Spoon Feeding Continuity & Differentiability

Simplified Knowledge Management Classes Bangalore

My name is Subhashish Chattopadhyay. I have been teaching for IIT-JEE, Various International Exams (such as IMO [International Mathematics Olympiad], IPhO [International Physics Olympiad], IChO [International Chemistry Olympiad]), IGCSE (IB), CBSE, I.Sc, Indian State Board exams such as WB-Board, Karnataka PU-II etc since 1989. As I write this book in 2016, it is my 25 th year of teaching. I was a Visiting Professor to BARC Mankhurd, Chembur, Mumbai, Homi Bhabha Centre for Science Education (HBCSE) Physics Olympics camp BARC Campus.

I am Life Member of …

- IAPT (Indian Association of Physics Teachers)

- IPA (Indian Physics Association)

- AMTI (Association of Mathematics Teachers of India)

- National Human Rights Association

- Men's Rights Movement (India and International)

- MGTOW Movement (India and International)

And also of

IACT (Indian Association of Chemistry Teachers)

The selection for National Camp (for Official Science Olympiads - Physics, Chemistry, Biology, Astronomy) happens in the following steps ….

1) **NSEP** (National Standard Exam in Physics) and **NSEC** (National Standard Exam in Chemistry) held around 24 rth November. Approx 35,000 students appear for these exams every year. The exam fees is Rs 100 each. Since 1998 the IIT JEE toppers have been topping these exams and they get to know their rank / performance ahead of others.

2) **INPhO** (Indian National Physics Olympiad) and **INChO** (Indian National Chemistry Olympiad). Around 300 students in each subject are allowed to take these exams. Students coming from outside cities are paid fair from the Govt of India.

3) The Top 35 students of each subject are invited at HBCSE (Homi Bhabha Center for Science Education) Mankhurd, near Chembur, BARC, Mumbai. After a 2-3 weeks camp the top 5 are selected to represent India. The flight tickets and many other expenses are taken care by Govt of India.

Since last 50 years there has been no dearth of "Good Books". Those who are interested in studies have been always doing well. This e-Book does not intend to replace any standard text book. These topics are very old and already standardized.

There are 3 kinds of Text Books

- The thin Books - Good students who want more details are not happy with these. Average students who need more examples are not happy with these. Most students who want to "Cram" quickly and pass somehow find the thin books "good" as they have to read less !!

- The Thick Books - Most students do not like these, as they want to read as less as possible. Average students are "busy" with many other things and have no time to read all these.

- The Average sized Books - Good students do not get all details in any one book. Most bad students do not want to read books of "this much thickness" also !!

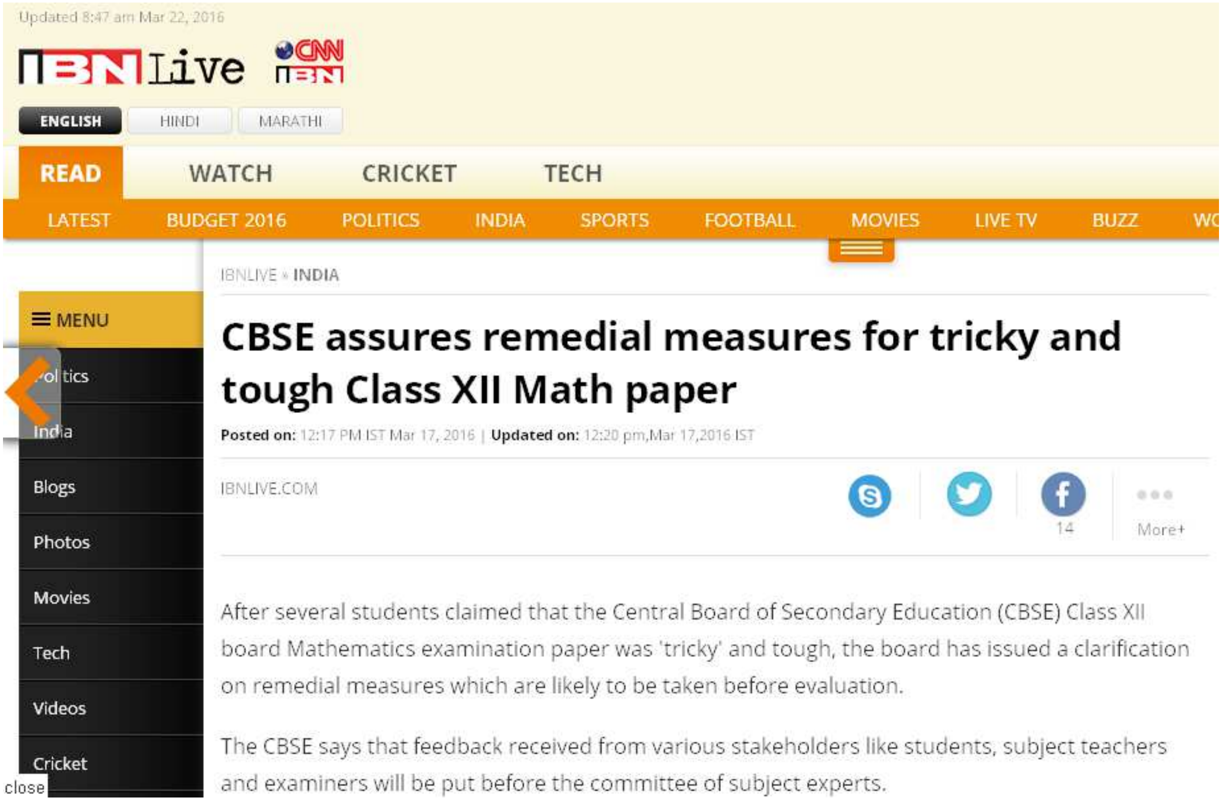
We know there can be no shoe that's fits in all.

Printed books are not e-Books! Can't be downloaded and kept in hard-disc for reading "later" ....

So if you read this book later, you will get all kinds of examples in a single place. This becomes a very good "Reference Material". I sincerely wish that all find this "very useful".

Students who do not practice lots of problems, do not do well. The rules of "doing well" had never changed .... Will never change !

After 2016 CBSE Mathematics exam, lots of students complained that the paper was tough!



Updated 8:47 am Mar 22, 2016

IBNLive CNN IBN

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IBNLIVE » INDIA

CBSE assures remedial measures for tricky and tough Class XII Math paper

Posted on: 12:17 PM IST Mar 17, 2016 | **Updated on:** 12:20 pm,Mar 17,2016 IST

IBNLIVE.COM

After several students claimed that the Central Board of Secondary Education (CBSE) Class XII board Mathematics examination paper was 'tricky' and tough, the board has issued a clarification on remedial measures which are likely to be taken before evaluation.

The CBSE says that feedback received from various stakeholders like students, subject teachers and examiners will be put before the committee of subject experts.

In 2015 also the same complain was there by many students

हिन्दी मराठी বাংলা

INDIA | STATES | WORLD | S ASIA | BIZ | SPORTS | CRICKET | SCI-TECH | SHOWBIZ | HEALTH | BLOG | EXCLUSIVE

News » India News

CBSE Class 12 exam: Issue of tough maths paper raised in Parliament

A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".

Last Updated: Thursday, March 19, 2015 - 14:41

2547 SHARES Facebook Twitter Share 16 33 Comments

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New Delhi: A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".

In March 2016, students of Karnataka PU-II also complained the same, regarding standard 12 (PU-II Mathematics Exam). Even though the Math Paper was identical to previous year, most students had not even solved the 2015 Question Paper.


Friday, March 25, 2016 - 13:28

The NEWS Minute

HOME NEWS ANDHRA KARNATAKA KERALA TAMIL NADU TELANGANA CULTURE MEDIA BLOG

Exams

Online petition for lenient evaluation of K'taka II PU math paper gets over 8000 supporters

The campaign, which was launched on Monday, has garnered over 8000 supporters

TNM Staff | Wednesday, March 16, 2016 - 09:32

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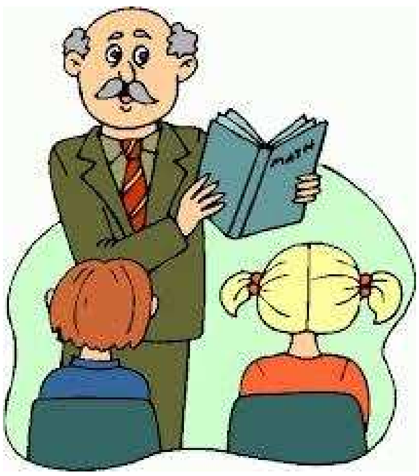
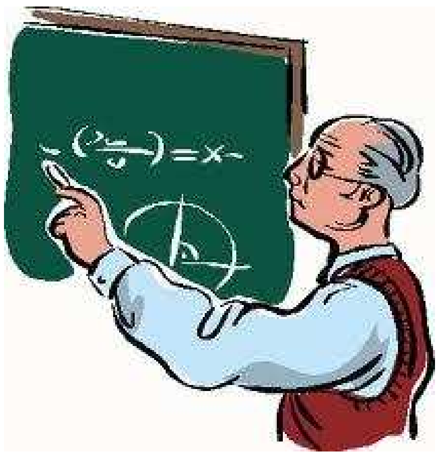
Following a "very tough" math paper that left many II PU students in tears, Saket Ravindran a student launched an online campaign demanding lenient evaluation.

These complains are not new. In fact since last 40 years, (since my childhood), I always see this; every year the same setback, same complain!

In this e-Book I am trying to solve this problem. Those students who practice can learn.

No one can help those who are not studying, or practicing.



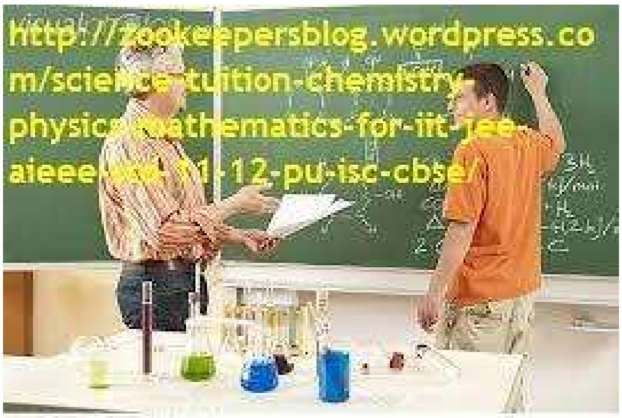


Learn more at http://skmclasses.weebly.com/iit-jee-home-tuitions-bangalore.html

Twitter - https://twitter.com/ZookeeperPhy

Facebook - https://www.facebook.com/IIT.JEE.by.Prof.Subhashish/

Blog - http://skmclasses.kinja.com



A very polite request :

I wish these e-Books are read only by Boys and Men. Girls and Women, better read something else; learn from somewhere else.

Preface

We all know that in the species "Homo Sapiens ", males are bigger than females. The reasons are explained in standard 10, or 11 (high school) Biology texts. **This shapes or size, influences all of our culture**. Before we recall / understand the reasons once again, let us see some random examples of the influence

Random - 1

If there is a Road rage, then who all fight ? (generally ?). Imagine two cars driven by adult drivers. Each car has a woman of similar age as that of the Man. The cars " touch " or " some issue happens". Who all comes out and fights ? Who all are most probable to drive the cars ?

(Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win)

Random - 2

Heavy metal music artists are all Men. Metallica, Black Sabbath, Motley Crue, Megadeth, Motorhead, AC/DC, Deep Purple, Slayer, Guns & Roses, Led Zeppelin, Aerosmith ….. the list can be in thousands. All these are grown-up Boys, known as Men.

(Men strive for perfection. Men are eager to excel. Men work hard. Men want to win.)

Random - 3

Apart from Marie Curie, only one more woman got Nobel Prize in Physics. (Maria Goeppert Mayer - 1963). So, … almost all are men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 4

The best Tabla Players are all Men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 5

History is all about, which Kings ruled. Kings, their men, and Soldiers went for wars. History is all about wars, fights, and killings by men.

Boys start fighting from school days. Girls do not fight like this

(Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.)

Random - 6

The highest award in Mathematics, the “ Fields Medal “ is around since decades. Till date only one woman could get that. (Maryam Mirzakhani - 2014). So, … almost all are men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 7

Actor is a gender neutral word. Could the movie like “ Top Gun “ be made with Female actors ? The best pilots, astronauts, Fighters are all Men.

Random - 8

In my childhood had seen a movie named “ The Tower in Inferno “. In the movie when the tall tower is in fire, women were being saved first, as only one lift was working….

Many decades later another movie is made. A box office hit. “ The Titanic “. In this also …. As the ship is sinking women are being saved. **Men are disposable**. Men may get their turn later…

Movies are not training programs. Movies do not teach people what to do, or not to do. Movies only reflect the prevalent culture. Men are disposable, is the culture in the society. Knowingly, unknowingly, the culture is depicted in Movies, Theaters, Stories, Poems, Rituals, etc. I or you can’t write a story, or make a movie in which after a minor car accident the Male passengers keep seating in the back seat, while the both the women drivers come out of the car and start fighting very bitterly on the road. There has been no story in this world, or no movie made, where after an accident or calamity, Men are being helped for safety first, and women are told to wait.

Random - 9

Artists generally follow the prevalent culture of the Society. In paintings, sculptures, stories, poems, movies, cartoon, Caricatures, knowingly / unknowingly, “ the prevalent Reality “ is depicted. The opposite will not go well with people. If deliberately “ the opposite “ is shown then it may only become a special art, considered as a special mockery.

Random - 10

Men go to “girl / woman’s house” to marry / win, and bring her to his home. That is a sort of winning her. When a boy gets a “ Girl-Friend “, generally he and his friends consider that as an achievement. The boy who “ got / won “ a girl-friend feels proud. His male friends feel, jealous, competitive and envious. Millions of stories have been written on these themes. Lakhs of movies show this. Boys / Men go for “ bike race “, or say “ Car Race “, where the winner “ gets “ the most beautiful girl of the college.




(Men want to excel. Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.)

Prithviraj Chauhan ‘ went ` to “ pickup “ or “ abduct “ or “ win “ or “ bring “ his love. There was a Hindi movie (hit) song … “ Pasand ho jaye, to ghar se utha laye “. It is not other way round. Girls do not go to Boy’s house or man’s house to marry. Nor the girls go in a gang to “ pick-up “ the boy / man and bring him to their home / place / den.

Random - 11

Rich people; often are very hard working. Successful business men, establish their business (empire), amass lot of wealth, with lot of difficulty. Lots of sacrifice, lots of hard work, gets into this. Rich people's wives had no contribution in this wealth creation. Women are smart, and successful upto the extent to choose the right/rich man to marry. So generally what happens in case of Divorces ? Search the net on " most costly divorces " and you will know. The women;(who had no contribution at all, in setting up the business / empire), often gets in Billions, or several Millions in divorce settlements.

Number 1

Rupert & Anna Murdoch -- $1.7 billion

One of the richest men in the world, **Rupert Murdoch** developed his worldwide media empire when he inherited his father's Australian newspaper in 1952. He married Anna Murdoch in the '60s and they remained together for 32 years, springing off three children.

They split amicably in 1998 but soon Rupert forced Anna off the board of News Corp and the gloves came off. The divorce was finalized in June 1999 when Rupert agreed to let his ex-wife leave with $1.7 billion worth of his assets, $110 million of it in cash. Seventeen days later, Rupert married Wendi Deng, one of his employees.

Ted Danson & Casey Coates -- $30 million

Ted Danson's claim to fame is undoubtedly his decade-long stint as Sam Malone on NBC's celebrated sitcom Cheers . While he did other TV shows and movies, he will always be known as the bartender of that place where everybody knows your name. He met his future first bride Casey, a designer, in 1976 while doing Erhard Seminars Training.

Ten years his senior, she suffered a paralyzing stroke while giving birth to their first child in 1979. In order to nurse her back to health, Danson took a break from acting for six months. But after two children and 15 years of marriage, the infatuation fell to pieces. Danson had started seeing Whoopi Goldberg while filming the comedy, Made in America and this precipitated the 1992 divorce. Casey got $30 million for her trouble.

See https://zookeepersblog.wordpress.com/misandry-and-men-issues-a-short-summary-at-single-place/

See http://skmclasses.kinja.com/save-the-male-1761788732

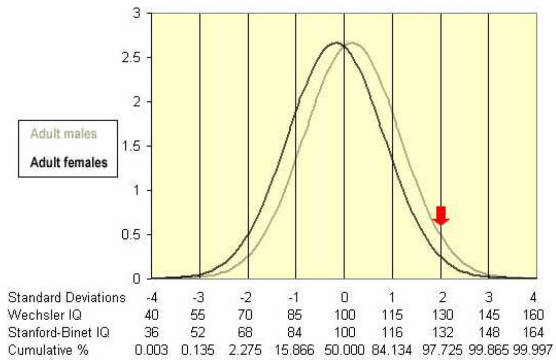
It was Boys and Men, who brought the girls / women home. The Laws are biased, completely favoring women. The men are paying for their own mistakes.

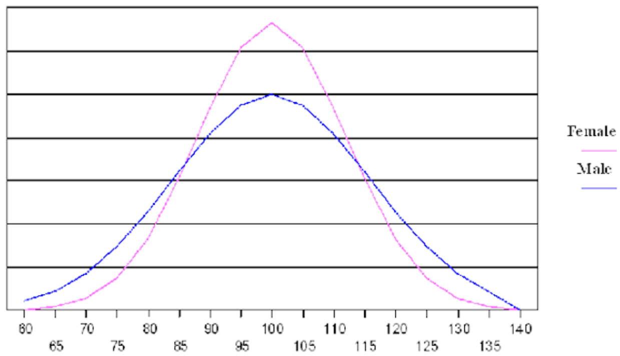
See https://zookeepersblog.wordpress.com/biased-laws/

(Man brings the Woman home. When she leaves, takes away her share of big fortune!)

Random - 12

A standardized test of Intelligence will never be possible. It never happened before, nor ever will happen in future; where the IQ test results will be acceptable by all. In the net there are thousands of charts which show that the intelligence scores of girls / women are lesser. Debates of Trillion words, does not improve performance of Girls.





I am not wasting a single second debating or discussing with anyone, on this. I am simply accepting ALL the results. IQ is only one of the variables which is required for success in life. Thousands of books have been written on “ Networking Skills “, EQ (Emotional Quotient), Drive, Dedication, Focus, “ Tenacity towards the end goal “ … etc. In each criteria, and in all together, women (in general) do far worse than men. Bangalore is known as “ ….. capital of India “. [Fill in the blanks]. The blanks are generally filled as “ Software Capital “, “ IT Capital “, “ Startup Capital “, etc. I am member in several startup eco-systems / groups. I have attended hundreds of meetings, regarding “ technology startups “, or “ idea startups “. These meetings have very few women. Starting up new companies are all “ Men’s Game “ / “ Men’s business “. Only in Divorce settlements women will take their goodies, due to Biased laws. There is no dedication, towards wealth creation, by women.

Random - 13

Many men, as fathers, very unfortunately treat their daughters as “ Princess “. Every “ non-performing “ woman / wife was “ princess daughter “ of some loving father. Pampering the girls, in name of “ equal opportunity “, or “ women empowerment “, have led to nothing.

See http://skmclasses.kinja.com/progressively-daughters-become-monsters-1764484338

See http://skmclasses.kinja.com/vivacious-vixens-1764483974

There can be thousands of more such random examples, where " Bigger Shape / size " of males have influenced our culture, our Society. **Let us recall the reasons**, that we already learned in standard 10 - 11, Biology text Books. In humans, women have a long gestation period, and also spends many years (almost a decade) to grow, nourish, and stabilize the child. (Million years of habit) Due to survival instinct Males want to inseminate. Boys and Men fight for the " facility (of womb + care) " the girl / woman may provide. Bigger size for males, has a winning advantage. Whoever wins, gets the " woman / facility ". The male who is of " Bigger Size ", has an advantage to win…. Leading to Natural selection over millions of years. In general " Bigger Males "; the " fighting instinct " in men; have led to wars, and solving tough problems (Mathematics, Physics, Technology, startups of new businesses, Wealth creation, Unreasonable attempts to make things [such as planes], Hard work ….)

So let us see the IIT-JEE results of girls. Statistics of several years show that there are around 17, (or less than 20) girls in top 1000 ranks, at all India level. Some people will yet not understand the performance, till it is said that … year after year we have around 980 boys in top 1000 ranks. Generally we see only 4 to 5 girls in top 500. In last 50 years not once any girl topped in IIT-JEE advanced. Forget about Single digit ranks, double digit ranks by girls have been extremely rare. It is all about " good boys ", " hard working ", " focused ", "**Bel-esprit**" **boys**.

In 2015, Only 2.6% of total candidates who qualified are girls (upto around 12,000 rank). while 20% of the Boys, amongst all candidates qualified. The Total number of students who appeared for the exam were around 1.4 million for IIT-JEE main. Subsequently 1.2 lakh (around 120 thousands) appeared for IIT-JEE advanced.

IIT-JEE results and analysis, of many years is given at https://zookeepersblog.wordpress.com/iit-jee-iseet-main-and-advanced-results/

In Bangalore it is rare to see a girl with rank better than 1000 in IIT-JEE advanced. We hardly see 6-7 boys with rank better than 1000. Hardly 2-3 boys get a rank better than 500.

See http://skmclasses.weebly.com/everybody-knows-so-you-should-also-know.html

Thousands of people are exposing the heinous crimes that Motherly Women are doing, or Female Teachers are committing. See https://www.facebook.com/WomenCriminals/

Some Random Examples must be known by all

Mother Admits On Facebook to Sleeping with 15 Yr Old Son, They Have a Baby Together - Alwayzturntup

Sometimes it hard to believe w From Alwayzturntup

ALWAYZTURNTUP.ME

It is extremely unfortunate that the " woman empowerment " has created. This is the kind of society and women we have now. I and many other sensible Men hate such women. Be away from such women, be aware of reality.

'Sex with my son is incredible - we're in love and we want a baby'

Ben Ford, who ditched his wife when he met his mother Kim West after 30 years, claims what the couple are doing 'isn't incest'

MIRROR.CO.UK

Former Shelbyville ISD teacher who had sex with underage student gets 3 years in prison

After a two day break over the weekend, A Shelby County jury was back in the courtroom looking to conclude the trial of a former Shelbyville ISD teacher who had...

KLTV.COM | BY CALEB BEAMES

Woman sent to jail for the rest of her life after raping her four grandchildren is described as the 'most evil person' the judge has ever seen

Edwina Louis rape...

See More

Woman sent to jail for raping her four grandchildren

A Ohio grandmother has been sentenced to four consecutive life terms after being found guilty of the rape of her own grandchildren. Edwina Louis, 53, will spend the rest of her life behind bars.

DAILYMAIL.CO.UK

http://www.thenativecanadian.com/.../eastern-ontario-teacher-...

The N.C. Chronicles.: Eastern Ontario teacher charged with 36 sexual offences

anti feminism, Child abuse, children's rights, Feminist hypocrisy,

THENATIVECANADIAN.COM | BY BLACKWOLF

Hyd woman kills newborn boy as she wanted daughter - Times of India

Having failed to bear a daughter for the third time, a shopkeeper's wife slit the throat of her 24day-old son with a shaving blade and left him to die in a street on Tuesday night.Purnima's first child was a stillborn boy, followed by another boy born five years ago.

TIMESOFINDIA.INDIATIMES.COM

Montgomery's son, Alan Vonn Webb, took the stand and was a key witness in her conviction.

"I want to see her placed somewhere she can never do that to children ...

See More

Woman sentenced to 40 years in prison for raping her children

A Murfreesboro mother found guilty of raping her own children learned her fate on Wednesday.

WAFF.COM | BY DENNIS FERRIER

gentler sex? Violence against men.'s photo.

Women, the gentler sex? Violence against men.

April 8 at 1:38am

Like Page

In fact, the past decade has seen a dramatic increase in the number of incidents of women raping and sexually assaulting boys and men. On May 2014, Jezebel repo...

End violence against women . . . .

North Carolina Grandma Eats Her Daughter's New Born Baby After Smoking Bath Salts

Henderson, North Carolina– A North Carolina grandmother of 4 and recovering drug addict, is now in custody after she allegedly ate her daughter's newborn baby....

AZ-365.TOP

28-Year-Old Texas Teacher Accused of Sending Nude Picture to 14-Year-Old Former Student

BREITBART.COM

http://latest.com/.../attractive-girl-gang-lured-men-alleywa.../

Attractive Girl Gang Lured Men Into Alleyways Where Female Body Builder Would Attack Them

A Mexican street gang made up entirely of women has been accused of using their feminine wiles to lure men into alleyways and then beating them up and...

LATEST.COM

http://www.wfmj.com/.../youngstown-woman-convicted-of-raping-...

Youngstown woman convicted of raping a 1 year old is back in jail

A Youngstown woman who went to prison for raping a 1-year-old boy fifteen years ago is in trouble with the law again.

WFMJ.COM

End violence against women . . . .

Women are raping boys and young men

Rape advocacy has been maligned and twisted into a political agenda controlled by radicalized activists. Tim Patten takes a razor keen and well supported look into the manufactured rape culture and...

AVOICEFORMEN.COM | BY TIM PATTEN

Bronx Woman Convicted of Poisoning and Drowning Her Children

Lisette Bamenga researched methods on the Internet before she killed her son and daughter in 2012.

NYTIMES.COM | BY MARC SANTORA

A Russian-born newlywed slowly butchered her German husband — feeding strips of his flesh to their dog until he took his last breath. Svetlana Batukova, 46, was...

See More

She killed her husband and then fed him to her dog: police

A Russian-born newlywed butchered her German hubby — and fed strips of his flesh to her pooch, authorities said. Svetlana Batukova offed Horst Hans Henkels at their...

NYPOST.COM

Daily Mail
January 15, 2015 ·

Mother charged with rape and sodomy of her son's 12-year-old friend

Mom, 30, 'raped and had oral sex with her son's 12-year-old friend'

Nicole Marie Smith, 30, (pictured) of St Charles County, Missouri, has been jailed after she allegedly targeted the 12-year-old boy at her home.

DAILYM.AI

April 4 at 4:48am ·

Female prison officers commit 90pc of sex assaults on male teens in US juvenile detention centres

Lawsuit in Idaho highlights the prevalence of sexual victimization of juvenile offenders.

IBTIMES.CO.UK | BY NICOLE ROJAS

This mother filmed herself raping her own son and then sold it to a man for $300. The courts just decide her fate. When you see what she got, you're going to be outraged.

Mother Who Filmed Herself Raping Her 1-Year-Old Son Receives Shocking Sentence

"...then used the money to buy herself a laptop..."

AMERICANEWS.COM

In several countries or rather in several regions of the world, family system has collapsed, due to bad nature and naughty acts of women. Particularly in Britain, and America, almost 50% people are alone, lonely, separated, divorced or failed marriages. In 2013, 48% children were born out of wedlock. It was projected that by 2016, more than 51% children will be born, to unmarried mothers. In these developed countries " paternity fraud " by women, are close to 20%. You can see several articles in the net, and in wikipedia etc. This means 1 out of 5 children are calling a wrong man as dad. The lonely, alone “ mothers " are frustrated. They see the children as burden. Love in the Society in general is lost, long time ago. The types of “ Mothers “ and “ Women “ we have now …………


This is the type of women we have in this world. These kind of women were also someones daughter
Mother Stabs Her Baby 90 Times With Scissors After He Bit Her While Breastfeeding Him!
Eight-month-old Xiao Bao was discovered by his uncle in a pool of blood Needed 100 stitches after the incident; he is now recovering in hospital Reports say his...
MOMMABUZZ.COM

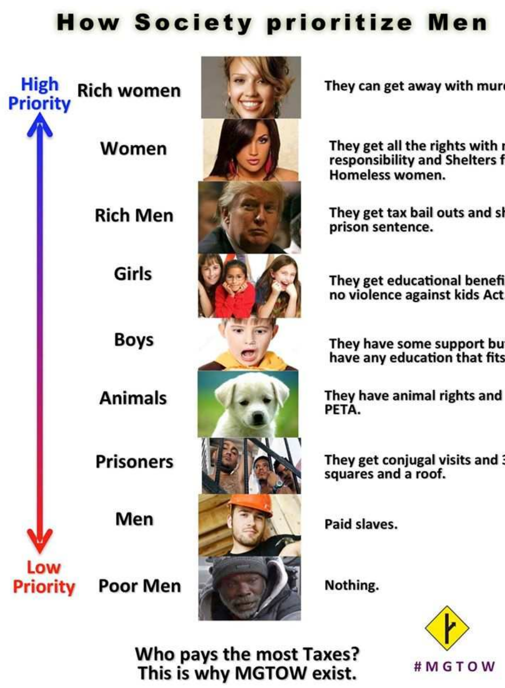

MY PUSSY MY RULES
NO GOD IN MY VAGINA
MA CHATTE MES DROITS


NATIONAL GO TOPLESS DAY
AUG 23 @ NOON - 3 PM
NYC - CENTRAL PARK
GOING TOPLESS IS LEGAL IN NY !!!
(since 1992)


NUDITY ISN'T SEXUAL


TOPLESS : ÉGALITÉ POUR TOUS OU POUR PERSONNE !
SORRY TO BOTHER YOU, BUT WE ARE CHANGING THE WORLD!
LES SEINS DES FEMMES NE SONT PAS INDÉCENTS


FOR WOMEN



Professor Subhashish Chattopadhyay

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Spoon Feeding Series - Continuity & Differentiability

Question

Prove that the function $f(x) = 5x - 3$ is continuous at $x = 0$, at $x = -3$ and at $x = 5$

Answer :

The given function is $f(x) = 5x - 3$

At $x = 0$, $f(0) = 5 \times 0 - 3 = 3$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} (5x - 3) = 5 \times 0 - 3 = -3$

$\therefore \lim_{x \to 0} f(x) = f(0)$

Therefore, f is continuous at x= 0

At $x = -3$, $f(-3) = 5 \times (-3) - 3 = -18$

$\lim_{x \to -3} f(x) = \lim_{x \to -3} (5x - 3) = 5 \times (-3) - 3 = -18$

$\therefore \lim_{x \to -3} f(x) = f(-3)$

Therefore, f is continuous at x= - 3

At $x = 5$, $f(x) = f(5) = 5 \times 5 - 3 = 25 - 3 = 22$

$\lim_{x \to 5} f(x) = \lim_{x \to 5} (5x - 3) = 5 \times 5 - 3 = 22$

$\therefore \lim_{x \to 5} f(x) = f(5)$

Therefore f(x) is continuous around x=5

Question

Examine the continuity of the function $f(x) = 2x^2 - 1$ at $x = 3$

Answer :

The given function is $f(x) = 2x^2 - 1$

At $x = 3$, $f(x) = f(3) = 2\times 3^2 - 1 = 17$

$\lim_{x\to 3} f(x) = \lim_{x\to 3}(2x^2 - 1) = 2\times 3^2 - 1 = 17$

$\therefore \lim_{x\to 3} f(x) = f(3)$

Thus, f is continuous at $x = 3$

Question

Examine the following functions for continuity.

(a) $f(x) = x - 5$ (b) $f(x) = \frac{1}{x-5}, x \neq 5$

(c) $f(x) = \frac{x^2 - 25}{x+5}, x \neq -5$ (d) $f(x) = |x - 5|$

Answer :

(a) The given function is $f(x) = x - 5$

It is evident that f is defined at every real number k and its value at k is $k - 5$.

It is also observed that, $\lim_{x\to k} f(x) = \lim_{x\to k}(x - 5) = k - 5 = f(k)$

$\therefore \lim_{x\to k} f(x) = f(k)$

Hence, f is continuous at every real number and therefore, it is a continuous function.

(b) The given function is $f(x) = \frac{1}{x-5}, x \neq 5$

For any real number $k \neq 5$, we obtain

$$\lim_{x\to k} f(x) = \lim_{x\to k} \frac{1}{x-5} = \frac{1}{k-5}$$
$$\text{Also, } f(k) = \frac{1}{k-5} \quad (\text{As } k \neq 5)$$
$$\therefore \lim_{x\to k} f(x) = f(k)$$

Hence, f is continuous at every point in the domain of f and therefore, it is a continuous function.

(c) The given function is $f(x) = \frac{x^2-25}{x+5}, x \neq -5$

For any real number $c \neq -5$, we obtain

$$\lim_{x\to c} f(x) = \lim_{x\to c} \frac{x^2-25}{x+5} = \lim_{x\to c} \frac{(x+5)(x-5)}{x+5} = \lim_{x\to c}(x-5) = (c-5)$$
$$\text{Also, } f(c) = \frac{(c+5)(c-5)}{c+5} = (c-5) \quad (\text{as } c \neq -5)$$
$$\therefore \lim_{x\to c} f(x) = f(c)$$

Hence, f is continuous at every point in the domain of f and therefore, it is a continuous function.

(d) The given function is $f(x) = |x-5| = \begin{cases} 5-x, & \text{if } x < 5 \\ x-5, & \text{if } x \geq 5 \end{cases}$

This function f is defined at all points of the real line.

Let c be a point on a real line. Then, $c < 5$ or $c = 5$ or $c > 5$

Case I: $c < 5$

Then, $f(c) = 5 - c$

$$\lim_{x\to c} f(x) = \lim_{x\to c}(5-x) = 5-c$$
$$\therefore \lim_{x\to c} f(x) = f(c)$$

Therefore, f is continuous at all real numbers less than 5.

Case II : $c = 5$

Then, $f(c) = f(5) = (5-5) = 0$

$$\lim_{x\to 5^-} f(x) = \lim_{x\to 5}(5-x) = (5-5) = 0$$

$$\lim_{x\to 5^+} f(x) = \lim_{x\to 5}(x-5) = 0$$
$$\therefore \lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$$

Therefore, f is continuous at $x = 5$

Case III: $c > 5$

Then, $f(c) = f(5) = c - 5$

$\lim_{x \to c} f(x) = \lim_{x \to c}(x - 5) = c - 5$

$\therefore \lim_{x \to c} f(x) = f(c)$

Question

Prove that the function $f(x) = x^n$ is continuous at $x = n$, where n is a positive integer.

Answer :

The given function is $f(x) = x^n$

It is evident that f is defined at all positive integers, n, and its value at n is n^n.

Then, $\lim_{x \to n} f(n) = \lim_{x \to n}(x^n) = n^n$

$\therefore \lim_{x \to n} f(x) = f(n)$

Therefore, f is continuous at n, where n is a positive integer.

Question

Is the function *f* defined by

$$f(x)=\begin{cases}x, & \text{if } x\le 1\\ 5, & \text{if } x>1\end{cases}$$

continuous at $x = 0$? At $x = 1$? At $x = 2$?

Answer :

The given function *f* is $f(x)=\begin{cases}x, & \text{if } x\le 1\\ 5, & \text{if } x>1\end{cases}$

At $x = 0$,

It is evident that *f* is defined at 0 and its value at 0 is 0.

Then, $\lim_{x\to 0} f(x) = \lim_{x\to 0} x = 0$

$\therefore \lim_{x\to 0} f(x) = f(0)$

Therefore, *f* is continuous at $x = 0$

At $x = 1$,

f is defined at 1 and its value at 1 is 1.

The left hand limit of f at $x = 1$ is,

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x = 1$$

The right hand limit of f at $x = 1$ is,

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (5) = 5$$

$$\therefore \lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$$

Therefore, f is not continuous at $x = 1$

At $x = 2$,

f is defined at 2 and its value at 2 is 5.

Then, $\lim_{x\to 2} f(x) = \lim_{x\to 2} (5) = 5$

$$\therefore \lim_{x\to 2} f(x) = f(2)$$

Therefore, f is continuous at $x = 2$

Question

Find all points of discontinuity of *f*, where *f* is defined by

$$f(x)=\begin{cases}2x+3, & \text{if } x\le 2\\ 2x-3, & \text{if } x>2\end{cases}$$

Answer :

The given function *f* is $f(x)=\begin{cases}2x+3, & \text{if } x\le 2\\ 2x-3, & \text{if } x>2\end{cases}$

It is evident that the given function *f* is defined at all the points of the real line.

Let *c* be a point on the real line. Then, three cases arise.

(i) $c < 2$

(ii) $c > 2$

(iii) $c = 2$

Case (i) $c < 2$

$$\text{Then, } f(c) = 2c+3$$
$$\lim_{x\to c} f(x) = \lim_{x\to c}(2x+3) = 2c+3$$

$\therefore \lim_{x \to c} f(x) = f(c)$

Therefore, *f* is continuous at all points *x*, such that *x* < 2

Case (ii) *c* > 2

Then, $f(c) = 2c - 3$

$\lim_{x \to c} f(x) = \lim_{x \to c} (2x - 3) = 2c - 3$

$\therefore \lim_{x \to c} f(x) = f(c)$

Therefore, *f* is continuous at all points *x*, such that *x* > 2

Case (iii) *c* = 2

Then, the left hand limit of *f* at *x* = 2 is,

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2x + 3) = 2 \times 2 + 3 = 7$

The right hand limit of *f* at *x* = 2 is,

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (2x - 3) = 2 \times 2 - 3 = 1$

It is observed that the left and right hand limit of *f* at *x* = 2 do not coincide.

Therefore, *f* is not continuous at *x* = 2

Hence, *x* = 2 is the only point of discontinuity of *f*.

Question

Find all points of discontinuity of *f*, where *f* is defined by

$$f(x)=\begin{cases}|x|+3, \text{if } x\le -3\\ -2x, \text{if } -3<x<3\\ 6x+2, \text{if } x\ge 3\end{cases}$$

Answer :

The given function *f* is $f(x)=\begin{cases}|x|+3=-x+3, \text{ if } x\le -3\\ -2x, \text{ if } -3<x<3\\ 6x+2, \text{ if } x\ge 3\end{cases}$

The given function *f* is defined at all the points of the real line.

Let *c* be a point on the real line.

Case I:

If $c<-3$, then $f(c)=-c+3$

$\lim\limits_{x\to c} f(x)=\lim\limits_{x\to c}(-x+3)=-c+3$

$\therefore \lim\limits_{x\to c} f(x)=f(c)$

Therefore, f is continuous at all points x, such that $x < -3$

Case II:

If $c = -3$, then $f(-3) = -(-3) + 3 = 6$

$$\lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} (-x + 3) = -(-3) + 3 = 6$$
$$\lim_{x \to -3^+} f(x) = \lim_{x \to -3^+} (-2x) = -2 \times (-3) = 6$$
$$\therefore \lim_{x \to -3} f(x) = f(-3)$$

Therefore, f is continuous at $x = -3$

Case III:

If $-3 < c < 3$, then $f(c) = -2c$ and $\lim_{x \to c} f(x) = \lim_{x \to c}(-2x) = -2c$

$$\therefore \lim_{x \to c} f(x) = f(c)$$

Therefore, f is continuous in $(-3, 3)$.

Case IV:

If $c = 3$, then the left hand limit of f at $x = 3$ is,

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (-2x) = -2 \times 3 = -6$$

The right hand limit of f at $x = 3$ is,

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (6x + 2) = 6 \times 3 + 2 = 20$$

It is observed that the left and right hand limit of f at $x = 3$ do not coincide.

