Spoon Feeding Indefinite Integrals

Simplified Knowledge Management Classes Bangalore

My name is Subhashish Chattopadhyay. I have been teaching for IIT-JEE, Various International Exams (such as IMO [International Mathematics Olympiad], IPhO [International Physics Olympiad], IChO [International Chemistry Olympiad]), IGCSE (IB), CBSE, I.Sc, Indian State Board exams such as WB-Board, Karnataka PU-II etc since 1989. As I write this book in 2016, it is my 25 th year of teaching. I was a Visiting Professor to BARC Mankhurd, Chembur, Mumbai, Homi Bhabha Centre for Science Education (HBCSE) Physics Olympics camp BARC Campus.

I am Life Member of …

- IAPT (Indian Association of Physics Teachers)
- IPA (Indian Physics Association)
- AMTI (Association of Mathematics Teachers of India)
- National Human Rights Association
- Men's Rights Movement (India and International)
- MGTOW Movement (India and International)

And also of

IACT (Indian Association of Chemistry Teachers)

The selection for National Camp (for Official Science Olympiads - Physics, Chemistry, Biology, Astronomy) happens in the following steps ….

1) **NSEP** (National Standard Exam in Physics) and **NSEC** (National Standard Exam in Chemistry) held around 24 rth November. Approx 35,000 students appear for these exams every year. The exam fees is Rs 100 each. Since 1998 the IIT JEE toppers have been topping these exams and they get to know their rank / performance ahead of others.

2) **INPhO** (Indian National Physics Olympiad) and **INChO** (Indian National Chemistry Olympiad). Around 300 students in each subject are allowed to take these exams. Students coming from outside cities are paid fair from the Govt of India.

3) The Top 35 students of each subject are invited at HBCSE (Homi Bhabha Center for Science Education) Mankhurd, near Chembur, BARC, Mumbai. After a 2-3 weeks camp the top 5 are selected to represent India. The flight tickets and many other expenses are taken care by Govt of India.

Since last 50 years there has been no dearth of "Good Books". Those who are interested in studies have been always doing well. This e-Book does not intend to replace any standard text book. These topics are very old and already standardized.

There are 3 kinds of Text Books

- The thin Books - Good students who want more details are not happy with these. Average students who need more examples are not happy with these. Most students who want to “Cram” quickly and pass somehow find the thin books “good” as they have to read less !!

- The Thick Books - Most students do not like these, as they want to read as less as possible. Average students are “busy” with many other things and have no time to read all these.

- The Average sized Books - Good students do not get all details in any one book. Most bad students do not want to read books of “this much thickness“ also !!

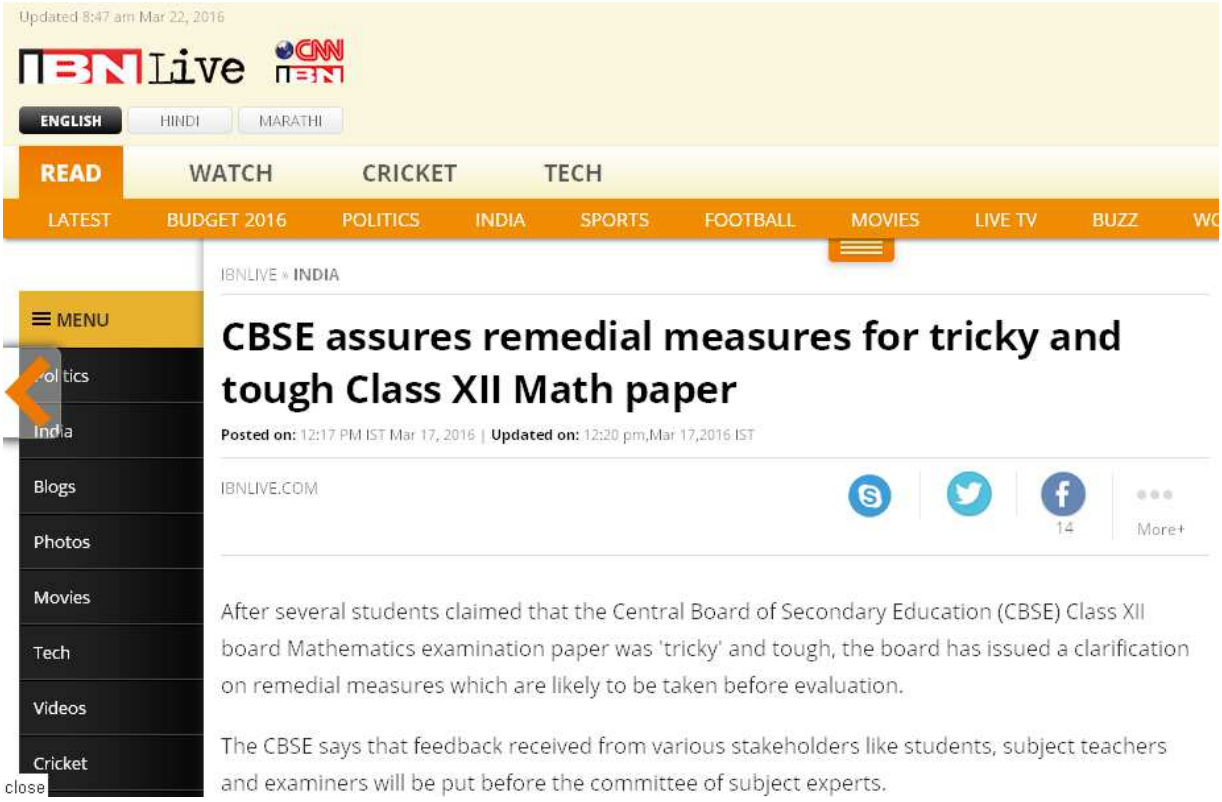
We know there can be no shoe that’s fits in all.

Printed books are not e-Books! Can’t be downloaded and kept in hard-disc for reading “later” ....

So if you read this book later, you will get all kinds of examples in a single place. This becomes a very good “Reference Material”. I sincerely wish that all find this “very useful”.

Students who do not practice lots of problems, do not do well. The rules of “doing well” had never changed .... Will never change !

After 2016 CBSE Mathematics exam, lots of students complained that the paper was tough!



Updated 8:47 am Mar 22, 2016

ENGLISH HINDI MARATHI

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IBNLIVE » INDIA

CBSE assures remedial measures for tricky and tough Class XII Math paper

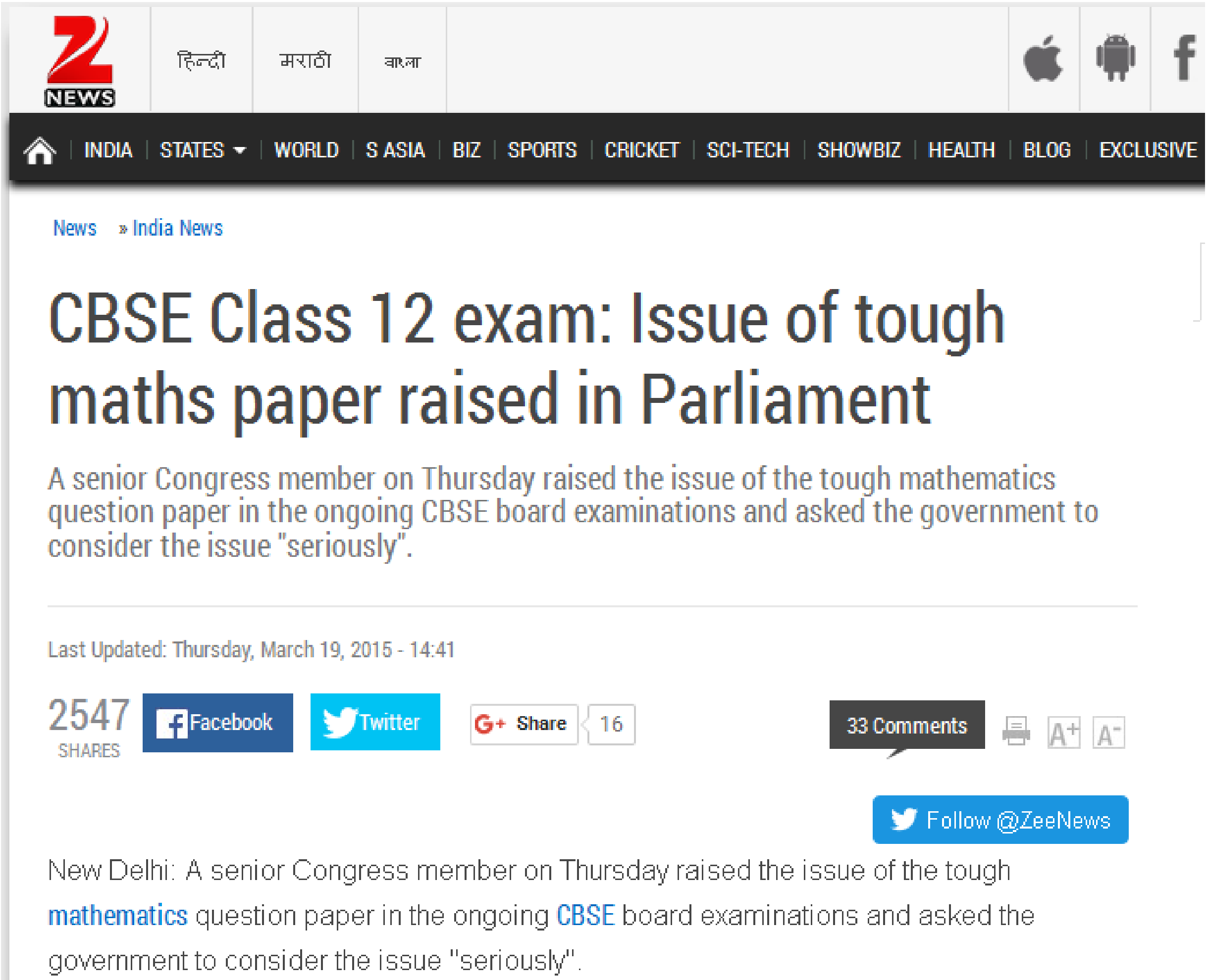
Posted on: 12:17 PM IST Mar 17, 2016 | **Updated on:** 12:20 pm,Mar 17,2016 IST

IBNLIVE.COM

After several students claimed that the Central Board of Secondary Education (CBSE) Class XII board Mathematics examination paper was 'tricky' and tough, the board has issued a clarification on remedial measures which are likely to be taken before evaluation.

The CBSE says that feedback received from various stakeholders like students, subject teachers and examiners will be put before the committee of subject experts.

In 2015 also the same complain was there by many students



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News » India News

CBSE Class 12 exam: Issue of tough maths paper raised in Parliament

A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".

Last Updated: Thursday, March 19, 2015 - 14:41

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New Delhi: A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".

In March 2016, students of Karnataka PU-II also complained the same, regarding standard 12 (PU-II Mathematics Exam). Even though the Math Paper was identical to previous year, most students had not even solved the 2015 Question Paper.

Friday, March 25, 2016 - 13:28

The NEWS Minute

HOME NEWS ANDHRA KARNATAKA KERALA TAMIL NADU TELANGANA CULTURE MEDIA BLOG

Exams

Online petition for lenient evaluation of K'taka II PU math paper gets over 8000 supporters

The campaign, which was launched on Monday, has garnered over 8000 supporters

TNM Staff | Wednesday, March 16, 2016 - 09:32

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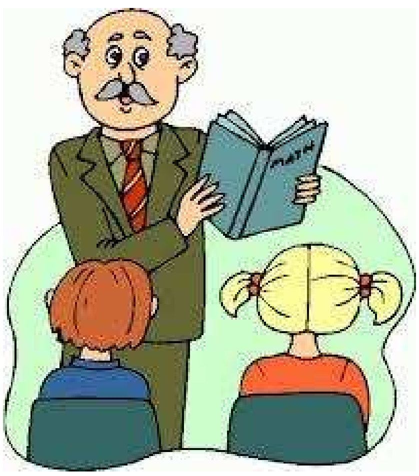
Share Tweet Email Share Reddit

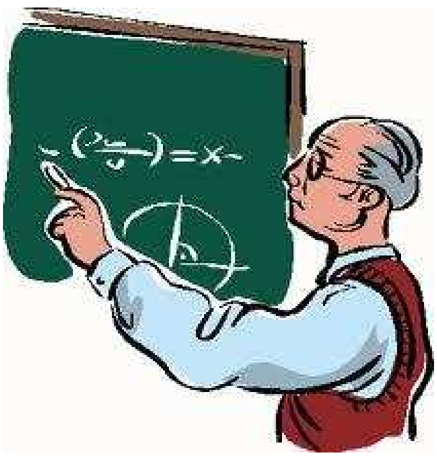
Following a "very tough" math paper that left many II PU students in tears, Saket Ravindran a student launched an online campaign demanding lenient evaluation.

These complains are not new. In fact since last 40 years, (since my childhood), I always see this; every year the same setback, same complain!

In this e-Book I am trying to solve this problem. Those students who practice can learn.

No one can help those who are not studying, or practicing.





Learn more at http://skmclasses.weebly.com/iit-jee-home-tuitions-bangalore.html

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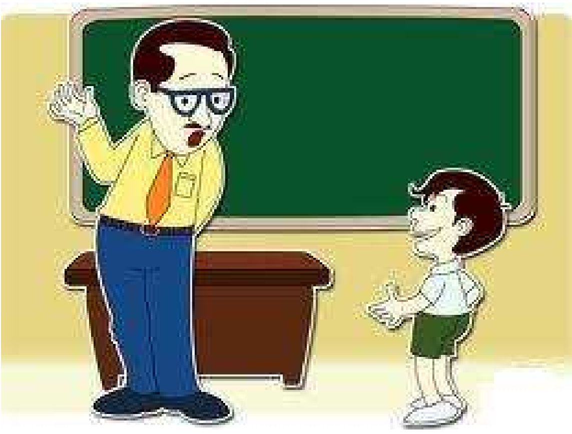


A very polite request :

I wish these e-Books are read only by Boys and Men. Girls and Women, better read something else; learn from somewhere else.

Preface

We all know that in the species "Homo Sapiens ", males are bigger than females. The reasons are explained in standard 10, or 11 (high school) Biology texts. **This shapes or size, influences all of our culture**. Before we recall / understand the reasons once again, let us see some random examples of the influence

Random - 1

If there is a Road rage, then who all fight ? (generally ?). Imagine two cars driven by adult drivers. Each car has a woman of similar age as that of the Man. The cars " touch " or " some issue happens". Who all comes out and fights ? Who all are most probable to drive the cars ?

(Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win)

Random - 2

Heavy metal music artists are all Men. Metallica, Black Sabbath, Motley Crue, Megadeth, Motorhead, AC/DC, Deep Purple, Slayer, Guns & Roses, Led Zeppelin, Aerosmith ….. the list can be in thousands. All these are grown-up Boys, known as Men.

(Men strive for perfection. Men are eager to excel. Men work hard. Men want to win.)

Random - 3

Apart from Marie Curie, only one more woman got Nobel Prize in Physics. (Maria Goeppert Mayer - 1963). So, … almost all are men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 4

The best Tabla Players are all Men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 5

History is all about, which Kings ruled. Kings, their men, and Soldiers went for wars. History is all about wars, fights, and killings by men.

Boys start fighting from school days. Girls do not fight like this

(Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.)

Random - 6

The highest award in Mathematics, the " Fields Medal " is around since decades. Till date only one woman could get that. (Maryam Mirzakhani - 2014). So, … almost all are men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 7

Actor is a gender neutral word. Could the movie like “ Top Gun “ be made with Female actors ? The best pilots, astronauts, Fighters are all Men.

Random - 8

In my childhood had seen a movie named “ The Tower in Inferno “. In the movie when the tall tower is in fire, women were being saved first, as only one lift was working….

Many decades later another movie is made. A box office hit. " The Titanic ". In this also …. As the ship is sinking women are being saved. **Men are disposable**. Men may get their turn later…

Movies are not training programs. Movies do not teach people what to do, or not to do. Movies only reflect the prevalent culture. Men are disposable, is the culture in the society. Knowingly, unknowingly, the culture is depicted in Movies, Theaters, Stories, Poems, Rituals, etc. I or you can't write a story, or make a movie in which after a minor car accident the Male passengers keep seating in the back seat, while the both the women drivers come out of the car and start fighting very bitterly on the road. There has been no story in this world, or no movie made, where after an accident or calamity, Men are being helped for safety first, and women are told to wait.

Random - 9

Artists generally follow the prevalent culture of the Society. In paintings, sculptures, stories, poems, movies, cartoon, Caricatures, knowingly / unknowingly, " the prevalent Reality " is depicted. The opposite will not go well with people. If deliberately " the opposite " is shown then it may only become a special art, considered as a special mockery.

Random - 10

Men go to “girl / woman’s house” to marry / win, and bring her to his home. That is a sort of winning her. When a boy gets a “ Girl-Friend “, generally he and his friends consider that as an achievement. The boy who “ got / won “ a girl-friend feels proud. His male friends feel, jealous, competitive and envious. Millions of stories have been written on these themes. Lakhs of movies show this. Boys / Men go for “ bike race “, or say “ Car Race “, where the winner “ gets “ the most beautiful girl of the college.

(Men want to excel. Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.)

Prithviraj Chauhan ‘ went ` to “ pickup “ or “ abduct “ or “ win “ or “ bring “ his love. There was a Hindi movie (hit) song … “ Pasand ho jaye, to ghar se utha laye “. It is not other way round. Girls do not go to Boy’s house or man’s house to marry. Nor the girls go in a gang to “ pick-up “ the boy / man and bring him to their home / place / den.

Random - 11

Rich people; often are very hard working. Successful business men, establish their business (empire), amass lot of wealth, with lot of difficulty. Lots of sacrifice, lots of hard work, gets into this. Rich people’s wives had no contribution in this wealth creation. Women are smart, and successful upto the extent to choose the right/rich man to marry. So generally what happens in case of Divorces ? Search the net on “ most costly divorces “ and you will know. The women;(who had no contribution at all, in setting up the business / empire), often gets in Billions, or several Millions in divorce settlements.

Number 1

Rupert & Anna Murdoch -- $1.7 billion

One of the richest men in the world, Rupert Murdoch developed his worldwide media empire when he inherited his father's Australian newspaper in 1952. He married Anna Murdoch in the '60s and they remained together for 32 years, springing off three children.

They split amicably in 1998 but soon Rupert forced Anna off the board of News Corp and the gloves came off. The divorce was finalized in June 1999 when Rupert agreed to let his ex-wife leave with $1.7 billion worth of his assets, $110 million of it in cash. Seventeen days later, Rupert married Wendi Deng, one of his employees.

Ted Danson & Casey Coates -- $30 million

Ted Danson's claim to fame is undoubtedly his decade-long stint as Sam Malone on NBC's celebrated sitcom Cheers . While he did other TV shows and movies, he will always be known as the bartender of that place where everybody knows your name. He met his future first bride Casey, a designer, in 1976 while doing Erhard Seminars Training.

Ten years his senior, she suffered a paralyzing stroke while giving birth to their first child in 1979. In order to nurse her back to health, Danson took a break from acting for six months. But after two children and 15 years of marriage, the infatuation fell to pieces. Danson had started seeing Whoopi Goldberg while filming the comedy, Made in America and this precipitated the 1992 divorce. Casey got $30 million for her trouble.

See https://zookeepersblog.wordpress.com/misandry-and-men-issues-a-short-summary-at-single-place/

See http://skmclasses.kinja.com/save-the-male-1761788732

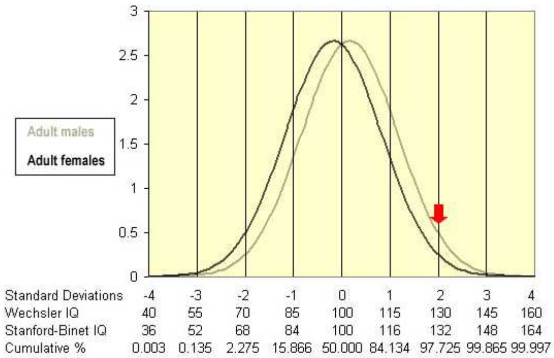
It was Boys and Men, who brought the girls / women home. The Laws are biased, completely favoring women. The men are paying for their own mistakes.

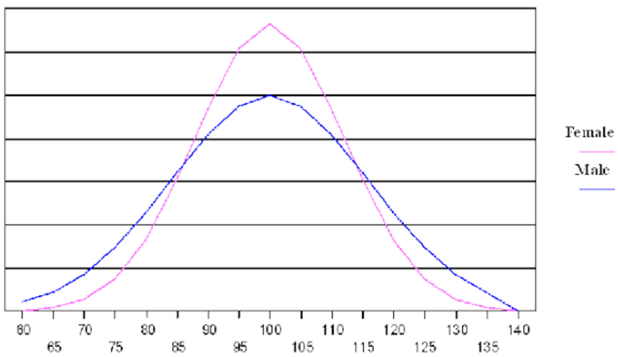
See https://zookeepersblog.wordpress.com/biased-laws/

(Man brings the Woman home. When she leaves, takes away her share of big fortune!)

Random - 12

A standardized test of Intelligence will never be possible. It never happened before, nor ever will happen in future; where the IQ test results will be acceptable by all. In the net there are thousands of charts which show that the intelligence scores of girls / women are lesser. Debates of Trillion words, does not improve performance of Girls.

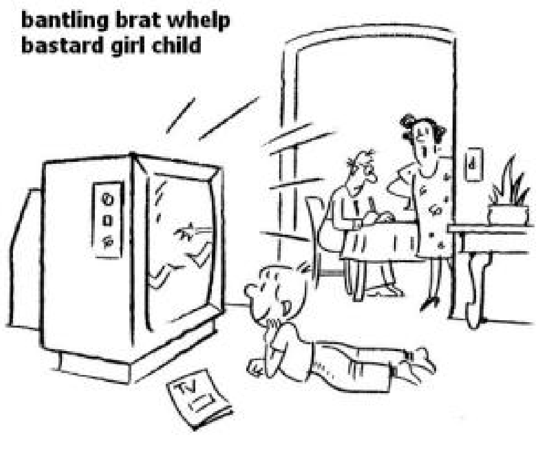




I am not wasting a single second debating or discussing with anyone, on this. I am simply accepting ALL the results. IQ is only one of the variables which is required for success in life. Thousands of books have been written on " Networking Skills ", EQ (Emotional Quotient), Drive, Dedication, Focus, " Tenacity towards the end goal " … etc. In each criteria, and in all together, women (in general) do far worse than men. Bangalore is known as " ….. capital of India ". [Fill in the blanks]. The blanks are generally filled as " Software Capital ", " IT Capital ", " Startup Capital ", etc. I am member in several startup eco-systems / groups. I have attended hundreds of meetings, regarding " technology startups ", or " idea startups ". These meetings have very few women. Starting up new companies are all " Men's Game " / " Men's business ". Only in Divorce settlements women will take their goodies, due to Biased laws. There is no dedication, towards wealth creation, by women.

Random - 13

Many men, as fathers, very unfortunately treat their daughters as " Princess ". Every " non-performing " woman / wife was " princess daughter " of some loving father. Pampering the girls, in name of " equal opportunity ", or " women empowerment ", have led to nothing.



"Please turn it down - Daddy is trying to do your homework."

See http://skmclasses.kinja.com/progressively-daughters-become-monsters-1764484338

See http://skmclasses.kinja.com/vivacious-vixens-1764483974

There can be thousands of more such random examples, where " Bigger Shape / size " of males have influenced our culture, our Society. Let us recall the reasons, that we already learned in standard 10 - 11, Biology text Books. In humans, women have a long gestation period, and also spends many years (almost a decade) to grow, nourish, and stabilize the child. (Million years of habit) Due to survival instinct Males want to inseminate. Boys and Men fight for the " facility (of womb + care) " the girl / woman may provide. Bigger size for males, has a winning advantage. Whoever wins, gets the " woman / facility ". The male who is of " Bigger Size ", has an advantage to win…. Leading to Natural selection over millions of years. In general " Bigger Males "; the " fighting instinct " in men; have led to wars, and solving tough problems (Mathematics, Physics, Technology, startups of new businesses, Wealth creation, Unreasonable attempts to make things [such as planes], Hard work ….)

So let us see the IIT-JEE results of girls. Statistics of several years show that there are around 17, (or less than 20) girls in top 1000 ranks, at all India level. Some people will yet not understand the performance, till it is said that … year after year we have around 980 boys in top 1000 ranks. Generally we see only 4 to 5 girls in top 500. In last 50 years not once any girl topped in IIT-JEE advanced. Forget about Single digit ranks, double digit ranks by girls have been extremely rare. It is all about " good boys ", " hard working ", " focused ", "**Bel-esprit**" boys.

In 2015, Only 2.6% of total candidates who qualified are girls (upto around 12,000 rank). while 20% of the Boys, amongst all candidates qualified. The Total number of students who appeared for the exam were around 1.4 million for IIT-JEE main. Subsequently 1.2 lakh (around 120 thousands) appeared for IIT-JEE advanced.

IIT-JEE results and analysis, of many years is given at https://zookeepersblog.wordpress.com/iit-jee-iseet-main-and-advanced-results/

In Bangalore it is rare to see a girl with rank better than 1000 in IIT-JEE advanced. We hardly see 6-7 boys with rank better than 1000. Hardly 2-3 boys get a rank better than 500.

See http://skmclasses.weebly.com/everybody-knows-so-you-should-also-know.html

Thousands of people are exposing the heinous crimes that Motherly Women are doing, or Female Teachers are committing. See https://www.facebook.com/WomenCriminals/

Some Random Examples must be known by all

Mother Admits On Facebook to Sleeping with 15 Yr Old Son, They Have a Baby Together - Alwayzturntup

Sometimes it hard to believe w From Alwayzturntup

ALWAYZTURNTUP.ME

It is extremely unfortunate that the " woman empowerment " has created. This is the kind of society and women we have now. I and many other sensible Men hate such women. Be away from such women, be aware of reality.

'Sex with my son is incredible - we're in love and we want a baby'

Ben Ford, who ditched his wife when he met his mother Kim West after 30 years, claims what the couple are doing 'isn't incest'

MIRROR.CO.UK

Former Shelbyville ISD teacher who had sex with underage student gets 3 years in prison

After a two day break over the weekend, A Shelby County jury was back in the courtroom looking to conclude the trial of a former Shelbyville ISD teacher who had...

KLTV.COM | BY CALEB BEAMES

Woman sent to jail for the rest of her life after raping her four grandchildren is described as the 'most evil person' the judge has ever seen

Edwina Louis rape...

See More

Woman sent to jail for raping her four grandchildren

A Ohio grandmother has been sentenced to four consecutive life terms after being found guilty of the rape of her own grandchildren. Edwina Louis, 53, will spend the rest of her life behind bars.

DAILYMAIL.CO.UK

http://www.thenativecanadian.com/…/eastern-ontario-teacher-…

The N.C. Chronicles.: Eastern Ontario teacher charged with 36 sexual offences

anti feminism, Child abuse, children's rights, Feminist hypocrisy,

THENATIVECANADIAN.COM | BY BLACKWOLF

Hyd woman kills newborn boy as she wanted daughter - Times of India

Having failed to bear a daughter for the third time, a shopkeeper's wife slit the throat of her 24day-old son with a shaving blade and left him to die in a street on Tuesday night.Purnima's first child was a stillborn boy, followed by another boy born five years ago.

TIMESOFINDIA.INDIATIMES.COM

Montgomery's son, Alan Vonn Webb, took the stand and was a key witness in her conviction.

"I want to see her placed somewhere she can never do that to children …

See More

Woman sentenced to 40 years in prison for raping her children

A Murfreesboro mother found guilty of raping her own children learned her fate on Wednesday.

WAFF.COM | BY DENNIS FERRIER

gentler sex? Violence against men.'s photo.

Women, the gentler sex? Violence against men. Like Page

April 8 at 1:38am

In fact, the past decade has seen a dramatic increase in the number of incidents of women raping and sexually assaulting boys and men. On May 2014, Jezebel repo…

End violence against women . . . .

North Carolina Grandma Eats Her Daughter's New Born Baby After Smoking Bath Salts

Henderson, North Carolina– A North Carolina grandmother of 4 and recovering drug addict, is now in custody after she allegedly ate her daughter's newborn baby....

AZ-365.TOP

28-Year-Old Texas Teacher Accused of Sending Nude Picture to 14-Year-Old Former Student

BREITBART.COM

http://latest.com/.../attractive-girl-gang-lured-men-alleywa.../

Attractive Girl Gang Lured Men Into Alleyways Where Female Body Builder Would Attack Them

A Mexican street gang made up entirely of women has been accused of using their feminine wiles to lure men into alleyways and then beating them up and...

LATEST.COM

http://www.wfmj.com/.../youngstown-woman-convicted-of-raping-...

Youngstown woman convicted of raping a 1 year old is back in jail

A Youngstown woman who went to prison for raping a 1-year-old boy fifteen years ago is in trouble with the law again.

WFMJ.COM

Women are raping boys and young men

Rape advocacy has been maligned and twisted into a political agenda controlled by radicalized activists. Tim Patten takes a razor keen and well supported look into the manufactured rape culture and...

AVOICEFORMEN.COM | BY TIM PATTEN

End violence against women . . . .

Bronx Woman Convicted of Poisoning and Drowning Her Children

Lisette Bamenga researched methods on the Internet before she killed her son and daughter in 2012.

NYTIMES.COM | BY MARC SANTORA

A Russian-born newlywed slowly butchered her German husband — feeding strips of his flesh to their dog until he took his last breath. Svetlana Batukova, 46, was...

See More

She killed her husband and then fed him to her dog: police

A Russian-born newlywed butchered her German hubby — and fed strips of his flesh to her pooch, authorities said. Svetlana Batukova offed Horst Hans Henkels at their...

NYPOST.COM

Daily Mail
January 15, 2015 ·

Mother charged with rape and sodomy of her son's 12-year-old friend

Mom, 30, 'raped and had oral sex with her son's 12-year-old friend'

Nicole Marie Smith, 30, (pictured) of St Charles County, Missouri, has been jailed after she allegedly targeted the 12-year-old boy at her home.

DAILYM.AI

April 4 at 4:48am ·

Female prison officers commit 90pc of sex assaults on male teens in US juvenile detention centres

Lawsuit in Idaho highlights the prevalence of sexual victimization of juvenile offenders.

IBTIMES.CO.UK | BY NICOLE ROJAS

This mother filmed herself raping her own son and then sold it to a man for $300. The courts just decide her fate. When you see what she got, you're going to be outraged.

Mother Who Filmed Herself Raping Her 1-Year-Old Son Receives Shocking Sentence

"...then used the money to buy herself a laptop..."

AMERICANEWS.COM

In several countries or rather in several regions of the world, family system has collapsed, due to bad nature and naughty acts of women. Particularly in Britain, and America, almost 50% people are alone, lonely, separated, divorced or failed marriages. In 2013, 48% children were born out of wedlock. It was projected that by 2016, more than 51% children will be born, to unmarried mothers. In these developed countries " paternity fraud " by women, are close to 20%. You can see several articles in the net, and in wikipedia etc. This means 1 out of 5 children are calling a wrong man as dad. The lonely, alone " mothers " are frustrated. They see the children as burden. Love in the Society in general is lost, long time ago. The types of " Mothers " and " Women " we have now …………


This is the type of women we have in this world. These kind of women were also someones daughter
Mother Stabs Her Baby 90 Times With Scissors After He Bit Her While Breastfeeding Him!
Eight-month-old Xiao Bao was discovered by his uncle in a pool of blood Needed 100 stitches after the incident; he is now recovering in hospital Reports say his...
MOMMABUZZ.COM


MY PUSSY MY RULES
NO GOD IN MY VAGINA


NATIONAL GO TOPLESS DAY
AUG 23 @ NOON - 3 PM
NYC - CENTRAL PARK
GOING TOPLESS IS LEGAL IN NY !!!


NUDITY ISN'T SEXUAL


TOPLESS : ÉGALITÉ POUR TOUS OU POUR PERSONNE !
SORRY TO BOTHER YOU, BUT WE ARE CHANGING THE WORLD!

Professor Subhashish Chattopadhyay

Spoon Feeding - Indefinite Integrals

Recall the various tricks, formulae, and rules of solving Indefinite Integrals

(i) $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

(ii) $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C$

(iii) $\int \frac{dx}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C = -\frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C$

(iv) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$

(v) $\int \frac{dx}{\sqrt{x^2-a^2}} = \log|x+\sqrt{x^2-a^2}| + C = \cosh^{-1}\left(\frac{x}{a}\right) + C$

(vi) $\int \frac{dx}{\sqrt{x^2+a^2}} = \log|x+\sqrt{x^2+a^2}| + C = \sinh^{-1}\left(\frac{x}{a}\right) + C$

(vii) $\int \sqrt{x^2+a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2+a^2} + a^2\log|x+\sqrt{x^2+a^2}|\right] + C$

(viii) $\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\sin^{-1}\left(\frac{x}{a}\right)\right] + C$

(ix) $\int \sqrt{x^2-a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2-a^2} - a^2\log|x+\sqrt{x^2-a^2}|\right] + C$

(x) $\int (px+q)\sqrt{ax^2+bx+c}\,dx = \frac{p}{2a}\int (2ax+b)\sqrt{ax^2+bx+c}\,dx$
$+\left(\frac{q-pb}{2a}\right)\int \sqrt{ax^2+bx+c}\,dx$

- $\int e^x dx = e^x$
- $\int e^{ax} dx = \frac{1}{a} e^{ax}$
- $\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2+b^2} (a \cos bx + b \sin bx)$
- $\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2+b^2} (a \sin bx - b \cos bx)$
- $\int a^x dx = \frac{a^x}{lna} + c$

- $\int \log x dx = x(\log x - 1) + c$
- $\int \frac{1}{x} dx = \log |x| + c$
- $\int a^x dx = a^x \log x + c$
- $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$
- $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \frac{x-a}{x+a} + c$

- $\int \csc x \cot x dx = -\csc x + c$
- $\int \csc^2 x dx = -\cot x + c$
- $\int \sec x \tan x dx = \sec x + c$
- $\int \sec^2 x dx = \tan x + c$
- $\int \sin x dx$ = - \cos x + c$
- $\int \cos x dx = \sin x + c$

- $\int (ax+b)^n = \frac{1}{a} \frac{(ax+b)^{n+1}}{n+1} + C$, $n \neq 1
- $\int \frac{dx}{(ax+b)} = \frac{1}{a} \log |ax+b| + C$
- $\int e^{ax+b} = \frac{1}{a} e^{ax+b} + C$
- $\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b) + C$
- $\int \sec^2(ax+b) dx = \frac{1}{a} \tan(ax+b) + C$
- $\int \csc^2(ax+b) dx = \frac{-1}{a} \cot(ax+b) + C$
- $\int \csc(ax+b) \cot(ax+b) dx = \frac{-1}{a} \csc(ax+b) + C$

For Integrals of the form

(i) $\int \frac{dx}{a + b \sin x}$ (ii) $\int \frac{dx}{a + b \cos x}$ (iii) $\int \frac{dx}{a \sin x + b \cos x + c}$

Put $\cos x = \frac{1 - \tan^2 x/2}{1 + \tan^2 x/2}$, $\sin x - \frac{2 \tan x/2}{1 + \tan^2 x/2}$

Some advanced procedures….

$$\int \frac{x^m}{(a+bx)^p}\,dx$$

m is a + ve integer — Put $a + bx = z$

$$\int \frac{dx}{x^m (a+bx)^p}.$$

where either (m and p positive integers) or (m and p are fractions, but $m + p$ = integers > 1) — Put $a + bx = zx$

$$\int x^m (a+bx^n)^p\,dx,$$

where m, n, p are rationals.

(i) p is a + ve integer — Apply Binomial theorem to $(a + bx^n)^p$

(ii) p is a − ve integer — Put $x = z^k$ where k = common denominator of m and n.

(iii) $\frac{m+1}{n}$ is an integer — Put $(a + bx^n) = z^k$ where k = denominator of p.

(iv) $\frac{m+1}{n} + p$ is an integer — Put $a + bx^n = x^n z^k$ where k = denominator of fraction p.

$$\int \frac{x^2\,dx}{x^4 + kx^2 + a^4} = \frac{1}{2}\int \frac{(x^2+a^2)\,dx}{(x^4+kx^2+x^4)} + \frac{1}{2}\int \frac{(x^2-a^2)\,dx}{(x^4+kx^2+a^4)}$$

$$\int \frac{dx}{(x^4+kx^2+a^4)} = \frac{1}{2a^2}\int \frac{(x^2+a^2)\,dx}{(x^4+kx^2+a^4)} - \frac{1}{2a^2}\int \frac{(x^2-a^2)\,dx}{(x^4+kx^2+a^4)}$$

$$\int \frac{dx}{(x^2+k)^n} = \frac{x}{k(2n-2)(x^2+k)^{n-1}} + \frac{(2n-3)}{k(2n-2)}\int \frac{dx}{(x^2+k)^{n-1}}.$$

For $\int \frac{dx}{(Ax^2+Bx+C)\sqrt{(ax^2+bx+c)}}$ we need to substitute $\frac{ax^2+bx+c}{Ax^2+Bx+C} = t^2$

We have hundreds of books for Calculus or Integration (both Indefinite or Definite). Then why am I writing another book ?

In general one of the limitations of books is to miss the discussions. While teaching in a class, a teacher faces many kinds of questions, many kinds of discussions happen. All of that cannot be given in a book. Even if someone gives all the discussions possible, the book will become very fat. The students will not read such a thick book. A video with explanations is one way better than books as we see the intermediate steps being written. There are several examples where in a book the final step is written, while the parts how it was developed is not shown or cannot be shown.

This book is backed-up with hundreds of videos. That makes the book uniquely different from previous or other books.

In the Industry the Indefinite Integrals that we get are generally very complicated. All of those are solved after expanding. We will cover Tayler series expansion, McLaurin's expansion (or Maclaurin series) etc later. Since 1970s, i.e. beginning of modern computers; we have Mathematical packages which can give you the series expansion of any function in milliseconds.

The following problems have to be solved by series expansions only

$\int \frac{dx}{\sqrt{x^3 + a}}$ or say $\int \sqrt{\text{Sec } x}\, dx$

Though definite integral of 0 to π/2 of root sec x can be analytically done.

The Questions students get in books, or exams, or class discussions are easy, cooked ones. These are known to be solved, rather easily. The Teacher will not able to solve any random integral.

There are around 30 or say max 40 " Patterns " which we know how to solve. **Indefinite Integration is a " Pattern Matching " approach. A given problem is modified towards a known pattern, and then finally we say the solution is** …..

If an Indefinite Integral can be solved in x then it also can be solved for x being replaced by x + k or ax+b etc.

So if we know how to solve $\int \sqrt{x^2 + a^2}\, dx$ then we can also solve $\int \sqrt{(x + k)^2 + a^2}\, dx$ as well

There have been several integration problems which depend on the trick of x being interchanged with ax+b Integration of Cos (ax + b) / Cos x is easy. Expand the Numerator and then simplify to integrate. But I have met lot of senior people in my life who just could not integrate cos x by cos (ax + b). Even after several hours of trying, or open book attempts. ☺

$$\int \frac{1}{\sqrt{x^2+2x+2}}dx = \int \frac{1}{\sqrt{(x+1)^2+(1)^2}}dx$$

Let $x+1=t$

$\therefore\ dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{x^2+2x+2}}dx = \int \frac{1}{\sqrt{t^2+1}}dt$$

$$= \log\left|t+\sqrt{t^2+1}\right|+C$$

$$= \log\left|(x+1)+\sqrt{(x+1)^2+1}\right|+C$$

$$= \log\left|(x+1)+\sqrt{x^2+2x+2}\right|+C$$

Some examples are highlighted in the video given below

https://archive.org/details/4Integrations6CosSquareIITJEEMath

https://archive.org/details/AIEEEIntegralCalculusSinXBySinXPiBy4FlipByLinearSubstitution2008

IIT JEE 1979 Integration x square by a plus bx whole square

https://archive.org/details/IITJEE1979IntegrationXSquareByAPlusBxSquare

See the following Videos to learn the basic steps.

I taught several batches in my life. I know how repetitive and boring it is. These are easy concepts and we gain nothing by typing the whole thing once again. I am keeping the study videos, which were recorded while I was doing classroom teaching, at archive.org

Concept of unknown constant. In Indefinite Integral there is an unknown constant which we always write as + c

Integration of Sin6x dx can have infinite possible values

https://archive.org/details/1IntegrationOfSin6xDxCanHaveInfinitePossibleValues

https://archive.org/details/2IntegrationOfSin6xDxCanHaveInfinitePossibleValues

-

How do you integrate Sin 2x ?

We know that

$$\frac{d}{dx}(\cos 2x) = -2\sin 2x$$
$$\Rightarrow \sin 2x = -\frac{1}{2}\frac{d}{dx}(\cos 2x)$$
$$\therefore \sin 2x = \frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right)$$

So ∫Sin 2x dx = (-1/2) Cos 2x + c

-

How do you integrate Cos 3x ?

We know that

$$\frac{d}{dx}(\sin 3x) = 3\cos 3x$$
$$\Rightarrow \cos 3x = \frac{1}{3}\frac{d}{dx}(\sin 3x)$$
$$\therefore \cos 3x = \frac{d}{dx}\left(\frac{1}{3}\sin 3x\right)$$

So ∫Cos 3x dx = (1/3) Sin 3x + c

-

Find $\int e^{2x}$ dx ?

