &= (2)^2 - 2 \times 2 - 3 \\ &= 4 - 4 - 3 \\ &= -3 \end{aligned}$$

(a) $\text{Rang}(f) = \{-4, -3, 0, 5\}$

(b) Clearly, pre-images of $6, -3$ and 5 is ϕ, $\{0, 2\}$, -2 respectively.

We have,

$$f(x) = \begin{cases} 3x - 2, & x < 0 \\ 1, & x = 0 \\ 4x + 1, & x > 0 \end{cases}$$

Now,

$$f(1) = 4 \times 1 + 1 = 5,$$

$$f(-1) = 3 \times (-1) - 2 = -3 - 2 = -5,$$

$$f(0) = 1,$$

and, $f(2) = 4 \times 2 + 1 = 9$

$\therefore$ $f(1) = 5,\quad f(-1) = -5,$

$f(0) = 1,\quad f(2) = 9,$

We have,

$$f(x) = x^2 \qquad \text{---(i)}$$

(a) clearly range of $f = R^+$ (set of all real numbers greater than or equal to zero)

(b) we have,

$\{x : f(x) = 4\}$

$\Rightarrow \quad f(x) = 4 \qquad$ ---(ii)

Using equation (i) and equation (ii), we get

$x^2 = 4$

$\Rightarrow \quad x = \pm 2$

$\therefore \quad \{x : f(x) = 4\} = \{-2, 2\}$

(c) $\{y : f(y) = -1\}$

$\Rightarrow \quad f(y) = -1 \qquad$ ---(iii)

Clearly, $x^2 \neq -1$ or $x^2 \geq 0$

$\Rightarrow \quad f(y) \neq -1$

$\therefore \quad \{y : f(y) = -1\} = \phi$

We have,

$f = R^{+} \to R$

and $f(x) = \log_e x$ ---(i)

(a) Now,

$f = R^{+} \to R$

$\therefore$ the image set of the domain of $f = R$

(b) Now,

$\{x : f(x) = -2\}$

$\Rightarrow$ $f(x) = -2$ ---(ii)

Using equation (i) and equation (ii), we get

$\log_e x = -2$

$\Rightarrow$ $x = e^{-2}$ $[\because \log_a b = c \Rightarrow b = a^c]$

$\therefore$ $\{x : f(x) = -2\} = \{e^{-2}\}$

(c) Now,

$f(xy) = \log_e (xy)$ $[f(x) = \log_e x]$

$= \log_e x + \log_e y$ $[\because \log mn = \log m + \log n]$

$f(x) + f(y)$

Yes, $f(xy) = f(x) + f(y)$

(a) we have,

$$\{(x,y) = y = 3x, x \in \{1,2,3\}, y \in \{3,6,9,12\}\}$$

Putting $x = 1,2,3$ in $y = 3x$, we get

$$y = 3,6,9 \text{ respectively}$$

$$\therefore \quad R = \{(1,3),\ (2,6),\ (3,9)\}$$

Yes, it is a function.

(b) we have,

$$\{(x,y) : y > x+1, x = 1,2 \text{ and } y = 2,4,6\}$$

Putting $x = 1,2$ in $y > x+1$, we get

$$y > 2,\ y > 3 \text{ respectively.}$$

$$\therefore \quad R = \{(1,4),\ (1,6),\ (2,4),\ (2,6)\}$$

It is not a function from A to B because two ordered pairs in R have the same first element.

(c) we have,

$$\{(x,y) = x+y = 3,\ x,y \in \{0,1,2,3\}\}$$

Now,

$$y = 3 - x$$

Putting $x = 0,1,2,3$, we get

$$y = 3,2,1,0 \text{ respectively}$$

$$\therefore \quad R = \{(0,3),\ (1,2),\ (2,1),\ (3,0)\}$$

Yes, this relation is a function.

We have,

$$f : R \to R \text{ and } g : c \to c$$

$\therefore$ Domain $(f) = R$ and Domain $(g) = c$

$\therefore$ Domain $(f) \neq$ Domain $(g) = c$

$\therefore$ $f(x)$ and $g(x)$ are not equal functions.

(i) We have,

$$f(x) = x^2$$

Range of $f(x) = R^+$ (set of all real numbers greater than or equal to zero)

$$= \{x \in R \mid x \geq 0\}$$

(ii) We have,

$$g(x) = \sin x$$

Range of $g(x) = \{x \in R : -1 \leq x \leq 1\}$

(iii) We have,

$$h(x) = x^2 + 1$$

Range of $h(x) = \{x \in R : x \geq 1\}$

(a) We have,

$$f_1 = \{(1,1),\ (2,11),\ (3,1),\ (4,15)\}$$

f_1 is a function from X to Y.

(b) We have,

$$f_2 = \{(1,1),\ (2,7),\ (3,5)\}$$

f_2 is not a function from X to Y because there is an element $4 \in x$ which is not associated to any element of Y.

(c) We have,

$$f_3 = \{(1,5),\ (2,9),\ (3,1),\ (4,5),\ (2,11)\}$$

f_3 is not a function from X to Y because an element $2 \in x$ is associated to two elements 9 and 11 in Y.

We have,

$$f(x) = \text{highest prime factor of } x.$$

$$\therefore \quad 12 = 3 \times 4,$$
$$13 = 13 \times 1,$$
$$14 = 7 \times 2,$$
$$15 = 5 \times 3,$$
$$16 = 2 \times 8,$$
$$17 = 17 \times 1$$

$$\therefore \quad f = \{(12,3),\ (13,13),\ (14,7),\ (15,5),\ (16,2),\ (17,17)\}$$

$\therefore$ Range$(f) = \{3, 13, 7, 5, 2, 17\}$

We know that,

if $f : A \to 13$

such that $y \in 3$. Then,

$f^{-1}(y) = \{x \in A : f(x) = y\}$. In other words, $f^{-1}(y)$ is the set of pre-images of y.

Let $f^{-1}\{17\} = x$. Then, $f(x) = 17$

$\Rightarrow \quad x^2 + 1 = 17$

$\Rightarrow \quad x^2 = 17 - 1 = 16$

$\Rightarrow \quad x = \pm 4$

Let $f^{-1}\{-3\} = x$. Then, $f(x) = -3$

$\Rightarrow \quad x^2 + 1 = -3$

$\Rightarrow \quad x^2 = -3 - 1 = -4$

$\Rightarrow \quad x = \sqrt{-4}$

$\therefore \quad f^{-1}\{-3\} = \theta$

We have,

$A = \{p, q, r, s\}$ and $B = \{1, 2, 3\}$

(a) Now,

$R_1 = \{(p,1), (q,2), (r,1), (s,2)\}$

R_1 is a function

(b) Now,

$R_2 = \{(p,1), (q,1), (r,1), (s,1)\}$

R_2 is a function

(c) Now,

$R_3 = \{(p,1), (q,2), (p,2), (s,3)\}$

R_3 is not a function because an element $p \in A$ is associated to two elements 1 and 2 in B.

(d) Now,

$R_4 = \{(p,2), (q,3), (r,2), (s,2)\}$

R_4 is a function.

We have,

$f(n)$ = the highest prime factor of n.

Now,

$9 = 3 \times 3,$
$10 = 5 \times 2,$
$11 = 11 \times 1,$
$12 = 3 \times 4,$
$13 = 13 \times 1$

$\therefore \quad f = \{(9,3),\ (10,5),\ (11,11),\ (12,3),\ (13,13)\}$

Clearly, range$(f) = \{3, 5, 11, 13\}$

We have,

$$f(x) = \begin{cases} x^2, & 0 \le x \le 3 \\ 3x, & 3 \le x \le 10 \end{cases}$$

and, $$g(x) = \begin{cases} x^2, & 0 \le x \le 2 \\ 3x, & 2 \le x \le 10 \end{cases}$$

Now, $f(3) = (3)^2 = 9$ and $f(3) = 3 \times 3 = 9$

and, $g(2) = (2)^2 = 4$ and $g(2) = 3 \times 2 = 6$

We observe that $f(x)$ takes unique value at each point in its domain $[0, 10]$. However $g(x)$ does not takes unique value at each point in its domain $[0, 10]$.

Hence, $g(x)$ is not a function.

Determine whether each of the following relations are reflexive, symmetric and transitive:

(i)Relation R in the set $A = \{1, 2, 3...13, 14\}$ defined as

$R = \{(x, y): 3x - y = 0\}$

(ii) Relation R in the set **N** of natural numbers defined as

$R = \{(x, y): y = x + 5 \text{ and } x < 4\}$

(iii) Relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as

$R = \{(x, y): y \text{ is divisible by } x\}$

(iv) Relation R in the set **Z** of all integers defined as

$R = \{(x, y): x - y \text{ is as integer}\}$

(v) Relation R in the set A of human beings in a town at a particular time given by

(a) $R = \{(x, y): x \text{ and } y \text{ work at the same place}\}$

(b) $R = \{(x, y): x \text{ and } y \text{ live in the same locality}\}$

(c) $R = \{(x, y): x \text{ is exactly 7 cm taller than } y\}$

(d) $R = \{(x, y): x \text{ is wife of } y\}$

(e) $R = \{(x, y): x \text{ is father of } y\}$

Answer

(i) $A = \{1, 2, 3 \dots 13, 14\}$

$R = \{(x, y): 3x - y = 0\}$

$\therefore R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$

R is not reflexive since $(1, 1), (2, 2) \dots (14, 14) \notin R$.

Also, R is not symmetric as $(1, 3) \in R$, but $(3, 1) \notin R$. $[3(3) - 1 \neq 0]$

Also, R is not transitive as $(1, 3), (3, 9) \in R$, but $(1, 9) \notin R$.

$[3(1) - 9 \neq 0]$

Hence, R is neither reflexive, nor symmetric, nor transitive.

(ii) $R = \{(x, y): y = x + 5 \text{ and } x < 4\} = \{(1, 6), (2, 7), (3, 8)\}$

It is seen that $(1, 1) \notin R$.

$\therefore R$ is not reflexive.

$(1, 6) \in R$

But,

$(1, 6) \notin R$.

$\therefore R$ is not symmetric.

Now, since there is no pair in R such that (x, y) and $(y, z) \in R$, then (x, z) cannot belong to R.

$\therefore$ R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(iii) $A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(x, y): y \text{ is divisible by } x\}$

We know that any number (x) is divisible by itself.

$\Rightarrow (x, x) \in R$

$\therefore$R is reflexive.

Now,

$(2, 4) \in R$ [as 4 is divisible by 2]

But,

$(4, 2) \notin R$. [as 2 is not divisible by 4]

$\therefore$R is not symmetric.

Let $(x, y), (y, z) \in R$. Then, y is divisible by x and z is divisible by y.

$\therefore z$ is divisible by x.

$\Rightarrow (x, z) \in R$

$\therefore$R is transitive.

Hence, R is reflexive and transitive but not symmetric.

(iv) $R = \{(x, y): x - y \text{ is an integer}\}$

Now, for every $x \in \mathbf{Z}$, $(x, x) \in R$ as $x - x = 0$ is an integer.

$\therefore$R is reflexive.

Now, for every $x, y \in \mathbf{Z}$ if $(x, y) \in R$, then $x - y$ is an integer.

$\Rightarrow -(x - y)$ is also an integer.

$\Rightarrow (y - x)$ is an integer.

$\therefore (y, x) \in R$

$\therefore$R is symmetric.

Now,

Let (x, y) and $(y, z) \in R$, where $x, y, z \in \mathbf{Z}$.

$\Rightarrow (x - y)$ and $(y - z)$ are integers.

$\Rightarrow x - z = (x - y) + (y - z)$ is an integer.

$\therefore (x, z) \in R$

$\therefore$R is transitive.

Hence, R is reflexive, symmetric, and transitive.

(v) (a) R = {(x, y): x and y work at the same place}

$\Rightarrow (x, x) \in R$

$\therefore$ R is reflexive.

If $(x, y) \in R$, then x and y work at the same place.

$\Rightarrow$ y and x work at the same place.

$\Rightarrow (y, x) \in R$.

$\therefore$R is symmetric.

Now, let $(x, y), (y, z) \in R$

$\Rightarrow$ x and y work at the same place and y and z work at the same place.

$\Rightarrow$ x and z work at the same place.

$\Rightarrow (x, z) \in R$

$\therefore$ R is transitive.

Hence, R is reflexive, symmetric, and transitive.

(b) R = {(x, y): x and y live in the same locality}

Clearly $(x, x) \in R$ as x and x is the same human being.

$\therefore$ R is reflexive.

If $(x, y) \in R$, then x and y live in the same locality.

$\Rightarrow$ y and x live in the same locality.

$\Rightarrow (y, x) \in R$

$\therefore$R is symmetric.

Now, let $(x, y) \in R$ and $(y, z) \in R$.

$\Rightarrow$ x and y live in the same locality and y and z live in the same locality.

$\Rightarrow$ x and z live in the same locality.

$\Rightarrow (x, z) \in R$

$\therefore$ R is transitive.

Hence, R is reflexive, symmetric, and transitive.

(c) R = {(x, y): x is exactly 7 cm taller than y}

Now,

$(x, x) \notin R$

Since human being x cannot be taller than himself.

$\therefore$R is not reflexive.

Now, let $(x, y) \in R$.

$\Rightarrow$ x is exactly 7 cm taller than y.

Then, y is not taller than x.

$\therefore (y, x) \notin R$

Indeed if x is exactly 7 cm taller than y, then y is exactly 7 cm shorter than x.

$\therefore$R is not symmetric.

Now,

Let $(x, y), (y, z) \in R$.

$\Rightarrow$ x is exactly 7 cm taller than y and y is exactly 7 cm taller than z.

$\Rightarrow$ x is exactly 14 cm taller than z .

$\therefore (x, z) \notin R$

$\therefore$ R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(d) $R = \{(x, y)$: x is the wife of $y\}$

Now,

$(x, x) \notin R$

Since x cannot be the wife of herself.

∴R is not reflexive.

Now, let $(x, y) \in R$

$\Rightarrow x$ is the wife of y.

Clearly y is not the wife of x.

$\therefore (y, x) \notin R$

Indeed if x is the wife of y, then y is the husband of x.

∴ R is not transitive.

Let $(x, y), (y, z) \in R$

$\Rightarrow x$ is the wife of y and y is the wife of z.

This case is not possible. Also, this does not imply that x is the wife of z.

$\therefore (x, z) \notin R$

∴R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(e) $R = \{(x, y): x$ is the father of $y\}$

$(x, x) \notin R$

As x cannot be the father of himself.

∴R is not reflexive.

Now, let $(x, y) \in R$.

$\Rightarrow x$ is the father of y.

$\Rightarrow y$ cannot be the father of y.

Indeed, y is the son or the daughter of y.

$\therefore (y, x) \notin R$

$\therefore$ R is not symmetric.

Now, let $(x, y) \in R$ and $(y, z) \in R$.

$\Rightarrow x$ is the father of y and y is the father of z.

$\Rightarrow x$ is not the father of z.

Indeed x is the grandfather of z.

$\therefore (x, z) \notin R$

$\therefore$R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

Show that the relation R in the set **R** of real numbers, defined as

R = $\{(a, b): a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

Answer

R = $\{(a, b): a \leq b^2\}$

It can be observed that $\left(\frac{1}{2}, \frac{1}{2}\right) \notin \mathbf{R}$, since $\frac{1}{2} > \left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

∴R is not reflexive.

Now, (1, 4) ∈ R as $1 < 4^2$

But, 4 is not less than 1^2.

∴(4, 1) ∉ R

∴R is not symmetric.

Now,

(3, 2), (2, 1.5) ∈ R

(as $3 < 2^2 = 4$ and $2 < (1.5)^2 = 2.25$)

But, $3 > (1.5)^2 = 2.25$

∴(3, 1.5) ∉ R

∴ R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as

R = {(a, b): b = a + 1} is reflexive, symmetric or transitive.

Answer

Let A = {1, 2, 3, 4, 5, 6}.

A relation R is defined on set A as:

R = {(a, b): b = a + 1}

∴R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}

We can find (a, a) ∉ R, where a ∈ A.

For instance,

(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) ∉ R

∴R is not reflexive.

It can be observed that (1, 2) ∈ R, but (2, 1) ∉ R.

∴R is not symmetric.

Now, (1, 2), (2, 3) ∈ **R**

But,

(1, 3) ∉ R

∴R is not transitive

Hence, R is neither reflexive, nor symmetric, nor transitive.

Show that the relation R in **R** defined as R = {(a, b): $a \leq b$}, is reflexive and transitive but not symmetric.

Answer

R = {(a, b); $a \leq b$}

Clearly (a, a) ∈ R as $a = a$.

∴R is reflexive.

Now,

(2, 4) ∈ R (as 2 < 4)

But, (4, 2) ∉ R as 4 is greater than 2.

∴ R is not symmetric.

Now, let $(a, b), (b, c) \in R$.

Then,

$a \leq b$ and $b \leq c$

$\Rightarrow a \leq c$

$\Rightarrow (a, c) \in R$

$\therefore$R is transitive.

Hence,R is reflexive and transitive but not symmetric.

Check whether the relation R in **R** defined as $R = \{(a, b): a \leq b^3\}$ is reflexive, symmetric or transitive.

Answer

$R = \{(a, b): a \leq b^3\}$

It is observed that $\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$ as $\frac{1}{2} > \left(\frac{1}{2}\right)^3 = \frac{1}{8}$.

$\therefore$ R is not reflexive.

Now,

$(1, 2) \in R$ (as $1 < 2^3 = 8$)

But,

$(2, 1) \notin R$ (as $2^3 > 1$)

$\therefore$ R is not symmetric.

We have $\left(3, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{6}{5}\right) \in R$ as $3 < \left(\frac{3}{2}\right)^3$ and $\frac{3}{2} < \left(\frac{6}{5}\right)^3$.

But $\left(3, \frac{6}{5}\right) \notin R$ as $3 > \left(\frac{6}{5}\right)^3$.

$\therefore$ R is not transitive.

Hence R is neither Reflexive, nor Symmetric, nor Transitive

Show that the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor transitive.

Answer

Let A = {1, 2, 3}.

A relation R on A is defined as R = {(1, 2), (2, 1)}.

It is seen that (1, 1), (2, 2), (3, 3) ∉R.

∴ R is not reflexive.

Now, as (1, 2) ∈ R and (2, 1) ∈ R, then R is symmetric.

Now, (1, 2) and (2, 1) ∈ R

However,

(1, 1) ∉ R

∴ R is not transitive.

Hence, R is symmetric but neither reflexive nor transitive.

Show that the relation R in the set A of all the books in a library of a college, given by R = {(x, y): x and y have same number of pages} is an equivalence relation.

Answer

Set A is the set of all books in the library of a college.

R = {x, y): x and y have the same number of pages}

Now, R is reflexive since (x, x) ∈ R as x and x has the same number of pages.

Let (x, y) ∈ R ⇒ x and y have the same number of pages.

⇒ y and x have the same number of pages.

⇒ (y, x) ∈ R

∴R is symmetric.

Now, let (x, y) ∈R and (y, z) ∈ R.

⇒ x and y and have the same number of pages and y and z have the same number of pages.

⇒ x and z have the same number of pages.

⇒ (x, z) ∈ R

∴R is transitive.

Hence, R is an equivalence relation.

Show that the relation R in the set A = {1, 2, 3, 4, 5} given by $R = \{(a, b) : |a-b| \text{ is even}\}$, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of 2, 4}.

Answer

A = {1, 2, 3, 4, 5}

$R = \{(a, b) : |a-b| \text{ is even}\}$

It is clear that for any element $a \in A$, we have $|a-a| = 0$ (which is even).

∴R is reflexive.

Let $(a, b) \in$ R.

$\Rightarrow |a-b|$ is even.

$\Rightarrow |-(a-b)| = |b-a|$ is also even.

$\Rightarrow (b, a) \in$ R

∴R is symmetric.

Now, let $(a, b) \in$ R and $(b, c) \in$ R.

$\Rightarrow |a-b|$ is even and $|b-c|$ is even.

$\Rightarrow (a-b)$ is even and $(b-c)$ is even.

$\Rightarrow (a-c) = (a-b) + (b-c)$ is even. [Sum of two even integers is even]

$\Rightarrow |a-c|$ is even.

$\Rightarrow (a, c) \in$ R

∴R is transitive.

Hence, R is an equivalence relation.

Now, all elements of the set {1, 2, 3} are related to each other as all the elements of this subset are odd. Thus, the modulus of the difference between any two elements will be even.

Similarly, all elements of the set {2, 4} are related to each other as all the elements of this subset are even.

Also, no element of the subset {1, 3, 5} can be related to any element of {2, 4} as all elements of {1, 3, 5} are odd and all elements of {2, 4} are even. Thus, the modulus of the difference between the two elements (from each of these two subsets) will not be even.

Show that each of the relation R in the set $A = \{x \in \mathbf{Z} : 0 \le x \le 12\}$, given by

(i) $R = \{(a,\ b) : |a-b| \text{ is a multiple of } 4\}$

(ii) $R = \{(a,\ b) : a = b\}$

is an equivalence relation. Find the set of all elements related to 1 in each case.

Answer

$A = \{x \in \mathbf{Z} : 0 \le x \le 12\} = \{0,\ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

(i) $R = \{(a,\ b) : |a-b| \text{ is a multiple of } 4\}$

For any element a ∈A, we have (a, a) ∈ R as $|a-a| = 0$ is a multiple of 4.

∴R is reflexive.

Now, let (a, b) ∈ R ⇒ $|a-b|$ is a multiple of 4.

$\Rightarrow |-(a-b)| = |b-a|$ is a multiple of 4.

⇒ (b, a) ∈ R

∴R is symmetric.

Now, let $(a, b), (b, c) \in$ R.

$\Rightarrow |a-b|$ is a multiple of 4 and $|b-c|$ is a multiple of 4.

$\Rightarrow (a-b)$ is a multiple of 4 and $(b-c)$ is a multiple of 4.

$\Rightarrow (a-c)=(a-b)+(b-c)$ is a multiple of 4.

$\Rightarrow |a-c|$ is a multiple of 4.

$\Rightarrow (a, c) \in$R

∴ R is transitive.

Hence, R is an equivalence relation.

The set of elements related to 1 is {1, 5, 9} since

$|1-1|=0$ is a multiple of 4,

$|5-1|=4$ is a multiple of 4, and

$|9-1|=8$ is a multiple of 4.

(ii) R = $\{(a, b): a = b\}$

For any element $a \in$A, we have $(a, a) \in$ R, since $a = a$.

∴R is reflexive.

Now, let $(a, b) \in R$.

$\Rightarrow a = b$

$\Rightarrow b = a$

$\Rightarrow (b, a) \in R$

∴R is symmetric.

Now, let $(a, b) \in R$ and $(b, c) \in R$.

$\Rightarrow a = b$ and $b = c$

$\Rightarrow a = c$

$\Rightarrow (a, c) \in R$

∴ R is transitive.

Hence, R is an equivalence relation.

The elements in R that are related to 1 will be those elements from set A which are equal to 1.

Hence, the set of elements related to 1 is $\{1\}$.

Given an example of a relation. Which is

(i) Symmetric but neither reflexive nor transitive.

(ii) Transitive but neither reflexive nor symmetric.

(iii) Reflexive and symmetric but not transitive.

(iv) Reflexive and transitive but not symmetric.

(v) Symmetric and transitive but not reflexive.

Answer

(i) Let $A = \{5, 6, 7\}$.

Define a relation R on A as $R = \{(5, 6), (6, 5)\}$.

Relation R is not reflexive as $(5, 5), (6, 6), (7, 7) \notin R$.

Now, as $(5, 6) \in R$ and also $(6, 5) \in R$, R is symmetric.

$\Rightarrow (5, 6), (6, 5) \in R$, but $(5, 5) \notin R$

$\therefore$ R is not transitive.

Hence, relation R is symmetric but not reflexive or transitive.

(ii)Consider a relation R in **R** defined as:

$R = \{(a, b): a < b\}$

For any $a \in R$, we have $(a, a) \notin R$ since a cannot be strictly less than a itself. In fact, $a = a$.

$\therefore$ R is not reflexive.

Now,

$(1, 2) \in R$ (as $1 < 2$)

But, 2 is not less than 1.

$\therefore (2, 1) \notin R$

$\therefore$ R is not symmetric.

Now, let $(a, b), (b, c) \in R$.

$\Rightarrow a < b$ and $b < c$

$\Rightarrow a < c$

$\Rightarrow (a, c) \in R$

$\therefore$R is transitive.

Hence, relation R is transitive but not reflexive and symmetric.

(iii)Let $A = \{4, 6, 8\}$.

Define a relation R on A as:

$A = \{(4, 4), (6, 6), (8, 8), (4, 6), (6, 4), (6, 8), (8, 6)\}$

Relation R is reflexive since for every $a \in A$, $(a, a) \in$R i.e., $(4, 4), (6, 6), (8, 8)\} \in$ R.

Relation R is symmetric since $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in R$.

Relation R is not transitive since $(4, 6), (6, 8) \in R$, but $(4, 8) \notin R$.

Hence, relation R is reflexive and symmetric but not transitive.

(iv) Define a relation R in **R** as:

R = $\{a, b): a^3 \geq b^3\}$

Clearly $(a, a) \in$ R as $a^3 = a^3$.

∴R is reflexive.

Now,

$(2, 1) \in$ R (as $2^3 \geq 1^3$)

But,

$(1, 2) \notin$ R (as $1^3 < 2^3$)

∴ R is not symmetric.

Now,

Let $(a, b), (b, c) \in$ R.

$\Rightarrow a^3 \geq b^3$ and $b^3 \geq c^3$

$\Rightarrow a^3 \geq c^3$

$\Rightarrow (a, c) \in$ R

∴R is transitive.

Hence, relation R is reflexive and transitive but not symmetric.

(v) Let $A = \{-5, -6\}$.

Define a relation R on A as:

R = $\{(-5, -6), (-6, -5), (-5, -5)\}$

Relation R is not reflexive as $(-6, -6) \notin$ R.

Relation R is symmetric as $(-5, -6) \in$ R and $(-6, -5\}\in$R.

It is seen that $(-5, -6), (-6, -5) \in$ R. Also, $(-5, -5) \in$ R.

∴The relation R is transitive.

Hence, relation R is symmetric and transitive but not reflexive.

Show that the relation R in the set A of points in a plane given by R = {(P, Q): distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set of all point related to a point P ≠ (0, 0) is the circle passing through P with origin as centre.

Answer

R = {(P, Q): distance of point P from the origin is the same as the distance of point Q from the origin}

Clearly, (P, P) ∈ R since the distance of point P from the origin is always the same as the distance of the same point P from the origin.

∴R is reflexive.

Now,

Let (P, Q) ∈ R.

⇒ The distance of point P from the origin is the same as the distance of point Q from the origin.

⇒ The distance of point Q from the origin is the same as the distance of point P from the origin.

⇒ (Q, P) ∈ R

∴R is symmetric.

Now,

Let (P, Q), (Q, S) ∈ R.

⇒ The distance of points P and Q from the origin is the same and also, the distance of points Q and S from the origin is the same.

⇒ The distance of points P and S from the origin is the same.

⇒ (P, S) ∈ R

∴R is transitive.

Therefore, R is an equivalence relation.

The set of all points related to P ≠ (0, 0) will be those points whose distance from the origin is the same as the distance of point P from the origin.

In other words, if O (0, 0) is the origin and OP = k, then the set of all points related to P is at a distance of k from the origin.

Hence, this set of points forms a circle with the centre as the origin and this circle passes through point P.

Show that the relation R defined in the set A of all triangles as R = $\{(T_1, T_2): T_1$ is similar to $T_2\}$, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

Answer

R = $\{(T_1, T_2): T_1$ is similar to $T_2\}$

R is reflexive since every triangle is similar to itself.

Further, if $(T_1, T_2) \in$ R, then T_1 is similar to T_2.

$\Rightarrow T_2$ is similar to T_1.

$\Rightarrow (T_2, T_1) \in$R

$\therefore$R is symmetric.

Now,

Let $(T_1, T_2), (T_2, T_3) \in$ R.

$\Rightarrow T_1$ is similar to T_2 and T_2 is similar to T_3.

$\Rightarrow T_1$ is similar to T_3.

$\Rightarrow (T_1, T_3) \in$ R

$\therefore$ R is transitive.

Thus, R is an equivalence relation.

Now, we can observe that:

$$\frac{3}{6}=\frac{4}{8}=\frac{5}{10}\left(=\frac{1}{2}\right)$$

$\therefore$The corresponding sides of triangles T_1 and T_3 are in the same ratio.

Then, triangle T_1 is similar to triangle T_3.

Hence, T_1 is related to T_3.

Show that the relation R defined in the set A of all polygons as R = $\{(P_1, P_2): P_1$ and P_2 have same number of sides$\}$, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

Answer

R = $\{(P_1, P_2): P_1$ and P_2 have same the number of sides$\}$

R is reflexive since $(P_1, P_1) \in$ R as the same polygon has the same number of sides with itself.

Let $(P_1, P_2) \in$ R.

$\Rightarrow P_1$ and P_2 have the same number of sides.

$\Rightarrow P_2$ and P_1 have the same number of sides.

$\Rightarrow (P_2, P_1) \in$ R

$\therefore$R is symmetric.

Now,

Let $(P_1, P_2), (P_2, P_3) \in$ R.

$\Rightarrow P_1$ and P_2 have the same number of sides. Also, P_2 and P_3 have the same number of sides.

$\Rightarrow P_1$ and P_3 have the same number of sides.

$\Rightarrow (P_1, P_3) \in$ R

$\therefore$R is transitive.

Hence, R is an equivalence relation.

The elements in A related to the right-angled triangle (T) with sides 3, 4, and 5 are those polygons which have 3 sides (since T is a polygon with 3 sides).

Hence, the set of all elements in A related to triangle T is the set of all triangles.

Let L be the set of all lines in XY plane and R be the relation in L defined as R = $\{(L_1, L_2): L_1$ is parallel to $L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

Answer

R = $\{(L_1, L_2)$: L_1 is parallel to $L_2\}$

R is reflexive as any line L_1 is parallel to itself i.e., $(L_1, L_1) \in$ R.

Now,

Let $(L_1, L_2) \in$ R.

$\Rightarrow L_1$ is parallel to L_2.

$\Rightarrow L_2$ is parallel to L_1.

$\Rightarrow (L_2, L_1) \in$ R

$\therefore$ R is symmetric.

Now,

Let (L_1, L_2), $(L_2, L_3) \in$R.

$\Rightarrow L_1$ is parallel to L_2. Also, L_2 is parallel to L_3.

$\Rightarrow L_1$ is parallel to L_3.

$\therefore$R is transitive.

Hence, R is an equivalence relation.

The set of all lines related to the line $y = 2x + 4$ is the set of all lines that are parallel to the line $y = 2x + 4$.

Slope of line $y = 2x + 4$ is $m = 2$

It is known that parallel lines have the same slopes.

The line parallel to the given line is of the form $y = 2x + c$, where $c \in \mathbf{R}$.

Hence, the set of all lines related to the given line is given by $y = 2x + c$, where $c \in \mathbf{R}$.

Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer.

(A) R is reflexive and symmetric but not transitive.

(B) R is reflexive and transitive but not symmetric.

(C) R is symmetric and transitive but not reflexive.

(D) R is an equivalence relation.

Answer

R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}

It is seen that $(a, a) \in$ **R,** for every $a \in \{1, 2, 3, 4\}$.

∴ R is reflexive.

It is seen that $(1, 2) \in$ R, but $(2, 1) \notin$ R.

∴R is not symmetric.

Also, it is observed that $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in \{1, 2, 3, 4\}$.

∴ R is transitive.

Hence, R is reflexive and transitive but not symmetric.

The correct answer is B.

Let R be the relation in the set **N** given by $R = \{(a, b): a = b - 2, b > 6\}$. Choose the correct answer.

(A) $(2, 4) \in R$ (B) $(3, 8) \in R$ (C) $(6, 8) \in R$ (D) $(8, 7) \in R$

Answer

$R = \{(a, b): a = b - 2, b > 6\}$

Now, since $b > 6$, $(2, 4) \notin R$

Also, as $3 \neq 8 - 2$, $(3, 8) \notin R$

And, as $8 \neq 7 - 2$

$\therefore (8, 7) \notin R$

Now, consider (6, 8).

We have $8 > 6$ and also, $6 = 8 - 2$.

$\therefore (6, 8) \in R$

The correct answer is C.

Show that the function f: $\mathbf{R}_* \to \mathbf{R}_*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto, where $\mathbf{R}_*$ is the set of all non-zero real numbers. Is the result true, if the domain $\mathbf{R}_*$ is replaced by **N** with co-domain being same as $\mathbf{R}_*$?

It is given that f: $\mathbf{R}_* \to \mathbf{R}_*$ is defined by $f(x) = \frac{1}{x}$.

One-one:

$$f(x) = f(y)$$
$$\Rightarrow \frac{1}{x} = \frac{1}{y}$$
$$\Rightarrow x = y$$

$\therefore f$ is one-one.

Onto:

It is clear that for $y \in \mathbf{R}_*$, there exists $x = \frac{1}{y} \in \mathrm{R}_*$ (Exists as $y \neq 0$) such that

$$f(x) = \frac{1}{\left(\frac{1}{y}\right)} = y.$$

$\therefore f$ is onto.

Thus, the given function (f) is one-one and onto.

Now, consider function g: **N** → **R**$_*$defined by

$$g(x) = \frac{1}{x}.$$

We have,

$$g(x_1) = g(x_2) \Rightarrow \frac{1}{x_1} = \frac{1}{x_2} \Rightarrow x_1 = x_2$$

∴g is one-one.

Further, it is clear that g is not onto as for 1.2 ∈**R**$_*$ there does not exit any x in **N** such that $g(x) = \frac{1}{1.2}$.

Hence, function g is one-one but not onto.

Check the injectivity and surjectivity of the following functions:

(i) f: **N** → **N** given by $f(x) = x^2$

(ii) f: **Z** → **Z** given by $f(x) = x^2$

(iii) f: **R** → **R** given by $f(x) = x^2$

(iv) f: **N** → **N** given by $f(x) = x^3$

(v) f: **Z** → **Z** given by $f(x) = x^3$

Answer

(i) f: **N** → **N** is given by,

$f(x) = x^2$

It is seen that for x, y ∈**N**, $f(x) = f(y) \Rightarrow x^2 = y^2 \Rightarrow x = y$.

∴f is injective.

Now, 2 ∈ **N**. But, there does not exist any x in **N** such that $f(x) = x^2 = 2$.

∴ f is not surjective.

Hence, function f is injective but not surjective.

(ii) f: **Z** → **Z** is given by,

$f(x) = x^2$

It is seen that $f(-1) = f(1) = 1$, but $-1 \neq 1$.

$\therefore$ f is not injective.

Now, $-2 \in \mathbf{Z}$. But, there does not exist any element $x \in \mathbf{Z}$ such that $f(x) = x^2 = -2$.

$\therefore$ f is not surjective.

Hence, function f is neither injective nor surjective.

(iii) f: $\mathbf{R} \to \mathbf{R}$ is given by,

$f(x) = x^2$

It is seen that $f(-1) = f(1) = 1$, but $-1 \neq 1$.

$\therefore$ f is not injective.

Now, $-2 \in \mathbf{R}$. But, there does not exist any element $x \in \mathbf{R}$ such that $f(x) = x^2 = -2$.

$\therefore$ f is not surjective.

Hence, function f is neither injective nor surjective.

(iv) f: $\mathbf{N} \to \mathbf{N}$ given by,

$f(x) = x^3$

It is seen that for $x, y \in \mathbf{N}$, $f(x) = f(y) \Rightarrow x^3 = y^3 \Rightarrow x = y$.

$\therefore f$ is injective.

Now, $2 \in \mathbf{N}$. But, there does not exist any element x in domain $\mathbf{N}$ such that $f(x) = x^3 = 2$.

$\therefore$ f is not surjective

Hence, function f is injective but not surjective.

(v) f: $\mathbf{Z} \to \mathbf{Z}$ is given by,

$f(x) = x^3$

It is seen that for $x, y \in \mathbf{Z}$, $f(x) = f(y) \Rightarrow x^3 = y^3 \Rightarrow x = y$.

$\therefore$ f is injective.

Now, $2 \in \mathbf{Z}$. But, there does not exist any element x in domain $\mathbf{Z}$ such that $f(x) = x^3 = 2$.

$\therefore$ f is not surjective.

Hence, function f is injective but not surjective.

Prove that the Greatest Integer Function f: $\mathbf{R} \to \mathbf{R}$ given by $f(x) = [x]$, is neither one-once nor onto, where $[x]$ denotes the greatest integer less than or equal to x.

Answer

f: $\mathbf{R} \to \mathbf{R}$ is given by,

$f(x) = [x]$

It is seen that $f(1.2) = [1.2] = 1$, $f(1.9) = [1.9] = 1$.

$\therefore f(1.2) = f(1.9)$, but $1.2 \neq 1.9$.

$\therefore f$ is not one-one.

Now, consider $0.7 \in \mathbf{R}$.

It is known that $f(x) = [x]$ is always an integer. Thus, there does not exist any element $x \in \mathbf{R}$ such that $f(x) = 0.7$.

$\therefore f$ is not onto.

Hence, the greatest integer function is neither one-one nor onto.

Show that the Modulus Function f: $\mathbf{R} \to \mathbf{R}$ given by $f(x) = |x|$, is neither one-one nor onto, where $|x|$ is x, if x is positive or 0 and $|x|$ is $-x$, if x is negative.

f: $\mathbf{R} \to \mathbf{R}$ is given by,

$$f(x) = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

It is seen that $f(-1) = |-1| = 1, f(1) = |1| = 1$.

$\therefore f(-1) = f(1)$, but $-1 \neq 1$.

$\therefore f$ is not one-one.

Now, consider $-1 \in \mathbf{R}$.

It is known that $f(x) = |x|$ is always non-negative. Thus, there does not exist any element x in domain $\mathbf{R}$ such that $f(x) = |x| = -1$.

$\therefore f$ is not onto.

Hence, the modulus function is neither one-one nor onto.

Show that the Signum Function f: **R** → **R**, given by

$$f(x)=\begin{cases}1, \text{ if } x>0\\ 0, \text{ if } x=0\\ -1, \text{ if } x<0\end{cases}$$

is neither one-one nor onto.

Answer

f: **R** → **R** is given by,

$$f(x)=\begin{cases}1, \text{ if } x>0\\ 0, \text{ if } x=0\\ -1, \text{ if } x<0\end{cases}$$

It is seen that $f(1) = f(2) = 1$, but $1 \neq 2$.

$\therefore f$ is not one-one.

Now, as $f(x)$ takes only 3 values (1, 0, or −1) for the element −2 in co-domain **R**, there does not exist any x in domain **R** such that $f(x) = -2$.

$\therefore f$ is not onto.

Hence, the signum function is neither one-one nor onto.

Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Show that f is one-one.

Answer

It is given that $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$.

f: $A \to B$ is defined as $f = \{(1, 4), (2, 5), (3, 6)\}$.

$\therefore f(1) = 4, f(2) = 5, f(3) = 6$

It is seen that the images of distinct elements of A under f are distinct.

Hence, function f is one-one.

In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

(i) f: **R** → **R** defined by $f(x) = 3 - 4x$

(ii) f: **R** → **R** defined by $f(x) = 1 + x^2$

Answer

(i) f: **R** → **R** is defined as $f(x) = 3 - 4x$.

Let $x_1, x_2 \in \mathbf{R}$ such that $f(x_1) = f(x_2)$.

$$\Rightarrow 3 - 4x_1 = 3 - 4x_2$$
$$\Rightarrow -4x_1 = -4x_2$$
$$\Rightarrow x_1 = x_2$$

∴ f is one-one.

For any real number (y) in **R**, there exists $\frac{3-y}{4}$ in **R** such that

$$f\left(\frac{3-y}{4}\right) = 3 - 4\left(\frac{3-y}{4}\right) = y.$$

∴ f is onto.

Hence, f is bijective.

(ii) f: **R** → **R** is defined as

$f(x) = 1 + x^2$.

Let $x_1, x_2 \in \mathbf{R}$ such that $f(x_1) = f(x_2)$.

$\Rightarrow 1 + x_1^2 = 1 + x_2^2$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = \pm x_2$

∴ $f(x_1) = f(x_2)$ does not imply that $x_1 = x_2$.

For instance,

$f(1) = f(-1) = 2$

∴ f is not one-one.

Consider an element −2 in co-domain **R**.

It is seen that $f(x) = 1 + x^2$ is positive for all $x \in \mathbf{R}$.

Thus, there does not exist any x in domain **R** such that $f(x) = -2$.

∴ f is not onto.

Hence, f is neither one-one nor onto.

Let A and B be sets. Show that $f: A \times B \rightarrow B \times A$ such that $(a, b) = (b, a)$ is bijective function.

Answer

$f: A \times B \rightarrow B \times A$ is defined as $f(a, b) = (b, a)$.

Let $(a_1, b_1), (a_2, b_2) \in A \times B$ such that $f(a_1, b_1) = f(a_2, b_2)$.

$\Rightarrow (b_1, a_1) = (b_2, a_2)$

$\Rightarrow b_1 = b_2$ and $a_1 = a_2$

$\Rightarrow (a_1, b_1) = (a_2, b_2)$

$\therefore$ f is one-one.

Now, let $(b, a) \in B \times A$ be any element.

Then, there exists $(a, b) \in A \times B$ such that $f(a, b) = (b, a)$. [By definition of f]

$\therefore$ f is onto.

Hence, f is bijective.

Let f: **N** → **N** be defined by $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$ for all $n \in \mathbf{N}$.

State whether the function f is bijective. Justify your answer.

Answer

f: **N** → **N** is defined as $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$ for all $n \in \mathbf{N}$.

It can be observed that:

$$f(1) = \frac{1+1}{2} = 1 \text{ and } f(2) = \frac{2}{2} = 1 \qquad [\text{By definition of } f]$$

$$\therefore f(1) = f(2), \text{ where } 1 \neq 2.$$

∴ f is not one-one.

Consider a natural number (n) in co-domain **N**.

Case **I:** n is odd

$\therefore n = 2r + 1$ for some $r \in$ **N.** Then, there exists $4r + 1 \in$ **N** such that

$$f(4r+1) = \frac{4r+1+1}{2} = 2r+1.$$

Case **II:** n is even

$\therefore n = 2r$ for some $r \in$ **N.** Then,there exists $4r \in$ **N** such that $f(4r) = \frac{4r}{2} = 2r$.

$\therefore f$ is onto.

Hence, f is not a bijective function.

Question 10:

Let A = **R** − {3} and B = **R** − {1}. Consider the function f: A → B defined by

$f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

Answer

A = **R** − {3}, B = **R** − {1}

f: A → B is defined as $f(x) = \left(\frac{x-2}{x-3}\right)$.

Let $x,\ y \in$ A such that $f(x) = f(y)$.

$$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$$
$$\Rightarrow (x-2)(y-3) = (y-2)(x-3)$$
$$\Rightarrow xy - 3x - 2y + 6 = xy - 3y - 2x + 6$$
$$\Rightarrow -3x - 2y = -3y - 2x$$
$$\Rightarrow 3x - 2x = 3y - 2y$$
$$\Rightarrow x = y$$

∴ *f* is one-one.

Let $y \in$B = **R** − {1}. Then, $y \neq 1$.

The function *f* is onto if there exists $x \in$A such that $f(x) = y$.

Now,

$$f(x) = y$$

$$\Rightarrow \frac{x-2}{x-3} = y$$
$$\Rightarrow x-2 = xy-3y$$
$$\Rightarrow x(1-y) = -3y+2$$
$$\Rightarrow x = \frac{2-3y}{1-y} \in \text{A} \qquad [y \neq 1]$$

Thus, for any $y \in$ B, there exists $\frac{2-3y}{1-y} \in \text{A}$ such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right)-2}{\left(\frac{2-3y}{1-y}\right)-3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y.$$

$\therefore f$ is onto.

Hence, function f is one-one and onto.

Let f: **R** → **R** be defined as $f(x) = x^4$. Choose the correct answer.

(A) f is one-one onto (B) f is many-one onto

(C) f is one-one but not onto (D) f is neither one-one nor onto

Answer

f: **R** → **R** is defined as $f(x) = x^4$.

Let $x, y \in$ **R** such that $f(x) = f(y)$.

$\Rightarrow x^4 = y^4$

$\Rightarrow x = \pm y$

$\therefore f(x_1) = f(x_2)$ does not imply that $x_1 = x_2$.

For instance,

$f(1) = f(-1) = 1$

$\therefore f$ is not one-one.

Consider an element 2 in co-domain **R**. It is clear that there does not exist any x in domain **R** such that $f(x) = 2$.

$\therefore f$ is not onto.

Hence, function f is neither one-one nor onto.

The correct answer is D.

Let f: **R** → **R** be defined as $f(x) = 3x$. Choose the correct answer.