Therefore, f is not continuous at $x = 3$

Case V:

If $c > 3$, then $f(c) = 6c + 2$ and $\lim_{x \to c} f(x) = \lim_{x \to c}(6x + 2) = 6c + 2$

$$\therefore \lim_{x \to c} f(x) = f(c)$$

Therefore, f is continuous at all points x, such that $x > 3$

Hence, $x = 3$ is the only point of discontinuity of f.

Question

Find all points of discontinuity of *f*, where *f* is defined by

$$f(x)=\begin{cases}\frac{|x|}{x} & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$$

Answer :

The given function *f* is $f(x)=\begin{cases}\frac{|x|}{x} & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$

It is known that, $x<0 \Rightarrow |x|=-x$ and $x>0 \Rightarrow |x|=x$

Therefore, the given function can be rewritten as

$$f(x)=\begin{cases}\frac{|x|}{x}=\frac{-x}{x}=-1 & \text{if } x<0\\ 0, & \text{if } x=0\\ \frac{|x|}{x}=\frac{x}{x}=1, & \text{if } x>0\end{cases}$$

The given function *f* is defined at all the points of the real line.

Let c be a point on the real line.

Case I:

If $c < 0$, then $f(c) = -1$

$$\lim_{x \to c} f(x) = \lim_{x \to c}(-1) = -1$$

$$\therefore \lim_{x \to c} f(x) = f(c)$$

Therefore, f is continuous at all points $x < 0$

Case II:

If $c = 0$, then the left hand limit of f at $x = 0$ is,

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-}(-1) = -1$$

The right hand limit of f at $x = 0$ is,

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+}(1) = 1$$

It is observed that the left and right hand limit of f at $x = 0$ do not coincide.

Therefore, f is not continuous at $x = 0$

Case III:

If $c > 0$, then $f(c) = 1$

$$\lim_{x \to c} f(x) = \lim_{x \to c}(1) = 1$$

$$\therefore \lim_{x \to c} f(x) = f(c)$$

Therefore, f is continuous at all points x, such that $x > 0$

Hence, $x = 0$ is the only point of discontinuity of f.

Question

Find all points of discontinuity of *f*, where *f* is defined by

$$f(x)=\begin{cases}\frac{x}{|x|}, \text{ if } x<0\\ -1, \text{ if } x\geq 0\end{cases}$$

Answer :

The given function *f* is $f(x)=\begin{cases}\frac{x}{|x|}, \text{ if } x<0\\ -1, \text{ if } x\geq 0\end{cases}$

It is known that, $x<0 \Rightarrow |x|=-x$

Therefore, the given function can be rewritten as

$$f(x)=\begin{cases}\frac{x}{|x|}=\frac{x}{-x}=-1, \text{ if } x<0\\ -1, \text{ if } x\geq 0\end{cases}$$

$$\Rightarrow f(x)=-1 \text{ for all } x\in \mathbf{R}$$

Let *c* be any real number. Then, $\lim_{x\to c} f(x)=\lim_{x\to c}(-1)=-1$

Also, $f(c)=-1=\lim_{x\to c} f(x)$

Therefore, the given function is a continuous function.

Hence, the given function has no point of discontinuity.

Question

Find all points of discontinuity of *f*, where *f* is defined by

$$f(x)=\begin{cases} x+1, & \text{if } x\ge 1 \\ x^2+1, & \text{if } x<1 \end{cases}$$

Answer :

The given function *f* is $f(x)=\begin{cases} x+1, & \text{if } x\ge 1 \\ x^2+1, & \text{if } x<1 \end{cases}$

The given function *f* is defined at all the points of the real line.

Let *c* be a point on the real line.

Case I:

If $c<1$, then $f(c)=c^2+1$ and $\lim\limits_{x\to c} f(x)=\lim\limits_{x\to c}\left(x^2+1\right)=c^2+1$

$\therefore \lim\limits_{x\to c} f(x)=f(c)$

Therefore, *f* is continuous at all points *x*, such that *x*< 1

Case II:

If $c=1$, then $f(c)=f(1)=1+1=2$

The left hand limit of f at $x = 1$ is,

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x^2 + 1) = 1^2 + 1 = 2$$

The right hand limit of f at $x = 1$ is,

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x + 1) = 1 + 1 = 2$$

$$\therefore \lim_{x \to 1} f(x) = f(1)$$

Therefore, f is continuous at $x = 1$

Case III:

If $c > 1$, then $f(c) = c + 1$

$$\lim_{x \to c} f(x) = \lim_{x \to c} (x + 1) = c + 1$$

$$\therefore \lim_{x \to c} f(x) = f(c)$$

Therefore, f is continuous at all points x, such that $x > 1$

Hence,the given function f has no point of discontinuity.

Question

Find all points of discontinuity of f, where f is defined by

$$f(x)=\begin{cases} x^3-3, & \text{if } x\le 2 \\ x^2+1, & \text{if } x>2 \end{cases}$$

Answer :

The given function f is $f(x)=\begin{cases} x^3-3, & \text{if } x\le 2 \\ x^2+1, & \text{if } x>2 \end{cases}$

The given function f is defined at all the points of the real line.

Let c be a point on the real line.

Case I:

If $c<2$, then $f(c)=c^3-3$ and $\lim_{x\to c} f(x)=\lim_{x\to c}(x^3-3)=c^3-3$

$\therefore \lim_{x\to c} f(x)=f(c)$

Therefore, f is continuous at all points x, such that $x<2$

Case II:

If $c = 2$, then $f(c) = f(2) = 2^3 - 3 = 5$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x^3 - 3) = 2^3 - 3 = 5$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x^2 + 1) = 2^2 + 1 = 5$$

$$\therefore \lim_{x \to 2} f(x) = f(2)$$

Therefore, f is continuous at $x = 2$

Case III:

If $c > 2$, then $f(c) = c^2 + 1$

$$\lim_{x \to c} f(x) = \lim_{x \to c} (x^2 + 1) = c^2 + 1$$

$$\therefore \lim_{x \to c} f(x) = f(c)$$

Therefore, f is continuous at all points x, such that $x > 2$

Thus, the given function f is continuous at every point on the real line.

Hence, f has no point of discontinuity.

Question

Find all points of discontinuity of *f*, where *f* is defined by

$$f(x)=\begin{cases} x^{10}-1, & \text{if } x\le 1 \\ x^2, & \text{if } x>1 \end{cases}$$

Answer :

The given function *f* is $f(x)=\begin{cases} x^{10}-1, & \text{if } x\le 1 \\ x^2, & \text{if } x>1 \end{cases}$

The given function *f* is defined at all the points of the real line.

Let *c* be a point on the real line.

Case I:

If $c<1$, then $f(c)=c^{10}-1$ and $\lim\limits_{x\to c} f(x)=\lim\limits_{x\to c}\left(x^{10}-1\right)=c^{10}-1$

$\therefore \lim\limits_{x\to c} f(x)=f(c)$

Therefore, *f* is continuous at all points *x*, such that $x<1$

Case II:

If $c=1$, then the left hand limit of *f* at $x=1$ is,

$\lim\limits_{x\to 1^-} f(x)=\lim\limits_{x\to 1^-}\left(x^{10}-1\right)=1^{10}-1=1-1=0$

The right hand limit of *f* at $x=1$ is,

$\lim\limits_{x\to 1^+} f(x)=\lim\limits_{x\to 1^+}\left(x^2\right)=1^2=1$

It is observed that the left and right hand limit of *f* at $x=1$ do not coincide.

Therefore, *f* is not continuous at $x=1$

Case III:

If $c>1$, then $f(c)=c^2$

$\lim\limits_{x\to c} f(x)=\lim\limits_{x\to c}\left(x^2\right)=c^2$

$\therefore \lim\limits_{x\to c} f(x)=f(c)$

Therefore, *f* is continuous at all points *x*, such that $x>1$

Thus, from the above observation, it can be concluded that $x=1$ is the only point of discontinuity of *f*.

Question

Is the function defined by

$$f(x) = \begin{cases} x+5, & \text{if } x \le 1 \\ x-5, & \text{if } x > 1 \end{cases}$$

a continuous function?

Answer :

The given function is $f(x) = \begin{cases} x+5, & \text{if } x \le 1 \\ x-5, & \text{if } x > 1 \end{cases}$

The given function *f* is defined at all the points of the real line.

Let *c* be a point on the real line.

Case I:

If $c < 1$, then $f(c) = c+5$ and $\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c}(x+5) = c+5$

$\therefore \lim\limits_{x \to c} f(x) = f(c)$

Therefore, *f* is continuous at all points *x*, such that *x*< 1

Case II:

If $c = 1$, then $f(1) = 1 + 5 = 6$

The left hand limit of *f* at $x = 1$ is,

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x + 5) = 1 + 5 = 6$$

The right hand limit of *f* at $x = 1$ is,

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x - 5) = 1 - 5 = -4$$

It is observed that the left and right hand limit of *f* at $x = 1$ do not coincide.

Therefore, *f* is not continuous at $x = 1$

Case III:

If $c > 1$, then $f(c) = c - 5$ and $\lim_{x \to c} f(x) = \lim_{x \to c} (x - 5) = c - 5$

$$\therefore \lim_{x \to c} f(x) = f(c)$$

Therefore, *f* is continuous at all points *x*, such that $x > 1$

Thus, from the above observation, it can be concluded that $x = 1$ is the only point of discontinuity of *f*.

Question

Discuss the continuity of the function *f*, where *f* is defined by

$$f(x)=\begin{cases}3, \text{ if } 0\le x\le 1\\ 4, \text{ if } 1<x<3\\ 5, \text{ if } 3\le x\le 10\end{cases}$$

Answer :

The given function is $f(x)=\begin{cases}3, \text{ if } 0\le x\le 1\\ 4, \text{ if } 1<x<3\\ 5, \text{ if } 3\le x\le 10\end{cases}$

The given function is defined at all points of the interval [0, 10].

Let *c* be a point in the interval [0, 10].

Case I:

If $0\le c<1$, then $f(c)=3$ and $\lim_{x\to c} f(x)=\lim_{x\to c}(3)=3$

$\therefore \lim_{x\to c} f(x)=f(c)$

Therefore, *f* is continuous in the interval [0, 1).

Case II:

If $c = 1$, then $f(3) = 3$

The left hand limit of *f* at *x* = 1 is,

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (3) = 3$$

Theright hand limit of *f* at *x* = 1 is,

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (4) = 4$$

It is observed that the left and right hand limits of *f* at *x*= 1 do not coincide.

Therefore, *f* is not continuous at *x*= 1

Case III:

If $1 < c < 3$, then $f(c) = 4$ and $\lim_{x \to c} f(x) = \lim_{x \to c} (4) = 4$

$$\therefore \lim_{x \to c} f(x) = f(c)$$

Therefore, *f* is continuous at all points of the interval (1, 3).

Case IV:

If $c = 3$, then $f(c) = 5$

The left hand limit of f at $x = 3$ is,

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (4) = 4$$

The right hand limit of f at $x = 3$ is,

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (5) = 5$$

It is observed that the left and right hand limits of f at $x = 3$ do not coincide.

Therefore, f is not continuous at $x = 3$

Case V:

If $3 < c \leq 10$, then $f(c) = 5$ and $\lim_{x \to c} f(x) = \lim_{x \to c} (5) = 5$

$$\lim_{x \to c} f(x) = f(c)$$

Therefore, f is continuous at all points of the interval (3, 10].

Hence, f is not continuous at $x = 1$ and $x = 3$

Discuss the continuity of the function *f*, where *f* is defined by

$$f(x)=\begin{cases}2x, & \text{if } x<0\\ 0, & \text{if } 0\le x\le 1\\ 4x, & \text{if } x>1\end{cases}$$

Answer :

The given function is $f(x)=\begin{cases}2x, & \text{if } x<0\\ 0, & \text{if } 0\le x\le 1\\ 4x, & \text{if } x>1\end{cases}$

The given function is defined at all points of the real line.

Let *c* be a point on the real line.

Case I:

$$\text{If } c<0, \text{ then } f(c)=2c$$

$$\lim_{x\to c} f(x)=\lim_{x\to c}(2x)=2c$$

$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, f is continuous at all points x, such that $x < 0$

Case II:

If $c = 0$, then $f(c) = f(0) = 0$

The left hand limit of f at $x = 0$ is,

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (2x) = 2 \times 0 = 0$$

The right hand limit of f at $x = 0$ is,

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (0) = 0$$

$$\therefore \lim_{x \to 0} f(x) = f(0)$$

Therefore, f is continuous at $x = 0$

Case III:

If $0 < c < 1$, then $f(x) = 0$ and $\lim_{x \to c} f(x) = \lim_{x \to c} (0) = 0$

$$\therefore \lim_{x \to c} f(x) = f(c)$$

Therefore, f is continuous at all points of the interval (0, 1).

Case IV:

If $c = 1$, then $f(c) = f(1) = 0$

Theleft hand limit of f at $x = 1$ is,

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (0) = 0$$

The right hand limit of f at $x = 1$ is,

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (4x) = 4 \times 1 = 4$$

It is observed that the left and right hand limits of f at $x = 1$ do not coincide.

Therefore, f is not continuous at $x = 1$

Case V:

If $c < 1$, then $f(c) = 4c$ and $\lim_{x \to c} f(x) = \lim_{x \to c} (4x) = 4c$

$$\therefore \lim_{x \to c} f(x) = f(c)$$

Therefore, f is continuous at all points x, such that $x > 1$

Hence, f is not continuous only at $x = 1$

Question

Discuss the continuity of the function *f*, where *f* is defined by

$$f(x)=\begin{cases}-2, \text{ if } x\le -1\\ 2x, \text{ if } -1<x\le 1\\ 2, \text{ if } x>1\end{cases}$$

Answer :

The given function *f* is $f(x)=\begin{cases}-2, \text{ if } x\le -1\\ 2x, \text{ if } -1<x\le 1\\ 2, \text{ if } x>1\end{cases}$

The given function is defined at all points of the real line.

Let *c* be a point on the real line.

Case I:

If $c<-1$, then $f(c)=-2$ and $\lim\limits_{x\to c} f(x)=\lim\limits_{x\to c}(-2)=-2$

$\therefore \lim\limits_{x\to c} f(x)=f(c)$

Therefore, *f* is continuous at all points *x*, such that *x* < - 1

Case II:

If $c = -1$, then $f(c) = f(-1) = -2$

The left hand limit of f at $x = -1$ is,

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (-2) = -2$$

The right hand limit of f at $x = -1$ is,

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} (2x) = 2 \times (-1) = -2$$

$$\therefore \lim_{x \to -1} f(x) = f(-1)$$

Therefore, f is continuous at $x = -1$

Case III:

If $-1 < c < 1$, then $f(c) = 2c$

$$\lim_{x \to c} f(x) = \lim_{x \to c} (2x) = 2c$$

$$\therefore \lim_{x \to c} f(x) = f(c)$$

Therefore, f is continuous at all points of the interval (- 1, 1).

Case IV:

If $c = 1$, then $f(c) = f(1) = 2 \times 1 = 2$

The left hand limit of f at $x = 1$ is,

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (2x) = 2 \times 1 = 2$$

The right hand limit of f at $x = 1$ is,

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 2 = 2$$

$$\therefore \lim_{x \to 1} f(x) = f(c)$$

Therefore, f is continuous at $x = 2$

Case V:

If $c > 1$, then $f(c) = 2$ and $\lim_{x \to c} f(x) = \lim_{x \to c} (2) = 2$

$$\lim_{x \to c} f(x) = f(c)$$

Therefore, f is continuous at all points x, such that $x > 1$

Thus, from the above observations, it can be concluded that f is continuous at all points of the real line.

Question

Find the relationship between *a* and *b* so that the function *f* defined by

$$f(x)=\begin{cases} ax+1, & \text{if } x\le 3 \\ bx+3, & \text{if } x>3 \end{cases}$$

is continuous at $x = 3$.

Answer :

The given function *f* is $f(x)=\begin{cases} ax+1, & \text{if } x\le 3 \\ bx+3, & \text{if } x>3 \end{cases}$

If *f* is continuous at $x = 3$, then

$$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^+} f(x)=f(3) \qquad ...(1)$$

Also,

$$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^-}(ax+1)=3a+1$$

$$\lim_{x\to 3^+} f(x)=\lim_{x\to 3^+}(bx+3)=3b+3$$

$$f(3)=3a+1$$

Therefore, from (1), we obtain

$$3a+1=3b+3=3a+1$$

$$\Rightarrow 3a+1=3b+3$$

$$\Rightarrow 3a=3b+2$$

$$\Rightarrow a=b+\frac{2}{3}$$

Therefore, the required relationship is given by, $a=b+\frac{2}{3}$

Question

For what value of λ is the function defined by

$$f(x)=\begin{cases}\lambda(x^2-2x), & \text{if } x\le 0\\ 4x+1, & \text{if } x>0\end{cases}$$

continuous at $x = 0$? What about continuity at $x = 1$?

Answer :

The given function f is $f(x)=\begin{cases}\lambda(x^2-2x), & \text{if } x\le 0\\ 4x+1, & \text{if } x>0\end{cases}$

If f is continuous at $x = 0$, then

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=f(0)$$

$$\Rightarrow \lim_{x\to 0^-} \lambda(x^2-2x)=\lim_{x\to 0^+}(4x+1)=\lambda(0^2-2\times 0)$$

$$\Rightarrow \lambda(0^2-2\times 0)=4\times 0+1=0$$

$$\Rightarrow 0=1=0, \text{ which is not possible}$$

Therefore, there is no value of λ for which f is continuous at $x = 0$

At $x = 1$,

$f(1) = 4x + 1 = 4 \times 1 + 1 = 5$

$$\lim_{x\to 1}(4x+1)=4\times 1+1=5$$

$$\therefore \lim_{x\to 1} f(x)=f(1)$$

Therefore for all values of λ f is continuous at x = 1

Question

Show that the function defined by $g(x) = x - [x]$ is discontinuous at all integral point. Here $[x]$ denotes the greatest integer less than or equal to *x*.

Answer :

The given function is $g(x) = x - [x]$

It is evident that *g* is defined at all integral points.

Let *n* be an integer.

Then,

$g(n) = n - [n] = n - n = 0$

The left hand limit of *f* at *x* = *n* is,

$\lim_{x \to n^-} g(x) = \lim_{x \to n^-} (x - [x]) = \lim_{x \to n^-} (x) - \lim_{x \to n^-} [x] = n - (n-1) = 1$

The right hand limit of *f* at *x* = *n* is,

$\lim_{x \to n^+} g(x) = \lim_{x \to n^+} (x - [x]) = \lim_{x \to n^+} (x) - \lim_{x \to n^+} [x] = n - n = 0$

It is observed that the left and right hand limits of *f* at *x* = *n* do not coincide.

Therefore, *f* is not continuous at *x* = *n*

Hence g is discontinuous at all integral points

Question

Is the function defined by $f(x) = x^2 - \sin x + 5$ continuous at $x =$?

Answer :

The given function is $f(x) = x^2 - \sin x + 5$

It is evident that f is defined at $x =$.

At $x = \pi$, $f(x) = f(\pi) = \pi^2 - \sin\pi + 5 = \pi^2 - 0 + 5 = \pi^2 + 5$

Consider $\lim\limits_{x\to\pi} f(x) = \lim\limits_{x\to\pi}\left(x^2 - \sin x + 5\right)$

Put $x = \pi + h$

If $x \to \pi$, *then it is evident that* $h \to 0$

$$\begin{aligned}
\therefore \lim_{x\to\pi} f(x) &= \lim_{x\to\pi}\left(x^2 - \sin x + 5\right) \\
&= \lim_{h\to 0}\left[(\pi+h)^2 - \sin(\pi+h) + 5\right] \\
&= \lim_{h\to 0}(\pi+h)^2 - \lim_{h\to 0}\sin(\pi+h) + \lim_{h\to 0} 5 \\
&= (\pi+0)^2 - \lim_{h\to 0}\left[\sin\pi\cosh + \cos\pi\sinh\right] + 5 \\
&= \pi^2 - \lim_{h\to 0}\sin\pi\cosh - \lim_{h\to 0}\cos\pi\sinh + 5 \\
&= \pi^2 - \sin\pi\cos 0 - \cos\pi\sin 0 + 5 \\
&= \pi^2 - 0\times 1 - (-1)\times 0 + 5 \\
&= \pi^2 + 5
\end{aligned}$$

$\therefore \lim\limits_{x\to\pi} f(x) = f(\pi)$

Therefore, the given function f is continuous at $x = \pi$

Question

Discuss the continuity of the following functions.

(a) $f(x) = \sin x + \cos x$

(b) $f(x) = \sin x - \cos x$

(c) $f(x) = \sin x \times \cos x$

Answer :

It is known that if g and h are two continuous functions, then

$g+h,\ g-h,$ and $g.h$ are also continuous.

It has to proved first that $g(x) = \sin x$ and $h(x) = \cos x$ are continuous functions.

Let $g(x) = \sin x$

It is evident that $g(x) = \sin x$ is defined for every real number.

Let c be a real number. Put $x = c + h$

If $x \longrightarrow c$, then $h \longrightarrow 0$

$g(c) = \sin c$

$\lim_{x \to c} g(x) = \lim_{x \to c} \sin x$

$$= \lim_{h \to 0} \sin(c+h)$$
$$= \lim_{h \to 0} [\sin c \cos h + \cos c \sin h]$$
$$= \lim_{h \to 0} (\sin c \cos h) + \lim_{h \to 0} (\cos c \sin h)$$
$$= \sin c \cos 0 + \cos c \sin 0$$
$$= \sin c + 0$$
$$= \sin c$$
$$\therefore \lim_{x \to c} g(x) = g(c)$$

Therefore, g is a continuous function.

Let $h(x) = \cos x$

It is evident that $h(x) = \cos x$ is defined for every real number.

Let c be a real number. Put $x = c + h$

If x Ã¢â€ ' c, then h Ã¢â€ '0

$h(c) = \cos c$

$$\lim_{x \to c} h(x) = \lim_{x \to c} \cos x$$
$$= \lim_{h \to 0} \cos(c+h)$$
$$= \lim [\cos c \cos h - \sin c \sin h]$$
$$= \lim_{h \to 0} \cos c \cos h - \lim_{h \to 0} \sin c \sin h$$
$$= \cos c \cos 0 - \sin c \sin 0$$
$$= \cos c \times 1 - \sin c \times 0$$
$$= \cos c$$
$$\therefore \lim_{x \to c} h(x) = h(c)$$

Therefore, h is a continuous function.

Therefore, it can be concluded that

(a) $f(x) = g(x) + h(x) = \sin x + \cos x$ is a continuous function

(b) $f(x) = g(x) - h(x) = \sin x - \cos x$ is a continuous function

(c) $f(x) = g(x) \times h(x) = \sin x \times \cos x$ is a continuous function

Question

Discuss the continuity of the cosine, cosecant, secant and cotangent functions,

Answer :

It is known that if g and h are two continuous functions, then

(i) $\frac{h(x)}{g(x)}$, $g(x) \neq 0$ is continuous

(ii) $\frac{1}{g(x)}$, $g(x) \neq 0$ is continuous

(iii) $\frac{1}{h(x)}$, $h(x) \neq 0$ is continuous

It has to be proved first that $g(x) = \sin x$ and $h(x) = \cos x$ are continuous functions.

Let $g(x) = \sin x$

It is evident that $g(x) = \sin x$ is defined for every real number.

Let c be a real number. Put $x = c + h$

If $x \to c$, then $h \to 0$

$g(c) = \sin c$

$$\begin{aligned}\lim_{x\to c} g(x) &= \lim_{x\to c} \sin x \\ &= \lim_{h\to 0} \sin(c+h) \\ &= \lim_{h\to 0}[\sin c \cos h + \cos c \sin h] \\ &= \lim_{h\to 0}(\sin c \cos h) + \lim_{h\to 0}(\cos c \sin h) \\ &= \sin c \cos 0 + \cos c \sin 0 \\ &= \sin c + 0 \\ &= \sin c\end{aligned}$$

$$\therefore \lim_{x\to c} g(x) = g(c)$$

Therefore, *g* is a continuous function.

Let $h(x) = \cos x$

It is evident that $h(x) = \cos x$ is defined for every real number.

Let c be a real number. Put $x = c + h$

If $x \to c$, then $h \to 0$

$h(c) = \cos c$

$$\begin{aligned}\lim_{x\to c} h(x) &= \lim_{x\to c}\cos x\\ &= \lim_{h\to 0}\cos(c+h)\\ &= \lim_{h\to 0}[\cos c\cos h - \sin c\sin h]\\ &= \lim_{h\to 0}\cos c\cos h - \lim_{h\to 0}\sin c\sin h\\ &= \cos c\cos 0 - \sin c\sin 0\\ &= \cos c\times 1 - \sin c\times 0\\ &= \cos c\end{aligned}$$

$$\therefore \lim_{x\to c} h(x) = h(c)$$

Therefore, $h(x) = \cos x$is continuous function.

It can be concluded that,

$$\operatorname{cosec} x = \frac{1}{\sin x},\ \sin x \neq 0 \text{ is continuous}$$
$$\Rightarrow \operatorname{cosec} x,\ x \neq n\pi\ (n \in Z) \text{ is continuous}$$

Therefore, cosecant is continuous except at $x = n$p, $n \in Z$

$$\sec x = \frac{1}{\cos x},\ \cos x \neq 0 \text{ is continuous}$$

$$\Rightarrow \sec x,\ x \neq (2n+1)\frac{\pi}{2}\ (n \in \mathbf{Z}) \text{ is continuous}$$

Therefore, secant is continuous except at $x = (2n+1)\frac{\pi}{2}\ (n \in \mathbf{Z})$

$$\cot x = \frac{\cos x}{\sin x},\ \sin x \neq 0 \text{ is continuous}$$
$$\Rightarrow \cot x,\ x \neq n\pi\ (n \in Z) \text{ is continuous}$$

Therefore, cotangent is continuous except at $x = n$p, $n \in Z$

Question

Find the points of discontinuity of *f*, where

$$f(x)=\begin{cases}\frac{\sin x}{x}, \text{ if } x<0\\ x+1, \text{ if } x\geq 0\end{cases}$$

Answer :

The given function *f* is $f(x)=\begin{cases}\frac{\sin x}{x}, \text{ if } x<0\\ x+1, \text{ if } x\geq 0\end{cases}$

It is evident that *f* is defined at all points of the real line.

Let *c* be a real number.

Case I:

If $c<0$, then $f(c)=\frac{\sin c}{c}$ and $\lim_{x\to c} f(x)=\lim_{x\to c}\left(\frac{\sin x}{x}\right)=\frac{\sin c}{c}$

$\therefore \lim_{x\to c} f(x)=f(c)$

Therefore, *f* is continuous at all points *x*, such that *x* < 0

Case II:

If $c > 0$, then $f(c) = c + 1$ and $\lim_{x \to c} f(x) = \lim_{x \to c}(x+1) = c + 1$

$\therefore \lim_{x \to c} f(x) = f(c)$

Therefore, f is continuous at all points x, such that $x > 0$

Case III:

If $c = 0$, then $f(c) = f(0) = 0 + 1 = 1$

The left hand limit of f at $x = 0$ is,

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0} \frac{\sin x}{x} = 1$$

The right hand limit of f at $x = 0$ is,

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (x+1) = 1$$

$$\therefore \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$$

Therefore, f is continuous at $x = 0$

From the above observations, it can be concluded that f is continuous at all points of the real line.

Thus f has no point of discontinuity

Question

Determine if *f* defined by

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

is a continuous function?

Answer :

The given function *f* is $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

It is evident that *f* is defined at all points of the real line.

Let *c* be a real number.

Case I:

If $c \neq 0$, then $f(c) = c^2 \sin\frac{1}{c}$

$$\lim_{x \to c} f(x) = \lim_{x \to c}\left(x^2 \sin\frac{1}{x}\right) = \left(\lim_{x \to c} x^2\right)\left(\lim_{x \to c} \sin\frac{1}{x}\right) = c^2 \sin\frac{1}{c}$$

$$\therefore \lim_{x \to c} f(x) = f(c)$$

Therefore, f is continuous at all points $x \neq 0$

Case II:

If $c = 0$, then $f(0) = 0$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \left(x^2 \sin \frac{1}{x} \right) = \lim_{x \to 0} \left(x^2 \sin \frac{1}{x} \right)$$

It is known that, $-1 \le \sin \frac{1}{x} \le 1,\ x \neq 0$

$$\Rightarrow -x^2 \le \sin \frac{1}{x} \le x^2$$

$$\Rightarrow \lim_{x \to 0} \left(-x^2 \right) \le \lim_{x \to 0} \left(x^2 \sin \frac{1}{x} \right) \le \lim_{x \to 0} x^2$$

$$\Rightarrow 0 \le \lim_{x \to 0} \left(x^2 \sin \frac{1}{x} \right) \le 0$$

$$\Rightarrow \lim_{x \to 0} \left(x^2 \sin \frac{1}{x} \right) = 0$$

$$\therefore \lim_{x \to 0^-} f(x) = 0$$

Similarly, $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \left(x^2 \sin \frac{1}{x} \right) = \lim_{x \to 0} \left(x^2 \sin \frac{1}{x} \right) = 0$

$$\therefore \lim_{x \to 0^-} f(x) = f(0) = \lim_{x \to 0^+} f(x)$$

Therefore, f is continuous at $x = 0$

From the above observations, it can be concluded that f is continuous at every point of the real line.

Thus, f is a continuous function.

Question

Examine the continuity of *f*, where *f* is defined by

$$f(x)=\begin{cases}\sin x-\cos x, \text{if } x\neq 0\\ -1 \qquad \text{if } x=0\end{cases}$$

Answer :

The given function *f* is $f(x)=\begin{cases}\sin x-\cos x, \text{if } x\neq 0\\ -1 \qquad \text{if } x=0\end{cases}$

It is evident that *f* is defined at all points of the real line.

Let *c* be a real number.

Case I:

$$\text{If } c\neq 0, \text{ then } f(c)=\sin c-\cos c$$
$$\lim_{x\to c} f(x)=\lim_{x\to c}(\sin x-\cos x)=\sin c-\cos c$$
$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, *f* is continuous at all points *x*, such that $x \neq 0$

Case II:

$$\text{If } c=0, \text{ then } f(0)=-1$$
$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0}(\sin x-\cos x)=\sin 0-\cos 0=0-1=-1$$
$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0}(\sin x-\cos x)=\sin 0-\cos 0=0-1=-1$$
$$\therefore \lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=f(0)$$

Therefore, *f* is continuous at $x = 0$

From the above observations, it can be concluded that *f* is continuous at every point of the real line.

Thus, *f* is a continuous function.

Question

Find the values of k so that the function f is continuous at the indicated point.

$$f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & \text{if } x\neq\frac{\pi}{2}\\ 3, & \text{if } x=\frac{\pi}{2}\end{cases} \quad \text{at } x=\frac{\pi}{2}$$

Answer :

The given function f is $f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & \text{if } x\neq\frac{\pi}{2}\\ 3, & \text{if } x=\frac{\pi}{2}\end{cases}$

The given function f is continuous at $x=\frac{\pi}{2}$, if f is defined at $x=\frac{\pi}{2}$ and if the value of the f at $x=\frac{\pi}{2}$ equals the limit of f at $x=\frac{\pi}{2}$.

It is evident that f is defined at $x=\frac{\pi}{2}$ and $f\left(\frac{\pi}{2}\right)=3$

$$\lim_{x\to\frac{\pi}{2}} f(x)=\lim_{x\to\frac{\pi}{2}}\frac{k\cos x}{\pi-2x}$$

Put $x=\frac{\pi}{2}+h$

Then, $x\to\frac{\pi}{2}\Rightarrow h\to 0$

$$\therefore \lim_{x\to\frac{\pi}{2}} f(x)=\lim_{x\to\frac{\pi}{2}}\frac{k\cos x}{\pi-2x}=\lim_{h\to 0}\frac{k\cos\left(\frac{\pi}{2}+h\right)}{\pi-2\left(\frac{\pi}{2}+h\right)}$$

$$=k\lim_{h\to 0}\frac{-\sin h}{-2h}=\frac{k}{2}\lim_{h\to 0}\frac{\sin h}{h}=\frac{k}{2}.1=\frac{k}{2}$$

$$\therefore \lim_{x\to\frac{\pi}{2}} f(x)=f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \frac{k}{2}=3$$

$$\Rightarrow k=6$$

Therefore, the required value of k is 6.

Question

Find the values of k so that the function f is continuous at the indicated point.

$$f(x)=\begin{cases} kx^2, & \text{if } x \le 2 \\ 3, & \text{if } x > 2 \end{cases} \quad \text{at } x=2$$

Answer :

The given function is $f(x)=\begin{cases} kx^2, & \text{if } x \le 2 \\ 3, & \text{if } x > 2 \end{cases}$

The given function f is continuous at $x = 2$, if f is defined at $x = 2$ and if the value of f at $x = 2$ equals the limit of f at $x = 2$

It is evident that f is defined at $x = 2$ and $f(2) = k(2)^2 = 4k$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$$

$$\Rightarrow \lim_{x \to 2^-} (kx^2) = \lim_{x \to 2^+} (3) = 4k$$

$$\Rightarrow k \times 2^2 = 3 = 4k$$

$$\Rightarrow 4k = 3 = 4k$$

$$\Rightarrow 4k = 3$$

$$\Rightarrow k = \frac{3}{4}$$

Therefore, the required value of k is $\frac{3}{4}$.

Question

Find the values of k so that the function f is continuous at the indicated point.

$$f(x)=\begin{cases} kx+1, & \text{if } x\le\pi \\ \cos x, & \text{if } x>\pi \end{cases} \qquad \text{at } x=\pi$$

Answer :

The given function is $f(x)=\begin{cases} kx+1, & \text{if } x\le\pi \\ \cos x, & \text{if } x>\pi \end{cases}$

The given function f is continuous at x= p, if f is defined at x= pand if the value of f at x= pequals the limit of $f$at$x$= p

It is evident that f is defined at$x$= pand $f(\pi)=k\pi+1$

$$\lim_{x\to\pi^-} f(x)=\lim_{x\to\pi^+} f(x)=f(\pi)$$
$$\Rightarrow \lim_{x\to\pi^-}(kx+1)=\lim_{x\to\pi^+}\cos x=k\pi+1$$
$$\Rightarrow k\pi+1=\cos\pi=k\pi+1$$
$$\Rightarrow k\pi+1=-1=k\pi+1$$
$$\Rightarrow k=-\frac{2}{\pi}$$

Therefore, the required value of k is $-\frac{2}{\pi}$.

Question

Find the values of k so that the function f is continuous at the indicated point.

$$f(x)=\begin{cases} kx+1, & \text{if } x\le 5 \\ 3x-5, & \text{if } x>5 \end{cases} \qquad \text{at } x=5$$

Answer :

The given function f is $f(x)=\begin{cases} kx+1, & \text{if } x\le 5 \\ 3x-5, & \text{if } x>5 \end{cases}$

The given function f is continuous at x= 5, if f is defined at x= 5 and if the value of f at x = 5 equals the limit of f at x = 5

It is evident that f is defined at x= 5 and $f(5)=kx+1=5k+1$

$$\lim_{x\to 5^-} f(x)=\lim_{x\to 5^+} f(x)=f(5)$$
$$\Rightarrow \lim_{x\to 5^-}(kx+1)=\lim_{x\to 5^+}(3x-5)=5k+1$$
$$\Rightarrow 5k+1=15-5=5k+1$$
$$\Rightarrow 5k+1=10$$
$$\Rightarrow 5k=9$$

$$\Rightarrow k = \frac{9}{5}$$

Therefore, the required value of k is $\frac{9}{5}$.

Question

Find the values of a and b such that the function defined by

$$f(x) = \begin{cases} 5, & \text{if } x \le 2 \\ ax + b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \ge 10 \end{cases}$$

is a continuous function.

Answer :

The given function f is $f(x) = \begin{cases} 5, & \text{if } x \le 2 \\ ax + b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \ge 10 \end{cases}$

It is evident that the given function f is defined at all points of the real line.

If f is a continuous function, then f is continuous at all real numbers.

In particular, f is continuous at $x = 2$ and $x = 10$

Since f is continuous at $x = 2$, we obtain

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$$
$$\Rightarrow \lim_{x\to 2^-} (5) = \lim_{x\to 2^+} (ax+b) = 5$$
$$\Rightarrow 5 = 2a+b = 5$$
$$\Rightarrow 2a+b=5 \qquad ...(1)$$

Since f is continuous at $x = 10$, we obtain

$$\lim_{x\to 10^-} f(x) = \lim_{x\to 10^+} f(x) = f(10)$$
$$\Rightarrow \lim_{x\to 10^-} (ax+b) = \lim_{x\to 10^+} (21) = 21$$
$$\Rightarrow 10a+b = 21 = 21$$
$$\Rightarrow 10a+b = 21 \qquad ...(2)$$

On subtracting equation (1) from equation (2), we obtain

$8a = 16$

$\Rightarrow a = 2$

Byputting $a = 2$ in equation (1), we obtain

$2 \times 2 + b = 5$

$\Rightarrow 4 + b = 5$

$\Rightarrow b = 1$

Therefore, the values of a and b for which f is a continuous function are 2 and 1 respectively.

Question

Show that the function defined by $f(x) = \cos(x^2)$ is a continuous function.

Answer :

The given function is $f(x) = \cos(x^2)$

This function f is defined for every real number and f can be written as the composition of two functions as,

$f = g \circ h$, where $g(x) = \cos x$ and $h(x) = x^2$

$$\left[\because (goh)(x) = g(h(x)) = g(x^2) = \cos(x^2) = f(x)\right]$$

It has to be first proved that $g(x) = \cos x$ and $h(x) = x^2$ are continuous functions.

It is evident that g is defined for every real number.

Let c be a real number.

Then, $g(c) = \cos c$

Put $x = c + h$

If $x \to c$, then $h \to 0$

$$\lim_{x\to c} g(x) = \lim_{x\to c} \cos x$$

$$\begin{aligned}
&= \lim_{h\to 0} \cos(c+h) \\
&= \lim_{h\to 0} [\cos c \cos h - \sin c \sin h] \\
&= \lim_{h\to 0} \cos c \cos h - \lim_{h\to 0} \sin c \sin h \\
&= \cos c \cos 0 - \sin c \sin 0 \\
&= \cos c \times 1 - \sin c \times 0 \\
&= \cos c
\end{aligned}$$

$$\therefore \lim_{x\to c} g(x) = g(c)$$

Therefore, $g(x) = \cos x$ is continuous function.

$h(x) = x^2$

Clearly, h is defined for every real number.

Let k be a real number, then $h(k) = k^2$

$$\lim_{x\to k} h(x) = \lim_{x\to k} x^2 = k^2$$

$$\therefore \lim_{x\to k} h(x) = h(k)$$

Therefore, h is a continuous function.

It is known that for real valued functions g and h, such that $(g \circ h)$ is defined at c, if g is continuous at c and if f is continuous at $g(c)$, then $(f \circ g)$ is continuous at c.

Therefore, $f(x) = (goh)(x) = \cos(x^2)$ is a continuous function.

Question

Show that the function defined by $f(x) = |\cos x|$ is a continuous function.