We know that

$$\frac{d}{dx}(e^{2x}) = 2e^{2x}$$
$$\Rightarrow e^{2x} = \frac{1}{2}\frac{d}{dx}(e^{2x})$$
$$\therefore e^{2x} = \frac{d}{dx}\left(\frac{1}{2}e^{2x}\right)$$

So $\int e^{2x}$ dx = (½) e^{2x} + c

Find $\int (ax + b)^2 \, dx$

The anti derivative of $(ax+b)^2$ is the function of x whose derivative is $(ax+b)^2$.

It is known that,

$$\frac{d}{dx}(ax+b)^3 = 3a(ax+b)^2$$
$$\Rightarrow (ax+b)^2 = \frac{1}{3a}\frac{d}{dx}(ax+b)^3$$
$$\therefore (ax+b)^2 = \frac{d}{dx}\left(\frac{1}{3a}(ax+b)^3\right)$$

Therefore, the anti derivative of $(ax+b)^2$ is $\frac{1}{3a}(ax+b)^3$.

-

Find $\int (\sin 2x - 4e^{3x}) dx$

The anti derivative of $(\sin 2x - 4e^{3x})$ is the function of x whose derivative is $(\sin 2x - 4e^{3x})$

It is known that,

$$\frac{d}{dx}\left(-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}\right) = \sin 2x - 4e^{3x}$$

Therefore, the anti derivative of $(\sin 2x - 4e^{3x})$ is $\left(-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}\right)$.

-

$$\int (4e^{3x} + 1) dx$$
$$= 4\int e^{3x} dx + \int 1 dx$$
$$= 4\left(\frac{e^{3x}}{3}\right) + x + C$$
$$= \frac{4}{3}e^{3x} + x + C$$

Find $\int x^2\left(1-\frac{1}{x^2}\right)dx$

Answer :

$$\int x^2\left(1-\frac{1}{x^2}\right)dx$$
$$= \int\left(x^2-1\right)dx$$
$$= \int x^2dx - \int 1dx$$
$$= \frac{x^3}{3} - x + C$$

-

$$\int\left(ax^2+bx+c\right)dx$$
$$= a\int x^2dx + b\int xdx + c\int 1.dx$$
$$= a\left(\frac{x^3}{3}\right) + b\left(\frac{x^2}{2}\right) + cx + C$$
$$= \frac{ax^3}{3} + \frac{bx^2}{2} + cx + C$$

-

$$\int\left(2x^2+e^x\right)dx$$
$$= 2\int x^2dx + \int e^xdx$$
$$= 2\left(\frac{x^3}{3}\right) + e^x + C$$
$$= \frac{2}{3}x^3 + e^x + C$$

-

$$\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$$

$$= \int\left(x+\frac{1}{x}-2\right)dx$$

$$= \int x dx + \int\frac{1}{x}dx - 2\int 1.dx$$

$$= \frac{x^2}{2} + \log|x| - 2x + C$$

-

$$\int\frac{x^3+5x^2-4}{x^2}dx$$

$$= \int\left(x+5-4x^{-2}\right)dx$$

$$= \int x dx + 5\int 1.dx - 4\int x^{-2}dx$$

$$= \frac{x^2}{2} + 5x - 4\left(\frac{x^{-1}}{-1}\right) + C$$

$$= \frac{x^2}{2} + 5x + \frac{4}{x} + C$$

-

$$\int \frac{x^3+3x+4}{\sqrt{x}}dx$$

$$= \int \left(x^{\frac{5}{2}} + 3x^{\frac{1}{2}} + 4x^{-\frac{1}{2}} \right) dx$$

$$= \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{3\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}} + \frac{4\left(x^{\frac{1}{2}}\right)}{\frac{1}{2}} + C$$

$$= \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + C$$

$$= \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8\sqrt{x} + C$$

-

$$\int \frac{x^3-x^2+x-1}{x-1}dx$$

On dividing, we obtain

$$= \int \left(x^2+1\right)dx$$

$$= \int x^2 dx + \int 1 dx$$

$$= \frac{x^3}{3} + x + C$$

-

$$\int (1-x)\sqrt{x}\,dx$$

$$= \int \left(\sqrt{x} - x^{\frac{3}{2}} \right) dx$$

$$= \int x^{\frac{1}{2}} dx - \int x^{\frac{3}{2}} dx$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C$$

$$= \frac{2}{3} x^{\frac{3}{2}} - \frac{2}{5} x^{\frac{5}{2}} + C$$

-

$$\int \sqrt{x}\left(3x^2 + 2x + 3\right) dx$$

$$= \int \left(3x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 3x^{\frac{1}{2}} \right) dx$$

$$= 3\int x^{\frac{5}{2}} dx + 2\int x^{\frac{3}{2}} dx + 3\int x^{\frac{1}{2}} dx$$

$$= 3\left(\frac{x^{\frac{7}{2}}}{\frac{7}{2}} \right) + 2\left(\frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right) + 3\frac{\left(x^{\frac{3}{2}} \right)}{\frac{3}{2}} + C$$

$$= \frac{6}{7} x^{\frac{7}{2}} + \frac{4}{5} x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + C$$

-

$$\int \left(2x - 3\cos x + e^x\right) dx$$

$$= 2\int x\,dx - 3\int \cos x\,dx + \int e^x dx$$

$$= \frac{2x^2}{2} - 3(\sin x) + e^x + C$$

$$= x^2 - 3\sin x + e^x + C$$

-

$$\int\left(2x^2-3\sin x+5\sqrt{x}\right)dx$$

$$=2\int x^2dx-3\int\sin x dx+5\int x^{\frac{1}{2}}dx$$

$$=\frac{2x^3}{3}-3(-\cos x)+5\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$

$$=\frac{2}{3}x^3+3\cos x+\frac{10}{3}x^{\frac{3}{2}}+C$$

-

$$\int\sec x(\sec x+\tan x)dx$$

$$=\int\left(\sec^2 x+\sec x\tan x\right)dx$$

$$=\int\sec^2 xdx+\int\sec x\tan xdx$$

$$=\tan x+\sec x+C$$

-

$$\int\frac{\sec^2 x}{\operatorname{cosec}^2 x}dx$$

$$=\int\frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}}dx$$

$$=\int\frac{\sin^2 x}{\cos^2 x}dx$$

$$=\int\tan^2 xdx$$

$$=\int\left(\sec^2 x-1\right)dx$$

$$=\int\sec^2 xdx-\int 1dx$$

$$=\tan x-x+C$$

-

$$\int \frac{2 - 3\sin x}{\cos^2 x} dx$$

$$= \int \left(\frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x} \right) dx$$

$$= \int 2\sec^2 x dx - 3\int \tan x \sec x dx$$

$$= 2\tan x - 3\sec x + C$$

-

Find $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$

$$= \int x^{\frac{1}{2}} dx + \int x^{-\frac{1}{2}} dx$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= \frac{2}{3} x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$$

-

If $\frac{d}{dx} f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$, then $f(x)$ is

(A) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ (B) $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(C) $x^4 + \frac{1}{x^3} + \frac{129}{8}$ (D) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

It is given that

$$\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$$

$\therefore$Anti derivative of $4x^3 - \frac{3}{x^4} = f(x)$

$$\therefore f(x) = \int 4x^3 - \frac{3}{x^4}\,dx$$

$$f(x) = 4\int x^3 dx - 3\int (x^{-4})dx$$

$$\therefore f(x) = 4\left(\frac{x^4}{4}\right) - 3\left(\frac{x^{-3}}{-3}\right) + C$$

$$f(x) = x^4 + \frac{1}{x^3} + C$$

Also

$$f(2) = 0$$

$$\therefore f(2) = (2)^4 + \frac{1}{(2)^3} + C = 0$$

$$\Rightarrow 16 + \frac{1}{8} + C = 0$$

$$\Rightarrow C = -\left(16 + \frac{1}{8}\right)$$

$$\Rightarrow C = \frac{-129}{8}$$

$$\therefore f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$$

Hence, the correct answer is A.

-

Find $\int \frac{2x}{1+x^2}$ dx

Let $1+x^2=t$

$\therefore 2x\,dx = dt$

$\Rightarrow \int \frac{2x}{1+x^2}dx = \int \frac{1}{t}dt$

$= \log|t| + C$

$= \log|1+x^2| + C$

$= \log(1+x^2) + C$

-

Find $\int \frac{(\log x)^2}{x}$ dx

Let $\log|x| = t$

$\therefore \frac{1}{x}dx = dt$

$\Rightarrow \int \frac{(\log|x|)^2}{x}dx = \int t^2 dt$

$= \frac{t^3}{3} + C$

$= \frac{(\log|x|)^3}{3} + C$

-

Find $\int \frac{1}{x + x\log x}$ dx

$$\frac{1}{x + x\log x} = \frac{1}{x(1 + \log x)}$$

Let 1 + log x = t

$$\therefore \frac{1}{x}dx = dt$$

$$\Rightarrow \int \frac{1}{x(1 + \log x)}dx = \int \frac{1}{t}dt$$

$$= \log|t| + C$$
$$= \log|1 + \log x| + C$$

-

Find $\int$ Sin x Sin (Cos x) dx

Let cos x = t

$\therefore$ - sin x dx = dt

$$\Rightarrow \int \sin x \cdot \sin(\cos x)\,dx = -\int \sin t\,dt$$
$$= -[-\cos t] + C$$
$$= \cos t + C$$
$$= \cos(\cos x) + C$$

-

Find $\int$ Sin (ax + b) Cos (ax + b) dx

$$\sin(ax+b)\cos(ax+b) = \frac{2\sin(ax+b)\cos(ax+b)}{2} = \frac{\sin 2(ax+b)}{2}$$

Let $2(ax+b) = t$

$\therefore 2adx = dt$

$$\Rightarrow \int \frac{\sin 2(ax+b)}{2} dx = \frac{1}{2} \int \frac{\sin t \; dt}{2a}$$
$$= \frac{1}{4a}[-\cos t] + C$$
$$= \frac{-1}{4a}\cos 2(ax+b) + C$$

-

Find $\int \sqrt{ax+b}$ dx

Let $ax + b = t$

$\Rightarrow adx = dt$

$\therefore dx = \frac{1}{a} dt$

$$\Rightarrow \int (ax+b)^{\frac{1}{2}} dx = \frac{1}{a} \int t^{\frac{1}{2}} dt$$

$$= \frac{1}{a}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{2}{3a}(ax+b)^{\frac{3}{2}} + C$$

-

Linear under root not necessarily need to be substituted as t^2 In the problem below substituting t can exchange the complexity, and simplify the problem

Let $(x+2)=t$

$\therefore dx = dt$

$$\Rightarrow \int x\sqrt{x+2}dx = \int(t-2)\sqrt{t}dt$$
$$= \int\left(t^{\frac{3}{2}} - 2t^{\frac{1}{2}}\right)dt$$
$$= \int t^{\frac{3}{2}}dt - 2\int t^{\frac{1}{2}}dt$$
$$= \frac{t^{\frac{5}{2}}}{\frac{5}{2}} - 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$
$$= \frac{2}{5}t^{\frac{5}{2}} - \frac{4}{3}t^{\frac{3}{2}} + C$$
$$= \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$$

-

Find $\int x\sqrt{1+2x^2}$ dx

Let $1 + 2x^2 = t$

$\therefore 4xdx = dt$

$$\Rightarrow \int x\sqrt{1+2x^2}dx = \int\frac{\sqrt{t}dt}{4}$$
$$= \frac{1}{4}\int t^{\frac{1}{2}}dt$$
$$= \frac{1}{4}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$
$$= \frac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}} + C$$

Find $\int (4x+2)\sqrt{x^2+x+1}\,dx$

Let $x^2+x+1=t$

$\therefore (2x+1)dx = dt$

$$\int (4x+2)\sqrt{x^2+x+1}\,dx$$
$$= \int 2\sqrt{t}\,dt$$
$$= 2\int \sqrt{t}\,dt$$
$$= 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$
$$= \frac{4}{3}\left(x^2+x+1\right)^{\frac{3}{2}} + C$$

-

Find $\int \frac{dx}{x-\sqrt{x}}$

$$\frac{1}{x-\sqrt{x}} = \frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)}$$

Let $\left(\sqrt{x}-1\right) = t$

$$\therefore \frac{1}{2\sqrt{x}}dx = dt$$

$$\Rightarrow \int \frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)}dx = \int \frac{2}{t}\,dt$$

$$= 2\log|t| + C$$
$$= 2\log\left|\sqrt{x}-1\right| + C$$

-

Find $\int \frac{x}{\sqrt{x+4}}$ dx

Put x + 4 = t^2 => dx = 2t dt and x = t^2 - 4

So $\int$ (t^2 - 4)/t 2tdt = 2 $\int$ (t^2 - 4) dt = (1/3) t^3 -4t + c put t = root (x + 4)

-

Find $\int (x^3-1)^{\frac{1}{3}} x^5 dx$

Let $x^3 - 1 = t$

$\therefore 3x^2 dx = dt$

$$\Rightarrow \int (x^3-1)^{\frac{1}{3}} x^5 dx = \int (x^3-1)^{\frac{1}{3}} x^3 \cdot x^2 dx$$

$$= \int t^{\frac{1}{3}} (t+1) \frac{dt}{3}$$

$$= \frac{1}{3} \int \left(t^{\frac{4}{3}} + t^{\frac{1}{3}} \right) dt$$

$$= \frac{1}{3} \left[\frac{t^{\frac{7}{3}}}{\frac{7}{3}} + \frac{t^{\frac{4}{3}}}{\frac{4}{3}} \right] + C$$

$$= \frac{1}{3} \left[\frac{3}{7} t^{\frac{7}{3}} + \frac{3}{4} t^{\frac{4}{3}} \right] + C$$

$$= \frac{1}{7} (x^3-1)^{\frac{7}{3}} + \frac{1}{4} (x^3-1)^{\frac{4}{3}} + C$$

-

Find $\int \frac{x^2}{\left(2+3x^3\right)^3}dx$

Let $2+3x^3=t$

$\therefore 9x^2\,dx=dt$

$$\Rightarrow \int \frac{x^2}{\left(2+3x^3\right)^3}dx=\frac{1}{9}\int \frac{dt}{(t)^3}$$

$$=\frac{1}{9}\left[\frac{t^{-2}}{-2}\right]+C$$

$$=\frac{-1}{18}\left(\frac{1}{t^2}\right)+C$$

$$=\frac{-1}{18\left(2+3x^3\right)^2}+C$$

-

Find $\int \frac{1}{x(\log x)^m}dx$

Let $\log x=t$

$\therefore \frac{1}{x}dx=dt$

$$\Rightarrow \int \frac{1}{x(\log x)^m}dx=\int \frac{dt}{(t)^m}$$

$$=\left(\frac{t^{-m+1}}{1-m}\right)+C$$

$$=\frac{(\log x)^{1-m}}{(1-m)}+C$$

-

Find $\int \frac{x}{9-4x^2} dx$

Let $9-4x^2 = t$

$\therefore -8x\, dx = dt$

$$\Rightarrow \int \frac{x}{9-4x^2} dx = \frac{-1}{8} \int \frac{1}{t} dt$$
$$= \frac{-1}{8} \log|t| + C$$
$$= \frac{-1}{8} \log|9-4x^2| + C$$

-

Find $\int e^{2x+3}\, dx$

Let $2x+3 = t$

$\therefore 2dx = dt$

$$\Rightarrow \int e^{2x+3} dx = \frac{1}{2} \int e^t dt$$
$$= \frac{1}{2}(e^t) + C$$
$$= \frac{1}{2} e^{(2x+3)} + C$$

-

Find $\int \frac{x}{e^{x^2}} dx$

Let $x^2 = t$

$\therefore 2x dx = dt$

$\Rightarrow \int \frac{x}{e^{x^2}} dx = \frac{1}{2} \int \frac{1}{e^t} dt$

$= \frac{1}{2} \int e^{-t} dt$

$= \frac{1}{2} \left(\frac{e^{-t}}{-1} \right) + C$

$= -\frac{1}{2} e^{-x^2} + C$

$= \frac{-1}{2e^{x^2}} + C$

-

Find $\int \frac{e^{\tan^{-1} x}}{1+x^2} dx$

Let $\tan^{-1} x = t$

$\therefore \frac{1}{1+x^2} dx = dt$

$\Rightarrow \int \frac{e^{\tan^{-1} x}}{1+x^2} dx = \int e^t dt$

$= e^t + C$

$= e^{\tan^{-1} x} + C$

-

Find $\int \frac{e^{2x}-1}{e^{2x}+1} dx$

Dividing numerator and denominator by e^x, we obtain

$$\frac{\frac{(e^{2x}-1)}{e^x}}{\frac{(e^{2x}+1)}{e^x}} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Let $e^x + e^{-x} = t$

$\therefore (e^x - e^{-x})dx = dt$

$$\Rightarrow \int \frac{e^{2x}-1}{e^{2x}+1} dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + C$$

$$= \log\left|e^x + e^{-x}\right| + C$$

-

Find $\int\left(\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\right)dx$

Let $e^{2x}+e^{-2x}=t$

$\therefore\left(2e^{2x}-2e^{-2x}\right)dx=dt$

$\Rightarrow 2\left(e^{2x}-e^{-2x}\right)dx=dt$

$$\Rightarrow \int\left(\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\right)dx=\int\frac{dt}{2t}$$
$$=\frac{1}{2}\int\frac{1}{t}dt$$
$$=\frac{1}{2}\log|t|+C$$
$$=\frac{1}{2}\log\left|e^{2x}+e^{-2x}\right|+C$$

-

Find $\int\tan^2(2x-3)\,dx$

$\tan^2(2x-3)=\sec^2(2x-3)-1$

Let $2x-3=t$

$\therefore 2dx=dt$

$$\Rightarrow \int\tan^2(2x-3)\,dx=\int\left[\left(\sec^2(2x-3)\right)-1\right]dx$$
$$=\frac{1}{2}\int\left(\sec^2 t\right)dt-\int 1dx$$
$$=\frac{1}{2}\int\sec^2 t\,dt-\int 1dx$$
$$=\frac{1}{2}\tan t-x+C$$
$$=\frac{1}{2}\tan(2x-3)-x+C$$

Find $\int \sec^2(7-4x)\,dx$

Let 7 - 4x = t

$\therefore$ - 4dx = dt

$$\therefore \int \sec^2(7-4x)\,dx = \frac{-1}{4}\int \sec^2 t\,dt$$
$$= \frac{-1}{4}(\tan t) + C$$
$$= \frac{-1}{4}\tan(7-4x) + C$$

-

Find $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx$

Let $\sin^{-1} x = t$

$$\therefore \frac{1}{\sqrt{1-x^2}}\,dx = dt$$

$$\Rightarrow \int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx = \int t\,dt$$

$$= \frac{t^2}{2} + C$$
$$= \frac{(\sin^{-1} x)^2}{2} + C$$

-

Find $\int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}\, dx$

$$\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x} = \frac{2\cos x - 3\sin x}{2(3\cos x + 2\sin x)}$$

Let $3\cos x + 2\sin x = t$

$\therefore\ (-3\sin x + 2\cos x)dx = dt$

$$\begin{aligned}\int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}\, dx &= \int \frac{dt}{2t} \\ &= \frac{1}{2}\int \frac{1}{t}\, dt \\ &= \frac{1}{2}\log|t| + C \\ &= \frac{1}{2}\log|2\sin x + 3\cos x| + C\end{aligned}$$

-

Find $\int \frac{\sec^2 x}{(1-\tan x)^2}\, dx$

$$\frac{1}{\cos^2 x(1-\tan x)^2} = \frac{\sec^2 x}{(1-\tan x)^2}$$

Let $(1-\tan x) = t$

$\therefore\ -\sec^2 x dx = dt$

$$\begin{aligned}\Rightarrow \int \frac{\sec^2 x}{(1-\tan x)^2}\, dx &= \int \frac{-dt}{t^2} \\ &= -\int t^{-2} dt \\ &= +\frac{1}{t} + C \\ &= \frac{1}{(1-\tan x)} + C\end{aligned}$$

Find $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

Let $\sqrt{x} = t$

$\therefore\ \frac{1}{2\sqrt{x}}\,dx = dt$

$$\Rightarrow \int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx = 2\int \cos t\,dt$$
$$= 2\sin t + C$$
$$= 2\sin\sqrt{x} + C$$

-

Find $\int \sqrt{\sin 2x}\ \cos 2x\,dx$

Let $\sin 2x = t$

$\therefore\ 2\cos 2x\,dx = dt$

$$\Rightarrow \int \sqrt{\sin 2x}\ \cos 2x\,dx = \frac{1}{2}\int \sqrt{t}\,dt$$
$$= \frac{1}{2}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$
$$= \frac{1}{3}t^{\frac{3}{2}} + C$$
$$= \frac{1}{3}(\sin 2x)^{\frac{3}{2}} + C$$

-

Find $\int \frac{\cos x}{\sqrt{1+\sin x}} dx$

Let $1+\sin x = t$

$\therefore \cos x \, dx = dt$

$$\Rightarrow \int \frac{\cos x}{\sqrt{1+\sin x}} dx = \int \frac{dt}{\sqrt{t}}$$
$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$
$$= 2\sqrt{t} + C$$
$$= 2\sqrt{1+\sin x} + C$$

-

Find $\int \cot x \log \sin x \, dx$

Let $\log \sin x = t$

$$\Rightarrow \frac{1}{\sin x} \cdot \cos x \, dx = dt$$
$$\therefore \cot x \, dx = dt$$

$$\Rightarrow \int \cot x \log \sin x \, dx = \int t \, dt$$
$$= \frac{t^2}{2} + C$$
$$= \frac{1}{2}(\log \sin x)^2 + C$$

-

Find $\int \frac{\sin x}{1+\cos x}\,dx$

Let $1 + \cos x = t$

$\therefore -\sin x\,dx = dt$

$$\Rightarrow \int \frac{\sin x}{1+\cos x}\,dx = \int -\frac{dt}{t}$$
$$= -\log|t| + C$$
$$= -\log|1+\cos x| + C$$

-

Find $\int \frac{\sin x}{(1+\cos x)^2}\,dx$

Let $1 + \cos x = t$

$\therefore -\sin x\,dx = dt$

$$\Rightarrow \int \frac{\sin x}{(1+\cos x)^2}\,dx = \int -\frac{dt}{t^2}$$
$$= -\int t^{-2}dt$$
$$= \frac{1}{t} + C$$
$$= \frac{1}{1+\cos x} + C$$

-

$$
\begin{aligned}
\text{Let } I &= \int \frac{1}{1+\cot x}dx \\
&= \int \frac{1}{1+\frac{\cos x}{\sin x}}dx \\
&= \int \frac{\sin x}{\sin x+\cos x}dx \\
&= \frac{1}{2}\int \frac{2\sin x}{\sin x+\cos x}dx \\
&= \frac{1}{2}\int \frac{(\sin x+\cos x)+(\sin x-\cos x)}{(\sin x+\cos x)}dx \\
&= \frac{1}{2}\int 1\,dx+\frac{1}{2}\int \frac{\sin x-\cos x}{\sin x+\cos x}dx \\
&= \frac{1}{2}(x)+\frac{1}{2}\int \frac{\sin x-\cos x}{\sin x+\cos x}dx
\end{aligned}
$$

Let $\sin x + \cos x = t \Rightarrow (\cos x - \sin x)\, dx = dt$

$$
\begin{aligned}
\therefore I &= \frac{x}{2}+\frac{1}{2}\int \frac{-(dt)}{t} \\
&= \frac{x}{2}-\frac{1}{2}\log|t|+\mathrm{C} \\
&= \frac{x}{2}-\frac{1}{2}\log|\sin x+\cos x|+\mathrm{C}
\end{aligned}
$$

-

$$\text{Let } I = \int \frac{1}{1-\tan x}dx$$
$$= \int \frac{1}{1-\frac{\sin x}{\cos x}}dx$$
$$= \int \frac{\cos x}{\cos x - \sin x}dx$$
$$= \frac{1}{2}\int \frac{2\cos x}{\cos x - \sin x}dx$$
$$= \frac{1}{2}\int \frac{(\cos x - \sin x)+(\cos x + \sin x)}{(\cos x - \sin x)}dx$$
$$= \frac{1}{2}\int 1\,dx + \frac{1}{2}\int \frac{\cos x + \sin x}{\cos x - \sin x}dx$$
$$= \frac{x}{2} + \frac{1}{2}\int \frac{\cos x + \sin x}{\cos x - \sin x}dx$$

Put $\cos x - \sin x = t \Rightarrow (-\sin x - \cos x)\,dx = dt$

$$\therefore I = \frac{x}{2} + \frac{1}{2}\int \frac{-(dt)}{t}$$
$$= \frac{x}{2} - \frac{1}{2}\log|t| + C$$
$$= \frac{x}{2} - \frac{1}{2}\log|\cos x - \sin x| + C$$

-

$$\text{Let } I = \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$$
$$= \int \frac{\sqrt{\tan x} \times \cos x}{\sin x \cos x \times \cos x} dx$$
$$= \int \frac{\sqrt{\tan x}}{\tan x \cos^2 x} dx$$
$$= \int \frac{\sec^2 x\, dx}{\sqrt{\tan x}}$$

Let $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$$\therefore I = \int \frac{dt}{\sqrt{t}}$$
$$= 2\sqrt{t} + C$$
$$= 2\sqrt{\tan x} + C$$

-

Find $\int \frac{(1+\log x)^2}{x} dx$

Let $1 + \log x = t$

$$\therefore \frac{1}{x} dx = dt$$

$$\Rightarrow \int \frac{(1+\log x)^2}{x} dx = \int t^2 dt$$
$$= \frac{t^3}{3} + C$$
$$= \frac{(1+\log x)^3}{3} + C$$

-

Find $\int\left(1+\frac{1}{x}\right)(x+\log x)^2\,dx$

$$\frac{(x+1)(x+\log x)^2}{x}=\left(\frac{x+1}{x}\right)(x+\log x)^2=\left(1+\frac{1}{x}\right)(x+\log x)^2$$

Let $(x+\log x)=t$

$$\therefore\left(1+\frac{1}{x}\right)dx=dt$$

$$\Rightarrow\int\left(1+\frac{1}{x}\right)(x+\log x)^2\,dx=\int t^2\,dt$$

$$=\frac{t^3}{3}+C$$

$$=\frac{1}{3}(x+\log x)^3+C$$

-

Find $\int \frac{x^3 \sin\left(\tan^{-1} x^4\right)}{1+x^8} dx$

Let $x^4 = t$

$\therefore 4x^3 dx = dt$

$$\Rightarrow \int \frac{x^3 \sin\left(\tan^{-1} x^4\right)}{1+x^8} dx = \frac{1}{4} \int \frac{\sin\left(\tan^{-1} t\right)}{1+t^2} dt \quad (1)$$

Let $\tan^{-1} t = u$

$\therefore \frac{1}{1+t^2} dt = du$

From (1), we obtain

$$\int \frac{x^3 \sin\left(\tan^{-1} x^4\right) dx}{1+x^8} = \frac{1}{4} \int \sin u \, du$$

$$= \frac{1}{4}(-\cos u) + C$$

$$= \frac{-1}{4} \cos\left(\tan^{-1} t\right) + C$$

$$= \frac{-1}{4} \cos\left(\tan^{-1} x^4\right) + C$$

-

Find $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$ equals

(A) $10^x - x^{10} + C$ (B) $10^x + x^{10} + C$

(C) $\left(10^x - x^{10}\right)^{-1} + C$ (D) $\log\left(10^x + x^{10}\right) + C$

Answer :

Let $x^{10} + 10^x = t$

$\therefore \left(10x^9 + 10^x \log_e 10\right) dx = dt$

$\Rightarrow \int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx = \int \frac{dt}{t}$

$= \log t + C$

$= \log\left(10^x + x^{10}\right) + C$

Hence, the correct answer is D.

-

Find $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$ equals

(A) $10^x - x^{10} + C$ (B) $10^x + x^{10} + C$

(C) $\left(10^x - x^{10}\right)^{-1} + C$ (D) $\log\left(10^x + x^{10}\right) + C$

Answer :

Let $x^{10} + 10^x = t$

$\therefore \left(10x^9 + 10^x \log_e 10\right) dx = dt$

$$\Rightarrow \int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx = \int \frac{dt}{t}$$

$$= \log t + C$$

$$= \log\left(10^x + x^{10}\right) + C$$

Hence, the correct answer is D.

-

$$\sin^2(2x+5) = \frac{1-\cos 2(2x+5)}{2} = \frac{1-\cos(4x+10)}{2}$$

$$\Rightarrow \int \sin^2(2x+5)\,dx = \int \frac{1-\cos(4x+10)}{2} dx$$

$$= \frac{1}{2}\int 1\,dx - \frac{1}{2}\int \cos(4x+10)\,dx$$

$$= \frac{1}{2}x - \frac{1}{2}\left(\frac{\sin(4x+10)}{4}\right) + C$$

$$= \frac{1}{2}x - \frac{1}{8}\sin(4x+10) + C$$

-

It is known that, $\sin A\cos B=\frac{1}{2}\{\sin(A+B)+\sin(A-B)\}$

$$\begin{aligned}
\therefore \int \sin 3x\cos 4x\,dx &= \frac{1}{2}\int\{\sin(3x+4x)+\sin(3x-4x)\}\,dx\\
&=\frac{1}{2}\int\{\sin 7x+\sin(-x)\}\,dx\\
&=\frac{1}{2}\int\{\sin 7x-\sin x\}\,dx\\
&=\frac{1}{2}\int \sin 7x\,dx-\frac{1}{2}\int \sin x\,dx\\
&=\frac{1}{2}\left(\frac{-\cos 7x}{7}\right)-\frac{1}{2}(-\cos x)+C\\
&=\frac{-\cos 7x}{14}+\frac{\cos x}{2}+C
\end{aligned}$$

-

It is known that, $\cos A\cos B=\frac{1}{2}\{\cos(A+B)+\cos(A-B)\}$

$$\begin{aligned}
\therefore \int \cos 2x(\cos 4x\cos 6x)\,dx &= \int \cos 2x\left[\frac{1}{2}\{\cos(4x+6x)+\cos(4x-6x)\}\right]dx\\
&=\frac{1}{2}\int\{\cos 2x\cos 10x+\cos 2x\cos(-2x)\}\,dx\\
&=\frac{1}{2}\int\{\cos 2x\cos 10x+\cos^2 2x\}\,dx\\
&=\frac{1}{2}\int\left[\left\{\frac{1}{2}\cos(2x+10x)+\cos(2x-10x)\right\}+\left(\frac{1+\cos 4x}{2}\right)\right]dx\\
&=\frac{1}{4}\int(\cos 12x+\cos 8x+1+\cos 4x)\,dx\\
&=\frac{1}{4}\left[\frac{\sin 12x}{12}+\frac{\sin 8x}{8}+x+\frac{\sin 4x}{4}\right]+C
\end{aligned}$$

-

Let $I = \int \sin^3(2x+1)$

$$\Rightarrow \int \sin^3(2x+1)\,dx = \int \sin^2(2x+1)\cdot\sin(2x+1)\,dx$$
$$= \int \left(1-\cos^2(2x+1)\right)\sin(2x+1)\,dx$$

Let $\cos(2x+1) = t$

$$\Rightarrow -2\sin(2x+1)\,dx = dt$$
$$\Rightarrow \sin(2x+1)\,dx = \frac{-dt}{2}$$

$$\Rightarrow I = \frac{-1}{2}\int\left(1-t^2\right)dt$$
$$= \frac{-1}{2}\left\{t - \frac{t^3}{3}\right\}$$
$$= \frac{-1}{2}\left\{\cos(2x+1) - \frac{\cos^3(2x+1)}{3}\right\}$$
$$= \frac{-\cos(2x+1)}{2} + \frac{\cos^3(2x+1)}{6} + C$$

-

$$\text{Let } I = \int \sin^3 x \cos^3 x \cdot dx$$
$$= \int \cos^3 x \cdot \sin^2 x \cdot \sin x \cdot dx$$
$$= \int \cos^3 x \left(1 - \cos^2 x\right) \sin x \cdot dx$$

$$\text{Let} \cos x = t$$
$$\Rightarrow -\sin x \cdot dx = dt$$
$$\Rightarrow I = -\int t^3 \left(1 - t^2\right) dt$$
$$= -\int \left(t^3 - t^5\right) dt$$
$$= -\left\{\frac{t^4}{4} - \frac{t^6}{6}\right\} + C$$
$$= -\left\{\frac{\cos^4 x}{4} - \frac{\cos^6 x}{6}\right\} + C$$
$$= \frac{\cos^6 x}{6} - \frac{\cos^4 x}{4} + C$$

-

Find $\int \frac{1-\cos x}{1+\cos x}dx$

$$\left[2\sin^2\frac{x}{2} = 1-\cos x \text{ and } 2\cos^2\frac{x}{2} = 1+\cos x\right]$$

$$\frac{1-\cos x}{1+\cos x} = \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$= \tan^2\frac{x}{2}$$

$$= \left(\sec^2\frac{x}{2} - 1\right)$$

$$\therefore \int \frac{1-\cos x}{1+\cos x}dx = \int\left(\sec^2\frac{x}{2} - 1\right)dx$$

$$= \left[\frac{\tan\frac{x}{2}}{\frac{1}{2}} - x\right] + C$$

$$= 2\tan\frac{x}{2} - x + C$$

-

$$\left[\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} \text{ and } \cos x = 2\cos^2\frac{x}{2} - 1\right]$$

$$\frac{\cos x}{1+\cos x} = \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$= \frac{1}{2}\left[1 - \tan^2\frac{x}{2}\right]$$

$$\therefore \int \frac{\cos x}{1+\cos x}dx = \frac{1}{2}\int\left(1 - \tan^2\frac{x}{2}\right)dx$$

$$= \frac{1}{2}\int\left(1 - \sec^2\frac{x}{2} + 1\right)dx$$

$$= \frac{1}{2}\int\left(2 - \sec^2\frac{x}{2}\right)dx$$

$$= \frac{1}{2}\left[2x - \frac{\tan\frac{x}{2}}{\frac{1}{2}}\right] + C$$

$$= x - \tan\frac{x}{2} + C$$

-

$$\begin{aligned}\sin^4 x &= \sin^2 x \sin^2 x \\ &= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right) \\ &= \frac{1}{4}(1-\cos 2x)^2 \\ &= \frac{1}{4}\left[1+\cos^2 2x - 2\cos 2x\right] \\ &= \frac{1}{4}\left[1+\left(\frac{1+\cos 4x}{2}\right)-2\cos 2x\right] \\ &= \frac{1}{4}\left[1+\frac{1}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right] \\ &= \frac{1}{4}\left[\frac{3}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right]\end{aligned}$$

$$\begin{aligned}\therefore \int \sin^4 x\, dx &= \frac{1}{4}\int\left[\frac{3}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right] dx \\ &= \frac{1}{4}\left[\frac{3}{2}x+\frac{1}{2}\left(\frac{\sin 4x}{4}\right)-\frac{2\sin 2x}{2}\right]+C \\ &= \frac{1}{8}\left[3x+\frac{\sin 4x}{4}-2\sin 2x\right]+C\end{aligned}$$

-

$$\cos^4 2x = \left(\cos^2 2x\right)^2$$
$$= \left(\frac{1+\cos 4x}{2}\right)^2$$
$$= \frac{1}{4}\left[1+\cos^2 4x + 2\cos 4x\right]$$
$$= \frac{1}{4}\left[1+\left(\frac{1+\cos 8x}{2}\right)+2\cos 4x\right]$$
$$= \frac{1}{4}\left[1+\frac{1}{2}+\frac{\cos 8x}{2}+2\cos 4x\right]$$
$$= \frac{1}{4}\left[\frac{3}{2}+\frac{\cos 8x}{2}+2\cos 4x\right]$$
$$\therefore \int \cos^4 2x\, dx = \int\left(\frac{3}{8}+\frac{\cos 8x}{8}+\frac{\cos 4x}{2}\right)dx$$
$$= \frac{3}{8}x+\frac{\sin 8x}{64}+\frac{\sin 4x}{8}+C$$

-

$$\left[\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2};\ \cos x = 2\cos^2\frac{x}{2}-1\right]$$

$$\frac{\sin^2 x}{1+\cos x} = \frac{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}$$
$$= \frac{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$
$$= 2\sin^2\frac{x}{2}$$
$$= 1-\cos x$$
$$\therefore \int\frac{\sin^2 x}{1+\cos x}dx = \int(1-\cos x)dx$$
$$= x-\sin x+C$$

$$\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha} = \frac{-2\sin\frac{2x+2\alpha}{2}\sin\frac{2x-2\alpha}{2}}{-2\sin\frac{x+\alpha}{2}\sin\frac{x-\alpha}{2}} \quad \left[\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$= \frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= \frac{\left[2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right)\right]\left[2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)\right]}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= 4\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$$

$$= 2\left[\cos\left(\frac{x+\alpha}{2}+\frac{x-\alpha}{2}\right)+\cos\frac{x+\alpha}{2}-\frac{x-\alpha}{2}\right]$$

$$= 2\left[\cos(x)+\cos\alpha\right]$$

$$= 2\cos x + 2\cos\alpha$$

$$\therefore \int\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}dx = \int 2\cos x + 2\cos\alpha$$

$$= 2\left[\sin x + x\cos\alpha\right] + C$$

-

$$\frac{\cos x-\sin x}{1+\sin 2x}=\frac{\cos x-\sin x}{\left(\sin^2 x+\cos^2 x\right)+2\sin x\cos x}$$

$$\left[\sin^2 x+\cos^2 x=1;\ \ \sin 2x=2\sin x\cos x\right]$$

$$=\frac{\cos x-\sin x}{\left(\sin x+\cos x\right)^2}$$

Let $\sin x+\cos x=t$

$\therefore\left(\cos x-\sin x\right)dx=dt$

$$\Rightarrow\int\frac{\cos x-\sin x}{1+\sin 2x}dx=\int\frac{\cos x-\sin x}{\left(\sin x+\cos x\right)^2}dx$$

$$=\int\frac{dt}{t^2}$$

$$=\int t^{-2}dt$$

$$=-t^{-1}+C$$

$$=-\frac{1}{t}+C$$

$$=\frac{-1}{\sin x+\cos x}+C$$

-

$$\tan^3 2x \sec 2x = \tan^2 2x \tan 2x \sec 2x$$
$$= \left(\sec^2 2x - 1\right)\tan 2x \sec 2x$$
$$= \sec^2 2x \cdot \tan 2x \sec 2x - \tan 2x \sec 2x$$
$$\therefore \int \tan^3 2x \sec 2x \, dx = \int \sec^2 2x \tan 2x \sec 2x \, dx - \int \tan 2x \sec 2x \, dx$$
$$= \int \sec^2 2x \tan 2x \sec 2x \, dx - \frac{\sec 2x}{2} + C$$

Let $\sec 2x = t$

$\therefore 2 \sec 2x \tan 2x \, dx = dt$

$$\therefore \int \tan^3 2x \sec 2x \, dx = \frac{1}{2}\int t^2 dt - \frac{\sec 2x}{2} + C$$
$$= \frac{t^3}{6} - \frac{\sec 2x}{2} + C$$
$$= \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + C$$

-

$$\tan^4 x$$
$$= \tan^2 x \cdot \tan^2 x$$
$$= \left(\sec^2 x - 1\right)\tan^2 x$$
$$= \sec^2 x \tan^2 x - \tan^2 x$$
$$= \sec^2 x \tan^2 x - \left(\sec^2 x - 1\right)$$
$$= \sec^2 x \tan^2 x - \sec^2 x + 1$$

$$\therefore \int \tan^4 x\, dx = \int \sec^2 x \tan^2 x\, dx - \int \sec^2 x\, dx + \int 1 \cdot dx$$
$$= \int \sec^2 x \tan^2 x\, dx - \tan x + x + C \qquad \ldots(1)$$

Consider $\int \sec^2 x \tan^2 x\, dx$

Let $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$$\Rightarrow \int \sec^2 x \tan^2 x dx = \int t^2 dt = \frac{t^3}{3} = \frac{\tan^3 x}{3}$$

From equation (1), we obtain

$$\int \tan^4 x\, dx = \frac{1}{3}\tan^3 x - \tan x + x + C$$

-

$$\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} = \frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x}$$
$$= \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x}$$
$$= \tan x \sec x + \cot x \operatorname{cosec} x$$
$$\therefore \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}\, dx = \int \left(\tan x \sec x + \cot x \operatorname{cosec} x\right) dx$$
$$= \sec x - \operatorname{cosec} x + C$$

-

$$\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$$
$$= \frac{\cos 2x + (1 - \cos 2x)}{\cos^2 x} \qquad \left[\cos 2x = 1 - 2\sin^2 x\right]$$
$$= \frac{1}{\cos^2 x}$$
$$= \sec^2 x$$
$$\therefore \int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}\, dx = \int \sec^2 x\, dx = \tan x + C$$

-

$$\frac{\cos 2x}{(\cos x + \sin x)^2} = \frac{\cos 2x}{\cos^2 x + \sin^2 x + 2\sin x \cos x} = \frac{\cos 2x}{1 + \sin 2x}$$
$$\therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2}\, dx = \int \frac{\cos 2x}{(1 + \sin 2x)}\, dx$$
$$\text{Let } 1 + \sin 2x = t$$
$$\Rightarrow 2\cos 2x\, dx = dt$$
$$\therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2}\, dx = \frac{1}{2}\int \frac{1}{t} dt$$
$$= \frac{1}{2}\log|t| + C$$
$$= \frac{1}{2}\log|1 + \sin 2x| + C$$
$$= \frac{1}{2}\log\left|(\sin x + \cos x)^2\right| + C$$
$$= \log|\sin x + \cos x| + C$$

-

$\sin^{-1}(\cos x)$

Let $\cos x = t$

Then, $\sin x = \sqrt{1-t^2}$

$$\Rightarrow (-\sin x)dx = dt$$

$$dx = \frac{-dt}{\sin x}$$

$$dx = \frac{-dt}{\sqrt{1-t^2}}$$

$$\therefore \int \sin^{-1}(\cos x)dx = \int \sin^{-1} t\left(\frac{-dt}{\sqrt{1-t^2}}\right)$$

$$= -\int \frac{\sin^{-1} t}{\sqrt{1-t^2}}dt$$

$$\left[\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

Substituting in (1) we get

$$\int \sin^{-1}(\cos x)\, dx = \frac{-\left[\frac{\pi}{2}-x\right]^2}{2} + C$$

$$= -\frac{1}{2}\left(\frac{\pi^2}{2} + x^2 - \pi x\right) + C$$

$$= -\frac{\pi^2}{8} - \frac{x^2}{2} + \frac{1}{2}\pi x + C$$

$$= \frac{\pi x}{2} - \frac{x^2}{2} + \left(C - \frac{\pi^2}{8}\right)$$

$$= \frac{\pi x}{2} - \frac{x^2}{2} + C_1$$

-

$$\frac{1}{\cos(x-a)\cos(x-b)} = \frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin\left[(x-b)-(x-a)\right]}{\cos(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\frac{\left[\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)\right]}{\cos(x-a)\cos(x-b)}$$

$$= \frac{1}{\sin(a-b)}\left[\tan(x-b)-\tan(x-a)\right]$$

$$\Rightarrow \int\frac{1}{\cos(x-a)\cos(x-b)}dx = \frac{1}{\sin(a-b)}\int\left[\tan(x-b)-\tan(x-a)\right]dx$$

$$= \frac{1}{\sin(a-b)}\left[-\log\left|\cos(x-b)\right|+\log\left|\cos(x-a)\right|\right]$$

$$= \frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right]+C$$

-

$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$ is equal to

A. tan x + cot x + C

B. tan x + cosec x + C

C. - tan x + cot x + C

D. tan x + sec x + C

Answer :

$$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} - \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx$$
$$= \int \left(\sec^2 x - \operatorname{cosec}^2 x \right) dx$$
$$= \tan x + \cot x + C$$

Hence, the correct answer is A.