(A) f is one-one onto (B) f is many-one onto

(C) f is one-one but not onto (D) f is neither one-one nor onto

Answer

f: **R** → **R** is defined as $f(x) = 3x$.

Let $x, y \in$ **R** such that $f(x) = f(y)$.

$\Rightarrow 3x = 3y$

$\Rightarrow x = y$

$\therefore f$ is one-one.

Also, for any real number (y) in co-domain **R**, there exists $\frac{y}{3}$ in **R** such that

$$f\left(\frac{y}{3}\right) = 3\left(\frac{y}{3}\right) = y$$

$\therefore f$ is onto.

Hence, function f is one-one and onto.

The correct answer is A.

Let f: {1, 3, 4} → {1, 2, 5} and g: {1, 2, 5} → {1, 3} be given by f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}. Write down gof.

Answer

The functions f: {1, 3, 4} → {1, 2, 5} and g: {1, 2, 5} → {1, 3} are defined as f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}.

$$gof(1) = g(f(1)) = g(2) = 3 \qquad [f(1) = 2 \text{ and } g(2) = 3]$$

$$gof(3) = g(f(3)) = g(5) = 1 \qquad [f(3) = 5 \text{ and } g(5) = 1]$$

$$gof(4) = g(f(4)) = g(1) = 3 \qquad [f(4) = 1 \text{ and } g(1) = 3]$$

$$\therefore gof = \{(1,3),(3,1),(4,3)\}$$

Let f, g and h be functions from **R** to **R**. Show that

$$(f+g)oh = foh + goh$$

$$(f \cdot g)oh = (foh)\cdot(goh)$$

Answer

To prove:

$$(f+g)oh = foh + goh$$

Consider:

$$((f+g)oh)(x)$$
$$= (f+g)(h(x))$$
$$= f(h(x)) + g(h(x))$$
$$= (foh)(x) + (goh)(x)$$
$$= \{(foh)+(goh)\}(x)$$
$$\therefore ((f+g)oh)(x) = \{(foh)+(goh)\}(x) \qquad \forall x \in \mathbf{R}$$

Hence, $(f+g)oh = foh + goh$.

To prove:

$$(f \cdot g)oh = (foh)\cdot(goh)$$

Consider:

$$((f \cdot g)oh)(x)$$
$$= (f \cdot g)(h(x))$$
$$= f(h(x)).g(h(x))$$
$$= (foh)(x).(goh)(x)$$
$$= \{(foh).(goh)\}(x)$$
$$\therefore ((f \cdot g)oh)(x) = \{(foh).(goh)\}(x) \ \forall x \in \mathrm{R}$$

Hence, $(f \cdot g)oh = (foh)\cdot(goh)$.

Find *gof* and *fog*, if

(i) $f(x)=|x|$ and $g(x)=|5x-2|$

(ii) $f(x)=8x^3$ and $g(x)=x^{\frac{1}{3}}$

Answer

(i) $f(x)=|x|$ and $g(x)=|5x-2|$

$\therefore (gof)(x)=g(f(x))=g(|x|)=|5|x|-2|$

$(fog)(x)=f(g(x))=f(|5x-2|)=||5x-2||=|5x-2|$

(ii) $f(x)=8x^3$ and $g(x)=x^{\frac{1}{3}}$

$\therefore (gof)(x)=g(f(x))=g(8x^3)=(8x^3)^{\frac{1}{3}}=2x$

$(fog)(x)=f(g(x))=f\left(x^{\frac{1}{3}}\right)=8\left(x^{\frac{1}{3}}\right)^3=8x$

If $f(x)=\frac{(4x+3)}{(6x-4)}, x \neq \frac{2}{3}$, show that f o $f(x) = x$, for all $x \neq \frac{2}{3}$. What is the inverse of f?

Answer

It is given that $f(x)=\frac{(4x+3)}{(6x-4)}, x \neq \frac{2}{3}$.

$$(fof)(x)=f(f(x))=f\left(\frac{4x+3}{6x-4}\right)$$

$$=\frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}=\frac{16x+12+18x-12}{24x+18-24x+16}=\frac{34x}{34}=x$$

Therefore, $fof(x)=x$, for all $x \neq \frac{2}{3}$.

$\Rightarrow fof = \mathrm{I}$

Hence, the given function f is invertible and the inverse of f is f itself.

State with reason whether following functions have inverse

(i) f: {1, 2, 3, 4} → {10} with

f = {(1, 10), (2, 10), (3, 10), (4, 10)}

(ii) g: {5, 6, 7, 8} → {1, 2, 3, 4} with

g = {(5, 4), (6, 3), (7, 4), (8, 2)}

(iii) h: {2, 3, 4, 5} → {7, 9, 11, 13} with

h = {(2, 7), (3, 9), (4, 11), (5, 13)}

Answer

(i) f: {1, 2, 3, 4} → {10}defined as:

f = {(1, 10), (2, 10), (3, 10), (4, 10)}

From the given definition of f, we can see that f is a many one function as: $f(1) = f(2) = f(3) = f(4) = 10$

∴f is not one-one.

Hence, function f does not have an inverse.

(ii) g: {5, 6, 7, 8} → {1, 2, 3, 4} defined as:

g = {(5, 4), (6, 3), (7, 4), (8, 2)}

From the given definition of g, it is seen that g is a many one function as: $g(5) = g(7) = 4$.

∴g is not one-one,

Hence, function g does not have an inverse.

(iii) h: {2, 3, 4, 5} → {7, 9, 11, 13} defined as:

h = {(2, 7), (3, 9), (4, 11), (5, 13)}

It is seen that all distinct elements of the set {2, 3, 4, 5} have distinct images under h.

∴Function h is one-one.

Also, h is onto since for every element y of the set {7, 9, 11, 13}, there exists an element x in the set {2, 3, 4, 5}such that $h(x) = y$.

Thus, h is a one-one and onto function. Hence, h has an inverse.

Show that f: $[-1, 1] \rightarrow$ **R**, given by $f(x)=\frac{x}{(x+2)}$ is one-one. Find the inverse of the function f: $[-1, 1] \rightarrow$ Range f.

(Hint: For $y \in$Range f, $y = f(x)=\frac{x}{x+2}$, for some x in $[-1, 1]$, i.e., $x=\frac{2y}{(1-y)}$)

Answer

f: $[-1, 1] \rightarrow$ R is given as $f(x)=\frac{x}{(x+2)}$.

Let $f(x) = f(y)$.

$$\Rightarrow \frac{x}{x+2}=\frac{y}{y+2}$$
$$\Rightarrow xy+2x=xy+2y$$
$$\Rightarrow 2x=2y$$
$$\Rightarrow x=y$$

$\therefore$ f is a one-one function.

It is clear that f: $[-1, 1] \rightarrow$ Range f is onto.

$\therefore$ f: $[-1, 1] \rightarrow$ Range f is one-one and onto and therefore, the inverse of the function: f: $[-1, 1] \rightarrow$ Range f exists.

Let g: Range $f \rightarrow [-1, 1]$ be the inverse of f.

Let y be an arbitrary element of range f.

Since f: $[-1, 1] \rightarrow$ Range f is onto, we have:

$$y=f(x) \text{ for same } x \in [-1, 1]$$
$$\Rightarrow y=\frac{x}{x+2}$$
$$\Rightarrow xy+2y=x$$
$$\Rightarrow x(1-y)=2y$$
$$\Rightarrow x=\frac{2y}{1-y},\ y \neq 1$$

Now, let us define g: Range $f \rightarrow [-1, 1]$ as

$$g(y)=\frac{2y}{1-y},\ y \neq 1.$$

$$\text{Now, } (gof)(x) = g(f(x)) = g\left(\frac{x}{x+2}\right) = \frac{2\left(\frac{x}{x+2}\right)}{1-\frac{x}{x+2}} = \frac{2x}{x+2-x} = \frac{2x}{2} = x$$

$$(fog)(y) = f(g(y)) = f\left(\frac{2y}{1-y}\right) = \frac{\frac{2y}{1-y}}{\frac{2y}{1-y}+2} = \frac{2y}{2y+2-2y} = \frac{2y}{2} = y$$

$\therefore gof = I_{[-1,1]}$ and $fog = I_{\text{Range} f}$

$\therefore f^{-1} = g$

$\Rightarrow f^{-1}(y) = \frac{2y}{1-y}, y \neq 1$

Question

Consider f: **R** → **R** given by $f(x) = 4x + 3$. Show that f is invertible. Find the inverse of f.

Answer

f: **R** → **R** is given by,

$f(x) = 4x + 3$

One-one:

Let $f(x) = f(y)$.

$\Rightarrow 4x+3 = 4y+3$

$\Rightarrow 4x = 4y$

$\Rightarrow x = y$

$\therefore f$ is a one-one function.

Onto:

For $y \in \mathbf{R}$, let $y = 4x + 3$.

$\Rightarrow x = \frac{y-3}{4} \in \mathbf{R}$

Therefore, for any $y \in \mathbf{R}$, there exists $x = \frac{y-3}{4} \in \mathbf{R}$ such that

$$f(x) = f\left(\frac{y-3}{4}\right) = 4\left(\frac{y-3}{4}\right) + 3 = y.$$

$\therefore f$ is onto.

Thus, f is one-one and onto and therefore, f^{-1} exists.

Let us define g: $\mathbf{R} \to \mathbf{R}$ by $g(x) = \frac{y-3}{4}$.

$$\text{Now,} (gof)(x) = g(f(x)) = g(4x+3) = \frac{(4x+3)-3}{4} = x$$

$$(fog)(y) = f(g(y)) = f\left(\frac{y-3}{4}\right) = 4\left(\frac{y-3}{4}\right) + 3 = y - 3 + 3 = y$$

$\therefore gof = fog = I_{\mathbf{R}}$

Hence, f is invertible and the inverse of f is given by

$$f^{-1}(y) = g(y) = \frac{y-3}{4}.$$

Question

Consider f: $\mathbf{R}_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of given f by $f^{-1}(y) = \sqrt{y-4}$, where $\mathbf{R}_+$ is the set of all non-negative real numbers.

Answer

f: $\mathbf{R}_+ \rightarrow [4, \infty)$ is given as $f(x) = x^2 + 4$

One-one:

Let $f(x) = f(y)$.

$$\Rightarrow x^2 + 4 = y^2 + 4$$
$$\Rightarrow x^2 = y^2$$
$$\Rightarrow x = y \qquad [\text{as } x = y \in \mathbf{R}_+]$$

$\therefore f$ is a one-one function.

Onto:

For $y \in [4, \infty)$, let $y = x^2 + 4$.

$$\Rightarrow x^2 = y - 4 \geq 0 \qquad [\text{as } y \geq 4]$$
$$\Rightarrow x = \sqrt{y-4} \geq 0$$

Therefore, for any $y \in \mathbf{R}$, there exists $x = \sqrt{y-4} \in \mathbf{R}$ such that

$$f(x) = f\left(\sqrt{y-4}\right) = \left(\sqrt{y-4}\right)^2 + 4 = y - 4 + 4 = y.$$

$\therefore f$ is onto.

Thus, f is one-one and onto and therefore, f^{-1} exists.

Let us define g: $[4, \infty) \rightarrow \mathbf{R}_+$ by,

$$g(y) = \sqrt{y-4}$$

$$\text{Now, } gof(x) = g(f(x)) = g(x^2+4) = \sqrt{(x^2+4)-4} = \sqrt{x^2} = x$$

$$\text{And, } fog(y) = f(g(y)) = f\left(\sqrt{y-4}\right) = \left(\sqrt{y-4}\right)^2 + 4 = (y-4) + 4 = y$$

$$\therefore gof = fog = I_{\mathbf{R}_+}$$

Hence, f is invertible and the inverse of f is given by

$$f^{-1}(y) = g(y) = \sqrt{y-4}.$$

Question

Find the values of $\tan^{-1}\sqrt{3} - \cot^{-1}\left(-\sqrt{3}\right)$ is equal to

(A) π **(B)** $-\frac{\pi}{2}$ **(C)** 0 **(D)** $2\sqrt{3}$

Answer

Let $\tan^{-1}\sqrt{3} = x$. Then, $\tan x = \sqrt{3} = \tan\frac{\pi}{3}$ where $\frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\therefore \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

Let $\cot^{-1}\left(-\sqrt{3}\right) = y$.

Then, $\cot y = -\sqrt{3} = -\cot\left(\frac{\pi}{6}\right) = \cot\left(\pi - \frac{\pi}{6}\right) = \cot\frac{5\pi}{6}$ where $\frac{5\pi}{6} \in (0, \pi)$.

The range of the principal value branch of $\cot^{-1}$ is $(0, \pi)$.

$$\therefore \cot^{-1}\left(-\sqrt{3}\right) = \frac{5\pi}{6}$$

$$\therefore \tan^{-1}\sqrt{3} - \cot^{-1}\left(-\sqrt{3}\right) = \frac{\pi}{3} - \frac{5\pi}{6} = \frac{2\pi - 5\pi}{6} = \frac{-3\pi}{6} = -\frac{\pi}{2}$$

So correct answer is B

Question

Consider f: $\mathbf{R}_+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with

$$f^{-1}(y) = \left(\frac{\left(\sqrt{y+6}\right) - 1}{3}\right).$$

Answer

f: $\mathbf{R}_+ \to [-5, \infty)$ is given as $f(x) = 9x^2 + 6x - 5$.

Let y be an arbitrary element of $[-5, \infty)$.

Let $y = 9x^2 + 6x - 5$.

$$\Rightarrow y = (3x+1)^2 - 1 - 5 = (3x+1)^2 - 6$$

$$\Rightarrow (3x+1)^2 = y + 6$$

$$\Rightarrow 3x + 1 = \sqrt{y+6} \qquad [\text{as } y \geq -5 \Rightarrow y + 6 > 0]$$

$$\Rightarrow x = \frac{\sqrt{y+6} - 1}{3}$$

$\therefore f$ is onto, thereby range $f = [-5, \infty)$.

Let us define g: $[-5, \infty) \to \mathbf{R}_+$ as $g(y) = \dfrac{\sqrt{y+6} - 1}{3}$.

We now have:

$$(gof)(x) = g(f(x)) = g(9x^2 + 6x - 5)$$
$$= g\left((3x+1)^2 - 6\right)$$

$$= \frac{\sqrt{(3x+1)^2 - 6 + 6} - 1}{3}$$

$$= \frac{3x+1-1}{3} = x$$

$$\text{And, } (fog)(y) = f(g(y)) = f\left(\frac{\sqrt{y+6}-1}{3}\right)$$

$$= \left[3\left(\frac{\sqrt{y+6}-1}{3}\right)+1\right]^2 - 6$$

$$= \left(\sqrt{y+6}\right)^2 - 6 = y + 6 - 6 = y$$

$\therefore$ $gof = I_{\mathbf{R}}$. and $fog = I_{[-5,\infty)}$

Hence, *f* is invertible and the inverse of *f* is given by

$$f^{-1}(y) = g(y) = \frac{\sqrt{y+6}-1}{3}.$$

Question

Let $f: X \to Y$ be an invertible function. Show that f has unique inverse.

(Hint: suppose g_1 and g_2 are two inverses of f. Then for all $y \in Y$,

$fog_1(y) = I_Y(y) = fog_2(y)$. Use one-one ness of f).

Answer

Let $f: X \to Y$ be an invertible function.

Also, suppose f has two inverses (say g_1 and g_2).

Then, for all $y \in Y$, we have:

$$fog_1(y) = I_Y(y) = fog_2(y)$$
$$\Rightarrow f(g_1(y)) = f(g_2(y))$$
$$\Rightarrow g_1(y) = g_2(y) \qquad [f \text{ is invertible} \Rightarrow f \text{ is one-one}]$$
$$\Rightarrow g_1 = g_2 \qquad [g \text{ is one-one}]$$

Hence, f has a unique inverse.

Question

Consider $f: \{1, 2, 3\} \to \{a, b, c\}$ given by $f(1) = a$, $f(2) = b$ and $f(3) = c$. Find f^{-1} and show that $(f^{-1})^{-1} = f$.

Answer

Function $f: \{1, 2, 3\} \to \{a, b, c\}$ is given by,

$f(1) = a$, $f(2) = b$, and $f(3) = c$

If we define $g: \{a, b, c\} \to \{1, 2, 3\}$ as $g(a) = 1$, $g(b) = 2$, $g(c) = 3$, then we have:

$$(fog)(a) = f(g(a)) = f(1) = a$$
$$(fog)(b) = f(g(b)) = f(2) = b$$
$$(fog)(c) = f(g(c)) = f(3) = c$$

And,

$$(gof)(1) = g(f(1)) = g(a) = 1$$
$$(gof)(2) = g(f(2)) = g(b) = 2$$
$$(gof)(3) = g(f(3)) = g(c) = 3$$

$\therefore gof = I_X$ and $fog = I_Y$, where $X = \{1, 2, 3\}$ and $Y= \{a, b, c\}$.

Thus, the inverse of f exists and $f^{-1} = g$.

$\therefore f^{-1}: \{a, b, c\} \to \{1, 2, 3\}$ is given by,

$f^{-1}(a) = 1$, $f^{-1}(b) = 2$, $f^{-1}(c) = 3$

Let us now find the inverse of f^{-1} i.e., find the inverse of g.

If we define $h: \{1, 2, 3\} \to \{a, b, c\}$ as

$h(1) = a$, $h(2) = b$, $h(3) = c$, then we have:

$$(goh)(1) = g(h(1)) = g(a) = 1$$
$$(goh)(2) = g(h(2)) = g(b) = 2$$
$$(goh)(3) = g(h(3)) = g(c) = 3$$

And,

$$(hog)(a) = h(g(a)) = h(1) = a$$
$$(hog)(b) = h(g(b)) = h(2) = b$$
$$(hog)(c) = h(g(c)) = h(3) = c$$

$\therefore goh = I_X$ and $hog = I_Y$, where $X = \{1, 2, 3\}$ and $Y = \{a, b, c\}$.

Thus, the inverse of g exists and $g^{-1} = h \Rightarrow (f^{-1})^{-1} = h$.

It can be noted that $h = f$.

Hence, $(f^{-1})^{-1} = f$.

Question

Let f: $X \to Y$ be an invertible function. Show that the inverse of f^{-1} is f, i.e., $(f^{-1})^{-1} = f$.

Answer

Let f: $X \to Y$ be an invertible function.

Then, there exists a function g: $Y \to X$ such that $gof = I_X$ and $fog = I_Y$.

Here, $f^{-1} = g$.

Now, $gof = I_X$ and $fog = I_Y$

$\Rightarrow f^{-1}of = I_X$ and $fof^{-1} = I_Y$

Hence, f^{-1}: $Y \to X$ is invertible and f is the inverse of f^{-1}

i.e., $(f^{-1})^{-1} = f$.

Question

If f: **R** $\to$ **R** be given by $f(x) = (3 - x^3)^{\frac{1}{3}}$, then $fof(x)$ is

(A) $\frac{1}{x^3}$ (B) x^3 (C) x (D) $(3 - x^3)$

Answer

f: **R** $\to$ **R** is given as $f(x) = (3 - x^3)^{\frac{1}{3}}$.

$$f(x) = (3 - x^3)^{\frac{1}{3}}$$

$$\therefore fof(x) = f(f(x)) = f\left((3 - x^3)^{\frac{1}{3}}\right) = \left[3 - \left((3 - x^3)^{\frac{1}{3}}\right)^3\right]^{\frac{1}{3}}$$

$$= \left[3 - (3 - x^3)\right]^{\frac{1}{3}} = (x^3)^{\frac{1}{3}} = x$$

$$\therefore fof(x) = x$$

The correct answer is C.

Question

Let $f:\mathbf{R}-\left\{-\frac{4}{3}\right\}\to\mathbf{R}$ be a function defined as $f(x)=\frac{4x}{3x+4}$. The inverse of f is map g: Range $f\to\mathbf{R}-\left\{-\frac{4}{3}\right\}$ given by

(A) $g(y)=\frac{3y}{3-4y}$ (B) $g(y)=\frac{4y}{4-3y}$

(C) $g(y)=\frac{4y}{3-4y}$ (D) $g(y)=\frac{3y}{4-3y}$

Answer

It is given that $f:\mathbf{R}-\left\{-\frac{4}{3}\right\}\to\mathbf{R}$ is defined as $f(x)=\frac{4x}{3x+4}$.

Let y be an arbitrary element of Range f.

Then, there exists $x \in \mathbf{R}-\left\{-\frac{4}{3}\right\}$ such that $y=f(x)$.

$$\Rightarrow y=\frac{4x}{3x+4}$$
$$\Rightarrow 3xy+4y=4x$$
$$\Rightarrow x(4-3y)=4y$$
$$\Rightarrow x=\frac{4y}{4-3y}$$

Let us define g: Range $f\to\mathbf{R}-\left\{-\frac{4}{3}\right\}$ as $g(y)=\frac{4y}{4-3y}$.

Now, $(g\circ f)(x)=g(f(x))=g\left(\frac{4x}{3x+4}\right)$

$$=\frac{4\left(\frac{4x}{3x+4}\right)}{4-3\left(\frac{4x}{3x+4}\right)}=\frac{16x}{12x+16-12x}=\frac{16x}{16}=x$$

And, $(f\circ g)(y)=f(g(y))=f\left(\frac{4y}{4-3y}\right)$

$$=\frac{4\left(\frac{4y}{4-3y}\right)}{3\left(\frac{4y}{4-3y}\right)+4}=\frac{16y}{12y+16-12y}=\frac{16y}{16}=y$$

$\therefore gof = I_{\mathbf{R}-\left\{-\frac{4}{3}\right\}}$ and $fog = I_{\text{Range} f}$

Thus, g is the inverse of f i.e., $f^{-1} = g$.

Hence, the inverse of f is the map g: Range $f \to \mathbf{R}-\left\{-\frac{4}{3}\right\}$, which is given by

$g(y)=\frac{4y}{4-3y}$.

The correct answer is B.

Question

Determine whether or not each of the definition of given below gives a binary operation. In the event that * is not a binary operation, give justification for this.

(i) On $\mathbf{Z}^+$, define * by $a * b = a - b$

(ii) On $\mathbf{Z}^+$, define * by $a * b = ab$

(iii) On $\mathbf{R}$, define * by $a * b = ab^2$

(iv) On $\mathbf{Z}^+$, define * by $a * b = |a - b|$

(v) On $\mathbf{Z}^+$, define * by $a * b = a$

Answer

(i) On $\mathbf{Z}^+$, * is defined by $a * b = a - b$.

It is not a binary operation as the image of (1, 2) under * is $1 * 2 = 1 - 2$ $= -1 \notin \mathbf{Z}^+$.

(ii) On $\mathbf{Z}^+$, * is defined by $a * b = ab$.

It is seen that for each $a, b \in \mathbf{Z}^+$, there is a unique element ab in $\mathbf{Z}^+$.

This means that * carries each pair (a, b) to a unique element $a * b = ab$ in $\mathbf{Z}^+$.

Therefore, * is a binary operation.

(iii) On $\mathbf{R}$, * is defined by $a * b = ab^2$.

It is seen that for each $a, b \in \mathbf{R}$, there is a unique element ab^2 in $\mathbf{R}$.

This means that * carries each pair (a, b) to a unique element $a * b = ab^2$ in **R**.

Therefore, * is a binary operation.

(iii) On **R**, * is defined by $a * b = ab^2$.

It is seen that for each $a, b \in$ **R**, there is a unique element ab^2 in **R**.

This means that * carries each pair (a, b) to a unique element $a * b = ab^2$ in **R**.

Therefore, * is a binary operation.

(iv) On $\mathbf{Z}^+$, * is defined by $a * b = |a - b|$.

It is seen that for each $a, b \in \mathbf{Z}^+$, there is a unique element $|a - b|$ in $\mathbf{Z}^+$.

This means that * carries each pair (a, b) to a unique element $a * b =$
$|a - b|$ in $\mathbf{Z}^+$.

Therefore, * is a binary operation.

(v) On $\mathbf{Z}^+$, * is defined by $a * b = a$.

* carries each pair (a, b) to a unique element $a * b = a$ in $\mathbf{Z}^+$.

Therefore, * is a binary operation.

Question

For each binary operation * defined below, determine whether * is commutative or associative.

(i) On **Z**, define $a * b = a - b$

(ii) On **Q**, define $a * b = ab + 1$

(iii) On **Q**, define $a * b = \frac{ab}{2}$

(iv) On $\mathbf{Z}^+$, define $a * b = 2^{ab}$

(v) On $\mathbf{Z}^+$, define $a * b = a^b$

(vi) On $\mathbf{R} - \{-1\}$, define $a * b = \frac{a}{b+1}$

Answer

(i) On **Z**, * is defined by $a * b = a - b$.

It can be observed that $1 * 2 = 1 - 2 = 1$ and $2 * 1 = 2 - 1 = 1$.

$\therefore 1 * 2 \neq 2 * 1$; where $1, 2 \in \mathbf{Z}$

Hence, the operation * is not commutative.

Also we have:

$(1 * 2) * 3 = (1 - 2) * 3 = -1 * 3 = -1 - 3 = -4$

$1 * (2 * 3) = 1 * (2 - 3) = 1 * -1 = 1 - (-1) = 2$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$; where 1, 2, 3 $\in$ **Z**

Hence, the operation * is not associative.

(ii) On **Q**, * is defined by $a * b = ab + 1$.

It is known that:

$ab = ba$ □ $a, b \in$ **Q**

$\Rightarrow ab + 1 = ba + 1$ □ $a, b \in$ **Q**

$\Rightarrow a * b = a * b$ □ $a, b \in$ **Q**

Therefore, the operation * is commutative.

It can be observed that:

$(1 * 2) * 3 = (1 \times 2 + 1) * 3 = 3 * 3 = 3 \times 3 + 1 = 10$

$1 * (2 * 3) = 1 * (2 \times 3 + 1) = 1 * 7 = 1 \times 7 + 1 = 8$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$; where 1, 2, 3 $\in$ **Q**

Therefore, the operation * is not associative.

(iii) On **Q**, * is defined by $a * b = \frac{ab}{2}$.

It is known that:

$ab = ba \ \forall \ a, b \in \mathbf{Q}$

$\Rightarrow \frac{ab}{2} = \frac{ba}{2} \ \forall a, b \in \mathbf{Q}$

$\Rightarrow a * b = b * a \ \forall a, b \in \mathbf{Q}$

Therefore, the operation * is commutative.

For all $a, b, c \in \mathbf{Q}$, we have:

$$(a*b)*c = \left(\frac{ab}{2}\right)*c = \frac{\left(\frac{ab}{2}\right)c}{2} = \frac{abc}{4}$$

$$a*(b*c) = a*\left(\frac{bc}{2}\right) = \frac{a\left(\frac{bc}{2}\right)}{2} = \frac{abc}{4}$$

$\therefore (a*b)*c = a*(b*c)$

Therefore, the operation * is associative.

(iv) On $\mathbf{Z}^+$, * is defined by $a * b = 2^{ab}$.

It is known that:

$ab = ba$ □ $a, b \in \mathbf{Z}^+$

$\Rightarrow 2^{ab} = 2^{ba}$ □ $a, b \in \mathbf{Z}^+$

$\Rightarrow a * b = b * a$ □ $a, b \in \mathbf{Z}^+$

Therefore, the operation * is commutative.

It can be observed that:

$$(1*2)*3 = 2^{(1\times 2)}*3 = 4*3 = 2^{4\times 3} = 2^{12}$$

$$1*(2*3) = 1*2^{2\times 3} = 1*2^6 = 1*64 = 2^{64}$$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$; where $1, 2, 3 \in \mathbf{Z}^+$

Therefore, the operation * is not associative.

(v) On $\mathbf{Z}^+$, * is defined by $a * b = a^b$.

It can be observed that:

$1*2=1^2=1$ and $2*1=2^1=2$

$\therefore 1 * 2 \neq 2 * 1$; where $1, 2 \in \mathbf{Z}^+$

Therefore, the operation * is not commutative.

It can also be observed that:

$$(2*3)*4=2^3*4=8*4=8^4=\left(2^3\right)^4=2^{12}$$

$$2*(3*4)=2*3^4=2*81=2^{81}$$

$\therefore (2 * 3) * 4 \neq 2 * (3 * 4)$; where $2, 3, 4 \in \mathbf{Z}^+$

Therefore, the operation * is not associative.

(vi) On $\mathbf{R}$, * − {−1} is defined by $a*b=\frac{a}{b+1}$.

It can be observed that $1*2=\frac{1}{2+1}=\frac{1}{3}$ and $2*1=\frac{2}{1+1}=\frac{2}{2}=1$

$\therefore 1 * 2 \neq 2 * 1$; where $1, 2 \in \mathbf{R} - \{-1\}$

Therefore, the operation * is not commutative.

It can also be observed that:

$$(1*2)*3=\frac{1}{3}*3=\frac{\frac{1}{3}}{3+1}=\frac{1}{12}$$

$$1*(2*3)=1*\frac{2}{3+1}=1*\frac{2}{4}=1*\frac{1}{2}=\frac{1}{\frac{1}{2}+1}=\frac{1}{\frac{3}{2}}=\frac{2}{3}$$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$; where $1, 2, 3 \in \mathbf{R} - \{-1\}$

Therefore, the operation * is not associative.

Question

Consider the binary operation ∨ on the set {1, 2, 3, 4, 5} defined by $a \vee b = \min\{a, b\}$. Write the operation table of the operation∨.

Answer

The binary operation ∨ on the set {1, 2, 3, 4, 5} is defined as $a \vee b = \min\{a, b\}$ $\forall\, a, b \in \{1, 2, 3, 4, 5\}$.

Thus, the operation table for the given operation ∨ can be given as:

∨	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

Consider a binary operation * on the set {1, 2, 3, 4, 5} given by the following multiplication table.

(i) Compute (2 * 3) * 4 and 2 * (3 * 4)

(ii) Is * commutative?

(iii) Compute (2 * 3) * (4 * 5).

(Hint: use the following table)

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

Answer

(i) $(2 * 3) * 4 = 1 * 4 = 1$

$2 * (3 * 4) = 2 * 1 = 1$

(ii) For every $a, b \in \{1, 2, 3, 4, 5\}$, we have $a * b = b * a$. Therefore, the operation * is commutative.

(iii) $(2 * 3) = 1$ and $(4 * 5) = 1$

$\therefore (2 * 3) * (4 * 5) = 1 * 1 = 1$

Question

Let $*'$ be the binary operation on the set $\{1, 2, 3, 4, 5\}$ defined by $a *' b$ = H.C.F. of a and b. Is the operation $*'$ same as the operation * defined in Exercise 4 above? Justify your answer.

Answer

The binary operation $*'$ on the set $\{1, 2, 3 4, 5\}$ is defined as $a *' b$ = H.C.F of a and b.

The operation table for the operation $*'$ can be given as:

*′	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

We observe that the operation tables for the operations * and *′ are the same.

Thus, the operation *′ is same as the operation*.

Question

Let * be the binary operation on **N** given by $a * b$ = L.C.M. of a and b. Find

(i) 5 * 7, 20 * 16 (ii) Is * commutative?

(iii) Is * associative? (iv) Find the identity of * in **N**

(v) Which elements of **N** are invertible for the operation *?

Answer

The binary operation * on **N** is defined as $a * b$ = L.C.M. of a and b.

(i) 5 * 7 = L.C.M. of 5 and 7 = 35

20 * 16 = L.C.M of 20 and 16 = 80

(ii) It is known that:

L.C.M of a and b = L.C.M of b and a □ $a, b \in$ **N**.

$\therefore a * b = b * a$

Thus, the operation * is commutative.

(iii) For $a, b, c \in$ **N**, we have:

$(a * b) * c$ = (L.C.M of a and b) * c = LCM of a, b, and c

$a * (b * c) = a *$ (LCM of b and c) = L.C.M of a, b, and c

$\therefore (a * b) * c = a * (b * c)$

Thus, the operation * is associative.

(iv) It is known that:

L.C.M. of a and 1 = a = L.C.M. 1 and a □ $a \in$ **N**

$\Rightarrow a * 1 = a = 1 * a$ □ $a \in$ **N**

Thus, 1 is the identity of * in **N**.

(v) An element a in **N** is invertible with respect to the operation * if there exists an element b in **N**, such that $a * b = e = b * a$.

Here, $e = 1$

This means that:

L.C.M of a and b = 1 = L.C.M of b and a

This case is possible only when a and b are equal to 1.

Thus, 1 is the only invertible element of **N** with respect to the operation *.

Question

Is * defined on the set {1, 2, 3, 4, 5} by $a * b$ = L.C.M. of a and b a binary operation? Justify your answer.

Answer

The operation * on the set A = {1, 2, 3, 4, 5} is defined as

$a * b$ = L.C.M. of a and b.

Then, the operation table for the given operation * can be given as:

*	1	2	3	4	5
1	1	2	3	4	5
2	2	2	6	4	10
3	3	6	3	12	15
4	4	4	12	4	20
5	5	10	15	20	5

It can be observed from the obtained table that:

$3 * 2 = 2 * 3 = 6 \notin A$, $5 * 2 = 2 * 5 = 10 \notin A$, $3 * 4 = 4 * 3 = 12 \notin A$

$3 * 5 = 5 * 3 = 15 \notin A$, $4 * 5 = 5 * 4 = 20 \notin A$

Hence, the given operation * is not a binary operation.

Question

Let * be the binary operation on **N** defined by $a * b$ = H.C.F. of a and b. Is * commutative? Is * associative? Does there exist identity for this binary operation on **N**?

Answer

The binary operation * on **N** is defined as:

$a * b$ = H.C.F. of a and b

It is known that:

H.C.F. of a and b = H.C.F. of b and a □ $a, b \in$ **N**.

$\therefore a * b = b * a$

Thus, the operation * is commutative.

For $a, b, c \in$ **N**, we have:

$(a * b) * c$ = (H.C.F. of a and b) $* c$ = H.C.F. of a, b, and c

$a * (b * c) = a *$(H.C.F. of b and c) = H.C.F. of a, b, and c

$\therefore (a * b) * c = a * (b * c)$

Thus, the operation * is associative.

Now, an element $e \in$ **N** will be the identity for the operation * if $a * e = a = e * a \; \forall a \in$ **N**.

But this relation is not true for any $a \in$ **N**.

Thus, the operation * does not have any identity in **N**.

Question

Let * be a binary operation on the set **Q** of rational numbers as follows:

(i) $a * b = a - b$ (ii) $a * b = a^2 + b^2$

(iii) $a * b = a + ab$ (iv) $a * b = (a - b)^2$

(v) $a * b = \frac{ab}{4}$ (vi) $a * b = ab^2$

Find which of the binary operations are commutative and which are associative.

Answer

(i) On **Q**, the operation * is defined as $a * b = a - b$.

It can be observed that:

$\frac{1}{2} * \frac{1}{3} = \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$ and $\frac{1}{3} * \frac{1}{2} = \frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} = \frac{-1}{6}$

$\therefore \frac{1}{2} * \frac{1}{3} \neq \frac{1}{3} * \frac{1}{2}$; where $\frac{1}{2}, \frac{1}{3} \in \mathbf{Q}$

Thus, the operation * is not commutative.

It can also be observed that:

$$\left(\frac{1}{2}*\frac{1}{3}\right)*\frac{1}{4}=\left(\frac{1}{2}-\frac{1}{3}\right)*\frac{1}{4}=\frac{1}{6}*\frac{1}{4}=\frac{1}{6}-\frac{1}{4}=\frac{2-3}{12}=\frac{-1}{12}$$

$$\frac{1}{2}*\left(\frac{1}{3}*\frac{1}{4}\right)=\frac{1}{2}*\left(\frac{1}{3}-\frac{1}{4}\right)=\frac{1}{2}*\frac{1}{12}=\frac{1}{2}-\frac{1}{12}=\frac{6-1}{12}=\frac{5}{12}$$

$$\therefore\left(\frac{1}{2}*\frac{1}{3}\right)*\frac{1}{4}\neq\frac{1}{2}*\left(\frac{1}{3}*\frac{1}{4}\right)\text{; where }\frac{1}{2},\ \frac{1}{3},\ \frac{1}{4}\in\mathbf{Q}$$

Thus, the operation * is not associative.

(ii) On **Q**, the operation * is defined as $a * b = a^2 + b^2$.

For $a, b \in$ **Q**, we have:

$$a*b=a^2+b^2=b^2+a^2=b*a$$

$\therefore a * b = b * a$

Thus, the operation * is commutative.

It can be observed that:

$$(1*2)*3=(1^2+2^2)*3=(1+4)*4=5*4=5^2+4^2=41$$

$$1*(2*3)=1*(2^2+3^2)=1*(4+9)=1*13=1^2+13^2=169$$

$\therefore (1*2)*3 \neq 1*(2*3)$; where $1, 2, 3 \in \mathbf{Q}$

Thus, ,the operation * is not associative.

(iii) On **Q**, the operation * is defined as $a * b = a + ab$.

It can be observed that:

$1*2 = 1+1\times 2 = 1+2 = 3$

$2*1 = 2+2\times 1 = 2+2 = 4$

$\therefore 1*2 \neq 2*1$; where $1, 2 \in \mathbf{Q}$

Thus, the operation * is not commutative.

It can also be observed that:

$(1*2)*3 = (1+1\times 2)*3 = 3*3 = 3+3\times 3 = 3+9 = 12$

$1*(2*3) = 1*(2+2\times 3) = 1*8 = 1+1\times 8 = 9$

$\therefore (1*2)*3 \neq 1*(2*3)$; where $1, 2, 3 \in \mathbf{Q}$

Thus, the operation * is not associative.

(iv) On **Q**, the operation * is defined by $a * b = (a - b)^2$.

For $a, b \in \mathbf{Q}$, we have:

$a * b = (a - b)^2$

$b * a = (b - a)^2 = [-(a - b)]^2 = (a - b)^2$

$\therefore a * b = b * a$

Thus, the operation * is commutative.

It can be observed that:

$$(1*2)*3 = (1-2)^2 * 3 = (-1)^2 * 3 = 1*3 = (1-3)^2 = (-2)^2 = 4$$
$$1*(2*3) = 1*(2-3)^2 = 1*(-1)^2 = 1*1 = (1-1)^2 = 0$$

$\therefore (1*2)*3 \neq 1*(2*3)$; where $1, 2, 3 \in \mathbf{Q}$

Thus, the operation * is not associative.

(v) On **Q**, the operation * is defined as $a*b=\frac{ab}{4}$.

For $a, b \in$ **Q**, we have:

$$a*b=\frac{ab}{4}=\frac{ba}{4}=b*a$$

$\therefore a * b = b * a$

Thus, the operation * is commutative.

For $a, b, c \in$ **Q**, we have:

$$(a*b)*c=\frac{ab}{4}*c=\frac{\frac{ab}{4}\cdot c}{4}=\frac{abc}{16}$$

$$a*(b*c)=a*\frac{bc}{4}=\frac{a\cdot\frac{bc}{4}}{4}=\frac{abc}{16}$$

$\therefore (a * b) * c = a * (b * c)$

Thus, the operation * is associative.

(vi) On **Q**, the operation * is defined as $a * b = ab^2$

It can be observed that:

$$\frac{1}{2} * \frac{1}{3} = \frac{1}{2} \cdot \left(\frac{1}{3}\right)^2 = \frac{1}{2} \cdot \frac{1}{9} = \frac{1}{18}$$

$$\frac{1}{3} * \frac{1}{2} = \frac{1}{3} \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$$

$$\therefore \frac{1}{2} * \frac{1}{3} \neq \frac{1}{3} * \frac{1}{2} \text{ ; where } \frac{1}{2}, \frac{1}{3} \in \mathbf{Q}$$

Thus, the operation * is not commutative.

It can also be observed that:

$$\left(\frac{1}{2} * \frac{1}{3}\right) * \frac{1}{4} = \left[\frac{1}{2} \cdot \left(\frac{1}{3}\right)^2\right] * \frac{1}{4} = \frac{1}{18} * \frac{1}{4} = \frac{1}{18} \cdot \left(\frac{1}{4}\right)^2 = \frac{1}{18 \times 16}$$

$$\frac{1}{2} * \left(\frac{1}{3} * \frac{1}{4}\right) = \frac{1}{2} * \left[\frac{1}{3} \cdot \left(\frac{1}{4}\right)^2\right] = \frac{1}{2} * \frac{1}{48} = \frac{1}{2} \cdot \left(\frac{1}{48}\right)^2 = \frac{1}{2 \times (48)^2}$$

$$\therefore \left(\frac{1}{2} * \frac{1}{3}\right) * \frac{1}{4} \neq \frac{1}{2} * \left(\frac{1}{3} * \frac{1}{4}\right) \text{ ; where } \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \in \mathbf{Q}$$

Thus, the operation * is not associative.

Hence, the operations defined in (ii), (iv), (v) are commutative and the operation defined in (v) is associative.

Question

Find which of the operations given above has identity.

Answer

An element $e \in \mathbf{Q}$ will be the identity element for the operation * if

$a * e = a = e * a, \ \forall a \in \mathbf{Q}$.

However, there is no such element $e \in \mathbf{Q}$ with respect to each of the six operations satisfying the above condition.

Thus, none of the six operations has identity.

Question

Let A = **N** × **N** and * be the binary operation on A defined by

$(a, b) * (c, d) = (a + c, b + d)$

Show that * is commutative and associative. Find the identity element for * on A, if any.

Answer

A = **N** × **N**

* is a binary operation on A and is defined by:

$(a, b) * (c, d) = (a + c, b + d)$

Let $(a, b), (c, d) \in A$

Then, $a, b, c, d \in \mathbf{N}$

We have:

$(a, b) * (c, d) = (a + c, b + d)$

$(c, d) * (a, b) = (c + a, d + b) = (a + c, b + d)$

[Addition is commutative in the set of natural numbers]

$\therefore (a, b) * (c, d) = (c, d) * (a, b)$

Therefore, the operation * is commutative.

Now, let $(a, b), (c, d), (e, f) \in A$

Then, $a, b, c, d, e, f \in \mathbf{N}$

We have:

$$((a,b)*(c,d))*(e,f) = (a+c,\ b+d)*(e,f) = (a+c+e,\ b+d+f)$$
$$(a,b)*((c,d)*(e,f)) = (a,b)*(c+e,\ d+f) = (a+c+e,\ b+d+f)$$

$$\therefore ((a,b)*(c,d))*(e,f) = (a,b)*((c,d)*(e,f))$$

Therefore, the operation * is associative.

An element $e = (e_1, e_2) \in A$ will be an identity element for the operation * if

$a*e = a = e*a \ \forall\ a = (a_1, a_2) \in A$, i.e., $(a_1+e_1, a_2+e_2) = (a_1, a_2) = (e_1+a_1, e_2+a_2)$, which is not true for any element in A.

Therefore, the operation * does not have any identity element.

Question

State whether the following statements are true or false. Justify.

(i) For an arbitrary binary operation * on a set **N**, $a * a = a \ \forall a * \mathbf{N}$.

(ii) If * is a commutative binary operation on **N**, then $a * (b * c) = (c * b) * a$

Answer

(i) Define an operation * on **N** as:

$a * b = a + b \ \forall a, b \in \mathbf{N}$

Then, in particular, for $b = a = 3$, we have:

$3 * 3 = 3 + 3 = 6 \neq 3$

Therefore, statement (i) is false.

(ii) R.H.S. $= (c * b) * a$

$= (b * c) * a$ [* is commutative]

$= a * (b * c)$ [Again, as * is commutative]

= L.H.S.

$\therefore a * (b * c) = (c * b) * a$

Therefore, statement (ii) is true.

Question

Consider a binary operation * on **N** defined as $a * b = a^3 + b^3$. Choose the correct answer.

(A) Is * both associative and commutative?

(B) Is * commutative but not associative?

(C) Is * associative but not commutative?

(D) Is * neither commutative nor associative?

Answer

On **N**, the operation * is defined as $a * b = a^3 + b^3$.

For, $a, b, \in$ **N**, we have:

$a * b = a^3 + b^3 = b^3 + a^3 = b * a$ [Addition is commutative in **N**]

Therefore, the operation * is commutative.

It can be observed that:

$$(1*2)*3=\left(1^3+2^3\right)*3=9*3=9^3+3^3=729+27=756$$

$$1*(2*3)=1*\left(2^3+3^3\right)=1*(8+27)=1\times35=1^3+35^3=1+(35)^3=1+42875=42876$$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$; where $1, 2, 3 \in$ **N**

Therefore, the operation * is not associative.

Hence, the operation * is commutative, but not associative. Thus, the correct answer is B.

Question

Let f: **R** → **R** be defined as $f(x) = 10x + 7$. Find the function g: **R** → **R** such that $g \circ f = f \circ g = 1_{\mathbf{R}}$.

Answer

It is given that f: **R** → **R** is defined as $f(x) = 10x + 7$.

One-one:

Let $f(x) = f(y)$, where $x, y \in \mathbf{R}$.