Answer :

The given function is $f(x) = |\cos x|$

This function *f* is defined for every real number and *f* can be written as the composition of two functions as,

f = *g o h*, where $g(x) = |x|$ and $h(x) = \cos x$

$\left[\because (goh)(x) = g(h(x)) = g(\cos x) = |\cos x| = f(x)\right]$

It has to be first proved that $g(x) = |x|$ and $h(x) = \cos x$ are continuous functions.

$g(x) = |x|$ can be written as

$$g(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Clearly, *g* is defined for all real numbers.

Let *c* be a real number.

Case I:

$$\text{If } c < 0, \text{ then } g(c) = -c \text{ and } \lim_{x \to c} g(x) = \lim_{x \to c} (-x) = -c$$
$$\therefore \lim_{x \to c} g(x) = g(c)$$

Therefore, *g* is continuous at all points *x*, such that $x < 0$

Case II:

$$\text{If } c > 0, \text{ then } g(c) = c \text{ and } \lim_{x \to c} g(x) = \lim_{x \to c} x = c$$
$$\therefore \lim_{x \to c} g(x) = g(c)$$

Therefore, *g* is continuous at all points *x*, such that $x > 0$

Case III:

$$\text{If } c = 0, \text{ then } g(c) = g(0) = 0$$

$$\lim_{x \to 0^-} g(x) = \lim_{x \to 0^-} (-x) = 0$$
$$\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} (x) = 0$$
$$\therefore \lim_{x \to 0^-} g(x) = \lim_{x \to 0^+} (x) = g(0)$$

Therefore, *g* is continuous at $x = 0$

From the above three observations, it can be concluded that g is continuous at all points.

$h(x) = \cos x$

It is evident that $h(x) = \cos x$ is defined for every real number.

Let c be a real number. Put $x = c + h$

If $x \to c$, then $h \to 0$

$h(c) = \cos c$

$$\begin{aligned}
\lim_{x\to c} h(x) &= \lim_{x\to c} \cos x \\
&= \lim_{h\to 0} \cos(c+h) \\
&= \lim_{h\to 0} [\cos c \cos h - \sin c \sin h] \\
&= \lim_{h\to 0} \cos c \cos h - \lim_{h\to 0} \sin c \sin h \\
&= \cos c \cos 0 - \sin c \sin 0 \\
&= \cos c \times 1 - \sin c \times 0 \\
&= \cos c
\end{aligned}$$

$$\therefore \lim_{x\to c} h(x) = h(c)$$

Therefore, $h(x) = \cos x$ is a continuous function.

It is known that for real valued functions g and h, such that $(g \circ h)$ is defined at c, if g is continuous at c and if f is continuous at $g(c)$, then $(f \circ g)$ is continuous at c.

Therefore, $f(x) = (goh)(x) = g(h(x)) = g(\cos x) = |\cos x|$ is a continuous function.

Question

Examine that $\sin|x|$ is a continuous function.

Answer :

Let $f(x) = \sin|x|$

This function *f* is defined for every real number and *f* can be written as the composition of two functions as,

f = *g o h*, where $g(x) = |x|$ and $h(x) = \sin x$

$\left[\because (goh)(x) = g(h(x)) = g(\sin x) = |\sin x| = f(x)\right]$

It has to be proved first that $g(x) = |x|$ and $h(x) = \sin x$ are continuous functions.

$g(x) = |x|$ can be written as

$$g(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Clearly, *g* is defined for all real numbers.

Let *c* be a real number.

Case I:

If $c < 0$, then $g(c) = -c$ and $\lim_{x \to c} g(x) = \lim_{x \to c}(-x) = -c$

$\therefore \lim_{x \to c} g(x) = g(c)$

Therefore, g is continuous at all points x, such that $x < 0$

Case II:

If $c > 0$, then $g(c) = c$ and $\lim_{x \to c} g(x) = \lim_{x \to c} x = c$

$\therefore \lim_{x \to c} g(x) = g(c)$

Therefore, g is continuous at all points x, such that $x > 0$

Case III:

If $c = 0$, then $g(c) = g(0) = 0$

$$\lim_{x \to 0^-} g(x) = \lim_{x \to 0^-}(-x) = 0$$
$$\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+}(x) = 0$$
$$\therefore \lim_{x \to 0^-} g(x) = \lim_{x \to 0^+}(x) = g(0)$$

Therefore, g is continuous at $x = 0$

From the above three observations, it can be concluded that g is continuous at all points.

$h(x) = \sin x$

It is evident that $h(x) = \sin x$ is defined for every real number.

Let c be a real number. Put $x = c + k$

If $x \longrightarrow c$, then $k \longrightarrow 0$

$h(c) = \sin c$

$$h(c) = \sin c$$

$$\begin{aligned}\lim_{x\to c} h(x) &= \lim_{x\to c} \sin x \\ &= \lim_{k\to 0} \sin(c+k) \\ &= \lim_{k\to 0}\left[\sin c\cos k + \cos c\sin k\right] \\ &= \lim_{k\to 0}(\sin c\cos k) + \lim_{h\to 0}(\cos c\sin k) \\ &= \sin c\cos 0 + \cos c\sin 0 \\ &= \sin c + 0 \\ &= \sin c\end{aligned}$$

$$\therefore \lim_{x\to c} h(x) = g(c)$$

Therefore, h is a continuous function.

It is known that for real valued functions g and h, such that $(g \circ h)$ is defined at c, if g is continuous at c and if f is continuous at $g(c)$, then $(f \circ g)$ is continuous at c.

Therefore, $f(x) = (goh)(x) = g(h(x)) = g(\sin x) = |\sin x|$ is a continuous function.

Question

Find all the points of discontinuity of f defined by $f(x)=|x|-|x+1|$

Answer :

The given function is $f(x)=|x|-|x+1|$

Thetwo functions, $g$and h, are defined as

$g(x)=|x|$ and $h(x)=|x+1|$

Then, $f=g-h$

Thecontinuity of $g$and h is examined first.

$g(x)=|x|$ can be written as

$$g(x)=\begin{cases}-x, & \text{if } x<0 \\ x, & \text{if } x\geq 0\end{cases}$$

Clearly, g is defined for all real numbers.

Let c be a real number.

Case I:

If $c < 0$, then $g(c) = -c$ and $\lim_{x\to c} g(x) = \lim_{x\to c}(-x) = -c$

$\therefore \lim_{x\to c} g(x) = g(c)$

Therefore, g is continuous at all points x, such that $x < 0$

Case II:

If $c > 0$, then $g(c) = c$ and $\lim_{x\to c} g(x) = \lim_{x\to c} x = c$

$\therefore \lim_{x\to c} g(x) = g(c)$

Therefore, g is continuous at all points x, such that $x > 0$

Case III:

If $c = 0$, then $g(c) = g(0) = 0$

$$\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-}(-x) = 0$$
$$\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+}(x) = 0$$
$$\therefore \lim_{x\to 0^-} g(x) = \lim_{x\to 0^+}(x) = g(0)$$

Therefore, g is continuous at $x = 0$

From the above three observations, it can be concluded that *g* is continuous at all points.

$h(x) = |x+1|$ can be written as

$$h(x) = \begin{cases} -(x+1), & \text{if } x < -1 \\ x+1, & \text{if } x \geq -1 \end{cases}$$

Clearly, *h* is defined for every real number.

Let *c* be a real number.

Case I:

If $c < -1$, then $h(c) = -(c+1)$ and $\lim_{x \to c} h(x) = \lim_{x \to c} [-(x+1)] = -(c+1)$

$\therefore \lim_{x \to c} h(x) = h(c)$

Therefore, *h* is continuous at all points *x*, such that *x* < - 1

Case II:

If $c > -1$, then $h(c) = c+1$ and $\lim_{x \to c} h(x) = \lim_{x \to c} (x+1) = c+1$

$\therefore \lim_{x \to c} h(x) = h(c)$

Therefore, *h* is continuous at all points *x*, such that *x*> - 1

Case III:

If $c = -1$, then $h(c) = h(-1) = -1+1 = 0$

$\lim_{x \to -1^-} h(x) = \lim_{x \to -1^-} [-(x+1)] = -(-1+1) = 0$

$\lim_{x \to -1^+} h(x) = \lim_{x \to -1^+} (x+1) = (-1+1) = 0$

$\therefore \lim_{x \to -1^-} h(x) = \lim_{h \to -1^+} h(x) = h(-1)$

Therefore, *h* is continuous at *x*= - 1

From the above three observations, it can be concluded that *h* is continuous at all points of the real line.

Question

Verify Rolle's Theorem for the function $f(x) = x^2 + 2x - 8$, $x \in [-4, 2]$

Answer :

The given function, $f(x) = x^2 + 2x - 8$, being a polynomial function, is continuous in [- 4, 2] and is differentiable in (- 4, 2).

$$f(-4) = (-4)^2 + 2\times(-4) - 8 = 16 - 8 - 8 = 0$$
$$f(2) = (2)^2 + 2\times 2 - 8 = 4 + 4 - 8 = 0$$

$\therefore$ $f(- 4) = f(2) = 0$

$\Rightarrow$ The value of $f(x)$ at - 4 and 2 coincides.

Rolle's Theorem states that there is a point $c \in (- 4, 2)$ such that $f'(c) = 0$

$$f(x) = x^2 + 2x - 8$$
$$\Rightarrow f'(x) = 2x + 2$$
$$\therefore f'(c) = 0$$
$$\Rightarrow 2c + 2 = 0$$

$$\Rightarrow 2c + 2 = 0$$
$$\Rightarrow c = -1, \text{ where } c = -1 \in (-4, 2)$$

Hence, Rolle's Theorem is verified for the given function.

Question

Examine if Rolle's Theorem is applicable to any of the following functions. Can you say some thing about the converse of Rolle's Theorem from these examples?

(i) $f(x)=[x]$ for $x\in[5,\ 9]$

(ii) $f(x)=[x]$ for $x\in[-2,\ 2]$

(iii) $f(x)=x^2-1$ for $x\in[1,\ 2]$

Answer :

By Rolle's Theorem, for a function $f:[a,\ b]\to \mathbf{R}$, if

(a) *f* is continuous on [*a*, *b*]

(b) *f* is differentiable on (*a*, *b*)

(c) *f*(*a*) = *f*(*b*)

then, there exists some $c \in (a,\ b)$ such that $f'(c)=0$

Therefore, Rolle's Theorem is not applicable to those functions that do not satisfy any of the three conditions of the hypothesis.

(i) $f(x)=[x]$ for $x\in[5,\ 9]$

It is evident that the given function *f*(*x*) is not continuous at every integral point.

In particular, *f*(*x*) is not continuous at *x* = 5 and *x* = 9

⇒ *f*(*x*) is not continuous in [5, 9].

Also, $f(5)=[5]=5$ and $f(9)=[9]=9$

$\therefore f(5)\neq f(9)$

The differentiability of *f* in (5, 9) is checked as follows.

Let *n* be an integer such that $n \in (5, 9)$.

The left hand limit of f at $x=n$ is,

$$\lim_{h\to 0^-}\frac{f(n+h)-f(n)}{h}=\lim_{h\to 0^-}\frac{[n+h]-[n]}{h}=\lim_{h\to 0^-}\frac{n-1-n}{h}=\lim_{h\to 0^-}\frac{-1}{h}=\infty$$

The right hand limit of f at $x=n$ is,

$$\lim_{h\to 0^+}\frac{f(n+h)-f(n)}{h}=\lim_{h\to 0^+}\frac{[n+h]-[n]}{h}=\lim_{h\to 0^+}\frac{n-n}{h}=\lim_{h\to 0^+}0=0$$

Since the left and right hand limits of *f* at *x* = *n* are not equal, *f* is not differentiable at *x* = *n*

$\therefore f$ is not differentiable in (5, 9).

It is observed that f does not satisfy all the conditions of the hypothesis of Rolle's Theorem.

Hence, Rolle's Theorem is not applicable for $f(x)=[x]$ for $x \in [5,\ 9]$.

(ii) $f(x)=[x]$ for $x \in [-2,\ 2]$

It is evident that the given function $f(x)$ is not continuous at every integral point.

In particular, $f(x)$ is not continuous at $x = -2$ and $x = 2$

$\Rightarrow f(x)$ is not continuous in [- 2, 2].

Also, $f(-2)=[-2]=-2$ and $f(2)=[2]=2$

$\therefore f(-2) \neq f(2)$

The differentiability of f in (- 2, 2) is checked as follows.

Let n be an integer such that $n \in$ (- 2, 2).

The left hand limit of f at $x = n$ is,

$$\lim_{h\to 0^-} \frac{f(n+h)-f(n)}{h} = \lim_{h\to 0^-} \frac{[n+h]-[n]}{h} = \lim_{h\to 0^-} \frac{n-1-n}{h} = \lim_{h\to 0^-} \frac{-1}{h} = \infty$$

The right hand limit of f at $x = n$ is,

$$\lim_{h\to 0^+} \frac{f(n+h)-f(n)}{h} = \lim_{h\to 0^+} \frac{[n+h]-[n]}{h} = \lim_{h\to 0^+} \frac{n-n}{h} = \lim_{h\to 0^+} 0 = 0$$

Since the left and right hand limits of f at $x = n$ are not equal, f is not differentiable at $x = n$

$\therefore f$ is not differentiable in (- 2, 2).

It is observed that f does not satisfy all the conditions of the hypothesis of Rolle's Theorem.

Hence, Rolle's Theorem is not applicable for $f(x)=[x]$ for $x \in [-2,\ 2]$.

(iii) $f(x)=x^2-1$ for $x \in [1,\ 2]$

It is evident that f, being a polynomial function, is continuous in [1, 2] and is differentiable in (1, 2).

$f(1)=(1)^2-1=0$

$f(2)=(2)^2-1=3$

$\therefore f(1) \neq f(2)$

It is observed that f does not satisfy a condition of the hypothesis of Rolle's Theorem.

Hence, Rolle's Theorem is not applicable for $f(x)=x^2-1$ for $x \in [1,\ 2]$.

Question

If $f:[-5,5]\to \mathbf{R}$ is a differentiable function and if $f'(x)$ does not vanish anywhere, then prove that $f(-5)\neq f(5)$.

Answer :

It is given that $f:[-5,5]\to \mathbf{R}$ is a differentiable function.

Since every differentiable function is a continuous function, we obtain

(a) f is continuous on [- 5, 5].

(b) f is differentiable on (- 5, 5).

Therefore, by the Mean Value Theorem, there exists $c \in$ (- 5, 5) such that

$$f'(c)=\frac{f(5)-f(-5)}{5-(-5)}$$
$$\Rightarrow 10f'(c)=f(5)-f(-5)$$

It is also given that $f'(x)$ does not vanish anywhere.

$$\therefore f'(c)\neq 0$$
$$\Rightarrow 10f'(c)\neq 0$$

$$\Rightarrow f(5)-f(-5)\neq 0$$
$$\Rightarrow f(5)\neq f(-5)$$

Hence, proved.

Question

Verify Mean Value Theorem, if $f(x)=x^2-4x-3$ in the interval $[a, b]$, where $a=1$ and $b=4$.

Answer :

The given function is $f(x)=x^2-4x-3$

f, being a polynomial function, is continuous in [1, 4] and is differentiable in (1, 4) whose derivative is $2x - 4$.

$$f(1)=1^2-4\times1-3=-6,\ f(4)=4^2-4\times4-3=-3$$
$$\therefore \frac{f(b)-f(a)}{b-a}=\frac{f(4)-f(1)}{4-1}=\frac{-3-(-6)}{3}=\frac{3}{3}=1$$

Mean Value Theorem states that there is a point $c\in(1, 4)$ such that $f'(c)=1$

$$f'(c)=1$$
$$\Rightarrow 2c-4=1$$
$$\Rightarrow c=\frac{5}{2},\text{ where } c=\frac{5}{2}\in(1,\ 4)$$

Hence, Mean Value Theorem is verified for the given function.

Question

Verify Mean Value Theorem, if $f(x)=x^3-5x^2-3x$ in the interval [*a*, *b*], where *a* = 1 and *b* = 3. Find all $c\in(1,3)$ for which $f'(c)=0$

Answer :

The given function *f* is $f(x)=x^3-5x^2-3x$

f, being a polynomial function, is continuous in [1, 3] and is differentiable in (1, 3) whose derivative is $3x^2 - 10x - 3$.

$$f(1)=1^3-5\times1^2-3\times1=-7,\ \ f(3)=3^3-5\times3^2-3\times3=-27$$
$$\therefore \frac{f(b)-f(a)}{b-a}=\frac{f(3)-f(1)}{3-1}=\frac{-27-(-7)}{3-1}=-10$$

Mean Value Theorem states that there exist a point $c\in(1, 3)$ such that $f'(c)=-10$

$$f'(c)=-10$$
$$\Rightarrow 3c^2-10c-3=10$$
$$\Rightarrow 3c^2-10c+7=0$$
$$\Rightarrow 3c^2-3c-7c+7=0$$
$$\Rightarrow 3c(c-1)-7(c-1)=0$$

$\Rightarrow (c-1)(3c-7)=0$

$\Rightarrow c=1,\ \frac{7}{3}$, where $c=\frac{7}{3}\in(1,\ 3)$

Hence, Mean Value Theorem is verified for the given function and $c=\frac{7}{3}\in(1,\ 3)$ is the only point for which $f'(c)=0$

Question

Examine the applicability of Mean Value Theorem for all three functions given in the above exercise 2.

Answer :

Mean Value Theorem states that for a function $f:[a,\ b]\to\mathbf{R}$, if

(a) *f* is continuous on [*a*, *b*]

(b) *f* is differentiable on (*a*, *b*)

then, there exists some $c \in (a, b)$ such that $f'(c)=\frac{f(b)-f(a)}{b-a}$

Therefore, Mean Value Theorem is not applicable to those functions that do not satisfy any of the two conditions of the hypothesis.

(i) $f(x)=[x]$ for $x\in[5,\ 9]$

It is evident that the given function *f* (*x*) is not continuous at every integral point.

In particular, *f*(*x*) is not continuous at $x = 5$ and $x = 9$

$\Rightarrow f(x)$ is not continuous in [5, 9].

The differentiability of f in (5, 9) is checked as follows.

Let n be an integer such that $n \in (5, 9)$.

The left hand limit of f at $x = n$ is,

$$\lim_{h\to 0^-} \frac{f(n+h)-f(n)}{h} = \lim_{h\to 0^-} \frac{[n+h]-[n]}{h} = \lim_{h\to 0^-} \frac{n-1-n}{h} = \lim_{h\to 0^-} \frac{-1}{h} = \infty$$

The right hand limit of f at $x = n$ is,

$$\lim_{h\to 0^+} \frac{f(n+h)-f(n)}{h} = \lim_{h\to 0^+} \frac{[n+h]-[n]}{h} = \lim_{h\to 0^+} \frac{n-n}{h} = \lim_{h\to 0^+} 0 = 0$$

Since the left and right hand limits of f at $x = n$ are not equal, f is not differentiable at $x = n$

$\therefore f$ is not differentiable in (5, 9).

It is observed that f does not satisfy all the conditions of the hypothesis of Mean Value Theorem.

Hence, Mean Value Theorem is not applicable for $f(x) = [x]$ for $x \in [5, 9]$.

(ii) $f(x) = [x]$ for $x \in [-2, 2]$

It is evident that the given function $f(x)$ is not continuous at every integral point.

In particular, $f(x)$ is not continuous at $x = -2$ and $x = 2$

$\Rightarrow f(x)$ is not continuous in [- 2, 2].

The differentiability of f in (- 2, 2) is checked as follows.

Let n be an integer such that $n \in$ (- 2, 2).

The left hand limit of f at $x = n$ is,

$$\lim_{h\to 0^-} \frac{f(n+h)-f(n)}{h} = \lim_{h\to 0^-} \frac{[n+h]-[n]}{h} = \lim_{h\to 0^-} \frac{n-1-n}{h} = \lim_{h\to 0^-} \frac{-1}{h} = \infty$$

The right hand limit of f at $x = n$ is,

$$\lim_{h\to 0^+} \frac{f(n+h)-f(n)}{h} = \lim_{h\to 0^+} \frac{[n+h]-[n]}{h} = \lim_{h\to 0^+} \frac{n-n}{h} = \lim_{h\to 0^+} 0 = 0$$

Since the left and right hand limits of f at $x = n$ are not equal, f is not differentiable at $x = n$

$\therefore f$ is not differentiable in (- 2, 2).

It is observed that f does not satisfy all the conditions of the hypothesis of Mean Value Theorem.

Hence, Mean Value Theorem is not applicable for $f(x) = [x]$ for $x \in [-2, 2]$.

(iii) $f(x) = x^2 - 1$ for $x \in [1, 2]$

It is evident that f, being a polynomial function, is continuous in [1, 2] and is differentiable in (1, 2).

It is observed that f satisfies all the conditions of the hypothesis of Mean Value Theorem.

Hence, Mean Value Theorem is applicable for $f(x) = x^2 - 1$ for $x \in [1, 2]$.

It can be proved as follows.

$$f(1) = 1^2 - 1 = 0,\ f(2) = 2^2 - 1 = 3$$

$$\therefore \frac{f(b) - f(a)}{b - a} = \frac{f(2) - f(1)}{2 - 1} = \frac{3 - 0}{1} = 3$$

$$f'(x) = 2x$$

$$\therefore f'(c) = 3$$

$$\Rightarrow 2c = 3$$

$$\Rightarrow c = \frac{3}{2} = 1.5, \text{ where } 1.5 \in [1, 2]$$

-

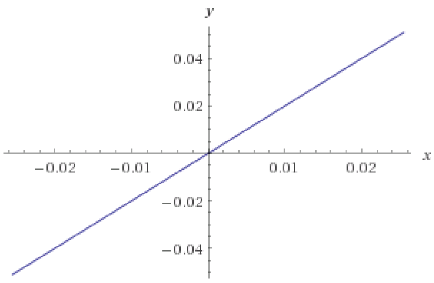
Review of Graphs

The student must be very good at Graphs of Various kinds of functions; to do well in Continuity, Differentiability, Area, Volume problems. Some limit problems also require concepts of Functions and Graphs. The graphs will not be given in the Questions. In case of Area problems, the student has to draw the graphs quickly, largely to scale; get the intersection points, and then plan for a piece-wise strategy to integrate and find the area.

Let us review the various graphs

y = mx will be a straight line passing through the origin. Positive m will make the line move upwards as we move in positive x i.e. towards right.

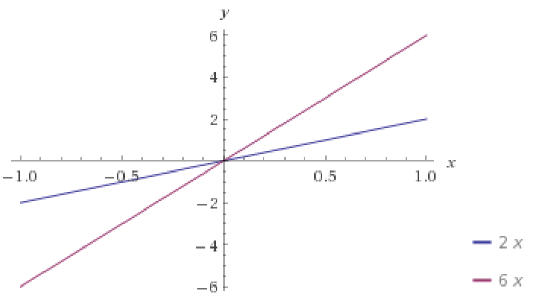
plot	$y = 2x$



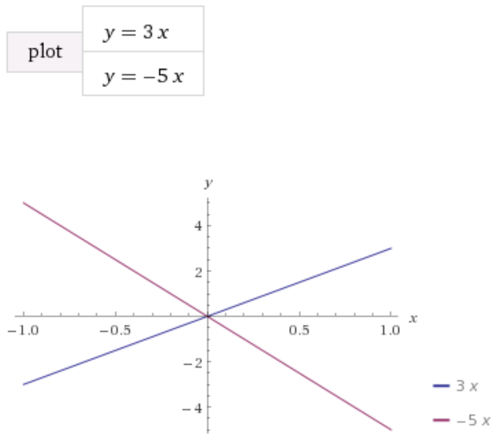
This is graph of y = 2 x   Don't get foxed by the angle being almost 45^0 The scales in y-axis and x-axis are not same.

If we compare two graphs then it becomes more clear.

plot	$y = 2x$
	$y = 6x$



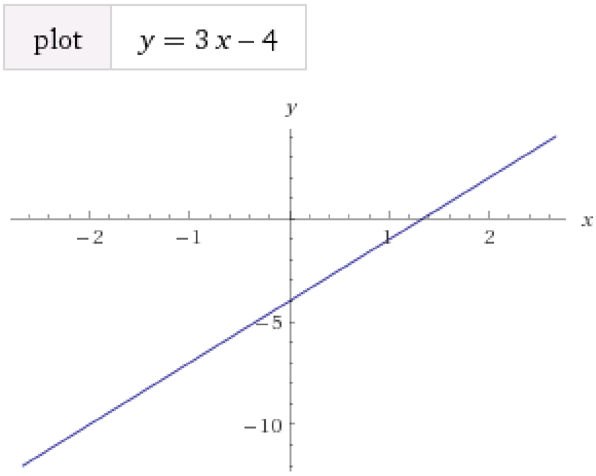
In this figure also scales of x-axis and y-axis are not same. But y = 6x has to be steeper than y = 2x



This is y = 3x and y = - 5 x graphs. For m = -5 the line moves down

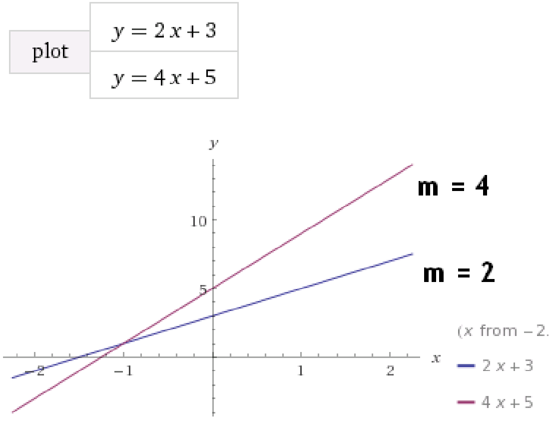
For y = m x + c the c becomes the intercept in the y axis

So y = 3x - 4 will look like



If c is a positive number then the intercept in y-axis will be on upper (positive) side.

Graphs of y = 2x + 3   and y = 4x + 5 will be



Again scales in x-axis and y-axis are different. But point made. See how the graphs pass through 3 and 5 respectively.

-

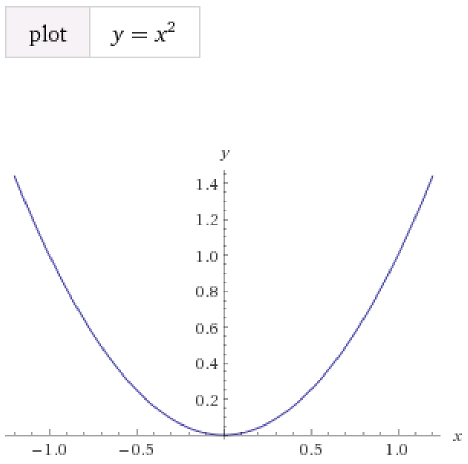
Nature of Curves, Types of Graphs, Shapes are explained / discussed at

https://archive.org/details/AreaDefiniteIntegralNatureOfCurvesTypesOfGraphsShapesDiscussions

-

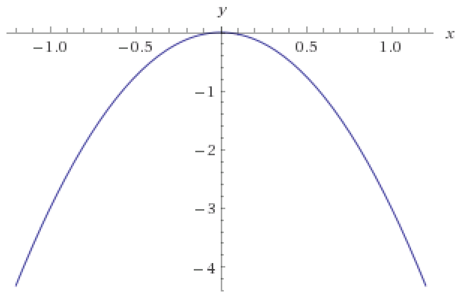
Now let us see graphs of Quadratic functions

Graph of $y = x^2$ will be

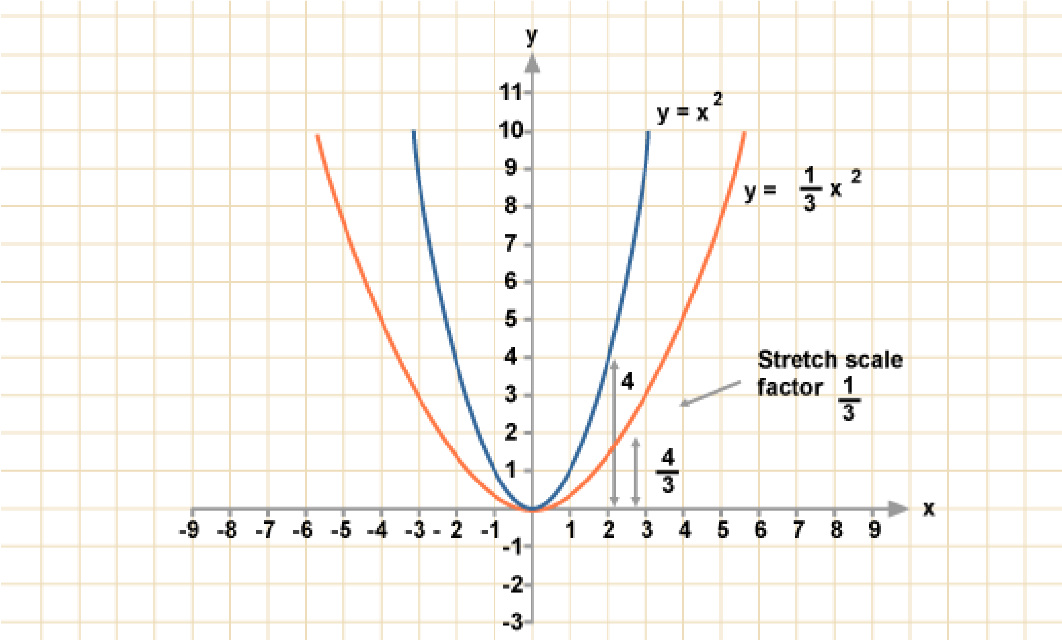


In contrast graph of $y = -3x^2$ will be downwards

plot	$y = -3x^2$

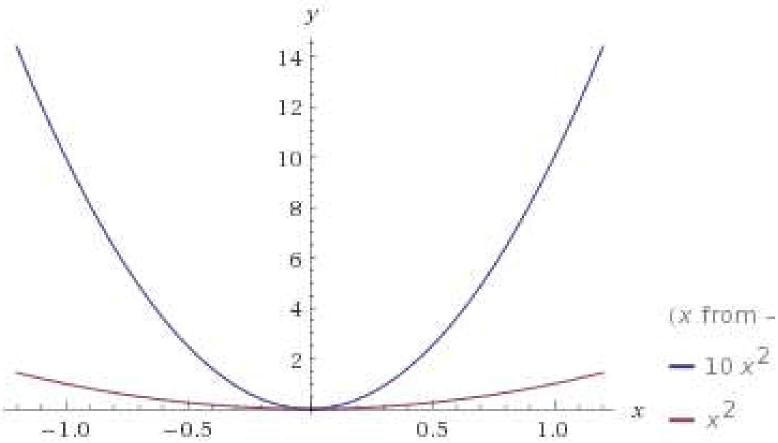


Graph of y = (1/3) x^2 will be flatter compared to y = x^2

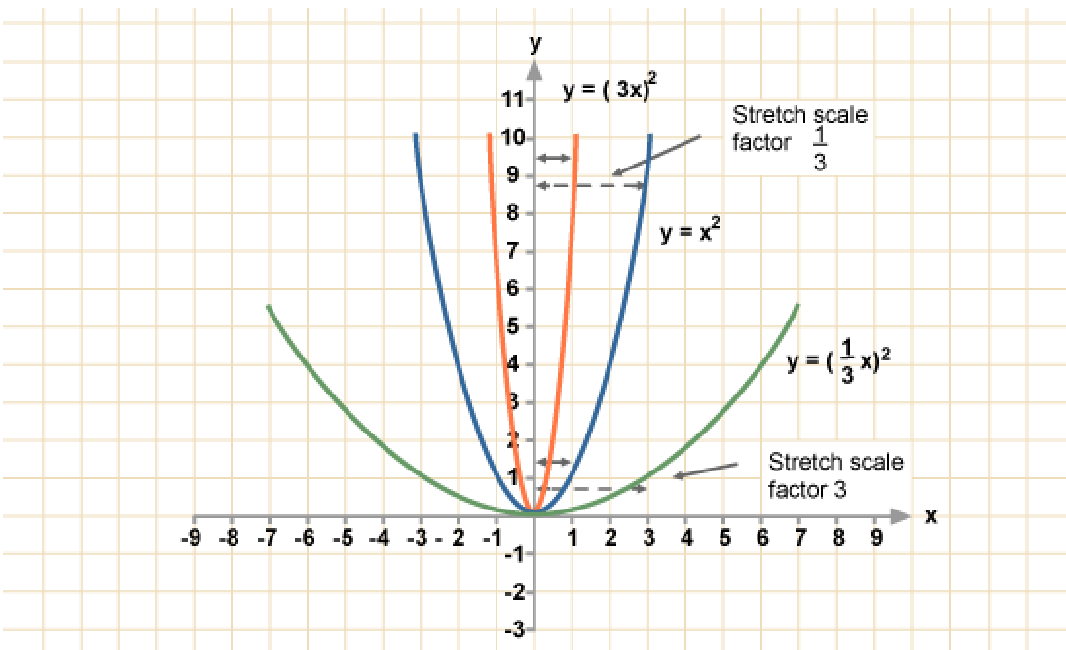


Similarly graph of y = $10x^2$ will be narrow and steeper compared to y = x^2

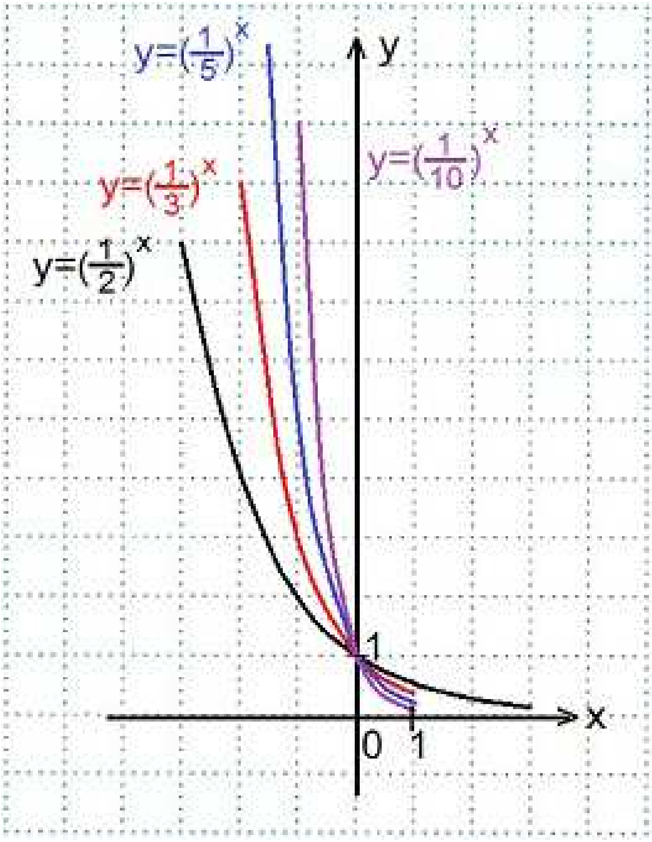
plot	$y = 10\,x^2$
	$y = x^2$



So see comparisons in a single image

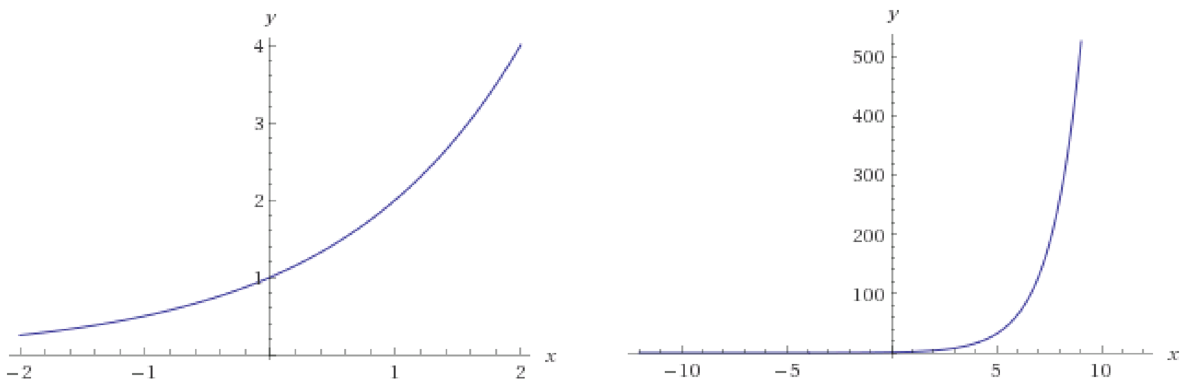


Similar things happen with power functions as well. Below we see fraction raised to power x



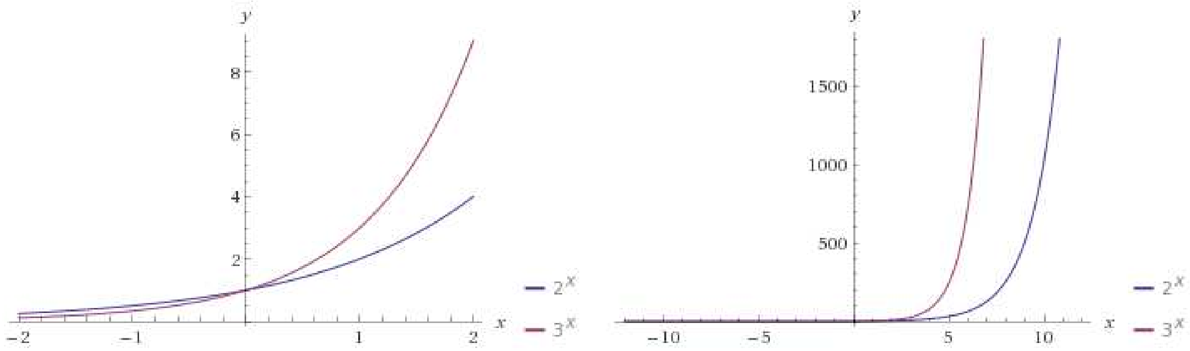
Let us see the graph of $y = 2^x$

plot	$y = 2^x$



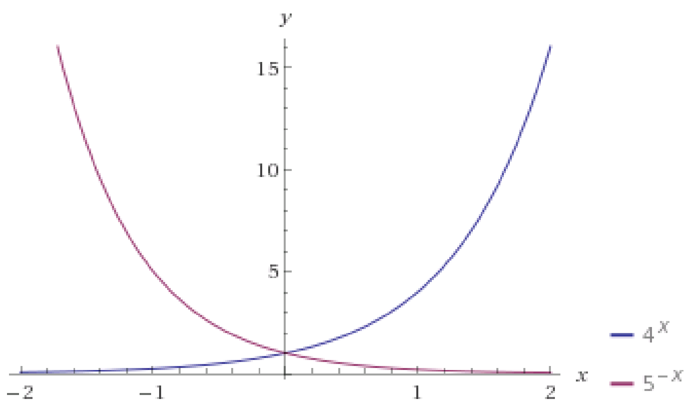
The graph of y = 3^x will be steeper and is understood easily by comparison

plot	$y = 2^x$
	$y = 3^x$

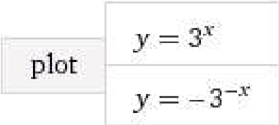


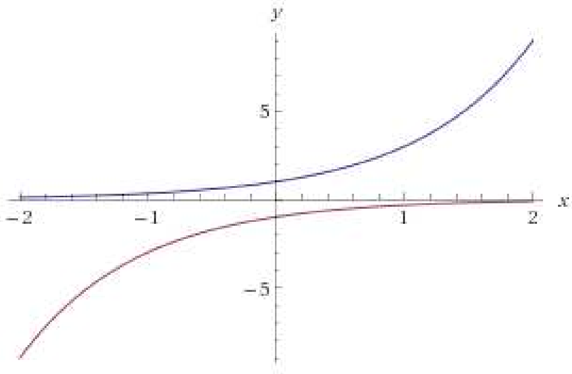
Now let us compare Integer to the power x and fraction to the power x

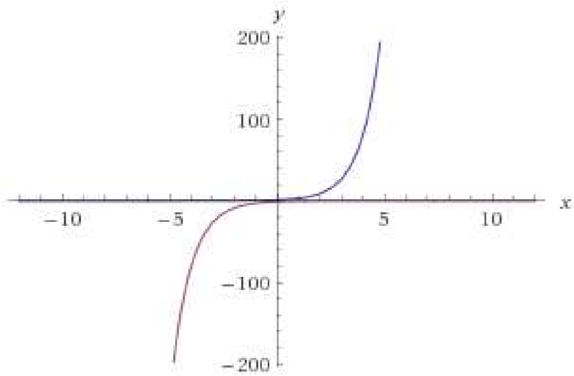
plot	$y = 4^x$
	$y = \left(\frac{1}{5}\right)^x$



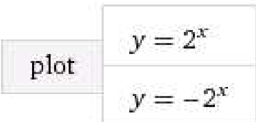
What about comparing y = 3^x and y = -3^{-x}

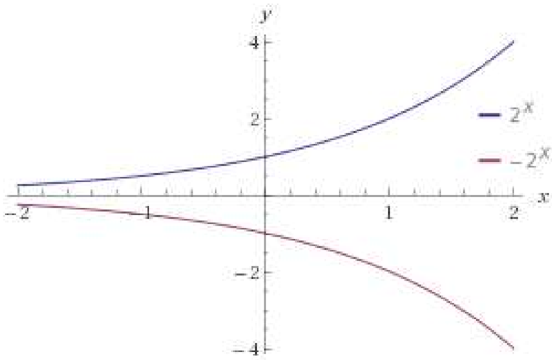


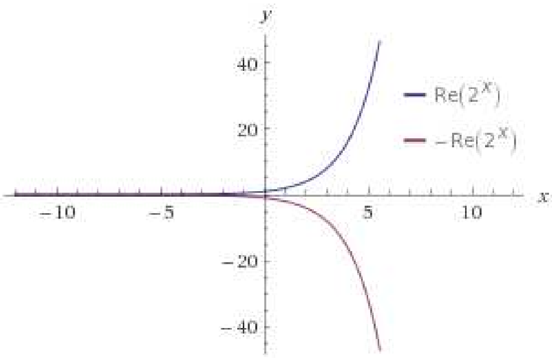




Spoon Feeding comparison of y = 2^x and y = -2^x

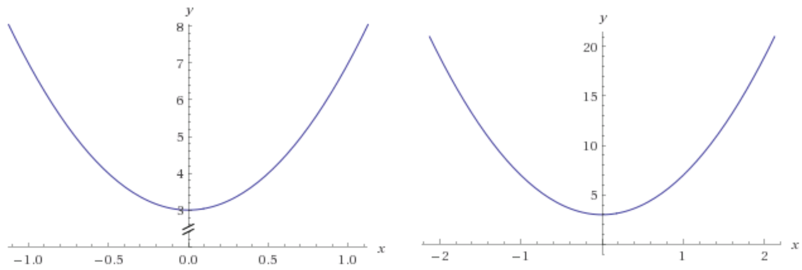






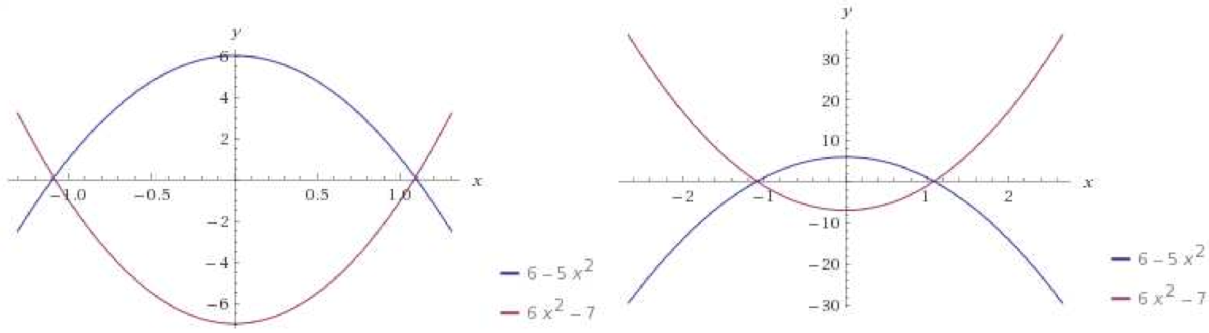
Graph of y = $4x^2 + 3$ will be 3 units above x-axis. So will pass through (0, 3) The parabola will look similar to y = x^2

plot $y = 4x^2 + 3$



Let us learn more with graphs of y = $-5x^2 + 6$ and y = $6x^2 - 7$

plot $y = -5x^2 + 6$
$y = 6x^2 - 7$



Don't quickly assume that the graphs are intersecting on x axis. The roots are very close.