-

$$\int \frac{e^x (1+x)}{\cos^2 (e^x x)} dx$$

Let $e^x x = t$

$$\Rightarrow \left(e^x \cdot x + e^x \cdot 1 \right) dx = dt$$
$$e^x (x+1) dx = dt$$

$$\therefore \int \frac{e^x (1+x)}{\cos^2 (e^x x)} dx = \int \frac{dt}{\cos^2 t}$$
$$= \int \sec^2 t \, dt$$
$$= \tan t + C$$
$$= \tan \left(e^x \cdot x \right) + C$$

-

$$\sin^2(2x+5) = \frac{1-\cos 2(2x+5)}{2} = \frac{1-\cos(4x+10)}{2}$$

$$\Rightarrow \int \sin^2(2x+5)\,dx = \int \frac{1-\cos(4x+10)}{2}dx$$

$$= \frac{1}{2}\int 1\,dx - \frac{1}{2}\int \cos(4x+10)\,dx$$

$$= \frac{1}{2}x - \frac{1}{2}\left(\frac{\sin(4x+10)}{4}\right) + C$$

$$= \frac{1}{2}x - \frac{1}{8}\sin(4x+10) + C$$

-

It is known that, $\sin A \cos B = \frac{1}{2}\{\sin(A+B) + \sin(A-B)\}$

$$\therefore \int \sin 3x \cos 4x\,dx = \frac{1}{2}\int \{\sin(3x+4x) + \sin(3x-4x)\}\,dx$$

$$= \frac{1}{2}\int \{\sin 7x + \sin(-x)\}\,dx$$

$$= \frac{1}{2}\int \{\sin 7x - \sin x\}\,dx$$

$$= \frac{1}{2}\int \sin 7x\,dx - \frac{1}{2}\int \sin x\,dx$$

$$= \frac{1}{2}\left(\frac{-\cos 7x}{7}\right) - \frac{1}{2}(-\cos x) + C$$

$$= \frac{-\cos 7x}{14} + \frac{\cos x}{2} + C$$

-

AIEEE 2007 - Integration of a Sin x + b Cos x form in denominator easy

$$\int \frac{dx}{a\,\mathrm{Sin}\,x + b\,\mathrm{Cos}\,x}$$

https://archive.org/details/AIEEEIntegrationOfASinXBCosXFormInDenominatorEasy2007

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AIEEE 2008 - Nature of cubic curve, x^3 - p x - q where is maxima, where is minima

https://archive.org/details/AIEEENatureOfCubicCurveX3PXQWhereMaximaWhereMinima2008

AIEEE 2008 - Tricks with monotonously increasing curves, How many real roots ?

https://archive.org/details/AIEEETricksWithMonotonouslyIncreasingCurvesHowManyRealRoots2008

AIEEE 2009 - Tricks with monotonously increasing curves, How many times crosses x-axis

https://archive.org/details/AIEEETricksWithMonotonouslyIncreasingCurvesHowManyTimesCrossesXAxis2009

-

It is known that, $\cos A \cos B = \frac{1}{2}\{\cos(A+B)+\cos(A-B)\}$

$$\therefore \int \cos 2x(\cos 4x \cos 6x)\,dx = \int \cos 2x\left[\frac{1}{2}\{\cos(4x+6x)+\cos(4x-6x)\}\right]dx$$

$$= \frac{1}{2}\int\{\cos 2x \cos 10x + \cos 2x \cos(-2x)\}\,dx$$

$$= \frac{1}{2}\int\{\cos 2x \cos 10x + \cos^2 2x\}\,dx$$

$$= \frac{1}{2}\int\left[\left\{\frac{1}{2}\cos(2x+10x)+\cos(2x-10x)\right\}+\left(\frac{1+\cos 4x}{2}\right)\right]dx$$

$$= \frac{1}{4}\int(\cos 12x + \cos 8x + 1 + \cos 4x)\,dx$$

$$= \frac{1}{4}\left[\frac{\sin 12x}{12}+\frac{\sin 8x}{8}+x+\frac{\sin 4x}{4}\right]+C$$

CBSE 2012 Integral Calculus Sin x Sin 2x Sin 3x dx Just convert Multiple angle and subtractions

$\int$ Sin x Sin 2x Sin 3x dx

It is known that, $\sin A \sin B = \frac{1}{2}\{\cos(A-B)-\cos(A+B)\}$

$$\therefore \int \sin x \sin 2x \sin 3x \, dx = \int \left[\sin x \cdot \frac{1}{2}\{\cos(2x-3x)-\cos(2x+3x)\} \right] dx$$

$$= \frac{1}{2}\int (\sin x \cos(-x) - \sin x \cos 5x) \, dx$$

$$= \frac{1}{2}\int (\sin x \cos x - \sin x \cos 5x) \, dx$$

$$= \frac{1}{2}\int \frac{\sin 2x}{2} \, dx - \frac{1}{2}\int \sin x \cos 5x \, dx$$

$$= \frac{1}{4}\left[\frac{-\cos 2x}{2}\right] - \frac{1}{2}\int \left\{\frac{1}{2}\sin(x+5x)+\sin(x-5x)\right\} dx$$

$$= \frac{-\cos 2x}{8} - \frac{1}{4}\int (\sin 6x + \sin(-4x)) \, dx$$

$$= \frac{-\cos 2x}{8} - \frac{1}{4}\left[\frac{-\cos 6x}{3} + \frac{\cos 4x}{4}\right] + C$$

$$= \frac{-\cos 2x}{8} - \frac{1}{8}\left[\frac{-\cos 6x}{3} + \frac{\cos 4x}{2}\right] + C$$

$$= \frac{1}{8}\left[\frac{\cos 6x}{3} - \frac{\cos 4x}{2} - \cos 2x\right] + C$$

https://archive.org/details/CBSE2012IntegralCalculusSinXSin2xSin3xDxJustConvertMultipleAngleSubtractions

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Differentiation of Sec inverse and then Integration. Integration as reverse of Differentiation.

https://archive.org/details/2DiffSecInverseVariousIntegrationsIITJEEMaths

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Integration of 1 by (Sin x + Cos x)

$$\int \frac{dx}{\sin x + \cos x}$$

https://archive.org/details/2Integration1BySinXCosXIITJEEMaths

-

Integration of 1 by (x^2 - a^2)

$$\int \frac{dx}{x^2 - a^2}$$

https://archive.org/details/2Integration2ByX2A2IITJEEMaths

https://archive.org/details/Integral1ByA2X2CanBeEasilySplitIntoPartialFractionsAndDone

Again Integration of 1 by root (x^2 - a^2)

Again $$\int \frac{dx}{\sqrt{x^2 - a^2}}$$

https://archive.org/details/4IntegrationRootXSquareASquareIITJEEMath

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Integration of Ln x by x plus 1 dx with limits and without limits

$$\int \frac{\ln x \, dx}{x + 1}$$

https://archive.org/details/1IntegrationOfLnXByXPlus1DxWithLimitsAndWithoutLimits

https://archive.org/details/2IntegrationOfLnXByXPlus1DxWithLimitsAndWithoutLimits

-

Integrate sec x by (sec x + tan x)

$$\int \frac{\text{Sec } x \quad dx}{\text{Sec } x + \tan x}$$

https://archive.org/details/IntegralStupidAndSmartWayOfDoingSecXBySecXTanX

https://archive.org/details/2Integration3BySecXTanXIITJEEMaths

-

Integration 1 by (Cos x + Cosec x)

$$\int \frac{dx}{\text{Cos } x + \text{Cosec } x}$$

https://archive.org/details/2Integration4ByCosXCosecXIITJEEMaths

-

Integration of 1 by (Cot x + Cosec x)

$$\int \frac{dx}{\text{Cot } x + \text{Cosec } x}$$

https://archive.org/details/2IntegrationByCotXCosecXIITJEEMaths

-

Integration 1 by (Cot x + Sec x)

$$\int \frac{dx}{\text{Cot } x + \sec x}$$

https://archive.org/details/2IntegrationByCotXSecXIITJEEMaths

-

Integration of 1 by (Tan x + Cosec x)

$$\int \frac{dx}{\tan x + \operatorname{Cosec} x}$$

https://archive.org/details/2IntegrationByTanXCosecXIITJEEMath1

https://archive.org/details/2VariousIntegrationsP5ByTanXCosecXIITJEEMaths

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CBSE 2012 Integrate ∫ Sin x Sin 2x Sin 3x dx

https://archive.org/details/CBSE2012IntegralSinXSin2xSin3xSplitIntoTerms

-

Many examples of Integrals discussed

https://archive.org/details/2VariousIntegrationsP4IITJEEMaths

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A student learns only if he practices a lot. No one else, say a teacher, can practice in his behalf. Only by repeated practice can the student remember.

$\int \frac{1}{x^4 - 1} dx$ can be solved by partial fraction or by dividing both Numerator and Denominator by x^2

Solution by partial fraction

$$\frac{1}{(x^4-1)}=\frac{1}{(x^2-1)(x^2+1)}=\frac{1}{(x+1)(x-1)(1+x^2)}$$

$$\text{Let } \frac{1}{(x+1)(x-1)(1+x^2)}=\frac{A}{(x+1)}+\frac{B}{(x-1)}+\frac{Cx+D}{(x^2+1)}$$

$$1=A(x-1)(x^2+1)+B(x+1)(x^2+1)+(Cx+D)(x^2-1)$$

$$1=A(x^3+x-x^2-1)+B(x^3+x+x^2+1)+Cx^3+Dx^2-Cx-D$$

$$1=(A+B+C)x^3+(-A+B+D)x^2+(A+B-C)x+(-A+B-D)$$

Equating the coefficient of x^3, x^2, x, and constant term, we obtain

$$A+B+C=0$$
$$-A+B+D=0$$
$$A+B-C=0$$
$$-A+B-D=1$$

On solving these equations, we obtain

$$A=-\frac{1}{4},B=\frac{1}{4},C=0,\text{ and } D=-\frac{1}{2}$$

$$\therefore \frac{1}{x^4-1}=\frac{-1}{4(x+1)}+\frac{1}{4(x-1)}-\frac{1}{2(x^2+1)}$$

$$\Rightarrow \int\frac{1}{x^4-1}dx=-\frac{1}{4}\log|x-1|+\frac{1}{4}\log|x-1|-\frac{1}{2}\tan^{-1}x+C$$

$$=\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|-\frac{1}{2}\tan^{-1}x+C$$

Important Integration of x^2 by (x^4 + 1)

$$\int\frac{x^2}{x^4+1}\,dx$$

https://archive.org/details/2VariousIntegrationsP6X2ByX41IITJEEMaths

https://archive.org/details/4IntegrationModificationOfDenominatorIITJEEMath

Find $\int \frac{1}{x(x^n+1)}dx$

Multiplying numerator and denominator by x^{n-1}, we obtain

$$\frac{1}{x(x^n+1)} = \frac{x^{n-1}}{x^{n-1}x(x^n+1)} = \frac{x^{n-1}}{x^n(x^n+1)}$$

Let $x^n = t \Rightarrow x^{n-1}dx = dt$

$$\therefore \int \frac{1}{x(x^n+1)}dx = \int \frac{x^{n-1}}{x^n(x^n+1)}dx = \frac{1}{n}\int \frac{1}{t(t+1)}dt$$

Let $\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{(t+1)}$

$$1 = A(1+t) + Bt \qquad ...(1)$$

Substituting $t = 0, -1$ in equation (1), we obtain

$A = 1$ and $B = -1$

$$\therefore \frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{(1+t)}$$

$$\Rightarrow \int \frac{1}{x(x^n+1)}dx = \frac{1}{n}\int\left\{\frac{1}{t} - \frac{1}{(t+1)}\right\}dx$$

$$= \frac{1}{n}\left[\log|t| - \log|t+1|\right] + C$$

$$= -\frac{1}{n}\left[\log|x^n| - \log|x^n+1|\right] + C$$

$$= \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + C$$

-

Rationalize and Simplify the Integral then split and solve individually

https://archive.org/details/RationalizeAndSimplifyTheIntegralThenSplitAndSolveIndividuallyPart1

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Slightly advanced …. Integration 1 by (a Cos x + b)

$$\int \frac{dx}{a \cos x + b}$$

https://archive.org/details/4Integrations7ByACosXBIITJEEMath

-

Slightly advanced …. Integration 1 by (Sec x + Cosec x)

$$\int \frac{dx}{\sec x + \operatorname{cosec} x}$$

https://archive.org/details/4Integrations8VariousStd1112IITJEEMath

-

Let $I = \int \sin^3(2x+1)$

$$\Rightarrow \int \sin^3(2x+1)\,dx = \int \sin^2(2x+1)\cdot\sin(2x+1)\,dx$$

$$= \int (1-\cos^2(2x+1))\sin(2x+1)\,dx$$

Let $\cos(2x+1) = t$

$$\Rightarrow -2\sin(2x+1)\,dx = dt$$

$$\Rightarrow \sin(2x+1)\,dx = \frac{-dt}{2}$$

$$\Rightarrow I = \frac{-1}{2}\int (1-t^2)\,dt$$

$$= \frac{-1}{2}\left\{t - \frac{t^3}{3}\right\}$$

$$= \frac{-1}{2}\left\{\cos(2x+1) - \frac{\cos^3(2x+1)}{3}\right\}$$

$$= \frac{-\cos(2x+1)}{2} + \frac{\cos^3(2x+1)}{6} + C$$

-

$$\begin{aligned}\text{Let } I &= \int \sin^3 x \cos^3 x \cdot dx \\ &= \int \cos^3 x \cdot \sin^2 x \cdot \sin x \cdot dx \\ &= \int \cos^3 x \left(1-\cos^2 x\right) \sin x \cdot dx\end{aligned}$$

$$\begin{aligned}&\text{Let } \cos x = t \\ &\Rightarrow -\sin x \cdot dx = dt \\ &\Rightarrow I = -\int t^3\left(1-t^2\right)dt \\ &\quad = -\int\left(t^3 - t^5\right)dt \\ &\quad = -\left\{\frac{t^4}{4} - \frac{t^6}{6}\right\} + C \\ &\quad = -\left\{\frac{\cos^4 x}{4} - \frac{\cos^6 x}{6}\right\} + C \\ &\quad = \frac{\cos^6 x}{6} - \frac{\cos^4 x}{4} + C\end{aligned}$$

-

$$\frac{\cos x}{1+\cos x} = \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} \qquad \left[\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} \text{ and } \cos x = 2\cos^2\frac{x}{2} - 1\right]$$

$$= \frac{1}{2}\left[1 - \tan^2\frac{x}{2}\right]$$

$$\begin{aligned}\therefore \int \frac{\cos x}{1+\cos x}dx &= \frac{1}{2}\int\left(1 - \tan^2\frac{x}{2}\right)dx \\ &= \frac{1}{2}\int\left(1 - \sec^2\frac{x}{2} + 1\right)dx \\ &= \frac{1}{2}\int\left(2 - \sec^2\frac{x}{2}\right)dx \\ &= \frac{1}{2}\left[2x - \frac{\tan\frac{x}{2}}{\frac{1}{2}}\right] + C\end{aligned}$$

$$= x - \tan\frac{x}{2} + C$$

-

$$\begin{aligned}
\sin^4 x &= \sin^2 x \sin^2 x \\
&= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right) \\
&= \frac{1}{4}(1-\cos 2x)^2 \\
&= \frac{1}{4}\left[1+\cos^2 2x - 2\cos 2x\right] \\
&= \frac{1}{4}\left[1+\left(\frac{1+\cos 4x}{2}\right) - 2\cos 2x\right] \\
&= \frac{1}{4}\left[1+\frac{1}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right] \\
&= \frac{1}{4}\left[\frac{3}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right]
\end{aligned}$$

$$\begin{aligned}
\therefore \int \sin^4 x \, dx &= \frac{1}{4}\int\left[\frac{3}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right] dx \\
&= \frac{1}{4}\left[\frac{3}{2}x + \frac{1}{2}\left(\frac{\sin 4x}{4}\right) - \frac{2\sin 2x}{2}\right] + C \\
&= \frac{1}{8}\left[3x + \frac{\sin 4x}{4} - 2\sin 2x\right] + C
\end{aligned}$$

-

$$\begin{aligned}\cos^4 2x &= \left(\cos^2 2x\right)^2\\ &= \left(\frac{1+\cos 4x}{2}\right)^2\\ &= \frac{1}{4}\left[1+\cos^2 4x+2\cos 4x\right]\\ &= \frac{1}{4}\left[1+\left(\frac{1+\cos 8x}{2}\right)+2\cos 4x\right]\\ &= \frac{1}{4}\left[1+\frac{1}{2}+\frac{\cos 8x}{2}+2\cos 4x\right]\\ &= \frac{1}{4}\left[\frac{3}{2}+\frac{\cos 8x}{2}+2\cos 4x\right]\\ \therefore \int\cos^4 2x\,dx &= \int\left(\frac{3}{8}+\frac{\cos 8x}{8}+\frac{\cos 4x}{2}\right)dx\\ &= \frac{3}{8}x+\frac{\sin 8x}{64}+\frac{\sin 4x}{8}+C\end{aligned}$$

-

$$\frac{\sin^2 x}{1+\cos x}=\frac{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}\quad\left[\sin x=2\sin\frac{x}{2}\cos\frac{x}{2};\ \cos x=2\cos^2\frac{x}{2}-1\right]$$

$$\begin{aligned}&= \frac{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}{2\cos^2\frac{x}{2}}\\ &= 2\sin^2\frac{x}{2}\\ &= 1-\cos x\\ \therefore \int\frac{\sin^2 x}{1+\cos x}dx &= \int(1-\cos x)dx\\ &= x-\sin x+C\end{aligned}$$

-

$$\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha} = \frac{-2\sin\frac{2x+2\alpha}{2}\sin\frac{2x-2\alpha}{2}}{-2\sin\frac{x+\alpha}{2}\sin\frac{x-\alpha}{2}} \qquad \left[\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$= \frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= \frac{\left[2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right)\right]\left[2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)\right]}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= 4\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$$

$$= 2\left[\cos\left(\frac{x+\alpha}{2}+\frac{x-\alpha}{2}\right)+\cos\frac{x+\alpha}{2}-\frac{x-\alpha}{2}\right]$$

$$= 2\left[\cos(x)+\cos\alpha\right]$$

$$= 2\cos x + 2\cos\alpha$$

$$\therefore \int\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}dx = \int 2\cos x + 2\cos\alpha$$

$$= 2\left[\sin x + x\cos\alpha\right] + C$$

-

$$\frac{\cos x-\sin x}{1+\sin 2x}=\frac{\cos x-\sin x}{\left(\sin^2 x+\cos^2 x\right)+2\sin x\cos x}$$

$$\left[\sin^2 x+\cos^2 x=1;\ \sin 2x=2\sin x\cos x\right]$$

$$=\frac{\cos x-\sin x}{\left(\sin x+\cos x\right)^2}$$

Let $\sin x+\cos x=t$

$\therefore\left(\cos x-\sin x\right)dx=dt$

$$\Rightarrow\int\frac{\cos x-\sin x}{1+\sin 2x}dx=\int\frac{\cos x-\sin x}{\left(\sin x+\cos x\right)^2}dx$$

$$=\int\frac{dt}{t^2}$$

$$=\int t^{-2}dt$$

$$=-t^{-1}+C$$

$$=-\frac{1}{t}+C$$

$$=\frac{-1}{\sin x+\cos x}+C$$

-

$$\tan^3 2x \sec 2x = \tan^2 2x \tan 2x \sec 2x$$
$$= \left(\sec^2 2x - 1\right) \tan 2x \sec 2x$$
$$= \sec^2 2x \cdot \tan 2x \sec 2x - \tan 2x \sec 2x$$
$$\therefore \int \tan^3 2x \sec 2x \, dx = \int \sec^2 2x \tan 2x \sec 2x \, dx - \int \tan 2x \sec 2x \, dx$$
$$= \int \sec^2 2x \tan 2x \sec 2x \, dx - \frac{\sec 2x}{2} + C$$

Let $\sec 2x = t$

$\therefore 2 \sec 2x \tan 2x \, dx = dt$

$$\therefore \int \tan^3 2x \sec 2x \, dx = \frac{1}{2} \int t^2 dt - \frac{\sec 2x}{2} + C$$
$$= \frac{t^3}{6} - \frac{\sec 2x}{2} + C$$
$$= \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + C$$

-

$\tan^4 x$

$= \tan^2 x \cdot \tan^2 x$

$= \left(\sec^2 x - 1\right) \tan^2 x$

$= \sec^2 x \tan^2 x - \tan^2 x$

$= \sec^2 x \tan^2 x - \left(\sec^2 x - 1\right)$

$= \sec^2 x \tan^2 x - \sec^2 x + 1$

$$\therefore \int \tan^4 x \, dx = \int \sec^2 x \tan^2 x \, dx - \int \sec^2 x \, dx + \int 1 \cdot dx$$
$$= \int \sec^2 x \tan^2 x \, dx - \tan x + x + C \qquad \ldots(1)$$

Consider $\int \sec^2 x \tan^2 x \, dx$

Let $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

$$\Rightarrow \int \sec^2 x \tan^2 x dx = \int t^2 dt = \frac{t^3}{3} = \frac{\tan^3 x}{3}$$

From equation (1), we obtain

$$\int \tan^4 x\, dx = \frac{1}{3}\tan^3 x - \tan x + x + C$$

-

$$\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} = \frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x}$$
$$= \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x}$$
$$= \tan x \sec x + \cot x \operatorname{cosec} x$$
$$\therefore \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}\, dx = \int (\tan x \sec x + \cot x \operatorname{cosec} x)\, dx$$
$$= \sec x - \operatorname{cosec} x + C$$

-

$$\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$$
$$= \frac{\cos 2x + (1 - \cos 2x)}{\cos^2 x} \qquad [\cos 2x = 1 - 2\sin^2 x]$$
$$= \frac{1}{\cos^2 x}$$
$$= \sec^2 x$$
$$\therefore \int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}\, dx = \int \sec^2 x\, dx = \tan x + C$$

-

$$\frac{\cos 2x}{(\cos x+\sin x)^2}=\frac{\cos 2x}{\cos^2 x+\sin^2 x+2\sin x\cos x}=\frac{\cos 2x}{1+\sin 2x}$$

$$\therefore \int\frac{\cos 2x}{(\cos x+\sin x)^2}\,dx=\int\frac{\cos 2x}{(1+\sin 2x)}\,dx$$

Let $1+\sin 2x=t$

$\Rightarrow 2\cos 2x\,dx=dt$

$$\therefore \int\frac{\cos 2x}{(\cos x+\sin x)^2}\,dx=\frac{1}{2}\int\frac{1}{t}dt$$

$$=\frac{1}{2}\log|t|+C$$

$$=\frac{1}{2}\log|1+\sin 2x|+C$$

$$=\frac{1}{2}\log\left|(\sin x+\cos x)^2\right|+C$$

$$=\log|\sin x+\cos x|+C$$

-

$$\frac{1}{\cos(x-a)\cos(x-b)}=\frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$$

$$=\frac{1}{\sin(a-b)}\left[\frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)}\right]$$

$$=\frac{1}{\sin(a-b)}\frac{[\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)]}{\cos(x-a)\cos(x-b)}$$

$$=\frac{1}{\sin(a-b)}[\tan(x-b)-\tan(x-a)]$$

$$\Rightarrow \int\frac{1}{\cos(x-a)\cos(x-b)}dx=\frac{1}{\sin(a-b)}\int[\tan(x-b)-\tan(x-a)]dx$$

$$=\frac{1}{\sin(a-b)}\left[-\log|\cos(x-b)|+\log|\cos(x-a)|\right]$$

$$=\frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right]+C$$

$$\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx$$

Let $e^x x = t$

$$\Rightarrow (e^x \cdot x + e^x \cdot 1)dx = dt$$

$$e^x(x+1)dx = dt$$

$$\therefore \int \frac{e^x(1+x)}{\cos^2(e^x x)}dx = \int \frac{dt}{\cos^2 t}$$

$$= \int \sec^2 t \, dt$$

$$= \tan t + C$$

$$= \tan(e^x \cdot x) + C$$

-

Let $x^3 = t$

$\therefore 3x^2 \, dx = dt$

$$\Rightarrow \int \frac{3x^2}{x^6+1}dx = \int \frac{dt}{t^2+1}$$

$$= \tan^1 t + C$$

$$= \tan^{-1}(x^3) + C$$

-

Let $2x = t$

$\therefore 2dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{1+4x^2}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{1+t^2}}$$

$$= \frac{1}{2}\left[\log\left|t + \sqrt{t^2+1}\right|\right] + C \qquad \left[\int \frac{1}{\sqrt{x^2+a^2}}dt = \log\left|x + \sqrt{x^2+a^2}\right|\right]$$

$$= \frac{1}{2}\log\left|2x + \sqrt{4x^2+1}\right| + C$$

Let $2 - x = t$

$\Rightarrow - dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{(2-x)^2+1}}dx = -\int \frac{1}{\sqrt{t^2+1}}dt$$

$$= -\log\left|t+\sqrt{t^2+1}\right| + C \qquad \left[\int \frac{1}{\sqrt{x^2+a^2}}dt = \log\left|x+\sqrt{x^2+a^2}\right|\right]$$

$$= -\log\left|2-x+\sqrt{(2-x)^2+1}\right| + C$$

$$= \log\left|\frac{1}{(2-x)+\sqrt{x^2-4x+5}}\right| + C$$

-

Let $5x = t$

$\therefore 5dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{9-25x^2}}\, dx = \frac{1}{5}\int \frac{1}{9-t^2}\, dt$$

$$= \frac{1}{5}\int \frac{1}{\sqrt{3^2-t^2}}\, dt$$

$$= \frac{1}{5}\sin^{-1}\left(\frac{t}{3}\right) + C$$

$$= \frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right) + C$$

-

Let $\sqrt{2}x^2 = t$

$\therefore\ 2\sqrt{2}x\ dx = dt$

$$\Rightarrow \int \frac{3x}{1+2x^4} dx = \frac{3}{2\sqrt{2}} \int \frac{dt}{1+t^2}$$

$$= \frac{3}{2\sqrt{2}} \left[\tan^{-1} t\right] + C$$

$$= \frac{3}{2\sqrt{2}} \tan^{-1}\left(\sqrt{2}x^2\right) + C$$

-

Let $x^3 = t$

$\therefore 3x^2\ dx = dt$

$$\Rightarrow \int \frac{x^2}{1-x^6} dx = \frac{1}{3} \int \frac{dt}{1-t^2}$$

$$= \frac{1}{3}\left[\frac{1}{2}\log\left|\frac{1+t}{1-t}\right|\right] + C$$

$$= \frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right| + C$$

-

$$\int \frac{x-1}{\sqrt{x^2-1}}\,dx = \int \frac{x}{\sqrt{x^2-1}}\,dx - \int \frac{1}{\sqrt{x^2-1}}\,dx \qquad ...(1)$$

For $\int \frac{x}{\sqrt{x^2-1}}dx$, let $x^2-1=t \Rightarrow 2x\,dx = dt$

$$\therefore \int \frac{x}{\sqrt{x^2-1}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$
$$= \frac{1}{2}\int t^{-\frac{1}{2}}dt$$
$$= \frac{1}{2}\left[2t^{\frac{1}{2}}\right]$$
$$= \sqrt{t}$$
$$= \sqrt{x^2-1}$$

From (1), we obtain

$$\int \frac{x-1}{\sqrt{x^2-1}}dx = \int \frac{x}{\sqrt{x^2-1}}dx - \int \frac{1}{\sqrt{x^2-1}}dx \qquad \left[\int \frac{1}{\sqrt{x^2-a^2}}dt = \log\left|x+\sqrt{x^2-a^2}\right|\right]$$
$$= \sqrt{x^2-1} - \log\left|x+\sqrt{x^2-1}\right| + C$$

-

Let $x^3 = t$

$\Rightarrow 3x^2\,dx = dt$

$$\therefore \int \frac{x^2}{\sqrt{x^6+a^6}}dx = \frac{1}{3}\int \frac{dt}{\sqrt{t^2+\left(a^3\right)^2}}$$
$$= \frac{1}{3}\log\left|t+\sqrt{t^2+a^6}\right| + C$$
$$= \frac{1}{3}\log\left|x^3+\sqrt{x^6+a^6}\right| + C$$

-

Let $\tan x = t$

$\therefore \sec^2 x \, dx = dt$

$$\Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx = \int \frac{dt}{\sqrt{t^2 + 2^2}}$$

$$= \log\left|t + \sqrt{t^2 + 4}\right| + C$$

$$= \log\left|\tan x + \sqrt{\tan^2 x + 4}\right| + C$$

-

$7 - 6x - x^2$ can be written as $7 - \left(x^2 + 6x + 9 - 9\right)$.

Therefore,

$7 - \left(x^2 + 6x + 9 - 9\right)$

$= 16 - \left(x^2 + 6x + 9\right)$

$= 16 - (x+3)^2$

$= (4)^2 - (x+3)^2$

$$\therefore \int \frac{1}{\sqrt{7 - 6x - x^2}} dx = \int \frac{1}{\sqrt{(4)^2 - (x+3)^2}} dx$$

Let $x + 3 = t$

$\Rightarrow dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{(4)^2 - (x+3)^2}} dx = \int \frac{1}{\sqrt{(4)^2 - (t)^2}} dt$$

$$= \sin^{-1}\left(\frac{t}{4}\right) + C$$

$$= \sin^{-1}\left(\frac{x+3}{4}\right) + C$$

-

$(x-1)(x-2)$ can be written as x^2-3x+2.

Therefore,

$$x^2-3x+2$$
$$=x^2-3x+\frac{9}{4}-\frac{9}{4}+2$$
$$=\left(x-\frac{3}{2}\right)^2-\frac{1}{4}$$
$$=\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2$$
$$\therefore \int \frac{1}{\sqrt{(x-1)(x-2)}}dx=\int \frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx$$

-

Let $x-\frac{3}{2}=t$

$\therefore\ dx=dt$

$$\Rightarrow \int \frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx=\int \frac{1}{\sqrt{t^2-\left(\frac{1}{2}\right)^2}}dt$$
$$=\log\left|t+\sqrt{t^2-\left(\frac{1}{2}\right)^2}\right|+C$$
$$=\log\left|\left(x-\frac{3}{2}\right)+\sqrt{x^2-3x+2}\right|+C$$

-

$8+3x-x^2$ can be written as $8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$.

Therefore,

$$8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$$

$$=\frac{41}{4}-\left(x-\frac{3}{2}\right)^2$$

$$\Rightarrow \int \frac{1}{\sqrt{8+3x-x^2}}dx=\int \frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}dx$$

Let $x-\frac{3}{2}=t$

$\therefore\ dx=dt$

$$\Rightarrow \int \frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}dx=\int \frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2-t^2}}dt$$

$$=\sin^{-1}\left(\frac{t}{\frac{\sqrt{41}}{2}}\right)+C$$

$$=\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{41}}{2}}\right)+C$$

$$=\sin^{-1}\left(\frac{2x-3}{\sqrt{41}}\right)+C$$

-

$(x-a)(x-b)$ can be written as $x^2-(a+b)x+ab$.

Therefore,

$$x^2-(a+b)x+ab$$

$$=x^2-(a+b)x+\frac{(a+b)^2}{4}-\frac{(a+b)^2}{4}+ab$$

$$=\left[x-\left(\frac{a+b}{2}\right)\right]^2-\frac{(a-b)^2}{4}$$

$$\Rightarrow\int\frac{1}{\sqrt{(x-a)(x-b)}}dx=\int\frac{1}{\sqrt{\left\{x-\left(\frac{a+b}{2}\right)\right\}^2-\left(\frac{a-b}{2}\right)^2}}dx$$

Let $x-\left(\frac{a+b}{2}\right)=t$

$\therefore\ dx=dt$

$$\Rightarrow\int\frac{1}{\sqrt{\left\{x-\left(\frac{a+b}{2}\right)\right\}^2-\left(\frac{a-b}{2}\right)^2}}dx=\int\frac{1}{\sqrt{t^2-\left(\frac{a-b}{2}\right)^2}}dt$$

$$=\log\left|t+\sqrt{t^2-\left(\frac{a-b}{2}\right)^2}\right|+C$$

$$=\log\left|\left\{x-\left(\frac{a+b}{2}\right)\right\}+\sqrt{(x-a)(x-b)}\right|+C$$

-

$$\text{Let } 4x+1 = A\frac{d}{dx}\left(2x^2+x-3\right)+B$$
$$\Rightarrow 4x+1 = A\left(4x+1\right)+B$$
$$\Rightarrow 4x+1 = 4Ax+A+B$$

Equating the coefficients of x and constant term on both sides, we obtain

$4A = 4 \Rightarrow A = 1$

$A + B = 1 \Rightarrow B = 0$

Let $2x^2 + x - 3 = t$

$\therefore (4x + 1)\ dx = dt$

$$\Rightarrow \int \frac{4x+1}{\sqrt{2x^2+x-3}}dx = \int \frac{1}{\sqrt{t}}dt$$
$$= 2\sqrt{t} + C$$
$$= 2\sqrt{2x^2+x-3} + C$$

-

$$\text{Let } x+2 = A\frac{d}{dx}\left(x^2-1\right)+B \qquad ...(1)$$
$$\Rightarrow x+2 = A(2x)+B$$

Equating the coefficients of x and constant term on both sides, we obtain

$$2A = 1 \Rightarrow A = \frac{1}{2}$$
$$B = 2$$

From (1), we obtain

$$(x+2) = \frac{1}{2}(2x)+2$$
$$\text{Then, } \int \frac{x+2}{\sqrt{x^2-1}}dx = \int \frac{\frac{1}{2}(2x)+2}{\sqrt{x^2-1}}dx$$
$$= \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx + \int \frac{2}{\sqrt{x^2-1}}dx \qquad ...(2)$$
$$\text{In } \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx, \text{ let } x^2-1 = t \Rightarrow 2x\,dx = dt$$
$$\frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$

$$= \frac{1}{2}\left[2\sqrt{t}\right]$$
$$= \sqrt{t}$$
$$= \sqrt{x^2-1}$$
$$\text{Then, } \int \frac{2}{\sqrt{x^2-1}}dx = 2\int \frac{1}{\sqrt{x^2-1}}dx = 2\log\left|x+\sqrt{x^2-1}\right|$$

From equation (2), we obtain

$$\int \frac{x+2}{\sqrt{x^2-1}}dx = \sqrt{x^2-1} + 2\log\left|x+\sqrt{x^2-1}\right| + \text{C}$$

-

$$\frac{6x+7}{\sqrt{(x-5)(x-4)}} = \frac{6x+7}{\sqrt{x^2-9x+20}}$$

Let $6x+7 = A\frac{d}{dx}\left(x^2-9x+20\right)+B$

$\Rightarrow 6x+7 = A(2x-9)+B$

Equating the coefficients of x and constant term, we obtain

$2A = 6 \Rightarrow A = 3$

$-9A + B = 7 \Rightarrow B = 34$

$\therefore 6x + 7 = 3(2x - 9) + 34$

$$\int\frac{6x+7}{\sqrt{x^2-9x+20}} = \int\frac{3(2x-9)+34}{\sqrt{x^2-9x+20}}dx$$

$$= 3\int\frac{2x-9}{\sqrt{x^2-9x+20}}dx + 34\int\frac{1}{\sqrt{x^2-9x+20}}dx$$

Let $I_1 = \int\frac{2x-9}{\sqrt{x^2-9x+20}}dx$ and $I_2 = \int\frac{1}{\sqrt{x^2-9x+20}}dx$

$$\therefore \int\frac{6x+7}{\sqrt{x^2-9x+20}} = 3I_1 + 34I_2 \qquad \ldots(1)$$

Then,

$$I_1 = \int \frac{2x-9}{\sqrt{x^2-9x+20}}dx$$

Let $x^2 - 9x + 20 = t$

$$\Rightarrow (2x-9)dx = dt$$

$$\Rightarrow I_1 = \frac{dt}{\sqrt{t}}$$

$$I_1 = 2\sqrt{t}$$

$$I_1 = 2\sqrt{x^2-9x+20} \qquad (2)$$

and $I_2 = \int \frac{1}{\sqrt{x^2-9x+20}}dx$

$x^2 - 9x + 20$ can be written as $x^2 - 9x + 20 + \frac{81}{4} - \frac{81}{4}$.