$\Rightarrow 10x + 7 = 10y + 7$

$\Rightarrow x = y$

∴ f is a one-one function.

Onto:

For $y \in \mathbf{R}$, let $y = 10x + 7$.

$$\Rightarrow x = \frac{y-7}{10} \in \mathbf{R}$$

Therefore, for any $y \in \mathbf{R}$, there exists $x = \frac{y-7}{10} \in \mathbf{R}$ such that

$$f(x) = f\left(\frac{y-7}{10}\right) = 10\left(\frac{y-7}{10}\right) + 7 = y - 7 + 7 = y.$$

∴ f is onto.

Therefore, f is one-one and onto.

Thus, f is an invertible function.

Let us define g: **R** → **R** as $g(y) = \frac{y-7}{10}$.

Now, we have:

$$gof(x) = g(f(x)) = g(10x+7) = \frac{(10x+7)-7}{10} = \frac{10x}{10} = 10$$

And,

$$fog(y) = f(g(y)) = f\left(\frac{y-7}{10}\right) = 10\left(\frac{y-7}{10}\right) + 7 = y - 7 + 7 = y$$

$$\therefore gof = I_{\mathbf{R}} \text{ and } fog = I_{\mathbf{R}}$$

Hence, the required function g: **R** → **R** is defined as $g(y) = \frac{y-7}{10}$.

Question

Let f: W → W be defined as $f(n) = n - 1$, if is odd and $f(n) = n + 1$, if n is even. Show that f is invertible. Find the inverse of f. Here, W is the set of all whole numbers.

Answer

It is given that:

f: W → W is defined as $f(n) = \begin{cases} n-1, \text{ if } n \text{ is odd} \\ n+1, \text{ if } n \text{ is even} \end{cases}$

One-one:

Let $f(n) = f(m)$.

It can be observed that if n is odd and m is even, then we will have $n - 1 = m + 1$.

$\Rightarrow n - m = 2$

However, this is impossible.

Similarly, the possibility of n being even and m being odd can also be ignored under a similar argument.

∴Both n and m must be either odd or even.

Now, if both n and m are odd, then we have:

$f(n) = f(m) \Rightarrow n - 1 = m - 1 \Rightarrow n = m$

Again, if both n and m are even, then we have:

$f(n) = f(m) \Rightarrow n + 1 = m + 1 \Rightarrow n = m$

∴f is one-one.

It is clear that any odd number $2r + 1$ in co-domain **N** is the image of $2r$ in domain **N** and any even number $2r$ in co-domain **N** is the image of $2r + 1$ in domain **N**.

∴f is onto.

Hence, f is an invertible function.

Let us define g: W → W as:

$$g(m) = \begin{cases} m+1, \text{ if } m \text{ is even} \\ m-1, \text{ if } m \text{ is odd} \end{cases}$$

Now, when n is odd:

$$gof(n) = g(f(n)) = g(n-1) = n-1+1 = n$$

And, when n is even:

$$gof(n) = g(f(n)) = g(n+1) = n+1-1 = n$$

Similarly, when m is odd:

$$fog(m) = f(g(m)) = f(m-1) = m-1+1 = m$$

When m is even:

$$fog(m) = f(g(m)) = f(m+1) = m+1-1 = m$$

$$\therefore gof = I_W \text{ and } fog = I_W$$

Thus, f is invertible and the inverse of f is given by $f^{-1} = g$, which is the same as f.

Hence, the inverse of f is f itself.

Question

If f: $\mathbf{R} \to \mathbf{R}$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.

Answer

It is given that f: $\mathbf{R} \to \mathbf{R}$ is defined as $f(x) = x^2 - 3x + 2$.

$$\begin{aligned} f(f(x)) &= f(x^2 - 3x + 2) \\ &= (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2 \\ &= x^4 + 9x^2 + 4 - 6x^3 - 12x + 4x^2 - 3x^2 + 9x - 6 + 2 \\ &= x^4 - 6x^3 + 10x^2 - 3x \end{aligned}$$

Question

Show that function f: $\mathbf{R} \to \{x \in \mathbf{R}: -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in \mathbf{R}$ is one-one and onto function.

Answer

It is given that f: $\mathbf{R} \to \{x \in \mathbf{R}: -1 < x < 1\}$ is defined as $f(x) = \frac{x}{1+|x|}$, $x \in \mathbf{R}$.

Suppose $f(x) = f(y)$, where $x, y \in \mathbf{R}$.

$$\Rightarrow \frac{x}{1+|x|} = \frac{y}{1+|y|}$$

It can be observed that if x is positive and y is negative, then we have:

$$\frac{x}{1+x} = \frac{y}{1-y} \Rightarrow 2xy = x - y$$

Since x is positive and y is negative:

$x > y \Rightarrow x - y > 0$

But, $2xy$ is negative.

Then, $2xy \neq x - y$.

Thus, the case of x being positive and y being negative can be ruled out.

Under a similar argument, x being negative and y being positive can also be ruled out

$\therefore$ x and y have to be either positive or negative.

When x and y are both positive, we have:

$$f(x) = f(y) \Rightarrow \frac{x}{1+x} = \frac{y}{1+y} \Rightarrow x + xy = y + xy \Rightarrow x = y$$

When x and y are both negative, we have:

$$f(x) = f(y) \Rightarrow \frac{x}{1-x} = \frac{y}{1-y} \Rightarrow x - xy = y - yx \Rightarrow x = y$$

$\therefore$ f is one-one.

Now, let $y \in \mathbf{R}$ such that $-1 < y < 1$.

If y is negative, then there exists $x = \frac{y}{1+y} \in \mathbf{R}$ such that

$$f(x)=f\left(\frac{y}{1+y}\right)=\frac{\left(\frac{y}{1+y}\right)}{1+\left|\frac{y}{1+y}\right|}=\frac{\frac{y}{1+y}}{1+\left(\frac{-y}{1+y}\right)}=\frac{y}{1+y-y}=y.$$

If y is positive, then there exists $x=\frac{y}{1-y}\in\mathbf{R}$ such that

$$f(x)=f\left(\frac{y}{1-y}\right)=\frac{\left(\frac{y}{1-y}\right)}{1+\left|\left(\frac{y}{1-y}\right)\right|}=\frac{\frac{y}{1-y}}{1+\frac{y}{1-y}}=\frac{y}{1-y+y}=y.$$

$\therefore$ f is onto.

Hence, f is one-one and onto.

Question

Show that the function f: $\mathbf{R}\rightarrow\mathbf{R}$ given by $f(x) = x^3$ is injective.

Answer

f: $\mathbf{R}\rightarrow\mathbf{R}$ is given as $f(x) = x^3$.

Suppose $f(x) = f(y)$, where $x, y \in \mathbf{R}$.

$\Rightarrow x^3 = y^3$ ... (1)

Now, we need to show that $x = y$.

Suppose $x \neq y$, their cubes will also not be equal.

$\Rightarrow x^3 \neq y^3$

However, this will be a contradiction to (1).

$\therefore x = y$

Hence, f is injective.

Question

Give examples of two functions f: **N** → **Z** and g: **Z** → **Z** such that $g \circ f$ is injective but g is not injective.

(Hint: Consider $f(x) = x$ and $g(x) = |x|$)

Answer

Define f: **N** → **Z** as $f(x) = x$ and g: **Z** → **Z** as $g(x) = |x|$.

We first show that g is not injective.

It can be observed that:

$g(-1) = |-1| = 1$

$g(1) = |1| = 1$

$\therefore g(-1) = g(1)$, but $-1 \neq 1$.

$\therefore g$ is not injective.

Now, gof: **N** → **Z** is defined as $gof(x) = g(f(x)) = g(x) = |x|$.

Let $x, y \in$ **N** such that $gof(x) = gof(y)$.

Let $x, y \in$ **N** such that $gof(x) = gof(y)$.

$\Rightarrow |x| = |y|$

Since x and $y \in$ **N,** both are positive.

$\therefore |x| = |y| \Rightarrow x = y$

Hence, gof is injective

Question

Given examples of two functions f: **N** → **N** and g: **N** → **N** such that gof is onto but f is not onto.

(Hint: Consider $f(x) = x + 1$ and $g(x) = \begin{cases} x-1 & \text{if } x>1 \\ 1 & \text{if } x=1 \end{cases}$

Answer

Define f: **N** → **N** by,

$f(x) = x + 1$

And, g: **N** → **N** by,

$$g(x) = \begin{cases} x-1 & \text{if } x>1 \\ 1 & \text{if } x=1 \end{cases}$$

We first show that g is not onto.

For this, consider element 1 in co-domain **N**. It is clear that this element is not an image of any of the elements in domain **N**.

∴ f is not onto.

Now, gof: **N** → **N** is defined by,

$$gof(x) = g(f(x)) = g(x+1) = (x+1) - 1 \qquad [x \in \mathbf{N} \Rightarrow (x+1) > 1]$$
$$= x$$

Then, it is clear that for $y \in$ **N**, there exists $x = y \in$ **N** such that $gof(x) = y$.

Hence, gof is onto.

Question

Given a non empty set X, consider P(X) which is the set of all subsets of X.

Define the relation R in P(X) as follows:

For subsets A, B in P(X), ARB if and only if $A \subset B$. Is R an equivalence relation on P(X)? Justify you answer:

Answer

Since every set is a subset of itself, ARA for all $A \in$ P(X).

$\therefore$R is reflexive.

Let $ARB \Rightarrow A \subset B$.

This cannot be implied to $B \subset A$.

For instance, if $A = \{1, 2\}$ and $B = \{1, 2, 3\}$, then it cannot be implied that B is related to A.

$\therefore$ R is not symmetric.

Further, if ARB and BRC, then $A \subset B$ and $B \subset C$.

$\Rightarrow A \subset C$

$\Rightarrow ARC$

$\therefore$ R is transitive.

Hence R is not an equivalence relation since it is not symmetric

Question

Given a non-empty set X, consider the binary operation $*: P(X) \times P(X) \to P(X)$ given by $A * B = A \cap B$ $\forall A, B$ in $P(X)$ is the power set of X. Show that X is the identity element for this operation and X is the only invertible element in $P(X)$ with respect to the operation$*$.

Answer

It is given that $*: P(X) \times P(X) \to P(X)$ is defined as $A * B = A \cap B \ \forall A, B \in P(X)$.

We know that $A \cap X = A = X \cap A \ \forall A \in P(X)$.

$\Rightarrow A * X = A = X * A \ \forall A \in P(X)$

Thus, X is the identity element for the given binary operation $*$.

Now, an element $A \in P(X)$ is invertible if there exists $B \in P(X)$ such that

$A * B = X = B * A$. (As X is the identity element)

i.e.,

$A \cap B = X = B \cap A$

This case is possible only when $A = X = B$.

Thus, X is the only invertible element in $P(X)$ with respect to the given operation$*$.

Hence, the given result is proved.

Question

Find the number of all onto functions from the set $\{1, 2, 3, \ldots, n)$ to itself.

Answer

Onto functions from the set $\{1, 2, 3, \ldots, n\}$ to itself is simply a permutation on n symbols $1, 2, \ldots, n$.

Thus, the total number of onto maps from $\{1, 2, \ldots, n\}$ to itself is the same as the total number of permutations on n symbols $1, 2, \ldots, n$, which is n.

Question

Let $S = \{a, b, c\}$ and $T = \{1, 2, 3\}$. Find F^{-1} of the following functions F from S to T, if it exists.

(i) $F = \{(a, 3), (b, 2), (c, 1)\}$ (ii) $F = \{(a, 2), (b, 1), (c, 1)\}$

Answer

$S = \{a, b, c\}$, $T = \{1, 2, 3\}$

(i) F: $S \to T$ is defined as:

$F = \{(a, 3), (b, 2), (c, 1)\}$

$\Rightarrow F(a) = 3, F(b) = 2, F(c) = 1$

Therefore, $F^{-1}: T \to S$ is given by

$F^{-1} = \{(3, a), (2, b), (1, c)\}$.

(ii) F: $S \to T$ is defined as:

$F = \{(a, 2), (b, 1), (c, 1)\}$

Since $F(b) = F(c) = 1$, F is not one-one.

Hence, F is not invertible i.e., F^{-1} does not exist.

Question

Consider the binary operations*: $\mathbf{R} \times \mathbf{R} \to$ and o: $\mathbf{R} \times \mathbf{R} \to \mathbf{R}$ defined as $a*b=|a-b|$ and $a \circ b = a, \forall a, b \in \mathbf{R}$. Show that * is commutative but not associative, o is associative but not commutative. Further, show that $\forall a, b, c \in \mathbf{R}$, $a*(b \circ c) = (a * b) \circ (a * c)$. [If it is so, we say that the operation * distributes over the operation o]. Does o distribute over *? Justify your answer.

Answer

It is given that *: $\mathbf{R} \times \mathbf{R} \to$ and o: $\mathbf{R} \times \mathbf{R} \to \mathbf{R}$ isdefined as

$a*b=|a-b|$ and $a \circ b = a, \forall a, b \in \mathbf{R}$.

For $a, b \in \mathbf{R}$, we have:

$a*b=|a-b|$

$b*a=|b-a|=|-(a-b)|=|a-b|$

$\therefore a * b = b * a$

$\therefore$ The operation * is commutative.

It can be observed that,

$$(1*2)*3=(|1-2|)*3=1*3=|1-3|=2$$
$$1*(2*3)=1*(|2-3|)=1*1=|1-1|=0$$
$$\therefore (1*2)*3 \neq 1*(2*3) \text{ (where } 1, 2, 3 \in \mathbf{R})$$

∴The operation * is not associative.

Now, consider the operation o:

It can be observed that 1 o 2 = 1 and 2 o 1 = 2

∴1 o 2 ≠ 2 o 1 (where 1, 2 ∈ **R**)

∴The operation o is not commutative.

Let a, b, c ∈ **R**. Then, we have:

(a o b) o c = a o c = a

a o (b o c) = a o b = a

⇒ a o b) o c = a o (b o c)

∴ The operation o is associative.

Now, let a, b, c ∈ **R**, then we have:

a * (b o c) = a * b = $|a-b|$

(a * b) o (a * c) = $(|a-b|) \text{o} (|a-c|) = |a-b|$

Hence, a * (b o c) = (a * b) o (a * c).

Now,

1 o (2 * 3) = $1 \text{ o} (|2-3|) = 1 \text{ o } 1 = 1$

(1 o 2) * (1 o 3) = 1 * 1 = $|1-1| = 0$

∴1 o (2 * 3) ≠ (1 o 2) * (1 o 3) (where 1, 2, 3 ∈ **R**)

∴ The operation o does not distribute over *.

Question

Given a non-empty set X, let $*$: $P(X) \times P(X) \to P(X)$ be defined as $A * B = (A - B) \cup (B - A)$, ☐ $A, B \in P(X)$. Show that the empty set Φ is the identity for the operation $*$ and all the elements A of $P(X)$ are invertible with $A^{-1} = A$. (Hint: $(A - \Phi) \cup (\Phi - A) = A$ and $(A - A) \cup (A - A) = A * A = \Phi$).

Answer

It is given that $*$: $P(X) \times P(X) \to P(X)$ is defined as

$A * B = (A - B) \cup (B - A)$ ☐ $A, B \in P(X)$.

Let $A \in P(X)$. Then, we have:

$A * \Phi = (A - \Phi) \cup (\Phi - A) = A \cup \Phi = A$

$\Phi * A = (\Phi - A) \cup (A - \Phi) = \Phi \cup A = A$

$\therefore A * \Phi = A = \Phi * A$. ☐ $A \in P(X)$

Thus, Φ is the identity element for the given operation$*$.

Now, an element $A \in P(X)$ will be invertible if there exists $B \in P(X)$ such that

$A * B = \Phi = B * A$. (As Φ is the identity element)

Now, we observed that $A * A = (A - A) \cup (A - A) = \phi \cup \phi = \phi \ \forall A \in P(X)$.

Hence, all the elements A of $P(X)$ are invertible with $A^{-1} = A$.

Question

Define a binary operation *on the set {0, 1, 2, 3, 4, 5} as

$$a*b=\begin{cases}a+b, & \text{if } a+b<6\\ a+b-6 & \text{if } a+b\geq 6\end{cases}$$

Show that zero is the identity for this operation and each element $a \neq 0$ of the set is invertible with $6 - a$ being the inverse of a.

Answer

Let $X = \{0, 1, 2, 3, 4, 5\}$.

The operation * on X is defined as:

$$a*b=\begin{cases}a+b & \text{if } a+b<6\\ a+b-6 & \text{if } a+b\geq 6\end{cases}$$

An element $e \in X$ is the identity element for the operation *, if $a*e=a=e*a\ \forall a\in X$.

For $a \in X$, we observed that:

$$a*0=a+0=a \qquad [a\in X \Rightarrow a+0<6]$$

$$0*a=0+a=a \qquad [a\in X \Rightarrow 0+a<6]$$

$$\therefore a*0=a=0*a\ \forall a\in X$$

Thus, 0 is the identity element for the given operation *.

An element $a \in X$ is invertible if there exists $b \in X$ such that $a * b = 0 = b * a$

$$\text{i.e.,}\begin{cases}a+b=0=b+a, & \text{if } a+b<6\\ a+b-6=0=b+a-6, & \text{if } a+b\geq 6\end{cases}$$

i.e.,

$a = -b$ or $b = 6 - a$

But, $X = \{0, 1, 2, 3, 4, 5\}$ and $a, b \in X$. Then, $a \neq -b$.

$\therefore b = 6 - a$ is the inverse of a □ $a \in X$.

Hence, the inverse of an element $a \in X$, $a \neq 0$ is $6 - a$ i.e., $a^{-1} = 6 - a$.

Question

Let $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$ and $f, g: A \to B$ be functions defined by $f(x) = x^2 - x, x \in A$ and $g(x) = 2\left|x - \frac{1}{2}\right| - 1, x \in A$. Are f and g equal?

Justify your answer. (Hint: One may note that two function $f: A \to B$ and g: $A \to$ B such that $f(a) = g(a)$ □$a \in A$, are called equal functions).

Answer

It is given that $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$.

Also, it is given that $f, g: A \to B$ are defined by $f(x) = x^2 - x, x \in A$ and

$$g(x) = 2\left|x - \frac{1}{2}\right| - 1, x \in A$$.

It is observed that:

$$f(-1) = (-1)^2 - (-1) = 1 + 1 = 2$$

$$g(-1) = 2\left|(-1) - \frac{1}{2}\right| - 1 = 2\left(\frac{3}{2}\right) - 1 = 3 - 1 = 2$$

$$\Rightarrow f(-1) = g(-1)$$

$$f(0)=(0)^2-0=0$$
$$g(0)=2\left|0-\frac{1}{2}\right|-1=2\left(\frac{1}{2}\right)-1=1-1=0$$
$$\Rightarrow f(0)=g(0)$$
$$f(1)=(1)^2-1=1-1=0$$
$$g(1)=2\left|1-\frac{1}{2}\right|-1=2\left(\frac{1}{2}\right)-1=1-1=0$$
$$\Rightarrow f(1)=g(1)$$
$$f(2)=(2)^2-2=4-2=2$$
$$g(2)=2\left|2-\frac{1}{2}\right|-1=2\left(\frac{3}{2}\right)-1=3-1=2$$
$$\Rightarrow f(2)=g(2)$$
$$\therefore f(a)=g(a)\ \forall a\in A$$

Hence, the functions f and g are equal.

Question

Let $A = \{1, 2, 3\}$. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is

(A) 1 (B) 2 (C) 3 (D) 4

Answer

The given set is A = {1, 2, 3}.

The smallest relation containing (1, 2) and (1, 3) which is reflexive and symmetric, but not transitive is given by:

R = {(1, 1), (2, 2), (3, 3), (1, 2), (1, 3), (2, 1), (3, 1)}

This is because relation R is reflexive as (1, 1), (2, 2), (3, 3) ∈ R.

Relation R is symmetric since (1, 2), (2, 1) ∈R and (1, 3), (3, 1) ∈R.

But relation R is not transitive as (3, 1), (1, 2) ∈ R, but (3, 2) ∉ R.

Now, if we add any two pairs (3, 2) and (2, 3) (or both) to relation R, then relation R will become transitive.

Hence, the total number of desired relations is one.

The correct answer is A.

Question

Let A = {1, 2, 3}. Then number of equivalence relations containing (1, 2) is

(A) 1 (B) 2 (C) 3 (D) 4

Answer

It is given that A = {1, 2, 3}.

The smallest equivalence relation containing (1, 2) is given by,

R_1 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)}

Now, we are left with only four pairs i.e., (2, 3), (3, 2), (1, 3), and (3, 1).

If we odd any one pair [say (2, 3)] to R_1, then for symmetry we must add (3, 2). Also, for transitivity we are required to add (1, 3) and (3, 1).

Hence, the only equivalence relation (bigger than R_1) is the universal relation.

This shows that the total number of equivalence relations containing (1, 2) is two.

The correct answer is B.

Question

Let f: **R** → **R** be the Signum Function defined as

$$f(x)=\begin{cases} 1, & x>0 \\ 0, & x=0 \\ -1, & x<0 \end{cases}$$

and g: **R** → **R** be the Greatest Integer Function given by $g(x)$ = [x], where [x] is greatest integer less than or equal to x. Then does fog and gof coincide in (0, 1]?

Answer

It is given that,

f: **R** → **R** is defined as
$$f(x)=\begin{cases} 1, & x>0 \\ 0, & x=0 \\ -1, & x<0 \end{cases}$$

Also, g: **R** → **R** is defined as $g(x)$ = [x], where [x] is the greatest integer less than or equal to x.

Now, let $x \in (0, 1]$.

Then, we have:

$[x] = 1$ if $x = 1$ and $[x] = 0$ if $0 < x < 1$.

$$\therefore fog(x) = f(g(x)) = f([x]) = \begin{cases} f(1), & \text{if } x = 1 \\ f(0), & \text{if } x \in (0,1) \end{cases} = \begin{cases} 1, & \text{if } x = 1 \\ 0, & \text{if } x \in (0,1) \end{cases}$$

$$\begin{aligned} gof(x) &= g(f(x)) \\ &= g(1) \qquad [x > 0] \\ &= [1] = 1 \end{aligned}$$

Thus, when $x \in (0, 1)$, we have $fog(x) = 0$and $gof(x) = 1$.

Hence, fog and gof do not coincide in (0, 1].

Question

Number of binary operations on the set $\{a, b\}$ are

(A) 10 (B) 16 (C) 20 (D) 8

Answer

A binary operation * on $\{a, b\}$ is a function from $\{a, b\} \times \{a, b\} \to \{a, b\}$

i.e., * is a function from $\{(a, a), (a, b), (b, a), (b, b)\} \to \{a, b\}$.

Hence, the total number of binary operations on the set $\{a, b\}$ is 2^4 i.e., 16.

The correct answer is B.

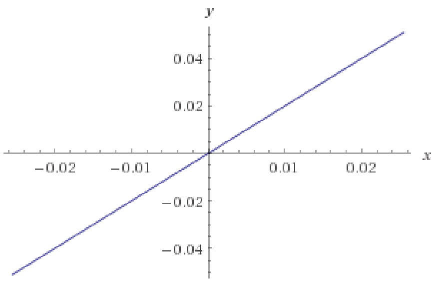
Review of Graphs

The student must be very good at Graphs of Various kinds of functions; to do well in Continuity, Differentiability, Area, Volume problems. Some limit problems also require concepts of Functions and Graphs. The graphs will not be given in the Questions. In case of Area problems, the student has to draw the graphs quickly, largely to scale; get the intersection points, and then plan for a piece-wise strategy to integrate and find the area.

Let us review the various graphs

y = mx will be a straight line passing through the origin. Positive m will make the line move upwards as we move in positive x i.e. towards right.

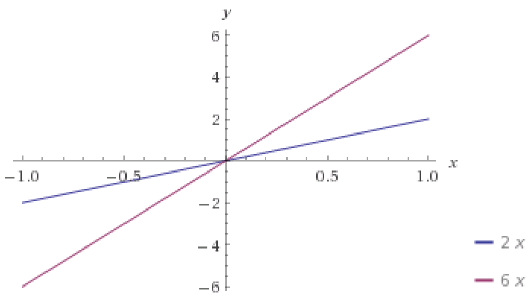
plot	$y = 2x$



This is graph of y = 2 x Don't get foxed by the angle being almost 45^0 The scales in y-axis and x-axis are not same.

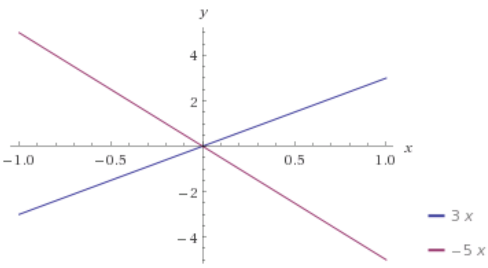
If we compare two graphs then it becomes more clear.

plot	$y = 2x$
	$y = 6x$



In this figure also scales of x-axis and y-axis are not same. But y = 6x has to be steeper than y = 2x

plot	$y = 3x$
	$y = -5x$

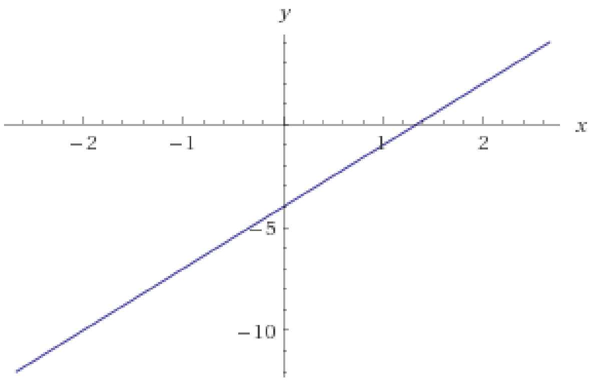


This is y = 3x and y = - 5 x graphs. For m = -5 the line moves down

For y = m x + c the c becomes the intercept in the y axis

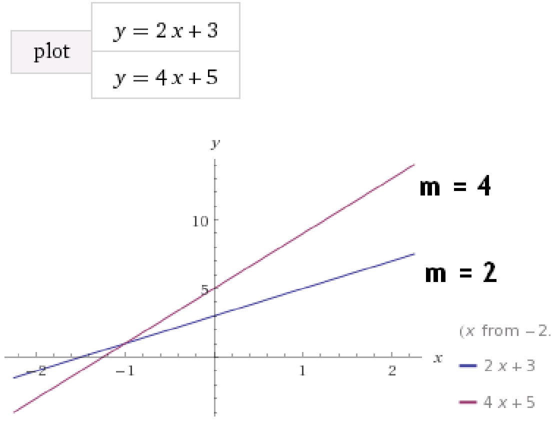
So y = 3x - 4 will look like

plot	$y = 3x - 4$



If c is a positive number then the intercept in y-axis will be on upper (positive) side.

Graphs of y = 2x + 3   and y = 4x + 5 will be



Again scales in x-axis and y-axis are different. But point made. See how the graphs pass through 3 and 5 respectively.

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Nature of Curves, Types of Graphs, Shapes are explained / discussed at

https://archive.org/details/AreaDefiniteIntegralNatureOfCurvesTypesOfGraphsShapesDiscussions

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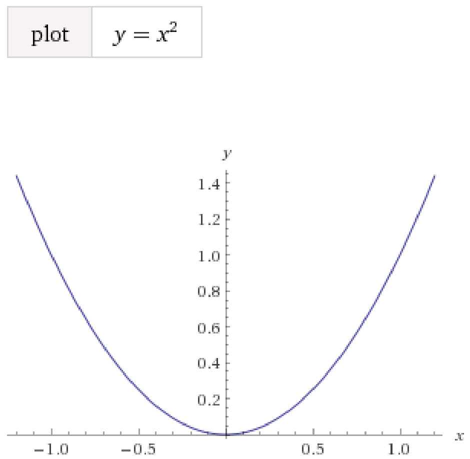
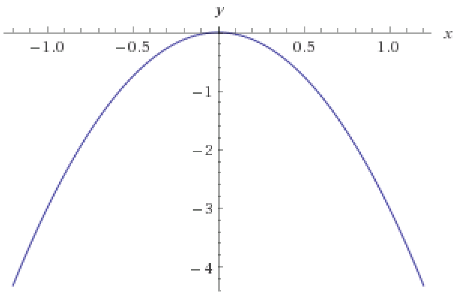
Now let us see graphs of Quadratic functions

Graph of $y = x^2$ will be

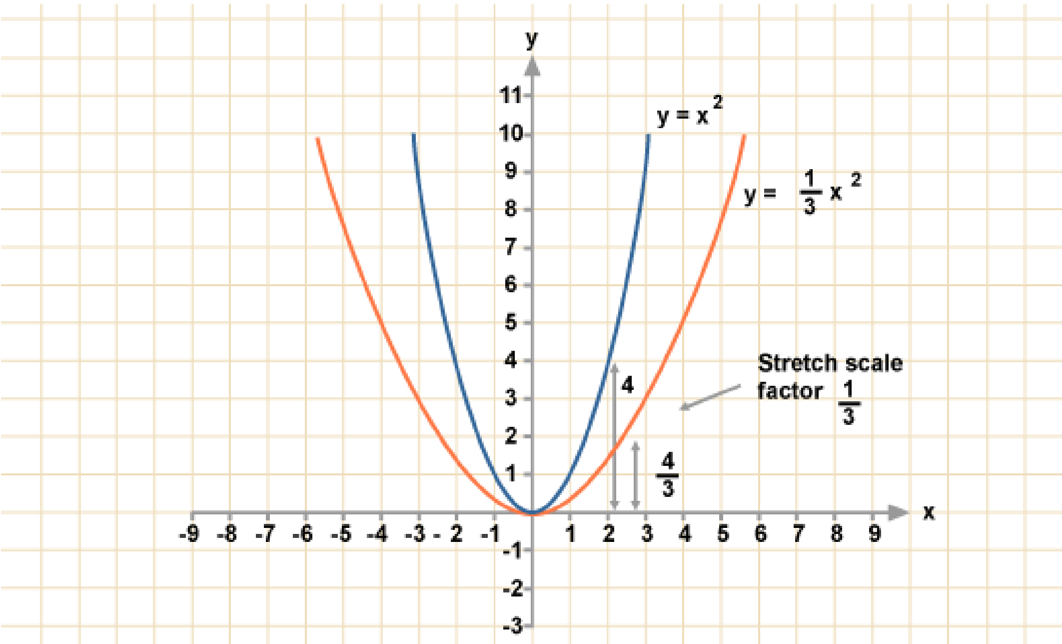


In contrast graph of $y = -3x^2$ will be downwards

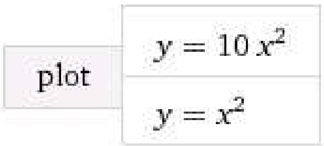
plot	$y = -3x^2$

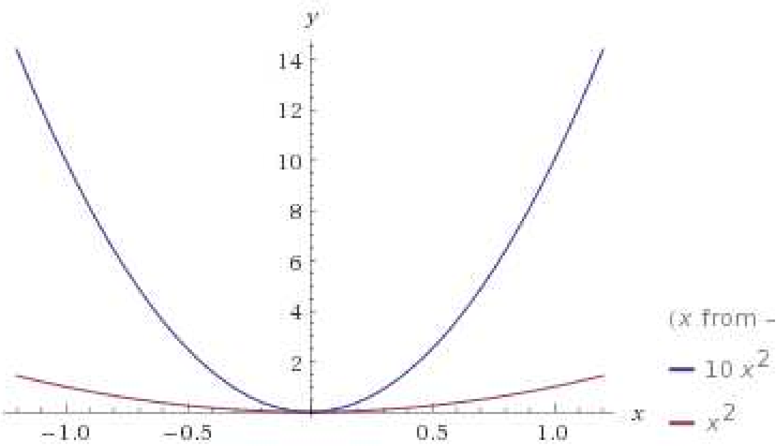


Graph of y = (1/3) x^2 will be flatter compared to y = x^2

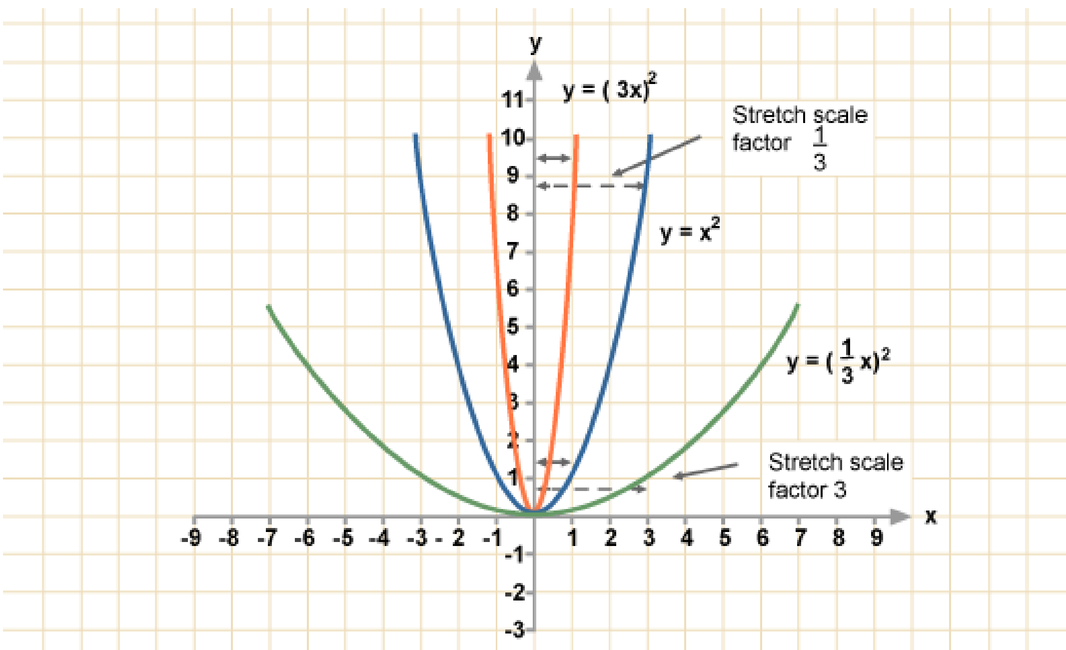


Similarly graph of y = $10x^2$ will be narrow and steeper compared to y = x^2

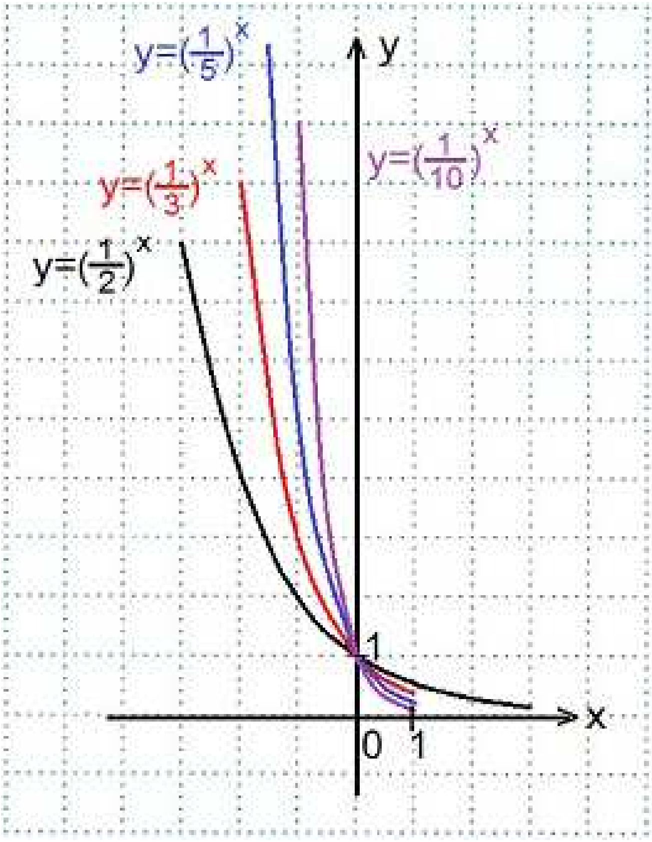




So see comparisons in a single image

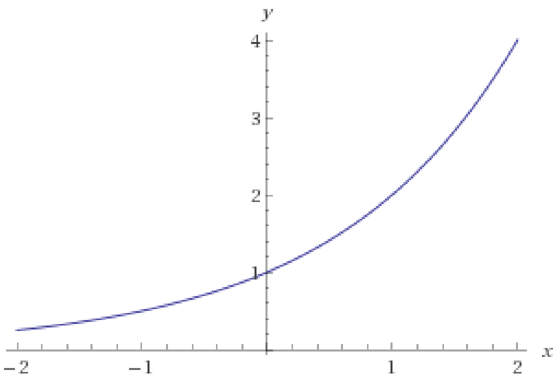


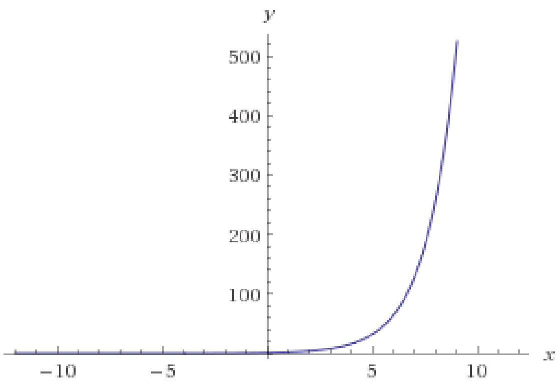
Similar things happen with power functions as well. Below we see fraction raised to power x



Let us see the graph of $y = 2^x$

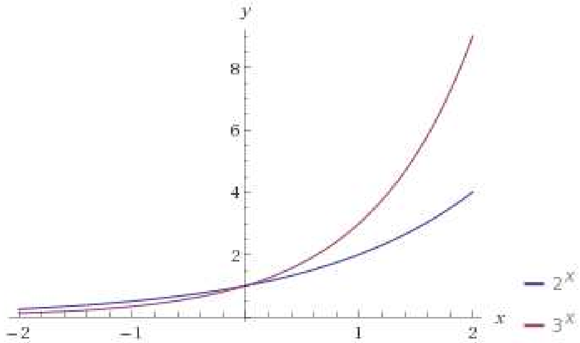
plot	$y = 2^x$

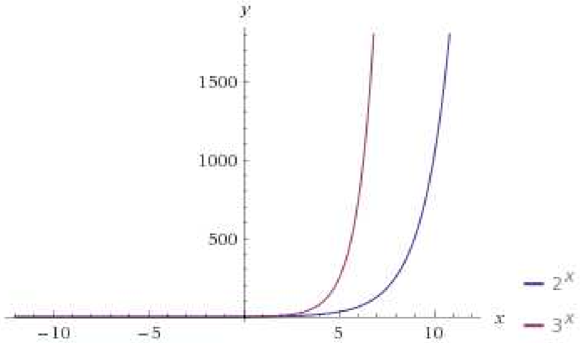




The graph of y = 3^x will be steeper and is understood easily by comparison

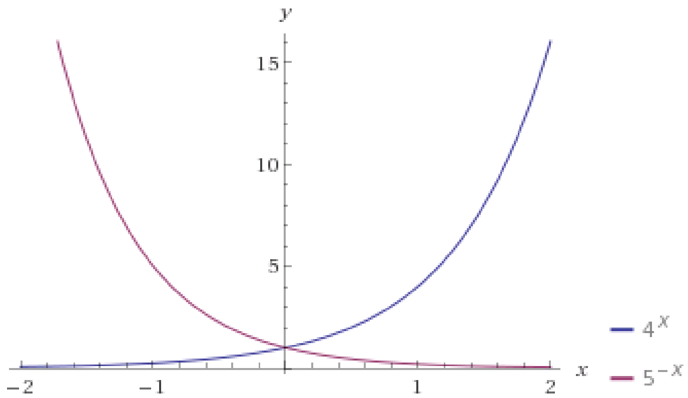
plot	$y = 2^x$
	$y = 3^x$





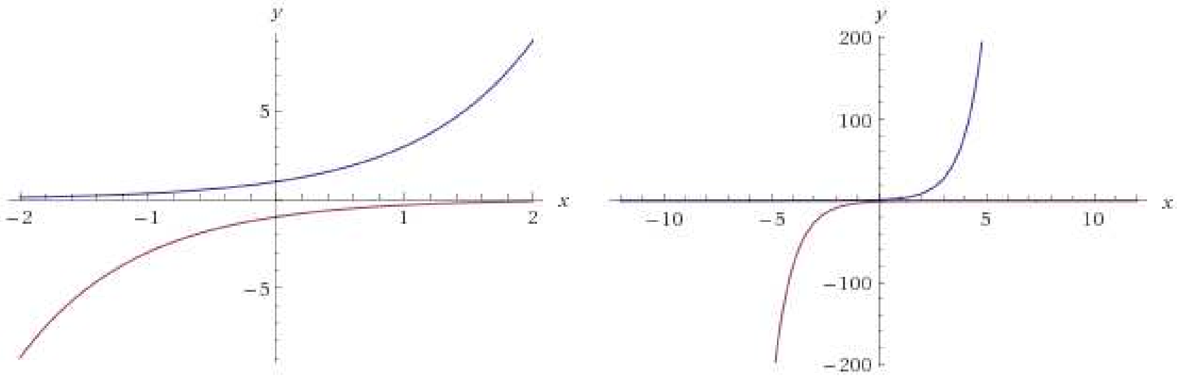
Now let us compare Integer to the power x and fraction to the power x

plot	$y = 4^x$
	$y = \left(\frac{1}{5}\right)^x$



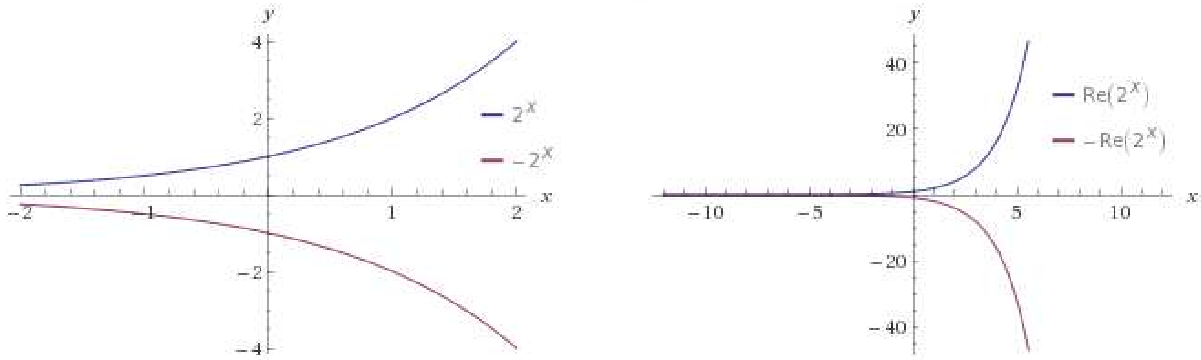
What about comparing y = 3^x and y = -3^{-x}

plot	$y = 3^x$
	$y = -3^{-x}$



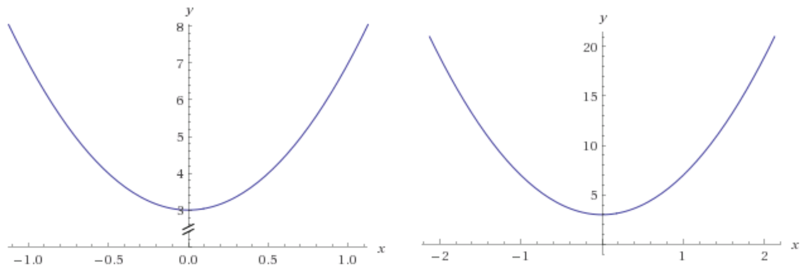
Spoon Feeding comparison of y = 2^x and y = -2^x

plot	$y = 2^x$
	$y = -2^x$



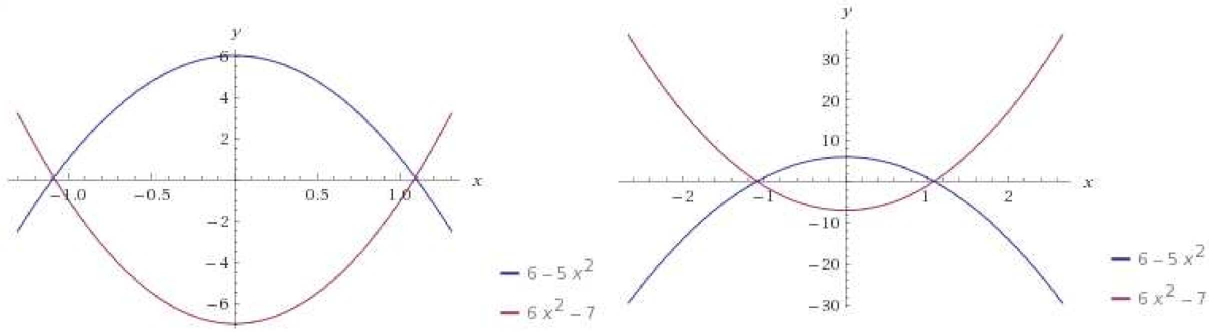
Graph of y = $4x^2$ + 3 will be 3 units above x-axis. So will pass through (0, 3) The parabola will look similar to y = x^2

plot	$y = 4x^2 + 3$



Let us learn more with graphs of y = $-5x^2$ + 6 and y = $6x^2$ -7

plot	$y = -5x^2 + 6$
	$y = 6x^2 - 7$



Don't quickly assume that the graphs are intersecting on x axis. The roots are very close.