$5x^2 = 6 \Rightarrow x = \pm\sqrt{6/5} = \pm 1.095$

While $6x^2 = 7 \Rightarrow x = \pm\sqrt{7/6} = \pm 1.0801$

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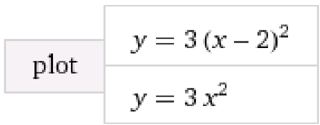
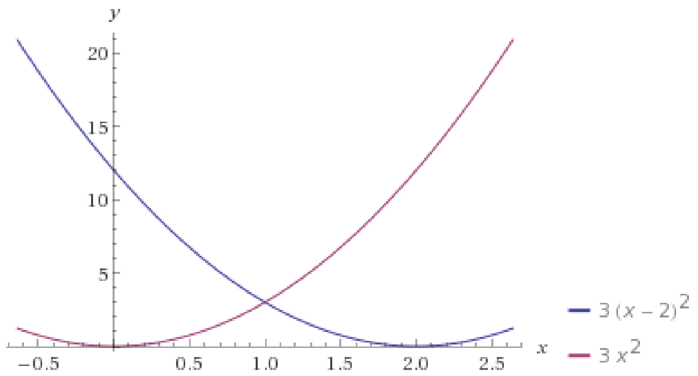
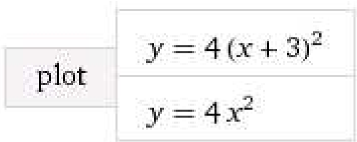
Concept of Shifting of graphs

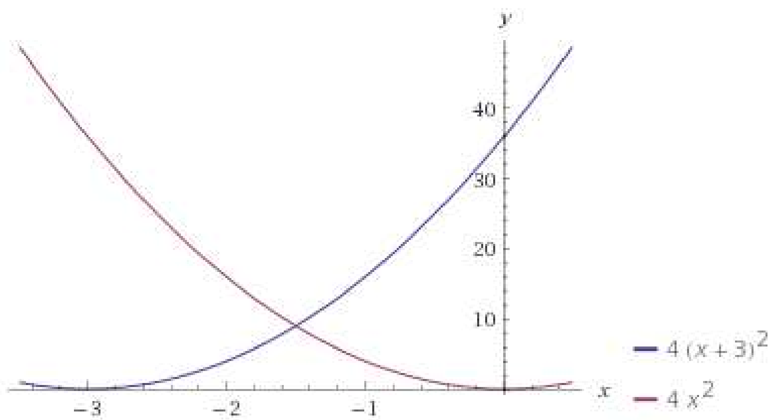
The graph of $y = 3(x - 2)^2$ will be same as $y = 3x^2$ while shifted by 2 units towards right





Similarly graph of $y = 4(x + 3)^2$ will be shifted by 3 units on left compared to $y = 4x^2$ which is through the origin





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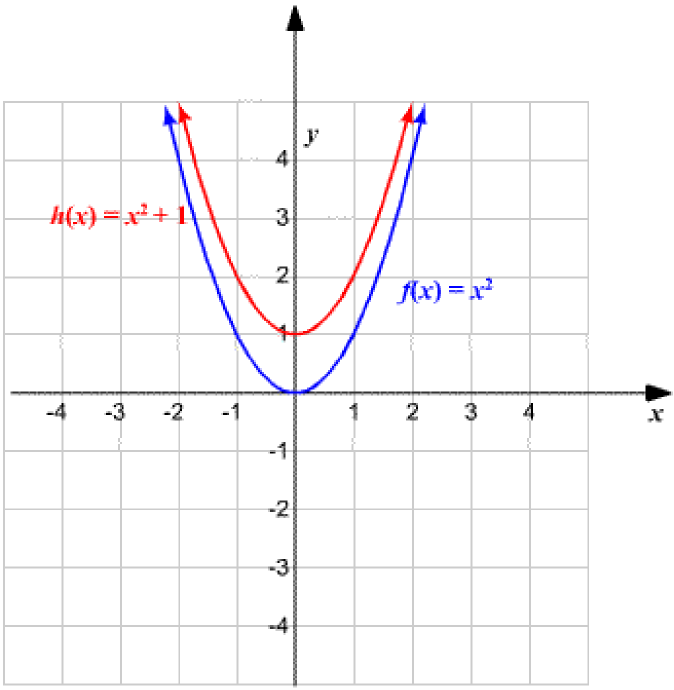
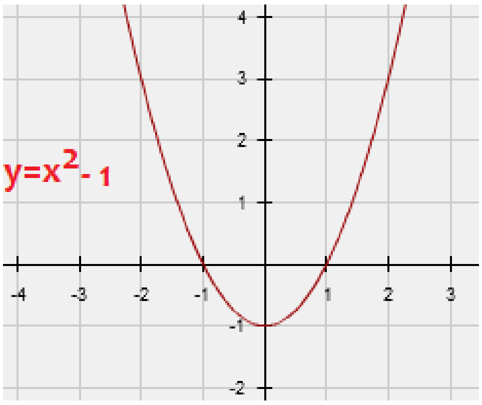
IIT-JEE 2005 Shifting a Parabola and then finding the area is discussed / explained at

https://archive.org/details/AreaDefiniteIntegralIITJEE2005ShiftingParabolasLeftOrRight



In the above image see how the upper graph is shifted up by 1 due to +1

In the image below the graph is shifted down by -1



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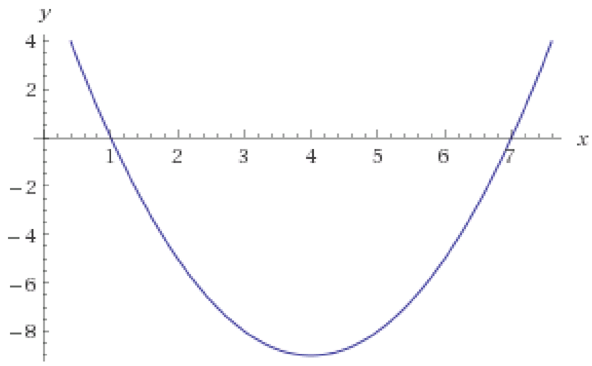
The parabola that passes through (1,0) and (7,0) will be (x - 1)(x - 7)

In simple words the Quadratic expression that has roots 1 and 7 is a parabola through 1 and 7

So graph of y = (x - 1)(x - 7) = x^2 - 8x + 7 is

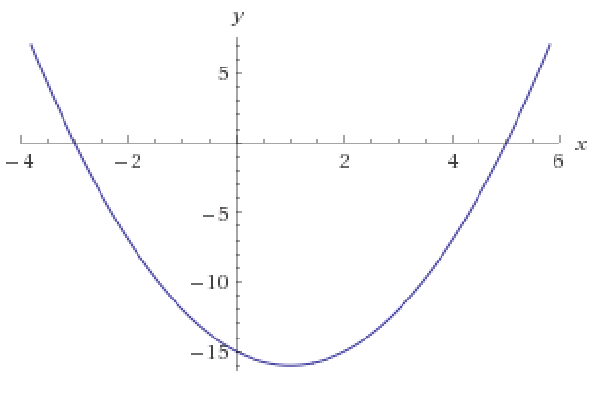
plot	$y = x^2 - 8x + 7$



If a Quadratic expression has roots -3, 5 then it will be a parabola passing through -3 and 5

So graph of y = (x + 3) (x - 5) = x^2 -2x - 15 is

plot	$y = x^2 - 2x - 15$

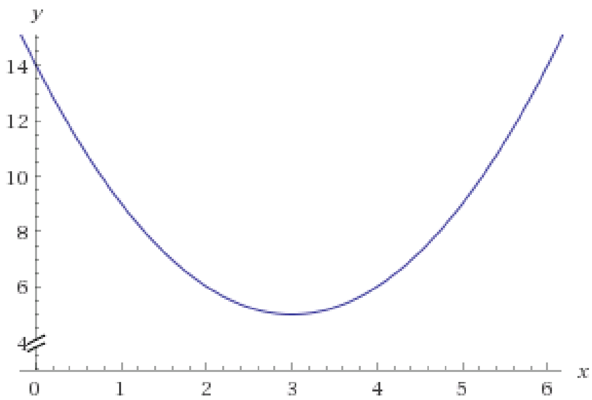


If the Discriminant D < 0 i.e. $b^2 < 4ac$ then the whole parabola is above x-axis signifying imaginary roots. As the parabola does not intersect the x-axis at all. For a > 0

If a is negative then the parabola will be downwards

So graph of $y = (x - 3)^2 + 5$ will be

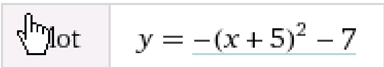
plot	$y = (x-3)^2 + 5$

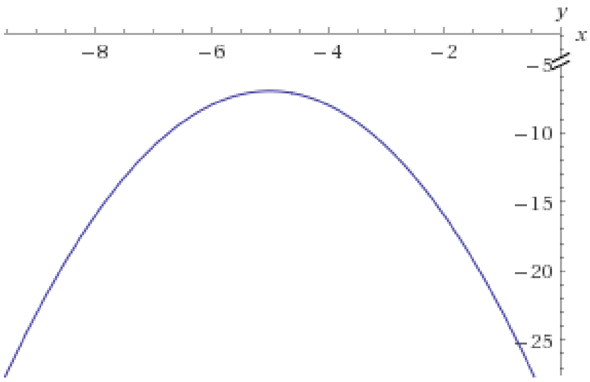


Meaning minima will be at x = 3 so x^2 graph shifted right by 3 and added 5 so moved up by 5 units

So we can easily guess the graph of $y = - (x + 5)^2 - 7$ …..

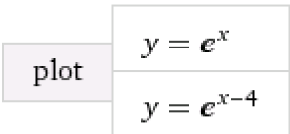
It will be shifted left by 5 units. So maxima will be at x = -5 and 7 units below x axis

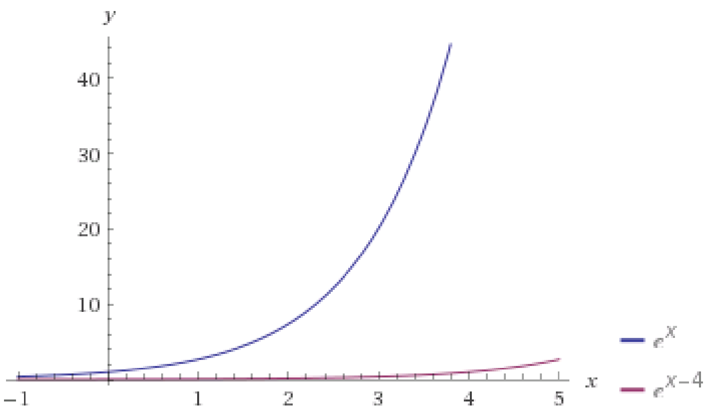




The parabola is downwards because coeff of x^2 is -ve

Don't use the idea of shift blindly ! The graph of y = e^{x-4} is not shifted by 4 units that of y = e^x



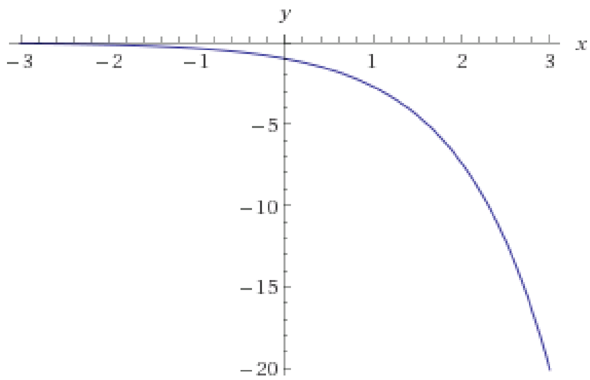


This is because $e^{(x-4)} = e^x / e^4$ means just divided by a value

Concept of Reflections

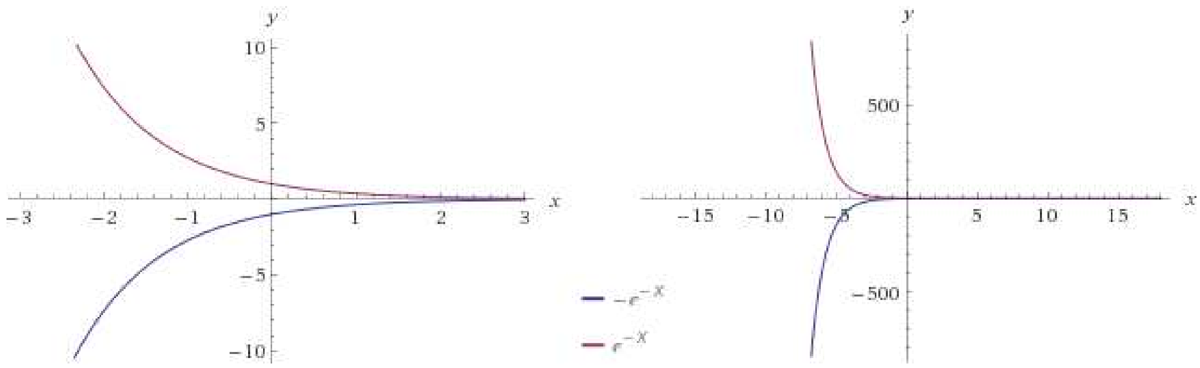
Guess the graph of y = - e^x

plot	$y = -e^x$



What about graph of y = $e^{(-x)}$ and y = - $e^{(-x)}$

plot	$y = -e^{-x}$
	$y = e^{-x}$



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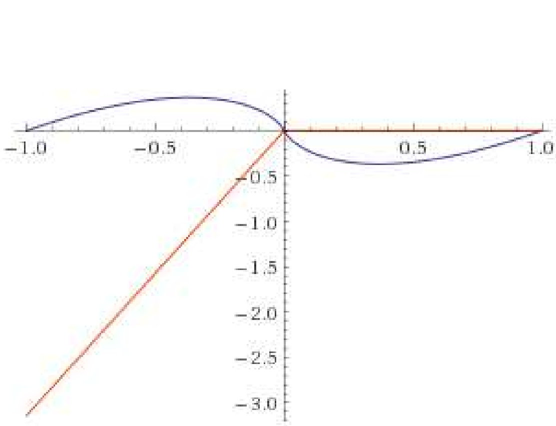
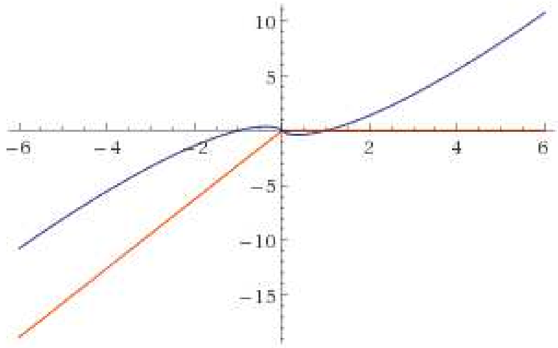
Graph of y = x ln x (Ignore the Imaginary part graph)

plot	$y = x \log(x)$

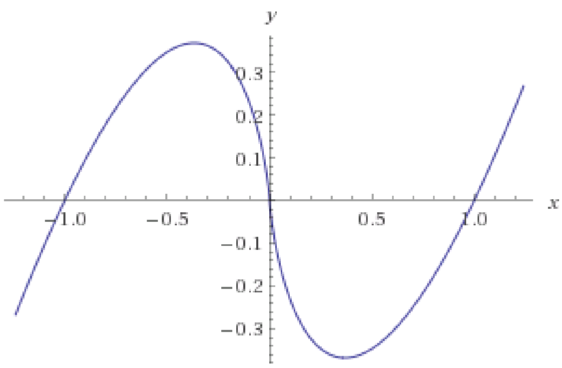


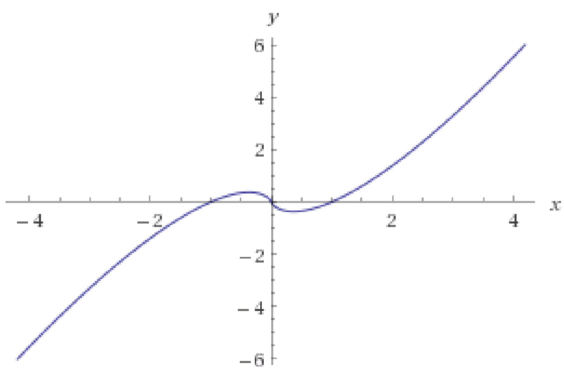


-

Graph of y = x ln | x |

plot	$y = x \log(\|x\|)$





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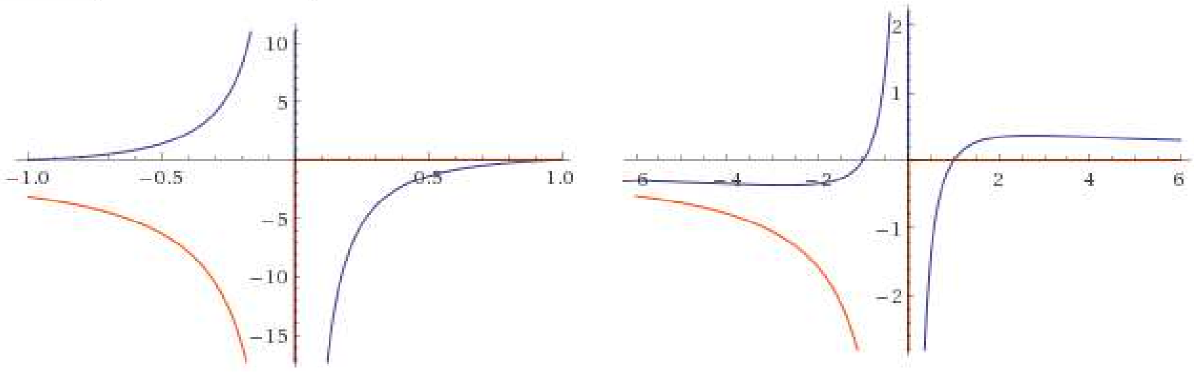
How will the graph of y = (ln x)/x look like ? (Ignore the Imaginary part)

plot	$y = \frac{1}{x}\log(x)$



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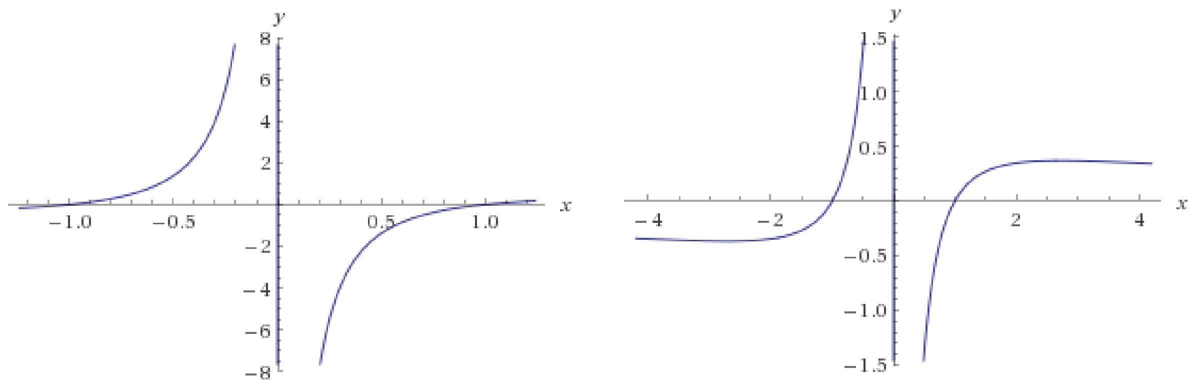
What about graph of y = (ln | x |)/x

plot	$y = \frac{1}{x}\log(\lvert x\rvert)$



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IIT-JEE 1990 problem and Solution on Area, Tricky graph of x Ln x is explained / Discussed at

https://archive.org/details/AreaDefiniteIntegralIITJEE1990TrickyGraphsOfXLnXAndLnXByX

IIT JEE 1984, 1992 Problems and Solutions as being discussed in the class. Explains various kinds of graphs at https://archive.org/details/AreaDefiniteIntegralIITJEE19841992TypesOfGraphs

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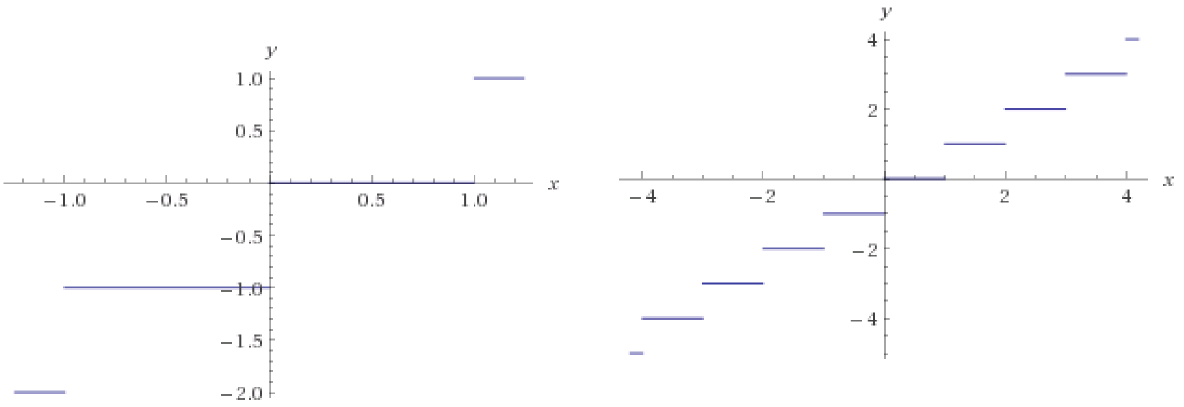
Graph of Floor x , i.e. greatest integer function x, y = [x]

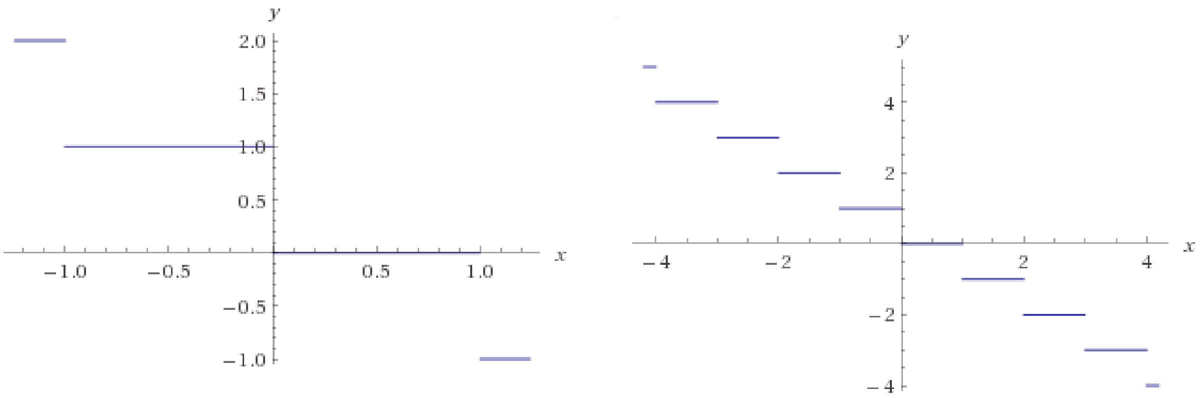
plot	$y = \lfloor x \rfloor$



Recall [-3.2] is -4 the integer less than -3.2 while [-3.99] is also -4

What about graph of y = - [x] (i.e. negative of Floor function)

plot	$y = -\lfloor x \rfloor$



Best way to learn is to " think " and try to plot it yourself, in rough.

There are many theorems related to " Floor or Greatest Integer functions ". Two theorems related to Floor function are discussed while solving a complicated Limit problem

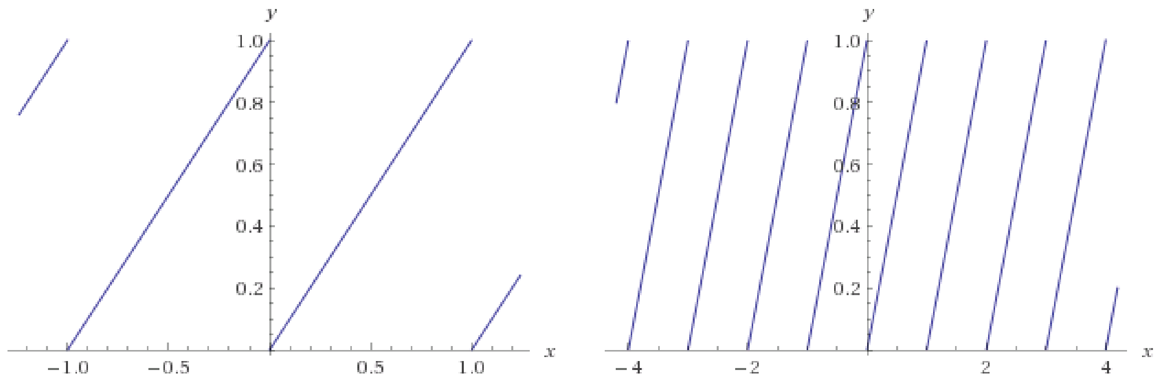
https://archive.org/details/VeryImportantTwoFloorTheoremsGreatestIntegerFunctionExplanationAndExample

-

Fraction x can be defined as x - [x] so graph of y = { x } will be

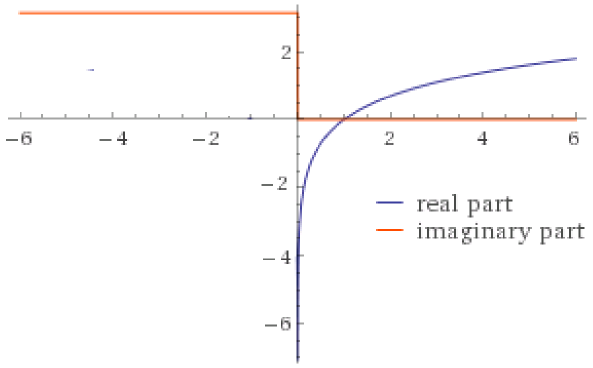
plot	$y = x - \lfloor x \rfloor$

{ 2.3 } = 0.3, { 2.4 } = 0.4, { 4.5 } = 0.5, { 4.6 } = 0.6

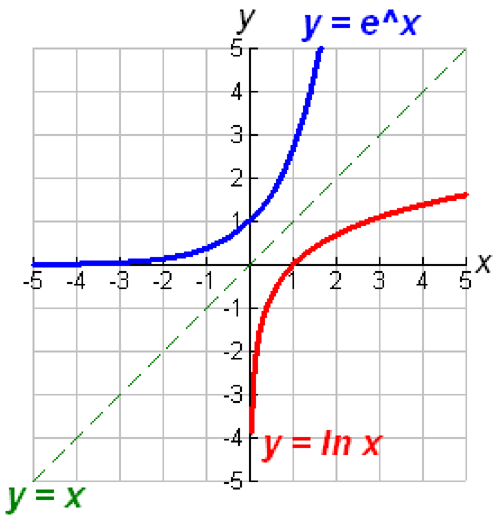


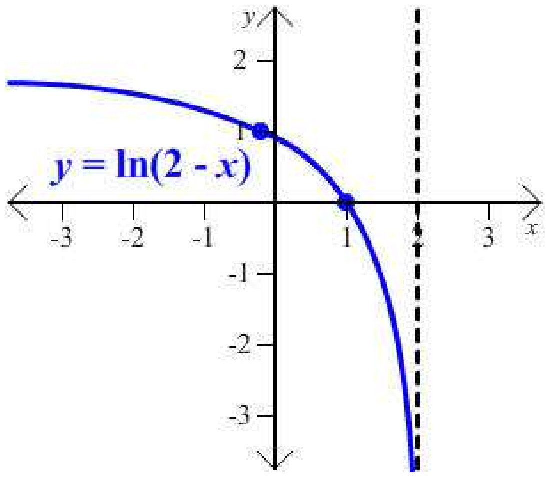
There are infinite number of discontinuities.

Graph of y = ln (x) Note : Log of negative number is imaginary as discussed in the complex number chapter.



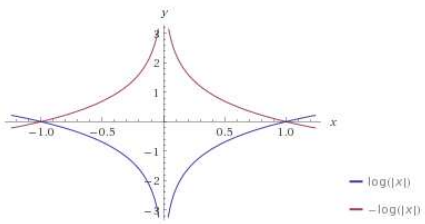
Ignore the graph of the imaginary part

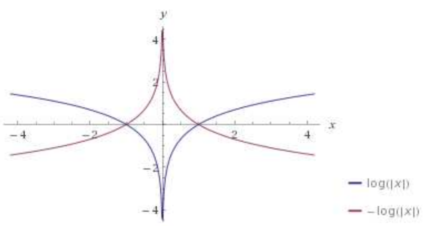


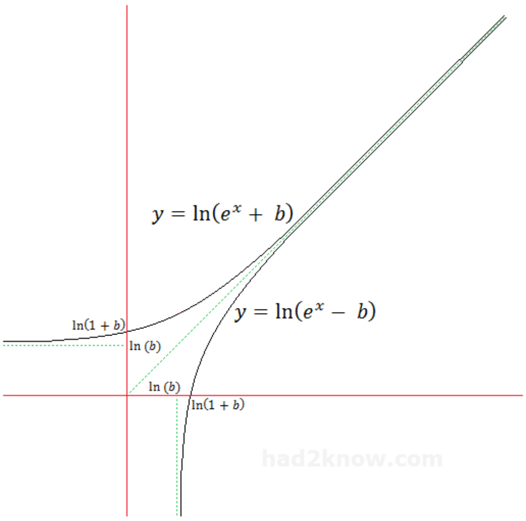


Graph of y = ln | x | and y = - ln | x |

plot	$y = \log(\lvert x \rvert)$
	$y = -\log(\lvert x \rvert)$

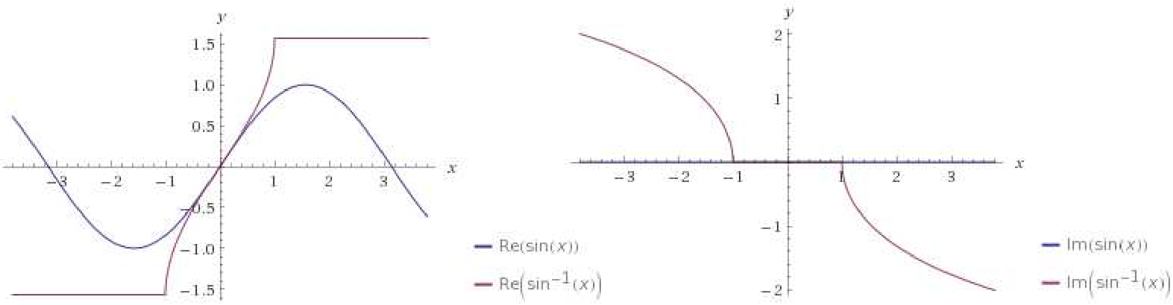






Graph of y = Sin x vs y = Sin^{-1} x

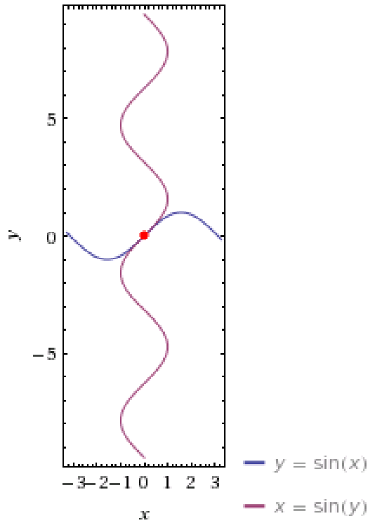
plot	$y = \sin(x)$
	$y = \sin^{-1}(x)$



Not sure if the above graph communicates well. Imaginary part of the graph to be ignored / avoided as of this discussion.

y = Sin^{-1} x means x = Sin y   The graph of which is drawn much easier.