Therefore,

$$x^2 - 9x + 20 + \frac{81}{4} - \frac{81}{4}$$

$$= \left(x - \frac{9}{2}\right)^2 - \frac{1}{4}$$

$$= \left(x - \frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$\Rightarrow I_2 = \int \frac{1}{\sqrt{\left(x-\frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}dx$$

$$I_2 = \log\left|\left(x - \frac{9}{2}\right) + \sqrt{x^2-9x+20}\right| \qquad \ldots(3)$$

Substituting equations (2) and (3) in (1), we obtain

$$\int \frac{6x+7}{\sqrt{x^2-9x+20}}dx = 3\left[2\sqrt{x^2-9x+20}\right] + 34\log\left[\left(x-\frac{9}{2}\right) + \sqrt{x^2-9x+20}\right] + C$$

$$= 6\sqrt{x^2-9x+20} + 34\log\left[\left(x-\frac{9}{2}\right) + \sqrt{x^2-9x+20}\right] + C$$

Find $\int \frac{x+2}{\sqrt{4x-x^2}}\,dx$

Let $x+2 = A\frac{d}{dx}\left(4x-x^2\right)+B$

$\Rightarrow x+2 = A\left(4-2x\right)+B$

Equating the coefficients of x and constant term on both sides, we obtain

$$-2A = 1 \Rightarrow A = -\frac{1}{2}$$

$$4A + B = 2 \Rightarrow B = 4$$

$$\Rightarrow (x+2) = -\frac{1}{2}(4-2x)+4$$

$$\therefore \int \frac{x+2}{\sqrt{4x-x^2}}dx = \int \frac{-\frac{1}{2}(4-2x)+4}{\sqrt{4x-x^2}}dx$$

$$= -\frac{1}{2}\int \frac{4-2x}{\sqrt{4x-x^2}}dx + 4\int \frac{1}{\sqrt{4x-x^2}}dx$$

Let $I_1 = \int \frac{4-2x}{\sqrt{4x-x^2}}dx$ and $I_2 \int \frac{1}{\sqrt{4x-x^2}}dx$

$$\therefore \int \frac{x+2}{\sqrt{4x-x^2}}dx = -\frac{1}{2}I_1 + 4I_2 \qquad ...(1)$$

Then, $I_1 = \int \frac{4-2x}{\sqrt{4x-x^2}}dx$

Let $4x - x^2 = t$

$\Rightarrow (4-2x)dx = dt$

$$\Rightarrow I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{4x-x^2} \qquad ...(2)$$

$I_2 = \int \frac{1}{\sqrt{4x-x^2}}dx$

$$\begin{aligned}\Rightarrow 4x - x^2 &= -\left(-4x + x^2\right)\\ &= \left(-4x + x^2 + 4 - 4\right)\\ &= 4 - (x-2)^2\\ &= (2)^2 - (x-2)^2\end{aligned}$$

$$\therefore I_2 = \int \frac{1}{\sqrt{(2)^2 - (x-2)^2}}dx = \sin^{-1}\left(\frac{x-2}{2}\right) \qquad ...(3)$$

Using equations (2) and (3) in (1), we obtain

$$\begin{aligned}\int \frac{x+2}{\sqrt{4x-x^2}}dx &= -\frac{1}{2}\left(2\sqrt{4x-x^2}\right) + 4\sin^{-1}\left(\frac{x-2}{2}\right) + C\\ &= -\sqrt{4x-x^2} + 4\sin^{-1}\left(\frac{x-2}{2}\right) + C\end{aligned}$$

-

$$\int \frac{(x+2)}{\sqrt{x^2+2x+3}}dx = \frac{1}{2}\int \frac{2(x+2)}{\sqrt{x^2+2x+3}}dx$$
$$= \frac{1}{2}\int \frac{2x+4}{\sqrt{x^2+2x+3}}dx$$
$$= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx + \frac{1}{2}\int \frac{2}{\sqrt{x^2+2x+3}}dx$$
$$= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx + \int \frac{1}{\sqrt{x^2+2x+3}}dx$$

Let $I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}}dx$ and $I_2 = \int \frac{1}{\sqrt{x^2+2x+3}}dx$

$$\therefore \int \frac{x+2}{\sqrt{x^2+2x+3}}dx = \frac{1}{2}I_1 + I_2 \quad \text{...(1)}$$

Then, $I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}}dx$

Let $x^2 + 2x + 3 = t$

$\Rightarrow (2x+2)\,dx = dt$

$$I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{x^2+2x+3} \quad \text{...(2)}$$

$$I_2 = \int \frac{1}{\sqrt{x^2+2x+3}}dx$$

$$\Rightarrow x^2+2x+3 = x^2+2x+1+2 = (x+1)^2 + \left(\sqrt{2}\right)^2$$

$$\therefore I_2 = \int \frac{1}{\sqrt{(x+1)^2+\left(\sqrt{2}\right)^2}}dx = \log\left|(x+1)+\sqrt{x^2+2x+3}\right| \quad \text{...(3)}$$

Using equations (2) and (3) in (1), we obtain

$$\int \frac{x+2}{\sqrt{x^2+2x+3}}dx = \frac{1}{2}\left[2\sqrt{x^2+2x+3}\right] + \log\left|(x+1)+\sqrt{x^2+2x+3}\right| + C$$
$$= \sqrt{x^2+2x+3} + \log\left|(x+1)+\sqrt{x^2+2x+3}\right| + C$$

-

Find $\int \frac{x+3}{x^2-2x-5}dx$

Let $(x+3) = A\frac{d}{dx}\left(x^2-2x-5\right)+B$

$(x+3) = A(2x-2)+B$

Equating the coefficients of x and constant term on both sides, we obtain

$$2A = 1 \Rightarrow A = \frac{1}{2}$$

$$-2A + B = 3 \Rightarrow B = 4$$

$$\therefore (x+3) = \frac{1}{2}(2x-2)+4$$

$$\Rightarrow \int \frac{x+3}{x^2-2x-5}dx = \int \frac{\frac{1}{2}(2x-2)+4}{x^2-2x-5}dx$$

$$= \frac{1}{2}\int \frac{2x-2}{x^2-2x-5}dx + 4\int \frac{1}{x^2-2x-5}dx$$

Let $I_1 = \int \frac{2x-2}{x^2-2x-5}dx$ and $I_2 = \int \frac{1}{x^2-2x-5}dx$

$$\therefore \int \frac{x+3}{\left(x^2-2x-5\right)}dx = \frac{1}{2}I_1 + 4I_2 \qquad ...(1)$$

Then, $I_1 = \int \frac{2x-2}{x^2-2x-5}dx$

Let $x^2 - 2x - 5 = t$

$\Rightarrow (2x-2)dx = dt$

$$\Rightarrow I_1 = \int \frac{dt}{t} = \log|t| = \log|x^2 - 2x - 5| \qquad ...(2)$$

$$I_2 = \int \frac{1}{x^2 - 2x - 5} dx$$

$$= \int \frac{1}{(x^2 - 2x + 1) - 6} dx$$

$$= \int \frac{1}{(x-1)^2 + (\sqrt{6})^2} dx$$

$$= \frac{1}{2\sqrt{6}} \log\left(\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right) \qquad ...(3)$$

Substituting (2) and (3) in (1), we obtain

$$\int \frac{x+3}{x^2-2x-5} dx = \frac{1}{2}\log|x^2 - 2x - 5| + \frac{4}{2\sqrt{6}} \log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$$

$$= \frac{1}{2}\log|x^2 - 2x - 5| + \frac{2}{\sqrt{6}} \log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$$

-

Find $\int \frac{5x+3}{\sqrt{x^2+4x+10}} dx$

$$\text{Let } 5x+3 = A\frac{d}{dx}\left(x^2+4x+10\right)+B$$
$$\Rightarrow 5x+3 = A\left(2x+4\right)+B$$

Equating the coefficients of x and constant term, we obtain

$$2A = 5 \Rightarrow A = \frac{5}{2}$$
$$4A+B = 3 \Rightarrow B = -7$$
$$\therefore 5x+3 = \frac{5}{2}(2x+4)-7$$
$$\Rightarrow \int \frac{5x+3}{\sqrt{x^2+4x+10}} dx = \int \frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}} dx$$
$$= \frac{5}{2}\int \frac{2x+4}{\sqrt{x^2+4x+10}} dx - 7\int \frac{1}{\sqrt{x^2+4x+10}} dx$$
$$\text{Let } I_1 = \int \frac{2x+4}{\sqrt{x^2+4x+10}} dx \text{ and } I_2 = \int \frac{1}{\sqrt{x^2+4x+10}} dx$$
$$\therefore \int \frac{5x+3}{\sqrt{x^2+4x+10}} dx = \frac{5}{2} I_1 - 7I_2 \qquad ...(1)$$
$$\text{Then, } I_1 = \int \frac{2x+4}{\sqrt{x^2+4x+10}} dx$$

Let $x^2+4x+10=t$

$\therefore (2x+4)dx=dt$

$$\Rightarrow I_1=\int\frac{dt}{t}=2\sqrt{t}=2\sqrt{x^2+4x+10} \qquad ...(2)$$

$$I_2=\int\frac{1}{\sqrt{x^2+4x+10}}dx$$

$$=\int\frac{1}{\sqrt{(x^2+4x+4)+6}}dx$$

$$=\int\frac{1}{(x+2)^2+(\sqrt{6})^2}dx$$

$$=\log\left|(x+2)\sqrt{x^2+4x+10}\right| \qquad ...(3)$$

Using equations (2) and (3) in (1), we obtain

$$\int\frac{5x+3}{\sqrt{x^2+4x+10}}dx=\frac{5}{2}\left[2\sqrt{x^2+4x+10}\right]-7\log\left|(x+2)+\sqrt{x^2+4x+10}\right|+C$$

$$=5\sqrt{x^2+4x+10}-7\log\left|(x+2)+\sqrt{x^2+4x+10}\right|+C$$

-

$$\int\frac{dx}{x^2+2x+2}=\int\frac{dx}{(x^2+2x+1)+1}$$

$$=\int\frac{1}{(x+1)^2+(1)^2}dx$$

$$=\left[\tan^{-1}(x+1)\right]+C$$

Hence, the correct answer is B.

-

$$\int \frac{dx}{\sqrt{9x-4x^2}}$$

$$= \int \frac{1}{\sqrt{-4\left(x^2-\frac{9}{4}x\right)}}dx$$

$$= \int \frac{1}{-4\left(x^2-\frac{9}{4}x+\frac{81}{64}-\frac{81}{64}\right)}dx$$

$$= \int \frac{1}{\sqrt{-4\left[\left(x-\frac{9}{8}\right)^2-\left(\frac{9}{8}\right)^2\right]}}dx$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{\left(\frac{9}{8}\right)^2-\left(x-\frac{9}{8}\right)^2}}dx$$

$$= \frac{1}{2}\left[\sin^{-1}\left(\frac{x-\frac{9}{8}}{\frac{9}{8}}\right)\right]+C \qquad \left(\int \frac{dy}{\sqrt{a^2-y^2}} = \sin^{-1}\frac{y}{a}+C\right)$$

$$= \frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+C$$

-

Find $\int \frac{x}{(x+1)(x+2)} dx$

Let $\frac{x}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$

$\Rightarrow x = A(x+2) + B(x+1)$

Equating the coefficients of x and constant term, we obtain

$A + B = 1$

$2A + B = 0$

On solving, we obtain

$A = -1$ and $B = 2$

$$\therefore \frac{x}{(x+1)(x+2)} = \frac{-1}{(x+1)} + \frac{2}{(x+2)}$$

$$\Rightarrow \int \frac{x}{(x+1)(x+2)} dx = \int \frac{-1}{(x+1)} + \frac{2}{(x+2)} dx$$

$$= -\log|x+1| + 2\log|x+2| + C$$

$$= \log(x+2)^2 - \log|x+1| + C$$

$$= \log\frac{(x+2)^2}{(x+1)} + C$$

-

Find $\int \frac{1}{(x^2-9)} dx$

Let $\frac{1}{(x+3)(x-3)} = \frac{A}{(x+3)} + \frac{B}{(x-3)}$

$1 = A(x-3) + B(x+3)$

Equating the coefficients of x and constant term, we obtain

$A + B = 0$

$-3A + 3B = 1$

On solving, we obtain

$A = -\frac{1}{6}$ and $B = \frac{1}{6}$

$$\therefore \frac{1}{(x+3)(x-3)} = \frac{-1}{6(x+3)} + \frac{1}{6(x-3)}$$

$$\Rightarrow \int \frac{1}{(x^2-9)} dx = \int \left(\frac{-1}{6(x+3)} + \frac{1}{6(x-3)} \right) dx$$

$$= -\frac{1}{6} \log|x+3| + \frac{1}{6} \log|x-3| + C$$

$$= \frac{1}{6} \log\left|\frac{(x-3)}{(x+3)}\right| + C$$

-

Find $\int \frac{3x-1}{(x-1)(x-2)(x-3)} dx$

Let $\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) \qquad ...(1)$

Substituting x = 1, 2, and 3 respectively in equation (1), we obtain

A = 1, B = - 5, and C = 4

$$\therefore \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)}$$

$$\Rightarrow \int \frac{3x-1}{(x-1)(x-2)(x-3)} dx = \int \left\{ \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)} \right\} dx$$

$$= \log|x-1| - 5\log|x-2| + 4\log|x-3| + C$$

-

Find $\int \frac{x}{(x-1)(x-2)(x-3)} dx$

Let $\frac{x}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$x = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) \qquad ...(1)$

Substituting x = 1, 2, and 3 respectively in equation (1), we obtain $A = \frac{1}{2}, B = -2, \text{ and } C = \frac{3}{2}$

$$\therefore \frac{x}{(x-1)(x-2)(x-3)} = \frac{1}{2(x-1)} - \frac{2}{(x-2)} + \frac{3}{2(x-3)}$$

$$\Rightarrow \int \frac{x}{(x-1)(x-2)(x-3)} dx = \int \left\{ \frac{1}{2(x-1)} - \frac{2}{(x-2)} + \frac{3}{2(x-3)} \right\} dx$$

$$= \frac{1}{2}\log|x-1| - 2\log|x-2| + \frac{3}{2}\log|x-3| + C$$

-

Find $\int \frac{2x}{(x+1)(x+2)} dx$

Let $\frac{2x}{x^2+3x+2} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$

$2x = A(x+2) + B(x+1) \qquad ...(1)$

Substituting $x = -1$ and -2 in equation (1), we obtain

$A = -2$ and $B = 4$

$$\therefore \frac{2x}{(x+1)(x+2)} = \frac{-2}{(x+1)} + \frac{4}{(x+2)}$$

$$\Rightarrow \int \frac{2x}{(x+1)(x+2)} dx = \int \left\{ \frac{4}{(x+2)} - \frac{2}{(x+1)} \right\} dx$$

$$= 4 \log|x+2| - 2 \log|x+1| + C$$

-

Find $\int \frac{1-x^2}{x(1-2x)} dx$

It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing $(1 - x^2)$ by $x(1 - 2x)$, we obtain

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2} + \frac{1}{2}\left(\frac{2-x}{x(1-2x)}\right)$$

Let $\frac{2-x}{x(1-2x)} = \frac{A}{x} + \frac{B}{(1-2x)}$

$$\Rightarrow (2-x) = A(1-2x) + Bx \qquad ...(1)$$

Substituting $x = 0$ and $\frac{1}{2}$ in equation (1), we obtain

$A = 2$ and $B = 3$

$$\therefore \frac{2-x}{x(1-2x)} = \frac{2}{x} + \frac{3}{1-2x}$$

Substituting in equation (1), we obtain

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2} + \frac{1}{2}\left\{\frac{2}{x} + \frac{3}{(1-2x)}\right\}$$

$$\Rightarrow \int \frac{1-x^2}{x(1-2x)} dx = \int \left\{\frac{1}{2} + \frac{1}{2}\left(\frac{2}{x} + \frac{3}{1-2x}\right)\right\} dx$$

$$= \frac{x}{2} + \log|x| + \frac{3}{2(-2)} \log|1-2x| + C$$

$$= \frac{x}{2} + \log|x| - \frac{3}{4} \log|1-2x| + C$$

-

Find $\int \frac{x \, \mathbf{dx}}{(x^2+1)(x-1)}$

Let $\frac{x}{(x^2+1)(x-1)} = \frac{Ax+B}{(x^2+1)} + \frac{C}{(x-1)}$

$x = (Ax+B)(x-1) + C(x^2+1)$

$x = Ax^2 - Ax + Bx - B + Cx^2 + C$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + C = 0$

$-A + B = 1$

$-B + C = 0$

On solving these equations, we obtain

$A = -\frac{1}{2}, B = \frac{1}{2}, \text{ and } C = \frac{1}{2}$

From equation (1), we obtain

$$\therefore \frac{x}{(x^2+1)(x-1)} = \frac{\left(-\frac{1}{2}x+\frac{1}{2}\right)}{x^2+1} + \frac{\frac{1}{2}}{(x-1)}$$

$$\Rightarrow \int \frac{x}{(x^2+1)(x-1)} = -\frac{1}{2}\int \frac{x}{x^2+1}dx + \frac{1}{2}\int \frac{1}{x^2+1}dx + \frac{1}{2}\int \frac{1}{x-1}dx$$

$$= -\frac{1}{4}\int \frac{2x}{x^2+1}dx + \frac{1}{2}\tan^{-1}x + \frac{1}{2}\log|x-1| + C$$

Consider $\int \frac{2x}{x^2+1}dx$, let $(x^2+1) = t \Rightarrow 2x\,dx = dt$

$$\Rightarrow \int \frac{2x}{x^2+1}dx = \int \frac{dt}{t} = \log|t| = \log|x^2+1|$$

$$\therefore \int \frac{x\ dx}{(x^2+1)(x-1)} = -\frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + \frac{1}{2}\log|x-1| + C$$

$$= \frac{1}{2}\log|x-1| - \frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + C$$

-

Find $\int \frac{x}{(x-1)^2(x+2)} dx$

Let $\frac{x}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$

$x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$

Substituting $x = 1$, we obtain

$B = \frac{1}{3}$

Equating the coefficients of x^2 and constant term, we obtain

$A + C = 0$

$-2A + 2B + C = 0$

On solving, we obtain

$A = \frac{2}{9}$ and $C = \frac{-2}{9}$

$$\therefore \frac{x}{(x-1)^2(x+2)} = \frac{2}{9(x-1)} + \frac{1}{3(x-1)^2} - \frac{2}{9(x+2)}$$

$$\Rightarrow \int \frac{x}{(x-1)^2(x+2)} dx = \frac{2}{9}\int \frac{1}{(x-1)} dx + \frac{1}{3}\int \frac{1}{(x-1)^2} dx - \frac{2}{9}\int \frac{1}{(x+2)} dx$$

$$= \frac{2}{9}\log|x-1| + \frac{1}{3}\left(\frac{-1}{x-1}\right) - \frac{2}{9}\log|x+2| + C$$

$$= \frac{2}{9}\log\left|\frac{x-1}{x+2}\right| - \frac{1}{3(x-1)} + C$$

-

Find $\int \frac{3x+5}{(x-1)^2(x+1)}dx$

$$\frac{3x+5}{x^3-x^2-x+1}=\frac{3x+5}{(x-1)^2(x+1)}$$

Let $\frac{3x+5}{(x-1)^2(x+1)}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x+1)}$

$$3x+5=A(x-1)(x+1)+B(x+1)+C(x-1)^2$$
$$3x+5=A(x^2-1)+B(x+1)+C(x^2+1-2x) \qquad ...(1)$$

Substituting $x = 1$ in equation (1), we obtain

$B = 4$

Equating the coefficients of x^2 and x, we obtain

$A + C = 0$

$B - 2C = 3$

On solving, we obtain

$A=-\frac{1}{2}$ and $C=\frac{1}{2}$

$$\therefore \frac{3x+5}{(x-1)^2(x+1)}=\frac{-1}{2(x-1)}+\frac{4}{(x-1)^2}+\frac{1}{2(x+1)}$$
$$\Rightarrow \int\frac{3x+5}{(x-1)^2(x+1)}dx=-\frac{1}{2}\int\frac{1}{x-1}dx+4\int\frac{1}{(x-1)^2}dx+\frac{1}{2}\int\frac{1}{(x+1)}dx$$
$$=-\frac{1}{2}\log|x-1|+4\left(\frac{-1}{x-1}\right)+\frac{1}{2}\log|x+1|+C$$
$$=\frac{1}{2}\log\left|\frac{x+1}{x-1}\right|-\frac{4}{(x-1)}+C$$

-

Find $\int \frac{2x-3}{(x^2-1)(2x+3)}dx$

$$\frac{2x-3}{(x^2-1)(2x+3)} = \frac{2x-3}{(x+1)(x-1)(2x+3)}$$

Let $\frac{2x-3}{(x+1)(x-1)(2x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{C}{(2x+3)}$

$$\Rightarrow (2x-3) = A(x-1)(2x+3) + B(x+1)(2x+3) + C(x+1)(x-1)$$
$$\Rightarrow (2x-3) = A(2x^2+x-3) + B(2x^2+5x+3) + C(x^2-1)$$
$$\Rightarrow (2x-3) = (2A+2B+C)x^2 + (A+5B)x + (-3A+3B-C)$$

Equating the coefficients of x^2 and x, we obtain

$B = -\frac{1}{10}$, $A = \frac{5}{2}$, and $C = -\frac{24}{5}$

$$\therefore \frac{2x-3}{(x+1)(x-1)(2x+3)} = \frac{5}{2(x+1)} - \frac{1}{10(x-1)} - \frac{24}{5(2x+3)}$$
$$\Rightarrow \int \frac{2x-3}{(x^2-1)(2x+3)}dx = \frac{5}{2}\int \frac{1}{(x+1)}dx - \frac{1}{10}\int \frac{1}{x-1}dx - \frac{24}{5}\int \frac{1}{(2x+3)}dx$$
$$= \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{24}{5\times 2}\log|2x+3|$$
$$= \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{12}{5}\log|2x+3| + C$$

-

Find $\int \frac{5x}{(x+1)(x^2-4)} dx$

$$\frac{5x}{(x+1)(x^2-4)} = \frac{5x}{(x+1)(x+2)(x-2)}$$

Let $\frac{5x}{(x+1)(x+2)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)} + \frac{C}{(x-2)}$

$$5x = A(x+2)(x-2) + B(x+1)(x-2) + C(x+1)(x+2) \qquad \ldots(1)$$

Substituting x = - 1, - 2, and 2 respectively in equation (1), we obtain

$$A = \frac{5}{3},\ B = -\frac{5}{2}, \text{ and } C = \frac{5}{6}$$

$$\therefore \frac{5x}{(x+1)(x+2)(x-2)} = \frac{5}{3(x+1)} - \frac{5}{2(x+2)} + \frac{5}{6(x-2)}$$

$$\Rightarrow \int \frac{5x}{(x+1)(x^2-4)} dx = \frac{5}{3}\int \frac{1}{(x+1)} dx - \frac{5}{2}\int \frac{1}{(x+2)} dx + \frac{5}{6}\int \frac{1}{(x-2)} dx$$

$$= \frac{5}{3}\log|x+1| - \frac{5}{2}\log|x+2| + \frac{5}{6}\log|x-2| + C$$

-

Find $\int \frac{x^3+x+1}{x^2-1}dx$

It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing ($x^3 + x + 1$) by $x^2 - 1$, we obtain

$$\frac{x^3+x+1}{x^2-1} = x + \frac{2x+1}{x^2-1}$$

Let $\frac{2x+1}{x^2-1} = \frac{A}{(x+1)} + \frac{B}{(x-1)}$

$$2x+1 = A(x-1) + B(x+1) \qquad ...(1)$$

Substituting $x = 1$ and -1 in equation (1), we obtain

$$A = \frac{1}{2} \text{ and } B = \frac{3}{2}$$

$$\therefore \frac{x^3+x+1}{x^2-1} = x + \frac{1}{2(x+1)} + \frac{3}{2(x-1)}$$

$$\Rightarrow \int \frac{x^3+x+1}{x^2-1}dx = \int x\,dx + \frac{1}{2}\int \frac{1}{(x+1)}dx + \frac{3}{2}\int \frac{1}{(x-1)}dx$$

$$= \frac{x^2}{2} + \frac{1}{2}\log|x+1| + \frac{3}{2}\log|x-1| + C$$

-

Find $\int \frac{2}{(1-x)(1+x^2)} dx$

Let $\frac{2}{(1-x)(1+x^2)} = \frac{A}{(1-x)} + \frac{Bx+C}{(1+x^2)}$

$2 = A(1+x^2) + (Bx+C)(1-x)$

$2 = A + Ax^2 + Bx - Bx^2 + C - Cx$

Equating the coefficient of x^2, x, and constant term, we obtain

$A - B = 0$

$B - C = 0$

$A + C = 2$

On solving these equations, we obtain

$A = 1$, $B = 1$, and $C = 1$

$$\therefore \frac{2}{(1-x)(1+x^2)} = \frac{1}{1-x} + \frac{x+1}{1+x^2}$$

$$\Rightarrow \int \frac{2}{(1-x)(1+x^2)} dx = \int \frac{1}{1-x} dx + \int \frac{x}{1+x^2} dx + \int \frac{1}{1+x^2} dx$$

$$= -\int \frac{1}{x-1} dx + \frac{1}{2} \int \frac{2x}{1+x^2} dx + \int \frac{1}{1+x^2} dx$$

$$= -\log|x-1| + \frac{1}{2} \log|1+x^2| + \tan^{-1} x + C$$

-

Find $\int \frac{3x-1}{(x+2)^2}\,dx$

Let $\frac{3x-1}{(x+2)^2} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2}$

$\Rightarrow 3x-1 = A(x+2)+B$

Equating the coefficient of x and constant term, we obtain

$A = 3$

$2A + B = -1 \Rightarrow B = -7$

$$\therefore \frac{3x-1}{(x+2)^2} = \frac{3}{(x+2)} - \frac{7}{(x+2)^2}$$

$$\Rightarrow \int \frac{3x-1}{(x+2)^2}\,dx = 3\int \frac{1}{(x+2)}dx - 7\int \frac{x}{(x+2)^2}\,dx$$

$$= 3\log|x+2| - 7\left(\frac{-1}{(x+2)}\right) + C$$

$$= 3\log|x+2| + \frac{7}{(x+2)} + C$$

-

Find $\int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx$

$$\frac{\cos x}{(1-\sin x)(2-\sin x)}$$

Let $\sin x = t \Rightarrow \cos x \, dx = dt$

$$\therefore \int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx = \int \frac{dt}{(1-t)(2-t)}$$

Let $\frac{1}{(1-t)(2-t)} = \frac{A}{(1-t)} + \frac{B}{(2-t)}$

$$1 = A(2-t) + B(1-t) \qquad ...(1)$$

Substituting $t = 2$ and then $t = 1$ in equation (1), we obtain

$A = 1$ and $B = -1$

$$\therefore \frac{1}{(1-t)(2-t)} = \frac{1}{(1-t)} - \frac{1}{(2-t)}$$

$$\Rightarrow \int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx = \int \left\{ \frac{1}{1-t} - \frac{1}{(2-t)} \right\} dt$$

$$= -\log|1-t| + \log|2-t| + C$$

$$= \log\left|\frac{2-t}{1-t}\right| + C$$

$$= \log\left|\frac{2-\sin x}{1-\sin x}\right| + C$$

-

Find $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx$

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 - \frac{(4x^2+10)}{(x^2+3)(x^2+4)}$$

Let $\frac{4x^2+10}{(x^2+3)(x^2+4)} = \frac{Ax+B}{(x^2+3)} + \frac{Cx+D}{(x^2+4)}$

$$4x^2+10 = (Ax+B)(x^2+4)+(Cx+D)(x^2+3)$$

$$4x^2+10 = Ax^3+4Ax+Bx^2+4B+Cx^3+3Cx+Dx^2+3D$$

$$4x^2+10 = (A+C)x^3+(B+D)x^2+(4A+3C)x+(4B+3D)$$

Equating the coefficients of x^3, x^2, x, and constant term, we obtain

$A + C = 0$

$B + D = 4$

$4A + 3C = 0$

$4B + 3D = 10$

On solving these equations, we obtain

$A = 0$, $B = -2$, $C = 0$, and $D = 6$

$$\therefore \frac{4x^2+10}{(x^2+3)(x^2+4)} = \frac{-2}{(x^2+3)} + \frac{6}{(x^2+4)}$$

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 - \left(\frac{-2}{(x^2+3)} + \frac{6}{(x^2+4)}\right)$$

$$\Rightarrow \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx = \int \left\{1 + \frac{2}{(x^2+3)} - \frac{6}{(x^2+4)}\right\} dx$$

$$= \int \left\{1 + \frac{2}{x^2+(\sqrt{3})^2} - \frac{6}{x^2+2^2}\right\}$$

$$= x + 2\left(\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}\right) - 6\left(\frac{1}{2}\tan^{-1}\frac{x}{2}\right) + C$$

$$= x + \frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} - 3\tan^{-1}\frac{x}{2} + C$$

-

Find $\int \frac{2x}{(x^2+1)(x^2+3)}dx$

$$\frac{2x}{(x^2+1)(x^2+3)}$$

Let $x^2 = t \Rightarrow 2x\,dx = dt$

$$\therefore \int \frac{2x}{(x^2+1)(x^2+3)}dx = \int \frac{dt}{(t+1)(t+3)} \qquad ...(1)$$

Let $\frac{1}{(t+1)(t+3)} = \frac{A}{(t+1)} + \frac{B}{(t+3)}$

$$1 = A(t+3) + B(t+1) \qquad ...(1)$$

Substituting $t = -3$ and $t = -1$ in equation (1), we obtain

$A = \frac{1}{2}$ and $B = -\frac{1}{2}$

$$\therefore \frac{1}{(t+1)(t+3)} = \frac{1}{2(t+1)} - \frac{1}{2(t+3)}$$

$$\Rightarrow \int \frac{2x}{(x^2+1)(x^2+3)}dx = \int\left\{\frac{1}{2(t+1)} - \frac{1}{2(t+3)}\right\}dt$$

$$= \frac{1}{2}\log|(t+1)| - \frac{1}{2}\log|t+3| + C$$

$$= \frac{1}{2}\log\left|\frac{t+1}{t+3}\right| + C$$

$$= \frac{1}{2}\log\left|\frac{x^2+1}{x^2+3}\right| + C$$

Find $\int \frac{1}{x\left(x^4-1\right)}dx$

Multiplying numerator and denominator by x^3, we obtain

$$\frac{1}{x\left(x^4-1\right)} = \frac{x^3}{x^4\left(x^4-1\right)}$$

$$\therefore \int \frac{1}{x\left(x^4-1\right)}dx = \int \frac{x^3}{x^4\left(x^4-1\right)}dx$$

Let $x^4 = t \Rightarrow 4x^3 dx = dt$

$$\therefore \int \frac{1}{x\left(x^4-1\right)}dx = \frac{1}{4}\int \frac{dt}{t(t-1)}$$

Let $\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{(t-1)}$

$1 = A(t-1) + Bt \qquad ...(1)$

Substituting $t = 0$ and 1 in (1), we obtain

$A = -1$ and $B = 1$

$$\Rightarrow \frac{1}{t(t+1)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\Rightarrow \int \frac{1}{x\left(x^4-1\right)}dx = \frac{1}{4}\int\left\{\frac{-1}{t} + \frac{1}{t-1}\right\}dt$$

$$= \frac{1}{4}\left[-\log|t| + \log|t-1|\right] + C$$

$$= \frac{1}{4}\log\left|\frac{t-1}{t}\right| + C$$

$$= \frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right| + C$$

-

Find ∫ dx / (e^x - 1)

Let $e^x = t \Rightarrow e^x\,dx = dt$

$$\Rightarrow \int \frac{1}{e^x - 1}dx = \int \frac{1}{t-1} \times \frac{dt}{t} = \int \frac{1}{t(t-1)}dt$$

$$\text{Let } \frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1}$$

$$1 = A(t-1) + Bt \qquad ...(1)$$

Substituting $t = 1$ and $t = 0$ in equation (1), we obtain

$A = -1$ and $B = 1$

$$\therefore \frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\Rightarrow \int \frac{1}{t(t-1)}dt = \log\left|\frac{t-1}{t}\right| + C$$

$$= \log\left|\frac{e^x - 1}{e^x}\right| + C$$

-

$$\int \frac{x\,dx}{(x-1)(x-2)} \text{ equals}$$

A. $\log\left|\frac{(x-1)^2}{x-2}\right| + C$

B. $\log\left|\frac{(x-2)^2}{x-1}\right| + C$

C. $\log\left|\left(\frac{x-1}{x-2}\right)^2\right| + C$

D. $\log\left|(x-1)(x-2)\right| + C$

Let $\frac{x}{(x-1)(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-2)}$

$x = A(x-2) + B(x-1)$ ...(1)

Substituting $x = 1$ and 2 in (1), we obtain

$A = -1$ and $B = 2$

$$\therefore \frac{x}{(x-1)(x-2)} = -\frac{1}{(x-1)} + \frac{2}{(x-2)}$$

$$\Rightarrow \int \frac{x}{(x-1)(x-2)} dx = \int \left\{ \frac{-1}{(x-1)} + \frac{2}{(x-2)} \right\} dx$$

$$= -\log|x-1| + 2\log|x-2| + C$$

$$= \log\left|\frac{(x-2)^2}{x-1}\right| + C$$

Hence, the correct answer is B.

-

- $\int \frac{dx}{x(x^2+1)}$ equals

A. $\log|x| - \frac{1}{2}\log(x^2+1) + C$

B. $\log|x| + \frac{1}{2}\log(x^2+1) + C$

C. $-\log|x| + \frac{1}{2}\log(x^2+1) + C$

D. $\frac{1}{2}\log|x| + \log(x^2+1) + C$

Let $\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$1 = A(x^2+1) + (Bx+C)x$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + B = 0$

$C = 0$

$A = 1$

On solving these equations, we obtain

$A = 1$, $B = -1$, and $C = 0$

$$\therefore \frac{1}{x(x^2+1)} = \frac{1}{x} + \frac{-x}{x^2+1}$$

$$\Rightarrow \int \frac{1}{x(x^2+1)} dx = \int \left\{ \frac{1}{x} - \frac{x}{x^2+1} \right\} dx$$

$$= \log|x| - \frac{1}{2}\log|x^2+1| + C$$

-

Let $I = \int x \sin x \, dx$

Taking x as first function and $\sin x$ as second function and integrating by parts, we obtain

$$I = x\int \sin x \, dx - \int \left\{ \left(\frac{d}{dx} x \right) \int \sin x \, dx \right\} dx$$

$$= x(-\cos x) - \int 1 \cdot (-\cos x) dx$$

$$= -x\cos x + \sin x + C$$

-

Let $I = \int x \sin 3x \, dx$

Taking x as first function and sin $3x$ as second function and integrating by parts, we obtain

$$I = x\int \sin 3x \ dx - \int\left\{\left(\frac{d}{dx}x\right)\int \sin 3x \ dx\right\}$$
$$= x\left(\frac{-\cos 3x}{3}\right) - \int 1\cdot\left(\frac{-\cos 3x}{3}\right) dx$$
$$= \frac{-x\cos 3x}{3} + \frac{1}{3}\int \cos 3x \ dx$$
$$= \frac{-x\cos 3x}{3} + \frac{1}{9}\sin 3x + C$$

-

Let $I = \int x^2 e^x dx$

Taking x^2 as first function and e^x as second function and integrating by parts, we obtain

$$I = x^2\int e^x \ dx - \int\left\{\left(\frac{d}{dx}x^2\right)\int e^x dx\right\}dx$$
$$= x^2e^x - \int 2x\cdot e^x dx$$
$$= x^2e^x - 2\int x\cdot e^x dx$$

Again integrating by parts, we obtain

$$= x^2e^x - 2\left[x\cdot\int e^x dx - \int\left\{\left(\frac{d}{dx}x\right)\cdot\int e^x dx\right\}dx\right]$$
$$= x^2e^x - 2\left[xe^x - \int e^x dx\right]$$
$$= x^2e^x - 2\left[xe^x - e^x\right]$$
$$= x^2e^x - 2xe^x + 2e^x + C$$
$$= e^x\left(x^2 - 2x + 2\right) + C$$

-

Let $I = \int x \log x dx$

Taking $\log x$ as first function and x as second function and integrating by parts, we obtain

$$I = \log x \int x\, dx - \int\left\{\left(\frac{d}{dx}\log x\right)\int x\, dx\right\}dx$$

$$= \log x \cdot \frac{x^2}{2} - \int \frac{1}{x}\cdot\frac{x^2}{2}dx$$

$$= \frac{x^2 \log x}{2} - \int \frac{x}{2}dx$$

$$= \frac{x^2 \log x}{2} - \frac{x^2}{4} + C$$

-

Let $I = \int x \log 2x dx$

Taking $\log 2x$ as first function and x as second function and integrating by parts, we obtain

$$I = \log 2x \int x\, dx - \int\left\{\left(\frac{d}{dx}2\log x\right)\int x\, dx\right\}dx$$

$$= \log 2x \cdot \frac{x^2}{2} - \int \frac{2}{2x}\cdot\frac{x^2}{2}dx$$

$$= \frac{x^2 \log 2x}{2} - \int \frac{x}{2}dx$$

$$= \frac{x^2 \log 2x}{2} - \frac{x^2}{4} + C$$

-

Let $I = \int x^2 \log x\, dx$

Taking $\log x$ as first function and x^2 as second function and integrating by parts, we obtain

$$I = \log x \int x^2\, dx - \int\left\{\left(\frac{d}{dx}\log x\right)\int x^2 dx\right\}dx$$

$$= \log x\left(\frac{x^3}{3}\right) - \int \frac{1}{x}\cdot\frac{x^3}{3}dx$$

$$= \frac{x^3 \log x}{3} - \int \frac{x^2}{3}dx$$

$$= \frac{x^3 \log x}{3} - \frac{x^3}{9} + C$$

Let $I = \int x \sin^{-1} x \, dx$

Taking $\sin^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$I = \sin^{-1} x \int x \, dx - \int \left\{ \left(\frac{d}{dx} \sin^{-1} x \right) \int x \, dx \right\} dx$$

$$= \sin^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} \, dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \frac{-x^2}{\sqrt{1-x^2}} \, dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \frac{1-x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} \right\} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}} \right\} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \int \sqrt{1-x^2} \, dx - \int \frac{1}{\sqrt{1-x^2}} \, dx \right\}$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x - \sin^{-1} x \right\} + C$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{x}{4} \sqrt{1-x^2} + \frac{1}{4} \sin^{-1} x - \frac{1}{2} \sin^{-1} x + C$$

$$= \frac{1}{4} \left(2x^2 - 1 \right) \sin^{-1} x + \frac{x}{4} \sqrt{1-x^2} + C$$

-

Let $I = \int x \tan^{-1} x \, dx$

Taking $\tan^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$I = \tan^{-1} x \int x\,dx - \int \left\{ \left(\frac{d}{dx} \tan^{-1} x \right) \int x\,dx \right\} dx$$
$$= \tan^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{1+x^2} \cdot \frac{x^2}{2} dx$$
$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \frac{x^2}{1+x^2} dx$$
$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \left(\frac{x^2+1}{1+x^2} - \frac{1}{1+x^2} \right) dx$$
$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \left(1 - \frac{1}{1+x^2} \right) dx$$
$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \left(x - \tan^{-1} x \right) + C$$
$$= \frac{x^2}{2} \tan^{-1} x - \frac{x}{2} + \frac{1}{2} \tan^{-1} x + C$$

-

Let $I = \int x \cos^{-1} x dx$

Taking $\cos^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$I = \cos^{-1} x \int x\,dx - \int \left\{ \left(\frac{d}{dx} \cos^{-1} x \right) \int x\,dx \right\} dx$$
$$= \cos^{-1} x \frac{x^2}{2} - \int \frac{-1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} dx$$
$$= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2} \int \frac{1-x^2-1}{\sqrt{1-x^2}} dx$$
$$= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2} \int \left\{ \sqrt{1-x^2} + \left(\frac{-1}{\sqrt{1-x^2}} \right) \right\} dx$$
$$= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2} \int \sqrt{1-x^2}\,dx - \frac{1}{2} \int \left(\frac{-1}{\sqrt{1-x^2}} \right) dx$$
$$= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2} I_1 - \frac{1}{2} \cos^{-1} x \qquad \ldots(1)$$

where, $I_1 = \int \sqrt{1-x^2}\,dx$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \int \frac{d}{dx}\sqrt{1-x^2}\int x\,dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \int \frac{-2x}{2\sqrt{1-x^2}}.x\,dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \int \frac{-x^2}{\sqrt{1-x^2}}dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \int \frac{1-x^2-1}{\sqrt{1-x^2}}dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \left\{\int \sqrt{1-x^2}\,dx + \int \frac{-dx}{\sqrt{1-x^2}}\right\}$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \left\{I_1 + \cos^{-1}x\right\}$$

$$\Rightarrow 2I_1 = x\sqrt{1-x^2} - \cos^{-1}x$$

$$\therefore I_1 = \frac{x}{2}\sqrt{1-x^2} - \frac{1}{2}\cos^{-1}x$$

Substituting in (1), we obtain

$$I = \frac{x\cos^{-1}x}{2} - \frac{1}{2}\left(\frac{x}{2}\sqrt{1-x^2} - \frac{1}{2}\cos^{-1}x\right) - \frac{1}{2}\cos^{-1}x$$

$$= \frac{\left(2x^2-1\right)}{4}\cos^{-1}x - \frac{x}{4}\sqrt{1-x^2} + C$$

Let $I = \int (\sin^{-1} x)^2 \cdot 1\, dx$

Taking $(\sin^{-1} x)^2$ as first function and 1 as second function and integrating by parts, we obtain

$$I = (\sin^{-1} x)\int 1 dx - \int\left\{\frac{d}{dx}(\sin^{-1} x)^2 \cdot \int 1 \cdot dx\right\}dx$$
$$= (\sin^{-1} x)^2 \cdot x - \int \frac{2\sin^{-1} x}{\sqrt{1-x^2}} \cdot x\, dx$$
$$= x(\sin^{-1} x)^2 + \int \sin^{-1} x \cdot \left(\frac{-2x}{\sqrt{1-x^2}}\right)dx$$
$$= x(\sin^{-1} x)^2 + \left[\sin^{-1} x \int \frac{-2x}{\sqrt{1-x^2}}dx - \int\left\{\left(\frac{d}{dx}\sin^{-1} x\right)\int \frac{-2x}{\sqrt{1-x^2}}dx\right\}dx\right]$$
$$= x(\sin^{-1} x)^2 + \left[\sin^{-1} x \cdot 2\sqrt{1-x^2} - \int \frac{1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2}\, dx\right]$$
$$= x(\sin^{-1} x)^2 + 2\sqrt{1-x^2}\sin^{-1} x - \int 2 dx$$
$$= x(\sin^{-1} x)^2 + 2\sqrt{1-x^2}\sin^{-1} x - 2x + C$$

-

Let $I = \int \frac{x\cos^{-1} x}{\sqrt{1-x^2}}dx$

$$I = \frac{-1}{2}\int \frac{-2x}{\sqrt{1-x^2}} \cdot \cos^{-1} x dx$$

Taking $\cos^{-1} x$ as first function and $\left(\frac{-2x}{\sqrt{1-x^2}}\right)$ as second function and integrating by parts, we obtain

$$I = \frac{-1}{2}\left[\cos^{-1} x \int \frac{-2x}{\sqrt{1-x^2}}dx - \int\left\{\left(\frac{d}{dx}\cos^{-1} x\right)\int \frac{-2x}{\sqrt{1-x^2}}dx\right\}dx\right]$$
$$= \frac{-1}{2}\left[\cos^{-1} x \cdot 2\sqrt{1-x^2} - \int \frac{-1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2}\, dx\right]$$
$$= \frac{-1}{2}\left[2\sqrt{1-x^2}\cos^{-1} x + \int 2 dx\right]$$
$$= \frac{-1}{2}\left[2\sqrt{1-x^2}\cos^{-1} x + 2x\right] + C$$
$$= -\left[\sqrt{1-x^2}\cos^{-1} x + x\right] + C$$