$5x^2 = 6 \Rightarrow x = \pm\sqrt{(6/5)} = \pm 1.095$

While $6x^2 = 7 \Rightarrow x = \pm\sqrt{(7/6)} = \pm 1.0801$

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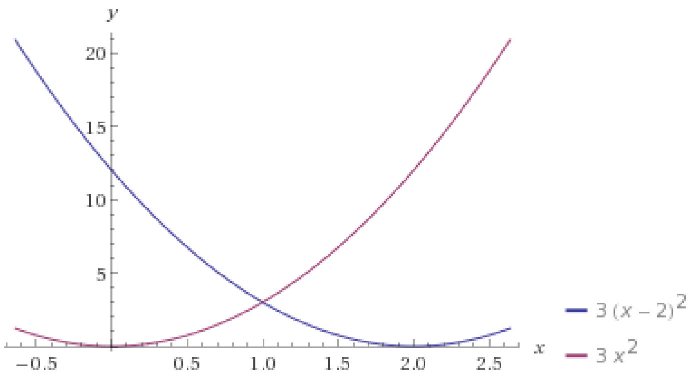
Concept of Shifting of graphs

The graph of $y = 3(x - 2)^2$ will be same as $y = 3x^2$ while shifted by 2 units towards right

plot $y = 3(x-2)^2$

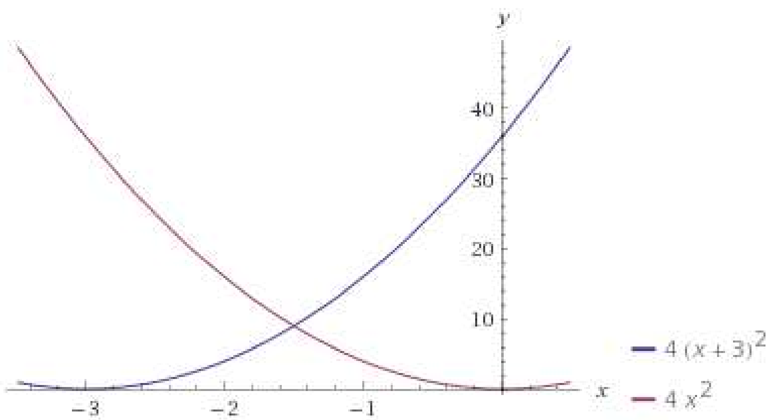
$y = 3x^2$



Similarly graph of $y = 4(x + 3)^2$ will be shifted by 3 units on left compared to $y = 4x^2$ which is through the origin

plot $y = 4(x+3)^2$

$y = 4x^2$



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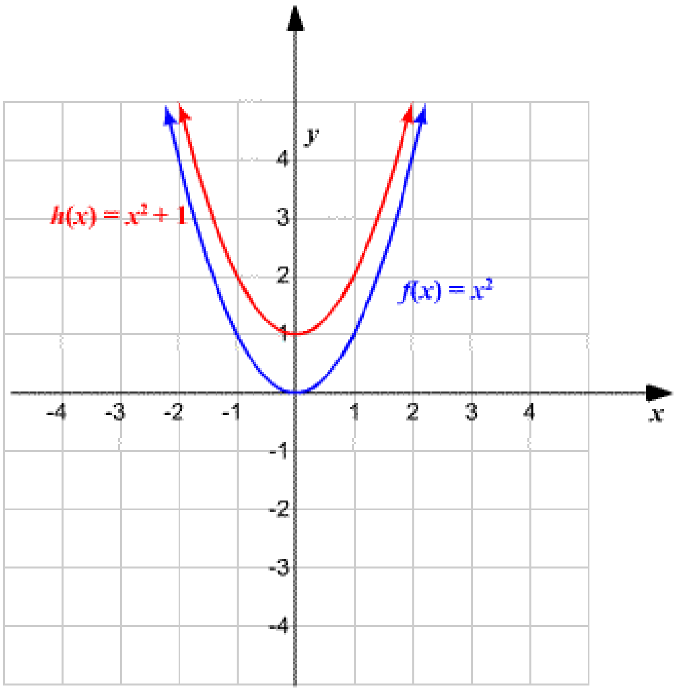
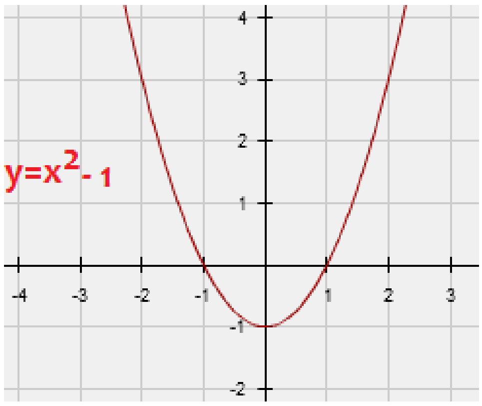
IIT-JEE 2005 Shifting a Parabola and then finding the area is discussed / explained at

https://archive.org/details/AreaDefiniteIntegralIITJEE2005ShiftingParabolasLeftOrRight



In the above image see how the upper graph is shifted up by 1 due to +1

In the image below the graph is shifted down by -1



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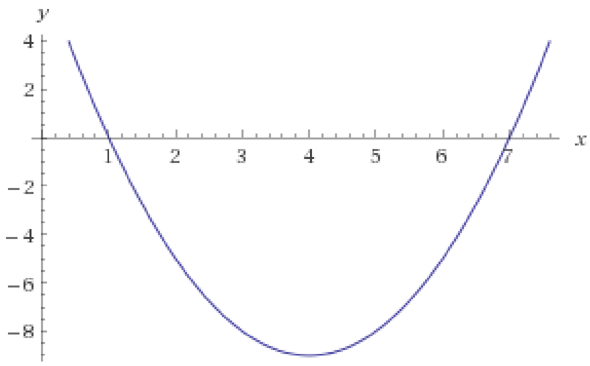
The parabola that passes through (1,0) and (7,0) will be (x - 1)(x - 7)

In simple words the Quadratic expression that has roots 1 and 7 is a parabola through 1 and 7

So graph of y = (x - 1)(x - 7) = x^2 - 8x + 7 is

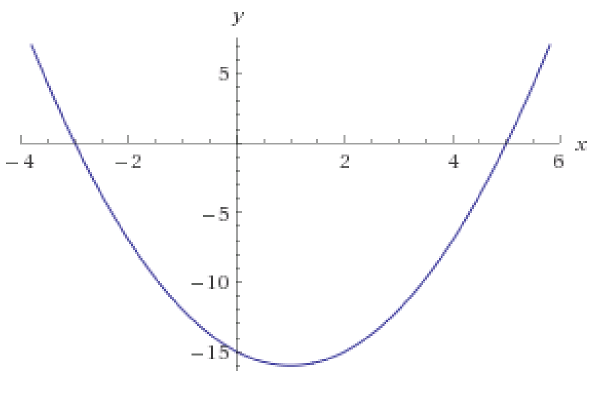
plot	$y = x^2 - 8\,x + 7$



If a Quadratic expression has roots -3, 5 then it will be a parabola passing through -3 and 5

So graph of y = (x + 3) (x - 5) = x^2 -2x - 15 is

plot	$y = x^2 - 2\,x - 15$

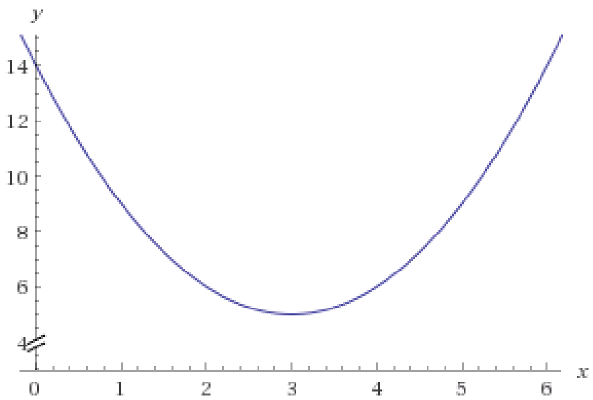


If the Discriminant D < 0 i.e. $b^2 < 4ac$ then the whole parabola is above x-axis signifying imaginary roots. As the parabola does not intersect the x-axis at all. For a > 0

If a is negative then the parabola will be downwards

So graph of $y = (x - 3)^2 + 5$ will be

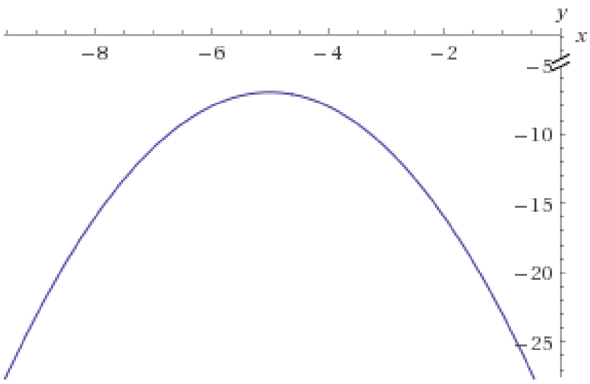
plot	$y = (x-3)^2 + 5$



Meaning minima will be at x = 3 so x^2 graph shifted right by 3 and added 5 so moved up by 5 units

So we can easily guess the graph of $y = - (x + 5)^2 -7$ …..

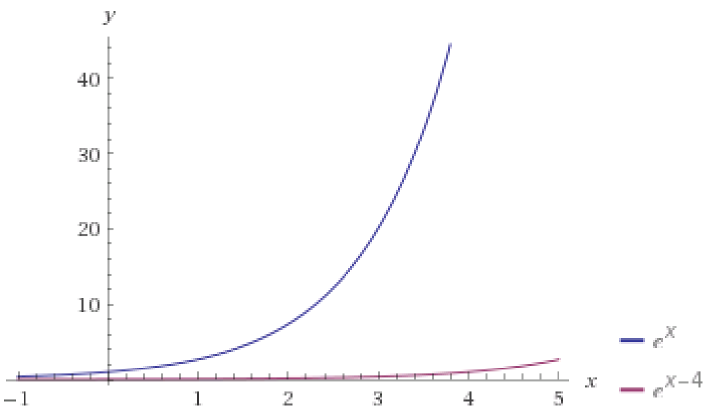
It will be shifted left by 5 units. So maxima will be at x = -5 and 7 units below x axis plot $y = -(x+5)^2 - 7$



The parabola is downwards because coeff of x^2 is -ve

Don't use the idea of shift blindly ! The graph of $y = e^{x-4}$ is not shifted by 4 units that of $y = e^x$

plot $y = e^x$
$y = e^{x-4}$

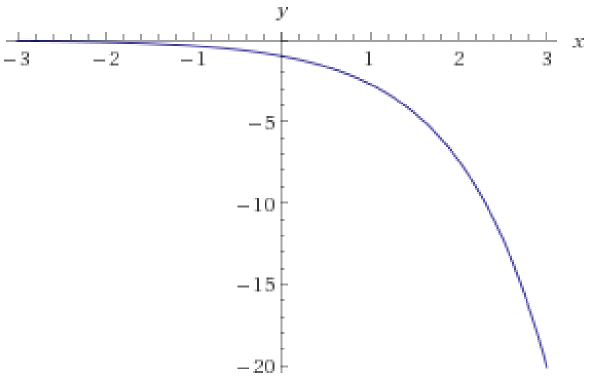


This is because $e^{(x-4)} = e^x / e^4$ means just divided by a value

Concept of Reflections

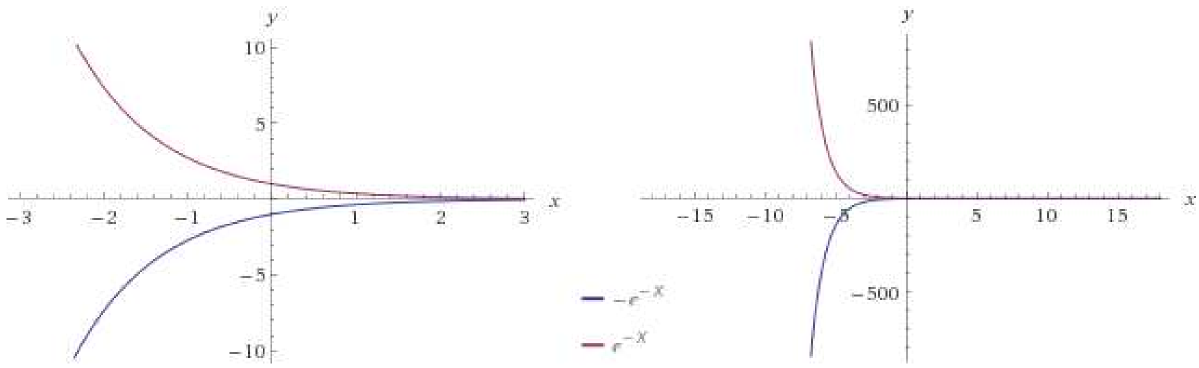
Guess the graph of $y = - e^x$

plot	$y = -e^x$



What about graph of y = $e^{(-x)}$ and y = - $e^{(-x)}$

plot	$y = -e^{-x}$
	$y = e^{-x}$



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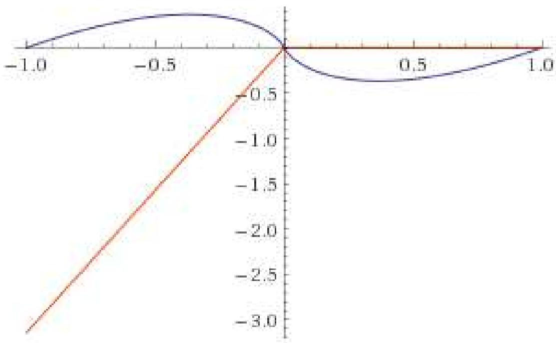
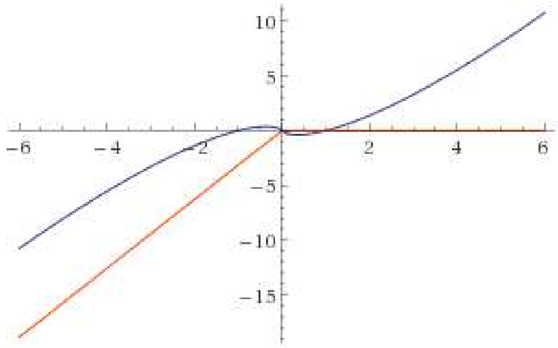
Graph of y = x ln x (Ignore the Imaginary part graph)

plot	$y = x \log(x)$





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Graph of y = x ln | x |

plot	$y = x \log(\lvert x \rvert)$

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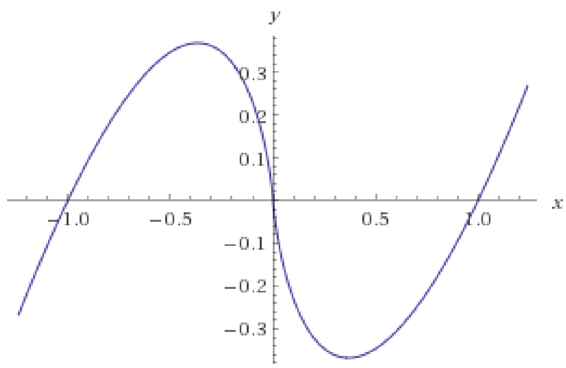

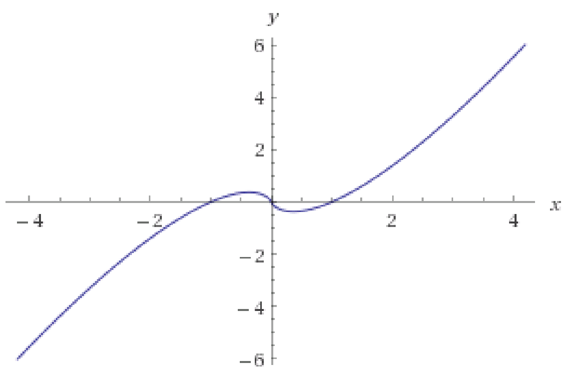

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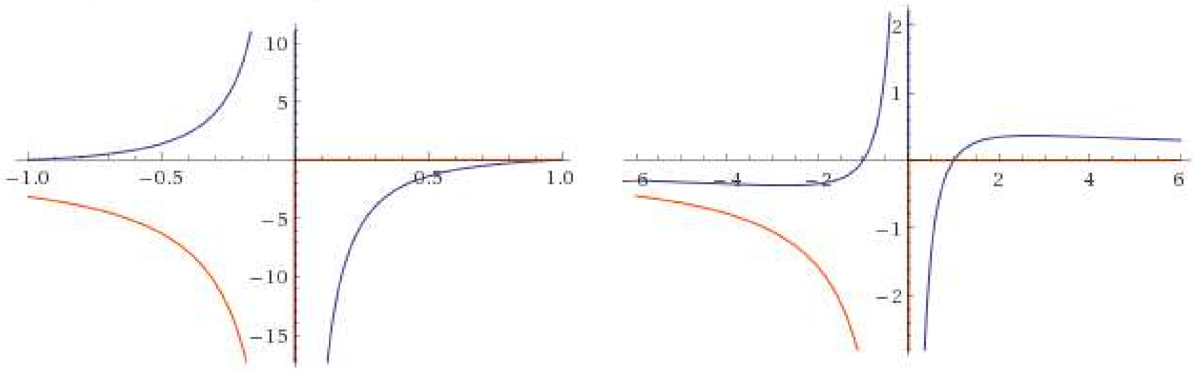
How will the graph of y = (ln x)/x look like ? (Ignore the Imaginary part)

plot	$y = \frac{1}{x}\log(x)$



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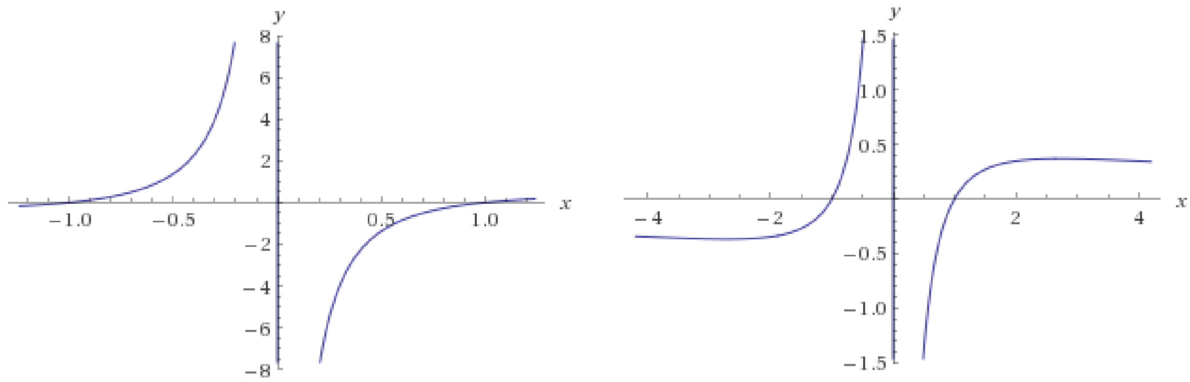
What about graph of y = (ln | x |)/x

plot	$y = \frac{1}{x}\log(\lvert x\rvert)$



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IIT-JEE 1990 problem and Solution on Area, Tricky graph of x Ln x is explained / Discussed at

https://archive.org/details/AreaDefiniteIntegralIITJEE1990TrickyGraphsOfXLnXAndLnXByX

IIT JEE 1984, 1992 Problems and Solutions as being discussed in the class. Explains various kinds of graphs at https://archive.org/details/AreaDefiniteIntegralIITJEE19841992TypesOfGraphs

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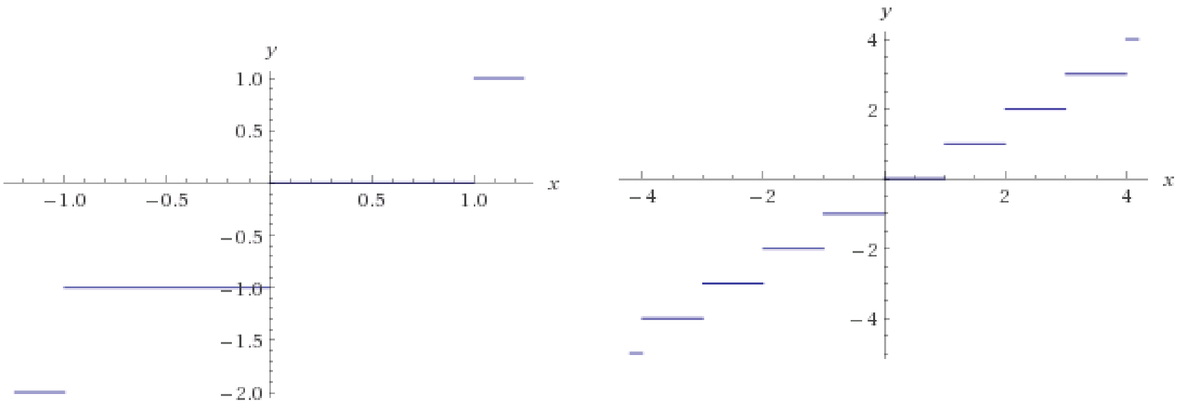
Graph of Floor x , i.e. greatest integer function x, y = [x]

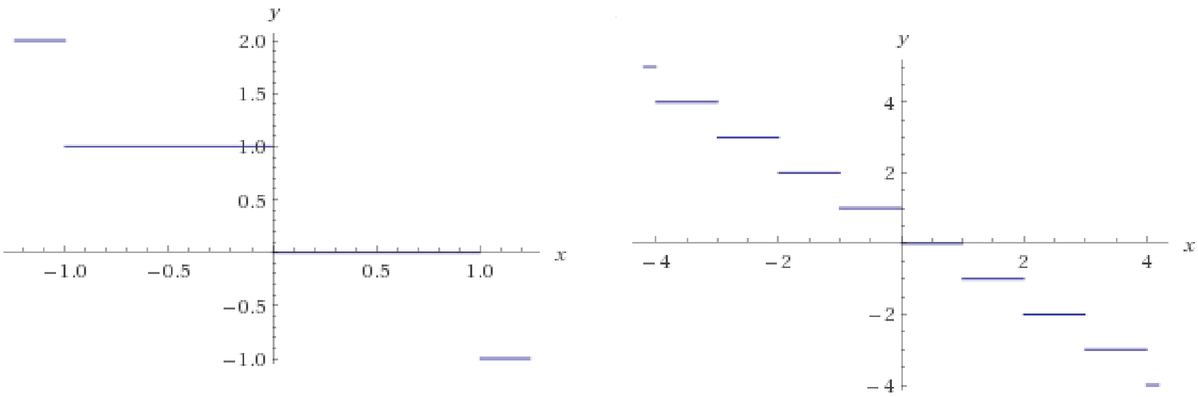
plot	$y = \lfloor x \rfloor$



Recall [-3.2] is -4 the integer less than -3.2 while [-3.99] is also -4

What about graph of y = - [x] (i.e. negative of Floor function)

plot	$y = -\lfloor x \rfloor$



Best way to learn is to " think " and try to plot it yourself, in rough.

There are many theorems related to " Floor or Greatest Integer functions ". Two theorems related to Floor function are discussed while solving a complicated Limit problem

https://archive.org/details/VeryImportantTwoFloorTheoremsGreatestIntegerFunctionExplanationAndExample

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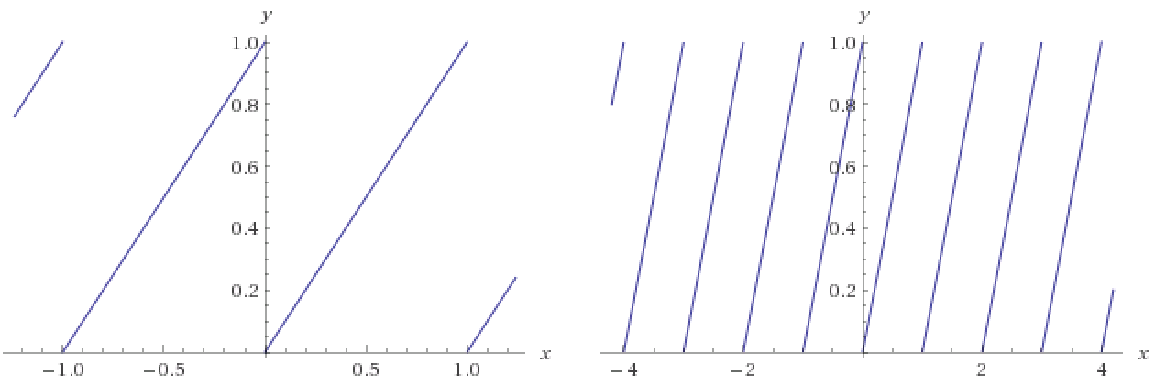
Fraction x can be defined as x - [x] so graph of y = { x } will be

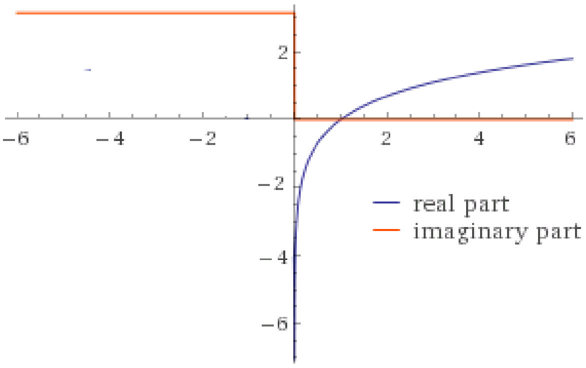
plot	$y = x - \lfloor x \rfloor$

{ 2.3 } = 0.3, { 2.4 } = 0.4, { 4.5 } = 0.5, { 4.6 } = 0.6

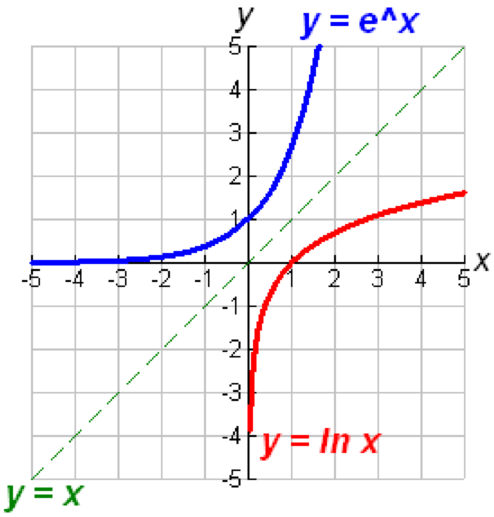


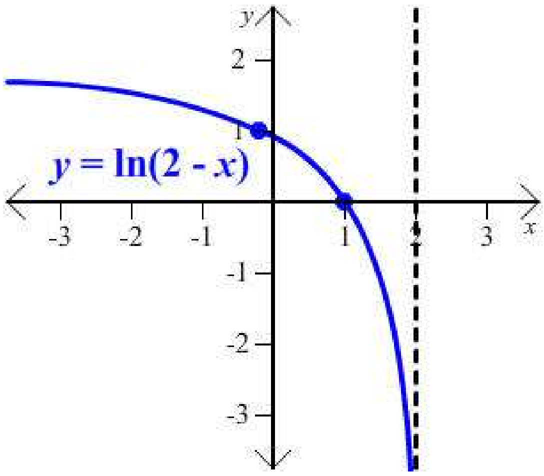
There are infinite number of discontinuities.

Graph of y = ln (x)   Note : Log of negative number is imaginary as discussed in the complex number chapter.



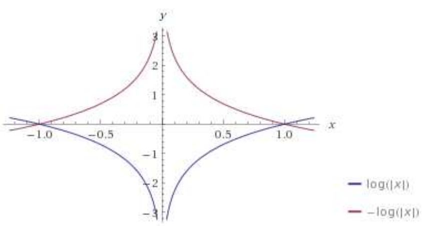
Ignore the graph of the imaginary part

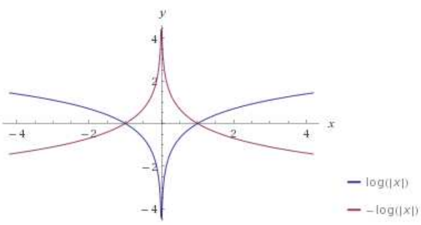


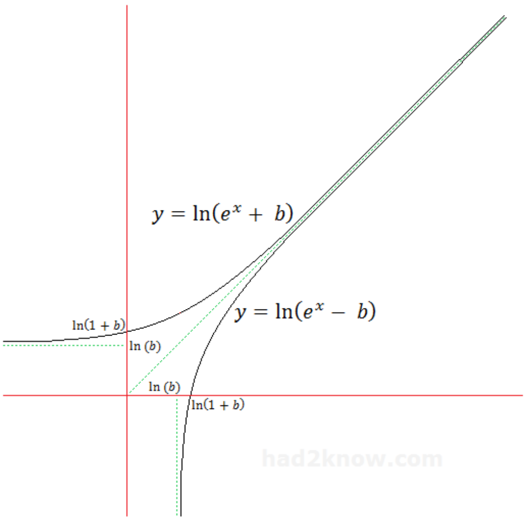


Graph of y = ln | x | and y = - ln | x |

plot	$y = \log(\lvert x \rvert)$
	$y = -\log(\lvert x \rvert)$

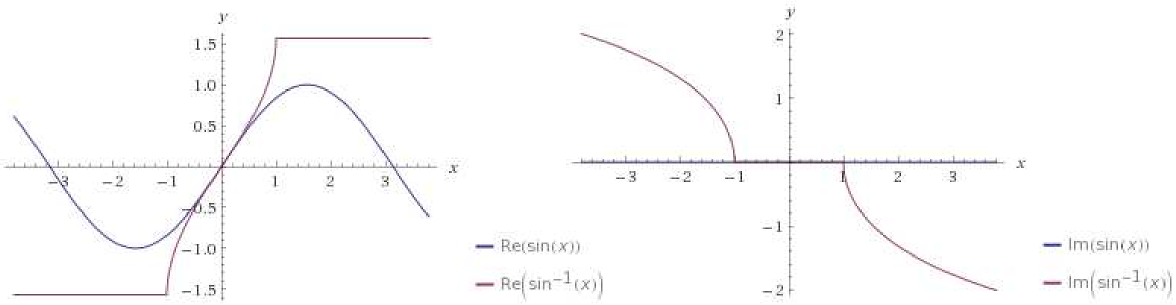






Graph of y = Sin x vs y = Sin^{-1} x

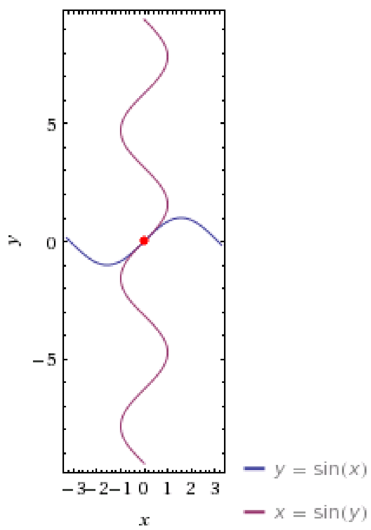
plot $y = \sin(x)$
$y = \sin^{-1}(x)$



Not sure if the above graph communicates well. Imaginary part of the graph to be ignored / avoided as of this discussion.

y = Sin^{-1} x means x = Sin y   The graph of which is drawn much easier.

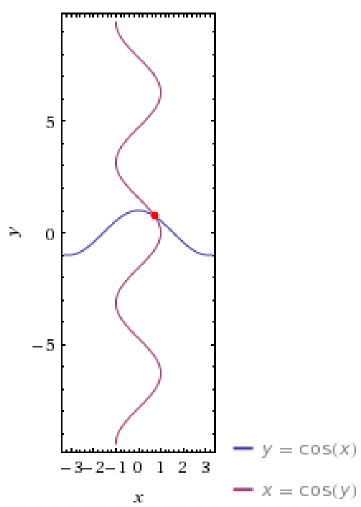
plot	$y = \sin(x)$
	$x = \sin(y)$



I am sure this is much better

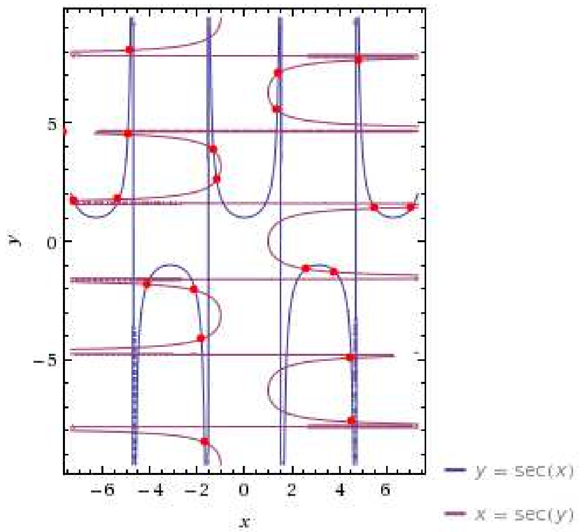
Graph of y = Cos x vs y = Cos^{-1} x

plot	$y = \cos(x)$
	$x = \cos(y)$

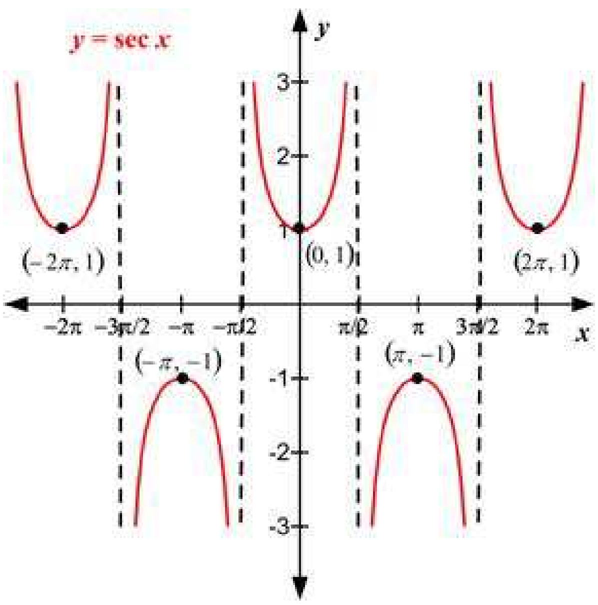


Graph of y = Sec x vs y = Sec^{-1} x

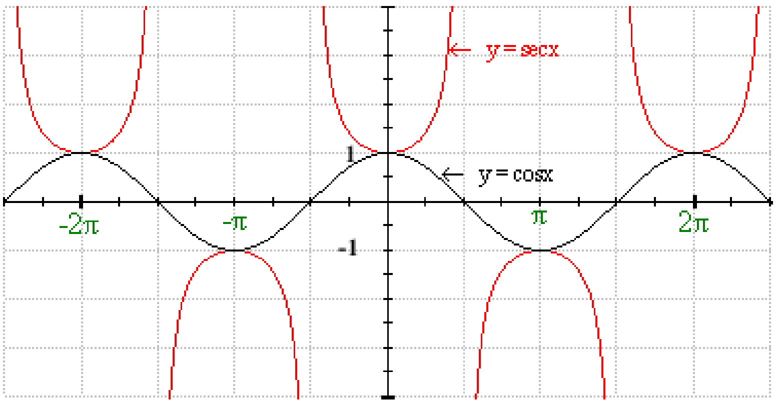
plot	$y = \sec(x)$
	$x = \sec(y)$



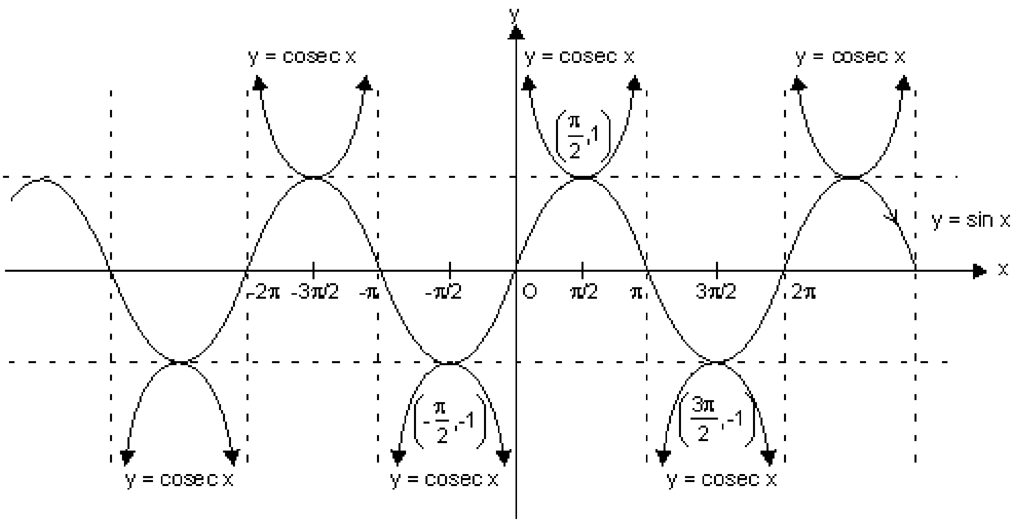
I guess we should see these graphs individually as these graphs are not commonly given in other text books



Actually Cos x can be drawn in the gap to fit-in well

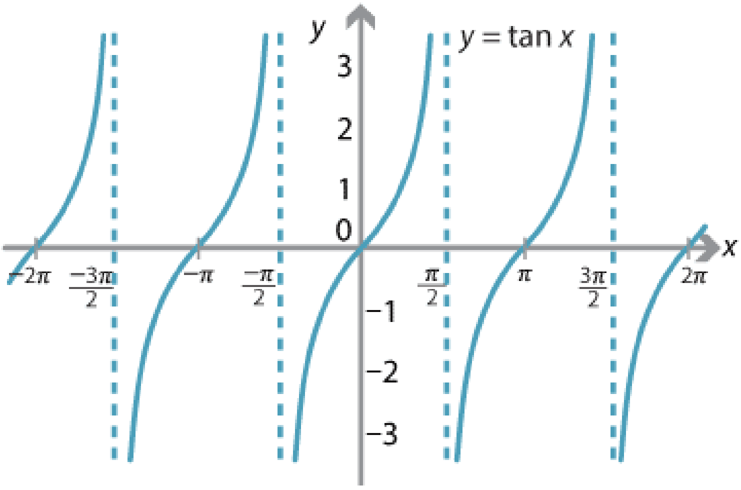


Graph of y = Cosec x

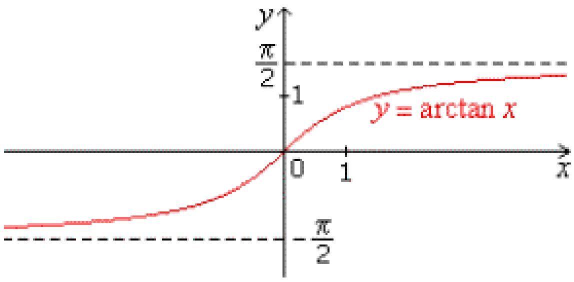


Y = Sin x has been fit into this

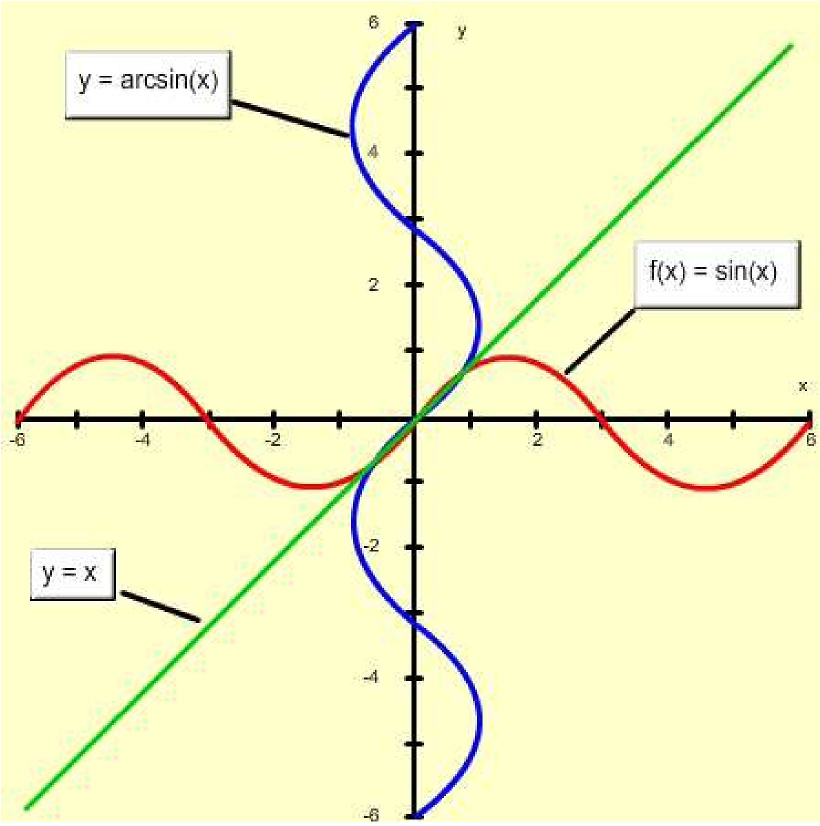
Graph of y = tan x

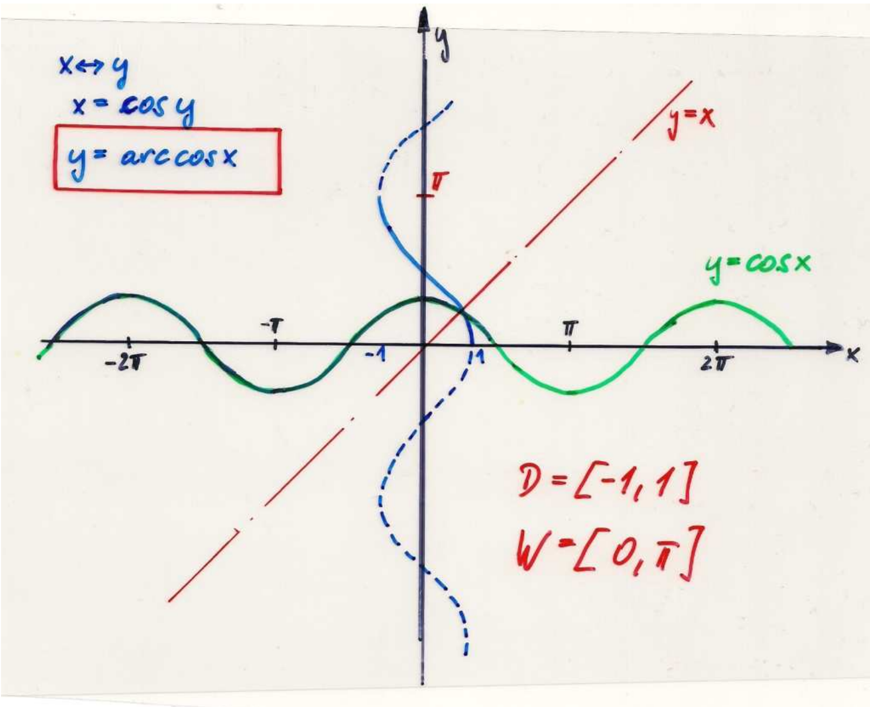


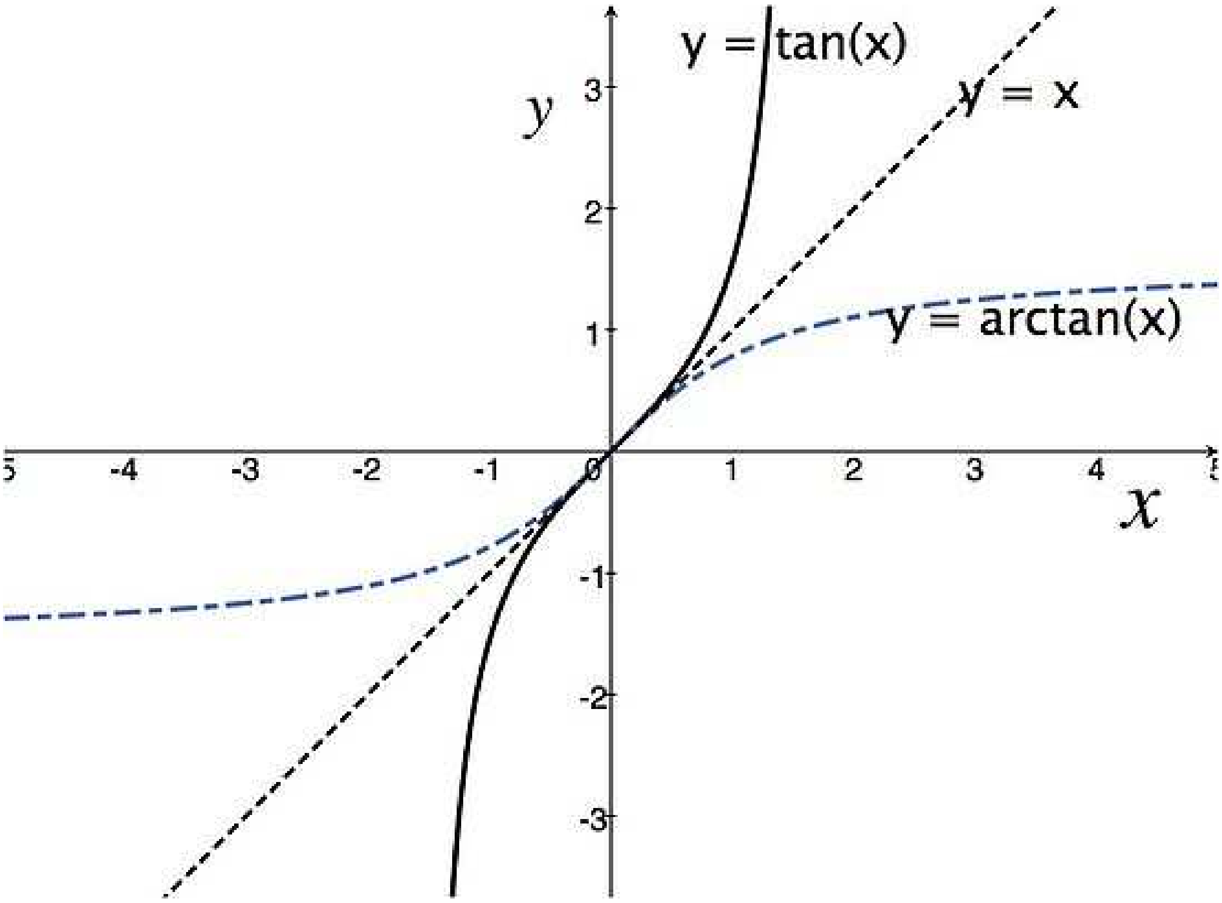
Graph of y = $\tan^{-1}$ x



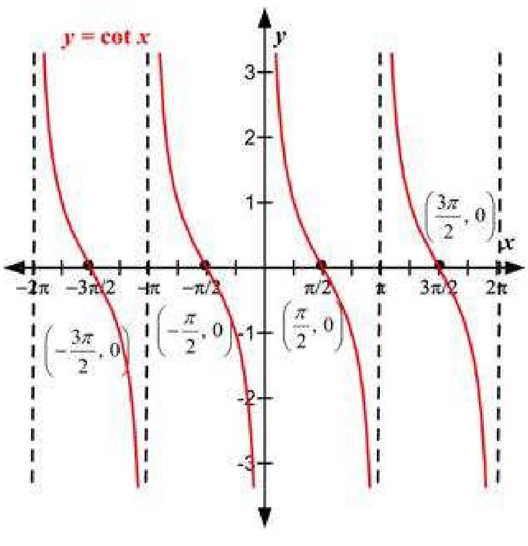
Let us compare these a few more times, so that we can remember