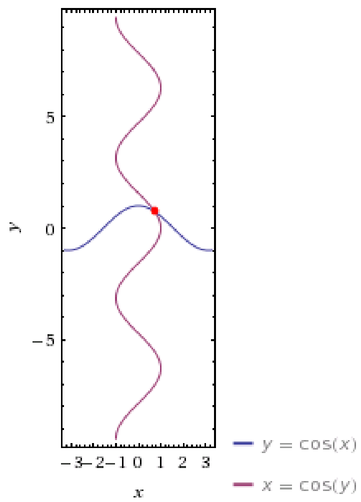
plot	$y = \sin(x)$
	$x = \sin(y)$



I am sure this is much better

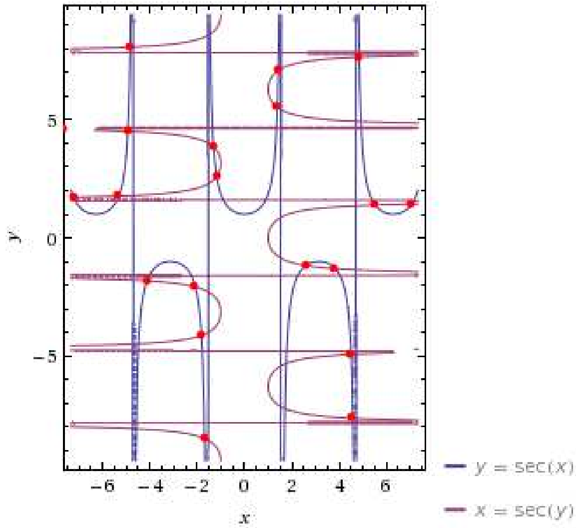
Graph of y = Cos x vs $y = \text{Cos}^{-1} x$

plot	$y = \cos(x)$
	$x = \cos(y)$

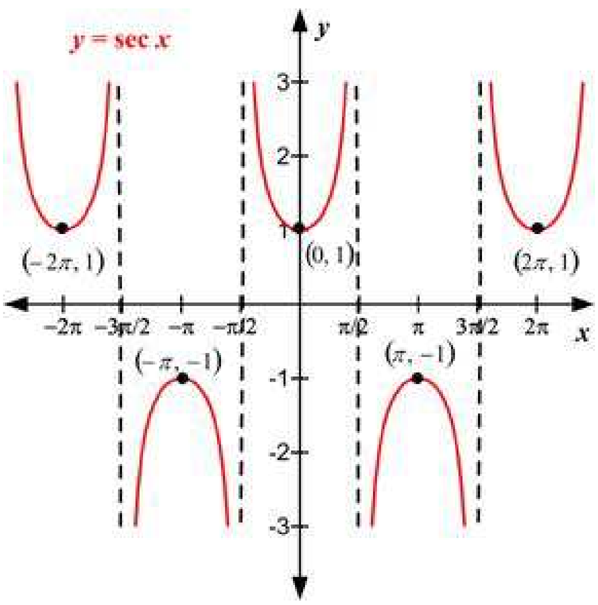


Graph of y = Sec x vs y = Sec^{-1} x

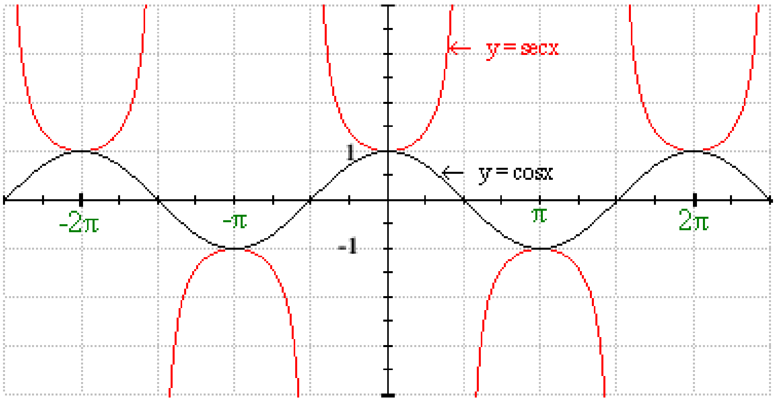
plot	$y = \sec(x)$
	$x = \sec(y)$



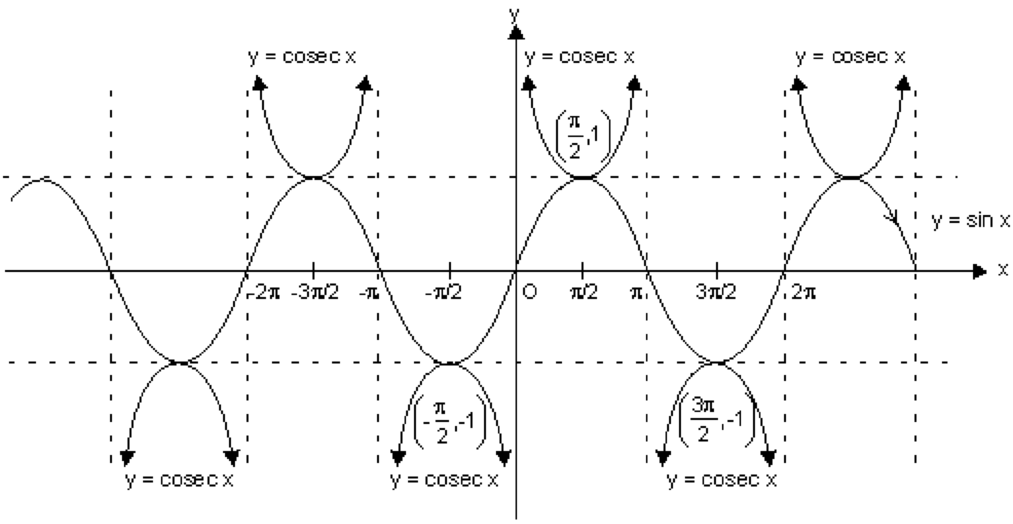
I guess we should see these graphs individually as these graphs are not commonly given in other text books



Actually Cos x can be drawn in the gap to fit-in well

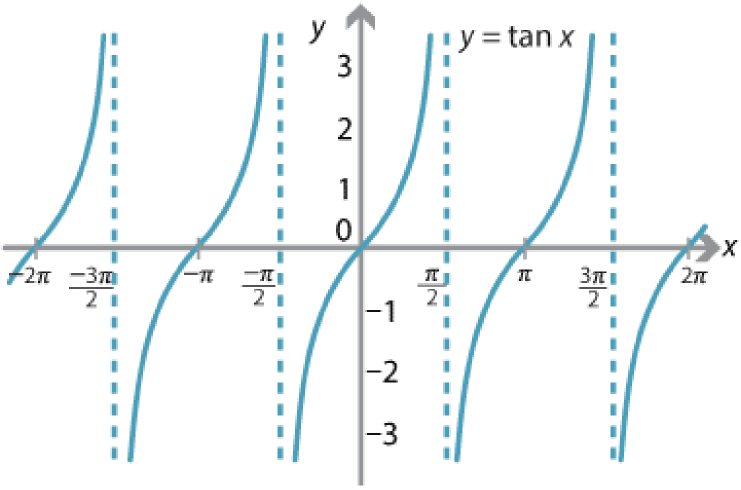


Graph of y = Cosec x

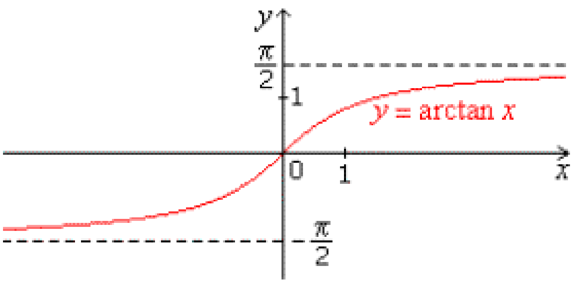


Y = Sin x has been fit into this

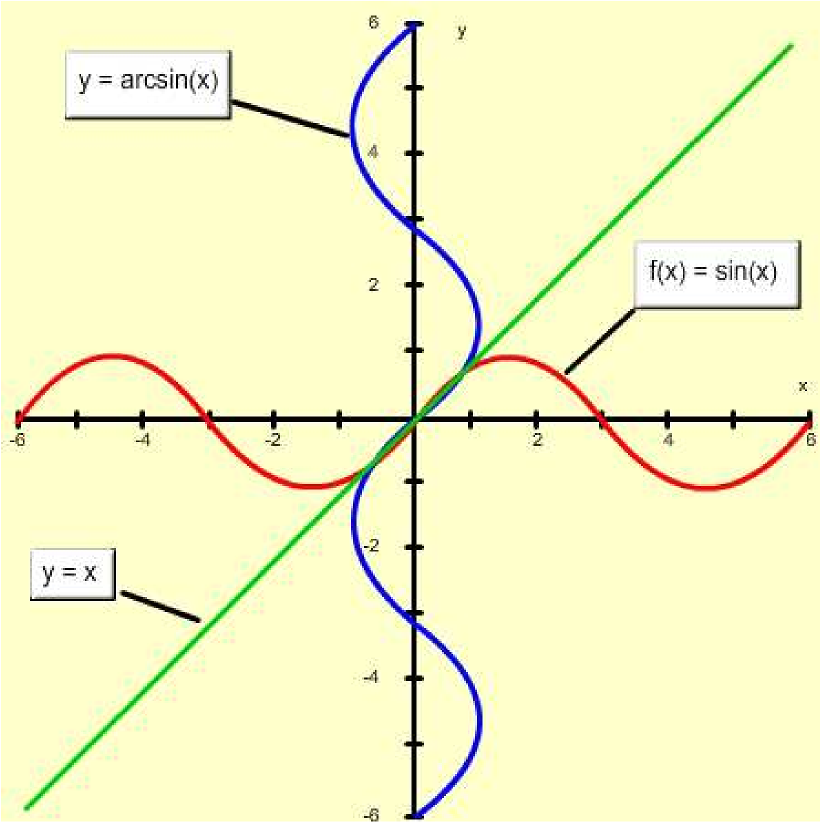
Graph of y = tan x

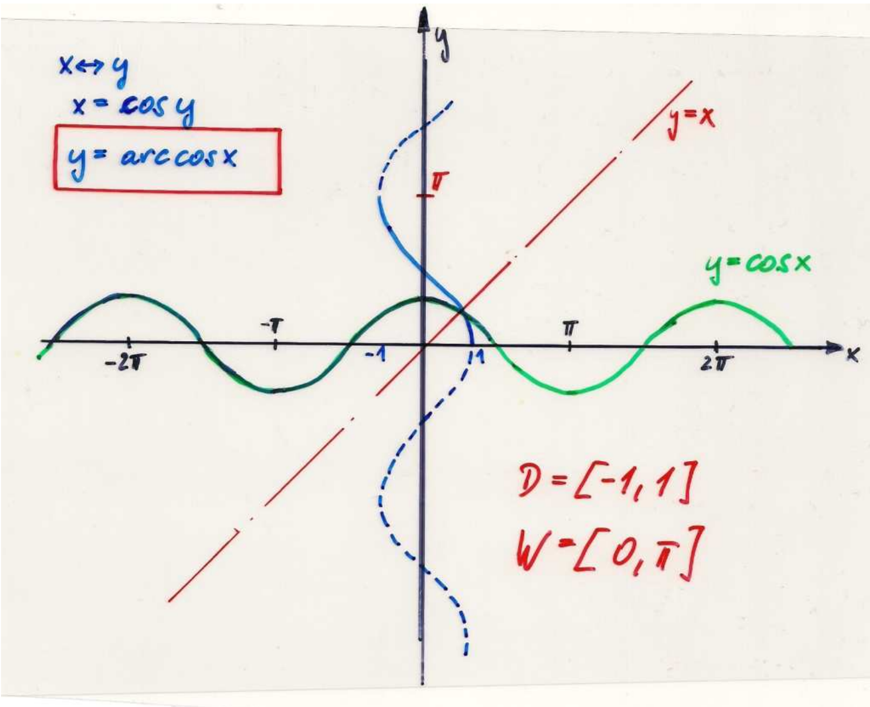


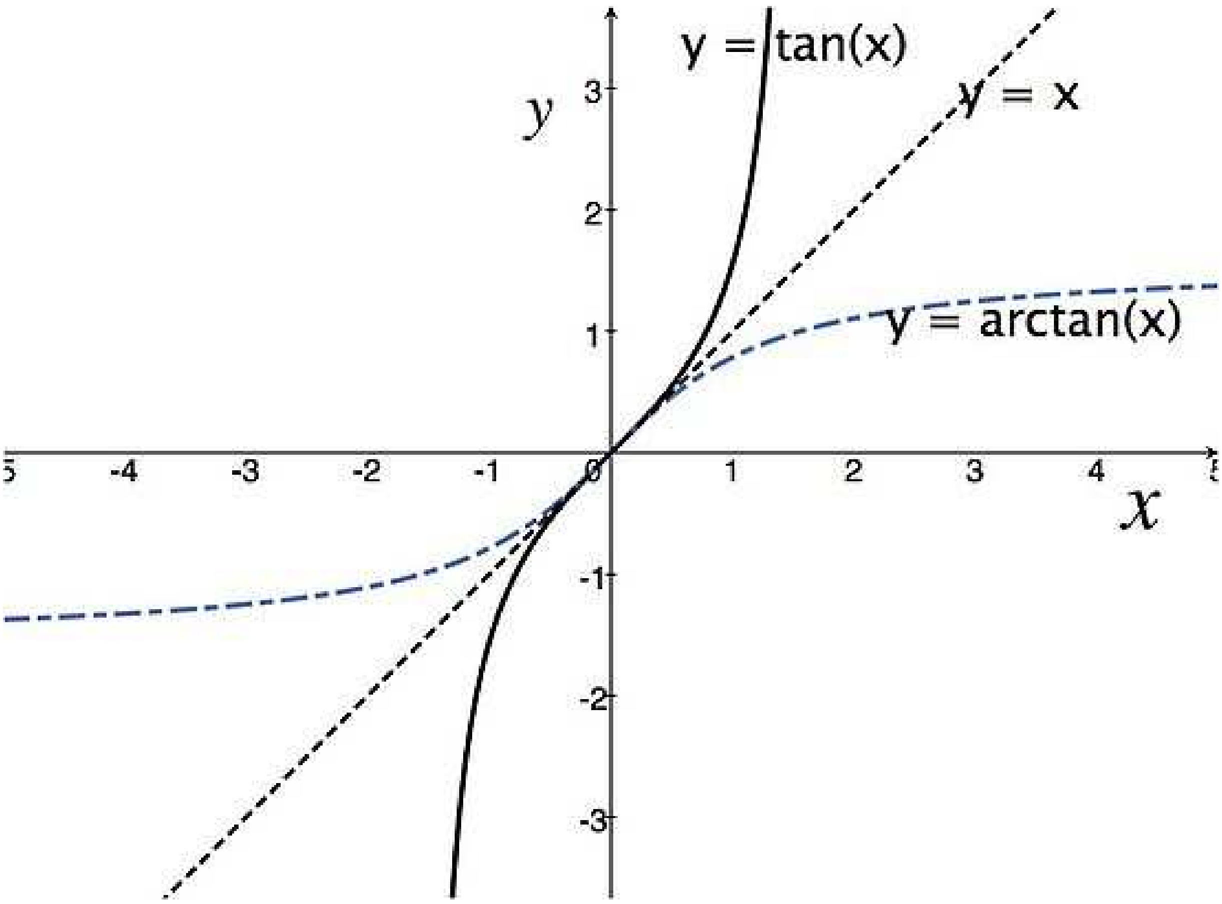
Graph of y = $\tan^{-1}$ x



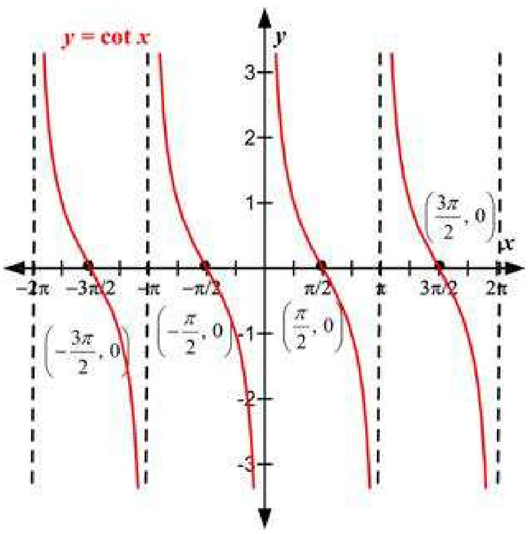
Let us compare these a few more times, so that we can remember



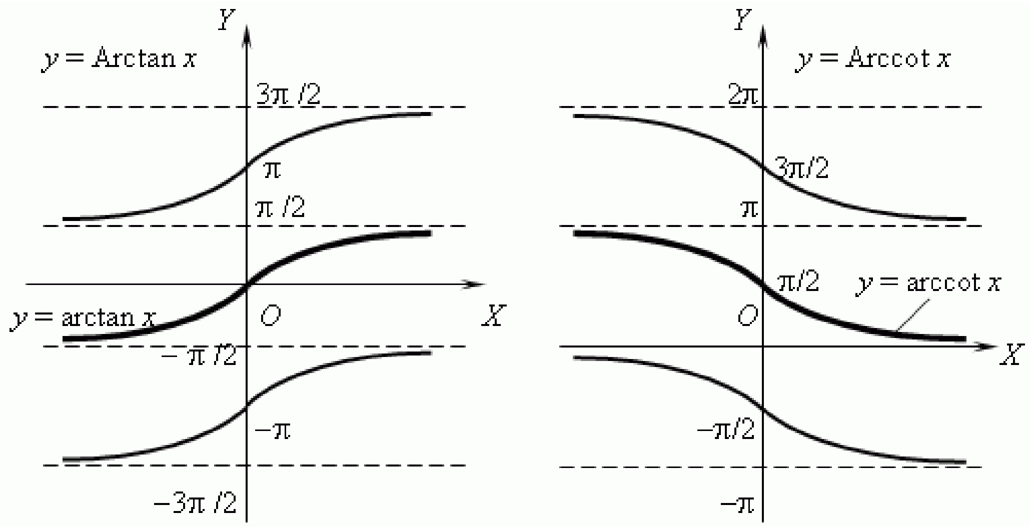


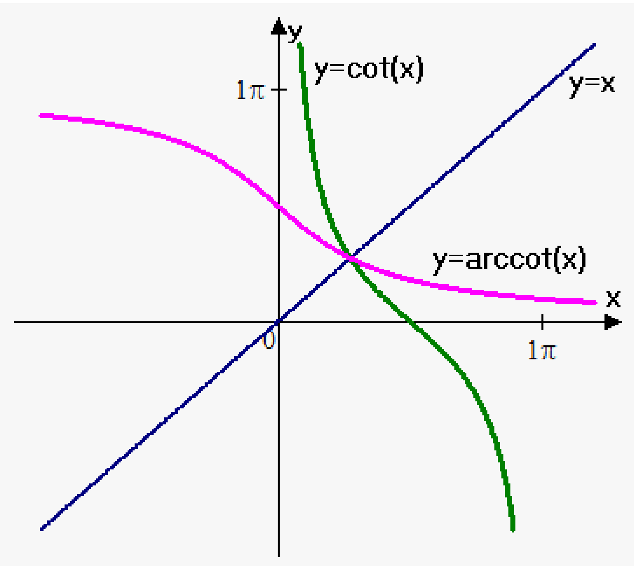


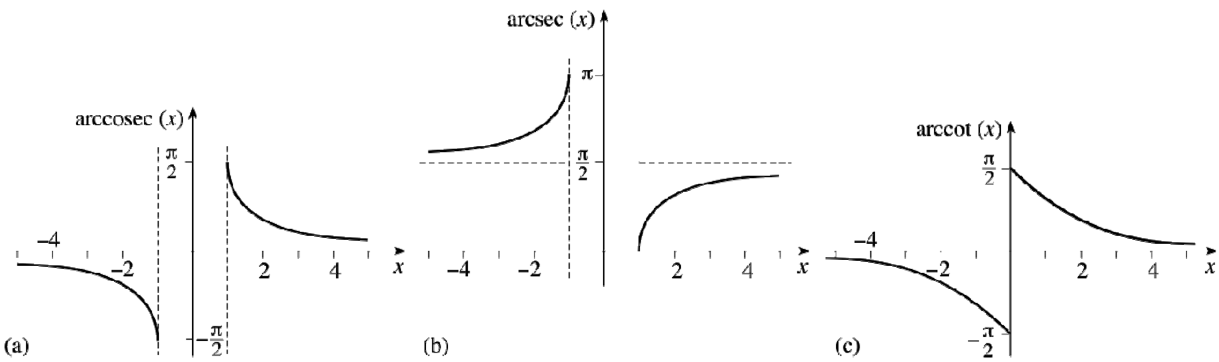
Graph of y = Cot x



Graph of y = Cot^{-1} x







An introduction to Periodic functions, Decision to Multiply or Divide is explained at

https://archive.org/details/PeriodicFunctionsAnIntroductionOfPeriodMultiplyOrDivide

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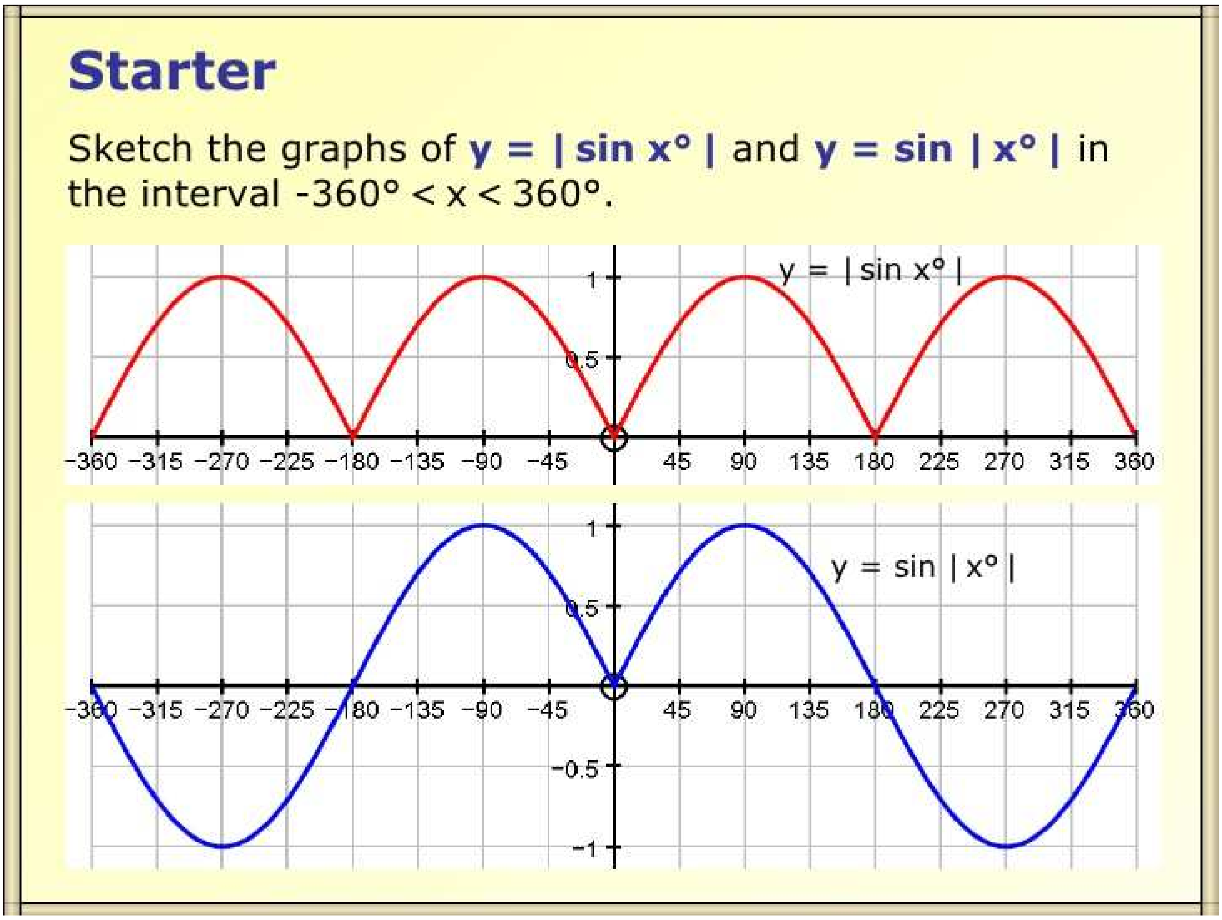
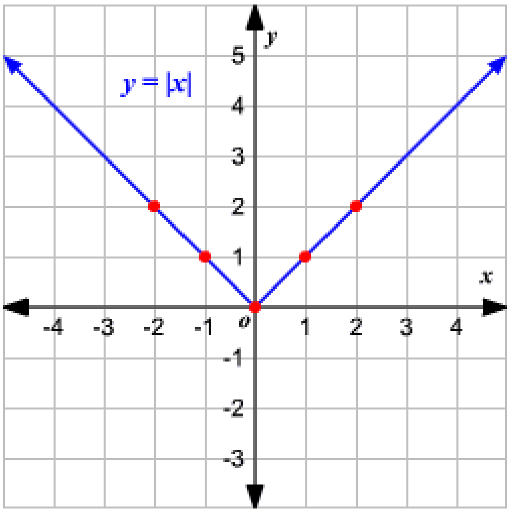
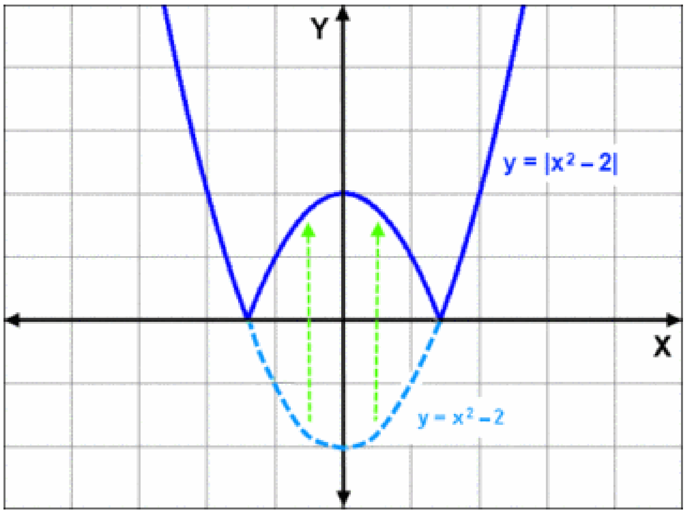
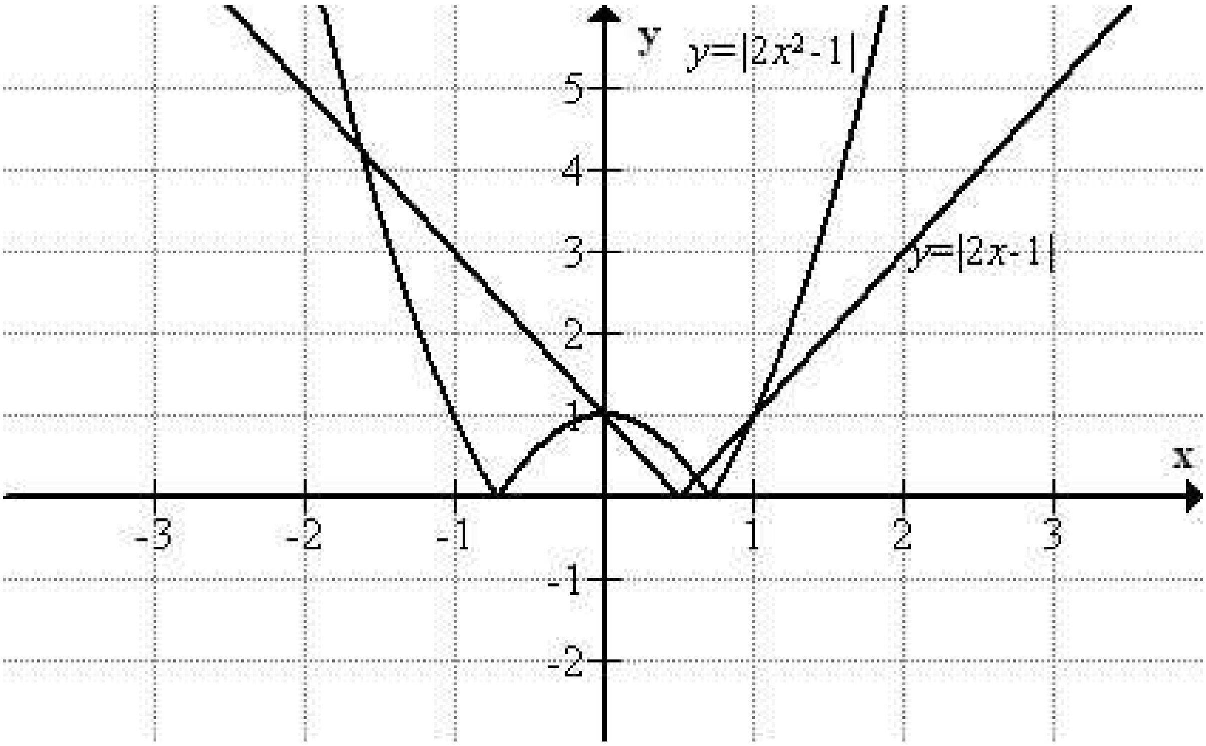
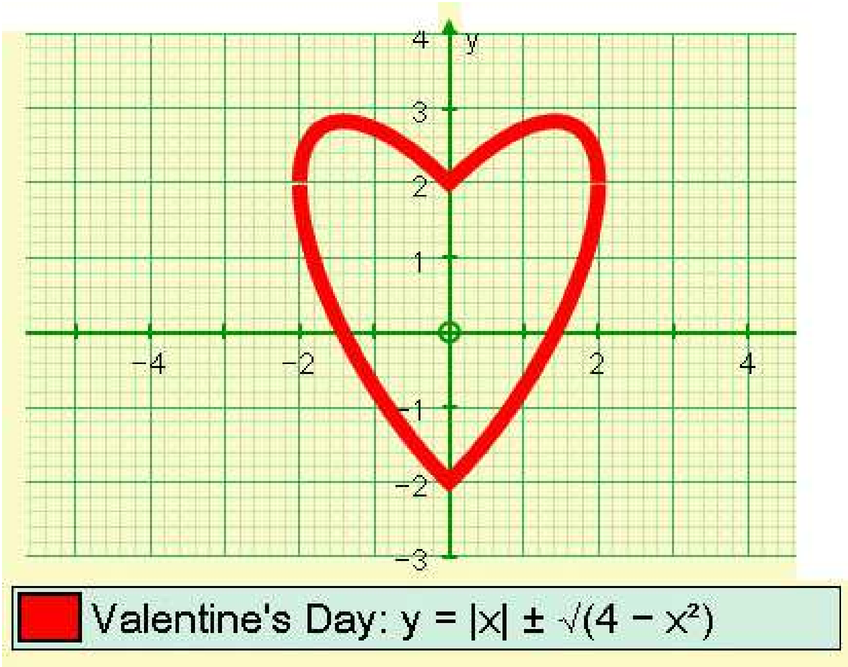
Graphs of modulus functions

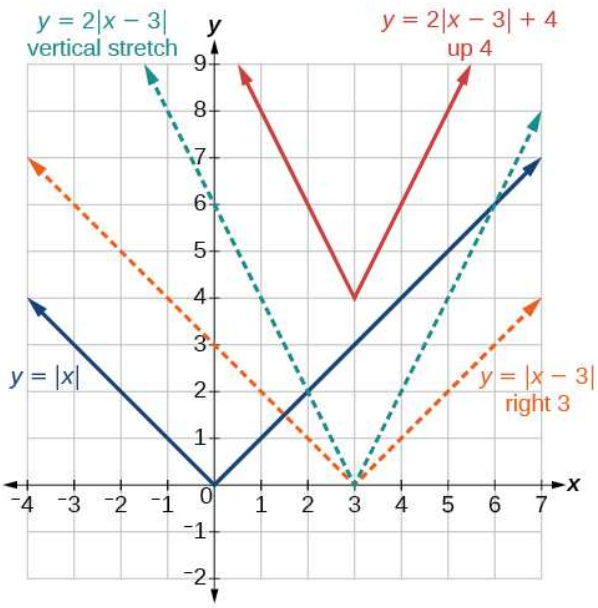



y = |x|
y
x

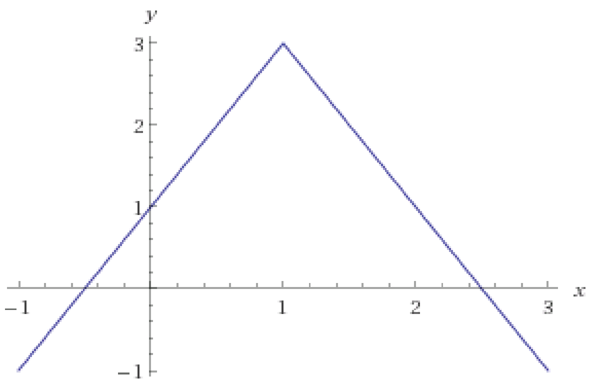

Y
X
y = |x² – 2|
y = x² – 2


y
y=|2x²-1|
y=|2x-1|
x


Valentine's Day: y = |x| ± √(4 − x²)

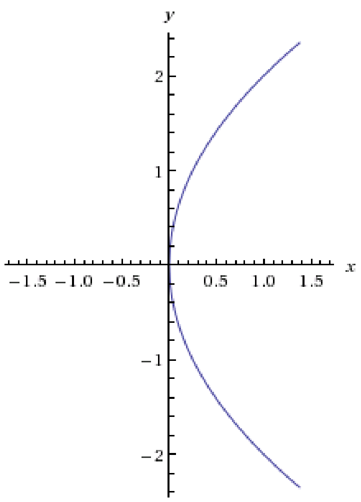

y = 2|x − 3|
vertical stretch
y = 2|x − 3| + 4
up 4
y = |x|
y = |x − 3|
right 3

plot	$-2\,|x-1|+3$

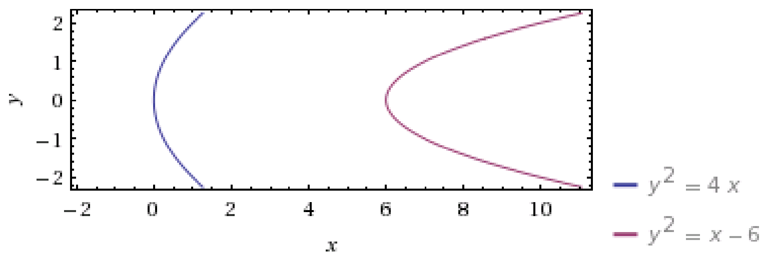


Now let us see Horizontal Parabolas

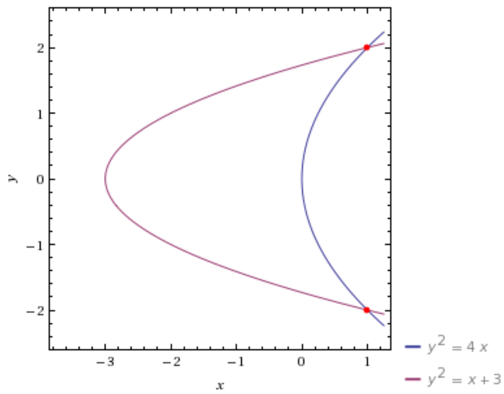
Graph of $y^2 = 4x$ is of the form $y^2 = 4\,a\,x$



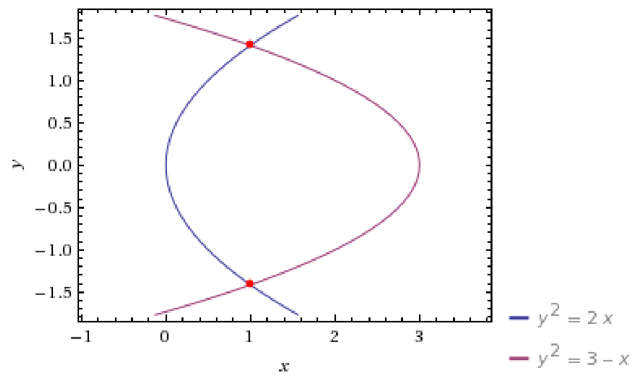
plot	$y^2 = 4x$
	$y^2 = x - 6$



plot $y^2 = 4x$, $y^2 = x + 3$



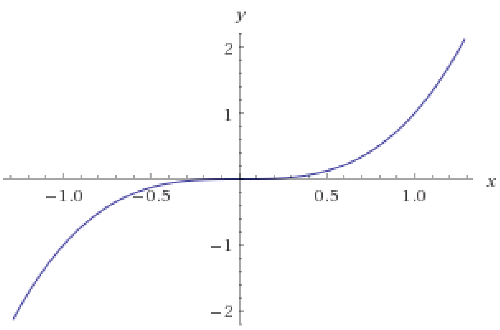
plot $y^2 = 2x$, $y^2 = -x + 3$



Graphs of Cubic Equations (y = x cube) and higher powers of x

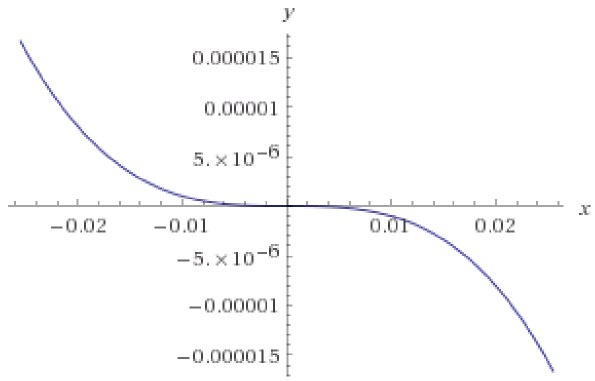
Graph of y = x^3 is

plot $y = x^3$

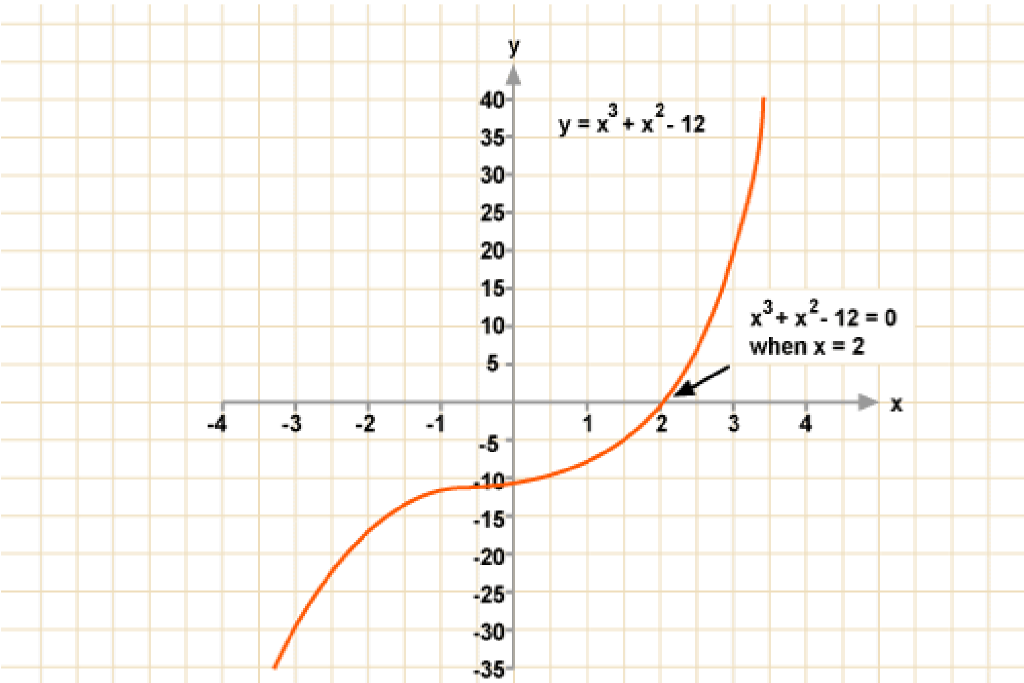


A good student can learn a lot by thinking how the graph of negative of the same function will look.

plot	$y = -x^3$



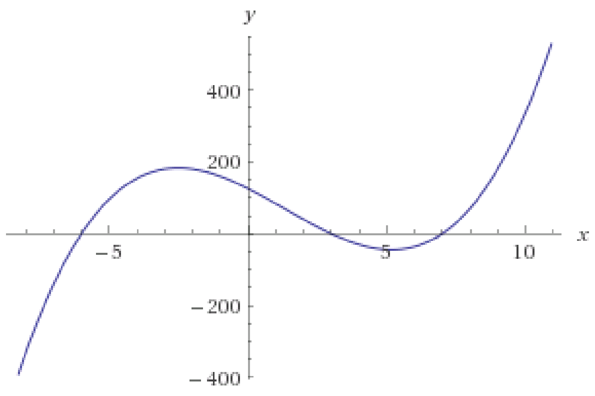
The previous graph flipped around x-axis



-

How will y = (x + 6)(x - 3)(x - 7) look like ? [x = -6, 3 and 7 will be roots. So the graph will pass through (-6, 0), (3,0) and (7,0)

plot	$y = (x + 6)(x - 3)(x - 7)$



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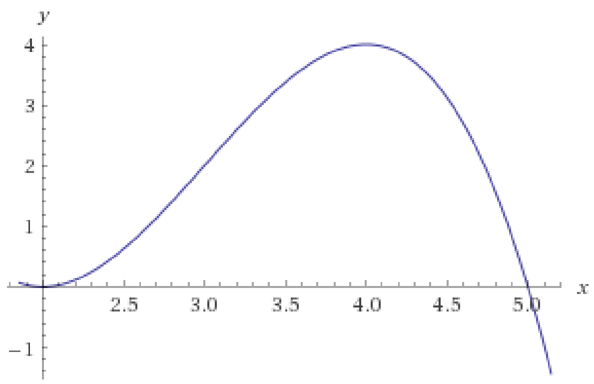
If coeff of x cube is negative then the graph will be downwards for increasing x. Also repeat roots can be there. Try to guess the graph of y = (5 - x)(2 - x)^2

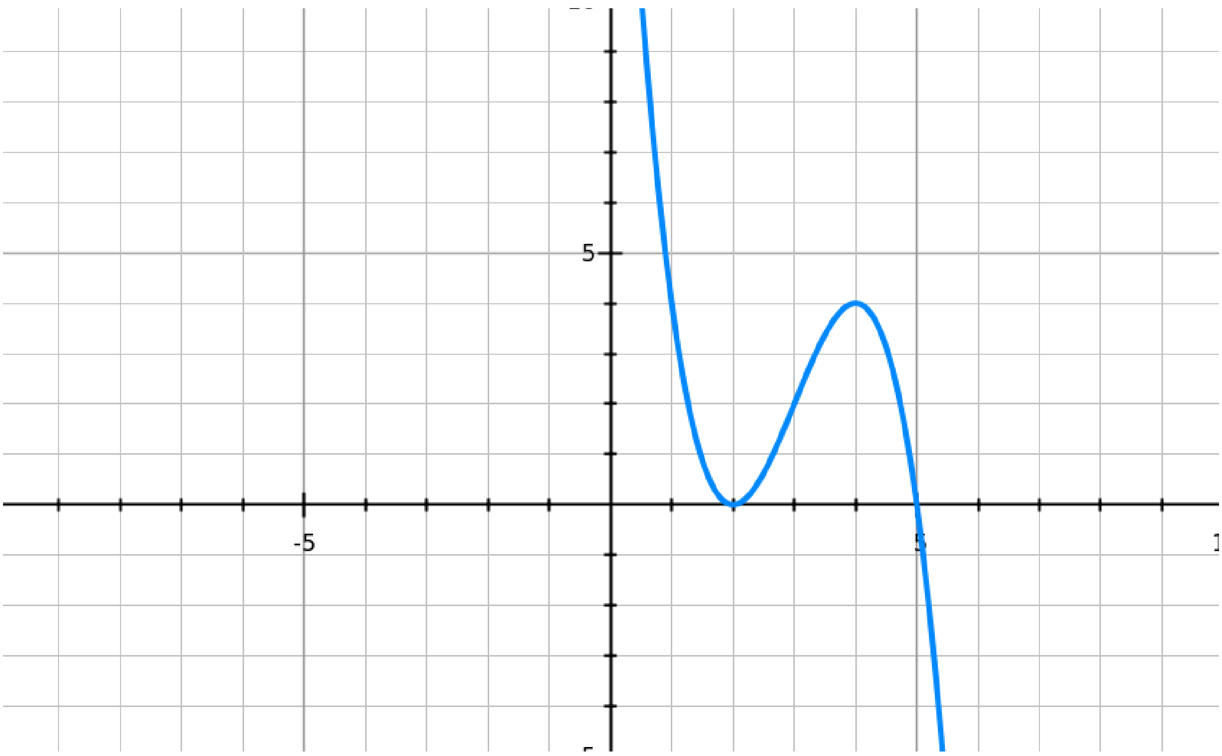
This will have roots at x = 5 and repeat roots (Two roots) at x = 2 so will touch x axis at x = 2

plot	$y = (5 - x)(2 - x)^2$



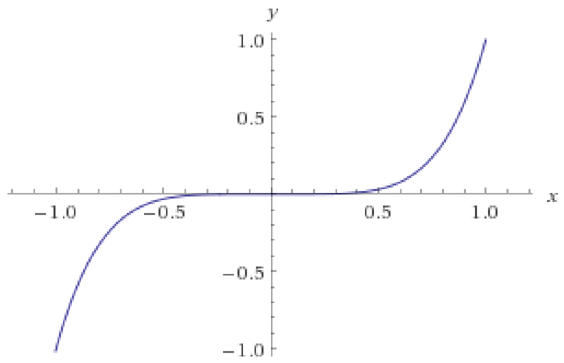
Because of distorted scale this graph is not a good one. The graph is correct but student must be mature to understand the distorted scale effects.