-

Let $I = \int x \sec^2 x dx$

Taking x as first function and $\sec^2 x$ as second function and integrating by parts, we obtain

$$I = x \int \sec^2 x \, dx - \int \left\{ \left\{ \frac{d}{dx} x \right\} \int \sec^2 x \, dx \right\} dx$$

$$= x \tan x - \int 1 \cdot \tan x dx$$

$$= x \tan x + \log |\cos x| + C$$

-

Let $I = \int 1 \cdot \tan^{-1} x dx$

Taking $\tan^{-1} x$ as first function and 1 as second function and integrating by parts, we obtain

$$I = \tan^{-1} x \int 1 dx - \int \left\{ \left(\frac{d}{dx} \tan^{-1} x \right) \int 1 \cdot dx \right\} dx$$

$$= \tan^{-1} x \cdot x - \int \frac{1}{1+x^2} \cdot x \, dx$$

$$= x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{1+x^2} dx$$

$$= x \tan^{-1} x - \frac{1}{2} \log |1+x^2| + C$$

$$= x \tan^{-1} x - \frac{1}{2} \log (1+x^2) + C$$

-

Let $I = \int (x^2+1)\log x\,dx = \int x^2 \log x\,dx + \int \log x\,dx$

Let $I = I_1 + I_2$ ... (1)

Where, $I_1 = \int x^2 \log x\,dx$ and $I_2 = \int \log x\,dx$

$I_1 = \int x^2 \log x dx$

Taking log x as first function and x^2 as second function and integrating by parts, we obtain

$$I_1 = \log x - \int x^2 dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x^2 dx \right\} dx$$
$$= \log x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$$
$$= \frac{x^3}{3} \log x - \frac{1}{3} \left(\int x^2 dx \right)$$
$$= \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 \qquad \text{... (2)}$$

$I_2 = \int \log x\,dx$

Taking log x as first function and 1 as second function and integrating by parts, we obtain

$$I_2 = \log x \int 1 \cdot dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int 1 \cdot dx \right\}$$
$$= \log x \cdot x - \int \frac{1}{x} \cdot x dx$$
$$= x \log x - \int 1 dx$$
$$= x \log x - x + C_2 \qquad \text{... (3)}$$

Using equations (2) and (3) in (1), we obtain

$$I = \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 + x \log x - x + C_2$$
$$= \frac{x^3}{3} \log x - \frac{x^3}{9} + x \log x - x + (C_1 + C_2)$$
$$= \left(\frac{x^3}{3} + x \right) \log x - \frac{x^3}{9} - x + C$$

-

Let $I = \int e^x (\sin x + \cos x) dx$

Let $f(x) = \sin x$

$\Rightarrow f'(x) = \cos x$

$\therefore I = \int e^x \{f(x) + f'(x)\} dx$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$\therefore I = e^x \sin x + C$

-

Let $I = \int \frac{xe^x}{(1+x)^2} dx = \int e^x \left\{ \frac{x}{(1+x)^2} \right\} dx$

$= \int e^x \left\{ \frac{1+x-1}{(1+x)^2} \right\} dx$

$= \int e^x \left\{ \frac{1}{1+x} - \frac{1}{(1+x)^2} \right\} dx$

Let $f(x) = \frac{1}{1+x} \Rightarrow f'(x) = \frac{-1}{(1+x)^2}$

$\Rightarrow \int \frac{xe^x}{(1+x)^2} dx = \int e^x \{f(x) + f'(x)\} dx$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$\therefore \int \frac{xe^x}{(1+x)^2} dx = \frac{e^x}{1+x} + C$

-

Find $\int \frac{e^x(1+\sin x)}{(1+\cos x)}dx$

$$e^x\left(\frac{1+\sin x}{1+\cos x}\right)$$

$$=e^x\left(\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$$

$$=\frac{e^x\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}$$

$$=\frac{1}{2}e^x\cdot\left(\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\cos\frac{x}{2}}\right)^2$$

$$=\frac{1}{2}e^x\left[\tan\frac{x}{2}+1\right]^2$$

$$=\frac{1}{2}e^2\left(1+\tan\frac{x}{2}\right)^2$$

$$= \frac{1}{2}e^x\left[1+\tan^2\frac{x}{2}+2\tan\frac{x}{2}\right]$$

$$= \frac{1}{2}e^x\left[\sec^2\frac{x}{2}+2\tan\frac{x}{2}\right]$$

$$\frac{e^x(1+\sin x)dx}{(1+\cos x)} = e^x\left[\frac{1}{2}\sec^2\frac{x}{2}+\tan\frac{x}{2}\right] \quad ...(1)$$

Let $\tan\frac{x}{2} = f(x) \Rightarrow f'(x) = \frac{1}{2}\sec^2\frac{x}{2}$

It is known that, $\int e^x\{f(x)+f'(x)\}dx = e^x f(x)+C$

From equation (1), we obtain

$$\int\frac{e^x(1+\sin x)}{(1+\cos x)}dx = e^x\tan\frac{x}{2}+C$$

-

Let $I = \int e^x\left[\frac{1}{x}-\frac{1}{x^2}\right]dx$

Also, let $\frac{1}{x} = f(x) \Rightarrow f'(x) = \frac{-1}{x^2}$

It is known that, $\int e^x\{f(x)+f'(x)\}dx = e^x f(x)+C$

$$\therefore I = \frac{e^x}{x}+C$$

-

$$\int e^x \left\{\frac{x-3}{(x-1)^3}\right\} dx = \int e^x \left\{\frac{x-1-2}{(x-1)^3}\right\} dx$$

$$= \int e^x \left\{\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}\right\} dx$$

Let $f(x) = \frac{1}{(x-1)^2} \Rightarrow f'(x) = \frac{-2}{(x-1)^3}$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$$\therefore \int e^x \left\{\frac{(x-3)}{(x-1)^2}\right\} dx = \frac{e^x}{(x-1)^2} + C$$

-

Let $I = \int e^{2x} \sin x \, dx \qquad ...(1)$

Integrating by parts, we obtain

$$I = \sin x \int e^{2x} dx - \int \left\{\left(\frac{d}{dx}\sin x\right) \int e^{2x} dx\right\} dx$$

$$\Rightarrow I = \sin x \cdot \frac{e^{2x}}{2} - \int \cos x \cdot \frac{e^{2x}}{2} dx$$

$$\Rightarrow I = \frac{e^{2x}\sin x}{2} - \frac{1}{2}\int e^{2x} \cos x \, dx$$

Again integrating by parts, we obtain

$$I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2}\left[\cos x \int e^{2x} dx - \int \left\{\left(\frac{d}{dx}\cos x\right) \int e^{2x} dx\right\} dx\right]$$

$$\Rightarrow I = \frac{e^{2x}\sin x}{2} - \frac{1}{2}\left[\cos x \cdot \frac{e^{2x}}{2} - \int (-\sin x) \frac{e^{2x}}{2} dx\right]$$

$$\Rightarrow I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2}\left[\frac{e^{2x}\cos x}{2} + \frac{1}{2}\int e^{2x} \sin x dx\right]$$

$$\Rightarrow I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{1}{4} I \qquad \text{[From (1)]}$$

$$\Rightarrow I + \frac{1}{4} I = \frac{e^{2x} \cdot \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

$$\Rightarrow \frac{5}{4} I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

$$\Rightarrow I = \frac{4}{5}\left[\frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4}\right] + C$$

$$\Rightarrow I = \frac{e^{2x}}{5}\left[2 \sin x - \cos x\right] + C$$

-

Find $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$

Let $x = \tan\theta \Rightarrow dx = \sec^2\theta \, d\theta$

$$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$$

$$\Rightarrow \int \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx = \int 2\theta \cdot \sec^2\theta \, d\theta = 2\int \theta \cdot \sec^2\theta \, d\theta$$

Integrating by parts, we obtain

$$2\left[\theta \cdot \int \sec^2\theta d\theta - \int\left\{\left(\frac{d}{d\theta}\theta\right) \int \sec^2\theta d\theta\right\} d\theta\right]$$

$$= 2\left[\theta \cdot \tan\theta - \int \tan\theta d\theta\right]$$

$$= 2\left[\theta \tan\theta + \log|\cos\theta|\right] + C$$

$$= 2\left[x \tan^{-1} x + \log\left|\frac{1}{\sqrt{1+x^2}}\right|\right] + C$$

$$= 2x \tan^{-1} x + 2\log\left(1+x^2\right)^{-\frac{1}{2}} + C$$

$$= 2x \tan^{-1} x + 2\left[-\frac{1}{2}\log\left(1+x^2\right)\right] + C$$

$$= 2x \tan^{-1} x - \log\left(1+x^2\right) + C$$

$\int x^2 e^{x^3} dx$ equals

(A) $\frac{1}{3}e^{x^3} + C$ (B) $\frac{1}{3}e^{x^2} + C$

(C) $\frac{1}{2}e^{x^3} + C$ (D) $\frac{1}{3}e^{x^2} + C$

Answer :

Let $I = \int x^2 e^{x^3} dx$

Also, let $x^3 = t \Rightarrow 3x^2 dx = dt$

$$\Rightarrow I = \frac{1}{3}\int e^t dt$$

$$= \frac{1}{3}\left(e^t\right) + C$$

$$= \frac{1}{3}e^{x^3} + C$$

Hence, the correct answer is A.

-

$\int e^x \sec x(1+\tan x)dx$ equals

(A) $e^x \cos x + C$ (B) $e^x \sec x + C$

(C) $e^x \sin x + C$ (D) $e^x \tan x + C$

Answer :

$\int e^x \sec x(1+\tan x)dx$

Let $I = \int e^x \sec x(1+\tan x)dx = \int e^x (\sec x + \sec x \tan x)dx$

Also, let $\sec x = f(x) \Rightarrow \sec x \tan x = f'(x)$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$\therefore I = e^x \sec x + C$

Hence, the correct answer is B.

-

Let $I = \int \sqrt{4-x^2}\,dx = \int \sqrt{(2)^2 - (x)^2}\,dx$

It is known that, $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

$\therefore I = \frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2} + C$

$= \frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2} + C$

-

Let $I = \int \sqrt{1-4x^2}\,dx = \int \sqrt{(1)^2 - (2x)^2}\,dx$

Let $2x = t \Rightarrow 2\,dx = dt$

$\therefore I = \frac{1}{2}\int \sqrt{(1)^2 - (t)^2}\,dt$

It is known that, $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

$$\Rightarrow I = \frac{1}{2}\left[\frac{t}{2}\sqrt{1-t^2} + \frac{1}{2}\sin^{-1} t\right] + C$$

$$= \frac{t}{4}\sqrt{1-t^2} + \frac{1}{4}\sin^{-1} t + C$$

$$= \frac{2x}{4}\sqrt{1-4x^2} + \frac{1}{4}\sin^{-1} 2x + C$$

$$= \frac{x}{2}\sqrt{1-4x^2} + \frac{1}{4}\sin^{-1} 2x + C$$

-

Let $I = \int \sqrt{x^2 + 4x + 6}\,dx$

$$= \int \sqrt{x^2 + 4x + 4 + 2}\,dx$$

$$= \int \sqrt{(x^2 + 4x + 4) + 2}\,dx$$

$$= \int \sqrt{(x+2)^2 + (\sqrt{2})^2}\,dx$$

It is known that, $\int \sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + C$

$$\therefore I = \frac{(x+2)}{2}\sqrt{x^2 + 4x + 6} + \frac{2}{2}\log\left|(x+2) + \sqrt{x^2 + 4x + 6}\right| + C$$

$$= \frac{(x+2)}{2}\sqrt{x^2 + 4x + 6} + \log\left|(x+2) + \sqrt{x^2 + 4x + 6}\right| + C$$

-

$$\text{Let } I = \int\sqrt{x^2+4x+1}\,dx$$
$$= \int\sqrt{\left(x^2+4x+4\right)-3}\,dx$$
$$= \int\sqrt{(x+2)^2-\left(\sqrt{3}\right)^2}\,dx$$

It is known that, $\int\sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$

$$\therefore I = \frac{(x+2)}{2}\sqrt{x^2+4x+1} - \frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right| + C$$

-

$$\text{Let } I = \int\sqrt{1-4x-x^2}\,dx$$
$$= \int\sqrt{1-\left(x^2+4x+4-4\right)}\,dx$$
$$= \int\sqrt{1+4-(x+2)^2}\,dx$$
$$= \int\sqrt{\left(\sqrt{5}\right)^2-(x+2)^2}\,dx$$

It is known that, $\int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

$$\therefore I = \frac{(x+2)}{2}\sqrt{1-4x-x^2} + \frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right) + C$$

-

Let $I = \int \sqrt{x^2 + 4x - 5}\, dx$

$$= \int \sqrt{(x^2 + 4x + 4) - 9}\, dx$$

$$= \int \sqrt{(x+2)^2 - (3)^2}\, dx$$

It is known that, $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$

$$\therefore I = \frac{(x+2)}{2}\sqrt{x^2 + 4x - 5} - \frac{9}{2}\log\left|(x+2) + \sqrt{x^2 + 4x - 5}\right| + C$$

-

Let $I = \int \sqrt{1 + 3x - x^2}\, dx$

$$= \int \sqrt{1 - \left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right)}\, dx$$

$$= \int \sqrt{\left(1 + \frac{9}{4}\right) - \left(x - \frac{3}{2}\right)^2}\, dx$$

$$= \int \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2}\, dx$$

It is known that, $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

$$\therefore I = \frac{x - \frac{3}{2}}{2}\sqrt{1 + 3x - x^2} + \frac{13}{4 \times 2}\sin^{-1}\left(\frac{x - \frac{3}{2}}{\frac{\sqrt{13}}{2}}\right) + C$$

$$= \frac{2x - 3}{4}\sqrt{1 + 3x - x^2} + \frac{13}{8}\sin^{-1}\left(\frac{2x - 3}{\sqrt{13}}\right) + C$$

-

Let $I = \int \sqrt{x^2 + 3x}\, dx$

$$= \int \sqrt{x^2 + 3x + \frac{9}{4} - \frac{9}{4}}\, dx$$

$$= \int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2}\, dx$$

It is known that, $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$

$$\therefore I = \frac{\left(x + \frac{3}{2}\right)}{2}\sqrt{x^2 + 3x} - \frac{\frac{9}{4}}{2}\log\left|\left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x}\right| + C$$

$$= \frac{(2x+3)}{4}\sqrt{x^2 + 3x} - \frac{9}{8}\log\left|\left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x}\right| + C$$

-

Let $I = \int \sqrt{1 + \frac{x^2}{9}}\, dx = \frac{1}{3}\int \sqrt{9 + x^2}\, dx = \frac{1}{3}\int \sqrt{(3)^2 + x^2}\, dx$

It is known that, $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + C$

$$\therefore I = \frac{1}{3}\left[\frac{x}{2}\sqrt{x^2 + 9} + \frac{9}{2}\log\left|x + \sqrt{x^2 + 9}\right|\right] + C$$

$$= \frac{x}{6}\sqrt{x^2 + 9} + \frac{3}{2}\log\left|x + \sqrt{x^2 + 9}\right| + C$$

-

$\int \sqrt{1+x^2}\, dx$ is equal to

A. $\frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\log\left|x+\sqrt{1+x^2}\right| + C$

B. $\frac{2}{3}\left(1+x^2\right)^{\frac{2}{3}} + C$

C. $\frac{2}{3}x\left(1+x^2\right)^{\frac{3}{2}} + C$

D. $\frac{x^2}{2}\sqrt{1+x^2} + \frac{1}{2}x^2\log\left|x+\sqrt{1+x^2}\right| + C$

Answer :

It is known that, $\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$

$\therefore \int \sqrt{1+x^2}\,dx = \frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\log\left|x+\sqrt{1+x^2}\right| + C$

Hence, the correct answer is A.

-

$\int\sqrt{x^2-8x+7}dx$ is equal to

A. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+9\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

B. $\frac{1}{2}(x+4)\sqrt{x^2-8x+7}+9\log\left|x+4+\sqrt{x^2-8x+7}\right|+C$

C. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

D. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

Answer :

Let $I=\int\sqrt{x^2-8x+7}\,dx$

$=\int\sqrt{(x^2-8x+16)-9}\,dx$

$=\int\sqrt{(x-4)^2-(3)^2}\,dx$

It is known that, $\int\sqrt{x^2-a^2}dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$\therefore I=\frac{(x-4)}{2}\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|(x-4)+\sqrt{x^2-8x+7}\right|+C$

Hence, the correct answer is D.

-

Solve a Problem

Evaluate $\int \cos 2x \log(1+\tan x)dx$.

Solution:

Integrating by parts taking cos 2x as the 2nd function, the given integral

$$= \{\log(1+\tan x)\}\frac{\sin 2x}{2} - \int \frac{\sec^2 x}{1+\tan x}\cdot\frac{\sin 2x}{2}dx$$

$$= \frac{1}{2}\sin 2x \log(1+\tan x) - \int \frac{\sin x}{\sin x + \cos x}dx.$$

Now $\int \frac{\sin x\, dx}{\sin x + \cos x}$

$$= \frac{1}{2}\int \frac{(\sin x + \cos x) - (\cos x - \sin x)}{\sin x + \cos x}dx,$$

$$= \frac{1}{2}\int \left[1 - \frac{\cos x - \sin x}{\sin x + \cos x}\right]dx = \frac{1}{2}[x - \log(\sin x + \cos x)].$$

Hence the given integral

$$= \frac{1}{2}\sin 2x \log(1+\tan x) - \frac{1}{2}[x - \log(\sin x + \cos x)].$$

Recall how to integrate Linear X root Quadratic in denominator

Find the value of the $\int \frac{dx}{(x+1)\sqrt{(1+2x-x^2)}}$

Putting $(x+1) = \frac{1}{t}$, so that $dx = -\frac{1}{t^2}\, dt$, $x = \frac{1-t}{t}$ and

$$(1+2x-x^2) = 1 + 2\left(\frac{1-t}{t}\right) - \frac{(1-t)^2}{t^2} = \frac{2}{t^2}\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (t-1)^2\right],$$

we get the value of the given integral transformed as

$$\int \frac{-\frac{1}{t^2}\,dt}{\frac{1}{t}\frac{2}{\sqrt{t}}\left[\left(\frac{1}{\sqrt{2}}\right)^2-(t-1)^2\right]} = -\frac{1}{\sqrt{2}}\sin^{-1}\frac{t-1}{\left(\frac{1}{\sqrt{2}}\right)} + C$$

$$= \frac{1}{\sqrt{2}}\sin^{-1}\frac{\sqrt{2}\,x}{(x+1)} + C$$

Remember -

For the form $\int \frac{dx}{(Ax+B)^r\sqrt{(ax^2+bx+c)}}$ where r is a positive integer

we can substitute $Ax+B = \frac{1}{t}$

But for $\int \frac{dx}{(Ax+B)\sqrt{(ax+b)}}$ $\int \frac{dx}{(Ax^2+Bx+C)\sqrt{(ax+b)}}$ we have to substitute ax + b = t^2

So the Linear expression that in inside the root will be substituted

See https://archive.org/details/4Integrations91ByUIsAPowerfulSubstitutionIITJEEMath

Another advanced example

Example Evaluate $\int \frac{dx}{x\sqrt{(1+x^n)}}$

Make the substitution $(1 + x^n) = t^2$ or $x^n = (t^2 - 1)$, so that $n\,x^{n-1}\,dx = 2t\,dt$, we get

$$\int \frac{2t\,dt}{n x^n t} = \frac{2}{n}\int \frac{dt}{(t^2-1)} = \frac{1}{n}\ln\left|\frac{t-1}{t+1}\right|$$

$$= \frac{1}{n}\ln\left|\frac{\sqrt{(1+x^n)}-1}{\sqrt{(1+x^n)}+1}\right| + C$$

Similarly

The value of integral $\int \frac{dx}{x\sqrt{1-x^3}}$ is given by

(a) $\frac{1}{3}\log\left|\frac{\sqrt{1-x^3}+1}{\sqrt{1-x^3}-1}\right| + C$ (b) $\frac{1}{3}\log\left|\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^2}+1}\right| + C$

(c) $\frac{2}{3}\log\left|\frac{1}{\sqrt{1-x^3}}\right| + C$ (d) $\frac{1}{3}\log|1-x^3| + C$

***Ans.* (b)**

Solution Put $1 - x^3 = t^2$. Then $-3x^2dx = 2t\,dt$ and the integral becomes

$$-\frac{1}{3}\int \frac{-3x^2dx}{x^3\sqrt{1-x^3}} = -\frac{1}{3}\int \frac{2t\,dt}{(1-t^2)t} = \frac{2}{3}\int \frac{dt}{t^2-1}$$

$$= \frac{2}{3}\left(\frac{1}{2}\log\left|\frac{t-1}{t+1}\right|\right) + C = \frac{1}{3}\log\left|\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right| + C$$

-

Integration of 1 by (Cos x + Cot x)

$$\int \frac{dx}{\text{Cot } x + \text{Cos } x}$$

https://archive.org/details/4Integrations10ByCosXCotXIITJEEMath

-

Solve a Problem

$\int \sqrt{\sec x - 1}\ dx$ is equal to

(a) $2\log\left(\cos\frac{x}{2} + \sqrt{\cos^2\frac{x}{2} - \frac{1}{2}}\right) + C$

(b) $\log\left(\cos\frac{x}{2} + \sqrt{\cos^2\frac{x}{2} - \frac{1}{2}}\right) + C$

(c) $-2\log\left(\cos\frac{x}{2} + \sqrt{\cos^2\frac{x}{2} - \frac{1}{2}}\right) + C$

(d) none of these

(c). $\int \sqrt{\sec x - 1}\ dx = \int \sqrt{\frac{1-\cos x}{\cos x}}\ dx$

$$= \sqrt{2}\int \frac{\sin\frac{x}{2}}{\sqrt{2\cos^2\frac{x}{2} - 1}}\,dx = -2\sqrt{2}\int \frac{dz}{\sqrt{2z^2 - 1}}$$

$$\left(\text{Putting } \cos\frac{x}{2} = z \Rightarrow \sin\frac{x}{2}\ dx = -2dz\right)$$

$$= -2\int \frac{dz}{\sqrt{z^2 - \left(\frac{1}{\sqrt{2}}\right)^2}}$$

$$= -2\log\left[z + \sqrt{z^2 - \left(\frac{1}{\sqrt{2}}\right)^2}\right] + C$$

$$= -2\log\left(\cos\frac{x}{2} + \sqrt{\cos^2\frac{x}{2} - \frac{1}{2}}\right) + C$$

Solve a tricky problem

Solve $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$

Solution: $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$

$$= \int \sqrt{\frac{\sin x}{\cos x \sin^2 x \cos^2 x} dx}$$

$$\int \frac{1}{\sqrt{\sin x \cos^3 x}} dx$$

$$\int \frac{1}{\sqrt{\sin^4 x \cot^3 x}} dx$$

$$-\int -cosec^2 x \; cot^{-3/2} x \, dx$$

$$= \frac{2}{\sqrt{\cot x}} + C$$

-

IIT JEE 2014 VIT Indefinite Integral e to the power x + 1 by x a tricky problem

$$\int \left(1 + x - \frac{1}{x}\right) e^{x + \frac{1}{x}} dx$$

https://archive.org/details/IITJEE2014VITIndefiniteIntegralEToThePowerX1ByXDezrinaBangaloreTricky

IIT-JEE-Integral with d (x - floor (x)) strange interpretation

$$\int d(x - [x])$$

https://archive.org/details/IITJEEIntegralWithDXFloorXStrangeInterpretation

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IIT-JEE-Integral x^2(0 to root 2) split into appropriate floors

$$\int_0^{\sqrt{2}} [\, x^2 \,]\, dx$$

https://archive.org/details/IITJEEIntegralX20ToRoot2SplitIntoAppropriateFloors

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Integration Tricks (63 examples)

$$\int \frac{1+x^4}{(1-x^4)^{3/2}}\, dx$$

https://archive.org/details/IndefiniteIntegralISEET2013QuestionLeakPowersOfXFactorisationSinLogXByParts

-

Integrate 1 by (x plus root x square plus 1) rationalize and proceed

$\int 1/(x + \sqrt{(x^2 + 1)})\ dx$

https://archive.org/details/Integrate1ByXPlusRootXSquarePlus1RationalizeAndProceed

-

Integrate 1 by x square into Cos 1 by x dx

$\int (1/x^2)\ \text{Cos}\ (1 / x)\ dx$

https://archive.org/details/Integrate1ByXSquareIntoCos1ByXDx

-

Beautiful Integrals explained step by step 1

$\int \frac{dx}{x^3 \sqrt[3]{x^3 + 5}}$ is easy but what about $\int \frac{dx}{x^3 \sqrt{x^2 - 1}}$

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard1

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Beautiful Integrals explained step by step 2

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard2

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Beautiful Integrals explained step by step 3

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard3

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Beautiful Integrals explained step by step 4

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard4

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Beautiful Integrals explained step by step 5

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard5

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Beautiful Integrals explained step by step 6

$$\int \frac{x^2 \quad dx}{\sqrt{x^2 - 1}}$$

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard6

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Beautiful Integrals explained step by step 7

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard7

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Beautiful Integrals explained step by step 8

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard8

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Beautiful Integrals explained step by step 9

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard9

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Beautiful Integrals explained step by step 10

$$\int \frac{2x \quad dx}{(1 - x^2)\sqrt{x^4 - 1}}$$

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Beautiful Integrals explained step by step 11

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard11

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Beautiful Integrals explained step by step 12

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard12

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Beautiful Integrals explained step by step 13

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard13

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Beautiful Integrals explained step by step 14

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard14

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Beautiful Integrals explained step by step 15

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard15

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Beautiful Integrals explained step by step 16

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard16

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Beautiful Integrals explained step by step 17

$$\int \frac{dx}{(x^2 - 1)\sqrt{x^2 + 1}}$$

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard17

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Beautiful Integrals explained step by step 18

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard18

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Beautiful Integrals explained step by step 19

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard19

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Beautiful Integrals explained step by step 20

$$\int \frac{(x + 2)\, dx}{(x^2 + 3x + 3)\sqrt{x + 1}}$$

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Beautiful Integrals explained step by step 21

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard21

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Beautiful Integrals explained step by step 22

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard22

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Beautiful Integrals explained step by step 23

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard23

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Beautiful Integrals explained step by step 24

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard24

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Beautiful Integrals explained step by step 25

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard25

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Beautiful Integrals explained step by step 26

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard26

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Beautiful Integrals explained step by step 27

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard27

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Beautiful Integrals explained step by step 28

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard28

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Beautiful Integrals explained step by step 29

e to the power Matrix …. A rare problem

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Beautiful Integrals explained step by step 30

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard30

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Beautiful Integrals explained step by step 31

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard31

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Beautiful Integrals explained step by step 32

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard32

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Beautiful Integrals explained step by step 33

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard33

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Beautiful Integrals explained step by step 34

Try solving $\int \sqrt{1 + \text{Cosec } x}\, dx$

$$I = \int \sqrt{1 + \operatorname{cosec} x}\,.\,dx$$

$$= \int \sqrt{1 + \frac{1}{\sin x}}\,.\,dx = \int \sqrt{\frac{\sin x + 1}{\sin x}}\,.\,dx$$

$$= \int \sqrt{\frac{(1 + \sin x)(1 - \sin x)}{\sin x\,(1 - \sin x)}}\,.\,dx \qquad \text{[On rationalization]}$$

$$= \int \sqrt{\frac{1 - \sin^2 x}{\sin x - \sin^2 x}}\,.\,dx \qquad [\because \quad (a + b)(a - b) = a^2 - b^2]$$

$$= \int \frac{\cos x}{\sqrt{\sin x - \sin^2 x}}\,.\,dx \qquad [\because \quad \sin^2 A + \cos^2 A = 1]$$

$$\sin x = z \quad \Rightarrow \quad \cos x\, dx = dz$$

$$I = \int \frac{1}{\sqrt{z - z^2}}\,.\,dz = \int \frac{1}{\sqrt{-(z^2 - z)}}\,.\,dz$$

$$= \int \frac{1}{\sqrt{\frac{1}{4} - \left(z^2 - z + \frac{1}{4}\right)}}\,.\,dz \qquad \left[\begin{array}{l}\text{Add and subtract } \frac{1}{4} \text{ to the denom.} \\ \because \left(\frac{1}{2} \text{ co-eff. of } x\right)^2 = \frac{1}{4}\end{array}\right]$$

$$= \int \frac{1}{\sqrt{\left(\frac{1}{2}\right)^2 - \left(z - \frac{1}{2}\right)^2}} \, . \, dz$$

$$\left(z - \frac{1}{2}\right) = y \quad \Rightarrow \quad dz = dy$$

$$\text{I} = \int \frac{1}{\sqrt{(1/2)^2 - y^2}} \, . \, dy \qquad \left[\text{By using} \int \frac{1}{\sqrt{a^2 - x^2}} \, . \, dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$= \sin^{-1}\left(\frac{y}{1/2}\right) + c$$

$$= \sin^{-1}\left(\frac{z - 1/2}{1/2}\right) + c \qquad [\because \quad y = z - 1/2]$$

See explanation video

https://archive.org/details/SeveralRelatedOrSimilarIntegrationProblemsDiscussedForBoysWithBeard34

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IIT JEE 2002 (Very interesting) Integral Calculus divide by x multiply by x to the power m

$$\int (x^{3m} + x^{2m} + x^{m})(2x^{2m} + 3x^{m} + 6)\, dx$$

https://archive.org/details/IITJEE2002IntegralCalculusDivideByXMultiplyByXToThePowerM2

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IIT JEE Integral Calculus modifications to play with x power divide and Multiply

https://archive.org/details/IITJEEIntegralCalculusModificationsToPlayWithXPowerDivideAndMultiply

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Integral Calculus Rationalize and proceed much easier if root is in Numerator

https://archive.org/details/IntegralCalculusRationalizeAndProceedMuchEasierIfRootIsInNumerator1

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Integration of e to the power x by x expand as series and then Integrate individual terms

https://archive.org/details/IntegrationOfEToThePowerXByXExpandAsSeriesAndThenIntegrateIndividualTerms

-

Solve another Integral

$$I = \int \sqrt{\frac{1+x}{x}}\,.\,dx$$

$$= \int \sqrt{\frac{1+x}{x} \times \frac{1+x}{1+x}}\,dx \qquad \text{[Multiply and divided by } (1+x)\text{]}$$

$$= \int \sqrt{\frac{(1+x)^2}{x(1+x)}}\,.\,dx = \int \frac{1+x}{\sqrt{x+x^2}}\,.\,dx$$

Let us write :

$$1 + x = \lambda\,.\,\frac{d}{dx}(x + x^2) + \mu$$

$$\Rightarrow \quad 1 + x = \lambda\,(1 + 2x) + \mu \qquad \text{...(1)}$$

$$\Rightarrow \quad 1 + x = 2\lambda x + \lambda + \mu$$

Comparing the coefficients of x and the constant terms, we have

$$1 = 2\lambda \quad \Rightarrow \quad \lambda = \frac{1}{2}$$

and

$$1 = \lambda + \mu \quad \Rightarrow \quad \mu = 1 - \lambda = 1 - \frac{1}{2} = \frac{1}{2}\,.$$

Putting the values of λ and μ in (1),

$$1 + x = \frac{1}{2}(1 + 2x) + \frac{1}{2}\,.$$

$\therefore \qquad I = \int \frac{\frac{1}{2}(1+2x)+\frac{1}{2}}{\sqrt{x+x^2}} \, . \, dx$

$= \frac{1}{2}\int \frac{1+2x}{\sqrt{x+x^2}} dx + \frac{1}{2}\int \frac{1}{\sqrt{x+x^2}} \, . \, dx$

$\Rightarrow \qquad I = \frac{1}{2} \; I_1 + \frac{1}{2} \; I_2 \qquad ...(2)$

Now $\qquad I_1 = \int \frac{1+2x}{\sqrt{x+x^2}} dx$

Put $\qquad x + x^2 = z \;\Rightarrow\; (1+2x)\,dx = dz$

$\therefore \qquad I_1 = \int \frac{1}{\sqrt{z}} . dz = \int z^{-1/2} . dz = \frac{z^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c_1 = 2\sqrt{z} + c_1$

$= 2\sqrt{x+x^2} + c_1 \qquad ...(3)$

and $\qquad I_2 = \int \frac{1}{\sqrt{x+x^2}} \, . \, dx$

$= \int \frac{1}{\sqrt{\left(x^2+x+\frac{1}{4}\right)-\frac{1}{4}}} \, . \, dx \qquad \left[\text{Add and subtract } \frac{1}{4} \text{ to the denom.} \;\; \because \left(\frac{1}{2}\text{ co-eff. of } x\right)^2 = \frac{1}{4}\right]$

$= \int \frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}} \, . \, dx$

Put $\qquad x + \frac{1}{2} = z \;\Rightarrow\; dx = dz$

$\therefore \qquad I_2 = \int \frac{1}{\sqrt{z^2-\left(\frac{1}{2}\right)^2}} \, . \, dz \qquad \left[\text{By using } \int \frac{1}{\sqrt{x^2-a^2}} . dx = \log\left|x+\sqrt{x^2-a^2}\right| + c\right]$

$= \log\left|z+\sqrt{z^2-\left(\frac{1}{2}\right)^2}\right| + c_2 = \log\left|\left(x+\frac{1}{2}\right)+\sqrt{\left(x+\frac{1}{2}\right)^2-\frac{1}{4}}\right| + c_2$

$= \log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x}\right| + c_2 \qquad ...(4)$

$\therefore$ From equation (2),

$I = \frac{1}{2} \, I_1 + \frac{1}{2} \, I_2 \qquad$ [Using (3) and (4)]

8 examples of Integration

https://archive.org/details/IntegrationExpandSin2xAAndTakeCosXCommonWithinTheRoot

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Solve another problem

$$I = \int \frac{ax^3 + bx}{x^4 + c^2}\,dx = \int \frac{ax^3}{x^4 + c^2}\,.dx + \int \frac{bx}{x^4 + c^2}\,.dx$$

$$= a\int \frac{x^3}{x^4 + c^2}\,.dx + b\int \frac{x}{x^4 + c^2}\,.dx$$

$\Rightarrow \quad I = a\,I_1 + b\,I_2$ ...(1)

Now $\quad I_1 = \int \frac{x^3}{x^4 + c^2}\,.dx$

$$= \frac{1}{4}\int \frac{4x^3}{x^4 + c^2}\,.dx$$ [Multiply and divided by 4]

$$= \frac{1}{4}\log\left|x^4 + c^2\right| + c_1$$ ...(2) $\left[\because \int \frac{f'(x)}{f(x)}\,.dx = \log|f(x)| + c\right]$

and $\quad I_2 = \int \frac{x}{x^4 + c^2}\,.dx$

$$= \frac{1}{2}\int \frac{2x}{(x^2)^2 + c^2}\,dx$$ [Multiply and divided by 2]

Put $\quad x^2 = z \Rightarrow 2x\,dx = dz$

$$= \frac{1}{2}\int \frac{1}{z^2 + c^2}\,dz$$ $\left[\text{By using} \int \frac{1}{x^2 + a^2}\,.dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c\right]$

Solve Integration root linear plus root linear in denominator

If $I = \int \frac{dx}{\sqrt{2x+3} + \sqrt{x+2}}$, then I equals

(a) $2(u - v) + \log \left|\frac{u-1}{u+1}\right| + \log\left|\frac{v-1}{v+1}\right| + C$

$u = \sqrt{2x+3}$, $v = \sqrt{x+2}$

(b) $\log\left|\frac{\sqrt{x+2} + \sqrt{2x+3}}{\sqrt{x+2} - \sqrt{2x+3}}\right| + C$

(c) $\log\left(\sqrt{x+2} + \sqrt{2x+3}\right) + C$

(d) is transcedental function in u and v, $u = \sqrt{2x+3}$

$v = \sqrt{x+2}$

Ans. (a), (d)

$$I = \int \frac{\sqrt{2x+3} - \sqrt{x+2}}{x+1}\,dx$$

$$= I_1 - I_2$$

where $I_1 = \int \frac{\sqrt{2x+3}}{x+1}\,dx$ and $I_2 = \int \frac{\sqrt{x+2}}{x+1}\,dx$

Put $2x + 3 = t^2$. in I_1, so that

$$I_1 = \int \frac{2t \cdot t}{t^2 - 1}\,dt = 2\int\left[1 + \frac{1}{t^2-1}\right]dt$$

$$= 2\left[t + \frac{1}{2}\log\left|\frac{t-1}{t+1}\right|\right]$$

In I_2, put $x + 2 = y^2$, so that

$$I_2 = \int \frac{2y^2}{y^2-1}\,dy = 2y + \log\left|\frac{y-1}{y+1}\right|$$

Thus,

$$I = 2\left(\sqrt{2x+3} - \sqrt{x+2}\right) + \log\left|\frac{\sqrt{2x+3}-1}{\sqrt{2x+3}+1}\right| + \log\left|\frac{\sqrt{x+2}-1}{\sqrt{x+2}+1}\right| + C$$

Solve another Problem

Evaluate $\int \frac{\sin 2x\, dx}{(a+b\cos x)^2}$.

Solution:

We have $I = \int \frac{\sin 2x dx}{(a+b\cos x)^2} = 2\int \frac{\sin x \cos x dx}{(a+b\cos x)^2}$

Now put $a + b \cos x = t$

so that $-b \sin x\, dx = dt$.

Also $\cos x = \frac{(t-a)}{b}$.

$$\therefore\ I = -\frac{2}{b}\int \frac{(t-a)/b}{t^2}dt = -\frac{2}{b^2}\int\left[\frac{t}{t^2}-\frac{a}{t^2}\right]dt$$

$$= -\frac{2}{b^2}\int\left[\frac{1}{t}-\frac{a}{t^2}\right]dt = -\frac{2}{b^2}\left[\log t + \frac{a}{t}\right]$$

$$= -\frac{2}{b^2}\left[\log(a+b\cos x) + \frac{a}{a+b\cos x}\right].$$

A special Integral

$$\int \frac{(1-\sqrt{1+x+x^2})^2}{x^2\sqrt{(1+x+x^2)}}\,dx$$

Here we set $\sqrt{1+x+x^2} = xt + 1$, so that

$$x = \frac{2t-1}{1-t^2},\ dx = \frac{2t^2-2t+2}{(1-t^2)^2}\,dt \text{ and}$$

$$(1-\sqrt{1+x+x^2}) = \frac{-2t^2+t}{(1-t^2)}$$

Substitution of these values in the given integral transforms the problem in the form

$$\int \frac{(-2t^2+t)^2\,(1-t^2)^2\,(1-t^2)\,(2t^2-2t+2)}{(1-t^2)^2\,(2t-1)^2\,(t^2-t+1)\,(1-t^2)^2}\,dt$$

$$= +2\int \frac{t^2}{1-t^2}\,dt = -2t + \ln\left|\frac{1+t}{1-t}\right| + C$$

7 very interesting and strange Integrals

https://archive.org/details/IntegrationRootWithinRootSoPutTheFullExpressionOfTheFirstRootAsT2

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An advanced example

$$I = \int \frac{(x+1)}{x(1+xe^x)^2}\,dx$$

$$I = \int \frac{e^x(x+1)}{x\,e^x(1+xe^x)^2}\,dx$$

put $1 + xc^x = t$, $(xe^x + e^x)\,dx = dt$

$$I = \int \frac{dt}{(t-1)t^2} = \int\left(\frac{1}{1-t} + \frac{1}{t} + \frac{1}{t^2}\right)dt$$

$$= -\log|1-t| + \log|t| - \frac{1}{t} + C = \log\left|\frac{t}{1-t}\right| - \frac{1}{t} + C$$

$$= \log\left|\frac{1+xe^x}{-xe^x}\right| - \frac{1}{1+xe^x} + C = \log\left(\frac{1+xe^x}{xe^x}\right) - \frac{1}{1+xe^x} + C$$

Practice Example

Let $f(x)$ be a function defined by $f(x) = \int_1^x x(x^2 - 3x + 2)\,dx$, $1 \le x \le 3$, then the range of $f(x)$ is

(a) $\left[-\frac{1}{4}, 2\right]$ (b) $\left[-\frac{1}{4}, 4\right]$

(c) $[0, 2]$ (d) none of these

Solution :

(a). We have,

$f'(x) = x(x^2 - 3x + 2) = x(x-1)(x-2)$

Clearly, $f'(x) \le 0$ in $1 \le x \le 2$ and $f'(x) \ge 0$ in $2 \le x \le 3$.