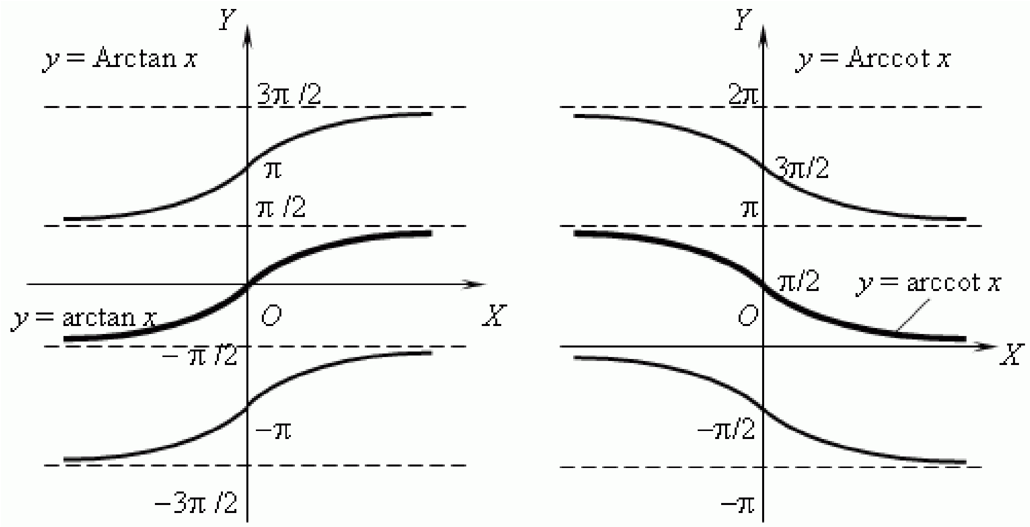


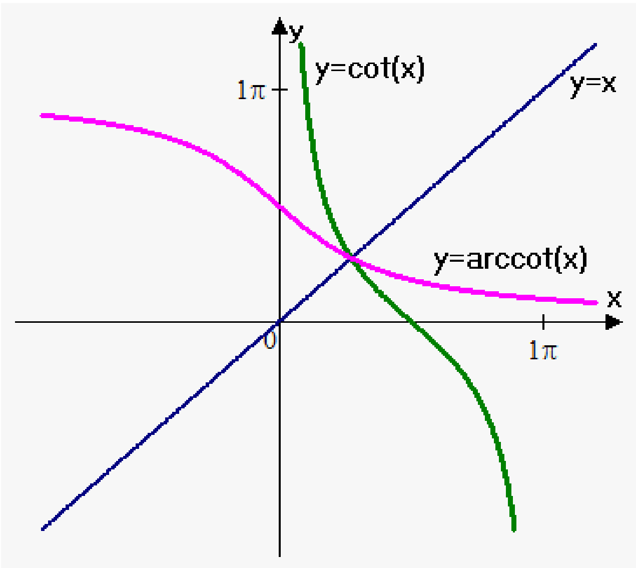


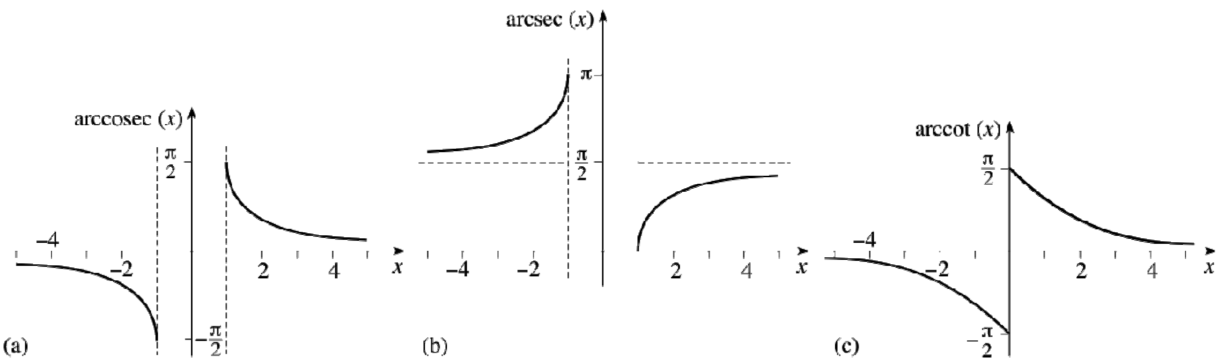
Graph of y = Cot x



Graph of y = Cot^{-1} x







An introduction to Periodic functions, Decision to Multiply or Divide is explained at

https://archive.org/details/PeriodicFunctionsAnIntroductionOfPeriodMultiplyOrDivide

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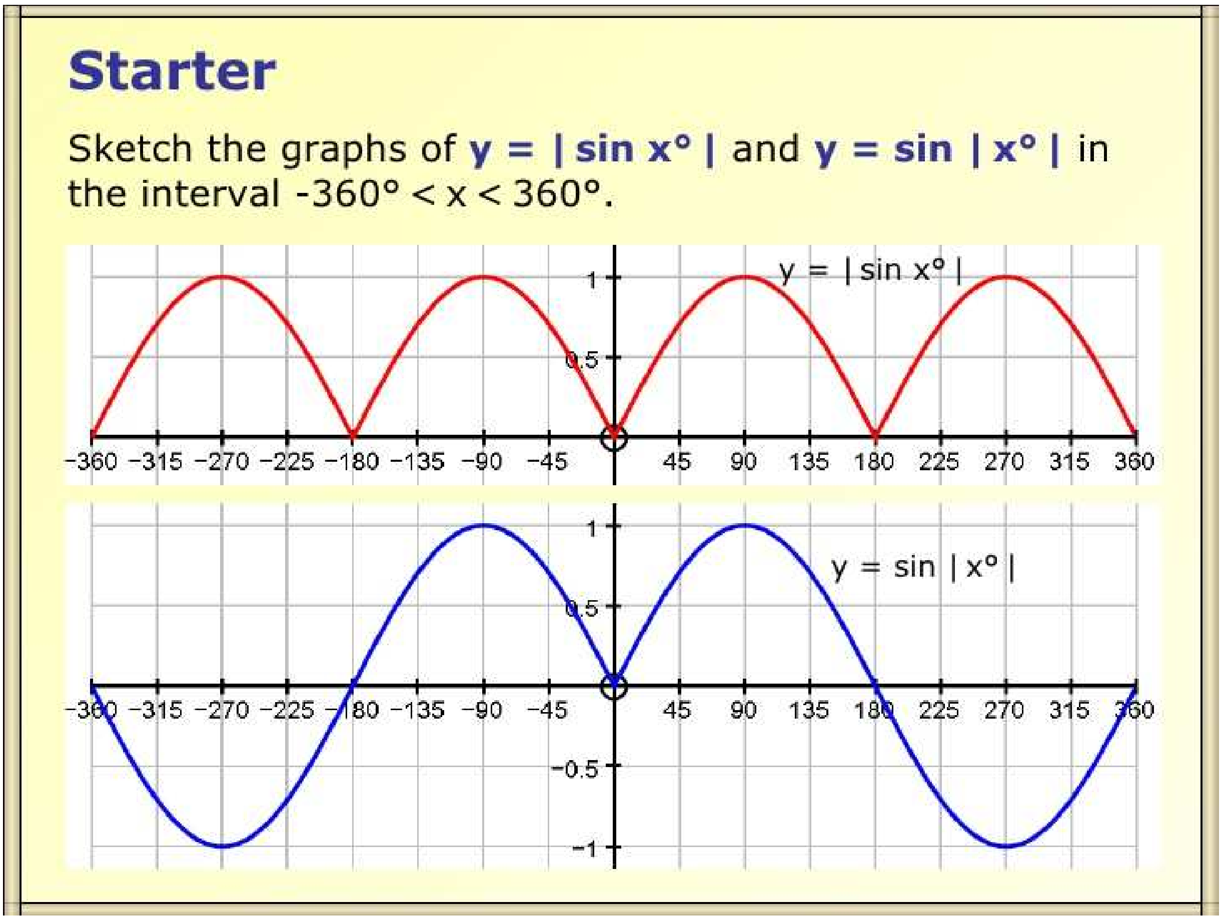
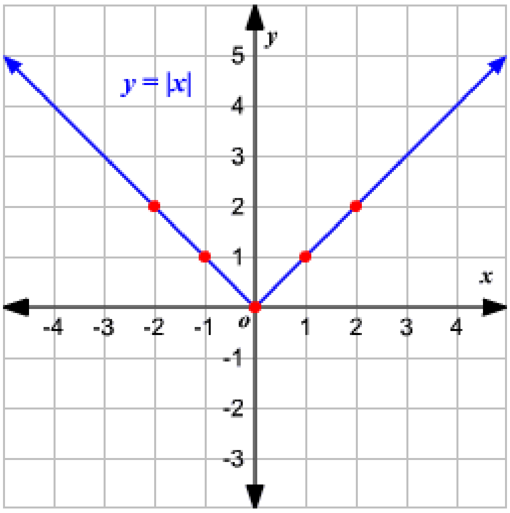
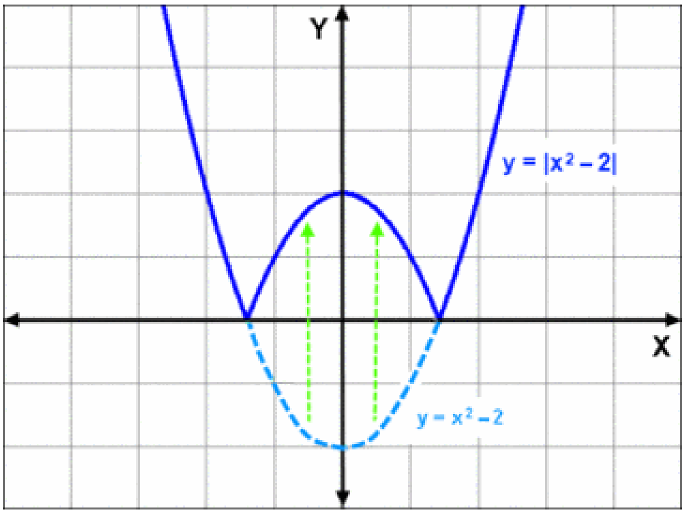
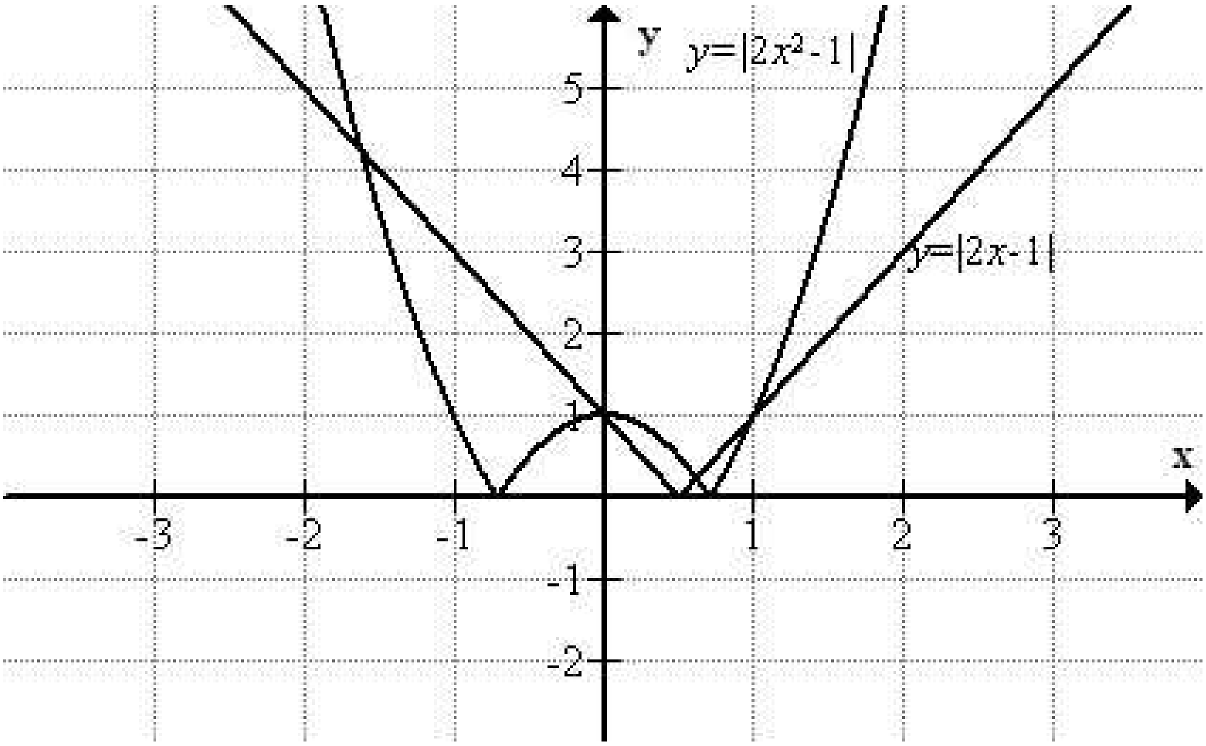
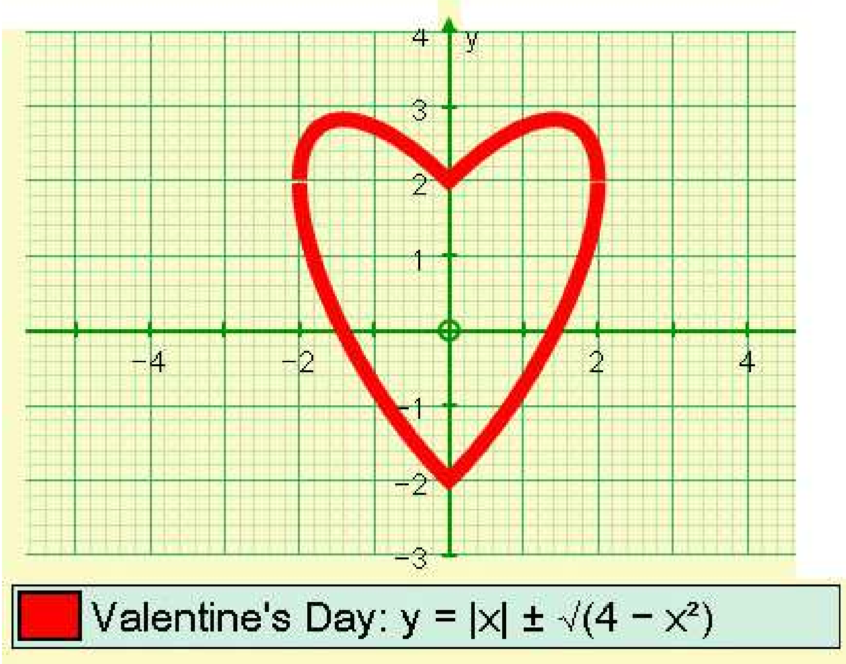
Graphs of modulus functions

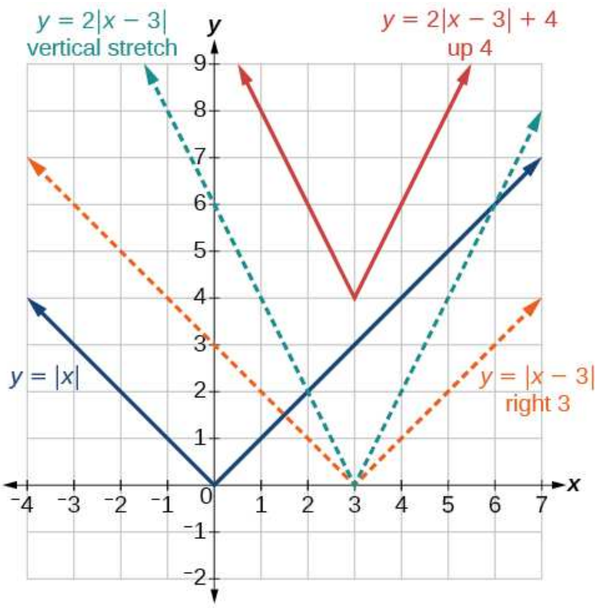



y = |x|
x
y

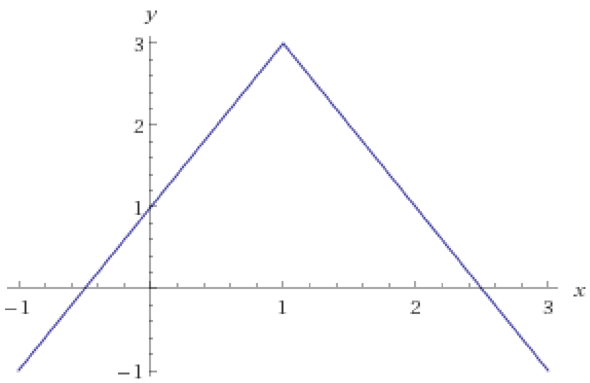

Y
X
y = |x² – 2|
y = x² – 2


y
x
y=|2x²-1|
y=|2x-1|


Valentine's Day: y = |x| ± √(4 − x²)

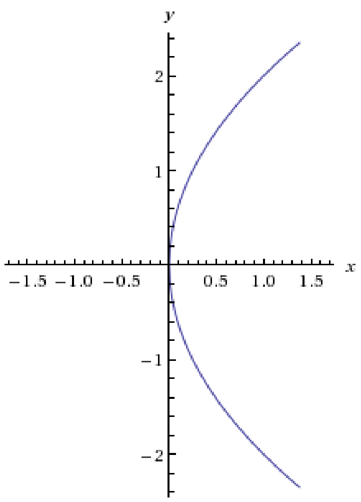

y = 2|x − 3|
vertical stretch
y = 2|x − 3| + 4
up 4
y = |x|
y = |x − 3|
right 3

plot	$-2\,\lvert x-1\rvert+3$

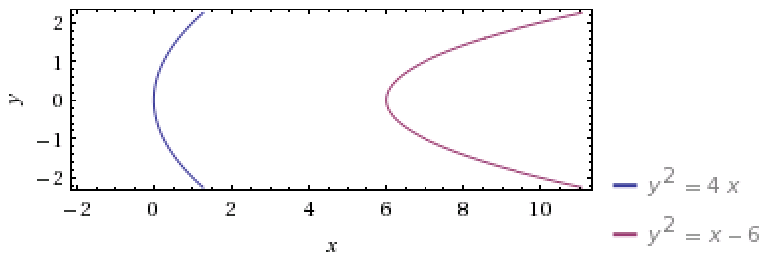


Now let us see Horizontal Parabolas

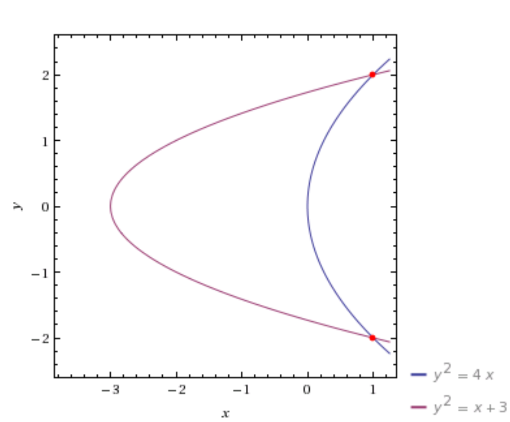
Graph of y^2 = 4x is of the form y^2 = 4 a x



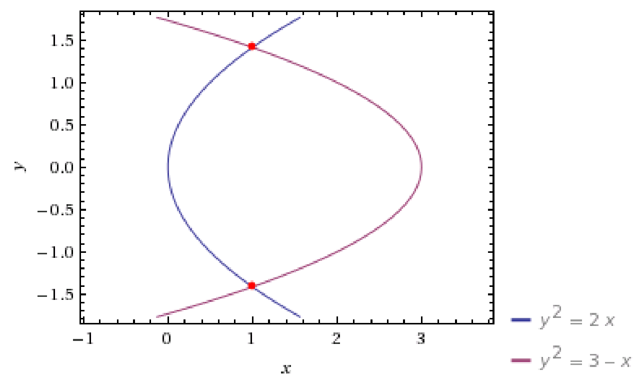
plot	$y^2 = 4x$
	$y^2 = x-6$



plot	$y^2 = 4x$ $y^2 = x + 3$



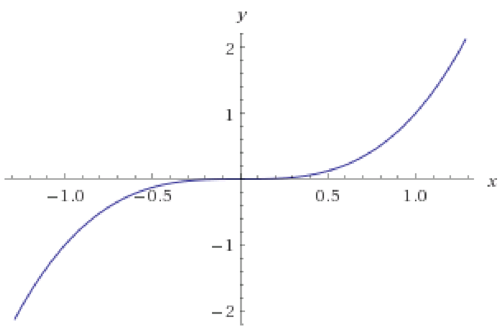
plot	$y^2 = 2x$ $y^2 = -x + 3$



Graphs of Cubic Equations (y = x cube) and higher powers of x

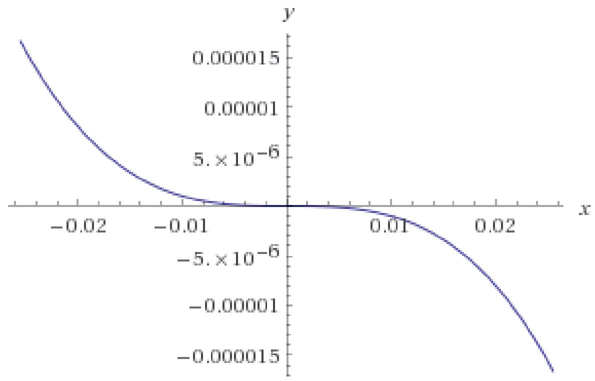
Graph of $y = x^3$ is

plot	$y = x^3$

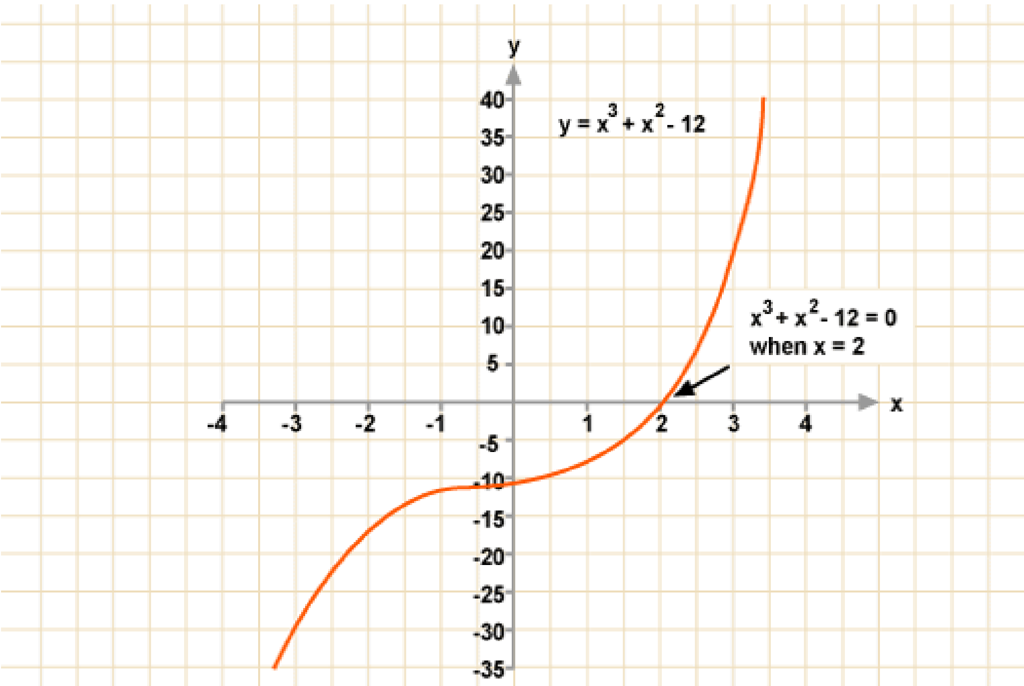


A good student can learn a lot by thinking how the graph of negative of the same function will look.

plot $y = -x^3$



The previous graph flipped around x-axis



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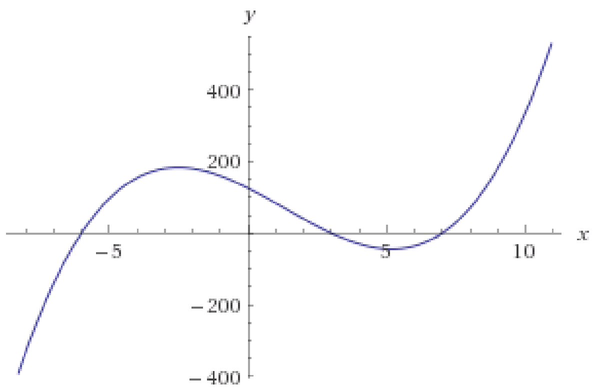
How will y = (x + 6)(x - 3)(x - 7) look like ? [x = -6, 3 and 7 will be roots. So the graph will pass through (-6, 0), (3,0) and (7,0)

plot	$y = (x+6)(x-3)(x-7)$

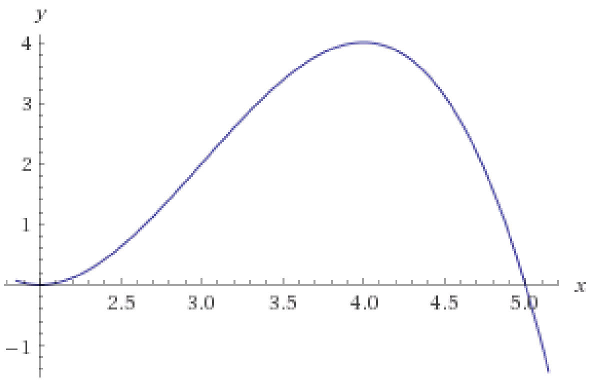


-

If coeff of x cube is negative then the graph will be downwards for increasing x. Also repeat roots can be there. Try to guess the graph of y = (5 - x)(2 - x)^2

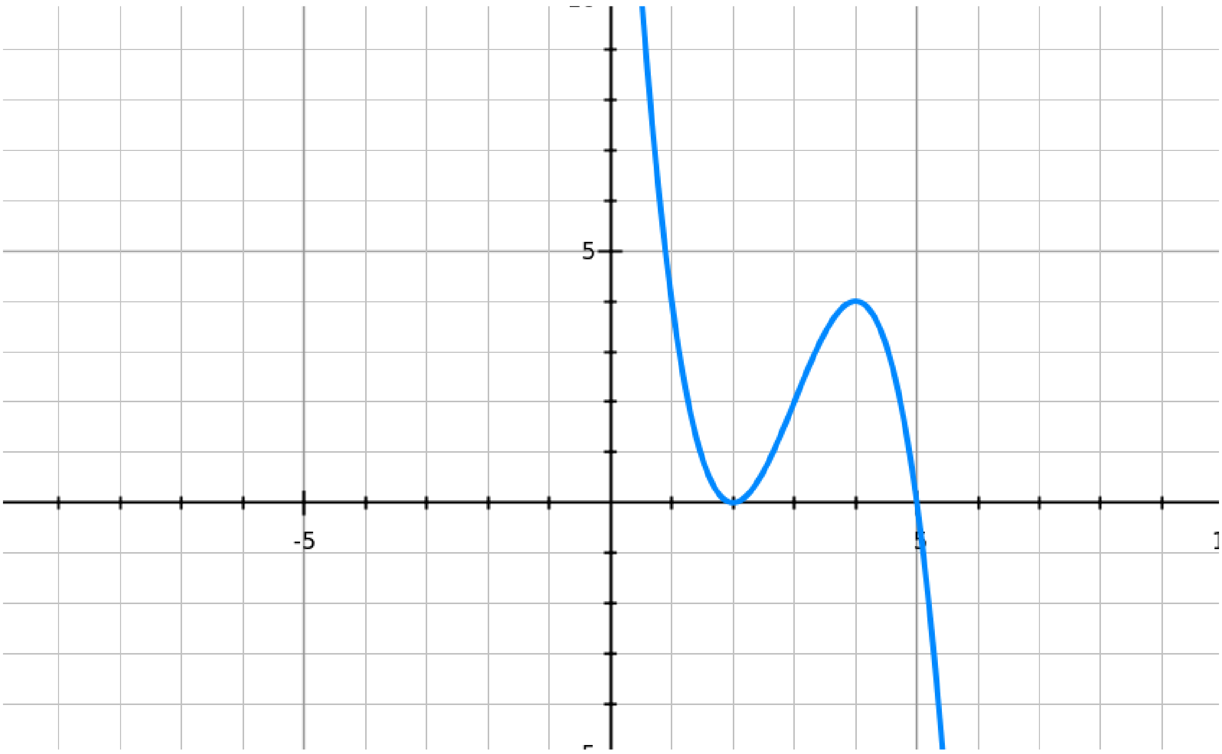
This will have roots at x = 5 and repeat roots (Two roots) at x = 2 so will touch x axis at x = 2

plot	$y = (5 - x)(2 - x)^2$



Because of distorted scale this graph is not a good one. The graph is correct but student must be mature to understand the distorted scale effects.

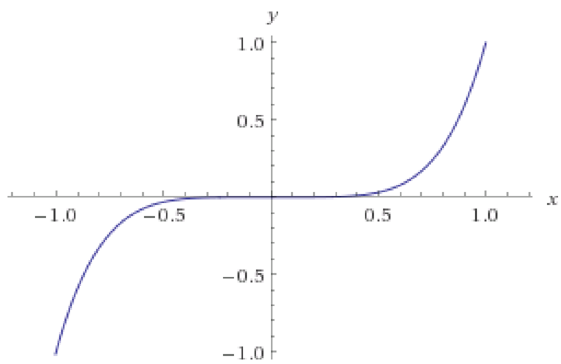
The graph below is a better one from a different plotter

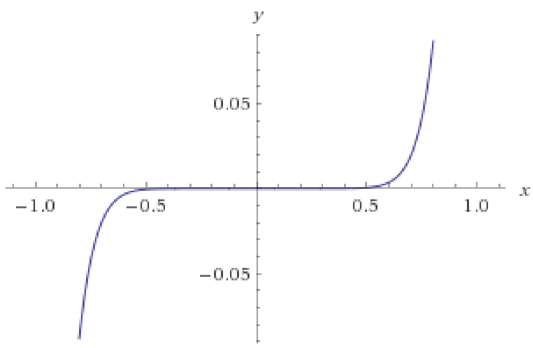


The graph of y = x^5 or say y = x^{11} will look very similar

plot	$y = x^5$

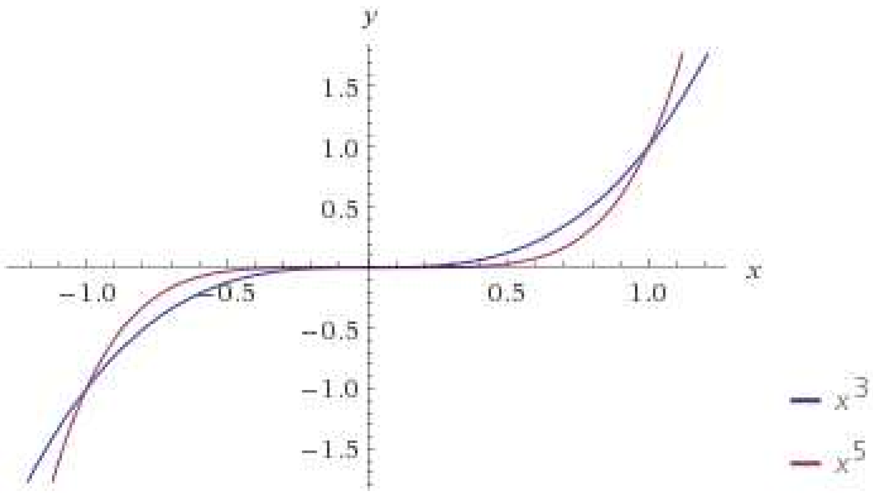
plot	$y = x^{11}$





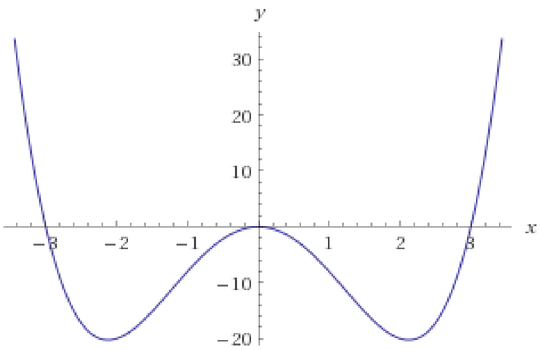
The difference is highlighted if the graphs are drawn together. All these graphs pass through (1, 1) and (-1, -1). While higher powered graph is flatter in between -1 to 1 and steeper after 1 or before -1

plot	$y = x^3$
	$y = x^5$



Graph of $y = x^2 (x + 3) (x - 3) = x^2 (x^2 - 9)$

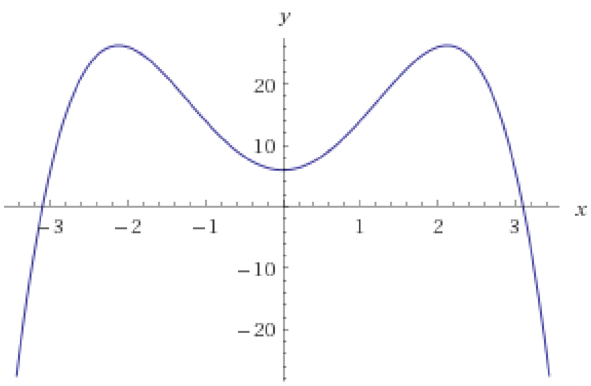
plot	$y = x^2 (x^2 - 9)$



X = 0 will be repeat root due to x square. Also x = 3 and x = -3 will be the roots

Graph of $y = - x^2 (x^2 - 9) + 6$

plot	$y = 6 - x^2 (x^2 - 9)$

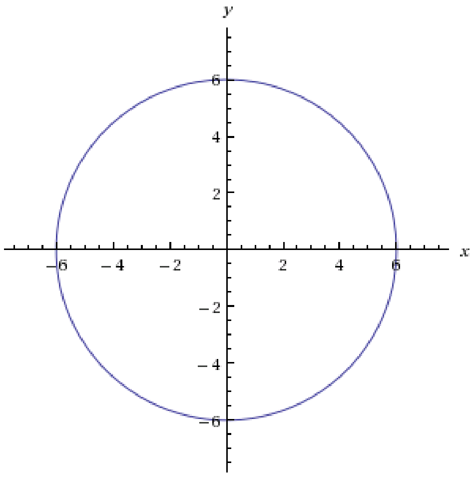


Now let us see graphs of Circles

Graph of $x^2 + y^2 = R^2$ will have the center at (0,0) and radius will be R

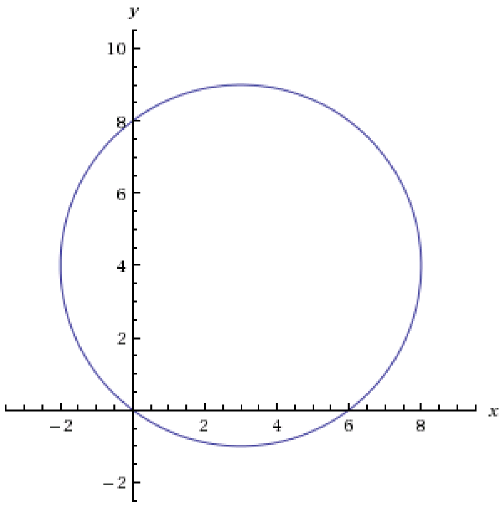
So graph of $x^2 + y^2 = 36$ is

plot	$x^2 + y^2 = 36$



Graph of (x - 3)^2 + (y - 4)^2 = 25 is

plot	$(x-3)^2 + (y-4)^2 = 25$



Center is at (3, 4)

Area problems, Graphs of Line, Circle, Triangle Areas discussed and explained at

https://archive.org/details/AreaDefiniteIntegralLineCircleModulusTriangleNatureAndType

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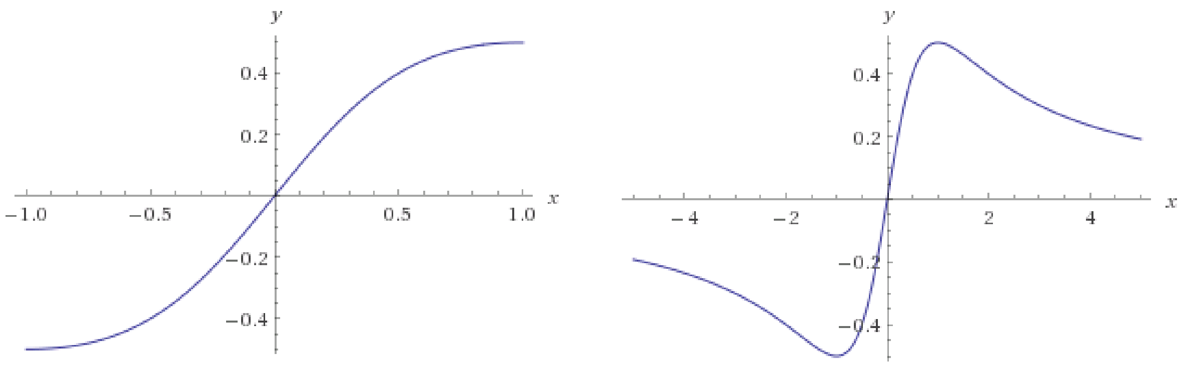
Some special graphs

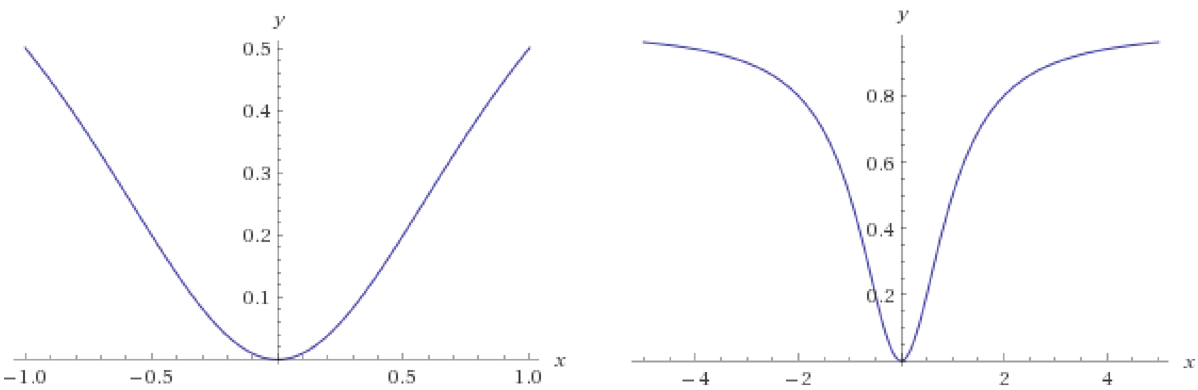
plot	$y = \frac{x}{x^2+1}$



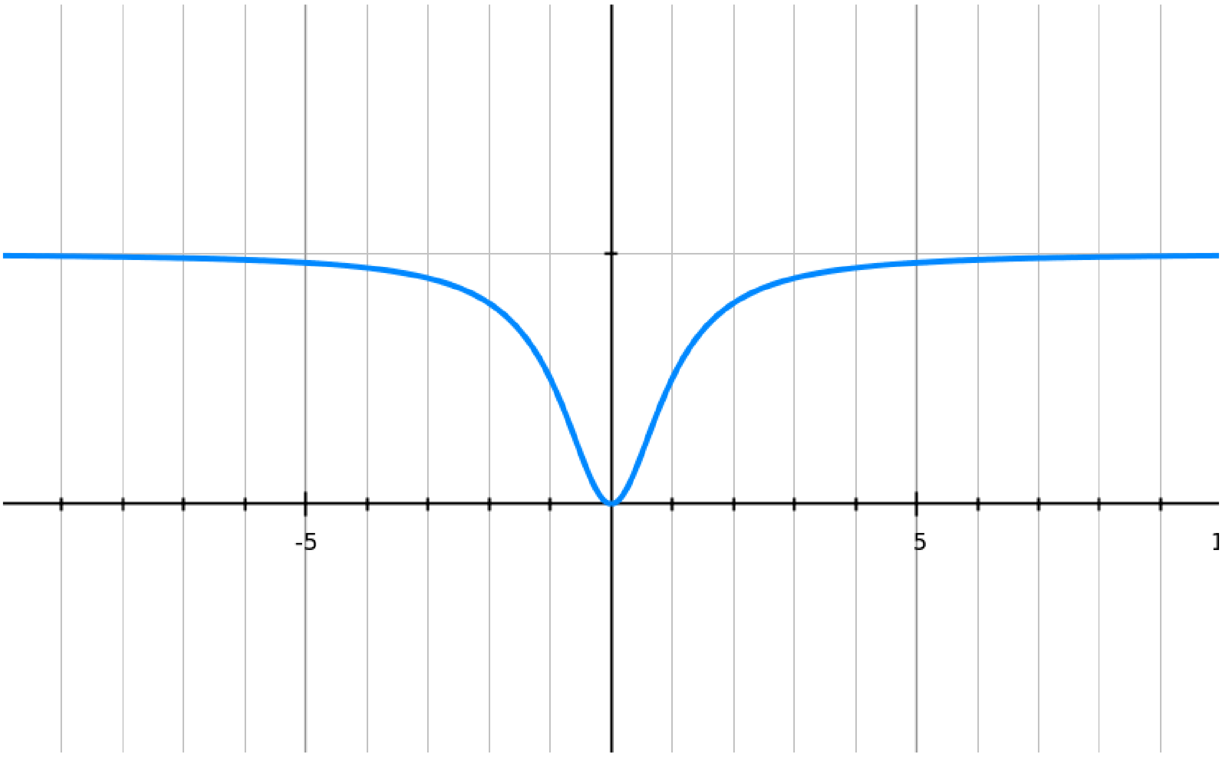
The graph become asymptotic to x-axis as we move towards right or left