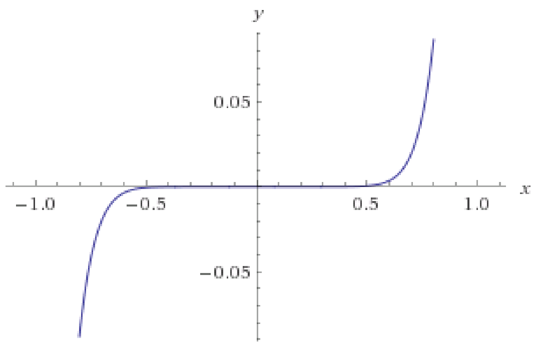
The graph below is a better one from a different plotter



The graph of $y = x^5$ or say $y = x^{11}$ will look very similar plot $y = x^5$

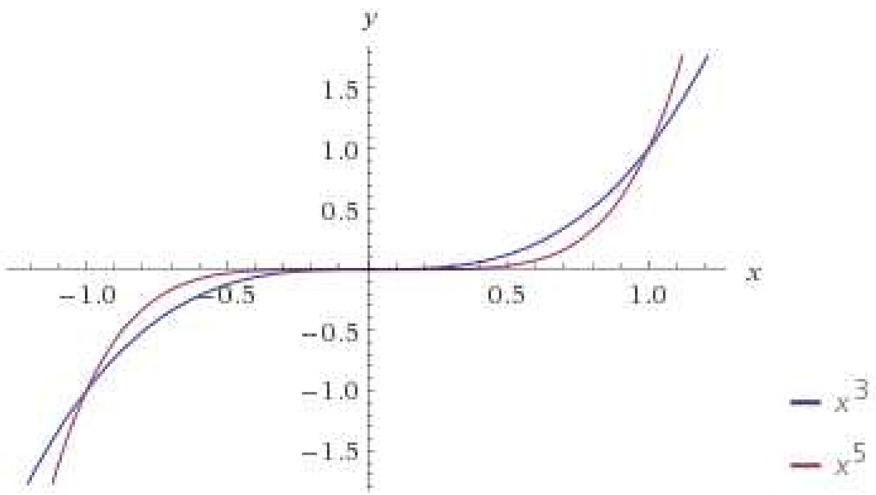
plot $y = x^{11}$



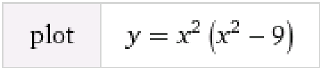


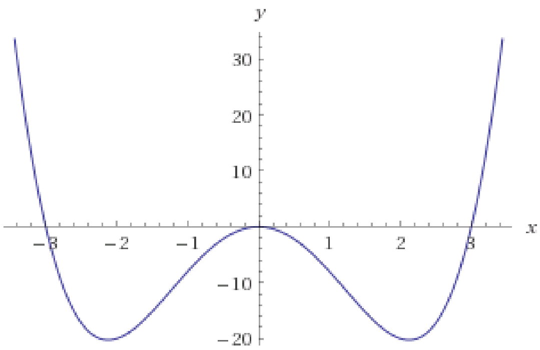
The difference is highlighted if the graphs are drawn together. All these graphs pass through (1, 1) and (-1, -1). While higher powered graph is flatter in between -1 to 1 and steeper after 1 or before -1

plot $y = x^3$, $y = x^5$



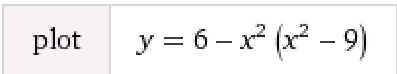
Graph of y = x^2 (x + 3) (x - 3) = x^2 (x^2 - 9)

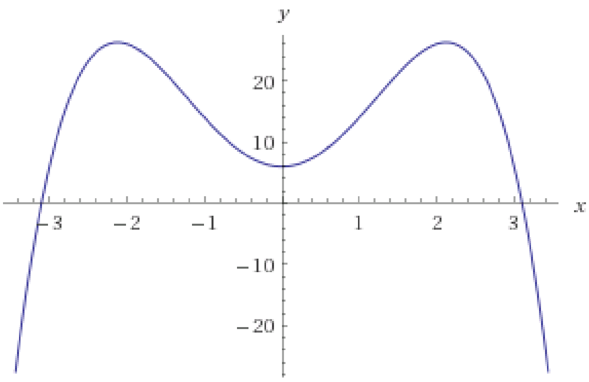




X = 0 will be repeat root due to x square. Also x = 3 and x = -3 will be the roots

Graph of y = - x^2 (x^2 - 9) + 6



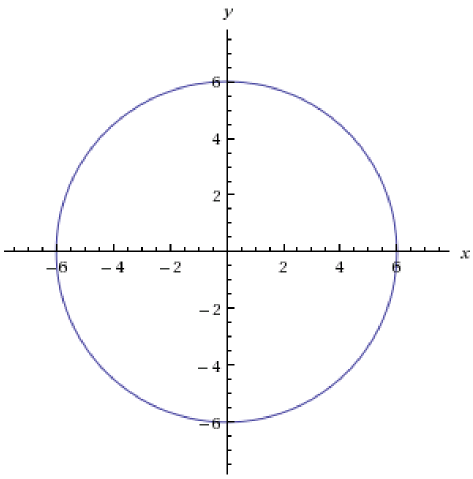


Now let us see graphs of Circles

Graph of $x^2 + y^2 = R^2$ will have the center at (0,0) and radius will be R

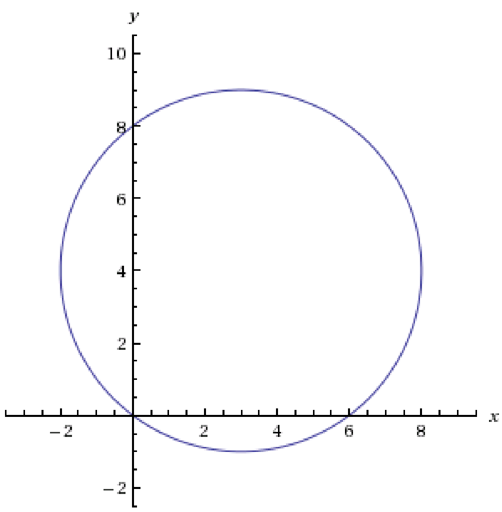
So graph of $x^2 + y^2 = 36$ is

plot	$x^2 + y^2 = 36$



Graph of $(x - 3)^2 + (y - 4)^2 = 25$ is

plot	$(x-3)^2 + (y-4)^2 = 25$



Center is at (3, 4)

Area problems, Graphs of Line, Circle, Triangle Areas discussed and explained at

https://archive.org/details/AreaDefiniteIntegralLineCircleModulusTriangleNatureAndType

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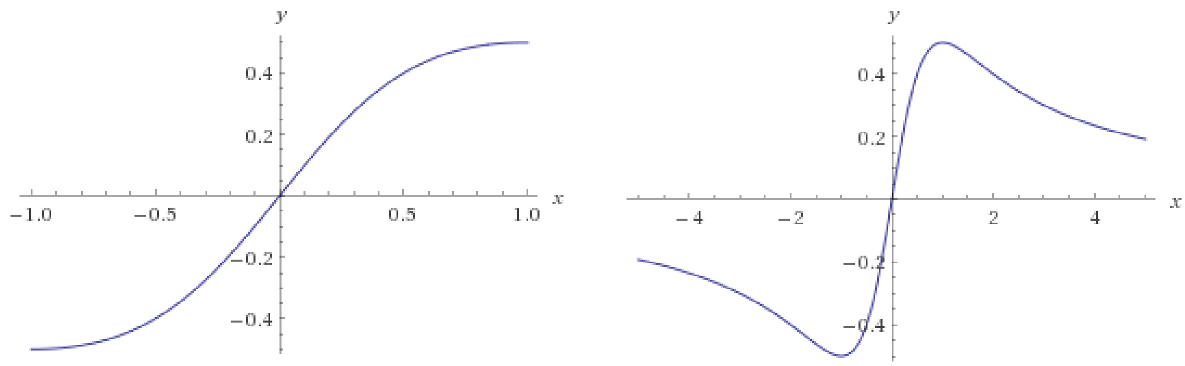
Some special graphs

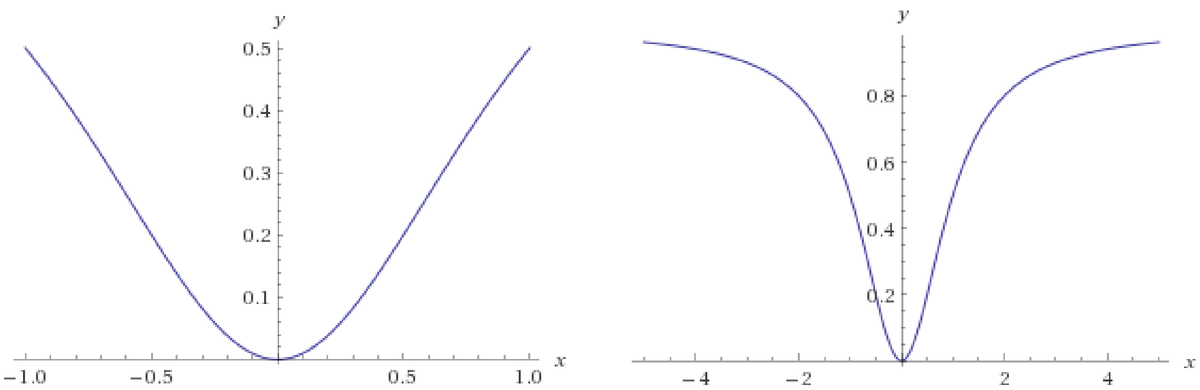
plot	$y = \dfrac{x}{x^2+1}$



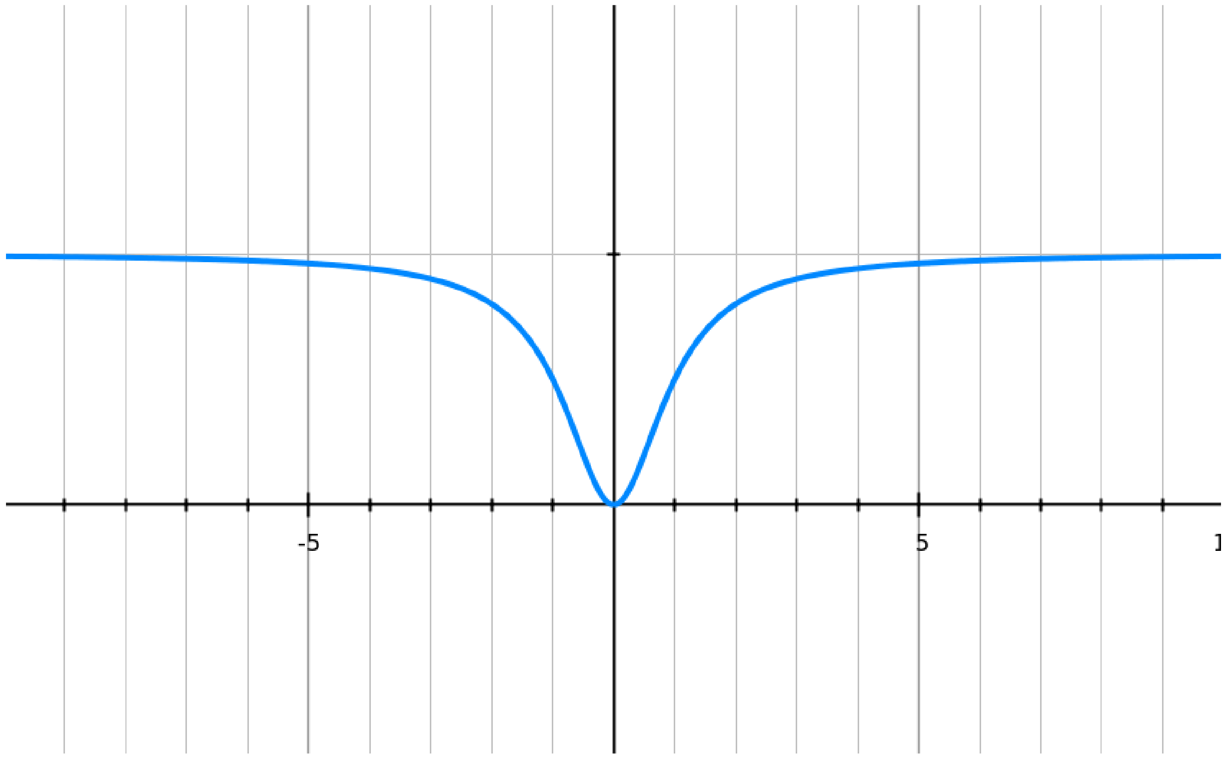
The graph become asymptotic to x-axis as we move towards right or left

The same will happen for $y = \dfrac{x^2}{x^2+1}$ though very slowly $\lim\limits_{x \to \pm\infty} \dfrac{x^2}{1+x^2} = 1$

plot	$y = \dfrac{x^2}{x^2+1}$

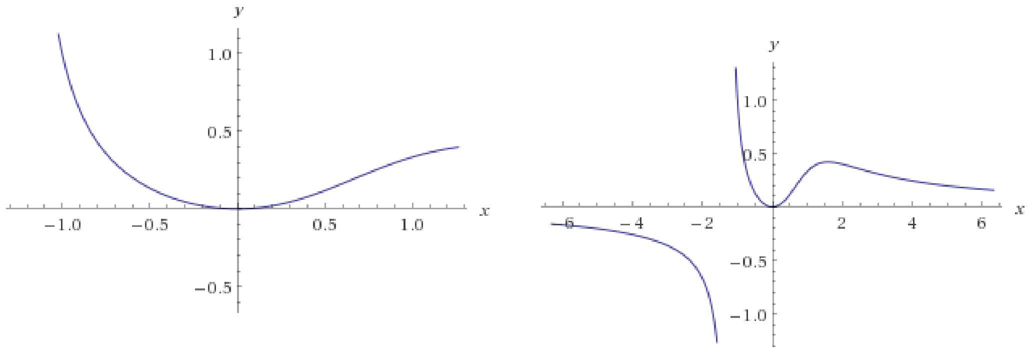


In this case the graph is asymptotic to 1 ($y = \dfrac{x^2}{x^2+1}$)



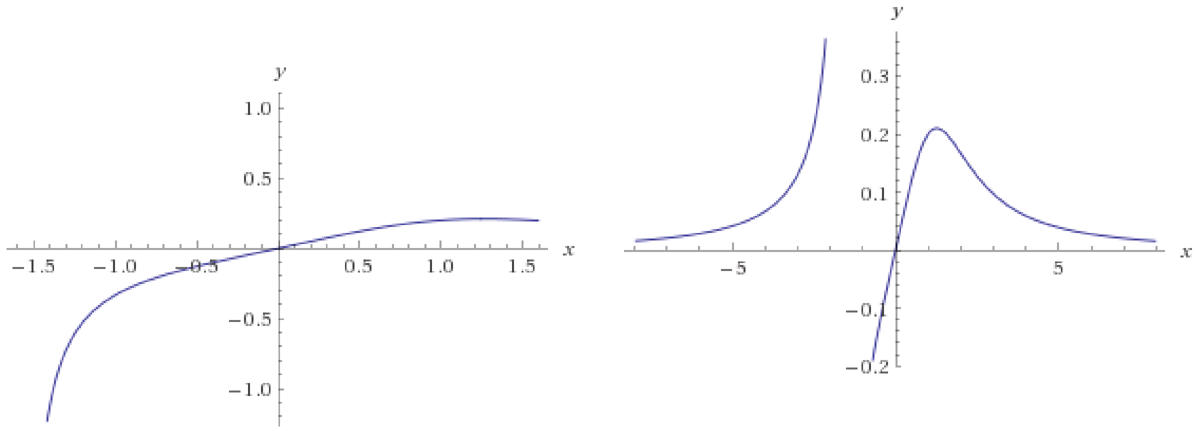
Can you guess what will happen in case of $y = \frac{x^2}{x^3 + 2}$? Did you notice the discontinuity around negative cuberoot of 2

plot $y = \frac{x^2}{x^3 + 2}$



Can you guess what will happen in case of $y = \frac{x}{x^3+4}$? Understand the discontinuity around negative cuberoot of 4

plot	$y = \frac{x}{x^3+4}$



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Find all asymptotes and sketch the function

$$f(x) = \frac{x^3 + 5}{x^2 + 3x + 1}$$

$x^2 + 3x + 1 = 0$

$$x = \frac{-3 \pm \sqrt{5}}{2} \quad \text{(2 vertical asymptotes)}$$

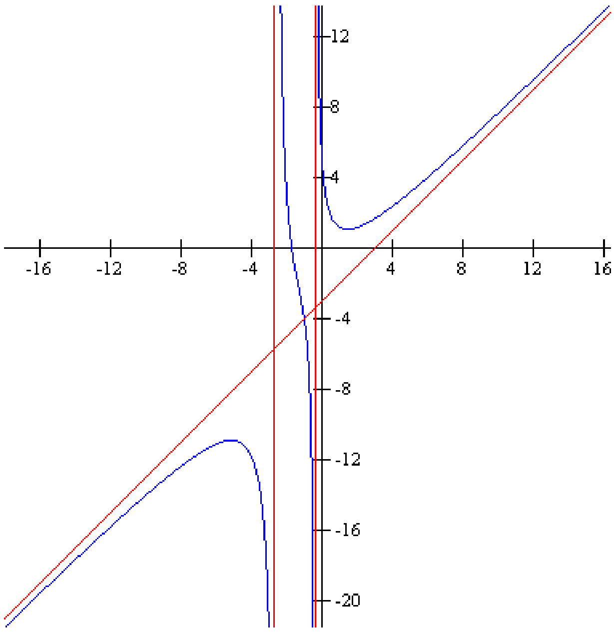
$$y = \frac{(x^3/x^3) + (5/x^3)}{(x^2/x^3) + (3x/x^3) + (1/x^3)} = \text{undefined (no horizontal asymptotes)}$$

```
                    x - 3 + ((8x + 8)/(x^2 + 3x + 1))
              -------------------
x^2 + 3x + 1 / x^3 + 0x^2 + 0x + 5
               x^3 + 3x^2 + x
               -----------
                  -3x^2 - x + 5
                  -3x^2 -9x - 3
                  -----------
                       8x + 8
```

$$y = x - 3 + \frac{8x/x^2 + 8/x^2}{x^2/x^2 + 3x/x^2 + 1/x^2}$$

$= x - 3 + 0$

$= x - 3$ (one oblique asymptote)



Find all asymptotes and sketch the function

$$g(x) = \frac{x^2}{x - 3}$$

$x - 3 = 0$

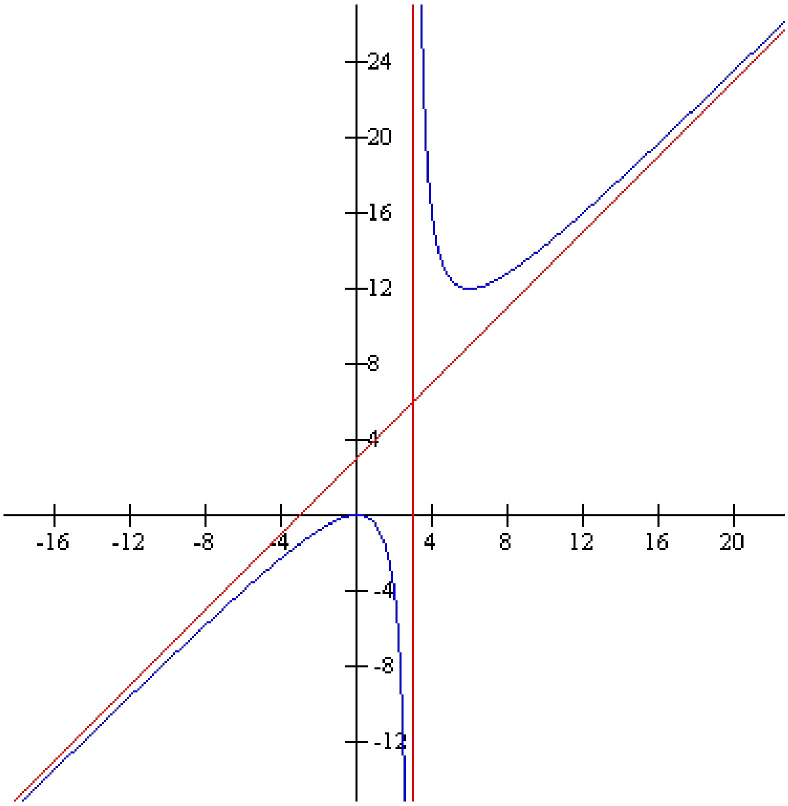
$x = 3$ (one vertical asymptote)

$$y = \frac{x^2/x^2}{x/x^2 - 3/x^2} = \text{undefined (no horizontal asymptotes)}$$

```
         x + ((3x)/(x - 3))
        -------------------
x - 3 / x^2 + 0x + 0
        x^2 - 3x
        ------------
            3x
```

$$y = x + \frac{3x/x}{\quad} = x + 3 \text{ (one oblique asymptote)}$$

$x/x - 3/x$



Find all asymptotes and sketch the function

$$y = \frac{x^3 - 4x^2 - 49x - 90}{2x^2 + 12x + 18}$$

$2x^2 + 12x + 18 = 2(x^2 + 6x + 9) = 0$
x = -3 (one vertical asymptote)

$$y = \frac{x^3/x^3 - 4x^2/x^3 - 49x/x^3 - 90/x^3}{2x^2/x^3 + 12x/x^3 + 18/x^3} = \text{undefined (no horizontal asymptotes)}$$

$0.5x - 5 + ((2x)/(2x^2 + 12x + 18))$

$2x^2 + 12x + 18 \ / \ x^3 - 4x^2 - 49x - 90$
$x^3 + 6x^2 + 9x$

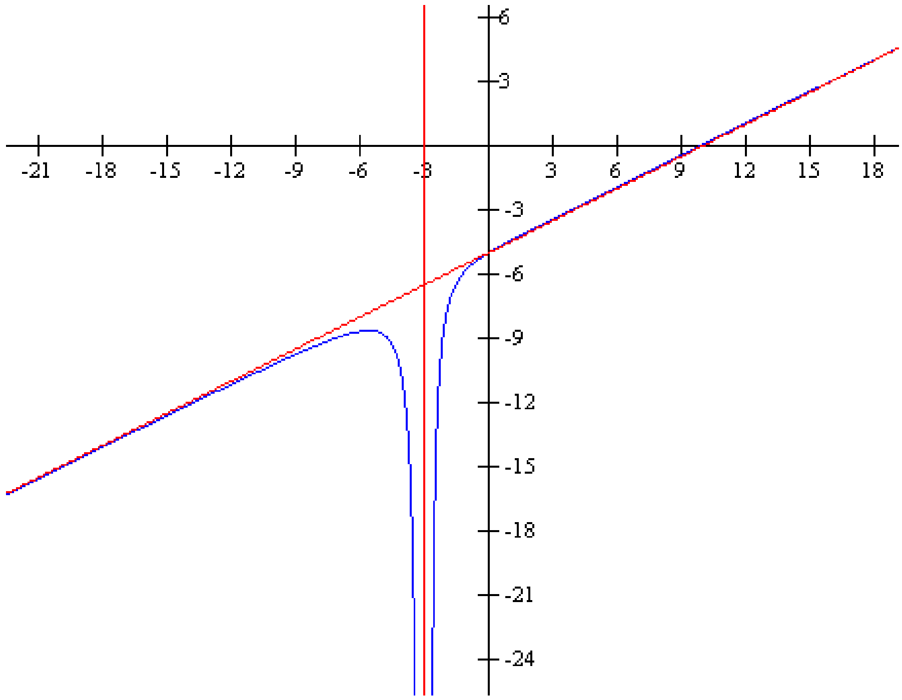
$-10x^2 - 58x - 90$
$-10x^2 - 60x - 90$

2x

$$y = 0.5x - 5 + \frac{2x/^2}{2x^2/x^2 + 12x/x^2 + 18/x^2}$$

$= 0.5x - 5 + 0$

$= 0.5x - 5$ (one oblique asymptote)



Find all asymptotes and sketch the function

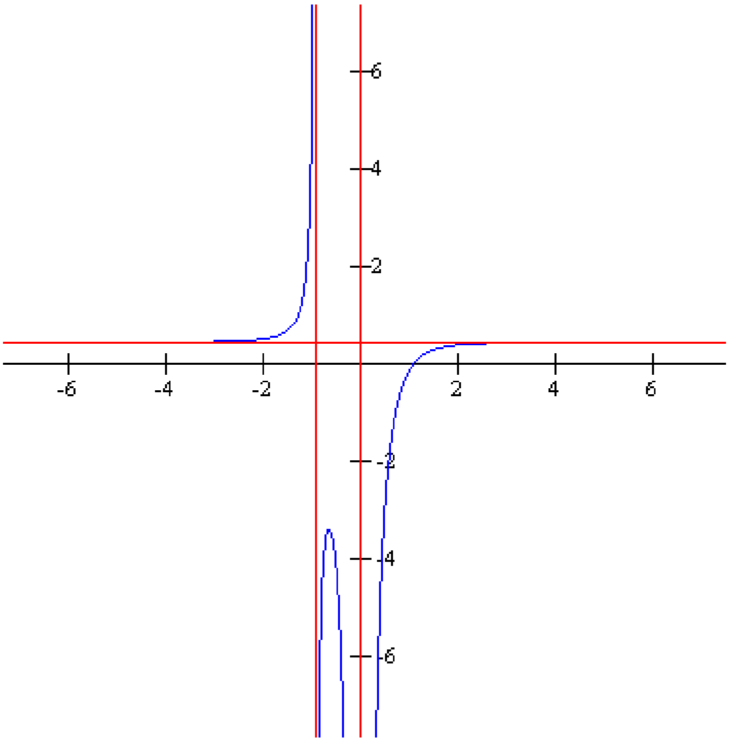
$$h(x) = \frac{4x^5 - 6}{9x^5 + 7x^2}$$

$9x^5 + 7x^2 = x^2(9x^3 + 7) = 0$

$x = (-7/9)^{1/3}$ or $x = 0$ (two vertical asymptotes)

$$y = \frac{4x^5/x^5 - 6/x^5}{9x^5/x^5 + 7x^2/x^5} = 4/9 \text{ (one horizontal asymptote)}$$

There are no oblique asymptotes, as the degree of the numerator is not one greater than the degree of the denominator



Find all asymptotes and sketch the function

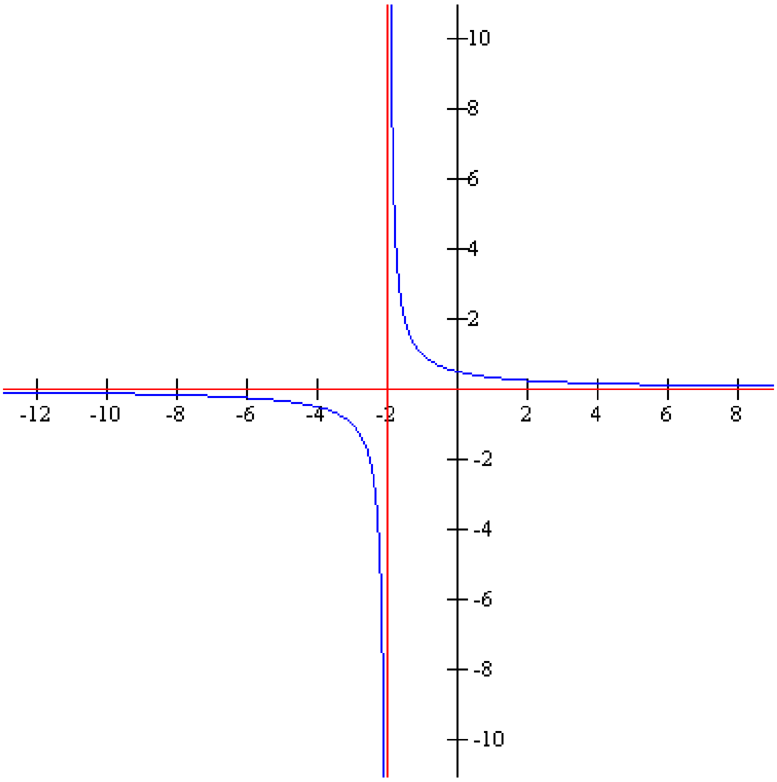
$$y = \frac{x^4 - 3x^3 + 5x^2 - 7x + 9}{x^5 - x^4 - x^3 + 3x^2 - 5x + 18}$$

First, reduce the equation to y = 1/(x + 2)

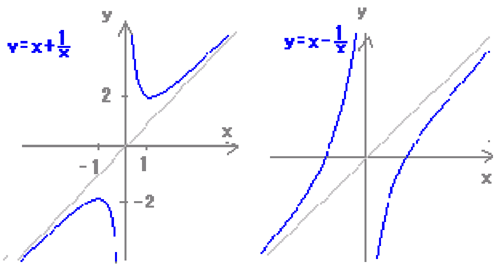
x + 2 = 0
x = -2 (one vertical asymptote)

$$y = \frac{1/x}{x/x + 2/x} = 0$$ (one horizontal asymptote)

There are no oblique asymptotes, as the degree of the numerator is not one greater than the degree of the denominator

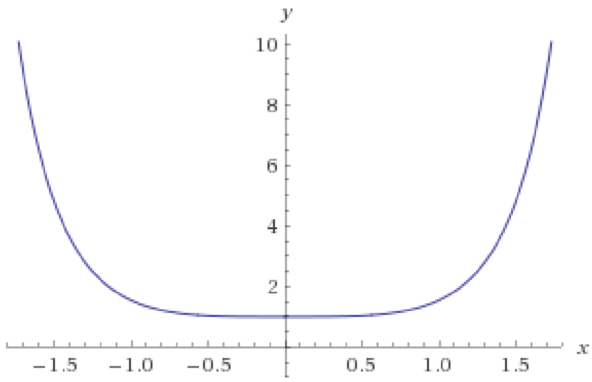


Graphs of y = x + 1/x   and   y = x - 1/x



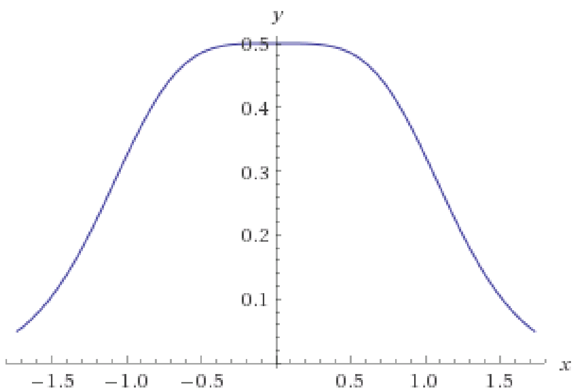
Graph of $y = \frac{1}{2}\left(e^{x^2} + e^{-x^2}\right)$

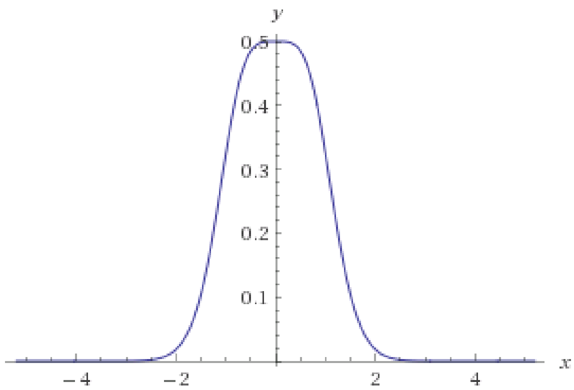
plot	$y = \frac{1}{2}\left(e^{x^2} + e^{-x^2}\right)$



So graph of $y = \frac{1}{e^{x^2} + e^{-x^2}}$

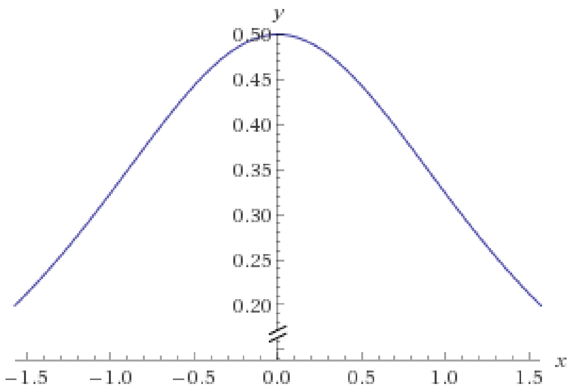
plot	$y = \frac{1}{e^{x^2} + e^{-x^2}}$

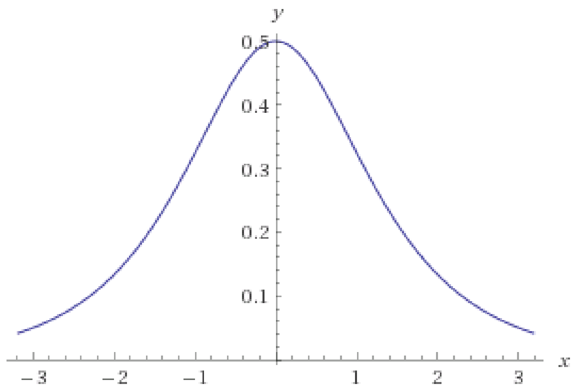


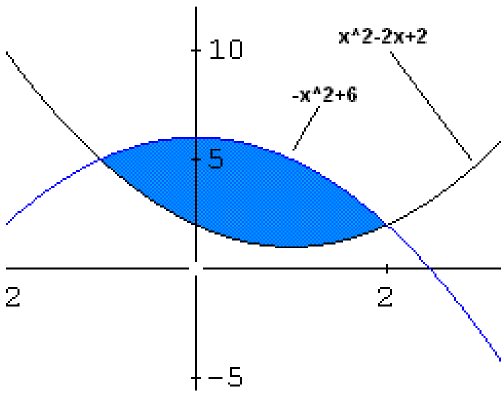


Spoon Feeding graph of $y = \dfrac{1}{e^x + e^{-x}}$

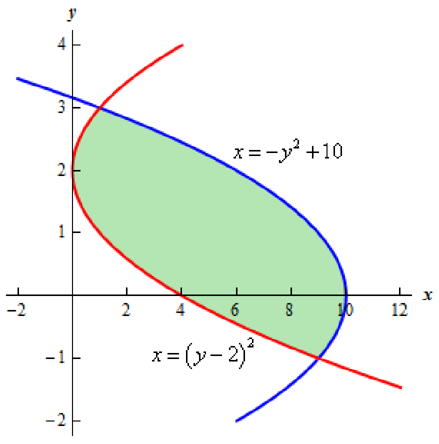
plot	$y = \dfrac{1}{e^x + e^{-x}}$



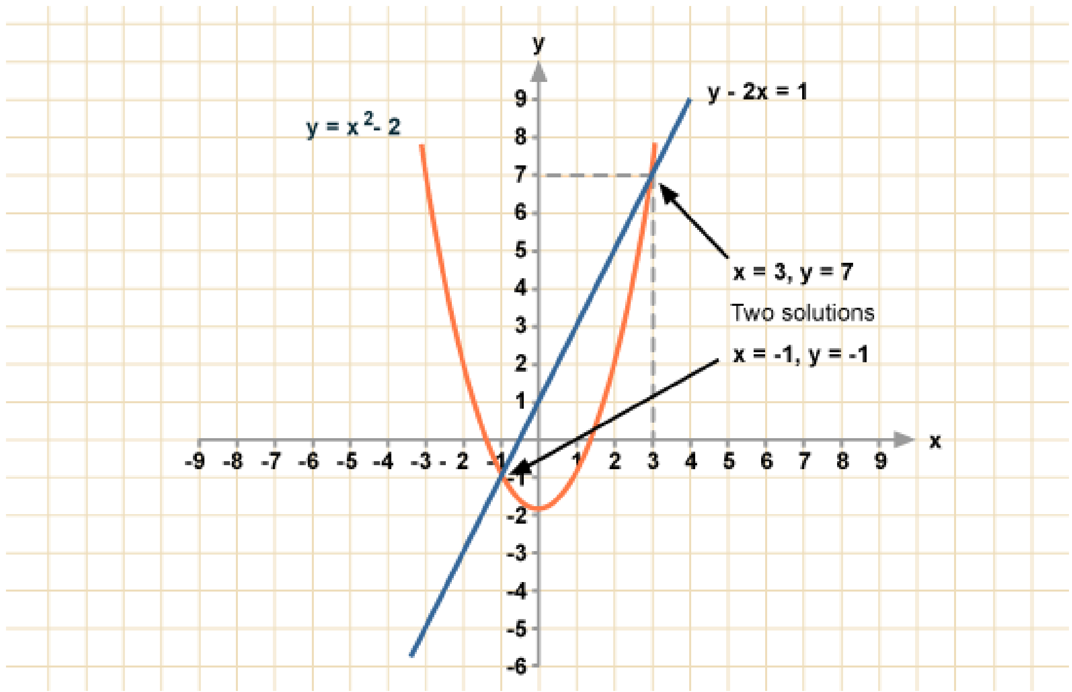




If area enclosed between two curves is needed; then the upper curve function minus the lower curve function needs to be integrated, between the two intersection points as limits.