$\therefore$ $f'(x)$ is monotonic decreasing in [1, 2] and monotonic increasing in [2, 3].

$$\therefore \text{Min. } f(x) = f(2) = \int_1^2 x(x^2 - 3x + 2)\,dx$$

$$= \left| \frac{x^4}{4} - x^3 + x^2 \right|_1^2 = \frac{-1}{4}$$

Max. $f(x)$ = the greatest among $(f(1), f(3))$

$$\text{Now, } f(1) = \int_1^1 x(x^2 - 3x + 2)\,dx = 0$$

$$f(3) = \int_1^3 x(x^2 - 3x + 2)\,dx$$

$$= \frac{x^4}{4} - x^3 + 2\Big|_1^3 = 2. \quad \therefore \text{ Max. } f(x) = 2$$

Hence, Range $= \left[\frac{-1}{4}, 2\right]$

Practice Example

If $I = \int_0^{\infty} \frac{\sqrt{x}\,dx}{(1+x)(2+x)(3+x)}$, then I equals

(a) $\frac{\pi}{2}(2\sqrt{2} - \sqrt{3} - 1)$ (b) $\frac{\pi}{2}(2\sqrt{2} + \sqrt{3} - 1)$

(c) $\frac{\pi}{2}(2\sqrt{2} - \sqrt{3} + 1)$ (d) none of these

Ans. (a)

Solution Put $\sqrt{x} = t$ or $x = t^2$, so that

$$I = 2\int_0^{\infty} \frac{t^2}{(1+t^2)(2+t^2)(3+t^2)}\,dt$$

$$= \int_0^{\infty} \left(-\frac{1}{1+t^2} + \frac{4}{2+t^2} - \frac{3}{3+t^2} \right) dt$$

$$= \left(-\tan^{-1} t + \frac{4}{\sqrt{2}} \tan^{-1}\left(\frac{t}{\sqrt{2}}\right) - \frac{3}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) \right) \Bigg]_0^{\infty}$$

$$= -\frac{\pi}{2} + 2\sqrt{2}\left(\frac{\pi}{2}\right) - \sqrt{3}\left(\frac{\pi}{2}\right)$$

$$= \frac{\pi}{2}(2\sqrt{2} - \sqrt{3} - 1).$$

Practice Example

The value $\int_0^1 \cot^{-1}(1 + x^2 - x)\, dx$ is

(a) $\pi/2 - \log 2$ (b) $\pi - \log 2$

(c) $\pi/4 - \log 2$ (d) $2\int_0^1 \tan^{-1} x\, dx$

Ans. (a), (d)

Solution $\cot^{-1}(1 + x^2 - x) = \tan^{-1}\left(\frac{x + 1 - x}{1 - x(1 - x)}\right)$

$$= \tan^{-1} x + \tan^{-1}(1 - x)$$

$$I = \int_0^1 \cot^{-1}(1+x^2-x)dx = \int_0^1 \tan^{-1} x\, dx + \int_0^1 \tan^{-1}(1-x)dx$$

$$= \int_0^1 \tan^{-1} x\, dx + \int_0^1 \tan^{-1} x\, dx = 2\int_0^1 \tan^{-1} x\, dx$$

$$= 2x \tan^{-1} x\Big]_0^1 - \int_0^1 \frac{2x}{1 + x^2} dx$$

$$= 2\tan^{-1}(1) - \log(1 + x^2)\Big]_0^1$$

$$= 2(\pi/4) - \log 2 = \pi/2 - \log 2$$

Practice Example

$$\int_{\sqrt{(3a^2+b^2)/4}}^{\sqrt{(a^2+b^2)/2}} \frac{x}{\sqrt{(x^2-a^2)(b^2-x^2)}}\,dx =$$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

Solution :

(d). Let $I = \int_{\sqrt{(3a^2+b^2)/4}}^{\sqrt{(a^2+b^2)/2}} \frac{x}{\sqrt{(x^2-a^2)(b^2-x^2)}}\,dx$

Put $x^2 = a^2\cos^2 t + b^2\sin^2 t$

$\Rightarrow \quad 2\,x\,dx = [2a^2\cos t\,(-\sin t) + 2\,b^2\sin t\,(\cos t)]\,dt$

$\Rightarrow \quad x\,dx = \frac{1}{2}(b^2-a^2)\sin 2t.dt$

For $\quad x^2 = \frac{a^2+b^2}{2} = a^2\cos^2 t + b^2\sin^2 t$

$\Rightarrow \quad a^2+b^2 = 2(1-\sin^2 t)\,a^2 + 2b^2\sin^2 t$

or, $\quad (a^2+b^2) = 2a^2 + 2(b^2-a^2)\sin^2 t$

$\Rightarrow \quad \sin^2 t = \frac{1}{2} \Rightarrow \cos 2t = 0 \quad \Rightarrow t = \pi/4$

For $\quad x^2 = \frac{3a^2+b^2}{4} = a^2\cos^2 t + b^2\sin^2 t$

$\Rightarrow \quad 3a^2+b^2 = 4a^2 + 4(b^2-a^2)\sin^2 t$

$\Rightarrow \quad \sin^2 t = \frac{1}{4} \Rightarrow \cos 2t = \frac{1}{2} \Rightarrow t = \frac{\pi}{4}$

$$\therefore \quad I = \int_{\pi/6}^{\pi/4} \frac{1}{2} \frac{(b^2-a^2)\sin 2t.dt}{\sqrt{(b^2-a^2)\sin^2 t(b^2-a)\cos^2 t}}$$

$$= \int_{\pi/6}^{\pi/4} dt = (t)_{\pi/6}^{\pi/4} = \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}$$

Practice Example

If $\int_0^{\pi/2} \frac{x^2 \cos x}{(1+\sin x)^2} dx = A\ \pi - \pi^2$ then A is

Ans. 2

Solution Integrating by parts, we have

$$\int_0^{\pi} \frac{x^2 \cos x}{(1+\sin x)^2}\, dx$$

$$= -\frac{x^2}{1+\sin x}\Bigg|_0^{\pi} + 2\int_0^{\pi} \frac{x}{1+\sin x} dx = -\pi^2 + 2I$$

where

$$I = \int_0^{\pi} \frac{x}{1+\sin x} dx = \int_0^{\pi} \frac{\pi - x}{1+\sin x} dx = \pi \int_0^{\pi} \frac{dx}{1+\sin x} - I$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{dx}{1+\sin x} = 2\pi \int_0^{\pi/2} \frac{dx}{1+\sin x}$$

$$\Rightarrow \quad I = \pi \int_0^{\pi/2} \frac{dx}{1+\sin x} = \pi \int_0^{\pi/2} \frac{dx}{1+\sin(\pi/2 - x)}$$

$$= \int_0^{\pi/2} \frac{dx}{1+\cos x}$$

$$= \frac{\pi}{2}\int_0^{\pi/2} \sec^2 (x/2) dx = \pi \tan (x/2)\Big]_0^{\pi/2} = \pi$$

Hence $\int_0^{\pi} \frac{x^2 \cos x}{(1+\sin x)^2} dx = -\pi^2 + 2\pi$

Practice example

Example $\int_0^{\pi}\frac{dx}{(1+a^2)-2a\cos x} = \frac{\pi}{1-a^2}$ or $\frac{\pi}{a^2-1}$

according as $a < 1$ or $a > 1$.

The given problem may be re-written in the form

$$\int_0^{\pi}\frac{dx}{(1+a^2)\left(\cos^2\frac{x}{2}+\sin^2\frac{x}{2}\right)-2a\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)}$$

which can be expressed in the forms

$$I = \frac{2}{(1+a^2)^2}\int\frac{dt}{\left(\frac{1-a}{1+a}\right)^2+t^2} \text{ or } \frac{2}{(1+a^2)^2}\int\frac{dt}{\left(\frac{a-1}{a+1}\right)^2+t^2}$$

according as $a < 1$ or > 1, where $t = \tan\frac{x}{2}$

Hence

Hence

$$I = \frac{2}{(1-a^2)}\left[\tan^{-1}\frac{t(1+a)}{(1-a)}\right]_0^{\infty} = \frac{\pi}{1-a^2} \text{ if } a < 1$$

Similarly in the other case the answer shall be $\frac{\pi}{a^2-1}$, $a > 1$

Practice example

$\int_0^{\sin^2 x}\sin^{-1}\left(\sqrt{t}\right)dt + \int_0^{\cos^2 x}\cos^{-1}\left(\sqrt{t}\right)dt$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$

(c) 0 (d) none of these

Solution :

(a). We have,

$$I = \int_0^{\sin^2 x} \sin^{-1}\left(\sqrt{t}\right) dt + \int_0^{\cos^2 x} \cos^{-1}\left(\sqrt{t}\right) dt$$

$$= \left[t \sin^{-1}\left(\sqrt{t}\right)\right]_0^{\sin^2 x} - \int_0^{\sin^2 x} \frac{\sqrt{t}}{\sqrt[2]{1-t}}\, dt$$

$$+ \left[t \cos^{-1}\left(\sqrt{t}\right)\right]_0^{\cos^2 x} - \int_0^{\cos^2 x} \frac{\sqrt{t}}{\sqrt[2]{1-t}}\, dt$$

$$= x \sin^2 x + \int_{\sin^2 x}^{0} \frac{\sqrt{t}}{\sqrt[2]{1-t}}\, dt + x \cos^2 x + \int_0^{\cos^2 x} \frac{\sqrt{t}}{\sqrt[2]{1-t}}\, dt$$

$$= x\,(\sin^2 x + \cos^2 x) + \int_{\sin^2 x}^{\cos^2 x} \frac{\sqrt{t}}{\sqrt[2]{1-t}}\, dt$$

Putting $t = \sin^2 \theta$ and $dt = 2 \sin \theta \cos \theta\, d\theta$, we get,

$$\int \frac{\sqrt{t}}{\sqrt[2]{1-t}}\, dt = \int \frac{\sin\theta}{\sqrt{1-\sin^2\theta}}\, 2 \sin\theta \cos\theta\, d\theta$$

$$= \int \sin^2\theta\, d\theta = \int \frac{1-\cos 2\theta}{2}\, d\theta$$

$$= \frac{\theta}{2} - \frac{\sin 2\theta}{4}$$

Also, when $t = \sin^2 x$, $\theta = x$ and when $t = \cos^2 x$, $\theta = \pi/2 - x$

$$\therefore\ I = x + \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_x^{\pi/2 - x}$$

$$= x + \left(\frac{\pi}{4} - \frac{x}{2} - \frac{\sin 2x}{4}\right) - \left(\frac{x}{2} - \frac{\sin 2x}{4}\right)$$

$$= x + \frac{\pi}{4} - x = \frac{\pi}{4}$$

Practice example

$$I = \int_0^{\pi/4} \frac{\sin 2\theta \, d\theta}{\sin^4\theta + \cos^4\theta} = \int_0^{\pi/4} \frac{2\sin\theta\cos\theta}{\sin^4\theta + \cos^4\theta} d\theta$$

$$= \int_0^{\pi/4} \frac{2\tan\theta\sec^2\theta\, d\theta}{1+\tan^4\theta},$$

dividing the numerator and denominator by $\cos^4\theta$

Put $\tan^2\theta = t$,

so that $2\tan\theta\sec^2\theta\, d\theta = dt$.

When $\theta = 0$,

$t = \tan^2 0 = 0$

and when $\theta = \frac{\pi}{4}$,

$t = \tan^2 \frac{1}{4}\pi = 1.$

$$\therefore \quad I = \int_0^1 \frac{dt}{1+t^2} = \left[\tan^{-1} t\right]_0^1$$

$$= \tan^{-1} 1 - \tan^{-1} 0 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

Practice example

If $f(x)$ satisfies the relation $\int_{-2}^{x} f(t)\,dt + xf''(3)$

$= \int_1^x x^3 dx + f'(1)\int_2^x x^2 dx + f''(2)\int_3^x x\,dx$, then

(a) $f(x) = x^3 + 5x^2 + 2x - 6$

(b) $f(x) = x^3 - 5x^2 + 2x + 6$

(c) $f(x) = x^3 + 5x^2 + 2x - 6$

(d) $f(x) = x^3 - 5x^2 + 2x - 6$

Solution :

(d). Differentiating the given equation w.r.t. x, we get

$$f(x)+f'''(3)=x^3+x^2f'(1)+xf''(2) \qquad ...(1)$$

Differentiating successively w.r.t. x, we get

$$f'(x)=3x^2+2xf'(1)+f''(2) \qquad ...(2)$$

$$f''(x)=6x+2f'(1) \qquad ...(3)$$

$$f'''(x)=6 \qquad ...(4)$$

Putting $x = 1$, 2 and 3 in equations (2), (3) and (4) respectively, we get

$$f'(1)=3+2f'(1)+f''(2), \quad f''(2)=12+2f'(1)$$

and, $f'''(3)=6$

Solving, we have

$$f'(1)=-5, f''(2)=2, f'''(3)=6$$

Putting the values in equation (1), we have

$$f(x)=x^3-5x^2+2x-6.$$

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Very Important Integral Calculus problem IIT JEE problems modification n th root

https://archive.org/details/VeryImportantIntegralCalculusProblemIITJEEProblemsModificationNThRoot

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Practice example

If $I_1 = \int_{1/e}^{\tan x} \frac{t}{1+t^2}\, dt$ and $I_2 = \int_{1/e}^{\cot x} \frac{dt}{t(1+t^2)}$ then the value of $I_1 + I_2$ is

(a) 1/2 (b) 1

(c) $e/2$ (d) $(1/2)(e + 1/e)$

Ans. (b)

Solution Putting $t = 1/u$ in I_2 we have

$$I_2 = -\int_{e}^{\tan x} \frac{u\, du}{1+u^2} = -\int_{1/e}^{\tan x} \frac{u\, du}{1+u^2} + \int_{1/e}^{e} \frac{u\, du}{1+u^2}$$

$$= -I_1 + \frac{1}{2}\int_{1/e}^{e} \frac{2u\, du}{1+u^2}$$

So $$I_1 + I_2 = \frac{1}{2}\log(u^2+1)\Big]_{1/e}^{e} = \frac{1}{2}\left[\log(e^2+1) - \log\left(\frac{e^2+1}{e^2}\right)\right]$$

$$= \frac{1}{2} \times 2 = 1.$$

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Integral dx by (x^2+a^2)^3 by 2 common important for many Physics Numericals

https://archive.org/details/8IntegralDxByx2a23By2CommonImportantForManyPhysicsNumericals

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Practice example

Evaluate $\int_0^a \left(a^2 + x^2\right)^{5/2} dx$.

Solution :

$$I = \int_0^a \left(a^2 + x^2\right)^{5/2} dx \qquad \text{Put} \quad x = a \tan\theta$$

$$\therefore \quad dx = a \sec^2\theta \, d\theta$$

$$= \int_0^{\pi/4} \left(a^2 + a^2 \tan^2\theta\right)^{5/2} \cdot a \sec^2\theta \, d\theta$$

$$= a^6 \int_0^{\pi/4} \sec^7\theta \, d\theta$$

$$= a^6 \left[\left(\frac{\sec^5\theta \tan\theta}{6} \right)_0^{\pi/4} + \frac{5}{6} \int_0^{\pi/4} \sec^5\theta \, d\theta \right]$$

$$= a^6 \left[\frac{2\sqrt{2}}{3} + \frac{5}{6} \int_0^{\pi/4} \sec^5\theta \, d\theta \right]$$

$$= a^6 \left[\frac{2\sqrt{2}}{3} + \frac{5}{6} \left\{ \left(\frac{\sec^3\theta + \tan\theta}{4} \right)_0^{\pi/4} + \frac{3}{4} \int_0^{\pi/4} \sec^3\theta \, d\theta \right\} \right]$$

$$= a^6 \left[\frac{2\sqrt{2}}{3} + \frac{5}{6} \left\{ \frac{2\sqrt{2}}{4} + \frac{3}{4} \int_0^{\pi/4} \sec^3\theta \, d\theta \right\} \right]$$

$$= a^6 \left[\frac{2\sqrt{2}}{3} + \frac{5\sqrt{2}}{12} + \frac{5}{8} \int_0^{\pi/4} \sec^3\theta \, d\theta \right]$$

$$= a^6 \left[\frac{2\sqrt{2}}{3} + \frac{5\sqrt{2}}{12} + \frac{5}{8} \left\{ \left(\frac{\sec\theta \tan\theta}{2} \right)_0^{\pi/4} + \frac{1}{2} \int_0^{\pi/4} \sec\theta \, d\theta \right\} \right]$$

$$= a^6 \left[\frac{2\sqrt{2}}{3} + \frac{5\sqrt{2}}{12} + \frac{5}{8} \left\{ \frac{\sqrt{2}}{2} + \frac{1}{2} \left\{ \log\left(\sec\theta + \tan\theta\right) \right\}_0^{\pi/4} \right\} \right]$$

$$= a^6 \left[\frac{2\sqrt{2}}{3} + \frac{5\sqrt{2}}{12} + \frac{5\sqrt{2}}{16} + \frac{5}{16} \log\left(\sqrt{2} + 1\right) \right]$$

$$= a^6\left[\frac{32\sqrt{2}}{48} + \frac{20\sqrt{2}}{48} + \frac{15\sqrt{2}}{48} + \frac{5}{16}\log\left(\sqrt{2}+1\right)\right]$$

$$= a^6\left[\frac{32\sqrt{2} + 20\sqrt{2} + 15\sqrt{2}}{48} + \frac{5}{16}\log\left(\sqrt{2}+1\right)\right]$$

$$= a^6\left[\frac{67\sqrt{2}}{48} + \frac{5}{16}\log\left(\sqrt{2}+1\right)\right]$$

$$= \frac{a^6}{48}\left[67\sqrt{2} + 15\log\left(\sqrt{2}+1\right)\right]$$

-

Practice example

$\int_0^5 \frac{\tan^{-1}(x-[x])}{1+(x-[x])^2}\,dx$, where $[\cdot]$ denotes the greatest integer function, is equal to

(a) $\frac{\pi^2}{32}$ (b) $\frac{3\pi^2}{32}$

(c) $\frac{5\pi^2}{32}$ (d) none of these

Solution :

(c). $\int_0^5 \frac{\tan^{-1}(x-[x])}{1+(x-[x])^2}\,dx$

$$= \int_0^5 \frac{\tan^{-1}(x-[x])}{1+(x-[x])^2}\,dx$$

$$= \int_0^1 \frac{\tan^{-1}x}{1+x^2}\,dx + \int_1^2 \frac{\tan^{-1}(x-1)}{1+(x-1)^2}\,dx + \ldots$$

$$+\int_4^5 \frac{\tan^{-1}(x-4)}{1+(x-4)^2}\,dx$$

$$=\int_0^1 \frac{\tan^{-1}x}{1+x^2}\,dx+\int_0^1 \frac{\tan^{-1}t}{1+t^2}\,dt+\ldots+\int_0^1 \frac{\tan^{-1}t}{1+t^2}\,dt$$

(Putting $x-1=t$) (Putting $x-4=t$)

$$=5\int_0^1 \frac{\tan^{-1}x}{1+x^2}\,dx=5\int_0^{\pi/4} u\,du \quad [\text{Putting } \tan^{-1}x=u]$$

$$=5\left[\frac{u^2}{2}\right]_0^{\pi/2}=\frac{5\pi^2}{32}$$

-

Practice example

Let $I_1=\int_{\sec^2 z}^{2-\tan^2 z} x f(x(3-x))\,dx$

and, $I_2=\int_{\sec^2 z}^{2-\tan^2 z} f(x(3-x))\,dx$,

where f is a continuous function and z is any real number, then $I_1/I_2=$

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$

(c) 1 (d) none of these

Solution

(a). We have, $I_1 = \int_{\sec^2 z}^{2-\tan^2 z} x\, f(x(3-x))\, dx$

$$= \int_{\sec^2 z}^{2-\tan^2 z} (3-x)\, f((3-x)\{3-(3-x)\})\, dx$$

$$\left[\because \int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx\right]$$

$$= \int_{\sec^2 z}^{2-\tan^2 z} (3-x)\, f(x(3-x))$$

$$= 3\int_{\sec^2 z}^{2-\tan^2 z} f(x(3-x))\, dx - \int_{\sec^2 z}^{2-\tan^2 z} x f(x(3-x))\, dx$$

$$= 3I_2 - I_1$$

$\therefore \quad 2I_1 = 3I_2$ and so $I_1/I_2 = \dfrac{3}{2}$

-

Practice example

Evaluate $\int_0^{\pi/4} \tan^5\theta\, d\theta$

$$I = \int_0^{\pi/4} \tan^5\theta\, d\theta$$

$$= \left(\frac{\tan^4\theta}{4}\right)_0^{\pi/4} - \int_0^{\pi/4} \tan^3\theta \, d\theta$$

$$= \frac{1}{4} - \int_0^{\pi/4} \tan^3\theta \, d\theta$$

$$= \frac{1}{4} - \left[\left(\frac{\tan^2\theta}{2}\right)_0^{\pi/4} - \int_0^{\pi/4} \tan\theta \, d\theta\right]$$

$$= \frac{1}{4} - \left[\frac{1}{2} - (\log\sec\theta)_0^{\pi/4}\right]$$

$$= \frac{1}{4} - \left[\frac{1}{2} - \log\sqrt{2}\right]$$

$$= -\frac{1}{4} + \log\sqrt{2}$$

$$= -\frac{1}{4} + \frac{1}{2}\log 2$$

-

Practice example

If $\varphi(n) = \int_0^{\pi/4} \tan^n x \, dx$, show that $\varphi(n) + \varphi(n-2) = \frac{1}{n-1}$ and deduce the value of $\varphi(5)$.

Solution :

$$\varphi(n) = \int_0^{\pi/4} \tan^n x\, dx$$

$$= \left(\frac{\tan^{n-1} x}{n-1}\right)_0^{\pi/4} - \int_0^{\pi/4} \tan^{n-2} x\, dx$$

$$= \frac{1}{n-1} - \varphi_{n-2}$$

$$\Rightarrow \varphi_n + \varphi_{n-2} = \frac{1}{n-1} \qquad \text{Proved}$$

Now $\varphi(5) = \frac{1}{4} - \varphi_3$

$$= \frac{1}{4} - \left[\frac{1}{2} - \varphi_1\right]$$

$$= -\frac{1}{4} + \varphi_1$$

$$= -\frac{1}{4} + \int_0^{\pi/4} \tan x\, dx$$

Practice Example

Prove that

$$\int_0^{\pi/2} \cos^m x \sin mx\, dx = \frac{1}{2^{m+1}}\left\{2 + \frac{2^2}{2} + \frac{2^3}{3} + \ldots\ldots + \frac{2^m}{m}\right\}$$

Solution :

We know that

$$\int_0^{\pi/2} \cos^m x \sin mx\, dx$$

$$= \left[-\frac{\cos^m x \cos mx}{m+m}\right]_0^{\pi/2} + \frac{m}{m+m} \int_0^{\pi/2} \cos^{m-1} x \sin(m-1)x\, dx$$

$$\Rightarrow \qquad I_{m,m} = \frac{1}{2m} + \frac{1}{2} I_{m-1,m-1}$$

Put $m-1$ for m,

$$I_{m-1,m-1} = \frac{1}{2(m-1)} + \frac{1}{2} I_{m-2,m-2}$$

$$I_{m,m} = \frac{1}{2m} + \frac{1}{2}\left[\frac{1}{2(m-1)} + \frac{1}{2} I_{m-2,m-2}\right]$$

$$= \frac{1}{2m} + \frac{1}{2^2(m-1)} + \frac{1}{2^2} I_{m-2,m-2}$$

$$= \frac{1}{2m} + \frac{1}{2^2(m-1)} + \frac{1}{2^3(m-2)} + \frac{1}{2^3} I_{m-3,m-3}$$

| Proceeding similarly

$$= \frac{1}{2m} + \frac{1}{2^2(m-1)} + \frac{1}{2^3(m-2)} + \ldots.. + \frac{1}{2^m\{m-(m-1)\}} + \frac{1}{2^m} I_{m-m,m-m}$$

$$= \frac{1}{2m} + \frac{1}{2^2(m-1)} + \frac{1}{2^3(m-2)} + \ldots.. + \frac{1}{2^m 1} + \frac{1}{2^m} I_{o,o}$$

$$= \frac{1}{2m} + \frac{1}{2^2(m-1)} + \frac{1}{2^3(m-2)} + \ldots.. + \frac{1}{2^m \cdot 1} + \frac{1}{2^m}\int_0^{\pi/2} o\, dx$$

Now $\int_0^{\pi/2} o\, dx = [c]_0^{\pi/2} = c - c = o$

$$\therefore \quad I_{m,m} = \frac{1}{2m} + \frac{1}{2^2(m-1)} + \frac{1}{2^3(m-2)} + \ldots.. + \frac{1}{2^m \cdot 1}$$

Writing the series in the reverse order

$$= \frac{1}{2^m \cdot 1} + \frac{1}{2^{m-1} \cdot 2} + \frac{1}{2^{m-2} \cdot 3} + \ldots.. + \frac{1}{2m}$$

$$= \frac{1}{2^{m+1}}\left[\frac{2^{m+1}}{2^m \cdot 1} + \frac{2^{m+1}}{2^{m-1} \cdot 2} + \frac{2^{m+1}}{2^{m-2} \cdot 3} + \ldots.. + \frac{2^{m+1}}{2m}\right]$$

Practice Example

Prove that $\int_0^{\pi/2} \cos^{n-2} x \sin nx \, dx = \frac{1}{n-1}$; n being an integer greater than unity.

Solution :

$$I = \int_0^{\pi/2} \cos^{n-2} x \sin nx \, dx$$

$$= \int_0^{\pi/2} \cos^{n-2} x \sin \{(n-1)x + x\} \, dx$$

$$= \int_0^{\pi/2} \cos^{n-2} x \{\sin (n-1) x \cos x + \cos (n-1) x \sin x\} \, dx$$

$$= \int_0^{\pi/2} \underset{\text{I}}{\cos^{n-1} x} \, \underset{\text{II}}{\sin (n-1) x} \, dx + \int_0^{\pi/2} \cos^{n-2} x \cos (n-1) x \sin x \, dx$$

Integrating the first integral only by parts

$$= \left\{\cos^{n-1} x \cdot -\frac{\cos (n-1) x}{n-1}\right\}_0^{\pi/2} - \int_0^{\pi/2} (n-1) \cos^{n-2} x (-\sin x) \cdot \left\{-\frac{\cos (n-1) x}{n-1}\right\} dx + \int_0^{\pi/2} \cos^{n-2} x \cos (n-1) x \sin x \, dx$$

$$= \frac{1}{n-1} - \int_0^{\pi/2} \cos^{n-2} x \cos (n-1) x \sin x \, dx + \int_0^{\pi/2} \cos^{n-2} x \cos (n-1) x \sin x \, dx$$

$$= \frac{1}{n-1}$$

Practice Example

If $I_{1,n} = \int_0^{\pi/2} \frac{\sin(2n-1)x}{\sin x} dx$ and $I_{2,n} = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin^2 x} dx$

$n \in N$, then

(a) $I_{2,n+1} - I_{2,n} = I_{1,n}$

(b) $I_{2,n+1} - I_{2,n} = I_{1,n+1}$

(c) $I_{2,n+1} + I_{1,n} = I_{2,n}$

(d) $I_{2,n+1} + I_{1,n+1} = I_{2,n}$

Solution

(b). $I_{2,n} - I_{2,n-1} = \int_0^{\pi/2} \frac{(\sin^2 nx - \sin^2(n-1)x)}{\sin^2 x} dx$

$$= \int_0^{\pi/2} \frac{\sin(2n-1)x \sin x}{\sin^2 x} dx$$

$$= \int_0^{\pi/2} \frac{\sin(2n-1)x}{\sin x} dx = I_{1,n}$$

$\therefore \quad I_{2,n+1} - I_{2,n} = I_{1,n+1}$

Reduction forms

Let $I_n = \int \sin^n x\,dx$ or $I_n = \int \sin^{n-1} x \sin x\,dx$.

Integrating by parts regarding sin x as the 2nd function, we have

$I_n = \sin^{n-1} x.(-\cos x) - \int (n-1) \sin^{n-2} x.\cos x.(-\cos x)dx$

$= -\sin^{n-1} x.\cos x + (n-1)\int \sin^{n-2} x.\cos^2 x\,dx$

$= -\sin^{n-1} x.\cos x + (n-1)\int \sin^{n-2} x.\left(1-\sin^2 x\right)dx$

$= -\sin^{n-1} x.\cos x + (n-1)\int \sin^{n-2} x\,dx - (n-1)\int \sin^n x\,dx$

$= -\sin^{n-1} x.\cos x + (n-1)\int \sin^{n-2} x\,dx - (n-1)I_n$.

Transposing the last term to the left, we have

$I_n (1 + n - 1) = -\sin^{n-1} x.\cos x + (n-1) I_{n-2}$,

$$\left[\because I_{n-2} = \int \sin^{n-2} x\,dx\right]$$

or $nI_n = -\sin^{n-1} x \cos x + (n-1) I_{n-2}$

or $I_n = -\dfrac{\sin^{n-1} x \cos x}{n} + \dfrac{n-1}{n} I_{n-2}$.

Let $I_n = \int \cos^n x\,dx$ or $I_n = \int \cos^{n-1} x.\cos x\,dx$.

Integrating by parts regarding cos x as the 2nd function, we have

$I_n = \cos^{n-1} x.\sin x - \int (n-1)\cos^{n-2} x.(\sin x).\sin x\,dx$

$= \cos^{n-1} x.\sin x + (n+1)\int \cos^{n-2} x.\sin^2 x\,dx$

$= \cos^{n-1} x.\sin x + (n-1)\int \cos^{n-2} x\left(1-\cos^2 x\right)dx$

$= \cos^{n-1} x.\sin x + (n-1)\int \cos^{n-2} x\,dx - (n-1)\int \cos^n x\,dx$

$= \cos^{n-1} x \sin x + (n-1) I_{n-2} - (n-1) I_n$.

Transposing the last term to the left, we have

$I_n (1 + n - 1) = \cos^{n-1} x.\sin x + (n-1) I_{n-2}$

or $n I_n = \cos^{n-1} x.\sin x + (n-1) I_{n-2}$.

$$\therefore \int \cos^n dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx.$$

We have $\int \tan^n x\,dx = \int \tan^{n-2} x.\tan^2 x\,dx$

$= \int \tan^{n-2} x.(\sec^2 x - 1)dx$

$= \int \tan^{n-2} x.\sec^2 x\,dx - \int \tan^{n-2} x\,dx$

$= \dfrac{(\tan x)^{n-2+1}}{n-2+1} - \int \tan^{n-2} x\,dx$

or $\int \tan^n x\,dx = \dfrac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx,$

We have $\int \cot^n x\,dx = \int \cot^{n-2} x.\cot^2 x\,dx$

$= \int \cot^{n-2} x(\operatorname{cosec}^2 x - 1)dx$

$= \int \cot^{n-2} x.\operatorname{cosec}^2 x\,dx - \int \cot^{n-2} x\,dx$

$= -\dfrac{(\cot x)^{n-1}}{n-1} - \int \cot^{n-2} x\,dx$

or $\cot^n x\,dx = -\dfrac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x\,dx,$

We have $I_n = \int \sec^n x\,dx = \int \sec^{n-2} x.\sec^2 x\,dx$

Integrating by parts regarding $\sec^2 x$ as the 2nd function, we have

$I_n = \sec^{n-2} x \tan x - \int (n-2)\sec^{n-3} x \sec x \tan^2 x\,dx$

$= \sec^{n-2} x \tan x - (n-2)\int \sec^{n-2} x(\sec^2 x - 1)dx$

$= \sec^{n-2} x \tan x - (n-2)\int \sec^n x\,dx + (n+2)\int \sec^{n-2} x\,dx.$

Transposing the term containing $\int \sec^n x\,dx$ to the left,

we have

$(n-2+1)\int \sec^n x\,dx = \sec^{n-2} x \tan x + (n-2)\int \sec^{n-2} x\,dx$

$$\int \sec^n x\, dx = \sec^{n-2} x \tan x - \int (n-2)\sec^{n-2} x \tan x \tan x\, dx$$

$$= \tan x \sec^{n-2} x - (n-2)\int \sec^{n-2} x\, (\sec^2 x - 1)\, dx$$

$$= \tan x \sec^{n-2} x - (n-2)\left(\sec^n x - \int \sec^{n-2} x\, dx\right)$$

$$[1+(n-2)]\int \sec^n x\, dx = \tan x \sec^{n-2} x + (n-2)\int \sec^{n-2} x\, dx$$

$$\int \sec^n x\, dx = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\, dx$$

$$\int \operatorname{cosec}^n x\, dx = \int \operatorname{cosec}^{n-2} x \operatorname{cosec}^2 x\, dx$$

Integrating by parts,

$$\int \operatorname{cosec}^n x\, dx = \operatorname{cosec}^{n-2} x\,(-\cot x) - \int (n-2)\operatorname{cosec}^{n-3} x\,(-\operatorname{cosec} x \cot x)(-\cot x)\, dx$$

$$= -\cot x \operatorname{cosec}^{n-2} x - (n-2)\int \operatorname{cosec}^{n-2} x\,(\operatorname{cosec}^2 x - 1)\, dx$$

$$= -\cot x \operatorname{cosec}^{n-2} x - (n-2)\left(\int \operatorname{cosec}^n x - \int \operatorname{cosec}^{n-2} x\, dx\right)$$

$$[1+(n-2)]\int \operatorname{cosec}^n x\, dx = -\cot x \operatorname{cosec}^{n-2} x + (n-2)\int \operatorname{cosec}^{n-2} x\, dx$$

$$\int \operatorname{cosec}^n x\, dx = \frac{-\cot x \operatorname{cosec}^{n-2} x}{n-1} + \frac{n-2}{n-1}\int \operatorname{cosec}^{n-2} x\, dx$$

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Landmark Integral Problem IIT JEE 2002 x becomes x to the power m inside the bracket

https://archive.org/details/LandmarkIntegralProblemIITJEE2002XBecomesXToThePowerMInsideTheBracket2

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To find the Area of a circle of Radius r

https://archive.org/details/7AreaOfCircleP1IITJEEPhyMath

Area of Circle Surface Area and Volume of Sphere

https://archive.org/details/7AreaOfCircleP2SurfaceAreaVolumeOfSphereIITJEEPhyMath

Very important concept of Edge Modeling Surface Area Volume of Sphere

https://archive.org/details/7EdgeModellingP2SurfaceAreaVolumeOfSphereIITJEEPhyMath

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Very important concept of solving differential equation of d2x by dt square equals f(x)

https://archive.org/details/7Steps2SolveD2xByDtSquarefxSHMIITJEEPhyMaths1

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A few Differential Equation Problems

https://archive.org/details/AIEEEDifferentialEquation2008CanBeSolvedAsHomogeneousAndAlsoLinear

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AIEEE 2009 - (Important) Shortest Distance between Graph and line by finding tangent

https://archive.org/details/AIEEEImportantShortestDistanceBetweenGraphAndLineByFindingTangent2009

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AIEEE-2002 Definite Integral Elimination of x by f(a-x) form even and odd functions

https://archive.org/details/AIEEE2002DefiniteIntegralEliminationOfXByFaXFormEvenAndOddFunctions

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AIEEE - Area by Integration - This one is easier to do by integrating f(y) dy seen from right

https://archive.org/details/AIEEEAreaByIntegrationThisOneIsEasierToDoByIntegratingFyDySeenFromRight

-

AIEEE 2007 - Expansion of e to the power -1 - must know the expansions

https://archive.org/details/AIEEEExpansionOfEToThePower1MustKnowTheExpansions2007

-

AIEEE 2008 - Inequality of Integrals - Sin x by root x and Cos x by root x

https://archive.org/details/AIEEEInequalityOfIntegralsSinXByRootXAndCosXByRootX2008

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AIEEE 2002 Problems in Calculus now known as IIT JEE main

https://archive.org/details/AIEEE2002MathsNiceIntegralWithTrickSimilarToAnIITProblemOf80s

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Very important recall in a definite integral tan inverse is in the denominator

https://archive.org/details/VeryImportantRecallInADefiniteIntegralTanInverseIsInTheDenominator

-

Miscellaneous Integration Problems solved by a very good Teacher, Mr Keshav Agarwal. (from youtube)

https://archive.org/details/INTEGRATIONPROBLEMSANDSOLUTIONS

-

Special extra problem in Limits solved by expansion

https://archive.org/details/VeryImportantLimitOfSinxToThePowerSinxLnSinXEtcSolvedByExpansions

AIEEE 2007 - Limit-Infinity- infinity form, expand e^x and do this, instead of L-Hospitals

https://archive.org/details/AIEEELimitInfinityInfinityFormExpandExAndDoThisInsteadOfLHospitals2007

Limits - Is this easy.... (1 + Sin x) to the power Cot x by 2

https://archive.org/details/LimitsIsThisEasy....1SinXToThePowerCotXBy2

-

AIEEE 2009 - Special extra problem in Implicit Differentiation-x to the power x and Cot y and x^2x

https://archive.org/details/AIEEEImplicitDifferentiationXToThePowerXAndCotYAndX2x2009

-

Solve by First Principle $\int_a^b x\,dx$

It is known that,

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+\ldots+f(a+(n-1)h)\right],\ \text{where } h=\frac{b-a}{n}$$

Here, $a=a$, $b=b$, and $f(x)=x$

$$\therefore \int_a^b x\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[a+(a+h)\ldots(a+2h)\ldots a+(n-1)h\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{a+a+a+\ldots+a}_{n\text{ times}}\right)+\left(h+2h+3h+\ldots+(n-1)h\right)\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+h\left(1+2+3+\ldots+(n-1)\right)\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+h\left\{\frac{(n-1)(n)}{2}\right\}\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+\frac{n(n-1)h}{2}\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{n}{n}\left[a+\frac{(n-1)h}{2}\right]$$

$$= (b-a)\lim_{n\to\infty}\left[a+\frac{(n-1)h}{2}\right]$$

$$= (b-a)\lim_{n\to\infty}\left[a+\frac{(n-1)(b-a)}{2n}\right]$$

$$= (b-a)\lim_{n\to\infty}\left[a+\frac{\left(1-\frac{1}{n}\right)(b-a)}{2}\right]$$

$$= (b-a)\left[a+\frac{(b-a)}{2}\right]$$

$$= (b-a)\left[\frac{2a+b-a}{2}\right]$$

$$= \frac{(b-a)(b+a)}{2}$$

$$= \frac{1}{2}\left(b^2-a^2\right)$$

-

Find $\int_0^5 (x+1)\,dx$

Let $I = \int_0^5 (x+1)\,dx$

It is known that,

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)\ldots f(a+(n-1)h)\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=0$, $b=5$, and $f(x)=(x+1)$

$$\Rightarrow h=\frac{5-0}{n}=\frac{5}{n}$$

$$\therefore \int_0^5 (x+1)\,dx = (5-0)\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+\ldots+f\left((n-1)\frac{5}{n}\right)\right]$$

$$=5\lim_{n\to\infty}\frac{1}{n}\left[1+\left(\frac{5}{n}+1\right)+\ldots\left\{1+\left(\frac{5(n-1)}{n}\right)\right\}\right]$$

$$=5\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{1+1+1\ldots1}_{n\text{ times}}\right)+\left[\frac{5}{n}+2\cdot\frac{5}{n}+3\cdot\frac{5}{n}+\ldots(n-1)\frac{5}{n}\right]\right]$$

$$=5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5}{n}\{1+2+3\ldots(n-1)\}\right]$$

$$=5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5}{n}\cdot\frac{(n-1)n}{2}\right]$$

$$=5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5(n-1)}{2}\right]$$

$$=5\lim_{n\to\infty}\left[1+\frac{5}{2}\left(1-\frac{1}{n}\right)\right]$$

$$=5\left[1+\frac{5}{2}\right]$$

$$=5\left[\frac{7}{2}\right]$$

$$=\frac{35}{2}$$

-

Find $\int_2^3 x^2 dx$

It is known that,

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+f(a+2h)\ldots f\{a+(n-1)h\}\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=2$, $b=3$, and $f(x)=x^2$

$$\Rightarrow h=\frac{3-2}{n}=\frac{1}{n}$$

$$\therefore \int_2^3 x^2 dx=(3-2)\lim_{n\to\infty}\frac{1}{n}\left[f(2)+f\left(2+\frac{1}{n}\right)+f\left(2+\frac{2}{n}\right)\ldots f\left\{2+(n-1)\frac{1}{n}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[(2)^2+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+\ldots\left(2+\frac{(n-1)}{n}\right)^2\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[2^2+\left\{2^2+\left(\frac{1}{n}\right)^2+2\cdot2\cdot\frac{1}{n}\right\}+\ldots+\left\{(2)^2+\frac{(n-1)^2}{n^2}+2\cdot2\cdot\frac{(n-1)}{n}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{2^2+\ldots+2^2}_{n\text{ times}}\right)+\left\{\left(\frac{1}{n}\right)^2+\left(\frac{2}{n}\right)^2+\ldots+\left(\frac{n-1}{n}\right)^2\right\}+2\cdot2\cdot\left\{\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\ldots+\frac{(n-1)}{n}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{1}{n^2}\left\{1^2+2^2+3^2\ldots+(n-1)^2\right\}+\frac{4}{n}\left\{1+2+\ldots+(n-1)\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{1}{n^2}\left\{\frac{n(n-1)(2n-1)}{6}\right\}+\frac{4}{n}\left\{\frac{n(n-1)}{2}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{n\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6}+\frac{4n-4}{2}\right]$$

$$=\lim_{n\to\infty}\left[4+\frac{1}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+2-\frac{2}{n}\right]$$

$$=4+\frac{2}{6}+2$$

$$=\frac{19}{3}$$

Find $\int_1^4 \left(x^2 - x\right) dx$

$$\text{Let } I = \int_1^4 \left(x^2 - x\right) dx$$
$$= \int_1^4 x^2\, dx - \int_1^4 x\, dx$$
$$\text{Let } I = I_1 - I_2, \text{ where } I_1 = \int_1^4 x^2 dx \text{ and } I_2 = \int_1^4 x\, dx \qquad ...(1)$$