The same will happen for $y = \frac{x^2}{x^2+1}$ though very slowly $\lim_{x \to \pm\infty} \frac{x^2}{1+x^2} = 1$

plot	$y = \frac{x^2}{x^2+1}$

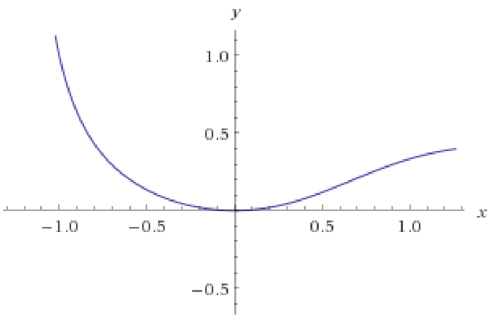


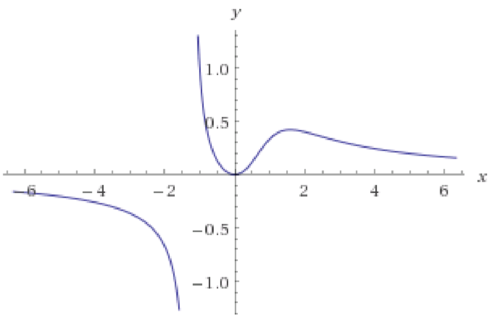
In this case the graph is asymptotic to 1 ($y = \frac{x^2}{x^2+1}$)



Can you guess what will happen in case of $y = \frac{x^2}{x^3+2}$? Did you notice the discontinuity around negative cuberoot of 2

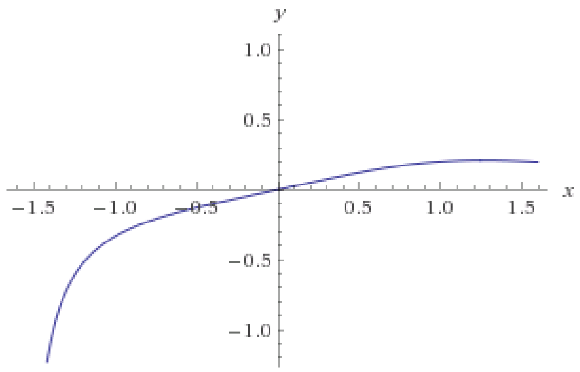
plot	$y = \frac{x^2}{x^3+2}$

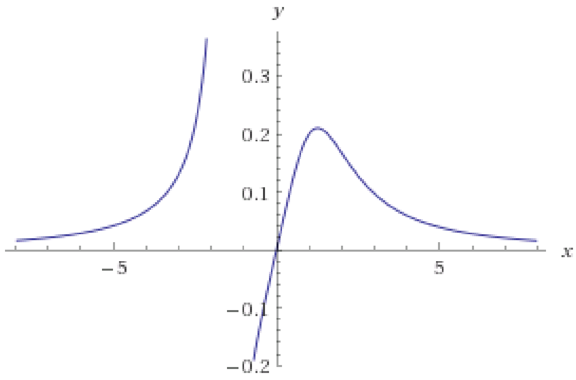




Can you guess what will happen in case of $y = \dfrac{x}{x^3 + 4}$? Understand the discontinuity around negative cuberoot of 4

plot	$y = \dfrac{x}{x^3+4}$





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Find all asymptotes and sketch the function

$$f(x) = \frac{x^3 + 5}{x^2 + 3x + 1}$$

$x^2 + 3x + 1 = 0$

$$x = \frac{-3 \pm \sqrt{5}}{2} \quad \text{(2 vertical asymptotes)}$$

$$y = \frac{(x^3/x^3) + (5/x^3)}{(x^2/x^3) + (3x/x^3) + (1/x^3)} = \text{undefined (no horizontal asymptotes)}$$

$x - 3 + ((8x + 8)/(x^2 + 3x + 1))$

$x^2 + 3x + 1$ / $x^3 + 0x^2 + 0x + 5$

$x^3 + 3x^2 + x$

$-3x^2 - x + 5$

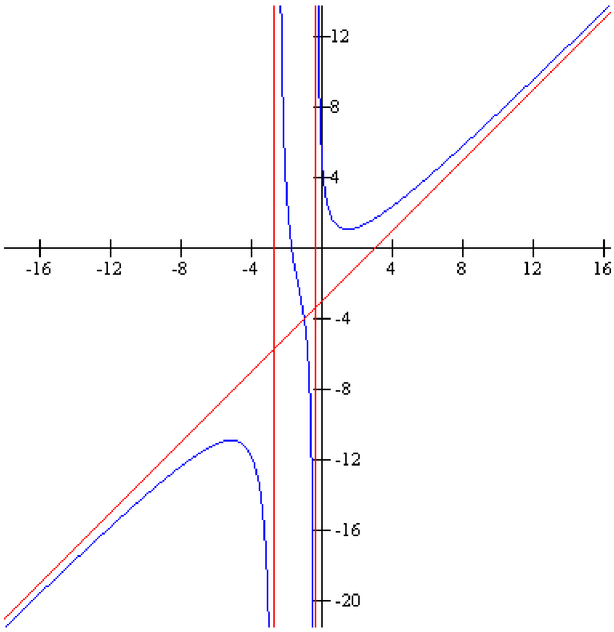
$-3x^2 - 9x - 3$

$8x + 8$

$$y = x - 3 + \frac{8x/x^2 + 8/x^2}{x^2/x^2 + 3x/x^2 + 1/x^2}$$

$= x - 3 + 0$

$= x - 3$ (one oblique asymptote)



Find all asymptotes and sketch the function

$$g(x) = \frac{x^2}{x - 3}$$

$x - 3 = 0$

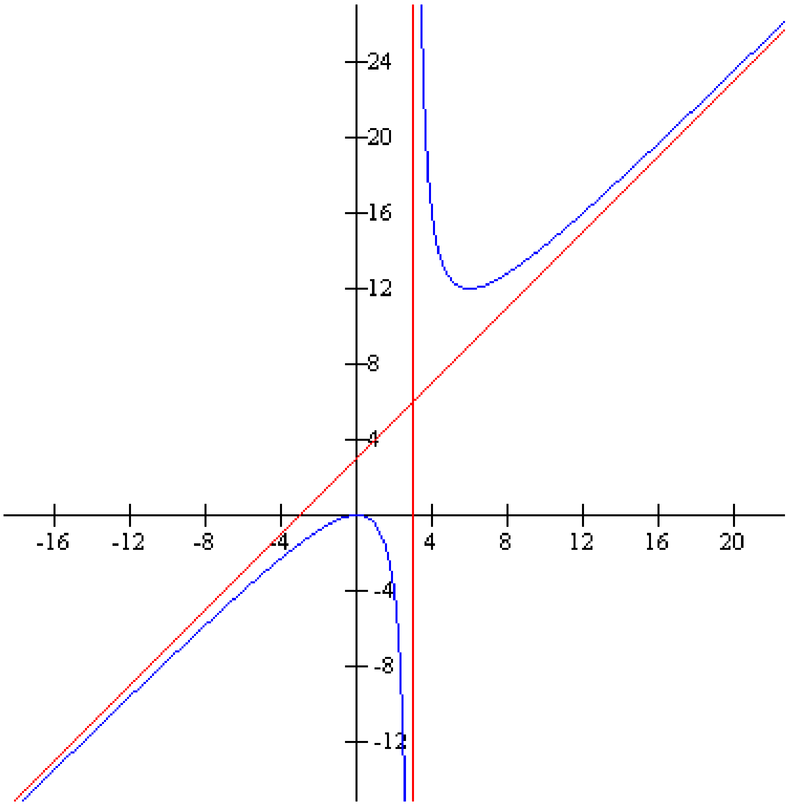
$x = 3$ (one vertical asymptote)

$$y = \frac{x^2/x^2}{x/x^2 - 3/x^2} = \text{undefined (no horizontal asymptotes)}$$

$$\begin{array}{l} \quad x + ((3x)/(x - 3)) \\ \quad \overline{} \\ x - 3 \;/\; x^2 + 0x + 0 \\ \quad x^2 - 3x \\ \quad \overline{} \\ \qquad 3x \end{array}$$

$$y = x + \frac{3x/x}{} = x + 3 \text{ (one oblique asymptote)}$$

x/x - 3/x



Find all asymptotes and sketch the function

$$y = \frac{x^3 - 4x^2 - 49x - 90}{2x^2 + 12x + 18}$$

$2x^2 + 12x + 18 = 2(x^2 + 6x + 9) = 0$
x = -3 (one vertical asymptote)

$$y = \frac{x^3/x^3 - 4x^2/x^3 - 49x/x^3 - 90/x^3}{2x^2/x^3 + 12x/x^3 + 18/x^3} = \text{undefined (no horizontal asymptotes)}$$

$0.5x - 5 + ((2x)/(2x^2 + 12x + 18))$

$2x^2 + 12x + 18 \ / \ x^3 - 4x^2 - 49x - 90$
$x^3 + 6x^2 + 9x$

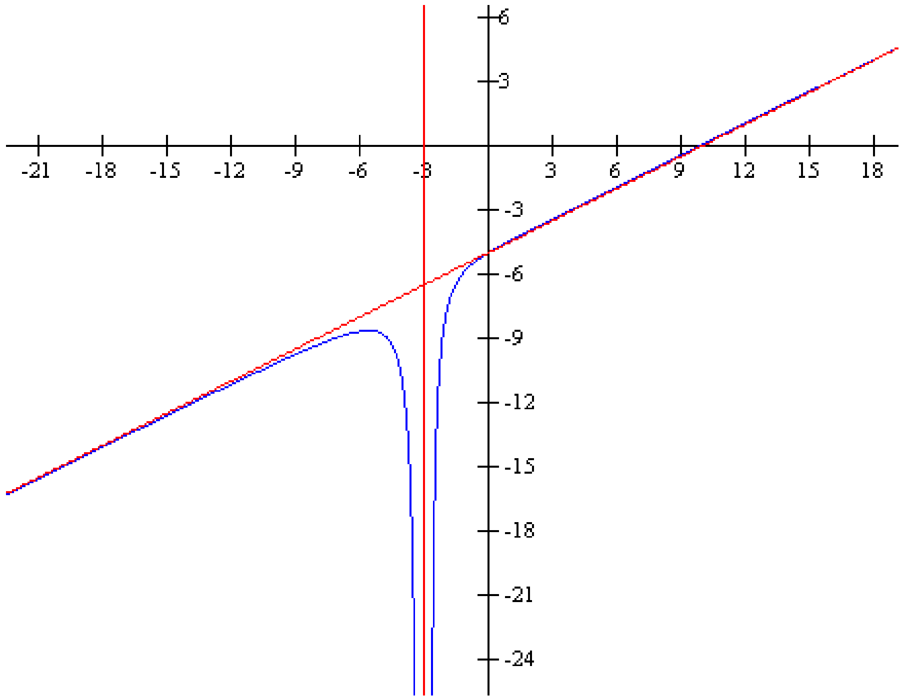
$-10x^2 - 58x - 90$
$-10x^2 - 60x - 90$

$$\overline{\quad 2x \quad}$$

$$y = 0.5x - 5 + \frac{2x/^2}{2x^2/x^2 + 12x/x^2 + 18/x^2}$$

$= 0.5x - 5 + 0$

$= 0.5x - 5$ (one oblique asymptote)



Find all asymptotes and sketch the function

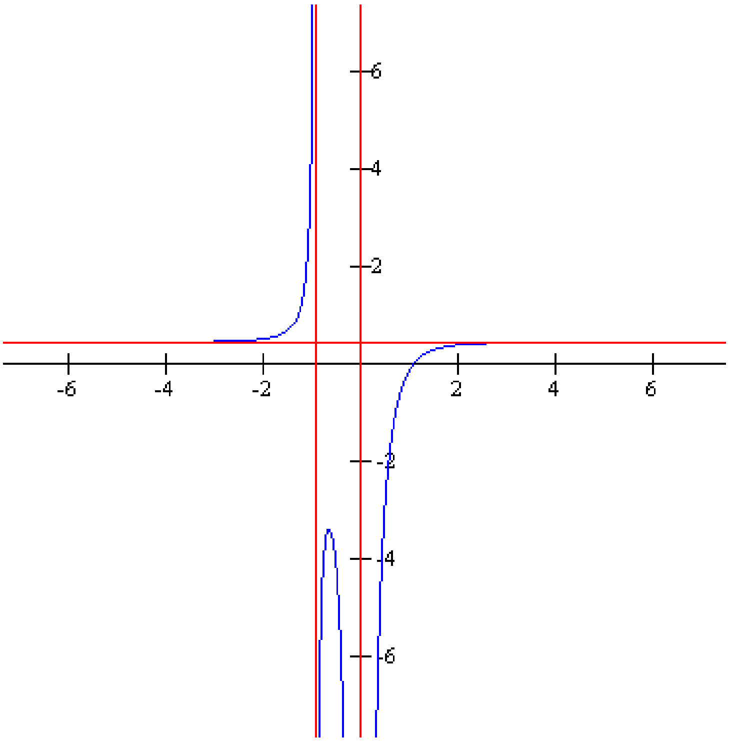
$$h(x) = \frac{4x^5 - 6}{9x^5 + 7x^2}$$

$9x^5 + 7x^2 = x^2(9x^3 + 7) = 0$

$x = (-7/9)^{1/3}$ or $x = 0$ (two vertical asymptotes)

$$y = \frac{4x^5/x^5 - 6/x^5}{9x^5/x^5 + 7x^2/x^5} = 4/9 \text{ (one horizontal asymptote)}$$

There are no oblique asymptotes, as the degree of the numerator is not one greater than the degree of the denominator



Find all asymptotes and sketch the function

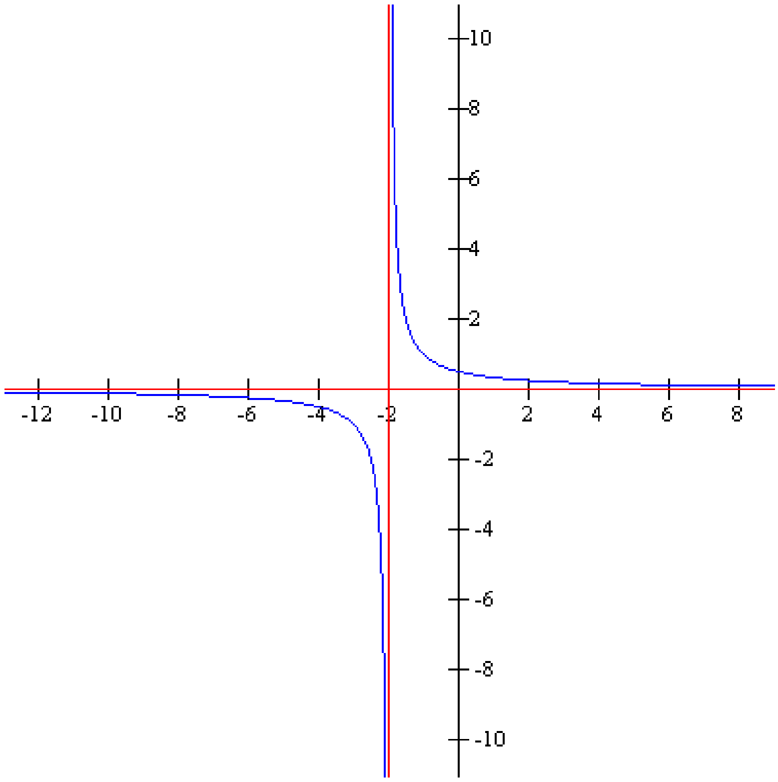
$$y = \frac{x^4 - 3x^3 + 5x^2 - 7x + 9}{x^5 - x^4 - x^3 + 3x^2 - 5x + 18}$$

First, reduce the equation to y = 1/(x + 2)

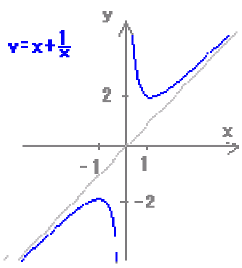
x + 2 = 0
x = -2 (one vertical asymptote)

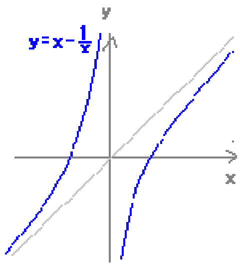
$$y = \frac{1/x}{x/x + 2/x} = 0$$ (one horizontal asymptote)

There are no oblique asymptotes, as the degree of the numerator is not one greater than the degree of the denominator



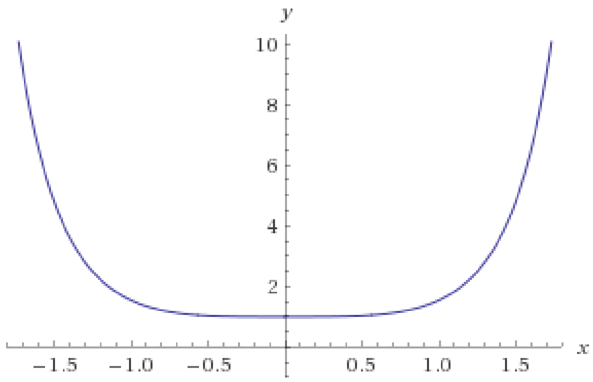
Graphs of y = x + 1/x   and   y = x - 1/x





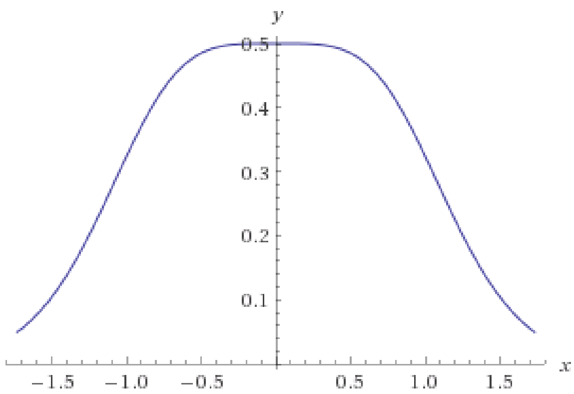
Graph of $y = \frac{1}{2}\left(e^{x^2} + e^{-x^2}\right)$

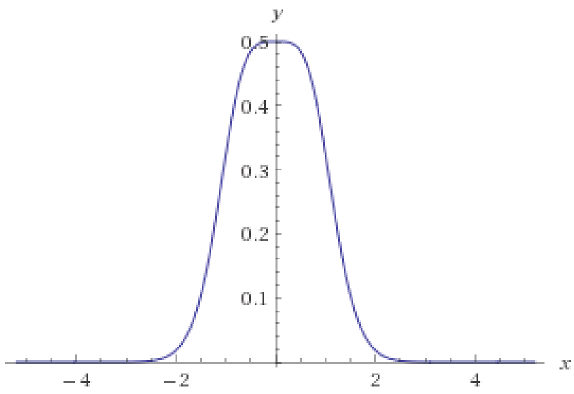
plot	$y = \frac{1}{2}\left(e^{x^2} + e^{-x^2}\right)$



So graph of $y = \dfrac{1}{e^{x^2} + e^{-x^2}}$

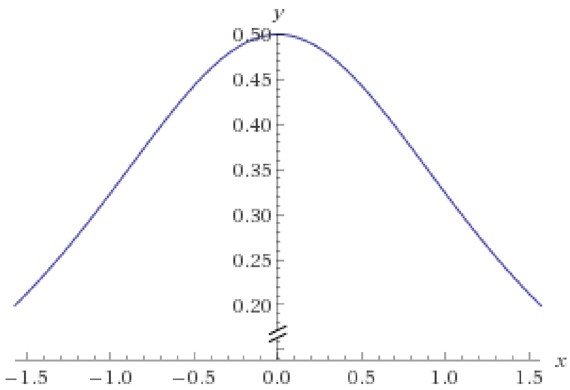
plot	$y = \dfrac{1}{e^{x^2} + e^{-x^2}}$

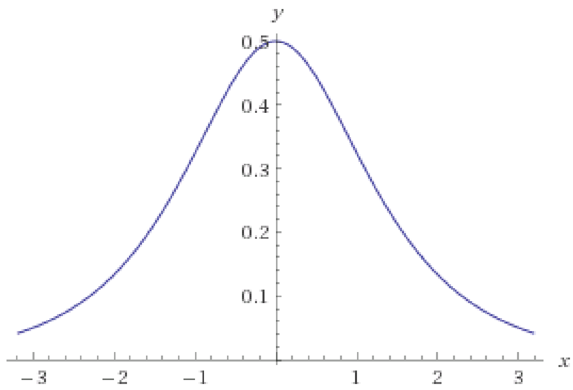


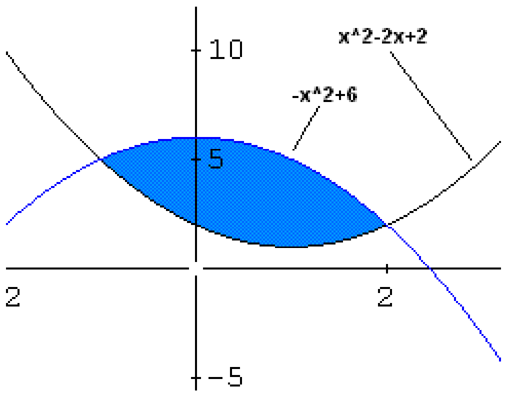


Spoon Feeding graph of $y = \dfrac{1}{e^x + e^{-x}}$

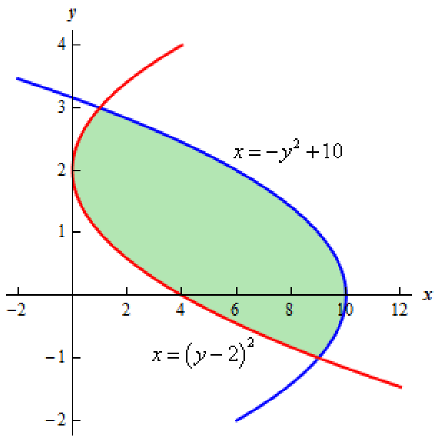
plot	$y = \dfrac{1}{e^x + e^{-x}}$



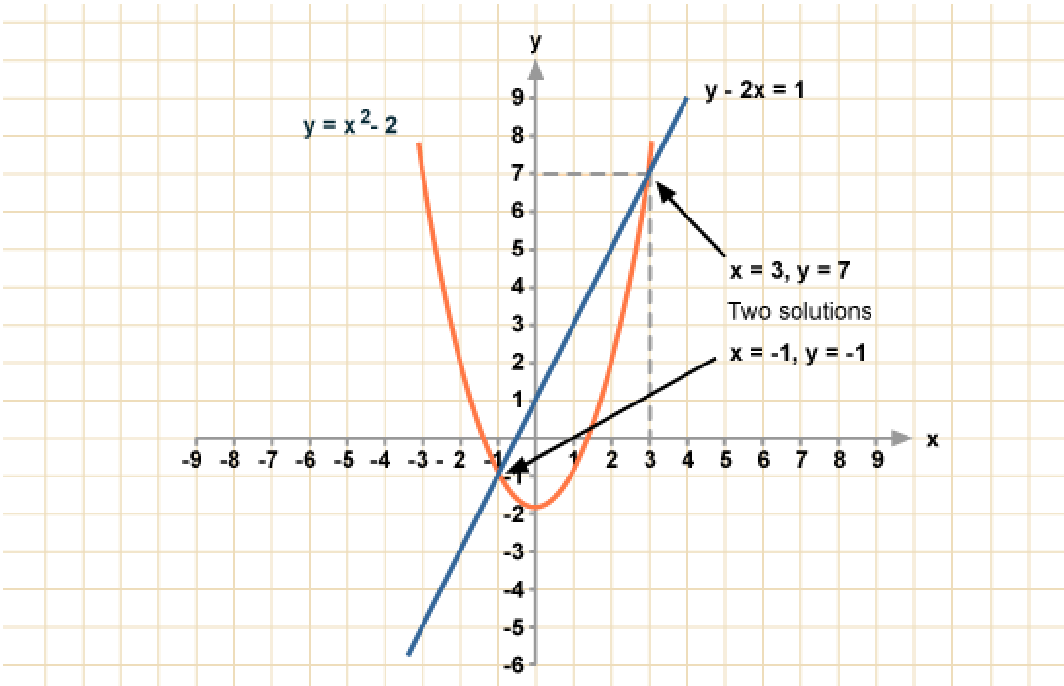




If area enclosed between two curves is needed; then the upper curve function minus the lower curve function needs to be integrated, between the two intersection points as limits.



We generally get questions with line intersecting a parabola kind …



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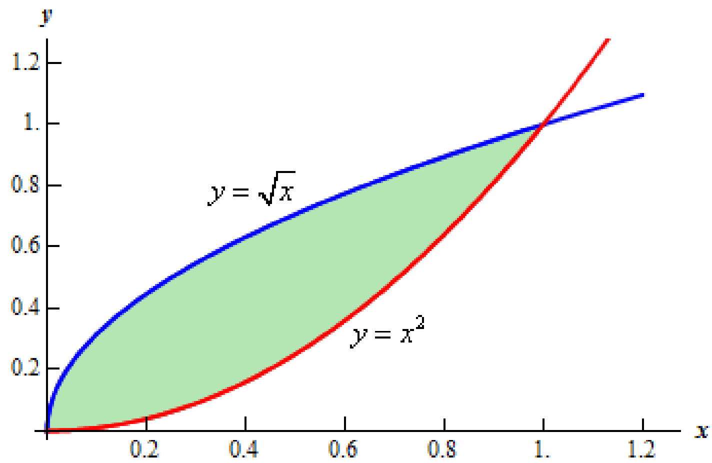
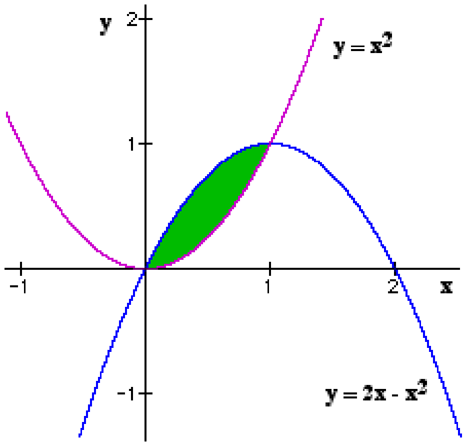


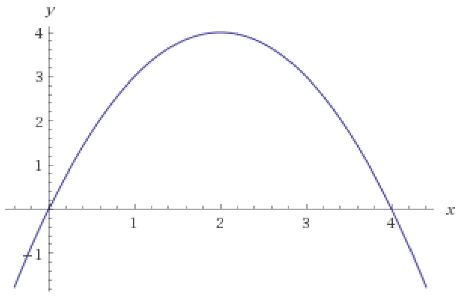
-

Draw the graphs of $y = x^2$ and $y = 2x - x^2$



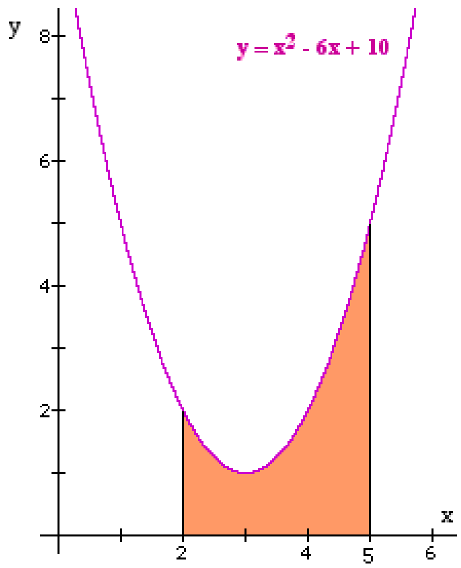
Draw the graph of the parabola y = $4x - x^2$

plot	$y = 4x - x^2$



-

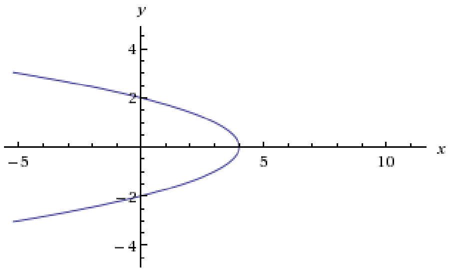
Draw the graph of f(x) = x^2-6x + 10 , the lines x = 2 and x = 5 and the x-axis



-

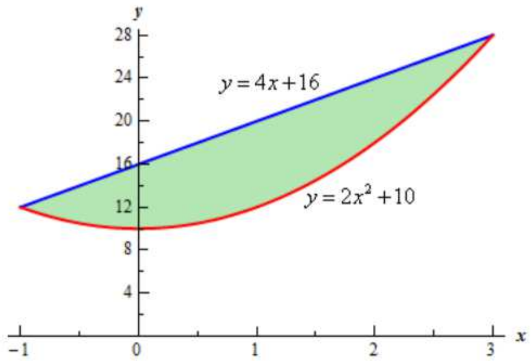
Draw the graph of $x = 4 - y^2$ => $y^2 = 4 - x$

plot	$y^2 = 4 - x$



-

Draw the graph of $y = 4x + 16$ and $y = 2x^2 + 10$

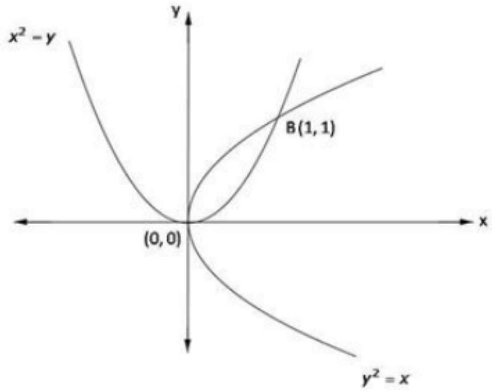


Solving these two given equations we get the intersection points as x = -1 and x = 3 (Quadratic equation $2x^2 + 10 = 4x + 16$ => $2x^2 - 4x - 6 = 0$

- ⇨ $x^2 - 2x - 3 = 0$ Factorize and you get x = -1 and x = 3)

-

Draw the graphs of $x = y^2$ and $y = x^2$

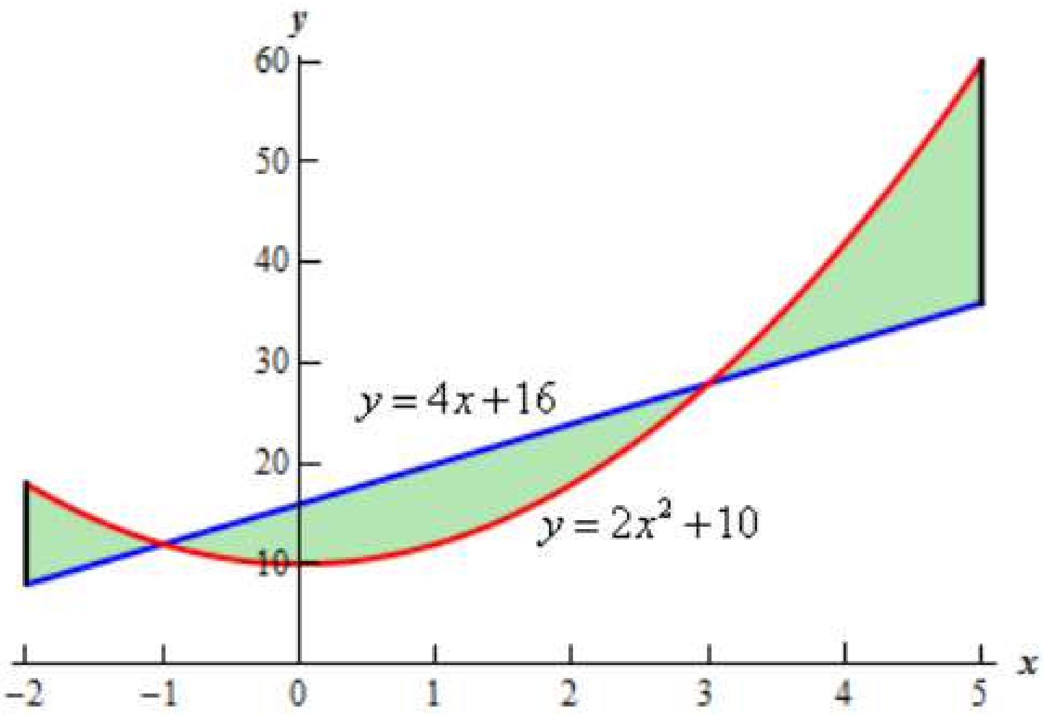


We can easily solve to see that the graphs intersect at (1, 1)

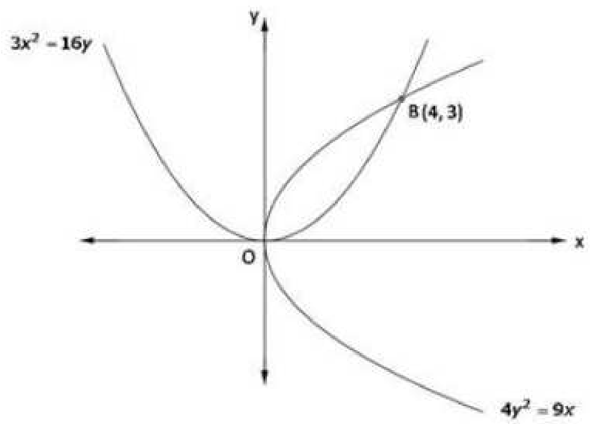
-

Draw the graphs of $y = 2x^2 + 10$, $y = 4x + 16$, $x = -2$, and $x = 5$

The regions in the graph needs to be plotted

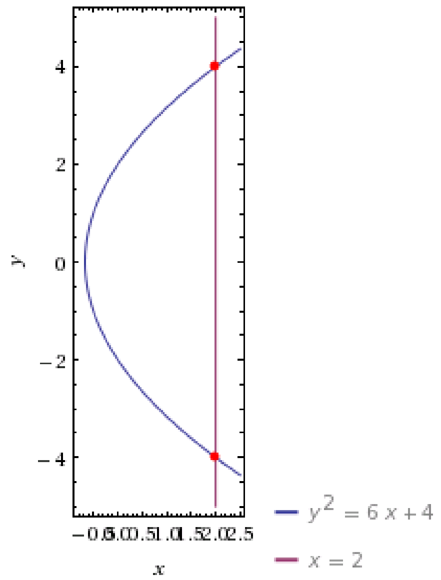


Draw the fraphs of $4y^2 = 9x$ and $3x^2 = 16y$

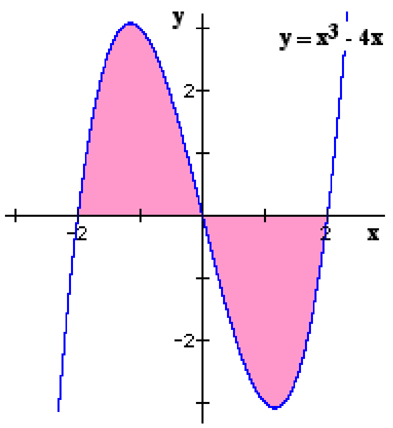


Draw the graph of $y^2 = 6x + 4$

plot	$y^2 = 6x + 4$
	$x = 2$



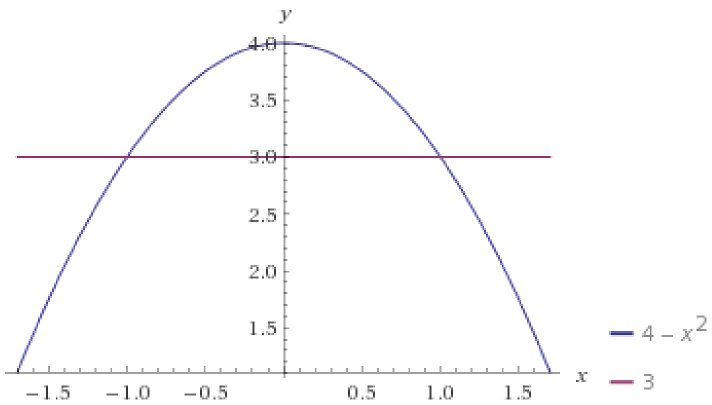
Draw the graph of $y = x^3 - 4x$



-

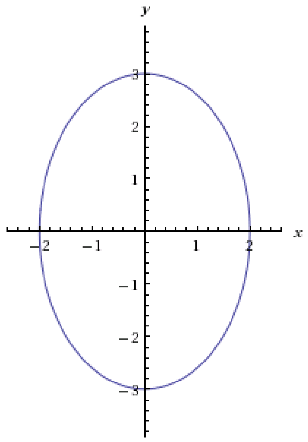
Graph of y = 4 - x^2

plot	$y = 4 - x^2$
	$y = 3$



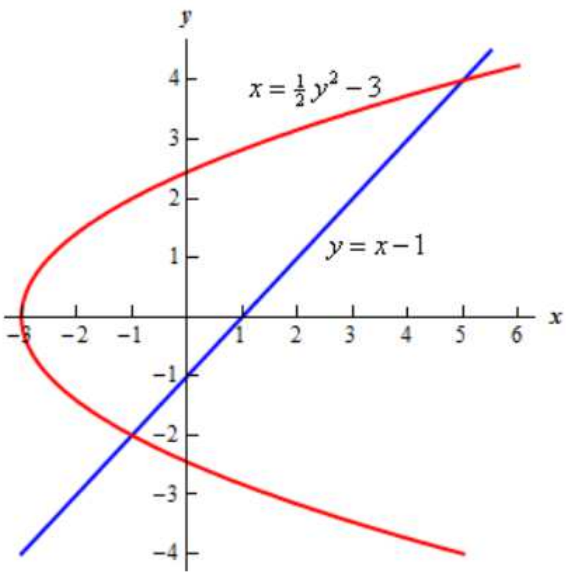
We need to know graphs of ellipse and problems related to those

plot	$\frac{x^2}{4} + \frac{y^2}{9} = 1$



-

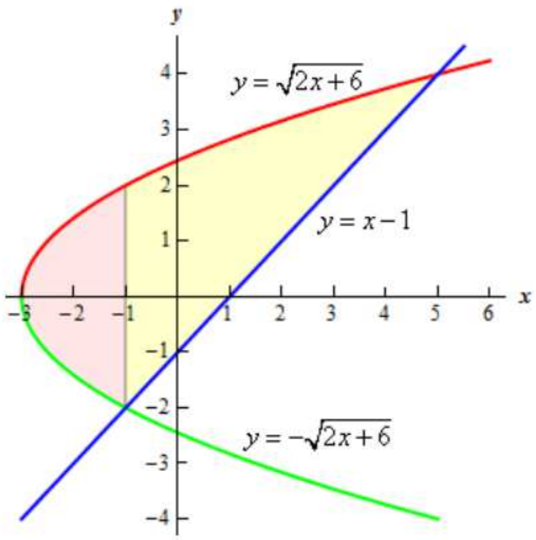
Graphs of $x = \frac{1}{2} y^2 - 3$ and y = x - 1



The line and the parabola intersect at y = -2 => x = y + 1 = -1 so (-1, -2)

and y = 4 => x = y + 1 = 5 so (5, 4)

the function becomes $y = \pm\sqrt{2x+6}$



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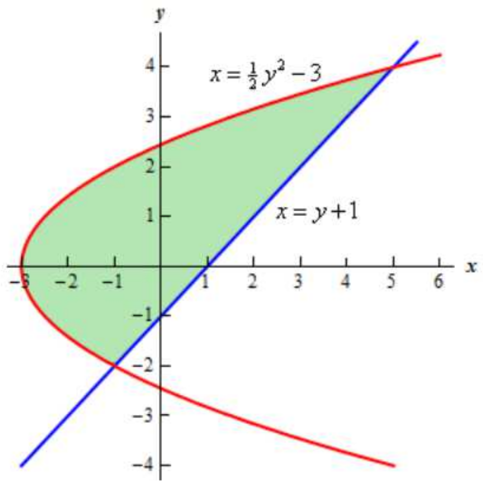
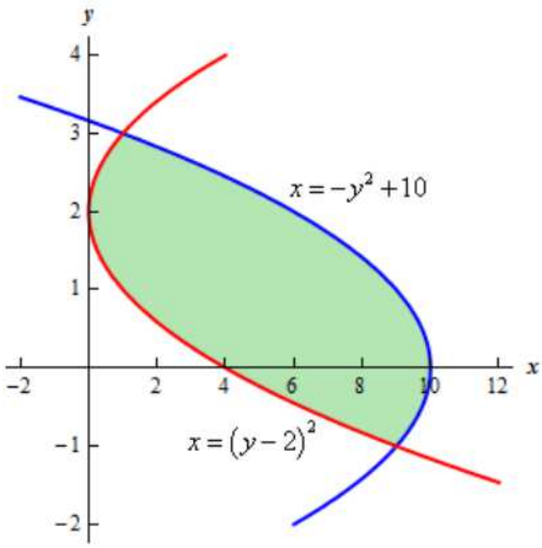


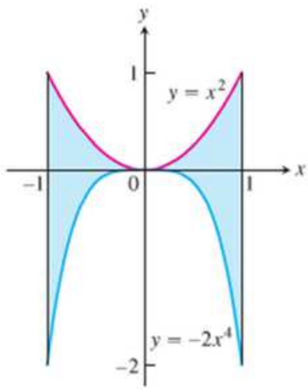
Graph of $x=-y^2+10$ and $x=(y-2)^2$.