We generally get questions with line intersecting a parabola kind …



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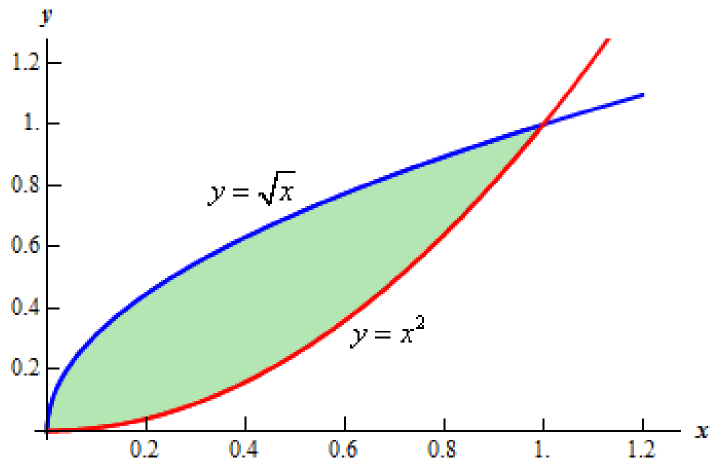

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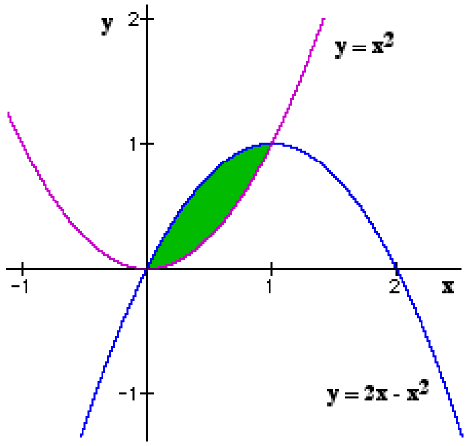
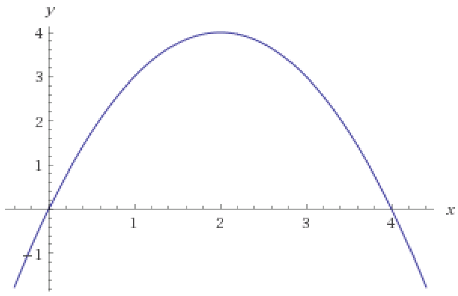
Draw the graphs of y = x^2 and y = 2x − x^2



Draw the graph of the parabola $y = 4x - x^2$

plot	$y = 4x - x^2$



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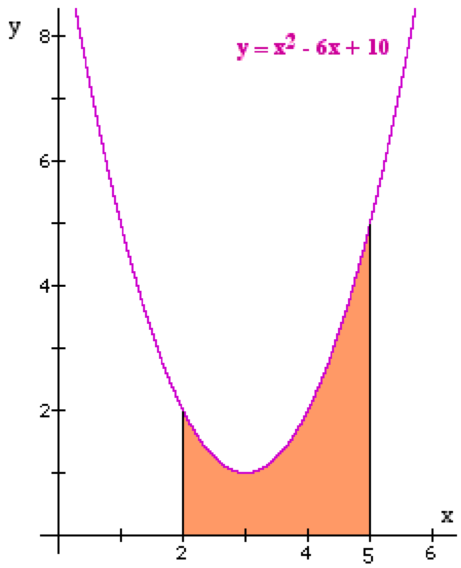
Draw the graph of $f(x) = x^2 - 6x + 10$, the lines $x = 2$ and $x = 5$ and the x-axis



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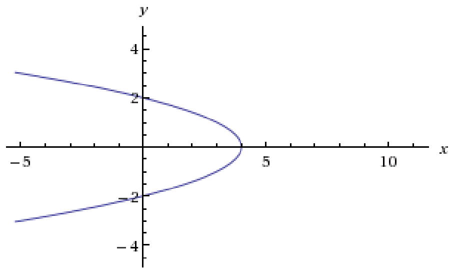
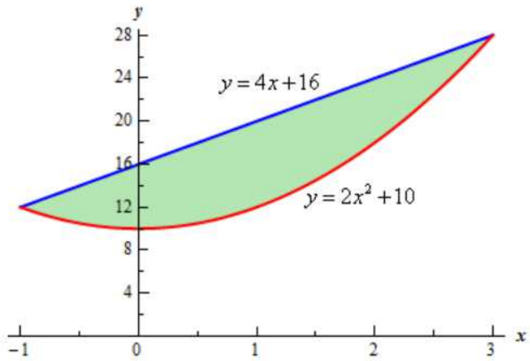
Draw the graph of $x = 4 - y^2$ => $y^2 = 4 - x$

plot	$y^2 = 4 - x$



-

Draw the graph of $y = 4x + 16$ and $y = 2x^2 + 10$

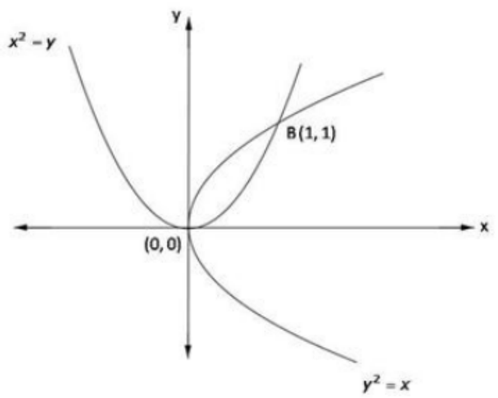


Solving these two given equations we get the intersection points as x = -1 and x = 3 (Quadratic equation $2x^2 + 10 = 4x + 16$ => $2x^2 - 4x - 6 = 0$

- ⇨ $x^2 - 2x - 3 = 0$ Factorize and you get x = -1 and x = 3)

-

Draw the graphs of $x = y^2$ and $y = x^2$

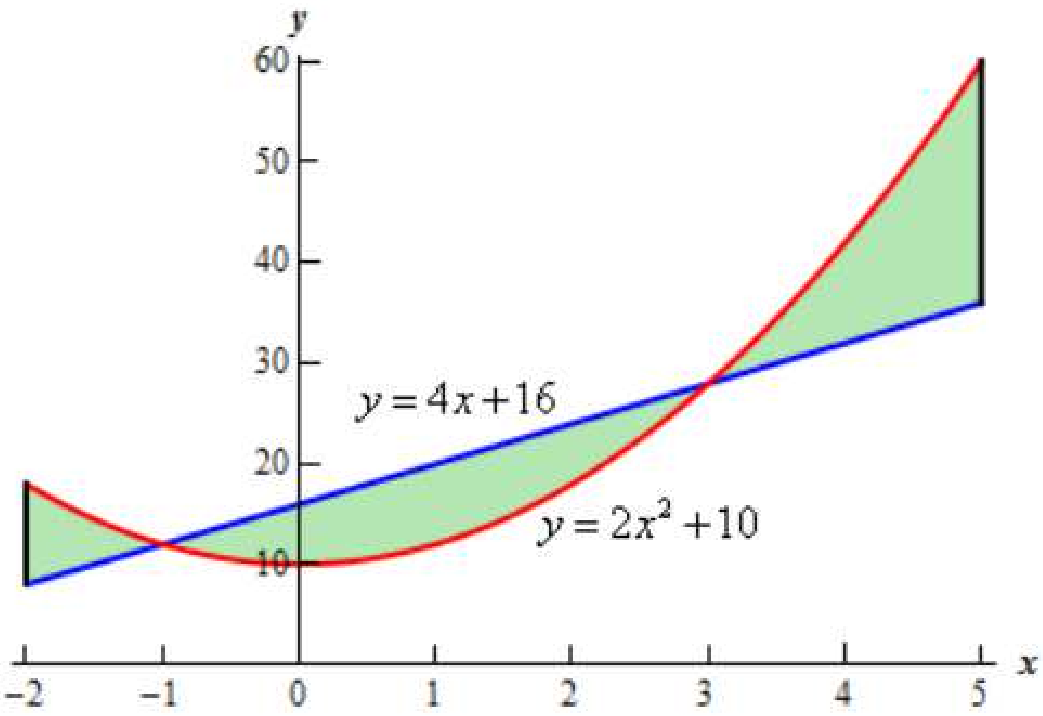


We can easily solve to see that the graphs intersect at (1, 1)

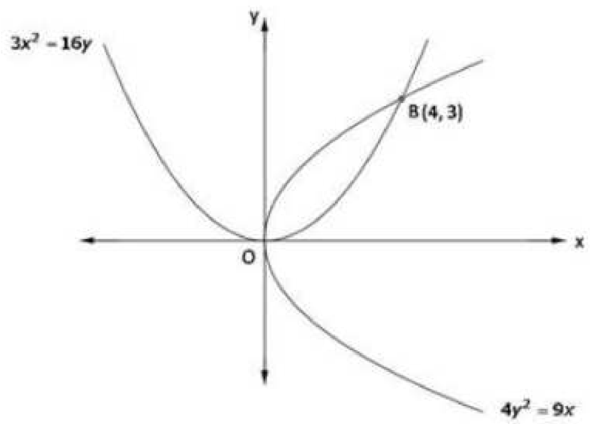
-

Draw the graphs of $y = 2x^2 + 10$, $y = 4x + 16$, $x = -2$, and $x = 5$

The regions in the graph needs to be plotted

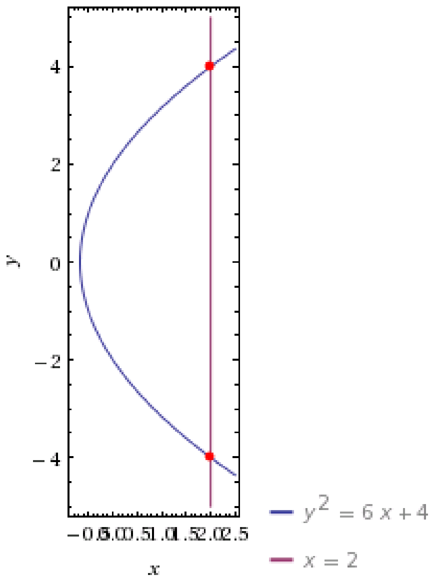


Draw the fraphs of $4y^2 = 9x$ and $3x^2 = 16y$

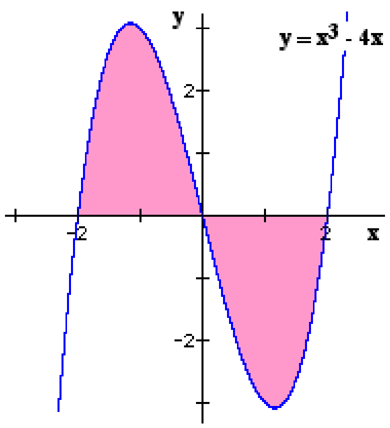


Draw the graph of $y^2 = 6x + 4$

plot	$y^2 = 6x + 4$
	$x = 2$



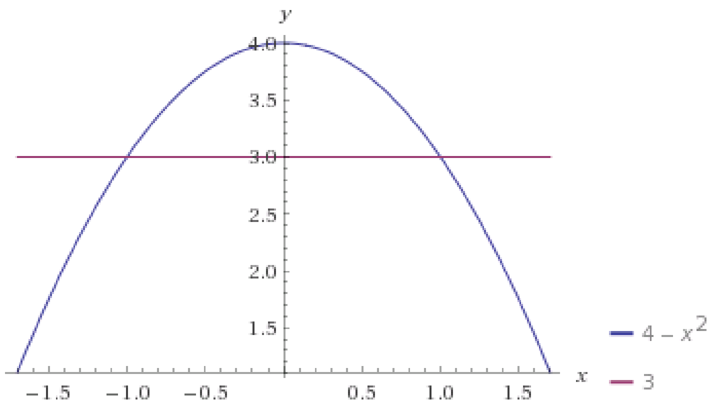
Draw the graph of y = x^3 - 4x



-

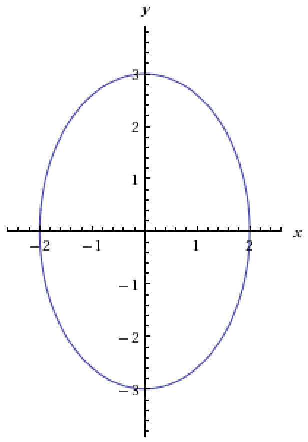
Graph of y = 4 - x^2

plot	$y = 4 - x^2$
	$y = 3$



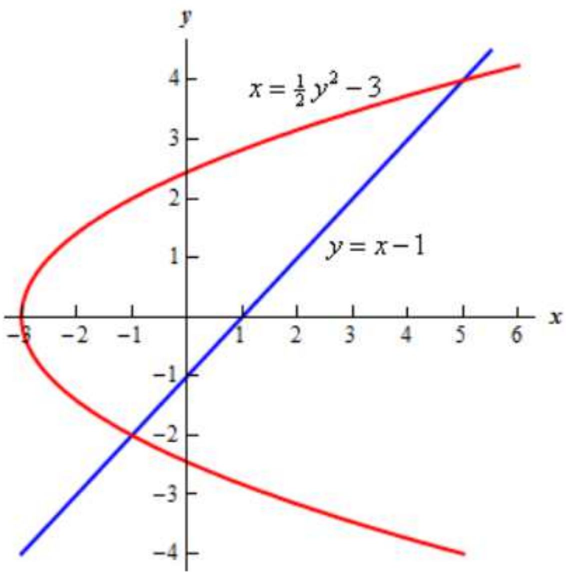
We need to know graphs of ellipse and problems related to those

plot	$\frac{x^2}{4} + \frac{y^2}{9} = 1$



-

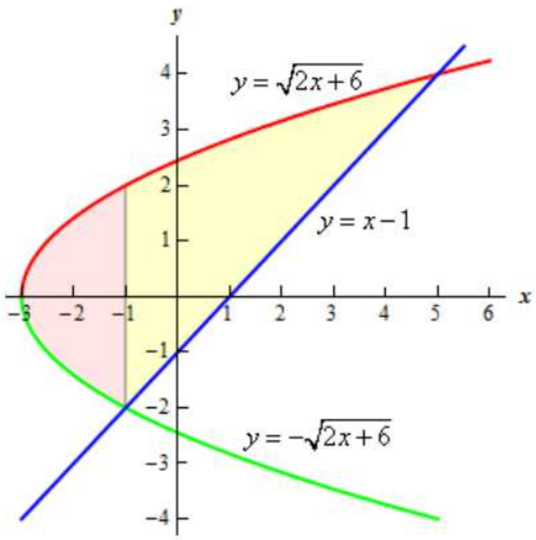
Graphs of $x = \frac{1}{2} y^2 - 3$ and y = x - 1



The line and the parabola intersect at y = -2 => x = y + 1 = -1 so (-1, -2)

and y = 4 => x = y + 1 = 5 so (5, 4)

the function becomes $y = \pm\sqrt{2x+6}$



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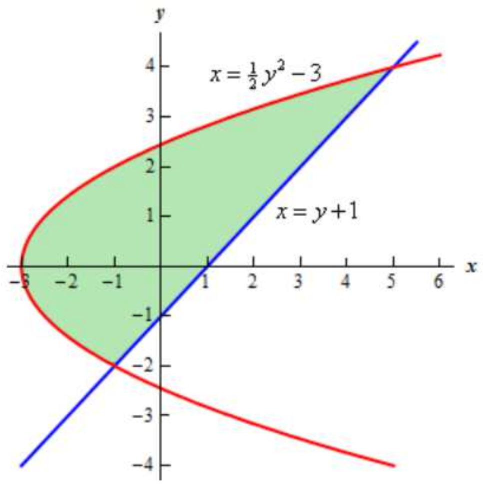
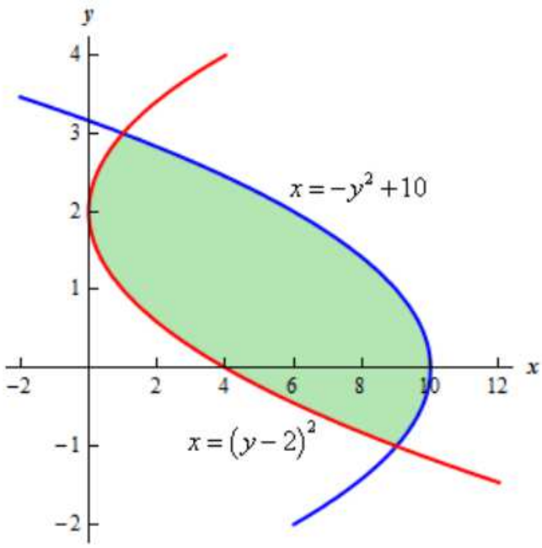


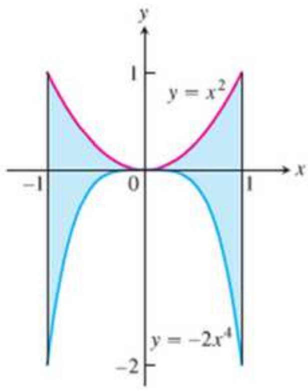
Graph of $x=-y^2+10$ and $x=(y-2)^2$.

The intersection points are y = -1 and y = 3

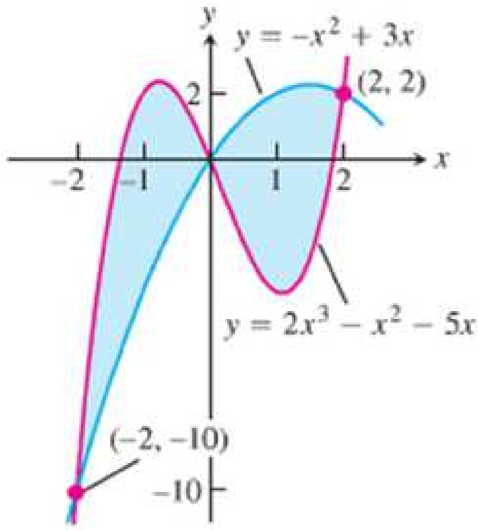


-

See the graphs



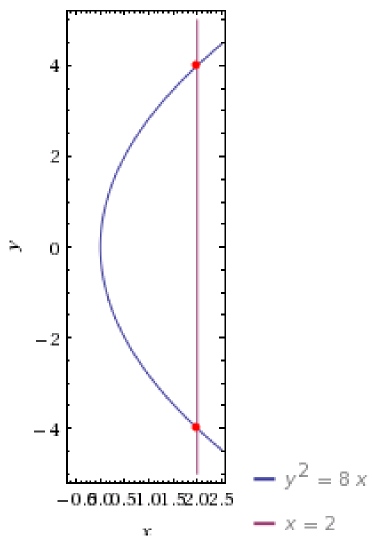
See the graphs



Spoon Feed

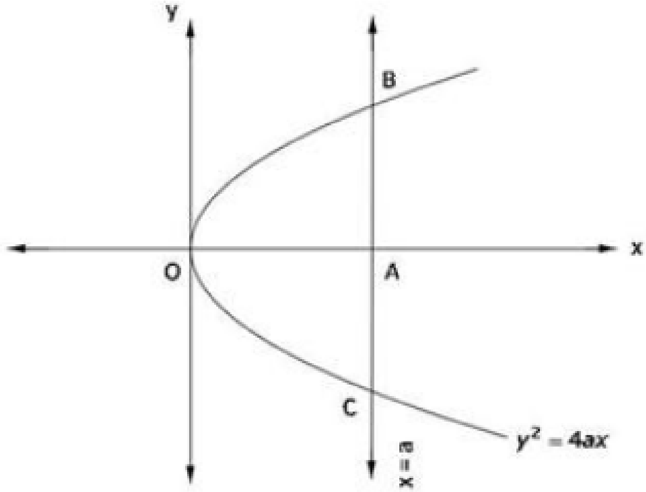
Draw the graphs of $x = 2$ and $y^2 = 8x$

plot	$y^2 = 8\,x$
	$x = 2$



-

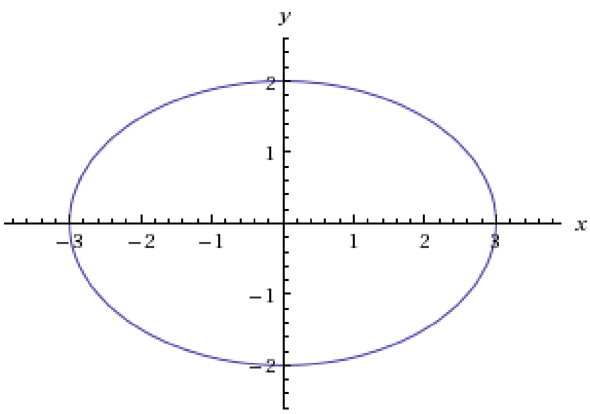
Graph of general $x = a$ and $y^2 = 4ax$



-

Graph of ellipse $4x^2 + 9y^2 = 36$

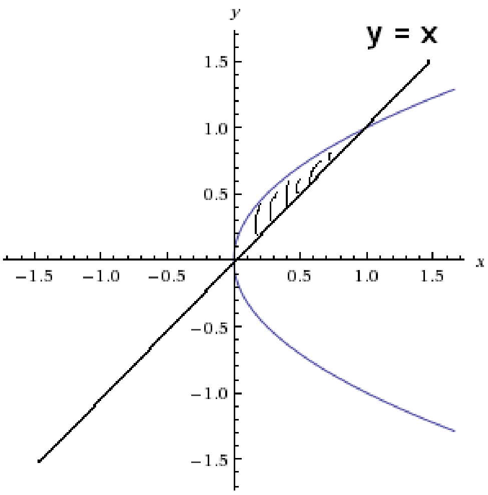
plot	$4x^2 + 9y^2 = 36$



-

Graphs of $y^2 = x$ and the line y = x

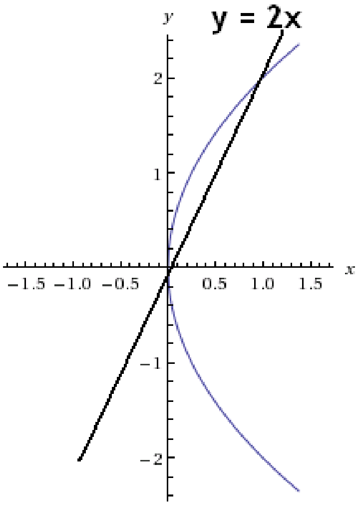
plot	$y^2 = x$



-

Spoon Feeding Graph of $y^2 = 4x$ and y = 2x

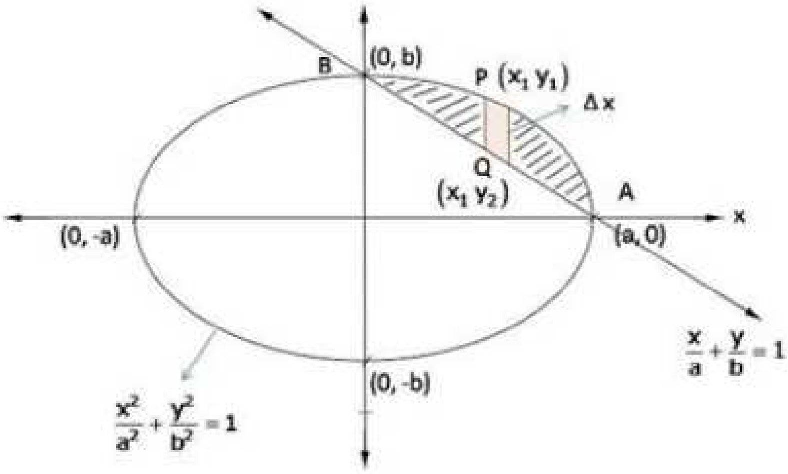
plot	$y^2 = 4x$



We see the intersection point is (1,2).

Graphs of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$

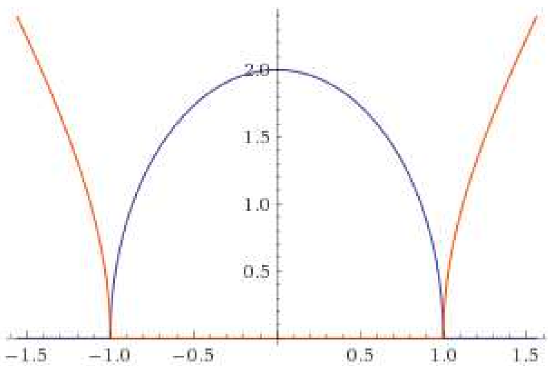
Assuming a > b



-

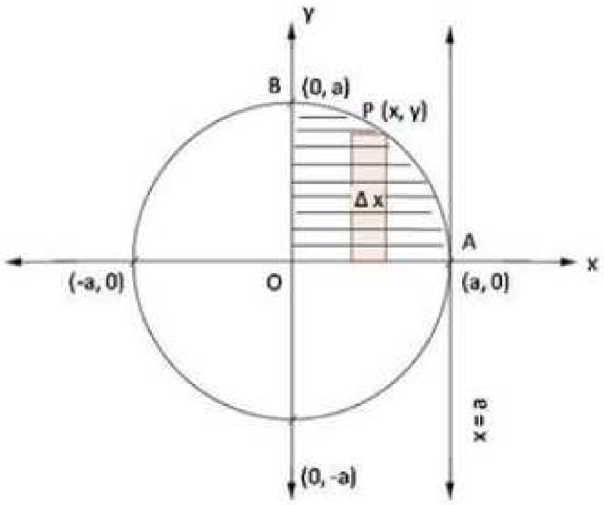
Graph of curve $y = 2\sqrt{1 - x^2}$

plot	$y = 2\sqrt{1 - x^2}$

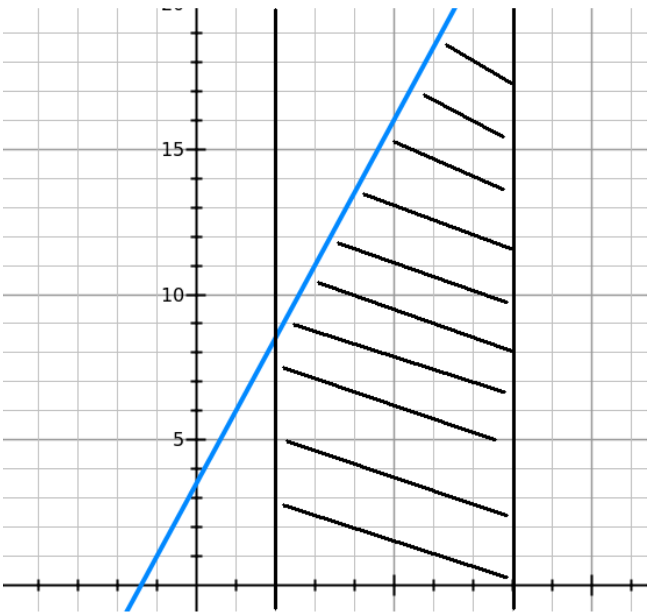


-

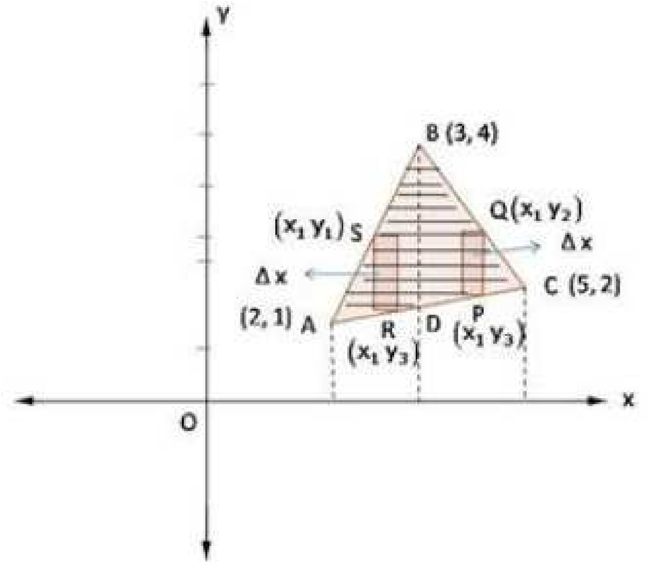
Graph of Circle. The equation of the circle will be $x^2 + y^2 = a^2$ so $y = \sqrt{a^2 - x^2}$



Graphs of 2y = 5x + 7, x = 2, and x = 8

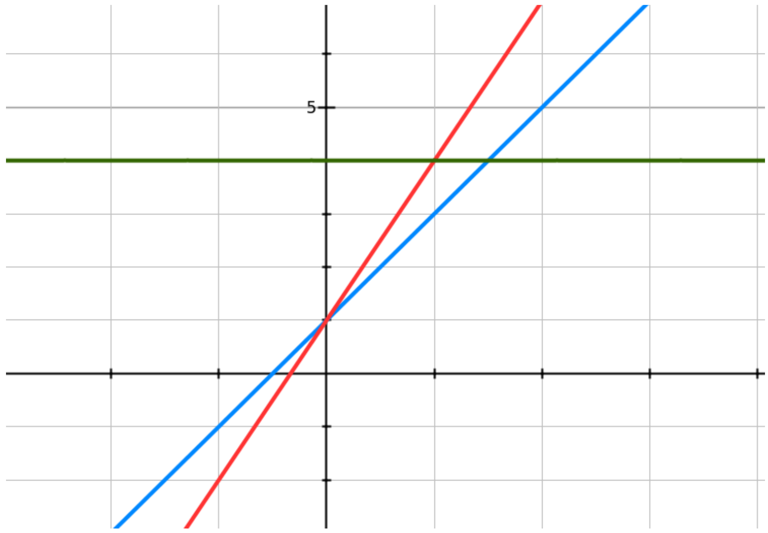


Draw the triangle. The vertices being A (2,1), B (3,4), C (5, 2)



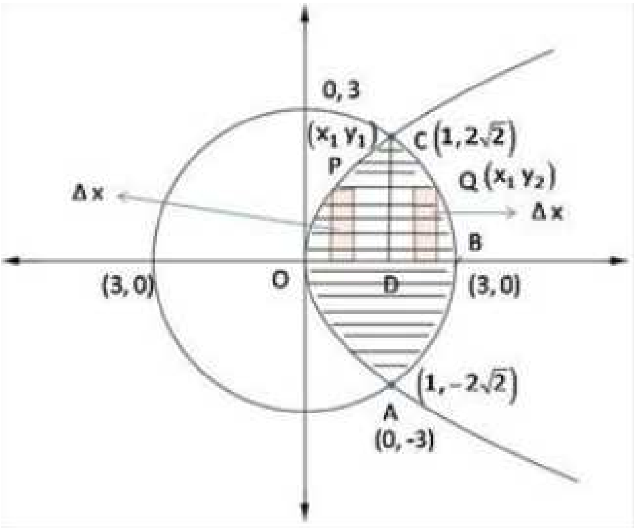
-

Draw the graphs y = 2x + 1 (line A), y = 3x + 1 (line B), y = 4 (line AC)



-

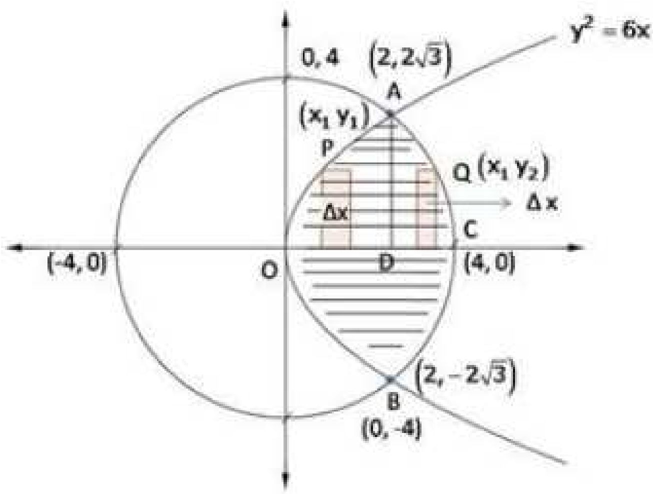
Draw graphs of $y^2 \leq 8x$, $x^2 + y^2 \leq 9$



-

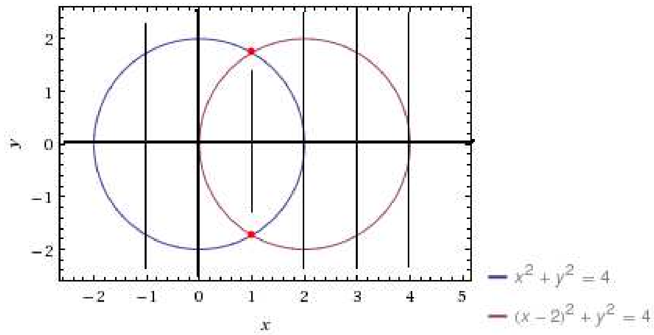
Graphs of $x^2 + y^2 = 16$, and $y^2 = 6x$

The graph will be



Graphs of $x^2 + y^2 = 4$, and $(x - 2)^2 + y^2 = 4$

plot	$x^2 + y^2 = 4$
	$(x - 2)^2 + y^2 = 4$



Equation (1) is a circle with centre O at eh origin and radius 2. Equation (2) is a circle with centre C (2,0) and radius 2. Solving equations (1) and (2), we have

$(x - 2)^2 + y^2 = x^2 + y^2$

Or $x^2 - 4x + 4 + y^2 = x^2 + y^2$

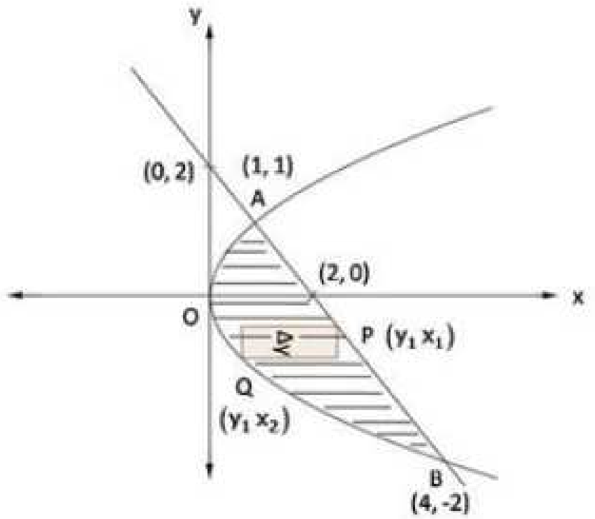
Or $x = 1$ which gives y $\pm\sqrt{3}$

Thus, the points of intersection of the given circles are A $\left(1, \sqrt{3}\right)$ and A' $\left(1, -\sqrt{3}\right)$

Graphs of $y^2 = x$ and $x + y = 2$

Equation (1) represents a parabola with vertex at origin and its axis as x-axis, equation (2) represents a line passing through (2,0) and (0,2). points of intersection of line and parabola are (1,1) and (4,−2).

A rough sketch of curves is as below:-

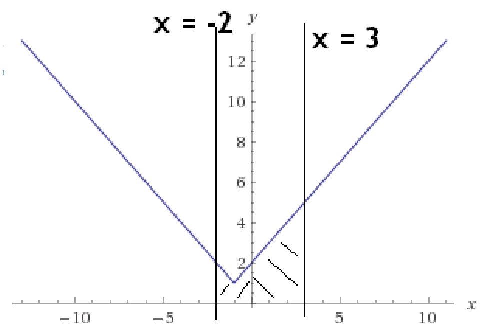


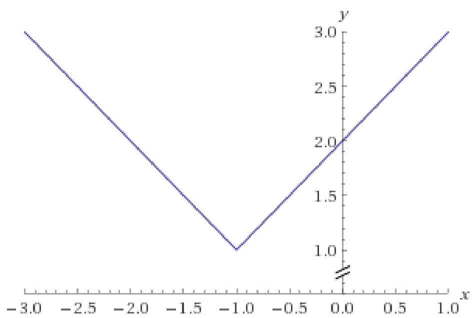
-

Graphs of x = -2, x = 3, x-axis (y = 0), and y = 1 + | x + 1 |

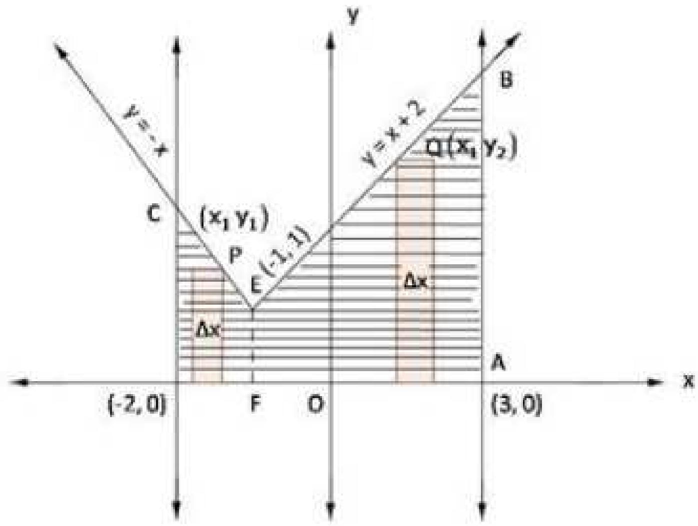
The straight lines for the mod function will flip around x = -1

plot	$y = 1 + \|x + 1\|$





So, equation (1) is a straight line that passes through (0,2) and (−1,1). Equation (2) is a line passing through (−1,1) and (−2,2) and it is enclosed by line $x = 2$ and $x = 3$ which are lines parallel to y-axis and pass through (2,0) and (3,0) respectively $y = 0$ is x-axis. So, a rough sketch of the curves is given as:-

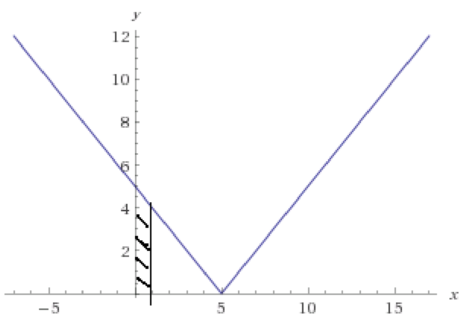


Shaded region represents the required area.