It is known that,

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a) + f(a+h) + f\left(a + (n-1)h\right)\right], \text{ where } h = \frac{b-a}{n}$$

$$\text{For } I_1 = \int_1^4 x^2 dx,$$
$$a = 1, b = 4, \text{ and } f(x) = x^2$$
$$\therefore h = \frac{4-1}{n} = \frac{3}{n}$$

$$I_1 = \int_1^4 x^2 dx = (4-1)\lim_{n\to\infty}\frac{1}{n}\left[f(1) + f(1+h) + \ldots + f\left(1 + (n-1)h\right)\right]$$
$$= 3\lim_{n\to\infty}\frac{1}{n}\left[1^2 + \left(1 + \frac{3}{n}\right)^2 + \left(1 + 2\cdot\frac{3}{n}\right)^2 + \ldots\left(1 + \frac{(n-1)3}{n}\right)^2\right]$$
$$= 3\lim_{n\to\infty}\frac{1}{n}\left[1^2 + \left\{1^2 + \left(\frac{3}{n}\right)^2 + 2\cdot\frac{3}{n}\right\} + \ldots + \left\{1^2 + \left(\frac{(n-1)3}{n}\right)^2 + \frac{2\cdot(n-1)\cdot 3}{n}\right\}\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{1^2+\ldots+1^2}_{n\text{ times}}\right)+\left(\frac{3}{n}\right)^2\left\{1^2+2^2+\ldots+(n-1)^2\right\}+2\cdot\frac{3}{n}\left\{1+2+\ldots+(n-1)\right\}\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{9}{n^2}\left\{\frac{(n-1)(n)(2n-1)}{6}\right\}+\frac{6}{n}\left\{\frac{(n-1)(n)}{2}\right\}\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{9n}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+\frac{6n-6}{2}\right]$$

$$= 3\lim_{n\to\infty}\left[1+\frac{9}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+3-\frac{3}{n}\right]$$

$$= 3[1+3+3]$$

$$= 3[7]$$

$$I_1 = 21 \qquad \ldots(2)$$

For $I_2 = \int_1^4 x\,dx$,

$a = 1, b = 4,$ and $f(x) = x$

$$\Rightarrow h = \frac{4-1}{n} = \frac{3}{n}$$

$$\therefore I_2 = (4-1)\lim_{n\to\infty}\frac{1}{n}\left[f(1)+f(1+h)+\ldots f(a+(n-1)h)\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[1+(1+h)+\ldots+(1+(n-1)h)\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[1+\left(1+\frac{3}{n}\right)+\ldots+\left\{1+(n-1)\frac{3}{n}\right\}\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{1+1+\ldots+1}_{n\text{ times}}\right)+\frac{3}{n}\left(1+2+\ldots+(n-1)\right)\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{3}{n}\left\{\frac{(n-1)n}{2}\right\}\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[1+\frac{3}{2}\left(1-\frac{1}{n}\right)\right]$$

$$= 3\left[1+\frac{3}{2}\right]$$

$$= 3\left[\frac{5}{2}\right]$$

$$I_2 = \frac{15}{2} \qquad \ldots(3)$$

From equations (2) and (3), we obtain

$$I = I_1 + I_2 = 21 - \frac{15}{2} = \frac{27}{2}$$

-

Find $\int_{-1}^{1} e^x dx$

$$\text{Let } I = \int_{-1}^{1} e^x dx \qquad ...(1)$$

It is known that,

$$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)...f(a+(n-1)h)\right], \text{ where } h = \frac{b-a}{n}$$

Here, $a=-1$, $b=1$, and $f(x)=e^x$

$$\therefore h = \frac{1+1}{n} = \frac{2}{n}$$

$$\therefore I = (1+1)\lim_{n\to\infty}\frac{1}{n}\left[f(-1)+f\left(-1+\frac{2}{n}\right)+f\left(-1+2\cdot\frac{2}{n}\right)+...+f\left(-1+\frac{(n-1)2}{n}\right)\right]$$

$$= 2\lim_{n\to\infty}\frac{1}{n}\left[e^{-1}+e^{\left(-1+\frac{2}{n}\right)}+e^{\left(-1+2\cdot\frac{2}{n}\right)}+...e^{\left(-1+(n-1)\frac{2}{n}\right)}\right]$$

$$= 2\lim_{n\to\infty}\frac{1}{n}\left[e^{-1}\left\{1+e^{\frac{2}{n}}+e^{\frac{4}{n}}+e^{\frac{6}{n}}+e^{(n-1)\frac{2}{n}}\right\}\right]$$

$$= 2\lim_{n\to\infty}\frac{e^{-1}}{n}\left[\frac{e^{\frac{2n}{n}}-1}{e^{\frac{2}{n}}-1}\right]$$

$$= e^{-1}\times 2\lim_{n\to\infty}\frac{1}{n}\left[\frac{e^2-1}{e^{\frac{2}{n}}-1}\right]$$

$$= \frac{e^{-1}\times 2(e^2-1)}{\lim_{\frac{2}{n}\to 0}\left(\frac{e^{\frac{2}{n}}-1}{\frac{2}{n}}\right)\times 2}$$

$$= e^{-1}\left[\frac{2(e^2-1)}{2}\right] \qquad \left[\lim_{h\to 0}\left(\frac{e^h-1}{h}\right)=1\right]$$

$$= \frac{e^2-1}{e}$$

$$= \left(e-\frac{1}{e}\right)$$

Find $\int_0^4 \left(x+e^{2x}\right)dx$

It is known that,

$$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+\ldots+f\left(a+(n-1)h\right)\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=0$, $b=4$, and $f(x)=x+e^{2x}$

$$\therefore h=\frac{4-0}{n}=\frac{4}{n}$$

$$\Rightarrow \int_0^4\left(x+e^{2x}\right)dx = (4-0)\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f(h)+f(2h)+\ldots+f\left((n-1)h\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\left(0+e^0\right)+\left(h+e^{2h}\right)+\left(2h+e^{2\cdot 2h}\right)+\ldots+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[1+\left(h+e^{2h}\right)+\left(2h+e^{4h}\right)+\ldots+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\left\{h+2h+3h+\ldots+(n-1)h\right\}+\left(1+e^{2h}+e^{4h}+\ldots+e^{2(n-1)h}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[h\left\{1+2+\ldots(n-1)\right\}+\left(\frac{e^{2hn}-1}{e^{2h}-1}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\frac{\left(h(n-1)n\right)}{2}+\left(\frac{e^{2hn}-1}{e^{2h}-1}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\frac{4}{n}.\frac{(n-1)n}{2}+\left(\frac{e^8-1}{e^{\frac{8}{n}}-1}\right)\right]$$

$$=4(2)+4\lim_{n\to\infty}\frac{\left(e^8-1\right)}{\left(\dfrac{e^{\frac{8}{n}}-1}{\frac{8}{n}}\right)8}$$

$$=8+\frac{4\cdot\left(e^8-1\right)}{8} \qquad \left(\lim_{x\to 0}\frac{e^x-1}{x}=1\right)$$

$$=8+\frac{e^8-1}{2}$$

$$=\frac{15+e^8}{2}$$

$$\int_{-1}^{1}(x+1)dx$$

Answer :

Let $I = \int_{-1}^{1}(x+1)dx$

$$\int(x+1)\,dx = \frac{x^2}{2} + x = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= \mathrm{F}(1) - \mathrm{F}(-1) \\ &= \left(\frac{1}{2}+1\right) - \left(\frac{1}{2}-1\right) \\ &= \frac{1}{2}+1-\frac{1}{2}+1 \\ &= 2 \end{aligned}$$

-

$$\int_{2}^{3}\frac{1}{x}dx$$

Answer :

Let $I = \int_{2}^{3}\frac{1}{x}\,dx$

$$\int\frac{1}{x}\,dx = \log|x| = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= \mathrm{F}(3) - \mathrm{F}(2) \\ &= \log|3| - \log|2| = \log\frac{3}{2} \end{aligned}$$

-

$$\int_1^2 \left(4x^3 - 5x^2 + 6x + 9\right) dx$$

Answer :

Let $I = \int_1^2 \left(4x^3 - 5x^2 + 6x + 9\right) dx$

$$\int \left(4x^3 - 5x^2 + 6x + 9\right) dx = 4\left(\frac{x^4}{4}\right) - 5\left(\frac{x^3}{3}\right) + 6\left(\frac{x^2}{2}\right) + 9(x)$$

$$= x^4 - \frac{5x^3}{3} + 3x^2 + 9x = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}(2) - \mathrm{F}(1)$$

$$I = \left\{2^4 - \frac{5\cdot(2)^3}{3} + 3(2)^2 + 9(2)\right\} - \left\{(1)^4 - \frac{5(1)^3}{3} + 3(1)^2 + 9(1)\right\}$$

$$= \left(16 - \frac{40}{3} + 12 + 18\right) - \left(1 - \frac{5}{3} + 3 + 9\right)$$

$$= 16 - \frac{40}{3} + 12 + 18 - 1 + \frac{5}{3} - 3 - 9$$

$$= 33 - \frac{35}{3}$$

$$= \frac{99 - 35}{3}$$

$$= \frac{64}{3}$$

-

$$\int_0^{\frac{\pi}{4}} \sin 2x dx$$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{4}} \sin 2x \, dx$$

$$\int \sin 2x \, dx = \left(\frac{-\cos 2x}{2}\right) = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}\left(\frac{\pi}{4}\right) - \mathrm{F}(0)$$

$$= -\frac{1}{2}\left[\cos 2\left(\frac{\pi}{4}\right) - \cos 0\right]$$

$$= -\frac{1}{2}\left[\cos\left(\frac{\pi}{2}\right) - \cos 0\right]$$

$$= -\frac{1}{2}[0-1]$$

$$= \frac{1}{2}$$

-

$$\int_0^{\frac{\pi}{2}} \cos 2x \, dx$$

Answer :

Let $I = \int_0^{\frac{\pi}{2}} \cos 2x \, dx$

$$\int \cos 2x \, dx = \left(\frac{\sin 2x}{2}\right) = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}\left(\frac{\pi}{2}\right) - \mathrm{F}(0)$$

$$= \frac{1}{2}\left[\sin 2\left(\frac{\pi}{2}\right) - \sin 0\right]$$

$$= \frac{1}{2}[\sin\pi - \sin 0]$$

$$= \frac{1}{2}[0-0] = 0$$

-

$$\int_4^5 e^x \, dx$$

Answer :

Let $I = \int_4^5 e^x \, dx$

$$\int e^x \, dx = e^x = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}(5) - \mathrm{F}(4)$$

$$= e^5 - e^4$$

$$= e^4(e-1)$$

-

$$\int_0^{\frac{\pi}{4}} \tan x \, dx$$

Answer :

Let $I = \int_0^{\frac{\pi}{4}} \tan x \, dx$

$\int \tan x \, dx = -\log|\cos x| = F(x)$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{4}\right) - F(0)$$

$$= -\log\left|\cos\frac{\pi}{4}\right| + \log|\cos 0|$$

$$= -\log\left|\frac{1}{\sqrt{2}}\right| + \log|1|$$

$$= -\log(2)^{-\frac{1}{2}}$$

$$= \frac{1}{2}\log 2$$

-

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$$

Answer :

Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$

$\int \operatorname{cosec} x \, dx = \log \left| \operatorname{cosec} x - \cot x \right| = F(x)$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{4}\right) - F\left(\frac{\pi}{6}\right)$$
$$= \log \left| \operatorname{cosec} \frac{\pi}{4} - \cot \frac{\pi}{4} \right| - \log \left| \operatorname{cosec} \frac{\pi}{6} - \cot \frac{\pi}{6} \right|$$
$$= \log \left| \sqrt{2} - 1 \right| - \log \left| 2 - \sqrt{3} \right|$$
$$= \log \left(\frac{\sqrt{2} - 1}{2 - \sqrt{3}} \right)$$

-

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

Answer :

Let $I = \int_0^1 \frac{dx}{\sqrt{1-x^2}}$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= \mathrm{F}(1) - \mathrm{F}(0) \\ &= \sin^{-1}(1) - \sin^{-1}(0) \\ &= \frac{\pi}{2} - 0 \\ &= \frac{\pi}{2} \end{aligned}$$

-

$$\int_0^1 \frac{dx}{1+x^2}$$

Answer :

Let $I = \int_0^1 \frac{dx}{1+x^2}$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= \mathrm{F}(1) - \mathrm{F}(0) \\ &= \tan^{-1}(1) - \tan^{-1}(0) \\ &= \frac{\pi}{4} \end{aligned}$$

-

$$\int_2^3 \frac{dx}{x^2-1}$$

Answer :

Let $I = \int_2^3 \frac{dx}{x^2-1}$

$$\int \frac{dx}{x^2-1} = \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(3) - F(2)$$

$$= \frac{1}{2}\left[\log\left|\frac{3-1}{3+1}\right| - \log\left|\frac{2-1}{2+1}\right|\right]$$

$$= \frac{1}{2}\left[\log\left|\frac{2}{4}\right| - \log\left|\frac{1}{3}\right|\right]$$

$$= \frac{1}{2}\left[\log\frac{1}{2} - \log\frac{1}{3}\right]$$

$$= \frac{1}{2}\left[\log\frac{3}{2}\right]$$

-

$$\int_0^{\frac{\pi}{2}} \cos^2 x\, dx$$

Answer :

Let $I = \int_0^{\frac{\pi}{2}} \cos^2 x\, dx$

$$\int \cos^2 x\, dx = \int \left(\frac{1+\cos 2x}{2}\right) dx = \frac{x}{2} + \frac{\sin 2x}{4} = \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \left[\mathrm{F}\left(\frac{\pi}{2}\right) - \mathrm{F}(0)\right]$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{2} - \frac{\sin \pi}{2}\right) - \left(0 + \frac{\sin 0}{2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2} + 0 - 0 - 0\right]$$

$$= \frac{\pi}{4}$$

-

$$\int_2^3 \frac{xdx}{x^2+1}$$

Answer :

Let $I = \int_2^3 \frac{x}{x^2+1}dx$

$$\int \frac{x}{x^2+1}dx = \frac{1}{2}\int \frac{2x}{x^2+1}dx = \frac{1}{2}\log\left(1+x^2\right) = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}(3) - \mathrm{F}(2)$$

$$= \frac{1}{2}\left[\log\left(1+(3)^2\right) - \log\left(1+(2)^2\right)\right]$$

$$= \frac{1}{2}\left[\log(10) - \log(5)\right]$$

$$= \frac{1}{2}\log\left(\frac{10}{5}\right) = \frac{1}{2}\log 2$$

-

$$\int_0^1 \frac{2x+3}{5x^2+1}dx$$

Answer :

$$\text{Let } I = \int_0^1 \frac{2x+3}{5x^2+1}dx$$

$$\int \frac{2x+3}{5x^2+1}dx = \frac{1}{5}\int \frac{5(2x+3)}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x+15}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x}{5x^2+1}dx + 3\int \frac{1}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x}{5x^2+1}dx + 3\int \frac{1}{5\left(x^2+\frac{1}{5}\right)}dx$$

$$= \frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{5}\cdot\frac{1}{\frac{1}{\sqrt{5}}}\tan^{-1}\frac{x}{\frac{1}{\sqrt{5}}}$$

$$= \frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}x\right)$$

= F(x)

By second fundamental theorem of calculus, we obtain

$$I = F(1) - F(0)$$

$$= \left\{\frac{1}{5}\log(5+1) + \frac{3}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}\right)\right\} - \left\{\frac{1}{5}\log(1) + \frac{3}{\sqrt{5}}\tan^{-1}(0)\right\}$$

$$= \frac{1}{5}\log 6 + \frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$$

-

$$\int_0^1 xe^{x^2}\,dx$$

Answer :

Let $I = \int_0^1 xe^{x^2}\,dx$

Put $x^2 = t \Rightarrow 2x\,dx = dt$

As $x \to 0, t \to 0$ and as $x \to 1, t \to 1$,

$\therefore I = \frac{1}{2}\int_0^1 e^t\,dt$

$\frac{1}{2}\int e^t\,dt = \frac{1}{2}e^t = \mathrm{F}(t)$

By second fundamental theorem of calculus, we obtain

$I = \mathrm{F}(1) - \mathrm{F}(0)$

$= \frac{1}{2}e - \frac{1}{2}e^0$

$= \frac{1}{2}(e-1)$

-

$$\int \frac{5x^2}{x^2+4x+3}$$

Answer :

Let $I = \int_1^2 \frac{5x^2}{x^2+4x+3}dx$

Dividing $5x^2$ by x^2+4x+3, we obtain

$$I = \int_1^2 \left\{5 - \frac{20x+15}{x^2+4x+3}\right\}dx$$

$$= \int_1^2 5dx - \int_1^2 \frac{20x+15}{x^2+4x+3}dx$$

$$= [5x]_1^2 - \int_1^2 \frac{20x+15}{x^2+4x+3}dx$$

$$I = 5 - I_1, \text{ where } I = \int_1^2 \frac{20x+15}{x^2+4x+3}dx \qquad ...(1)$$

Consider $I_1 = \int_1^2 \frac{20x+15}{x^2+4x+8}dx$

$$\text{Let } 20x+15 = \text{A}\frac{d}{dx}\left(x^2+4x+3\right)+\text{B}$$
$$= 2\text{A}x+\left(4\text{A}+\text{B}\right)$$

Equating the coefficients of x and constant term, we obtain

A = 10 and B = - 25

$$\Rightarrow I_1 = 10\int_1^2 \frac{2x+4}{x^2+4x+3}dx - 25\int_1^2 \frac{dx}{x^2+4x+3}$$

Let $x^2+4x+3=t$

$$\Rightarrow (2x+4)dx = dt$$

$$\Rightarrow I_1 = 10\int\frac{dt}{t} - 25\int\frac{dx}{(x+2)^2-1^2}$$
$$= 10\log t - 25\left[\frac{1}{2}\log\left(\frac{x+2-1}{x+2+1}\right)\right]$$
$$= \left[10\log\left(x^2+4x+3\right)\right]_1^2 - 25\left[\frac{1}{2}\log\left(\frac{x+1}{x+3}\right)\right]_1^2$$
$$= \left[10\log 15 - 10\log 8\right] - 25\left[\frac{1}{2}\log\frac{3}{5} - \frac{1}{2}\log\frac{2}{4}\right]$$

-

$$=\left[10\log(5\times3)-10\log(4\times2)\right]-\frac{25}{2}\left[\log3-\log5-\log2+\log4\right]$$

$$=\left[10\log5+10\log3-10\log4-10\log2\right]-\frac{25}{2}\left[\log3-\log5-\log2+\log4\right]$$
$$=\left[10+\frac{25}{2}\right]\log5+\left[-10-\frac{25}{2}\right]\log4+\left[10-\frac{25}{2}\right]\log3+\left[-10+\frac{25}{2}\right]\log2$$
$$=\frac{45}{2}\log5-\frac{45}{2}\log4-\frac{5}{2}\log3+\frac{5}{2}\log2$$
$$=\frac{45}{2}\log\frac{5}{4}-\frac{5}{2}\log\frac{3}{2}$$

Substituting the value of I_1 in (1), we obtain

$$I=5-\left[\frac{45}{2}\log\frac{5}{4}-\frac{5}{2}\log\frac{3}{2}\right]$$
$$=5-\frac{5}{2}\left[9\log\frac{5}{4}-\log\frac{3}{2}\right]$$

-

$$\int_0^{\frac{\pi}{4}}\left(2\sec^2 x + x^3 + 2\right)dx$$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{4}}\left(2\sec^2 x + x^3 + 2\right)dx$$

$$\int\left(2\sec^2 x + x^3 + 2\right)dx = 2\tan x + \frac{x^4}{4} + 2x = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}\left(\frac{\pi}{4}\right) - \mathrm{F}(0)$$

$$= \left\{\left(2\tan\frac{\pi}{4} + \frac{1}{4}\left(\frac{\pi}{4}\right)^4 + 2\left(\frac{\pi}{4}\right)\right) - \left(2\tan 0 + 0 + 0\right)\right\}$$

$$= 2\tan\frac{\pi}{4} + \frac{\pi^4}{4^5} + \frac{\pi}{2}$$

$$= 2 + \frac{\pi}{2} + \frac{\pi^4}{1024}$$

-

$$\int_0^\pi \left(\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2} \right) dx$$

Answer :

$$\text{Let } I = \int_0^\pi \left(\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2} \right) dx$$

$$= -\int_0^\pi \left(\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \right) dx$$

$$= -\int_0^\pi \cos x \, dx$$

$$\int \cos x \, dx = \sin x = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}(\pi) - \mathrm{F}(0)$$

$$= \sin\pi - \sin 0$$

$$= 0$$

-

$$\int_0^2 \frac{6x+3}{x^2+4}dx$$

Answer :

$$\text{Let } I = \int_0^2 \frac{6x+3}{x^2+4}dx$$

$$\int \frac{6x+3}{x^2+4}dx = 3\int \frac{2x+1}{x^2+4}dx$$

$$= 3\int \frac{2x}{x^2+4}dx + 3\int \frac{1}{x^2+4}dx$$

$$= 3\log\left(x^2+4\right) + \frac{3}{2}\tan^{-1}\frac{x}{2} = \text{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \text{F}(2) - \text{F}(0)$$

$$= \left\{3\log\left(2^2+4\right) + \frac{3}{2}\tan^{-1}\left(\frac{2}{2}\right)\right\} - \left\{3\log(0+4) + \frac{3}{2}\tan^{-1}\left(\frac{0}{2}\right)\right\}$$

$$= 3\log 8 + \frac{3}{2}\tan^{-1}1 - 3\log 4 - \frac{3}{2}\tan^{-1}0$$

$$= 3\log 8 + \frac{3}{2}\left(\frac{\pi}{4}\right) - 3\log 4 - 0$$

$$= 3\log\left(\frac{8}{4}\right) + \frac{3\pi}{8}$$

$$= 3\log 2 + \frac{3\pi}{8}$$

-

$$\int_0^1 \left(xe^x + \sin\frac{\pi x}{4} \right) dx$$

Answer :

Let $I = \int_0^1 \left(xe^x + \sin\frac{\pi x}{4} \right) dx$

$$\int \left(xe^x + \sin\frac{\pi x}{4} \right) dx = x \int e^x dx - \int \left\{ \left(\frac{d}{dx} x \right) \int e^x dx \right\} dx + \left\{ \frac{-\cos\frac{\pi x}{4}}{\frac{\pi}{4}} \right\}$$

$$= xe^x - \int e^x dx - \frac{4\pi}{\pi} \cos\frac{x}{4}$$

$$= xe^x - e^x - \frac{4\pi}{\pi} \cos\frac{x}{4}$$

$$= \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}(1) - \mathrm{F}(0)$$

$$= \left(1.e^1 - e^1 - \frac{4}{\pi} \cos\frac{\pi}{4} \right) - \left(0.e^0 - e^0 - \frac{4}{\pi} \cos 0 \right)$$

$$= e - e - \frac{4}{\pi} \left(\frac{1}{\sqrt{2}} \right) + 1 + \frac{4}{\pi}$$

$$= 1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}$$

-

$\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2}$ equals

A. $\frac{\pi}{3}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{12}$

Answer :

$$\int \frac{dx}{1+x^2} = \tan^{-1} x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2} = F(\sqrt{3}) - F(1)$$

$$= \tan^{-1}\sqrt{3} - \tan^{-1} 1$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{\pi}{12}$$

Hence, the correct answer is D.

-

$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$ equals

A. $\frac{\pi}{6}$

B. $\frac{\pi}{12}$

C. $\frac{\pi}{24}$

D. $\frac{\pi}{4}$

Answer :

$$\int \frac{dx}{4+9x^2} = \int \frac{dx}{(2)^2+(3x)^2}$$

Put $3x = t \Rightarrow 3dx = dt$

$$\therefore \int \frac{dx}{(2)^2+(3x)^2} = \frac{1}{3}\int \frac{dt}{(2)^2+t^2}$$

$$= \frac{1}{3}\left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]$$
$$= \frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right)$$
$$= F(x)$$

By second fundamental theorem of calculus, we obtain

$$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} = F\left(\frac{2}{3}\right) - F(0)$$
$$= \frac{1}{6}\tan^{-1}\left(\frac{3}{2}\cdot\frac{2}{3}\right) - \frac{1}{6}\tan^{-1} 0$$
$$= \frac{1}{6}\tan^{-1} 1 - 0$$
$$= \frac{1}{6} \times \frac{\pi}{4}$$
$$= \frac{\pi}{24}$$

Hence, the correct answer is C.

-

$$\int_0^1 \frac{x}{x^2+1}dx$$

Answer :

$$\int_0^1 \frac{x}{x^2+1}dx$$

Let $x^2+1=t \Rightarrow 2x\,dx = dt$

When $x=0$, $t=1$ and when $x=1$, $t=2$

$$\therefore \int_0^1 \frac{x}{x^2+1}dx = \frac{1}{2}\int_1^2 \frac{dt}{t}$$

$$= \frac{1}{2}\left[\log|t|\right]_1^2$$

$$= \frac{1}{2}\left[\log 2 - \log 1\right]$$

$$= \frac{1}{2}\log 2$$

-

$$\int_0^1 \frac{x}{x^2+1}\,dx$$

Answer :

$$\int_0^1 \frac{x}{x^2+1}\,dx$$

Let $x^2+1=t \Rightarrow 2x\,dx = dt$

When $x = 0$, $t = 1$ and when $x = 1$, $t = 2$

$$\therefore \int_0^1 \frac{x}{x^2+1}dx = \frac{1}{2}\int_1^2 \frac{dt}{t}$$

$$= \frac{1}{2}\left[\log|t|\right]_1^2$$

$$= \frac{1}{2}\left[\log 2 - \log 1\right]$$

$$= \frac{1}{2}\log 2$$

-

$$\int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\,\cos^5\phi\,d\phi$$

Answer :

Let $I = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^5\phi\,d\phi = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\,\cos^4\phi\cos\phi\,d\phi$

Also, let $\sin\phi = t \Rightarrow \cos\phi\,d\phi = dt$

When $\phi = 0$, $t = 0$ and when $\phi = \frac{\pi}{2}$, $t = 1$

$$\therefore I = \int_0^1 \sqrt{t}\left(1-t^2\right)^2 dt$$

$$= \int_0^1 t^{\frac{1}{2}}\left(1+t^4-2t^2\right)dt$$

$$= \int_0^1 \left[t^{\frac{1}{2}} + t^{\frac{9}{2}} - 2t^{\frac{5}{2}}\right]dt$$

$$= \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{11}{2}}}{\frac{11}{2}} - \frac{2t^{\frac{7}{2}}}{\frac{7}{2}} \right]_0^1$$

$$= \frac{2}{3} + \frac{2}{11} - \frac{4}{7}$$

$$= \frac{154 + 42 - 132}{231}$$

$$= \frac{64}{231}$$

-

- $\int_0^{\frac{\pi}{2}} \sqrt{\sin\phi} \cos^5\phi \, d\phi$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi} \cos^5\phi \, d\phi = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi} \cos^4\phi \cos\phi \, d\phi$$

Also, let $\sin\phi = t \Rightarrow \cos\phi \, d\phi = dt$

When $\phi = 0$, $t = 0$ and when $\phi = \frac{\pi}{2}$, $t = 1$

$$\therefore I = \int_0^1 \sqrt{t} \left(1 - t^2\right)^2 dt$$

$$= \int_0^1 t^{\frac{1}{2}} \left(1 + t^4 - 2t^2\right) dt$$

$$= \int_0^1 \left[t^{\frac{1}{2}} + t^{\frac{9}{2}} - 2t^{\frac{5}{2}} \right] dt$$

$$= \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{11}{2}}}{\frac{11}{2}} - \frac{2t^{\frac{7}{2}}}{\frac{7}{2}} \right]_0^1$$

$$= \frac{2}{3} + \frac{2}{11} - \frac{4}{7}$$
$$= \frac{154 + 42 - 132}{231}$$
$$= \frac{64}{231}$$

-

$$\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$$

Answer :

Let $I = \int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$

Also, let $x = \tan\theta \Rightarrow dx = \sec^2\theta \, d\theta$

When $x = 0$, $\theta = 0$ and when $x = 1$, $\theta = \frac{\pi}{4}$

$$I = \int_0^{\frac{\pi}{4}} \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) \sec^2\theta \, d\theta$$
$$= \int_0^{\frac{\pi}{4}} \sin^{-1}(\sin 2\theta) \sec^2\theta \, d\theta$$
$$= \int_0^{\frac{\pi}{4}} 2\theta \cdot \sec^2\theta \, d\theta$$
$$= 2\int_0^{\frac{\pi}{4}} \theta \cdot \sec^2\theta \, d\theta$$

Taking θ as first function and $\sec^2\theta$ as second function and integrating by parts, we obtain

$$I = 2\left[\theta \int \sec^2\theta \, d\theta - \int \left\{ \left(\frac{d}{dx}\theta \right) \int \sec^2\theta \, d\theta \right\} d\theta \right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\theta \tan\theta - \int \tan\theta \, d\theta\right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\theta \tan\theta + \log|\cos\theta|\right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\frac{\pi}{4}\tan\frac{\pi}{4} + \log\left|\cos\frac{\pi}{4}\right| - \log|\cos 0|\right]$$

$$= 2\left[\frac{\pi}{4} + \log\left(\frac{1}{\sqrt{2}}\right) - \log 1\right]$$

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\log 2\right]$$

$$= \frac{\pi}{2} - \log 2$$

-

$$\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$$

Answer :

Let $I = \int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

Also, let $x = \tan\theta \Rightarrow dx = \sec^2\theta \, d\theta$

When $x = 0$, $\theta = 0$ and when $x = 1$, $\theta = \frac{\pi}{4}$

$$I = \int_0^{\frac{\pi}{4}} \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)\sec^2\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \sin^{-1}(\sin 2\theta)\sec^2\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} 2\theta \cdot \sec^2\theta \, d\theta$$

$$= 2\int_0^{\frac{\pi}{4}} \theta \cdot \sec^2\theta \, d\theta$$

Taking θ as first function and $\sec^2\theta$ as second function and integrating by parts, we obtain

$$I = 2\left[\theta \int \sec^2\theta \, d\theta - \int\left\{\left(\frac{d}{dx}\theta\right)\int \sec^2\theta \, d\theta\right\}d\theta\right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\theta\tan\theta - \int \tan\theta \, d\theta\right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\theta\tan\theta + \log|\cos\theta|\right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\frac{\pi}{4}\tan\frac{\pi}{4} + \log\left|\cos\frac{\pi}{4}\right| - \log|\cos 0|\right]$$

$$= 2\left[\frac{\pi}{4} + \log\left(\frac{1}{\sqrt{2}}\right) - \log 1\right]$$

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\log 2\right]$$

$$= \frac{\pi}{2} - \log 2$$

-

$$\int_0^2 x\sqrt{x+2}\ \left(\text{Put } x+2=t^2\right)$$

Answer :

$$\int_0^2 x\sqrt{x+2}dx$$

Let $x + 2 = t^2 \Rightarrow dx = 2tdt$

When $x = 0$, $t=\sqrt{2}$ and when $x = 2$, $t = 2$

$$\therefore \int_0^2 x\sqrt{x+2}dx = \int_{\sqrt{2}}^2 \left(t^2-2\right)\sqrt{t^2}\,2tdt$$

$$= 2\int_{\sqrt{2}}^2 \left(t^2-2\right)t^2 dt$$

$$= 2\int_{\sqrt{2}}^2 \left(t^4-2t^2\right)dt$$

$$= 2\left[\frac{t^5}{5}-\frac{2t^3}{3}\right]_{\sqrt{2}}^2$$

$$= 2\left[\frac{32}{5}-\frac{16}{3}-\frac{4\sqrt{2}}{5}+\frac{4\sqrt{2}}{3}\right]$$

$$= 2\left[\frac{96-80-12\sqrt{2}+20\sqrt{2}}{15}\right]$$

$$= 2\left[\frac{16+8\sqrt{2}}{15}\right]$$

$$= \frac{16\left(2+\sqrt{2}\right)}{15}$$

$$= \frac{16\sqrt{2}\left(\sqrt{2}+1\right)}{15}$$

-

$$\int_0^2 x\sqrt{x+2} \quad \left(\text{Put } x+2=t^2\right)$$

Answer :

$$\int_0^2 x\sqrt{x+2}dx$$

Let $x + 2 = t^2 \Rightarrow dx = 2tdt$

When $x = 0$, $t=\sqrt{2}$ and when $x = 2$, $t = 2$

$$\therefore \int_0^2 x\sqrt{x+2}dx = \int_{\sqrt{2}}^2 \left(t^2-2\right)\sqrt{t^2}\,2tdt$$

$$= 2\int_{\sqrt{2}}^2 \left(t^2-2\right)t^2 dt$$

$$= 2\int_{\sqrt{2}}^2 \left(t^4-2t^2\right)dt$$

$$= 2\left[\frac{t^5}{5}-\frac{2t^3}{3}\right]_{\sqrt{2}}^2$$

$$= 2\left[\frac{32}{5}-\frac{16}{3}-\frac{4\sqrt{2}}{5}+\frac{4\sqrt{2}}{3}\right]$$

$$= 2\left[\frac{96-80-12\sqrt{2}+20\sqrt{2}}{15}\right]$$

$$= 2\left[\frac{16+8\sqrt{2}}{15}\right]$$

$$= \frac{16\left(2+\sqrt{2}\right)}{15}$$

$$= \frac{16\sqrt{2}\left(\sqrt{2}+1\right)}{15}$$

-

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx$$

Answer :

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx$$

Let cos $x = t \Rightarrow$ - sin$x$ $dx = dt$

When $x = 0$, $t = 1$ and when $x = \frac{\pi}{2}, t = 0$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx = -\int_1^0 \frac{dt}{1+t^2}$$

$$= -\left[\tan^{-1} t\right]_1^0$$

$$= -\left[\tan^{-1} 0 - \tan^{-1} 1\right]$$

$$= -\left[-\frac{\pi}{4}\right]$$

$$= \frac{\pi}{4}$$

-

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}dx$$

Answer :

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}dx$$

Let cos $x = t \Rightarrow$ - sin$x$ $dx = dt$

When $x = 0$, $t = 1$ and when $x = \frac{\pi}{2}, t = 0$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}dx = -\int_1^0 \frac{dt}{1+t^2}$$

$$= -\left[\tan^{-1} t\right]_1^0$$

$$= -\left[\tan^{-1} 0 - \tan^{-1} 1\right]$$

$$= -\left[-\frac{\pi}{4}\right]$$

$$= \frac{\pi}{4}$$

-

$$\int_0^2 \frac{dx}{x+4-x^2}$$

Answer :

$$\int_0^2 \frac{dx}{x+4-x^2} = \int_0^2 \frac{dx}{-\left(x^2-x-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left(x^2-x+\frac{1}{4}-\frac{1}{4}-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2-\frac{17}{4}\right]}$$

$$= \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}$$

Let $x-\frac{1}{2}=t \Rightarrow dx = dt$

When $x=0$, $t=-\frac{1}{2}$ and when $x=2$, $t=\frac{3}{2}$

$$\therefore \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}=\int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2-t^2}$$

$$=\left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)} \log \frac{\frac{\sqrt{17}}{2}+t}{\frac{\sqrt{17}}{2}-t}\right]_{-\frac{1}{2}}^{\frac{3}{2}}$$

$$=\frac{1}{\sqrt{17}}\left[\log \frac{\frac{\sqrt{17}}{2}+\frac{3}{2}}{\frac{\sqrt{17}}{2}-\frac{3}{2}}-\frac{\log \frac{\sqrt{17}}{2}-\frac{1}{2}}{\log \frac{\sqrt{17}}{2}+\frac{1}{2}}\right]$$

$$=\frac{1}{\sqrt{17}}\left[\log \frac{\sqrt{17}+3}{\sqrt{17}-3}-\log \frac{\sqrt{17}-1}{\sqrt{17}+1}\right]$$

$$= \frac{1}{\sqrt{17}} \log \frac{\sqrt{17}+3}{\sqrt{17}-3} \times \frac{\sqrt{17}+1}{\sqrt{17}-1}$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{20+4\sqrt{17}}{20-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{5+\sqrt{17}}{5-\sqrt{17}}\right)$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{\left(5+\sqrt{17}\right)\left(5+\sqrt{17}\right)}{25-17}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{25+17+10\sqrt{17}}{8}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{42+10\sqrt{17}}{8}\right)$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{21+5\sqrt{17}}{4}\right)$$

-

$$\int_0^2 \frac{dx}{x+4-x^2}$$

Answer :

$$\int_0^2 \frac{dx}{x+4-x^2} = \int_0^2 \frac{dx}{-\left(x^2-x-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left(x^2-x+\frac{1}{4}-\frac{1}{4}-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2-\frac{17}{4}\right]}$$

$$= \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}$$

Let $x-\frac{1}{2}=t \Rightarrow dx = dt$

When $x = 0,\ t = -\frac{1}{2}$ and when $x = 2,\ t = \frac{3}{2}$

$$\therefore \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2} = \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2 - t^2}$$

$$= \left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)} \log \frac{\frac{\sqrt{17}}{2} + t}{\frac{\sqrt{17}}{2} - t}\right]_{-\frac{1}{2}}^{\frac{3}{2}}$$

$$= \frac{1}{\sqrt{17}}\left[\log \frac{\frac{\sqrt{17}}{2} + \frac{3}{2}}{\frac{\sqrt{17}}{2} - \frac{3}{2}} - \frac{\log \frac{\sqrt{17}}{2} - \frac{1}{2}}{\log \frac{\sqrt{17}}{2} + \frac{1}{2}}\right]$$

$$= \frac{1}{\sqrt{17}}\left[\log \frac{\sqrt{17} + 3}{\sqrt{17} - 3} - \log \frac{\sqrt{17} - 1}{\sqrt{17} + 1}\right]$$

$$= \frac{1}{\sqrt{17}} \log \frac{\sqrt{17} + 3}{\sqrt{17} - 3} \times \frac{\sqrt{17} + 1}{\sqrt{17} - 1}$$

$$= \frac{1}{\sqrt{17}} \log\left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log\left[\frac{20+4\sqrt{17}}{20-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log\left(\frac{5+\sqrt{17}}{5-\sqrt{17}}\right)$$

$$= \frac{1}{\sqrt{17}} \log\left[\frac{\left(5+\sqrt{17}\right)\left(5+\sqrt{17}\right)}{25-17}\right]$$

$$= \frac{1}{\sqrt{17}} \log\left[\frac{25+17+10\sqrt{17}}{8}\right]$$

$$= \frac{1}{\sqrt{17}} \log\left(\frac{42+10\sqrt{17}}{8}\right)$$

$$= \frac{1}{\sqrt{17}} \log\left(\frac{21+5\sqrt{17}}{4}\right)$$

-

$$\int_{-1}^{1} \frac{dx}{x^2+2x+5}$$

Answer :

$$\int_{-1}^{1} \frac{dx}{x^2+2x+5} = \int_{-1}^{1} \frac{dx}{\left(x^2+2x+1\right)+4} = \int_{-1}^{1} \frac{dx}{(x+1)^2+(2)^2}$$

Let $x + 1 = t \Rightarrow dx = dt$

When $x = -1$, $t = 0$ and when $x = 1$, $t = 2$

$$\therefore \int_{-1}^{1} \frac{dx}{(x+1)^2+(2)^2} = \int_{0}^{2} \frac{dt}{t^2+2^2}$$

$$= \left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]_0^2$$

$$= \frac{1}{2}\tan^{-1}1 - \frac{1}{2}\tan^{-1}0$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{8}$$

-

$$\int_{-1}^{1} \frac{dx}{x^2+2x+5}$$

Answer :

$$\int_{-1}^{1} \frac{dx}{x^2+2x+5} = \int_{-1}^{1} \frac{dx}{\left(x^2+2x+1\right)+4} = \int_{-1}^{1} \frac{dx}{(x+1)^2+(2)^2}$$

Let $x + 1 = t \Rightarrow dx = dt$

When $x = -1$, $t = 0$ and when $x = 1$, $t = 2$

$$\therefore \int_{-1}^{1} \frac{dx}{(x+1)^2+(2)^2} = \int_{0}^{2} \frac{dt}{t^2+2^2}$$

$$= \left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]_0^2$$

$$= \frac{1}{2}\tan^{-1}1 - \frac{1}{2}\tan^{-1}0$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{8}$$

-

$$\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx$$

Answer :

$$\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx$$

Let $2x = t \Rightarrow 2dx = dt$

When $x = 1$, $t = 2$ and when $x = 2$, $t = 4$

$$\therefore \int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx = \frac{1}{2}\int_2^4 \left(\frac{2}{t} - \frac{2}{t^2}\right) e^t dt$$

$$= \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt$$

Let $\frac{1}{t} = f(t)$

Then, $f'(t) = -\frac{1}{t^2}$

$$\Rightarrow \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt = \int_2^4 e^t \left[f(t) + f'(t)\right] dt$$

$$= \left[e^t f(t)\right]_2^4$$

$$= \left[e^t \cdot \frac{2}{t}\right]_2^4$$

$$= \left[\frac{e^t}{t}\right]_2^4$$

$$= \frac{e^4}{4} - \frac{e^2}{2}$$

$$= \frac{e^2\left(e^2 - 2\right)}{4}$$

-

$$\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx$$

Answer :

$$\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx$$

Let $2x = t \Rightarrow 2dx = dt$

When $x = 1$, $t = 2$ and when $x = 2$, $t = 4$

$$\therefore \int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx = \frac{1}{2}\int_2^4 \left(\frac{2}{t} - \frac{2}{t^2}\right) e^t dt$$

$$= \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt$$

Let $\frac{1}{t} = f(t)$

Then, $f'(t) = -\frac{1}{t^2}$

$$\Rightarrow \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt = \int_2^4 e^t \left[f(t) + f'(t)\right] dt$$

$$= \left[e^t f(t)\right]_2^4$$

$$= \left[e^t \cdot \frac{2}{t}\right]_2^4$$

$$= \left[\frac{e^t}{t}\right]_2^4$$

$$= \frac{e^4}{4} - \frac{e^2}{2}$$

$$= \frac{e^2\left(e^2 - 2\right)}{4}$$

-

The value of the integral $\int_{\frac{1}{3}}^{1} \frac{\left(x-x^3\right)^{\frac{1}{3}}}{x^4} dx$ is

A. 6

B. 0

C. 3

D. 4

Answer :

Let $I = \int_{\frac{1}{3}}^{1} \frac{\left(x-x^3\right)^{\frac{1}{3}}}{x^4} dx$

Also, let $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

When $x = \frac{1}{3}$, $\theta = \sin^{-1}\left(\frac{1}{3}\right)$ and when $x = 1$, $\theta = \frac{\pi}{2}$

$$\Rightarrow I = \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{\left(\sin\theta - \sin^3\theta\right)^{\frac{1}{3}}}{\sin^4\theta} \cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin\theta)^{\frac{1}{3}}\left(1-\sin^2\theta\right)^{\frac{1}{3}}}{\sin^4\theta} \cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin\theta)^{\frac{1}{3}}(\cos\theta)^{\frac{2}{3}}}{\sin^4\theta} \cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin\theta)^{\frac{1}{3}}(\cos\theta)^{\frac{2}{3}}}{\sin^2\theta \sin^2\theta} \cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\cos\theta)^{\frac{5}{3}}}{(\sin\theta)^{\frac{5}{3}}} \operatorname{cosec}^2\theta \, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} (\cot\theta)^{\frac{5}{3}} \operatorname{cosec}^2\theta \, d\theta$$

Let $\cot\theta = t \Rightarrow$ - cosec2θ $d\theta = dt$

When $\theta = \sin^{-1}\left(\frac{1}{3}\right)$, $t = 2\sqrt{2}$ and when $\theta = \frac{\pi}{2}$, $t = 0$

$$\therefore I = -\int_{2\sqrt{2}}^{0} (t)^{\frac{5}{3}} dt$$

$$= -\left[\frac{3}{8}(t)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}$$

$$= -\frac{3}{8}\left[(t)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}$$

$$= -\frac{3}{8}\left[-\left(2\sqrt{2}\right)^{\frac{8}{3}}\right]$$

$$= \frac{3}{8}\left[\left(\sqrt{8}\right)^{\frac{8}{3}}\right]$$

$$= \frac{3}{8}\left[(8)^{\frac{4}{3}}\right]$$

$$= \frac{3}{8}[16]$$

$$= 3 \times 2$$

$$= 6$$

Hence, the correct answer is A.