The intersection points are y = -1 and y = 3

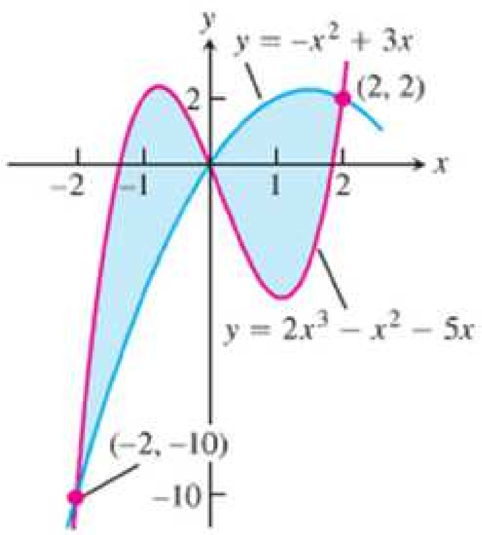


-

See the graphs



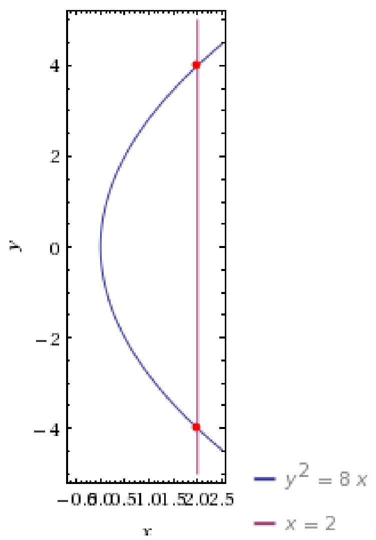
See the graphs



Spoon Feed

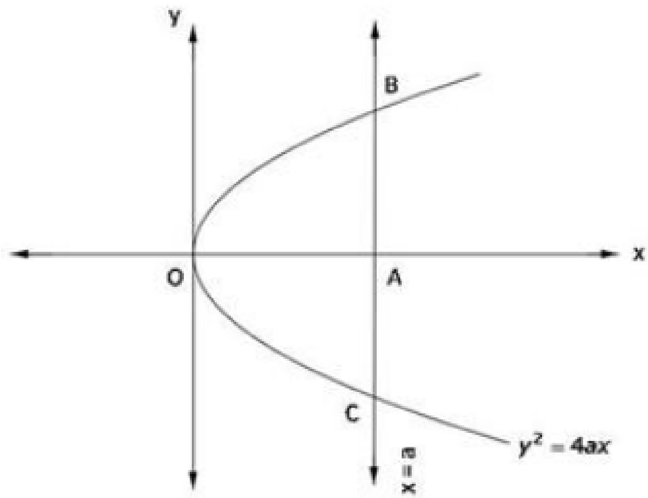
Draw the graphs of x = 2 and y^2 = 8x

plot	$y^2 = 8\,x$
	$x = 2$



-

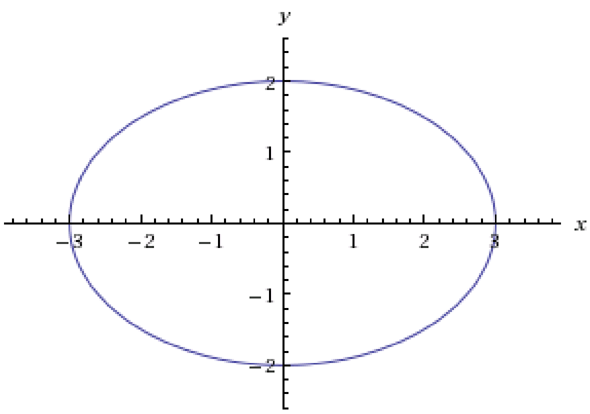
Graph of general $x = a$ and $y^2 = 4ax$



-

Graph of ellipse $4x^2 + 9y^2 = 36$

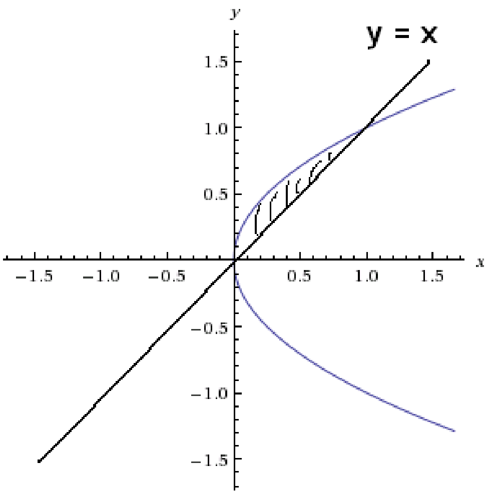
plot	$4x^2 + 9y^2 = 36$



-

Graphs of y^2 = x and the line y = x

plot	$y^2 = x$



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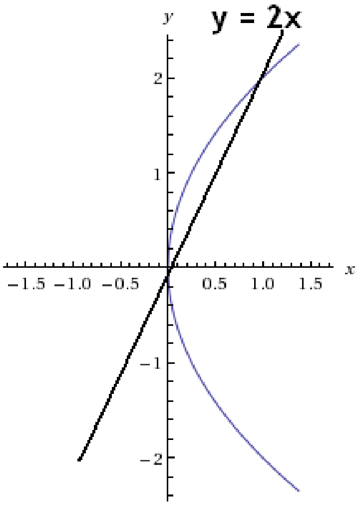
Spoon Feeding Graph of y^2 = 4x and y = 2x

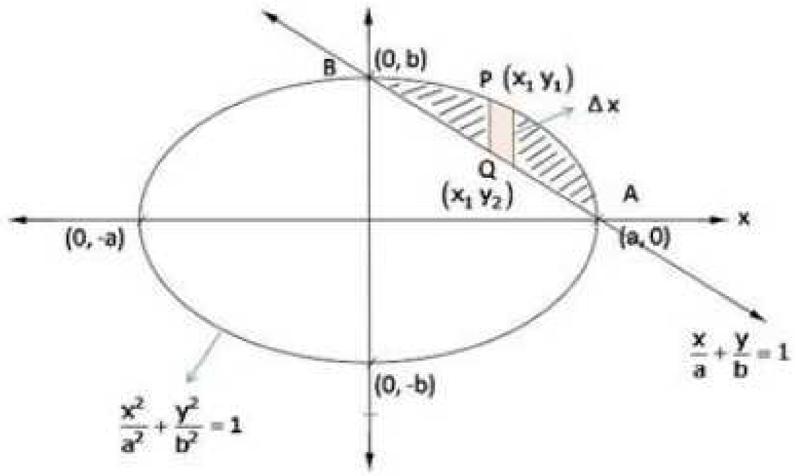
plot	$y^2 = 4x$



We see the intersection point is (1,2).

Graphs of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$

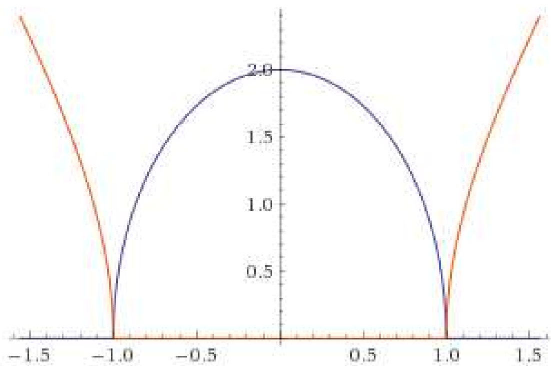
Assuming a > b



-

Graph of curve $y = 2\sqrt{1 - x^2}$

plot	$y = 2\sqrt{1 - x^2}$



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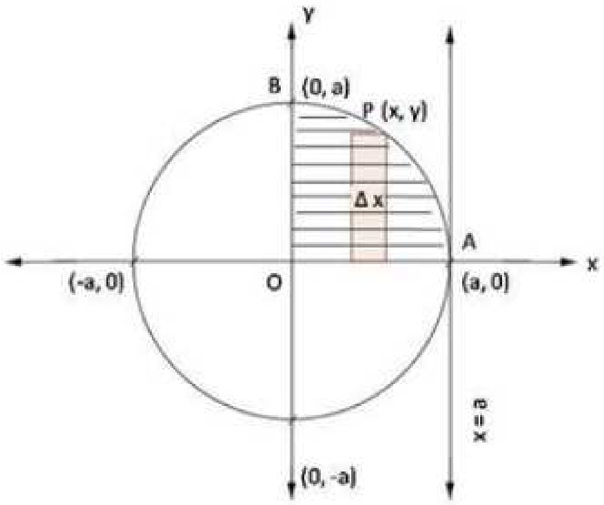
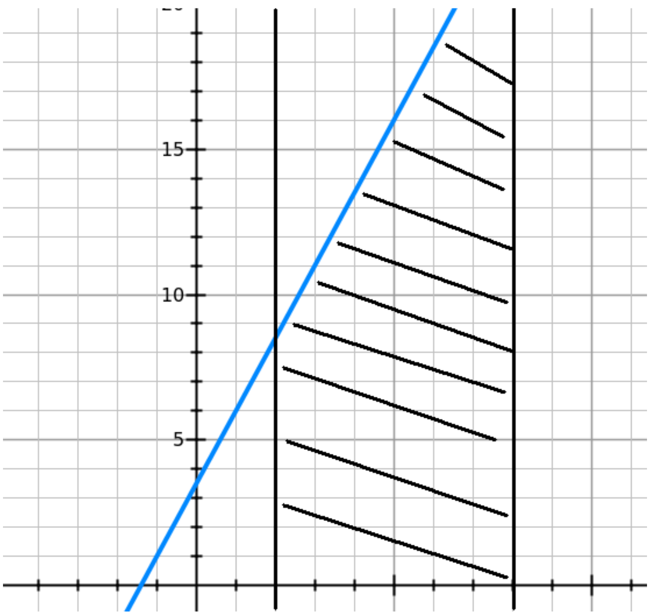
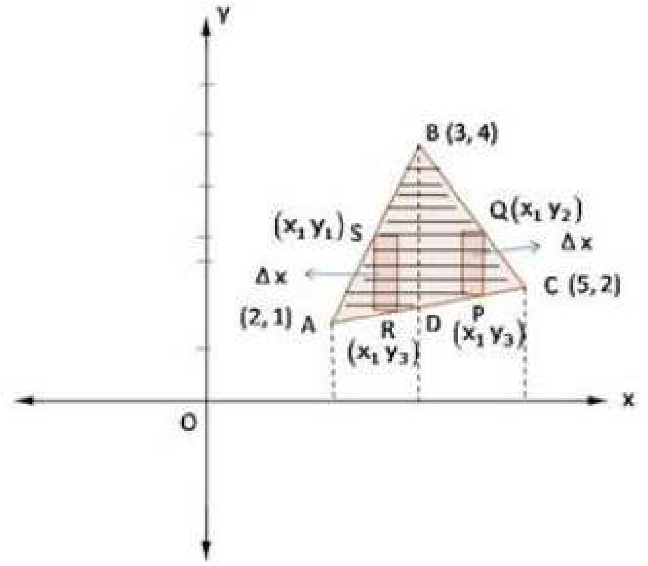
Graph of Circle. The equation of the circle will be $x^2 + y^2 = a^2$ so $y = \sqrt{a^2 - x^2}$



Graphs of 2y = 5x + 7, x = 2, and x = 8

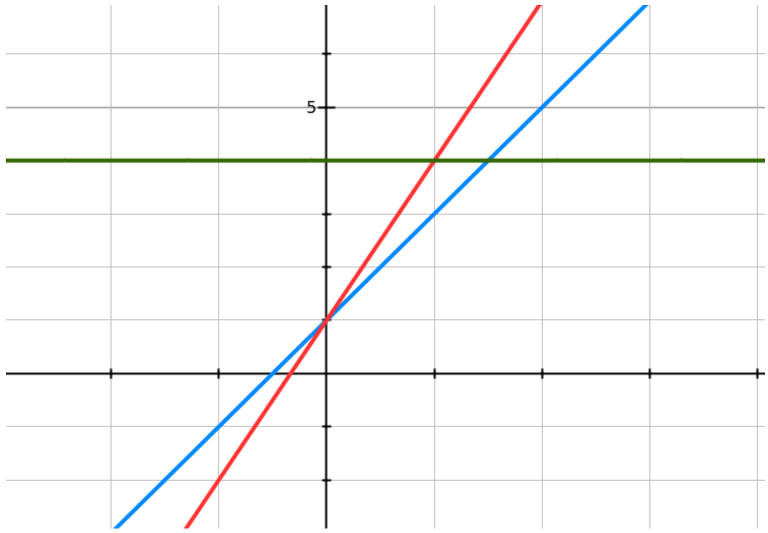


Draw the triangle. The vertices being A (2,1), B (3,4), C (5, 2)



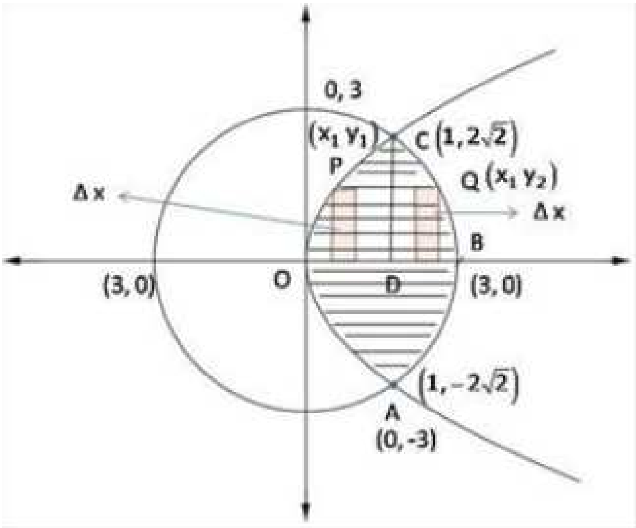
-

Draw the graphs y = 2x + 1 (line A), y = 3x + 1 (line B), y = 4 (line AC)



-

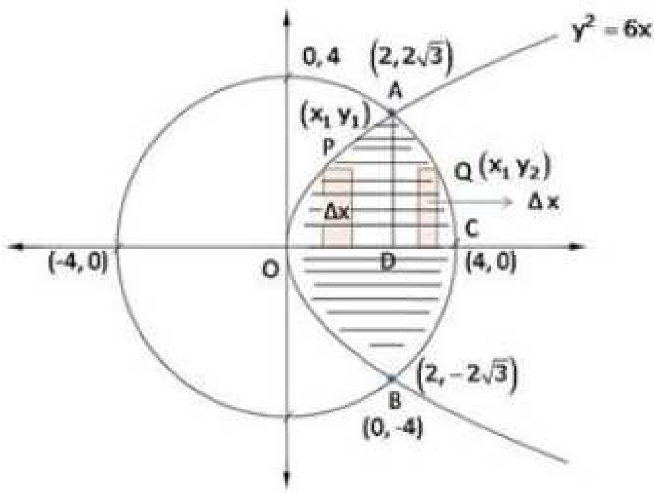
Draw graphs of $y^2 \le 8x$, $x^2 + y^2 \le 9$



-

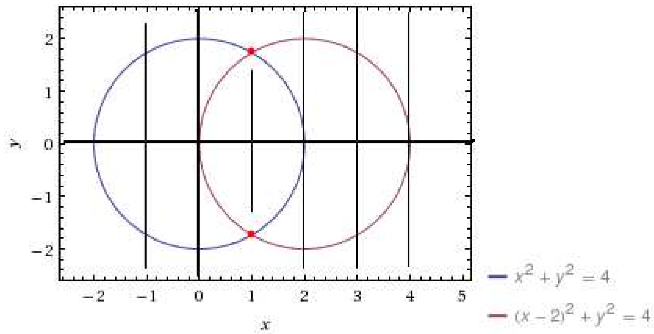
Graphs of $x^2 + y^2 = 16$, and $y^2 = 6x$

The graph will be



Graphs of $x^2 + y^2 = 4$, and $(x - 2)^2 + y^2 = 4$

plot	$x^2 + y^2 = 4$
	$(x-2)^2 + y^2 = 4$



Equation (1) is a circle with centre O at eh origin and radius 2. Equation (2) is a circle with centre C (2,0) and radius 2. Solving equations (1) and (2), we have

$$(x - 2)^2 + y^2 = x^2 + y^2$$

Or $\quad x^2 - 4x + 4 + y^2 = x^2 + y^2$

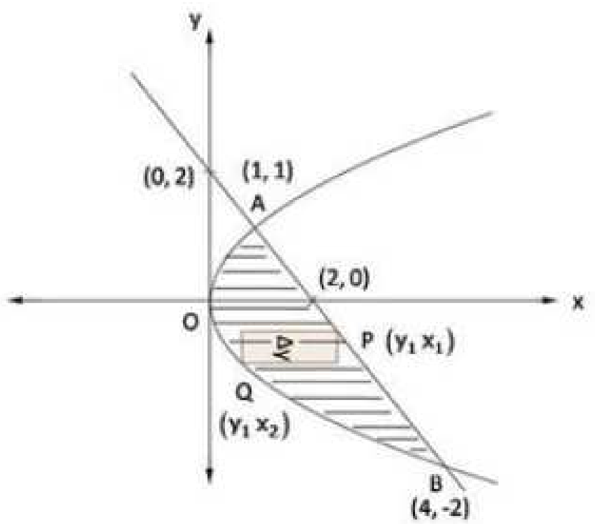
Or $\quad x = 1$ which gives y $\pm\sqrt{3}$

Thus, the points of intersection of the given circles are A $\left(1, \sqrt{3}\right)$ and A' $\left(1, -\sqrt{3}\right)$

Graphs of $y^2 = x$ and $x + y = 2$

Equation (1) represents a parabola with vertex at origin and its axis as x-axis, equation (2) represents a line passing through (2,0) and (0,2). points of intersection of line and parabola are (1,1) and (4,-2).

A rough sketch of curves is as below:-

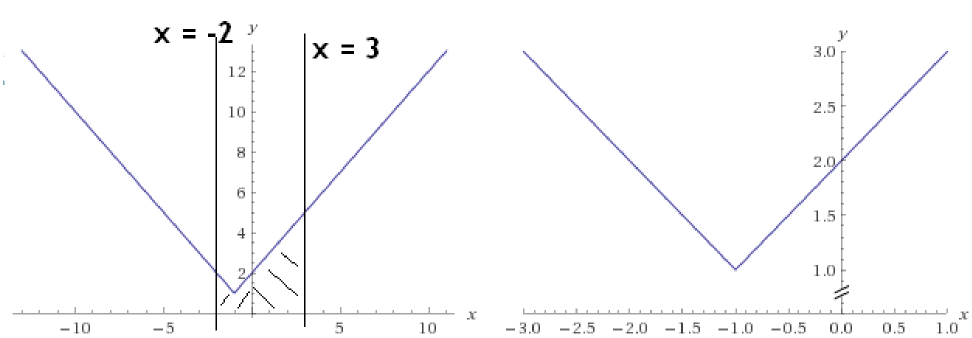


-

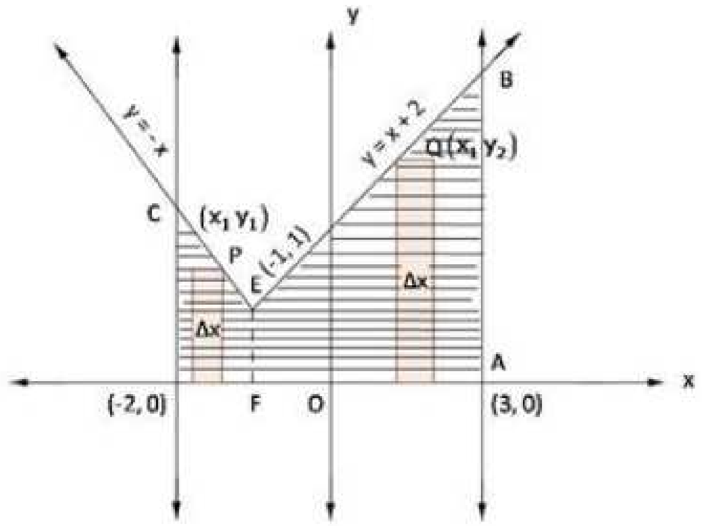
Graphs of x = -2, x = 3, x-axis (y = 0), and y = 1 + | x + 1 |

The straight lines for the mod function will flip around x = -1

plot	$y = 1 + \lvert x + 1 \rvert$



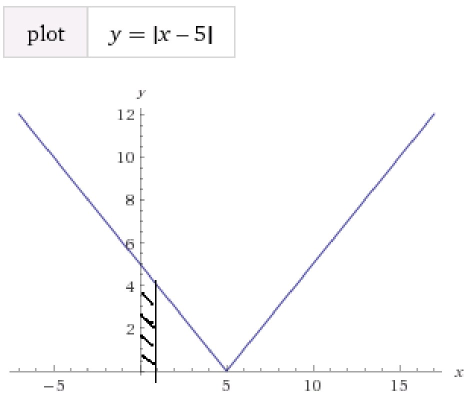
So, equation (1) is a straight line that passes through (0,2) and (−1,1). Equation (2) is a line passing through (−1,1) and (−2,2) and it is enclosed by line $x = 2$ and $x = 3$ which are lines parallel to y-axis and pass through (2,0) and (3,0) respectively $y = 0$ is x-axis. So, a rough sketch of the curves is given as:-



Shaded region represents the required area.

Draw 0 < x < 1 for y = | x - 5 |

The graph of the modulus function will flip around x = 5



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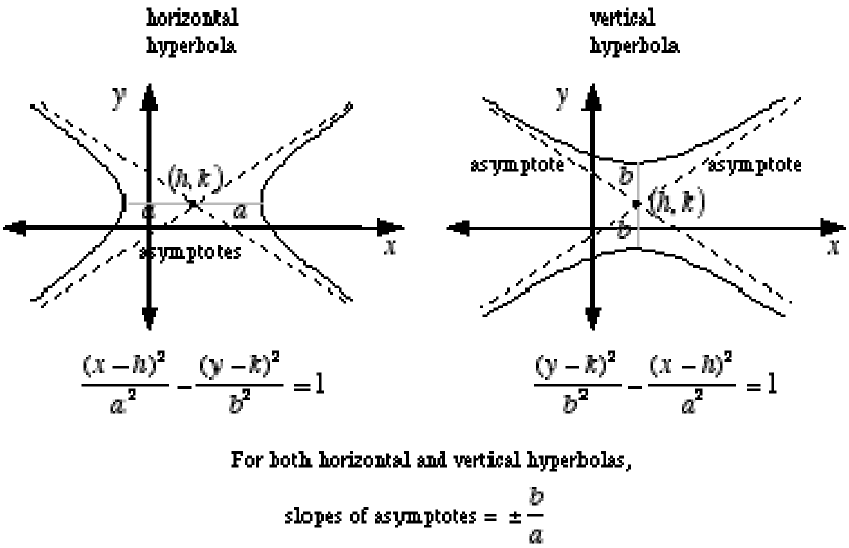
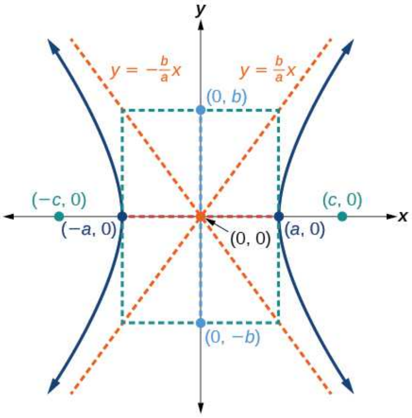
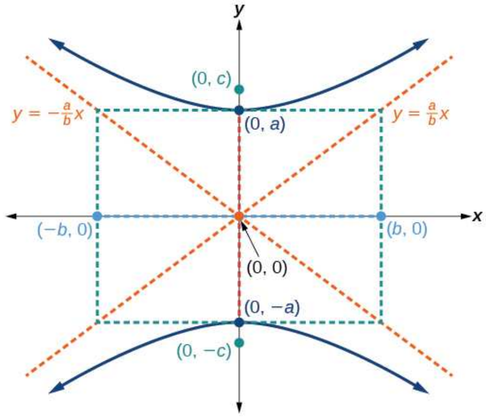
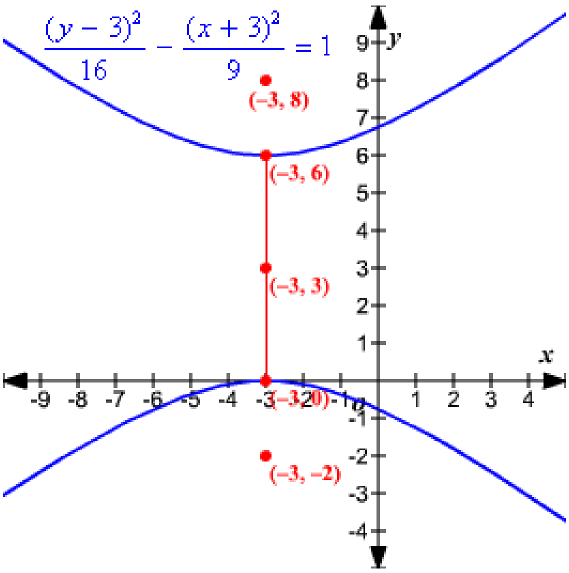
Graphs of Hyperbolas

.	x −axis	y −axis
Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$-\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$
Center	$C(h, k)$	$C(h, k)$
Semi − transverse axis	a	a
Semi − conjugate axis	b	b
Vertices	$V(h \pm a, k)$	$V(h, k \pm a)$
Foci	$F(h \pm a\,e, k)$	$F(h, k \pm a\,e)$
Directrices	$x = h \pm a / e$	$y = k \pm a / e$
Asymptotes	$b\,x \pm a\,y - (b\,h \pm a\,k) = 0$	$a\,x \pm b\,y - (a\,h \pm b\,k) = 0$
Focal chord length	$2b^2 / a$	$2b^2 / a$
Eccentricity	$e = \frac{\sqrt{a^2+b^2}}{a} > 1$	$e = \frac{\sqrt{a^2+b^2}}{a} > 1$

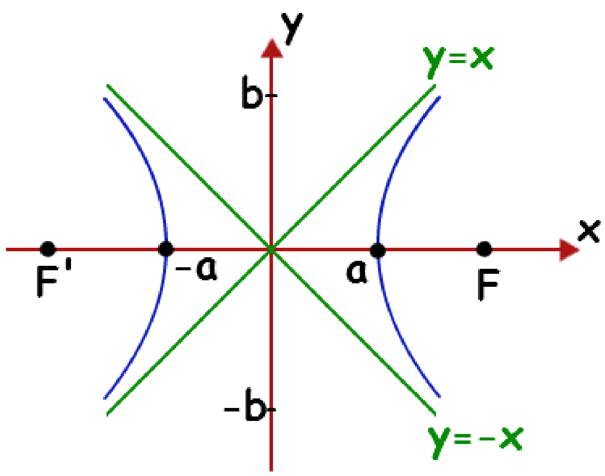


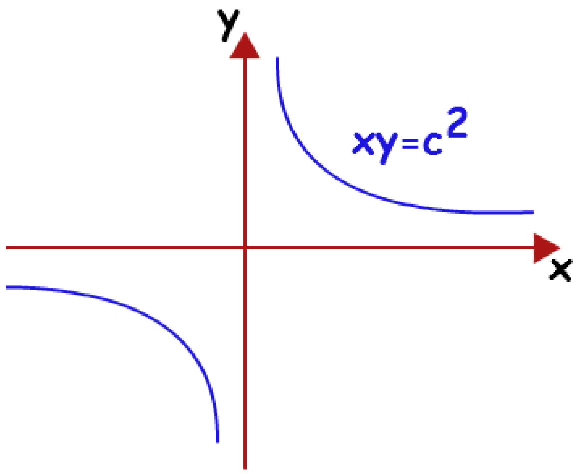


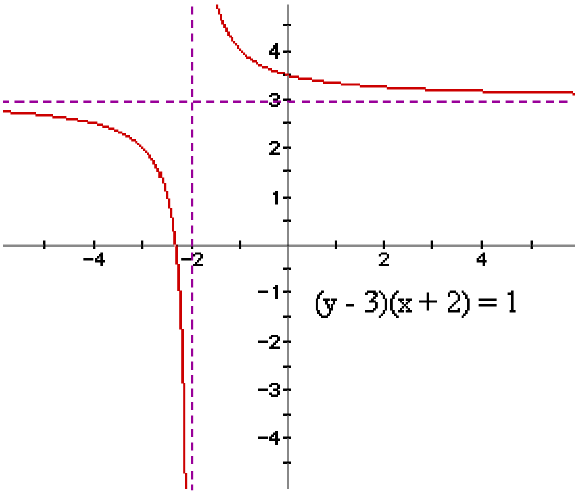




Rectangular Hyperbolas (where the eccentricity = √2 ($x^2 - y^2 = 1$) and ($xy = 1$) type



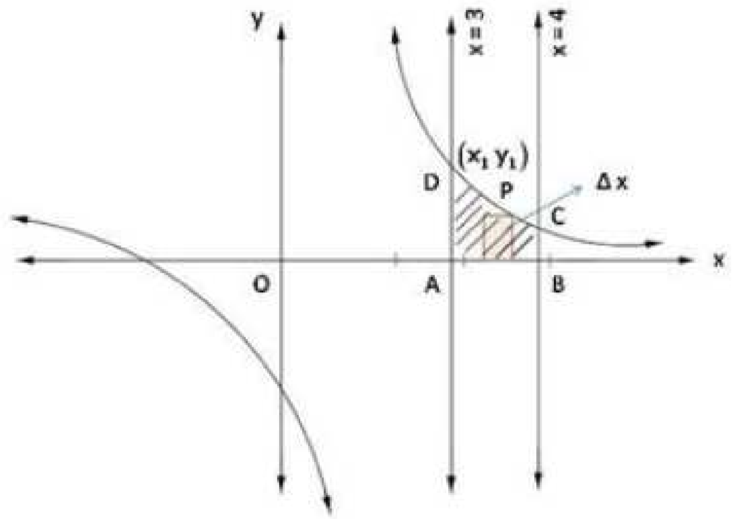



4
3
2
1
-1
-2
-3
-4
-4
-2
2
4
(y - 3)(x + 2) = 1

$\Rightarrow \quad y(x-2) = 3x + 10$

$\Rightarrow \quad y = \dfrac{3x+10}{x-2}$

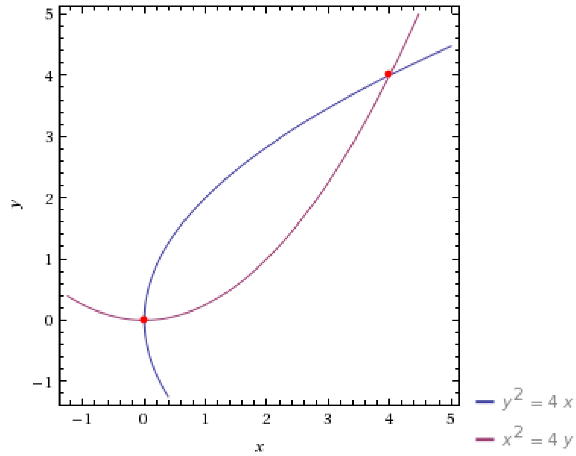
A rough sketch of the curves is given below:-



-

Draw $y^2 = 4x$ and $x^2 = 4y$

plot	$y^2 = 4x$
	$x^2 = 4y$

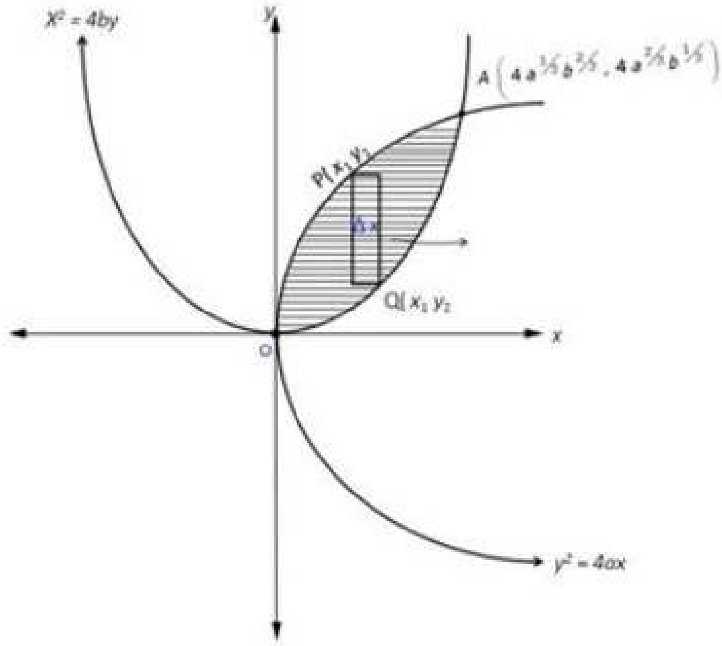


-

Draw abstract graph of $y^2 = 4ax$ and $x^2 = 4by$

Equation (1) represents a parabola with vertex (0,0) and axis as x-axis,
equation (2) represents a parabola with vertex (0,0) and axis as y-axis,
points of intersection of parabolas are (0,0) and $\left(4a\frac{1}{3}\, b\frac{2}{3},\ 4a\frac{2}{3}\, b\frac{1}{3}\right)$

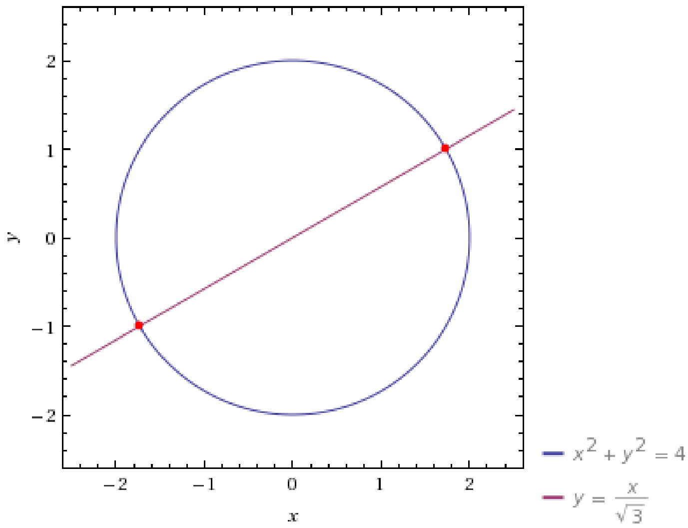
A rough sketch is given as:-



-

Draw graphs of $x^2 + y^2 = 4$ and $x = \sqrt{3}\, y$

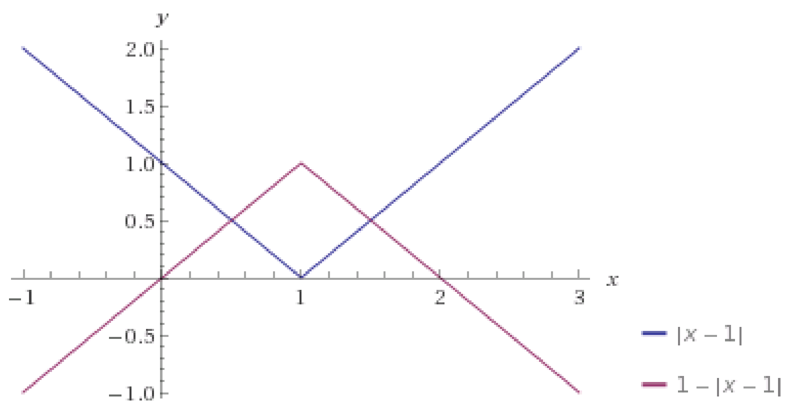
plot	$x^2 + y^2 = 4$
	$y = \frac{x}{\sqrt{3}}$



-

Draw Graphs of y = | x - 1 | and y = - | x - 1 | + 1

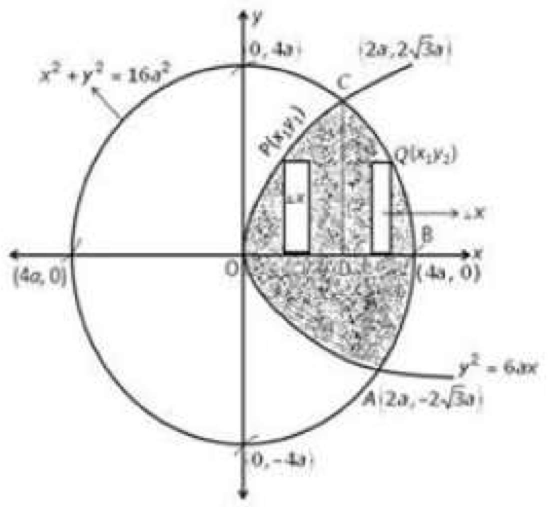
plot	$y = \lvert x-1 \rvert$
	$y = -\lvert x-1 \rvert + 1$



Draw $x^2 + y^2 = 16\,a^2$ and $y^2 = 6ax$

Equation (1) represents a circle with centre $(0,0)$ and meets axes $(\pm 4a,0),(0,\pm 4a)$. Equation(2) represents a parabola with vertex $(0,0)$ and axis as x-axis. Points of intersection of circle and parabola are $\left(2a, 2\sqrt{3}a\right), \left(2a, -2\sqrt{3}a\right)$.

A rough sketch of curves is given as:-

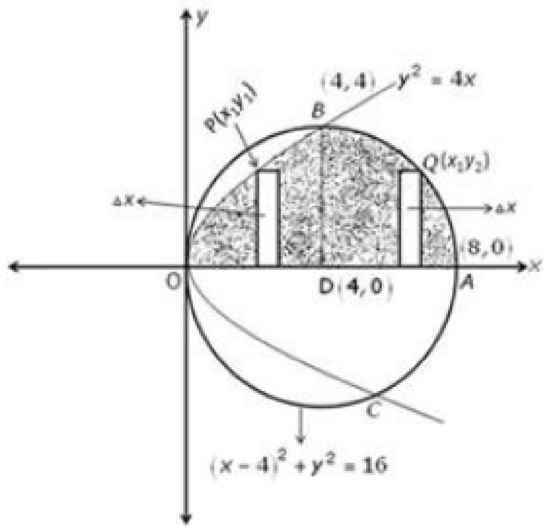


-

Draw $x^2 + y^2 = 8x$ and $(x - 4)^2 + y^2 = 16$ and $y^2 = 4x$

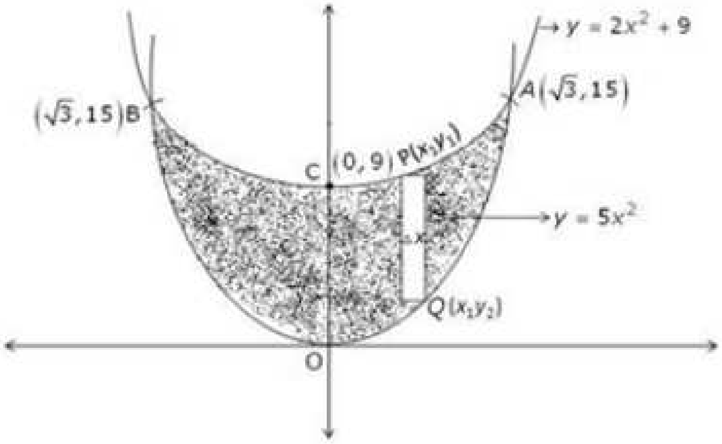
Equation (1) represents a circle with centre (4,0) and meets axes at (0,0) and (8,0). Equation (2) represent a parabola with vertex (0,0) and axis as x-axis. They intersect at (4,−4) and (4,4).

A rough sketch of the curves is as under:-



Shaded region is the required region

A rough sketch of curves is given as:-



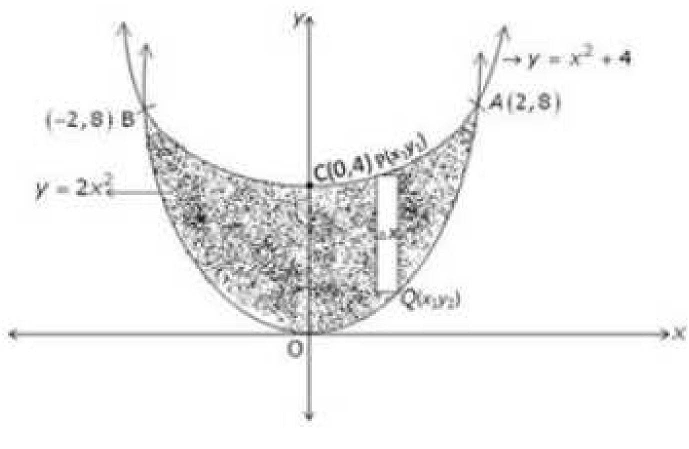
Region $AOCA$ is sliced into rectangles with area $(y_1 - y_2)\Delta x$. It slides from $x = 0$ to $x = \sqrt{3}$, so

-

Graph of $y = 2x^2$ and $y = x^2 + 4$

Equation (1) represents a parabola with vertex $(0,0)$ and axis as y-axis. Equation (2) represents a parabola with vertex $(0,4)$ and axis as y-axis. Points of intersection of parabolas are $(2,8)$ and $(-2,8)$.

A rough sketch of curves is given as:-



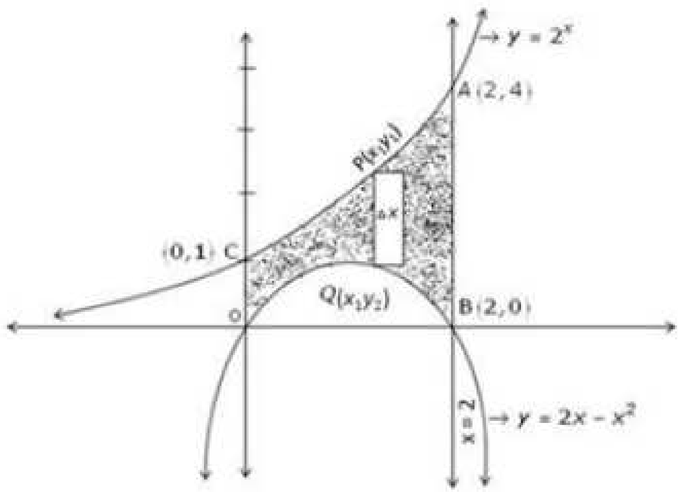
Region $AOCA$ is sliced into rectangles with area $(y_1 - y_2)\Delta x$. And it slides from $x = 0$ to $x = 2$

-

Graphs of x = 0, x = 2, y = 2^x , y = 2x - x^2

$$\Rightarrow \quad y = -\left(x^2 - 2x + 1 - 1\right)$$
$$= -\left[\left(x-1\right)^2 - 1\right]$$
$$\Rightarrow \quad y = -\left(x-1\right)^2 + 1$$
$$\Rightarrow \quad -\left(y-1\right) = \left(x-1\right)^2 \qquad ---(2)$$

Equation (2) represents a downward parabola with axis parallel to y-axis and vertex at (1, −1) table for equation (1) is



-

Graphs of $3x^2$ + 5y = 32 and y = | x - 2 |

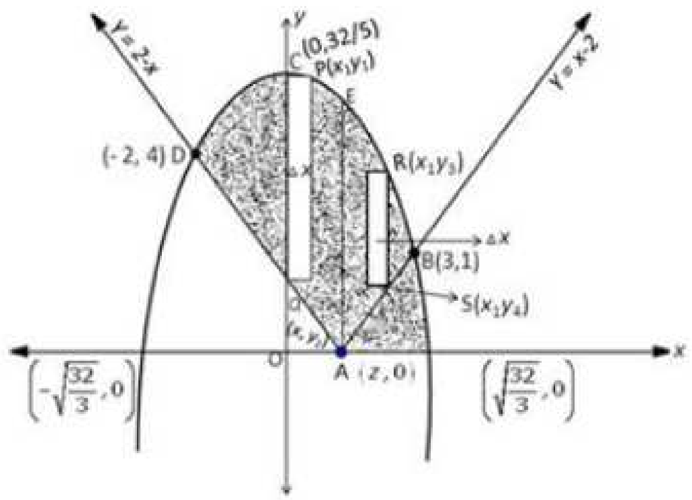
$$3x^2 = -5\left(y - \frac{32}{5}\right) \qquad ---(1)$$

$$y = |x - 2|$$

$$\Rightarrow \quad y = \begin{cases} -(x-2), \text{ if } x - 2 < 1 \\ (x-2), \text{ if } x - 2 \geq 1 \end{cases}$$

$$\Rightarrow \quad y = \begin{cases} 2 - x, \text{ if } x < 2 \\ x - 2, \text{ if } x \geq 2 \end{cases} \qquad ---(2)$$

Equation (1) represents a downward parabola with vertex $\left(0, \frac{32}{5}\right)$ and equation (2) represents lines. A rough sketch of curves is given as: -



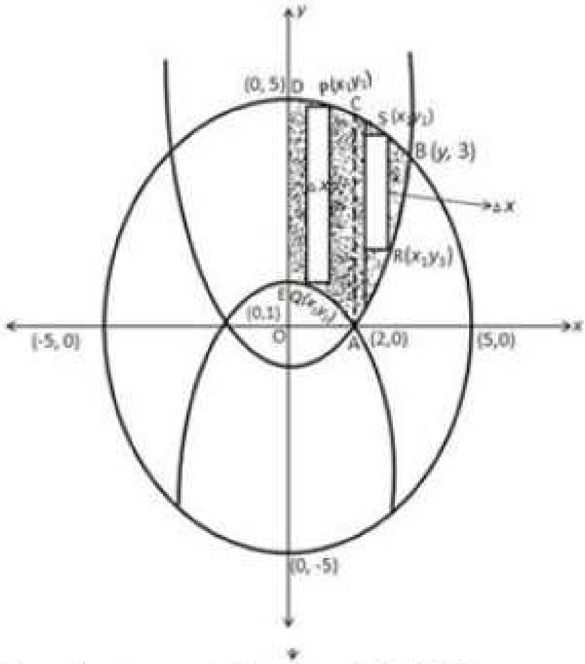
-

Graphs of y-axis (i.e. x = 0), and 4y = | 4 - x^2 |

$$4y = \begin{cases} 4 - x^2, \text{ if } -2 \le x \le 2 \\ x^2 - 4, \text{ if } x < -2, x > 2 \end{cases}$$

$$\Rightarrow \qquad x^2 = \begin{cases} -4(y-1), \text{ if } -2 \le x \le 2 & \textbf{(1)} \\ 4(y+1), \text{ if } x < -2, x > 2 & \textbf{(2)} \end{cases}$$

Equation (1) represents a parabola with vertex (0,0) and downward. Equation (2) represents an upward parabola with vertex (0, – 1) equation (3) represents a circle with centre (0,0) and meets axes at $(\pm 5,0),(\pm 0,5)$. A rough sketch is as follows:-

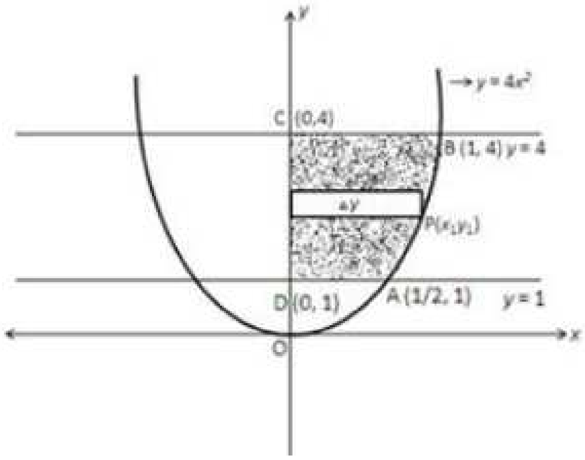


Required area = Region $EABCDE$

-

Graphs of x = 0, y = 1, y = 4, and y = $4x^2$

Equation (1) represents a parabola with vertex $(0,0)$ and axis as y-axis. $x = 0$ is y-axis and $y = 1$, $y = 4$ are lines parallel to x-axis passing through $(0,1)$ and $(0,4)$ respectively. A rough sketch of the curves is given as:-

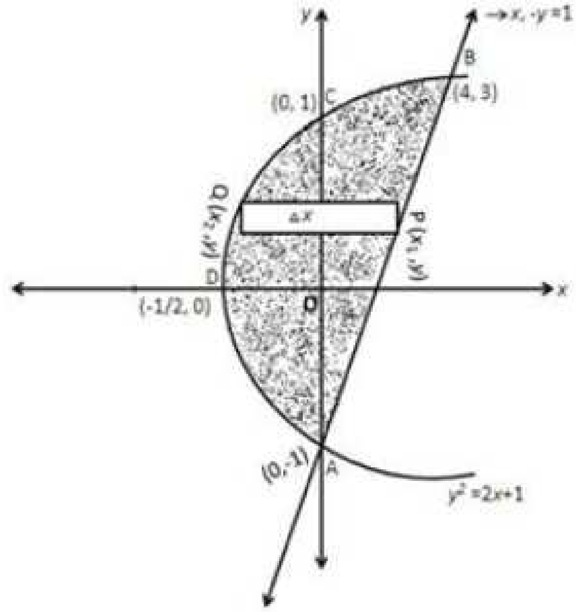


-

Graphs of $y^2 = 2x + 1$, - (1) and $x - y = 1$ - (2)

Equation (1) is a parabola with vertex $\left(-\frac{1}{2},0\right)$ and passes through $(0,1),(0,-1)$. Equation (2) is a line passing through $(1,0)$ and $(0,-1)$. Points of intersection of parabola and line are $(3,2)$ and $(0,-1)$.

A rough sketch of the curves is given as:-



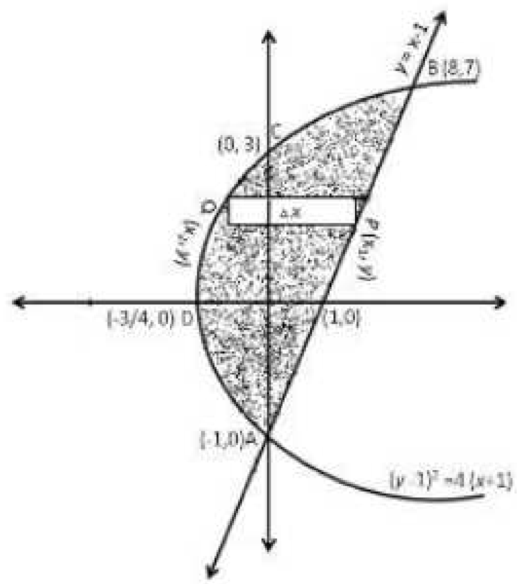
Shaded region represents the required area. It is sliced in rectangles of area $(x_1 - x_2)\Delta y$. It slides from $y = -1$ to $y = 3$, so

-

Graphs of y = x - 1, - (1) and $(y - 1)^2 = 4 (x + 1)$

Equation (1) represents a line passing through $(1,0)$ and $(0,-1)$ equation (2) represents a parabola with vertex $(-1,1)$ passes through $(0,3),(0,-1),\left(-\frac{3}{4},0\right)$.
Their points of intersection $(0,-1)$ and $(8,7)$.

A rough sketch of curves is given as:-



-

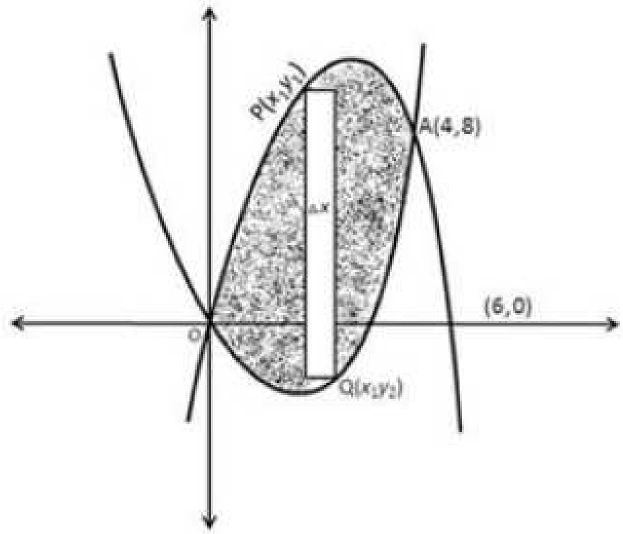
Draw graphs of

$$y = 6x - x^2$$
$$\Rightarrow \quad -y = x^2 - 6x$$
$$\Rightarrow \quad -y = x^2 - 6x + 9 - 9$$
$$\Rightarrow \quad -(y-9) = (x-3)^2 \;\textbf{(1)}$$

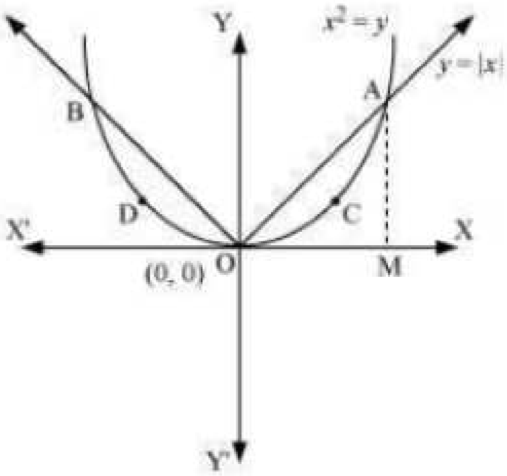
And

$$y = x^2 - 2x$$
$$y + 1 = x^2 - 2x + 1$$
$$(y+1) = (x-1)^2 \;\textbf{(2)}$$

Equation (1) represents a parabola with vertex (3,9) and downward. Equation (2) represents a parabola with vertex (1, – 1) and upward. Points of intersection of parabolas are (0,0) and (4,8). A rough sketch of the curves is given as:-



Graphs of $y = x^2$, and $y = | x |$



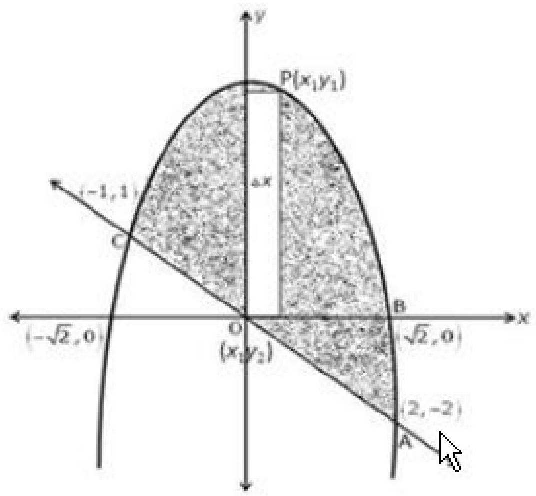
The given area is symmetrical about y-axis.

$\therefore$ Area OACO = Area ODBO

-

Graphs of y = 2 - x^2 -- (1) and y + x = 0 -- (2)

Equation (1) represents a parabola with vertex $(0,2)$ and downward, meets axes at $(\pm\sqrt{2},0)$.
Equation (2) represents a line passing through $(0,0)$ and $(2,-2)$. The points of intersection of line and parabola are $(2,-2)$ and $(-1,1)$.
A rough sketch of curves is as follows:-

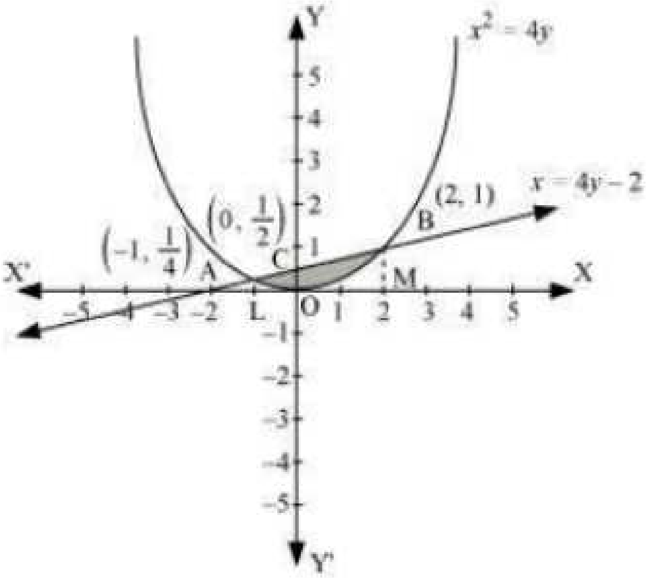


Shaded region is sliced into rectangles with area = $(y_1 - y_2)\Delta x$. It slides from $x = -1$ to $x = 2$, so

-

Graphs of $x^2 = 4y$ -- (1) and $x = 4y - 2$ -- (2)

shaded area OBAO.



Let A and B be the points of intersection of the line and parabola.

Coordinates of point A are $\left(-1, \frac{1}{4}\right)$.

-

Graphs of

$$y = 4x - x^2$$
$$\Rightarrow \quad -y = x^2 - 4x + 4 - 4$$
$$\Rightarrow \quad -y + 4 = (x - 2)^2$$
$$\Rightarrow \quad -(y - 4) = (x - 2)^2 \text{ (1)}$$

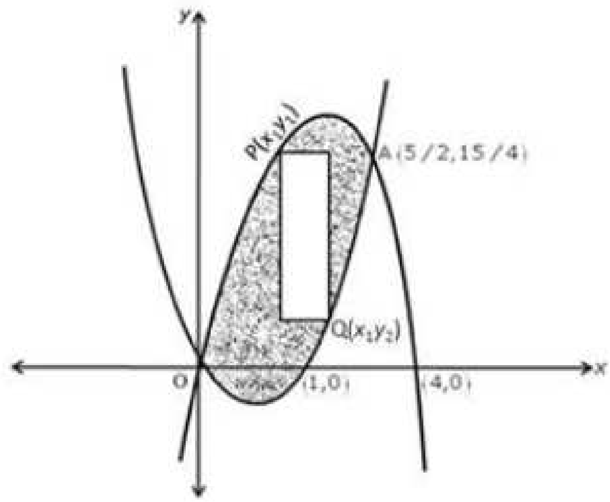
And

and $y = x^2 - x$

$$\left(y + \frac{1}{4}\right) = \left(x - \frac{1}{2}\right)^2 \text{ (2)}$$

Equation (1) represents a parabola downward with vertex at $(2,4)$ and meets axes at $(4,0),(0,0)$. Equation (2) represents a parabola upword whose vertex is $\left(\frac{1}{2},-\frac{1}{4}\right)$ and meets axes at $(1,0),(0,0)$. Points of intersection of parabolas are $(0,0)$ and $\left(\frac{5}{2},\frac{15}{4}\right)$.

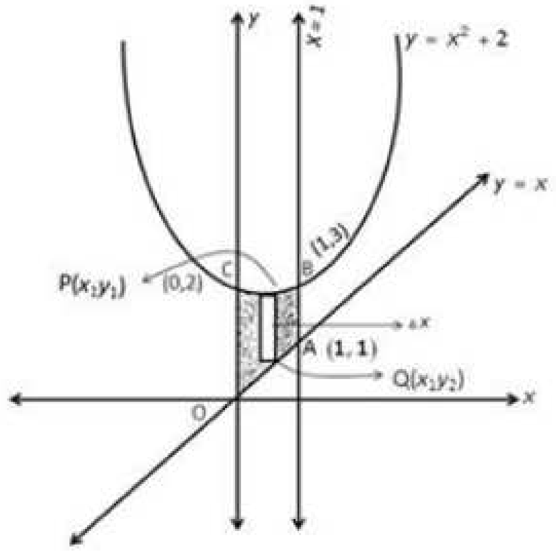
A rough sketch of the curves is as under:-



-

Graphs of x = 0, x = 1 and y = x -- (1)   and $y = x^2 + 2$ -- (2)

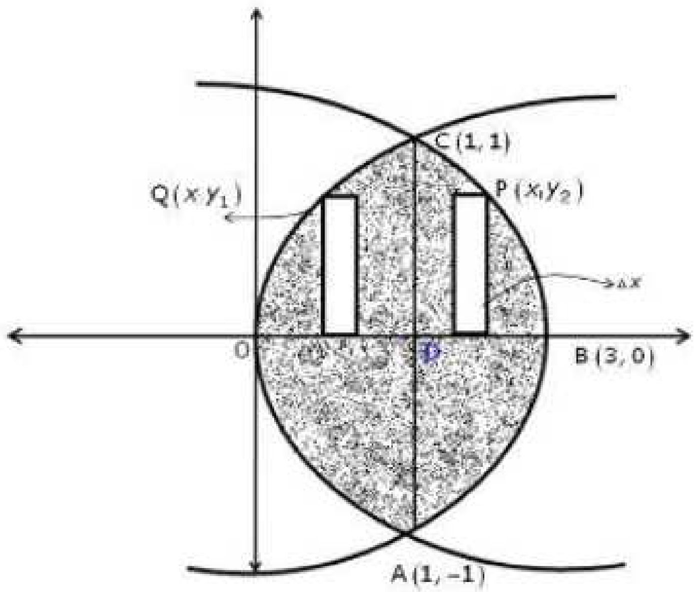
Equation (1) is a line passing through (2,2) and (0,0). Equation (2) is a parabola upward with vertex at (0,2). A rough sketch of curves is as under:-



-

Graphs of $x = y^2$ -- (1) and $x = 3 - 2y^2$ -- (2)

Equation (1) represents an upward parabola with vertex $(0,0)$ and axis -y. Equation (2) represents a parabola with vertex $(3,0)$ and axis as x-axis. They intersect at $(1,-1)$ and $(1,1)$. A rough sketch of the curves is as under:-



Graphs of $y = 4x - x^2$ -- (1) $y = x^2 - x$ -- (2)

Given curves are

$y = 4x - x^2$

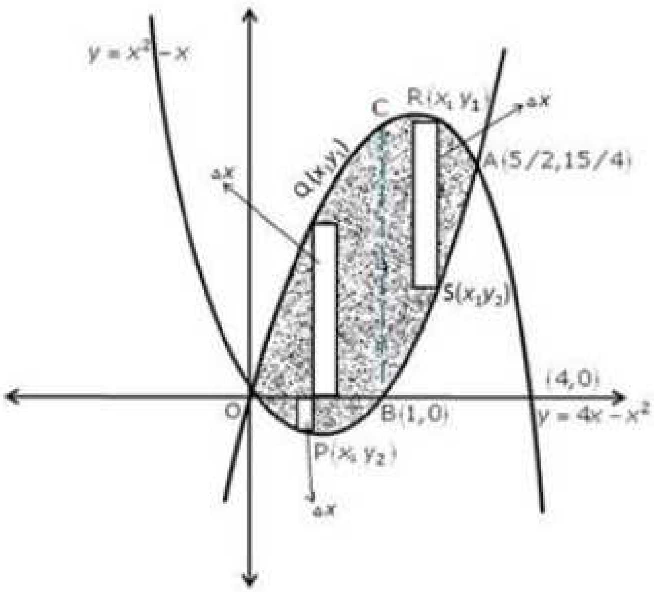
$\Rightarrow \quad -(y-4) = (x-2)^2 \qquad ---(1)$

and

$y = x^2 - x$

$\Rightarrow \quad \left(y + \frac{1}{4}\right)^2 = \left(x - \frac{1}{2}\right)^2 \qquad ---(2)$

Equation (1) represents a parabola downward with vertex at $(2,4)$ and meets axes at $(4,0),(0,0)$. Equation (2) represents a parabola upward whose vertex is $\left(\frac{1}{2},-\frac{1}{4}\right)$ and meets axes at $(1,0),(0,0)$ and $\left(\frac{5}{2},\frac{15}{4}\right)$. A rough sketch of the curves is as under:-



Area of the region above x-axis

-

Graphs of y = | x - 1 | -- (1) and y = 3 - | x | -- (2)

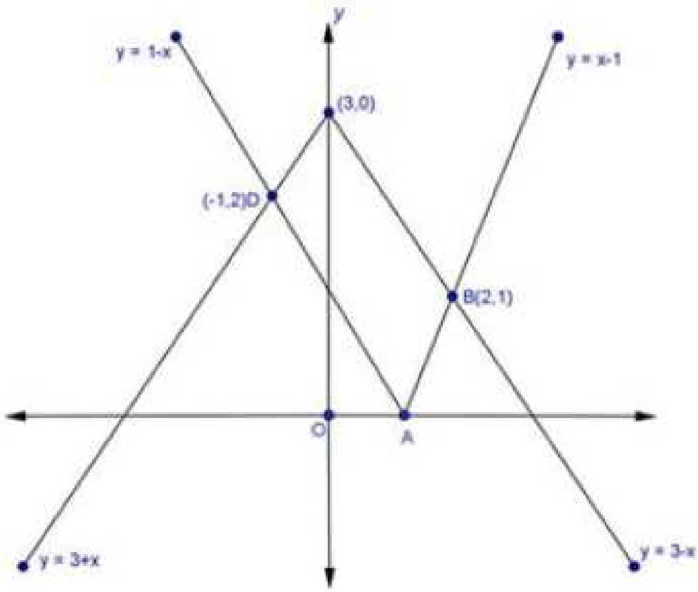
$$y = |x-1|$$

$$\Rightarrow \quad y = \begin{cases} 1-x, \text{ if } x < 1 & ---(1) \\ x-1, \text{ if } x \geq 1 & ---(2) \end{cases}$$

$$\text{and} \quad y = 3-|x|$$

$$\Rightarrow \quad y = \begin{cases} 3+x, \text{ if } x < 0 & ---(3) \\ 3-x, \text{ if } x \geq 0 & ---(4) \end{cases}$$

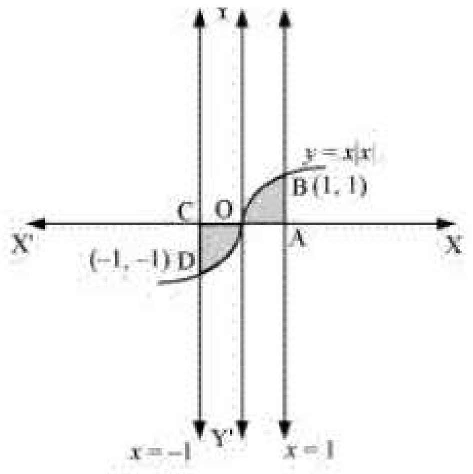
Drawing the rough sketch of lines (1), (2), (3) and (4) as under:-



Shaded region is the required area

-

Graphs of y = x|x| - (1) and x = -1 and x = 1

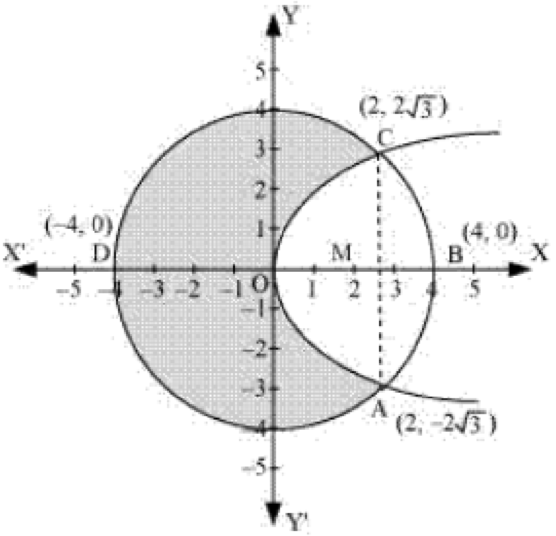


$$\text{Required area} = \int_{-1}^{1} y\,dx$$

$$= \int_{-1}^{1} x|x|\,dx$$
$$= \int_{-1}^{0} x^2 dx + \int_{0}^{1} x^2 dx$$
$$= \left[\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{1}$$
$$= -\left(-\frac{1}{3}\right) + \frac{1}{3}$$
$$= \frac{2}{3} \text{ sq. units}$$

-

Graphs of $x^2 + y^2 = 16$ -- (1) and $y^2 = 6x$ -- (2)



Area bounded by the circle and parabola

$$= 2\left[\text{Area}(\text{OADO}) + \text{Area}(\text{ADBA})\right]$$

$$= 2\left[\int_0^2 \sqrt{16x}\,dx + \int_2^4 \sqrt{16-x^2}\,dx\right]$$

-

Function

A function is a relation for which there is only one value of y corresponding to any value of x. We sometimes write y = f (x), which is notation meaning 'y is a function of x'.

Some very common mathematical constructions are not functions. For example, consider the relation $x^2 + y^2 = 4$ because multiple values can satisfy the equation. Ay put y = 0, then for x = 2 and x = -2 both the expression is 4

There is a simple test to check if a relation is a function, by looking at its graph. This test is called the
vertical line test. If it is possible to draw any vertical line (a line of constant x) which crosses the graph of the relation more than once, then the relation is not a function. If more than one intersection point exists, then the intersections correspond to multiple values of y for a single value of x.

An inverse function is a function which "does the reverse" of a given function. More formally, if f is a
function with domain X, then f^{-1} is its inverse function if and only if for every x ε X we have

$$f^{-1}(f(x)) = \mathbf{x}$$

A simple way to think about this is that a function, say y = f (x), gives you a y-value if you substitute an
x-value into f (x). The inverse function tells you tells you which x-value was used to get a particular y-valuewhen you substitue the y-value into f^{-1} (x).

If f(x) = 3x + 2 then find f^{-1} (x)

Solution :
Put y = 3x + 2 and solve for x => y - 2 = 3x or x = (y - 2)/3
Now exchange x and y
So f^{-1}(x) = (x - 2)/3

-

We say that a function g(x) is periodic if there is a positive or negative number T for which g(x + T) = g(x) for all x We call T a period of g(x)

A periodic function has many periods

Since the graph of g repeats after x increases by T, it also repeats after x increases by 2T, or −3T, or any integer multiple (positive or negative) of T. This means that a periodic function always has many periods. (That's why the definition refers to "a period" rather than "the period.")

The period of a periodic function is its smallest positive period. It is the size of a single cycle.

If the function g(x) is periodic, then its frequency is the number of cycles per unit x.

In general, if f is the frequency of a periodic function g(x) and T is its period, then we have

$$f = \frac{1}{T} \quad \textbf{and} \quad T = \frac{1}{f}$$

if the period is measured in seconds, then the frequency is measured in cycles per second.

The term Hertz is a special unit used to measure time frequencies; it equals one cycle per second. Hertz is abbreviated Hz; thus a kilohertz (kHz) and a megahertz (MHz) are 1,000 and 1,000,000 cycles per second, respectively. This unit is commonly used to describe sound, light, radio, and television waves.

Circular functions. While there are innumerable examples of periodic functions, two in particular are considered basic: the sine and the cosine. They are called circular functions because they are defined by means of a circle. To be specific, take the circle of radius 1 centered at the origin in the x, y-plane. Given any real number t, measure a distance of t units around the circumference of the circle.

Start on the positive x-axis, and measure counterclockwise if t is positive, clockwise if t is negative. The coordinates of the point you reach this way are, by definition, the cosine and the sine functions of t, respectively:
$x = \cos(t)$
$y = \sin(t)$

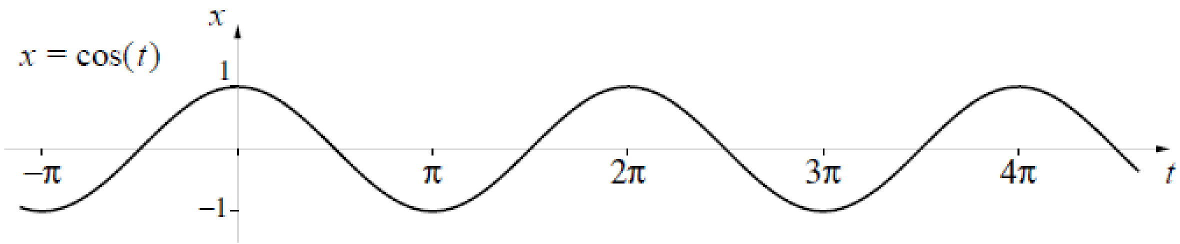
The whole circumference of the circle measures 2π units. Therefore, if we add 2π units to the t units we have already measured, we will arrive back at the same point on the circle. That is, we get to the same point on the circle by measuring either t or t + 2π units around the circumference. We can describe the coordinates of this point two ways:

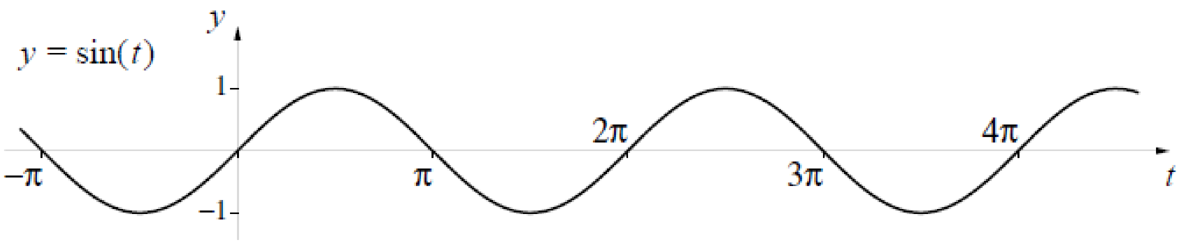
$(\cos(t), \sin(t))$ or $(\cos(t + 2\pi), \sin(t + 2\pi))$

Thus

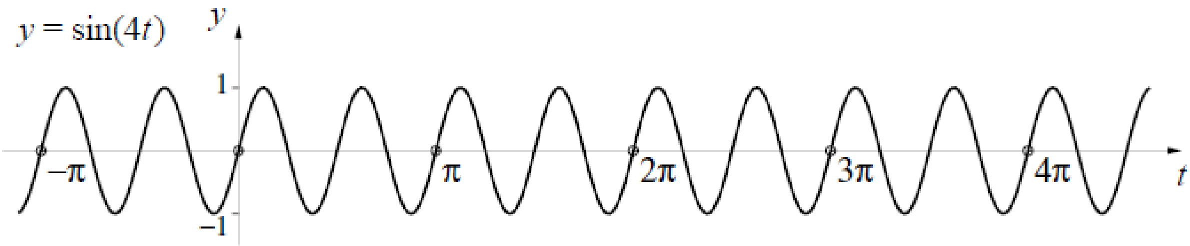
$\cos(t + 2\pi) = \cos(t)$ $\sin(t + 2\pi) = \sin(t)$,
so cos(t) and sin(t) are both periodic, and they have the same period, 2π.

Here are their graphs. By reading their slopes we can see $(\sin t)' = \cos t$ and $(\cos t)' = -\sin t$





While graph of $y = \sin(4t)$ will be



Their scales are identical, so it is clear that the frequency of sin(4t) is four times the frequency of sin(t). The general pattern is described in the following table.

function		period	frequency
$\sin(t)$	$\cos(t)$	2π	$1/2\pi$
$\sin(4t)$	$\cos(4t)$	$2\pi/4$	$4/2\pi$
$\sin(bt)$	$\cos(bt)$	$2\pi/b$	$b/2\pi$

Notice that it is the frequency—not the period—that is increased by a factor of b when we multiply the input variable by b.

Constructing a circular function with a given frequency

By using the information in the table, we can circular functions with any period or frequency whatsoever. For instance, suppose we wanted a cosine function x = cos(bt) with a frequency of 5 cycles per unit t. This means

$$5 = \text{frequency} = \frac{b}{2\pi}$$

which implies that we should set b = 10π and x = cos(10πt)

-

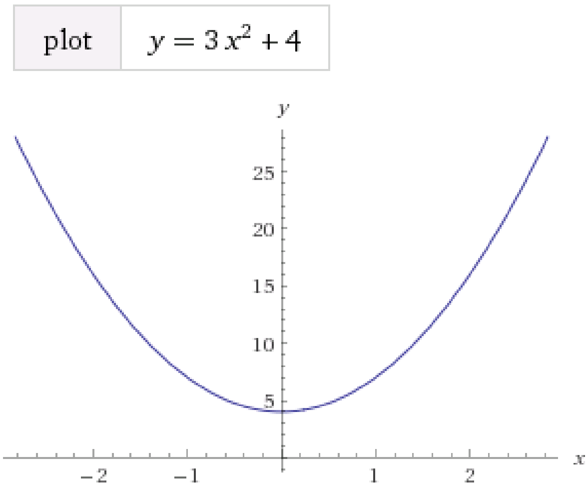
Domain and Range of functions (graphs)

In simple words - Given a function, the values of x that are allowed to be supplied to the function, so that all terms remain real, is the domain. So Domain is the values for x as it varies.

Range are the values y = f(x) takes, keeping the function as Real. So Range is the way y varies.

Consider the parabola $y = f(x) = 3x^2 + 4$

The graph will be



We see that y is minimum 5 or more. So Range is [5, ∞)   while Domain is (-∞ , ∞) or   ] -∞ , ∞ [

Note : the round brackets (or) means open bracket. Meaning close to that value but not equal to.

∞ is always written with open bracket ") " as we can have a very high value, but exactly infinity is undefined. Other way of writing the same thing is $5 \le y < \infty$

Thus the ≤ sign is giving the symbol " ["

In some statement if we get $6 < y < 9$ then same thing will be written as (6, 9)

But $1 \le y \le 8$ will be [1, 8]

For the above problem for all values (positive, negative, zero) of x the graph exists. So $-\infty < x < \infty$ watch the ≤ sign is not being used for infinity.

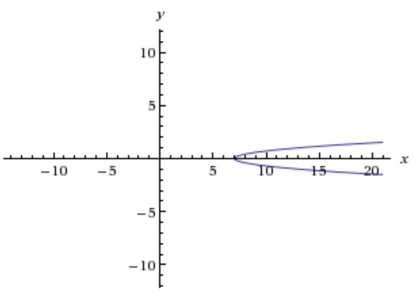
Now some Mathematicians take pleasure in writing the same thing as] -∞ , ∞ [as this is easier to print.

-

Spoon Feeding : Find Domain and Range for $x = 6y^2 + 7$ and write the answer in all methods

Graph will be

plot	$6y^2 = x - 7$



So Domain is : $7 \le x$ or $7 \le x < \infty$ or $[7, \infty)$ or $[7, \infty[$

And Range is : $]-\infty, \infty[$ or $(-\infty, \infty)$ or $-\infty < y < \infty$

-

Spoon Feeding : One of the most favorite questions by Math teachers, of standard 11 is to ask Domain and Range of $y = f(x) = \sqrt{\dfrac{9 - x}{x - 1}}$

Let me try to solve this without drawing the graph. If the student can guess the graph, of-course it becomes very easy to solve.

If x becomes more than 9 then Numerator(N) becomes negative while Denominator(D) remains positive. As imaginary value of y is not allowed, x has to be less than or equal to 9 so $x \le 9$

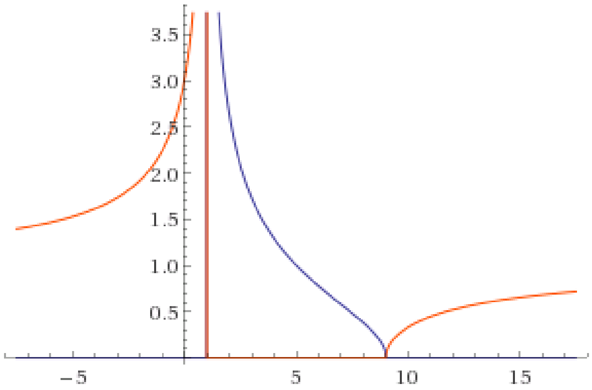
N can be zero, but D is not allowed to be zero as dividing by 0 is not defined. So $x = 1$ is not allowed. If $x < 1$ then the N remain +ve but D becomes -ve. As y cannot be imaginary, $1 < x$, meaning x has to be greater than 1. But x can be arbitrarily close to 1 say $1 + \delta$ where δ is very small positive number. In that case D becomes δ while $N = 9 - (1 + \delta) = 8 - \delta$

$(8 - \delta) / \delta$ tends to ∞ So Range for y will go upwards to infinity. But what will be the least value ?

We already saw that N can be zero (0) but not negative. So Range will be from 0 to infinity.

Now let us see the graph

plot	$y = \sqrt{\dfrac{9-x}{x-1}}$



Ignore the red lines of imaginary solutions.

Let us write the Solutions

Domain : $1 < x \le 9$ or $(1, 9]$ or $]1, 9]$ Happy ?

Range : $0 \le y < \infty$ or $[0, \infty)$ or $[0, \infty[$ ☺

Spoon Feeding : Find Domain and Range of $y = f(x) = \sqrt{\dfrac{8 - x}{1 - x}}$

If x is in between 1 and 8, say 6 then N is +ve while D is -ve, which is not allowed.

If x is greater than 8 ($8 < x$) then both N and D are negative so fine for us, as y the function is real. If x is less than 1 say 0 or -100 then also y is real. As both N, and D is positive.

We need to analyze the function when x is very close to 1 i.e. $x = 1 - \delta$ where δ is very small positive number. D becomes $1 - (1 - \delta) = \delta$

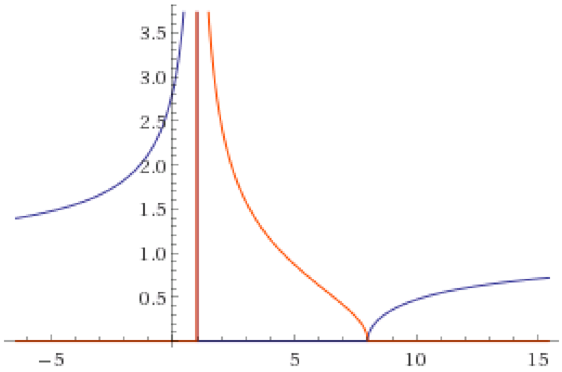
Positive D divided by δ will tend to infinity.

Now we can write the Domain : $8 \le x$ or $x < 1$

While Range : $0 \le y < \infty$ or $[0, \infty)$ or $[0, \infty[$ ☺

Let us confirm by drawing the graph. (Though in the exam you have to just guess the graph)

plot	$y = \sqrt{\dfrac{8-x}{1-x}}$



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To recall standard integrals

$f(x)$	$\int f(x)dx$	$f(x)$	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1}\quad (n \neq -1)$	$[g(x)]^n\, g'(x)$	$\frac{[g(x)]^{n+1}}{n+1}\quad (n \neq -1)$
$\frac{1}{x}$	$\ln\lvert x\rvert$	$\frac{g'(x)}{g(x)}$	$\ln\lvert g(x)\rvert$
e^x	e^x	a^x	$\frac{a^x}{\ln a}\quad (a > 0)$
$\sin x$	$-\cos x$	$\sinh x$	$\cosh x$
$\cos x$	$\sin x$	$\cosh x$	$\sinh x$
$\tan x$	$-\ln\lvert\cos x\rvert$	$\tanh x$	$\ln\cosh x$
$\operatorname{cosec} x$	$\ln\lvert\tan\frac{x}{2}\rvert$	$\operatorname{cosech} x$	$\ln\lvert\tanh\frac{x}{2}\rvert$
$\sec x$	$\ln\lvert\sec x + \tan x\rvert$	$\operatorname{sech} x$	$2\tan^{-1} e^x$
$\sec^2 x$	$\tan x$	$\operatorname{sech}^2 x$	$\tanh x$
$\cot x$	$\ln\lvert\sin x\rvert$	$\coth x$	$\ln\lvert\sinh x\rvert$
$\sin^2 x$	$\frac{x}{2} - \frac{\sin 2x}{4}$	$\sinh^2 x$	$\frac{\sinh 2x}{4} - \frac{x}{2}$
$\cos^2 x$	$\frac{x}{2} + \frac{\sin 2x}{4}$	$\cosh^2 x$	$\frac{\sinh 2x}{4} + \frac{x}{2}$

$f(x)$	$\int f(x)\,dx$	$f(x)$	$\int f(x)\,dx$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a}$	$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\|$ $(0<\|x\|<a)$
	$(a>0)$	$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\|$ $(\|x\|>a>0)$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\frac{x}{a}$	$\frac{1}{\sqrt{a^2+x^2}}$	$\ln\left\|\frac{x+\sqrt{a^2+x^2}}{a}\right\|$ $(a>0)$
	$(-a<x<a)$	$\frac{1}{\sqrt{x^2-a^2}}$	$\ln\left\|\frac{x+\sqrt{x^2-a^2}}{a}\right\|$ $(x>a>0)$
$\sqrt{a^2-x^2}$	$\frac{a^2}{2}\left[\sin^{-1}\left(\frac{x}{a}\right)\right.$	$\sqrt{a^2+x^2}$	$\frac{a^2}{2}\left[\sinh^{-1}\left(\frac{x}{a}\right)+\frac{x\sqrt{a^2+x^2}}{a^2}\right]$
	$\left.+\frac{x\sqrt{a^2-x^2}}{a^2}\right]$	$\sqrt{x^2-a^2}$	$\frac{a^2}{2}\left[-\cosh^{-1}\left(\frac{x}{a}\right)+\frac{x\sqrt{x^2-a^2}}{a^2}\right]$

Some series Expansions -

$$\frac{\pi}{2}=\left(\frac{2}{1}\frac{2}{3}\right)\left(\frac{4}{3}\frac{4}{5}\right)\left(\frac{6}{5}\frac{6}{7}\right)\left(\frac{8}{7}\frac{8}{9}\right)\dots$$

$$\pi=\frac{4}{1}-\frac{4}{3}+\frac{4}{5}-\frac{4}{7}+\frac{4}{9}-\frac{4}{11}+\frac{4}{13}-\cdots$$

$$\frac{\pi}{4}=\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\dots$$

$$\pi=\sqrt{12}\left(1-\frac{1}{3\cdot 3}+\frac{1}{5\cdot 3^2}-\frac{1}{7\cdot 3^3}+\cdots\right)$$

$$\frac{\pi^2}{6}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\dots=\sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\int_0^{\pi/2}\log\sin x\,dx=-\frac{\pi}{2}\log 2=\frac{\pi}{2}\log\frac{1}{2}$$

Solve a series problem

If $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots$ upto $\infty = \frac{\pi^2}{6}$, then value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots$ up to ∞ is

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{6}$ (c) $\frac{\pi^2}{8}$ (d) $\frac{\pi^2}{12}$

Ans. (c)

Solution We have $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots$ upto ∞

$$= \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}\cdots \text{ upto } \infty$$

$$-\frac{1}{2^2}\left[1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots\right]$$

$$= \frac{\pi^2}{6}-\frac{1}{4}\left(\frac{\pi^2}{6}\right)=\frac{\pi^2}{8}$$

$$1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\frac{1}{5^2}-\frac{1}{6^2}+\ldots\ \infty = \frac{\pi^2}{12}$$

$$\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+\ldots\ \infty = \frac{\pi^2}{24}$$

$$\frac{\sin\sqrt{x}}{\sqrt{x}} = 1-\frac{x}{3!}+\frac{x^2}{5!}-\frac{x^3}{7!}+\frac{x^4}{9!}-\frac{x^5}{11!}+\ldots.$$

$$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots = \sum_{k=0}^{n}\frac{(-1)^k x^{2k}}{(2k)!}$$

$$\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots = \sum_{k=0}^{n}\frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$\cosh x = 1+\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots = \sum_{k=0}^{n}\frac{x^{2k}}{(2k)!}$$

$$\sinh x = x+\frac{x^3}{3!}+\frac{x^5}{5!}+\cdots = \sum_{k=0}^{n}\frac{x^{2k+1}}{(2k+1)!}$$

$$\tan^{-1} x = x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots \qquad (-1\le x<1)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} \cdots + \frac{2^{2n}\left(2^{2n}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \qquad |x| < \frac{\pi}{2}$$

$$\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \cdots + \frac{E_n x^{2n}}{(2n)!} + \cdots \quad |x| < \frac{\pi}{2}$$

$$\csc x = \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^5}{15120} + \cdots + \frac{2\left(2^{2n-1}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \qquad 0 < |x| < \pi$$

$$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - - \cdots - \frac{2^{2n} B_n x^{2n-1}}{(2n)!} - \cdots \quad 0 < |x| < \pi$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$$

$$\sec x = 1 + \frac{x^2}{\lfloor 2} + \frac{5x^4}{\lfloor 4} + \cdots$$

$$\log(\cos x) = -\frac{x^2}{\lfloor 2} - \frac{2x^4}{\lfloor 4} - \cdots$$

$$\log(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \cdots$$

$$\sin^{-1} x = x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \cdots \quad |x| < 1$$

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$= \frac{\pi}{2} - \left(x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \cdots\right) \quad |x| < 1$$

$$\tan^{-1} x = \begin{cases} x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \cdots & |x| < 1 \\ \pm\dfrac{\pi}{2} - \dfrac{1}{x} + \dfrac{1}{3x^3} - \dfrac{1}{5x^5} + \cdots & \begin{cases} + \text{ if } x \geq 1 \\ - \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$$

$$= \frac{\pi}{2} - \left(\frac{1}{x} + \frac{1}{2\cdot 3x^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5x^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \cdots\right) \quad |x| > 1$$

$$\csc^{-1} x = \sin^{-1}(1/x)$$

$$= \frac{1}{x} + \frac{1}{2\cdot 3x^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5x^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \cdots \quad |x| > 1$$

$$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$$

$$= \begin{cases} \dfrac{\pi}{2} - \left(x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \cdots\right) & |x| < 1 \\ p\pi + \dfrac{1}{x} - \dfrac{1}{3x^3} + \dfrac{1}{5x^5} + \cdots & \begin{cases} p = 0 \text{ if } x \geq 1 \\ p = 1 \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\ln x = 2\left[\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \ldots\right]$$

$$= 2\sum_{n=1}^{\infty} \frac{1}{2n-1}\left(\frac{x-1}{x+1}\right)^{2n-1} \quad (x > 0)$$

$$\ln x = \frac{x-1}{x} + \frac{1}{2}\left(\frac{x-1}{x}\right)^2 + \frac{1}{3}\left(\frac{x-1}{x}\right)^3 + \ldots$$

$$= \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{x-1}{x}\right)^n \quad (x > \frac{1}{2})$$

$$\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}(x-1)^n \quad (0 < x \le 2)$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} x^n \quad (|x| < 1)$$

$$\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots\infty \, (-1 \le x < 1)$$

$$\log_e(1+x) - \log_e(1-x) =$$

$$\log_e \frac{1+x}{1-x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\infty\right)(-1 < x < 1)$$

$$\log_e\left(1+\frac{1}{n}\right) = \log_e \frac{n+1}{n} = 2 \left[\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \ldots\infty\right]$$

$$\log_e\,(1+x) + \log_e\,(1-x) = \log_e(1-x^2) = -2\left(\frac{x^2}{2} + \frac{x^4}{4} + \ldots\infty\right)(-1 < x < 1)$$

$$\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots \quad = \frac{1}{1\cdot 2} + \frac{1}{3.4} + \frac{1}{5.6} + \ldots$$

Important Results

(i) (a) $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cos^n x}{\sin^n x + \cos^n x} dx$

(b) $\int_0^{\pi/2} \frac{\tan^n x}{1 + \tan^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{dx}{1 + \tan^n x}$

(c) $\int_0^{\pi/2} \frac{dx}{1 + \cot^n x} = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{1 + \cot^n x} dx$

(d) $\int_0^{\pi/2} \frac{\tan^n x}{\tan^n x + \cot^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{\tan^n x + \cot^n x} dx$

(e) $\int_0^{\pi/2} \frac{\sec^n x}{\sec^n x + \operatorname{cosec}^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\operatorname{cosec}^n x}{\sec^n x + \operatorname{cosec}^n x} dx$ where, $n \in R$

(ii) $\int_0^{\pi/2} \frac{a^{\sin^n x}}{a^{\sin^n x} + a^{\cos^n x}} dx = \int_0^{\pi/2} \frac{a^{\cos^n x}}{a^{\sin^n x} + a^{\cos^n x}} dx = \frac{\pi}{4}$

(iii) (a) $\int_0^{\pi/2} \log \sin x \, dx = \int_0^{\pi/2} \log \cos x \, dx = -\frac{\pi}{2} \log 2$

(b) $\int_0^{\pi/2} \log \tan x \, dx = \int_0^{\pi/2} \log \cot x \, dx = 0$

(c) $\int_0^{\pi/2} \log \sec x \, dx = \int_0^{\pi/2} \log \operatorname{cosec} x \, dx = \frac{\pi}{2} \log 2$

(iv) (a) $\int_0^{\infty} e^{-ax} \sin bx \, dx = \frac{b}{a^2 + b^2}$

(b) $\int_0^{\infty} e^{-ax} \cos bx \, dx = \frac{a}{a^2 + b^2}$

(c) $\int_0^{\infty} e^{-ax} x^n \, dx = \frac{n!}{a^n + 1}$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left(x + \sqrt{x^2 - a^2}\right) + C$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left(\frac{x-a}{x+a}\right) + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left(\frac{a+x}{a-x}\right) + C$$

$$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{a^2 + x^2}\,dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right) + C$$

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Good Luck to you for your Preparations, References, and Exams

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