-

If $f(x)=\int_0^x t\sin t\,dt$, then $f'(x)$ is

A. $\cos x + x\sin x$

B. $x\sin x$

C. $x\cos x$

D. $\sin x + x\cos x$

$f(x)=\int_0^x t\sin t\,dt$

Integrating by parts, we obtain

$$f(x)=t\int_0^x \sin t\,dt - \int_0^x\left\{\left(\frac{d}{dt}t\right)\int \sin t\,dt\right\}dt$$
$$=\left[t(-\cos t)\right]_0^x - \int_0^x(-\cos t)\,dt$$
$$=\left[-t\cos t+\sin t\right]_0^x$$
$$=-x\cos x+\sin x$$

$$\Rightarrow f'(x)=-\left[\{x(-\sin x)\}+\cos x\right]+\cos x$$
$$=x\sin x-\cos x+\cos x$$
$$=x\sin x$$

Hence, the correct answer is B.

-

If $f(x)=\int_0^x t\sin t\,dt$, then $f'(x)$ is

A. $\cos x + x\sin x$

B. $x\sin x$

C. $x\cos x$

D. $\sin x + x\cos x$

$$f(x) = \int_0^x t \sin t dt$$

Integrating by parts, we obtain

$$f(x) = t \int_0^x \sin t \, dt - \int_0^x \left\{ \left(\frac{d}{dt} t \right) \int \sin t \, dt \right\} dt$$
$$= \left[t(-\cos t) \right]_0^x - \int_0^x (-\cos t) dt$$
$$= \left[-t \cos t + \sin t \right]_0^x$$
$$= -x \cos x + \sin x$$

$$\Rightarrow f'(x) = -\left[\left\{ x(-\sin x) \right\} + \cos x \right] + \cos x$$
$$= x \sin x - \cos x + \cos x$$
$$= x \sin x$$

Hence, the correct answer is B.

-

$$\int_0^{\frac{\pi}{2}} \cos^2 x\, dx$$

Answer :

$$I = \int_0^{\frac{\pi}{2}} \cos^2 x\, dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \cos^2\left(\frac{\pi}{2} - x\right) dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \left(\sin^2 x + \cos^2 x\right) dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

-

$$\int_0^{\frac{\pi}{2}} \cos^2 x\,dx$$

Answer :

$$I = \int_0^{\frac{\pi}{2}} \cos^2 x\,dx \qquad ...(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \cos^2\left(\frac{\pi}{2} - x\right)dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \left(\sin^2 x + \cos^2 x\right)dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1\,dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

-

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

Answer :

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos}}{\sqrt{\cos}+\sqrt{\sin x}}dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1\cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

-

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

Answer :

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \qquad ...(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos}}{\sqrt{\cos}+\sqrt{\sin x}}dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1\, dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

-

$$\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}$$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \quad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right) + \cos^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)} dx \quad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \quad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

-

$$\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}$$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \quad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right) + \cos^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)} dx \quad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \quad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \, dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

-

$$\int_0^{\frac{\pi}{2}} \frac{\cos^5 x dx}{\sin^5 x + \cos^5 x}$$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^5\left(\frac{\pi}{2} - x\right)}{\sin^5\left(\frac{\pi}{2} - x\right) + \cos^5\left(\frac{\pi}{2} - x\right)} dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\sin^5 x + \cos^5 x} dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x + \cos^5 x}{\sin^5 x + \cos^5 x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

-

$$\int_{-5}^{5} |x+2|\, dx$$

Answer :

Let $I = \int_{-5}^{5} |x+2| dx$

It can be seen that $(x + 2) \leq 0$ on $[-5, -2]$ and $(x + 2) \geq 0$ on $[-2, 5]$.

$$\therefore I = \int_{-5}^{-2} -(x+2) dx + \int_{-2}^{5} (x+2) dx \qquad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$

$$I = -\left[\frac{x^2}{2} + 2x\right]_{-5}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{5}$$

$$= -\left[\frac{(-2)^2}{2} + 2(-2) - \frac{(-5)^2}{2} - 2(-5)\right] + \left[\frac{(5)^2}{2} + 2(5) - \frac{(-2)^2}{2} - 2(-2)\right]$$

$$= -\left[2 - 4 - \frac{25}{2} + 10\right] + \left[\frac{25}{2} + 10 - 2 + 4\right]$$

$$= -2 + 4 + \frac{25}{2} - 10 + \frac{25}{2} + 10 - 2 + 4$$

$$= 29$$

-

$$\int_2^8 |x-5|\,dx$$

Answer :

Let $I = \int_2^8 |x-5|\,dx$

It can be seen that $(x - 5) \leq 0$ on [2, 5] and $(x - 5) \geq 0$ on [5, 8].

$$I = \int_2^5 -(x-5)\,dx + \int_2^8 (x-5)\,dx \qquad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$

$$= -\left[\frac{x^2}{2} - 5x\right]_2^5 + \left[\frac{x^2}{2} - 5x\right]_5^8$$

$$= -\left[\frac{25}{2} - 25 - 2 + 10\right] + \left[32 - 40 - \frac{25}{2} + 25\right]$$

$$= 9$$

-

$$\int_0^1 x(1-x)^n \, dx$$

Answer :

Let $I = \int_0^1 x(1-x)^n \, dx$

$\therefore I = \int_0^1 (1-x)(1-(1-x))^n \, dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$= \int_0^1 (1-x)(x)^n \, dx$

$= \int_0^1 \left(x^n - x^{n+1}\right) dx$

$= \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1$

$= \left[\frac{1}{n+1} - \frac{1}{n+2}\right]$

$= \frac{(n+2)-(n+1)}{(n+1)(n+2)}$

$= \frac{1}{(n+1)(n+2)}$

-

$$\int_0^{\frac{\pi}{4}} \log\left(1+\tan x\right)dx$$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{4}} \log\left(1+\tan x\right)dx \qquad \ldots(1)$$

$$\therefore I = \int_0^{\frac{\pi}{4}} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\left\{1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right\}dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\left\{1+\frac{1-\tan x}{1+\tan}\right\}dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\frac{2}{(1+\tan x)}dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log 2\,dx - \int_0^{\frac{\pi}{4}} \log\left(1+\tan x\right)dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log 2dx - I \qquad \left[\text{From } (1)\right]$$

$$\Rightarrow 2I = \left[x\log 2\right]_0^{\frac{\pi}{4}}$$

$$\Rightarrow 2I = \frac{\pi}{4}\log 2$$

$$\Rightarrow I = \frac{\pi}{8}\log 2$$

-

$$\int_0^2 x\sqrt{2-x}\,dx$$

Answer :

Let $I = \int_0^2 x\sqrt{2-x}\,dx$

$I = \int_0^2 (2-x)\sqrt{x}\,dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$

$= \int_0^2 \left\{2x^{\frac{1}{2}} - x^{\frac{3}{2}}\right\} dx$

$= \left[2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) - \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^2$

$= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^2$

$= \frac{4}{3}(2)^{\frac{3}{2}} - \frac{2}{5}(2)^{\frac{5}{2}}$

$= \frac{4\times 2\sqrt{2}}{3} - \frac{2}{5}\times 4\sqrt{2}$

$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}$

$= \frac{40\sqrt{2} - 24\sqrt{2}}{15}$

$= \frac{16\sqrt{2}}{15}$

-

$$\int_0^{\frac{\pi}{2}}(2\log\sin x-\log\sin 2x)\,dx$$

Answer :

Let $I=\int_0^{\frac{\pi}{2}}(2\log\sin x-\log\sin 2x)\,dx$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\left\{2\log\sin x-\log(2\sin x\cos x)\right\}dx$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\left\{2\log\sin x-\log\sin x-\log\cos x-\log 2\right\}dx$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\left\{\log\sin x-\log\cos x-\log 2\right\}dx \qquad \ldots(1)$

It is known that, $\left(\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx\right)$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\left\{\log\cos x-\log\sin x-\log 2\right\}dx \qquad \ldots(2)$

Adding (1) and (2), we obtain

$2I=\int_0^{\frac{\pi}{2}}(-\log 2-\log 2)\,dx$

$\Rightarrow 2I=-2\log 2\int_0^{\frac{\pi}{2}}1\,dx$

$\Rightarrow I=-\log 2\left[\frac{\pi}{2}\right]$

$\Rightarrow I=\frac{\pi}{2}(-\log 2)$

$\Rightarrow I=\frac{\pi}{2}\left[\log\frac{1}{2}\right]$

$\Rightarrow I=\frac{\pi}{2}\log\frac{1}{2}$

-

$$\int_0^{\pi} \frac{x\,dx}{1+\sin x}$$

Answer :

$$\text{Let } I = \int_0^{\pi} \frac{x\,dx}{1+\sin x} \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi-x)}{1+\sin(\pi-x)}dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi-x)}{1+\sin x}dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\pi} \frac{\pi}{1+\sin x}dx$$

$$\Rightarrow 2I = \pi\int_0^{\pi} \frac{(1-\sin x)}{(1+\sin x)(1-\sin x)}dx$$

$$\Rightarrow 2I = \pi\int_0^{\pi} \frac{1-\sin x}{\cos^2 x}dx$$

$$\Rightarrow 2I = \pi\int_0^{\pi} \left\{\sec^2 x - \tan x \sec x\right\}dx$$

$$\Rightarrow 2I = \pi\left[\tan x - \sec x\right]_0^{\pi}$$

$$\Rightarrow 2I = \pi[2]$$

$$\Rightarrow I = \pi$$

-

Series

F(x) in general can be expanded around a value. Often the Value around which it is expanded is chosen as x = 0

$$\begin{aligned} f(x) &= f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 \\ &\quad + \frac{f'''(x_0)}{3!}(x - x_0)^3 + \frac{f''''(x_0)}{4!}(x - x_0)^4 + \cdots \\ &= \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n. \end{aligned}$$

Taylor series: $f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$

Taylor polynomial: $f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$

Remainder: $R_n(x) = \frac{f^{(n+1)}(a)}{(n+1)!}(x-a)^{n+1} + \frac{f^{(n+2)}(a)}{(n+2)!}(x-a)^{n+2} + \frac{f^{(n+3)}(a)}{(n+3)!}(x-a)^{n+3} + \cdots$

Derivative form of remainder: $R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1}$ where z is a number between a and x.

Integral form of remainder: $R_n(x) = \frac{1}{n!}\int_a^x f^{(n+1)}(t)(x-t)^n \, dt$

Function	Maclaurin series
e^x	$\sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$
$\sin x$	$\sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$
$\cos x$	$\sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$
$\frac{1}{1-x}$	$\sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + \cdots$ (if $-1 < x < 1$)
$\ln(1+x)$	$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^k}{k} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ (if $-1 < x \le 1$)

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots$	$R = 1$
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$	$R = \infty$
$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$	$R = \infty$
$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$	$R = \infty$
$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$	$R = 1$
$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$	$R = 1$
$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!} x^2 + \frac{k(k-1)(k-2)}{3!} x^3 + \dots$	$R = 1$

We start by supposing that f is any function that can be represented by a power series:

$$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + c_4(x-a)^4 + \dots \qquad |x-a| < R \tag{1}$$

Let's try to determine what the coefficients c_n must be in terms of f. To begin, notice that if we put $x = a$ in Equation 1, then all terms after the first one are 0 and we get

$$f(a) = c_0$$

By Theorem 8.6.2, we can differentiate the series in Equation 1 term by term:

$$f'(x) = c_1 + 2c_2(x-a) + 3c_3(x-a)^2 + 4c_4(x-a)^3 + \dots \qquad |x-a| < R \tag{2}$$

and substitution of $x = a$ in Equation 2 gives $f'(a) = c_1$ Now we differentiate both sides of Equation 2 and obtain

$$f''(x) = 2c_2 + 2 \cdot 3c_3(x-a) + 3 \cdot 4c_4(x-a)^2 + \dots \qquad |x-a| < R \tag{3}$$

Again we put $x = a$ in Equation 3. The result is

$$f''(a) = 2c_2$$

Let's apply the procedure one more time. Differentiation of the series in Equation 3 gives

$$f'''(x) = 2 \cdot 3c_3 + 2 \cdot 3 \cdot 4c_4(x-a) + 3 \cdot 4 \cdot 5c_5(x-a)^2 + \ldots \qquad |x-a| < R \quad (4)$$

and substitution of $x = a$ in Equation 4 gives

$$f'''(a) = 2 \cdot 3c_3 = 3!c_3$$

By now you can see the pattern. If we continue to differentiate and substitute $x = a$, we obtain

$$f^{(n)}(a) = 2 \cdot 3 \cdot 4 \cdot \ldots \cdot nc_n = n!c_n$$

Solving this equation for the nth coefficient c_n, we get

$$c_n = \frac{f^{(n)}(a)}{n!}$$

This formula remains valid even for $n = 0$ if we adopt the conventions that $0! = 1$ and $f^{(0)} = f$.

Thus we have proved the following theorem.

THEOREM: If f has a power series representation (expansion) at a, that is, if

$$\boxed{f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n \quad \text{and} \quad |x-a| < R, \quad \text{then} \quad c_n = \frac{f^{(n)}(a)}{n!}} \quad (5)$$

Substituting this formula for c_n back into the series, we see that *if* f has a power series expansion at a, then it must be of the following form.

$$\boxed{\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \\ &= f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots \end{aligned}} \quad (6)$$

The series in Equation 6 is called the **Taylor series of the function** f at a (or about a **or centered at** a). For the special case $a = 0$ the Taylor series becomes

$$\boxed{f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots} \quad (7)$$

This case arises frequently enough that is is given the special name **Maclaurin series.**

NOTE: We have shown that *if* f can be represented as a power series about a, then f is equal to the sum of its Taylor series. But there exist functions that are not equal to the sum of their Taylor series. For example, one can show that the function defined by

$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is not equal to its Maclaurin series.

Let's investigate the more general question: Under what circumstances is a function equal to the sum of its Taylor series? In other words, if f has derivatives of all orders, when is it true that

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

As with any convergent series, this means that $f(x)$ is the limit of the sequence of partial sums. In the case of the Taylor series, the partial sums are

$$T_n(x) = \sum_{i=0}^{n} \frac{f^{(i)}(a)}{i!}(x-a)^i = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Notice that T_n is a polynomial of degree n called the **nth-degree Taylor polynomial of f at** a. For instance, for the exponential function $f(x) = e^x$, the result of Example 1 shows that the Taylor polynomials at 0 (or Maclaurin polynomials) with $n = 1, 2$, and 3 are

$$T_1(x) = 1 + x, \qquad T_2(x) = 1 + x + \frac{x^2}{2!}, \qquad T_3(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

In general, $f(x)$ is the sum of its Taylor series if

$$f(x) = \lim_{n\to\infty} T_n(x)$$

If we let

$$R_n(x) = f(x) - T_n(x) \quad \text{so that} \quad f(x) = T_n(x) + R_n(x)$$

then $R_n(x)$ is called the **remainder** of the Taylor series. If we can somehow show that $\lim_{n\to\infty} R_n(x) = 0$, then it follows that

$$\lim_{n\to\infty} T_n(x) = \lim_{n\to\infty} [f(x) - R_n(x)] = f(x) - \lim_{n\to\infty} R_n(x) = f(x)$$

We have therefore proved the following.

THEOREM: If $f(x) = T_n(x) + R_n(x)$, where T_n is the nth-degree Taylor polynomial of f at a and

$$\lim_{n\to\infty} R_n(x) = 0 \quad (8)$$

for $|x-a| < R$, then f is equal to the sum of its Taylor series on the interval $|x-a| < R$.

In trying to show that $\lim_{n\to\infty} R_n(x) = 0$ for a specific function f, we usually use the expression in the next theorem.

THEOREM (TAYLOR'S FORMULA): If f has $n+1$ derivatives in an interval I that contains the number a, then for x in I there is a number z strictly between x and a such that the remainder term in the Taylor series can be expressed as

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1}$$

NOTE 1: For the special case $n = 0$, if we put $x = b$ and $z = c$ in Taylor's Formula, we get

$$f(b) = f(a) + f'(c)(b-a) \quad (9)$$

which is the Mean Value Theorem. In fact, Theorem 9 can be proved by a method similar to the proof of the Mean Value Theorem.

NOTE 2: Notice that the remainder term

$$\boxed{R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1}} \quad (10)$$

is very similar to the terms in the Taylor series except that $f^{(n+1)}$ is evaluated at z instead of at a. All we say about the number z is that it lies somewhere between x and a. The expression for $R_n(x)$ in Equation 10 is known as **Lagrange's form of the remainder term**.

NOTE 3: In Section 8.8 we will explore the use of Taylor's Formula in approximating functions. Our immediate use of it is in conjunction with Theorem 8. In applying Theorems 8 and 9 it is often helpful to make use of the following fact:

$$\boxed{\lim_{n\to\infty} \frac{x^n}{n!} = 0 \quad \text{for every real number } x} \quad (11)$$

This is true because we know from Example 1 that the series $\sum \frac{x^n}{n!}$ converges for all x and so its n th term approaches 0.

EXAMPLE 2: Prove that e^x is equal to the sum of its Taylor series with $a = 0$ (Maclaurin series).

From Example 2 it follows that

$$\boxed{e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad \text{for all } x} \quad (12)$$

In particular, if we put $x = 1$ in Equation 12, we obtain

$$\boxed{e = \sum_{n=0}^{\infty} \frac{1}{n!} = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots} \quad (13)$$

EXAMPLE 3: Find the Taylor series for $f(x) = e^x$ at $a = 2$.

Solution: We have $f^{(n)}(2) = e^2$ and so, putting $a = 2$ in the definition of a Taylor series (6), we get

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(2)}{n!}(x-2)^n = \sum_{n=0}^{\infty} \frac{e^2}{n!}(x-2)^n$$

Again it can be verified, as in Example 1, that the radius of convergence is $R = \infty$. As in Example 2 we can verify that $\lim_{n\to\infty} R_n(x) = 0$, so

$$e^x = \sum_{n=0}^{\infty} \frac{e^2}{n!}(x-2)^n \quad \text{for all } x \quad (14)$$

We have two power series expansions for e^x, the Maclaurin series in Equation 12 and the Taylor series in Equation 14. The first is better if we are interested in values of x near 0 and the second is better if x is near 2.

EXAMPLE 4: Find the Maclaurin series for $\sin x$ and prove that it represents $\sin x$ for all x.

Solution: We arrange our computation in two columns as follows:

$$\begin{aligned} f(x) &= \sin x & f(0) &= 0 \\ f'(x) &= \cos x & f'(0) &= 1 \\ f''(x) &= -\sin x & f''(0) &= 0 \\ f'''(x) &= -\cos x & f'''(0) &= -1 \\ f^{(4)}(x) &= \sin x & f^{(4)}(0) &= 0 \end{aligned}$$

Since the derivatives repeat in a cycle of four, we can write the Maclaurin series as follows:

$$\begin{aligned} f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \ldots &= 0 + \frac{1}{1!}x + \frac{0}{2!}x^2 + \frac{-1}{3!}x^3 + \frac{0}{4!}x^4 + \ldots \\ &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \\ &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \end{aligned}$$

Using the remainder term (10) with $a = 0$, we have

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}x^{n+1}$$

where $f(x) = \sin x$ and z lies between 0 and x. But $f^{(n+1)}(z)$ is $\pm \sin z$ or $\pm \cos z$. In any case, $|f^{(n+1)}(z)| \leq 1$ and so

$$0 \leq |R_n(x)| = \frac{|f^{(n+1)}(z)|}{(n+1)!}|x^{n+1}| \leq \frac{1}{(n+1)!}|x^{n+1}| = \frac{|x|^{n+1}}{(n+1)!} \quad (15)$$

By Equation 11 the right side of this inequality approaches 0 as $n \to \infty$, so $R_n(x) \to 0$ by the Squeeze Theorem. It follows that $R_n(x) \to 0$ as $n \to \infty$, so $\sin x$ is equal to the sum of its Maclaurin series by Theorem 8. Thus

$$\boxed{\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \quad \text{for all } x} \quad (16)$$

EXAMPLE 5: Find the Maclaurin series for $\cos x$.

Solution 1: We arrange our computation in two columns as follows:

$$
\begin{aligned}
f(x) &= \cos x & f(0) &= 1 \\
f'(x) &= -\sin x & f'(0) &= 0 \\
f''(x) &= -\cos x & f''(0) &= -1 \\
f'''(x) &= \sin x & f'''(0) &= 0 \\
f^{(4)}(x) &= \cos x & f^{(4)}(0) &= 1
\end{aligned}
$$

Since the derivatives repeat in a cycle of four, we can write the Maclaurin series as follows:

$$
\begin{aligned}
f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \dots &= 1 + \frac{0}{1!}x + \frac{-1}{2!}x^2 + \frac{0}{3!}x^3 + \frac{1}{4!}x^4 + \dots \\
&= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \\
&= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}
\end{aligned}
$$

Solution 2: We differentiate the Maclaurin series for $\sin x$ given by Equation 16:

$$
\begin{aligned}
\cos x = \frac{d}{dx}(\sin x) &= \frac{d}{dx}\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots\right) \\
&= 1 - \frac{3x^2}{3!} + \frac{5x^4}{5!} - \frac{7x^6}{7!} + \dots \\
&= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots
\end{aligned}
$$

Since the Maclaurin series for $\sin x$ converges for all x, Theorem tells us that the differentiated series for $\cos x$ also converges for all x. Thus

$$
\boxed{\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \quad \text{for all } x} \qquad (17)
$$

The power series that we obtained by indirect methods in Examples 5 and 6 and in Section 8.6 are indeed the Taylor or Maclaurin series of the given functions because Theorem 5 asserts that, no matter how we obtain a power series representation

$$f(x) = \sum c_n (x-a)^n$$

it is always true that $c_n = f^{(n)}(a)/n!$. In other words, the coefficients are uniquely determined.

EXAMPLE 7: Find the Maclaurin series for $f(x) = (1+x)^k$, where k is any real number.

Solution: Arranging our work in columns, we have

$$\begin{aligned}
f(x) &= (1+x)^k & f(0) &= 1 \\
f'(x) &= k(1+x)^{k-1} & f'(0) &= k \\
f''(x) &= k(k-1)(1+x)^{k-2} & f''(0) &= k(k-1) \\
f'''(x) &= k(k-1)(k-2)(1+x)^{k-3} & f'''(0) &= k(k-1)(k-2) \\
&\vdots & &\vdots \\
f^{(n)}(x) &= k(k-1)\ldots(k-n+1)(1+x)^{k-n} & f^{(n)}(0) &= k(k-1)\ldots(k-n+1)
\end{aligned}$$

Therefore, the Maclaurin series of $f(x) = (1+x)^k$ is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = 1 + \sum_{n=1}^{\infty} \frac{k(k-1)\ldots(k-n+1)}{n!} x^n$$

This series is called the **binomial series**. If its nth term is a_n, then

$$\begin{aligned}
\left|\frac{a_{n+1}}{a_n}\right| &= \left|\frac{k(k-1)\ldots(k-n+1)(k-n)x^{n+1}}{(n+1)!} \cdot \frac{n!}{k(k-1)\ldots(k-n+1)x^n}\right| \\
&= \frac{|k-n|}{n+1}|x| = \frac{\left|\frac{k}{n}-1\right|}{1+\frac{1}{n}}|x| \to |x| \quad \text{as } n \to \infty
\end{aligned}$$

Thus by the Ratio Test the binomial series converges if $|x| < 1$ and diverges if $|x| > 1$.

The traditional notation for the coefficients in the binomial series is

$$\binom{k}{n} = \frac{k!}{n!(k-n)!} = \frac{(k-n)!(k-n+1)\ldots(k-2)(k-1)k}{n!(k-n)!} = \frac{k(k-1)(k-2)\ldots(k-n+1)}{n!}$$

and these numbers are called the **binomial coefficients**. The following theorem states that $(1+x)^k$ is equal to the sum of its Maclaurin series. It is possible to prove this by showing that the remainder term $R_n(x)$ approaches 0, but that turns out to be quite difficult.

THEOREM (THE BINOMIAL SERIES): If k is any real number and $|x| < 1$, then

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \dots \quad (18)$$

Although the binomial series always converges when $|x| < 1$, the question of whether or not it converges at the endpoints, ± 1, depends on the value of k. It turns out that the series converges at 1 if $-1 < k < 0$ and at both endpoints if $k \geq 0$. Notice that if k is a positive integer and $n > k$, then the expression for $\binom{k}{n}$ contains a factor $(k-k)$, so

$$\binom{k}{n} = 0$$

for $n > k$. This means that the series terminates and reduces to the ordinary Binomial Theorem when k is a positive integer.

Say we have to expand x Cos x then write general expansion of Cos x and multiply with x

$$x\cos x = x\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!} = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n)!}$$

Find the Maclaurin series for $f(x) = \dfrac{1}{\sqrt{4-x}}$ and its radius of convergence.

We write $f(x)$ in a form where we can use the binomial series:

$$\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{4\left(1-\frac{x}{4}\right)}} = \frac{1}{2\sqrt{1-\frac{x}{4}}} = \frac{1}{2}\left(1-\frac{x}{4}\right)^{-1/2}$$

Using the binomial series with $k = -\dfrac{1}{2}$ and with x replaced by $-\dfrac{x}{4}$, we have

$$\frac{1}{\sqrt{4-x}} = \frac{1}{2}\left(1-\frac{x}{4}\right)^{-1/2} = \frac{1}{2}\sum_{n=0}^{\infty}\binom{-\frac{1}{2}}{n}\left(-\frac{x}{4}\right)^n$$

$$= \frac{1}{2}\left[1+\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}\left(-\frac{x}{4}\right)^2+\frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}\left(-\frac{x}{4}\right)^3\right.$$

$$\left.+\ldots+\frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})\ldots(-\frac{1}{2}-n+1)}{n!}\left(-\frac{x}{4}\right)^n+\ldots\right]$$

$$= \frac{1}{2}\left[1+\frac{1}{8}x+\frac{1\cdot 3}{2!8^2}x^2+\frac{1\cdot 3\cdot 5}{3!8^3}x^3+\ldots+\frac{1\cdot 3\cdot 5\cdot\ldots\cdot(2n-1)}{n!8^n}x^n+\ldots\right]$$

We know from (18) that this series converges when $|-x/4| < 1$, that is, $|x| < 4$, so the radius of convergence is $R = 4$.

(a) Evaluate $\int e^{-x^2}dx$ as an infinite series.

(b) Evaluate $\int_0^1 e^{-x^2}dx$ correct to within an error of 0.001.

(a) First we find the Maclaurin series for $f(x) = e^{-x^2}$. Although it's possible to use the direct method, let's find it simply by replacing x with $-x^2$ in the series for e^x given in the table of Maclaurin series. Thus, for all values of x,

$$e^{-x^2} = \sum_{n=0}^{\infty}\frac{(-x^2)^n}{n!} = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{n!} = 1-\frac{x^2}{1!}+\frac{x^4}{2!}-\frac{x^6}{3!}+\ldots$$

Now we integrate term by term:

$$\int e^{-x^2}dx = \int\left(1-\frac{x^2}{1!}+\frac{x^4}{2!}-\frac{x^6}{3!}+\ldots+(-1)^n\frac{x^{2n}}{n!}+\ldots\right)dx$$

$$= C+x-\frac{x^3}{3\cdot 1!}+\frac{x^5}{5\cdot 2!}-\frac{x^7}{7\cdot 3!}+\ldots+(-1)^n\frac{x^{2n+1}}{(2n+1)n!}+\ldots$$

This series converges for all x because the original series for e^{-x^2} converges for all x.

(b) The Fundamental Theorem of Calculus gives

$$\int_0^1 e^{-x^2}dx = \left[x - \frac{x^3}{3\cdot 1!} + \frac{x^5}{5\cdot 2!} - \frac{x^7}{7\cdot 3!} + \frac{x^9}{9\cdot 4!} - \dots\right]_0^1$$

$$= 1 - \frac{1}{3} + \frac{1}{10} - \frac{1}{42} + \frac{1}{216} - \dots$$

$$\approx 1 - \frac{1}{3} + \frac{1}{10} - \frac{1}{42} + \frac{1}{216} \approx 0.7475$$

The Alternating Series Estimation Theorem shows that the error involved in this approximation is less than

$$\frac{1}{11\cdot 5!} = \frac{1}{1320} < 0.001$$

Evaluate $\lim\limits_{x\to 0}\frac{e^x - 1 - x}{x^2}$

Using the Maclaurin series for e^x, we have

$$\lim_{x\to 0}\frac{e^x-1-x}{x^2} = \lim_{x\to 0}\frac{\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots\right)-1-x}{x^2}$$

$$= \lim_{x\to 0}\frac{\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\dots}{x^2}$$

$$= \lim_{x\to 0}\left(\frac{1}{2}+\frac{x}{3!}+\frac{x^2}{4!}+\frac{x^3}{5!}+\dots\right) = \frac{1}{2}$$

because power series are continuous functions.

Thus the conclusive Remark :

Any random function which cannot be integrated by simple methods, can be expanded into power series and then integrated by individual terms. The number of terms to be integrated depends on the accuracy we require.

Practically we rarely need integration of indefinite functions. Definite integrals can be evaluated upto any degree of accuracy by numerical techniques.

-

To recall standard integrals

$f(x)$	$\int f(x)dx$	$f(x)$	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1}\quad (n \neq -1)$	$[g(x)]^n g'(x)$	$\frac{[g(x)]^{n+1}}{n+1}\quad (n \neq -1)$
$\frac{1}{x}$	$\ln\lvert x\rvert$	$\frac{g'(x)}{g(x)}$	$\ln\lvert g(x)\rvert$
e^x	e^x	a^x	$\frac{a^x}{\ln a}\quad (a>0)$
$\sin x$	$-\cos x$	$\sinh x$	$\cosh x$
$\cos x$	$\sin x$	$\cosh x$	$\sinh x$
$\tan x$	$-\ln\lvert\cos x\rvert$	$\tanh x$	$\ln\cosh x$
$\operatorname{cosec} x$	$\ln\left\lvert\tan\frac{x}{2}\right\rvert$	$\operatorname{cosech} x$	$\ln\left\lvert\tanh\frac{x}{2}\right\rvert$
$\sec x$	$\ln\lvert\sec x+\tan x\rvert$	$\operatorname{sech} x$	$2\tan^{-1} e^x$
$\sec^2 x$	$\tan x$	$\operatorname{sech}^2 x$	$\tanh x$
$\cot x$	$\ln\lvert\sin x\rvert$	$\coth x$	$\ln\lvert\sinh x\rvert$
$\sin^2 x$	$\frac{x}{2}-\frac{\sin 2x}{4}$	$\sinh^2 x$	$\frac{\sinh 2x}{4}-\frac{x}{2}$
$\cos^2 x$	$\frac{x}{2}+\frac{\sin 2x}{4}$	$\cosh^2 x$	$\frac{\sinh 2x}{4}+\frac{x}{2}$

$f(x)$	$\int f(x)\,dx$	$f(x)$	$\int f(x)\,dx$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a}$ $(a>0)$	$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert\ (0<\lvert x\rvert<a)$
		$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert\ (\lvert x\rvert>a>0)$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\frac{x}{a}$ $(-a<x<a)$	$\frac{1}{\sqrt{a^2+x^2}}$	$\ln\left\lvert\frac{x+\sqrt{a^2+x^2}}{a}\right\rvert\ (a>0)$
		$\frac{1}{\sqrt{x^2-a^2}}$	$\ln\left\lvert\frac{x+\sqrt{x^2-a^2}}{a}\right\rvert\ (x>a>0)$
$\sqrt{a^2-x^2}$	$\frac{a^2}{2}\left[\sin^{-1}\left(\frac{x}{a}\right)+\frac{x\sqrt{a^2-x^2}}{a^2}\right]$	$\sqrt{a^2+x^2}$	$\frac{a^2}{2}\left[\sinh^{-1}\left(\frac{x}{a}\right)+\frac{x\sqrt{a^2+x^2}}{a^2}\right]$
		$\sqrt{x^2-a^2}$	$\frac{a^2}{2}\left[-\cosh^{-1}\left(\frac{x}{a}\right)+\frac{x\sqrt{x^2-a^2}}{a^2}\right]$

Some series Expansions -

$$\frac{\pi}{2} = \left(\frac{2}{1}\frac{2}{3}\right)\left(\frac{4}{3}\frac{4}{5}\right)\left(\frac{6}{5}\frac{6}{7}\right)\left(\frac{8}{7}\frac{8}{9}\right)\ldots$$

$$\pi = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \frac{4}{13} - \cdots$$

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$$

$$\pi = \sqrt{12}\left(1 - \frac{1}{3\cdot 3} + \frac{1}{5\cdot 3^2} - \frac{1}{7\cdot 3^3} + \cdots\right)$$

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots = \sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\int_0^{\pi/2} \log \sin x \, dx = -\frac{\pi}{2}\log 2 = \frac{\pi}{2}\log\frac{1}{2}$$

Solve a series problem

If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$ upto $\infty = \frac{\pi^2}{6}$, then value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ up to ∞ is

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{6}$ (c) $\frac{\pi^2}{8}$ (d) $\frac{\pi^2}{12}$

Ans. (c)

Solution We have $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ upto ∞

$$= \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} \cdots \text{ upto } \infty$$

$$- \frac{1}{2^2}\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right]$$

$$= \frac{\pi^2}{6} - \frac{1}{4}\left(\frac{\pi^2}{6}\right) = \frac{\pi^2}{8}$$

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \ldots \infty = \frac{\pi^2}{12}$$

$$\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \ldots \infty = \frac{\pi^2}{24}$$

$$\frac{\sin\sqrt{x}}{\sqrt{x}} = 1 - \frac{x}{3!} + \frac{x^2}{5!} - \frac{x^3}{7!} + \frac{x^4}{9!} - \frac{x^5}{11!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = \sum_{k=0}^{n} \frac{(-1)^k x^{2k}}{(2k)!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots = \sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots = \sum_{k=0}^{n} \frac{x^{2k}}{(2k)!}$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots = \sum_{k=0}^{n} \frac{x^{2k+1}}{(2k+1)!}$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad (-1 \le x < 1)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} \cdots + \frac{2^{2n}\left(2^{2n}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \qquad |x| < \frac{\pi}{2}$$

$$\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \cdots + \frac{E_n x^{2n}}{(2n)!} + \cdots \quad |x| < \frac{\pi}{2}$$

$$\csc x = \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^5}{15120} + \cdots + \frac{2\left(2^{2n-1}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \qquad 0 < |x| < \pi$$

$$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - - \cdots - \frac{2^{2n} B_n x^{2n-1}}{(2n)!} - \cdots \quad 0 < |x| < \pi$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$$

$$\sec x = 1 + \frac{x^2}{\lfloor 2} + \frac{5x^4}{\lfloor 4} + \cdots$$

$$\log(\cos x) = -\frac{x^2}{\lfloor 2} - \frac{2x^4}{\lfloor 4} - \cdots$$

$$\log(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \cdots$$

$$\sin^{-1} x = x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \cdots \quad |x| < 1$$

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$= \frac{\pi}{2} - \left(x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \cdots\right) \quad |x| < 1$$

$$\tan^{-1} x = \begin{cases} x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots & |x| < 1 \\ \pm\frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \cdots & \begin{cases} + \text{ if } x \geq 1 \\ - \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$$

$$= \frac{\pi}{2} - \left(\frac{1}{x} + \frac{1}{2\cdot 3x^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5x^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \cdots\right) \quad |x| > 1$$

$$\csc^{-1} x = \sin^{-1}(1/x)$$

$$= \frac{1}{x} + \frac{1}{2\cdot 3x^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5x^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \cdots \quad |x| > 1$$

$$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$$

$$= \begin{cases} \frac{\pi}{2} - \left(x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots\right) & |x| < 1 \\ p\pi + \frac{1}{x} - \frac{1}{3x^3} + \frac{1}{5x^5} + \cdots & \begin{cases} p = 0 \text{ if } x \geq 1 \\ p = 1 \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\ln x = 2\left[\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \ldots\right]$$

$$= 2\sum_{n=1}^{\infty} \frac{1}{2n-1}\left(\frac{x-1}{x+1}\right)^{2n-1} \quad (x > 0)$$

$$\ln x = \frac{x-1}{x} + \frac{1}{2}\left(\frac{x-1}{x}\right)^2 + \frac{1}{3}\left(\frac{x-1}{x}\right)^3 + \ldots$$

$$= \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{x-1}{x}\right)^n \quad (x > \frac{1}{2})$$

$$\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}(x-1)^n \quad (0 < x \le 2)$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} x^n \quad (|x| < 1)$$

$$\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots\infty \, (-1 \le x < 1)$$

$$\log_e(1+x) - \log_e(1-x) =$$

$$\log_e \frac{1+x}{1-x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\infty\right)(-1 < x < 1)$$

$$\log_e\left(1 + \frac{1}{n}\right) = \log_e \frac{n+1}{n} = 2 \left[\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \ldots\infty\right]$$

$$\log_e (1 + x) + \log_e (1 - x) = \log_e(1 - x^2) = -2\left(\frac{x^2}{2} + \frac{x^4}{4} + \ldots\infty\right)(-1 < x < 1)$$

$$\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots = \frac{1}{1 \cdot 2} + \frac{1}{3.4} + \frac{1}{5.6} + \ldots$$

Important Results

(i) (a) $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x}\,dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cos^n x}{\sin^n x + \cos^n x}\,dx$

(b) $\int_0^{\pi/2} \frac{\tan^n x}{1+\tan^n x}\,dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{dx}{1+\tan^n x}$

(c) $\int_0^{\pi/2} \frac{dx}{1+\cot^n x} = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{1+\cot^n x}\,dx$

(d) $\int_0^{\pi/2} \frac{\tan^n x}{\tan^n x + \cot^n x}\,dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{\tan^n x + \cot^n x}\,dx$

(e) $\int_0^{\pi/2} \frac{\sec^n x}{\sec^n x + \operatorname{cosec}^n x}\,dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\operatorname{cosec}^n x}{\sec^n x + \operatorname{cosec}^n x}\,dx$ where, $n \in R$

(ii) $\int_0^{\pi/2} \frac{a^{\sin^n x}}{a^{\sin^n x} + a^{\cos^n x}}\,dx = \int_0^{\pi/2} \frac{a^{\cos^n x}}{a^{\sin^n x} + a^{\cos^n x}}\,dx = \frac{\pi}{4}$

(iii) (a) $\int_0^{\pi/2} \log \sin x\,dx = \int_0^{\pi/2} \log \cos x\,dx = -\frac{\pi}{2}\log 2$

(b) $\int_0^{\pi/2} \log \tan x\,dx = \int_0^{\pi/2} \log \cot x\,dx = 0$

(c) $\int_0^{\pi/2} \log \sec x\,dx = \int_0^{\pi/2} \log \operatorname{cosec} x\,dx = \frac{\pi}{2}\log 2$

(iv) (a) $\int_0^{\infty} e^{-ax} \sin bx\,dx = \frac{b}{a^2+b^2}$

(b) $\int_0^{\infty} e^{-ax} \cos bx\,dx = \frac{a}{a^2+b^2}$

(c) $\int_0^{\infty} e^{-ax} x^n\,dx = \frac{n!}{a^n+1}$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left(x + \sqrt{x^2 - a^2}\right) + C$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left(\frac{x-a}{x+a}\right) + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left(\frac{a+x}{a-x}\right) + C$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{a^2 + x^2}\, dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right) + C$$

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Good Luck to you for your Preparations, References, and Exams

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