Draw 0 < x < 1 for y = | x - 5 |

The graph of the modulus function will flip around x = 5

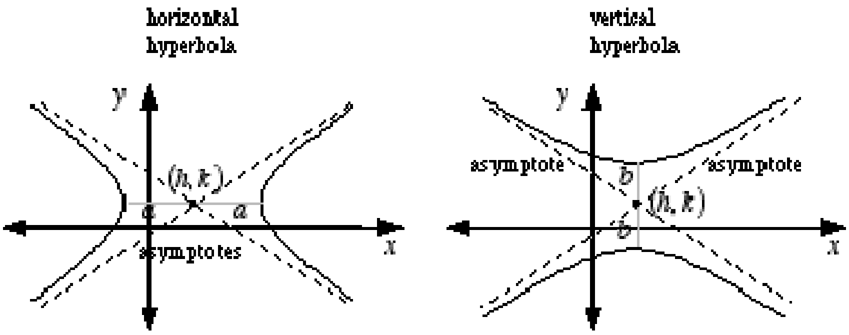
plot	$y = \lvert x - 5 \rvert$



-

Graphs of Hyperbolas

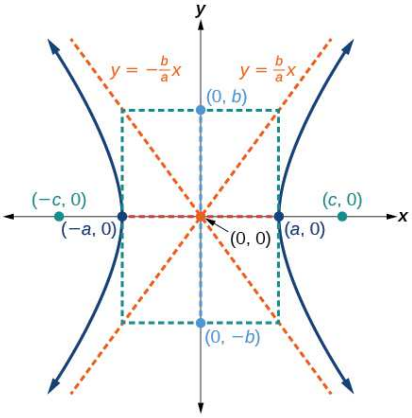
.	x – axis	y – axis
Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$-\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$
Center	$C(h, k)$	$C(h, k)$
Semi – transverse axis	a	a
Semi – conjugate axis	b	b
Vertices	$V(h \pm a, k)$	$V(h, k \pm a)$
Foci	$F(h \pm a e, k)$	$F(h, k \pm a e)$
Directrices	$x = h \pm a / e$	$y = k \pm a / e$
Asymptotes	$b x \pm a y - (b h \pm a k) = 0$	$a x \pm b y - (a h \pm b k) = 0$
Focal chord length	$2 b^2 / a$	$2 b^2 / a$
Eccentricity	$e = \frac{\sqrt{a^2+b^2}}{a} > 1$	$e = \frac{\sqrt{a^2+b^2}}{a} > 1$

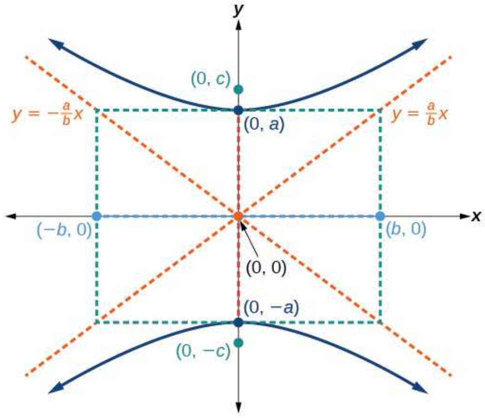


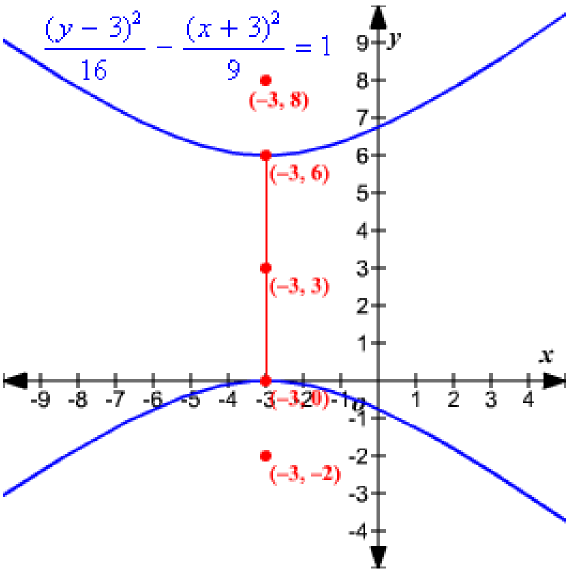
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad \frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

For both horizontal and vertical hyperbolas,

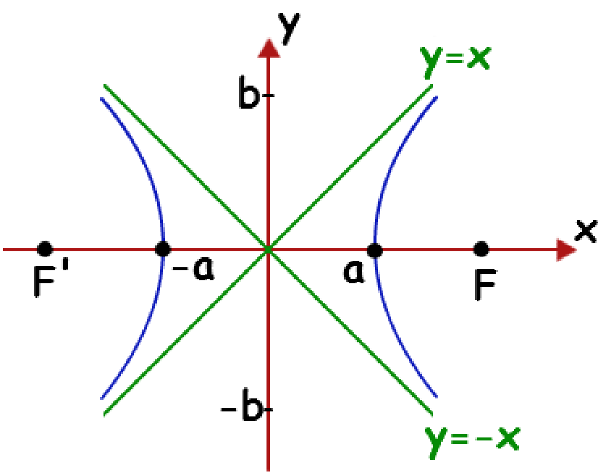
slopes of asymptotes $= \pm \frac{b}{a}$

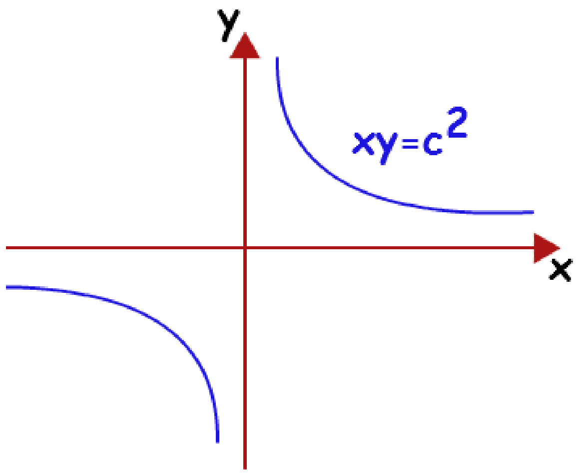


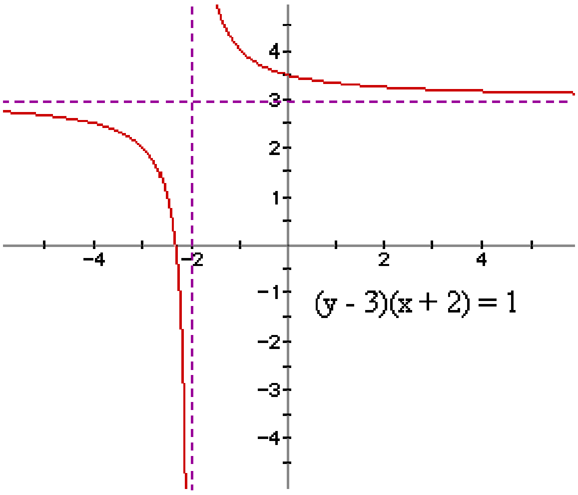




Rectangular Hyperbolas (where the eccentricity = √2 (x^2 - y^2 = 1) and (xy = 1) type



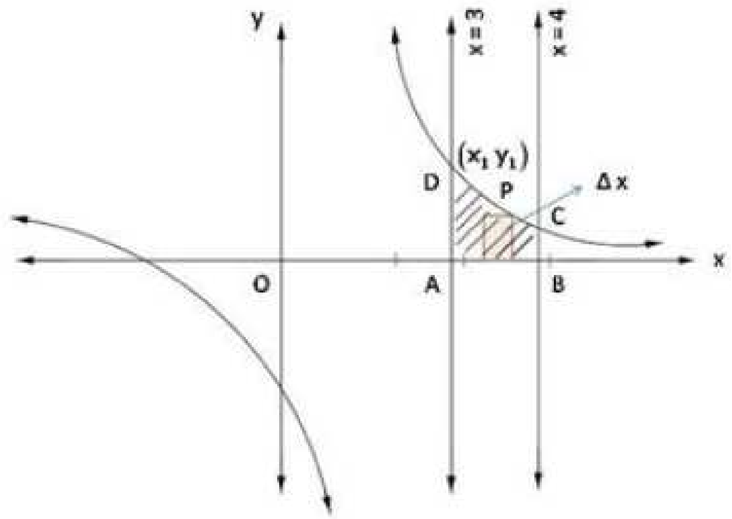



(y - 3)(x + 2) = 1

$\Rightarrow \quad y(x-2) = 3x + 10$

$\Rightarrow \quad y = \dfrac{3x+10}{x-2}$

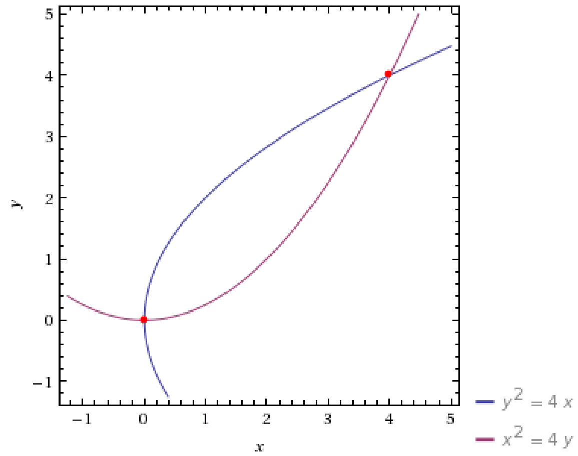
A rough sketch of the curves is given below:-



-

Draw $y^2 = 4x$ and $x^2 = 4y$

plot $y^2 = 4x$, $x^2 = 4y$



-

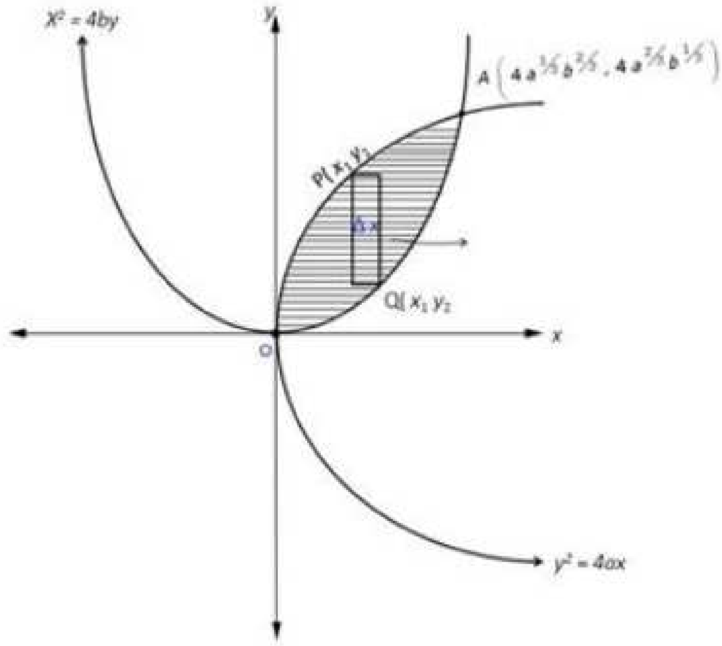
Draw abstract graph of $y^2 = 4ax$ and $x^2 = 4by$

Equation (1) represents a parabola with vertex (0,0) and axis as x-axis,

equation (2) represents a parabola with vertex (0,0) and axis as y-axis,

points of intersection of parabolas are (0,0) and $\left(4a\frac{1}{3}\,b\frac{2}{3},\ 4a\frac{2}{3}\,b\frac{1}{3}\right)$

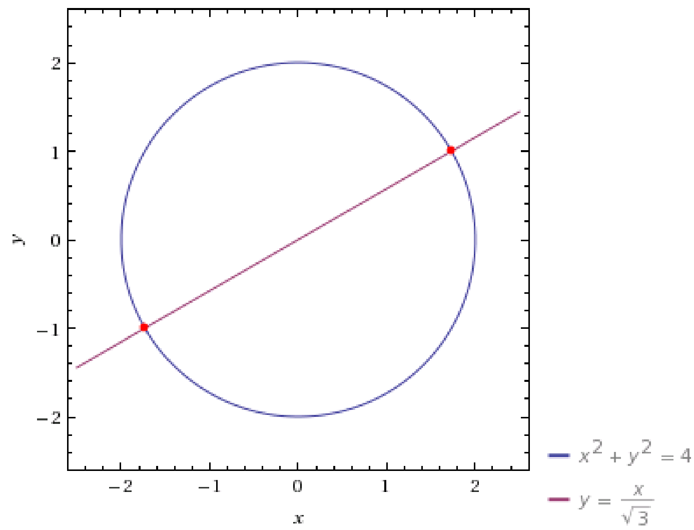
A rough sketch is given as:-



-

Draw graphs of $x^2 + y^2 = 4$ and $x = \sqrt{3}\ y$

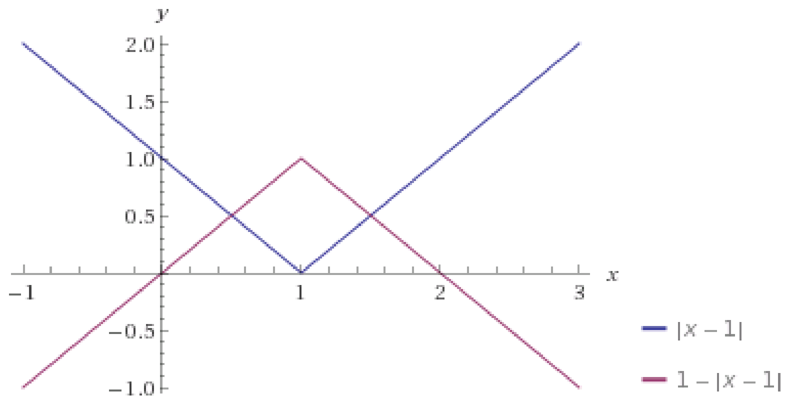
plot	$x^2 + y^2 = 4$
	$y = \frac{x}{\sqrt{3}}$



-

Draw Graphs of y = | x - 1 | and y = - | x - 1 | + 1

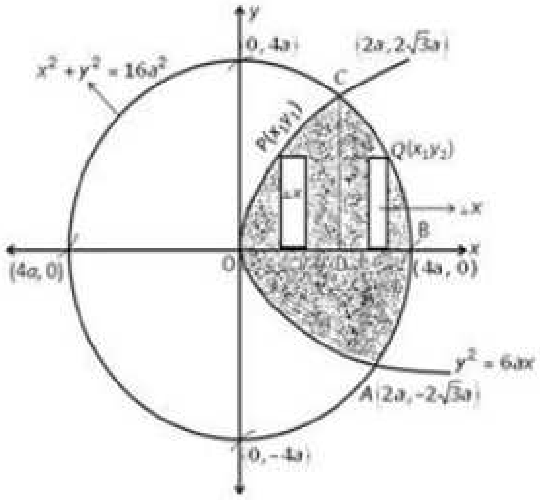
plot	$y = \lvert x-1 \rvert$
	$y = -\lvert x-1 \rvert + 1$



Draw $x^2 + y^2 = 16\,a^2$ and $y^2 = 6ax$

Equation (1) represents a circle with centre $(0,0)$ and meets axes $(\pm 4a, 0), (0, \pm 4a)$. Equation (2) represents a parabola with vertex $(0,0)$ and axis as x-axis. Points of intersection of circle and parabola are $(2a, 2\sqrt{3}a), (2a, -2\sqrt{3}a)$.

A rough sketch of curves is given as:-

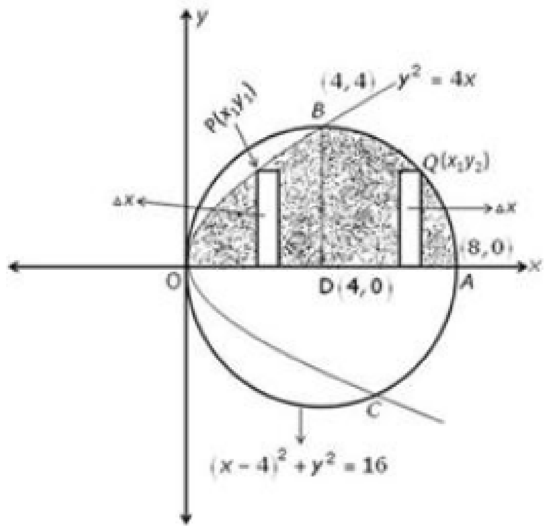


-

Draw $x^2 + y^2 = 8x$ and $(x - 4)^2 + y^2 = 16$ and $y^2 = 4x$

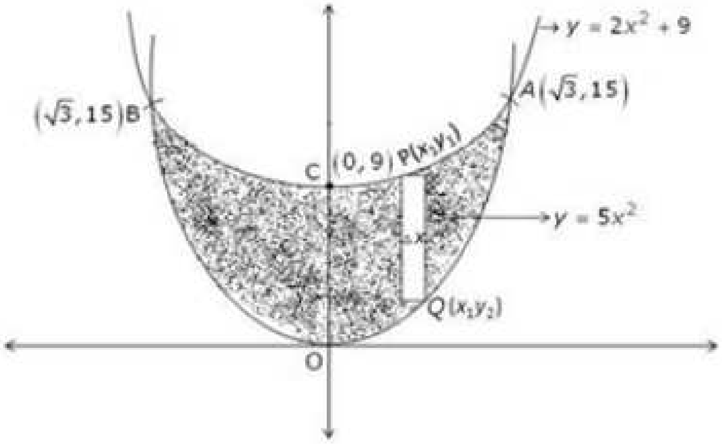
Equation (1) represents a circle with centre (4,0) and meets axes at (0,0) and (8,0). Equation(2) represent a parabola with vertex (0,0) and axis as x-axis. They intersect at (4,−4) and (4,4).

A rough sketch of the curves is as under:-



Shaded region is the required region

A rough sketch of curves is given as:-



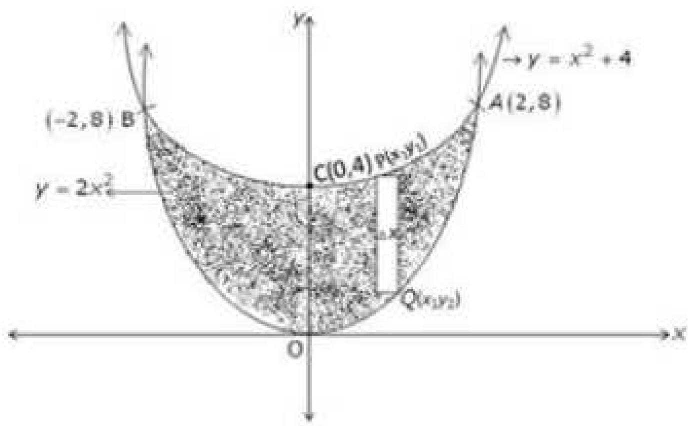
Region $AOCA$ is sliced into rectangles with area $(y_1 - y_2)\Delta x$. It slides from $x = 0$ to $x = \sqrt{3}$, so

-

Graph of $y = 2x^2$ and $y = x^2 + 4$

Equation (1) represents a parabola with vertex (0,0) and axis as y-axis. Equation (2) represents a parabola with vertex (0,4) and axis as y-axis. Points of intersection of parabolas are (2,8) and (−2,8).

A rough sketch of curves is given as:-



Region $AOCA$ is sliced into rectangles with area $(y_1 - y_2)\Delta x$. And it slides from $x = 0$ to $x = 2$

-

Graphs of $x = 0$, $x = 2$, $y = 2^x$, $y = 2x - x^2$

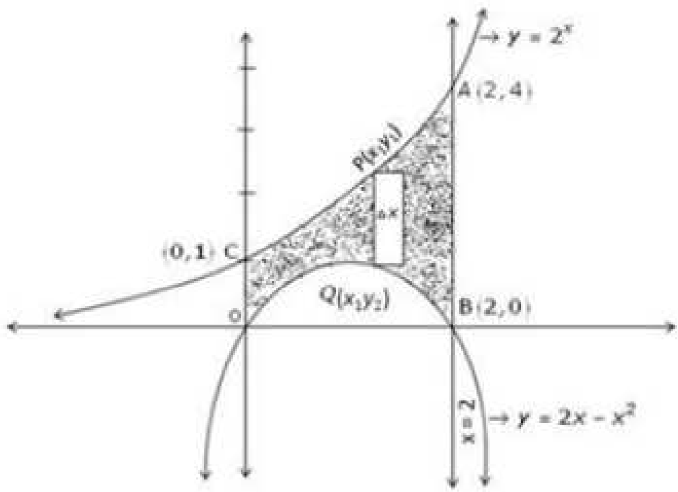
$$\Rightarrow \quad y = -\left(x^2 - 2x + 1 - 1\right)$$

$$= -\left[\left(x - 1\right)^2 - 1\right]$$

$$\Rightarrow \quad y = -\left(x - 1\right)^2 + 1$$

$$\Rightarrow \quad -\left(y - 1\right) = \left(x - 1\right)^2 \qquad ---(2)$$

Equation (2) represents a downward parabola with axis parallel to y-axis and vertex at $(1, -1)$ table for equation (1) is



-

Graphs of $3x^2 + 5y = 32$ and $y = | x - 2 |$

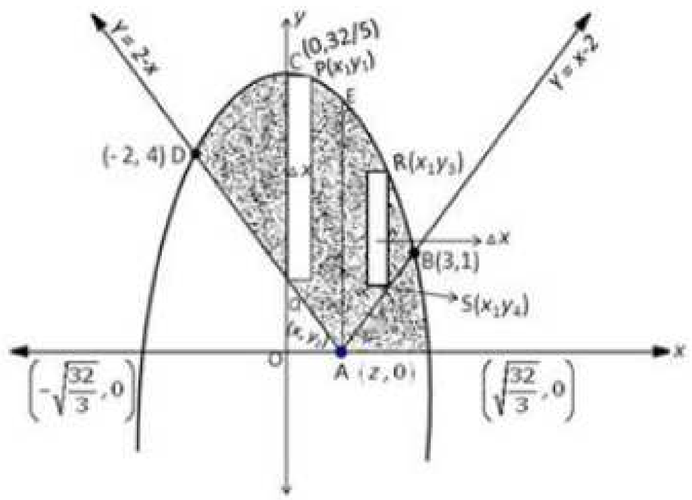
$$3x^2 = -5\left(y - \frac{32}{5}\right) \qquad ---(1)$$

$$y = |x - 2|$$

$$\Rightarrow \quad y = \begin{cases} -(x-2), \text{ if } x - 2 < 1 \\ (x-2), \text{ if } x - 2 \geq 1 \end{cases}$$

$$\Rightarrow \quad y = \begin{cases} 2 - x, \text{ if } x < 2 \\ x - 2, \text{ if } x \geq 2 \end{cases} \qquad ---(2)$$

Equation (1) represents a downward parabola with vertex $\left(0, \frac{32}{5}\right)$ and equation (2) represents lines. A rough sketch of curves is given as: -



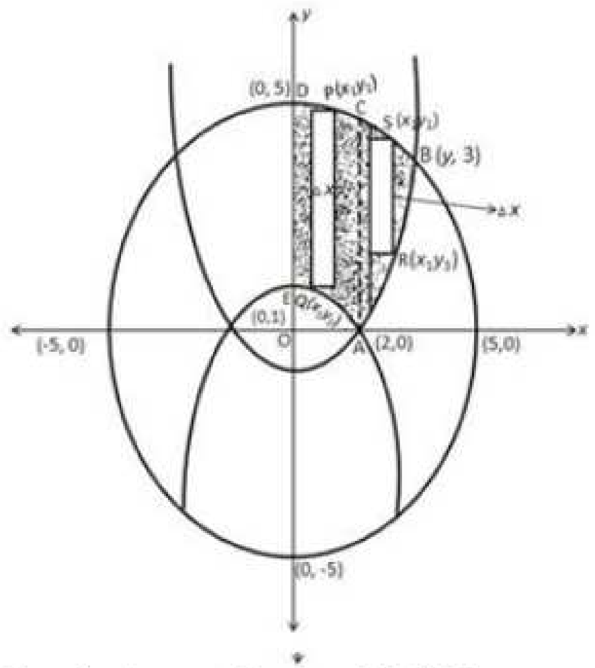
-

Graphs of y-axis (i.e. x = 0), and $4y = | 4 - x^2 |$

$$4y = \begin{cases} 4 - x^2, \text{ if } -2 \le x \le 2 \\ x^2 - 4, \text{ if } x < -2, x > 2 \end{cases}$$

$$\Rightarrow \quad x^2 = \begin{cases} -4(y-1), \text{ if } -2 \le x \le 2 \quad (1) \\ 4(y+1), \text{ if } x < -2, x > 2 \quad (2) \end{cases}$$

Equation (1) represents a parabola with vertex (0,0) and downward. Equation (2) represents an upward parabola with vertex (0, − 1) equation (3) represents a circle with centre (0,0) and meets axes at (±5,0),(±0,5). A rough sketch is as follows:-

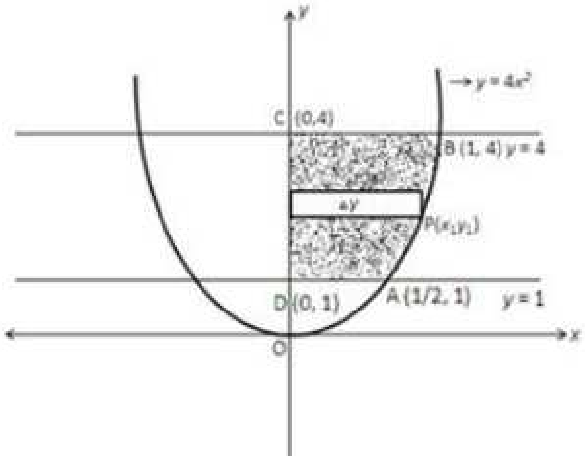


Required area = Region $EABCDE$

-

Graphs of x = 0, y = 1, y = 4, and y = $4x^2$

Equation (1) represents a parabola with vertex (0,0) and axis as y-axis. $x = 0$ is y-axis and $y = 1$, $y = 4$ are lines parallel to x-axis passing through (0,1) and (0, 4) respectively. A rough sketch of the curves is given as:-

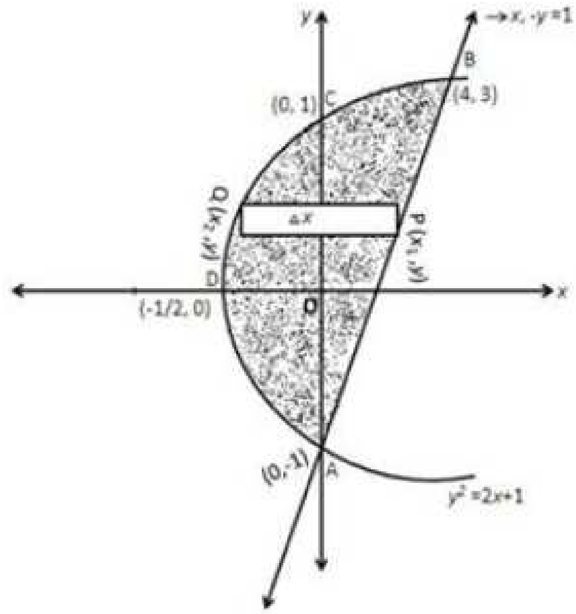


-

Graphs of $y^2 = 2x + 1$, - (1) and $x - y = 1$ - (2)

Equation (1) is a parabola with vertex $\left(-\frac{1}{2}, 0\right)$ and passes through (0,1), (0,−1). Equation (2) is a line passing through (1,0) and (0,−1). Points of intersection of parabola and line are (3,2) and (0,−1).

A rough sketch of the curves is given as:-



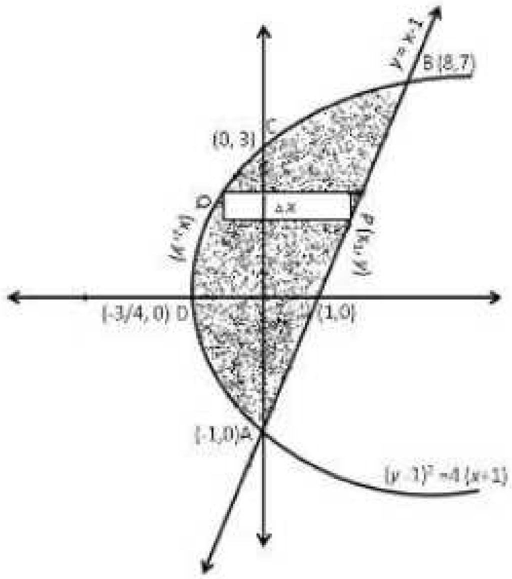
Shaded region represents the required area. It is sliced in rectangles of area $(x_1 - x_2)\Delta y$. It slides from $y = -1$ to $y = 3$, so

-

Graphs of y = x - 1, - (1) and $(y - 1)^2 = 4 (x + 1)$

Equation (1) represents a line passing through $(1,0)$ and $(0,-1)$ equation (2) represents a parabola with vertex $(-1,1)$ passes through $(0,3),(0,-1),\left(-\frac{3}{4},0\right)$. Their points of intersection $(0,-1)$ and $(8,7)$.

A rough sketch of curves is given as:-



-

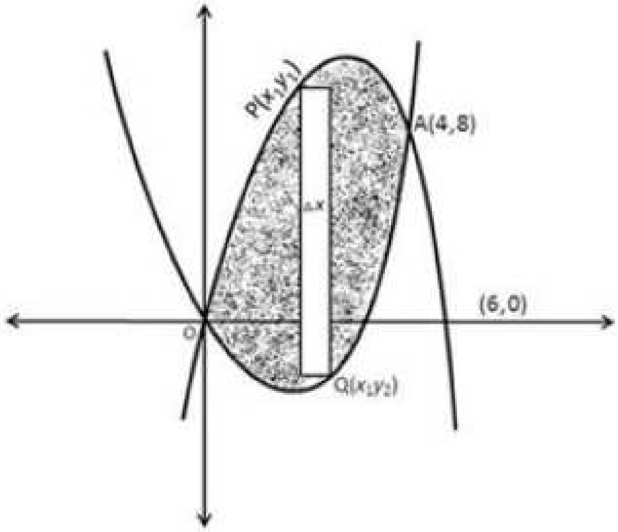
Draw graphs of

$$y = 6x - x^2$$
$$\Rightarrow \quad -y = x^2 - 6x$$
$$\Rightarrow \quad -y = x^2 - 6x + 9 - 9$$
$$\Rightarrow \quad -(y-9) = (x-3)^2 \quad \textbf{(1)}$$

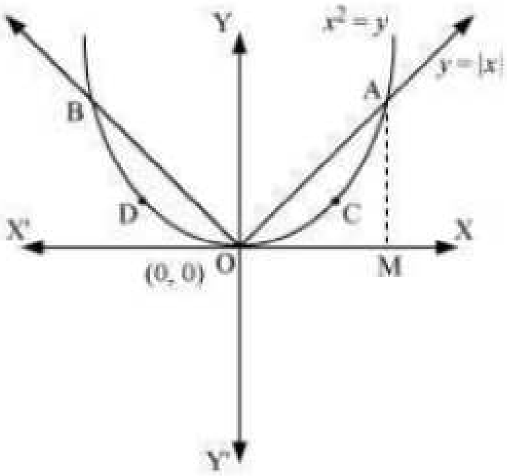
And

$$y = x^2 - 2x$$
$$y + 1 = x^2 - 2x + 1$$
$$(y+1) = (x-1)^2 \quad \textbf{(2)}$$

Equation (1) represents a parabola with vertex (3,9) and downward. Equation (2) represents a parabola with vertex (1, – 1) and upward. Points of intersection of parabolas are (0,0) and (4,8). A rough sketch of the curves is given as:-



Graphs of $y = x^2$, and y = | x |



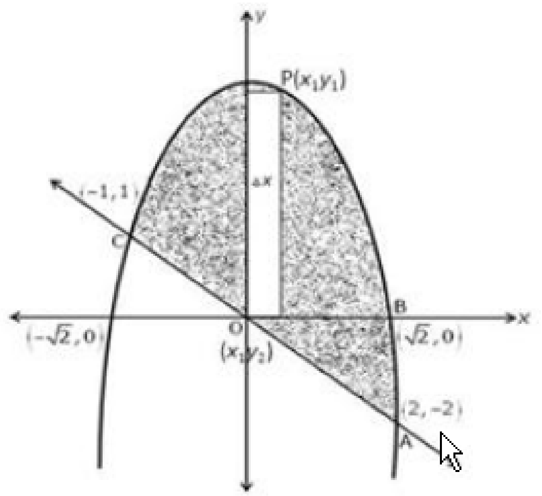
The given area is symmetrical about *y*-axis.

∴ Area OACO = Area ODBO

-

Graphs of y = 2 - x^2 -- (1) and y + x = 0 -- (2)

Equation (1) represents a parabola with vertex (0,2) and downward, meets axes at $(\pm\sqrt{2},0)$.
Equation (2) represents a line passing through (0,0) and (2, − 2). The points of intersection of line and parabola are (2, − 2) and (−1,1).
A rough sketch of curves is as follows:-

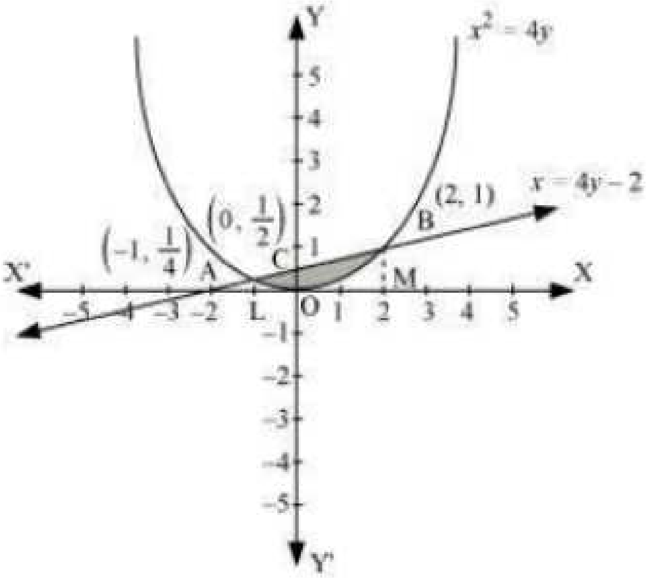


Shaded region is sliced into rectangles with area = $(y_1 - y_2)\Delta x$. It slides from $x = -1$ to $x = 2$, so

-

Graphs of $x^2 = 4y$ -- (1) and $x = 4y - 2$ -- (2)

shaded area OBAO.



Let A and B be the points of intersection of the line and parabola.

Coordinates of point A are $\left(-1, \frac{1}{4}\right)$.

-

Graphs of

$y = 4x - x^2$

$\Rightarrow \quad -y = x^2 - 4x + 4 - 4$

$\Rightarrow \quad -y + 4 = (x - 2)^2$

$\Rightarrow \quad -(y - 4) = (x - 2)^2$ **(1)**

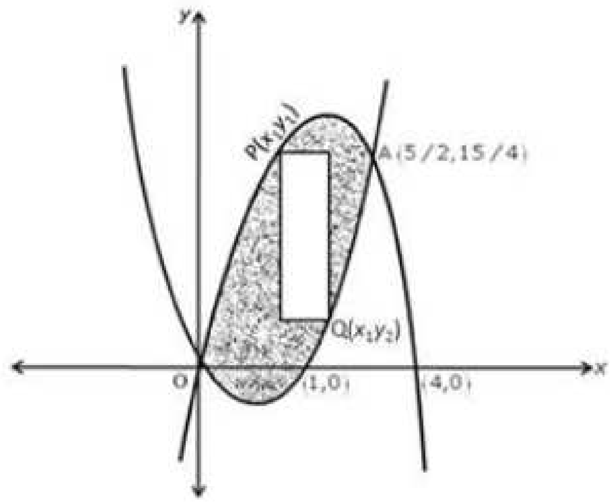
And

and $y = x^2 - x$

$\left(y + \frac{1}{4}\right) = \left(x - \frac{1}{2}\right)^2$ **(2)**

Equation (1) represents a parabola downward with vertex at (2,4) and meets axes at (4, 0), (0,0). Equation (2) represents a parabola upword whose vertex is $\left(\frac{1}{2}, -\frac{1}{4}\right)$ and meets axes at (1,0), (0, 0). Points of intersection of parabolas are (0,0) and $\left(\frac{5}{2}, \frac{15}{4}\right)$.

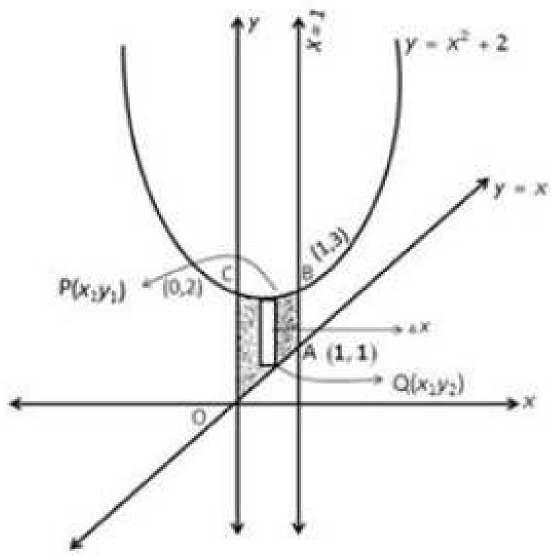
A rough sketch of the curves is as under:-



-

Graphs of x = 0, x = 1 and y = x -- (1) and $y = x^2 + 2$ -- (2)

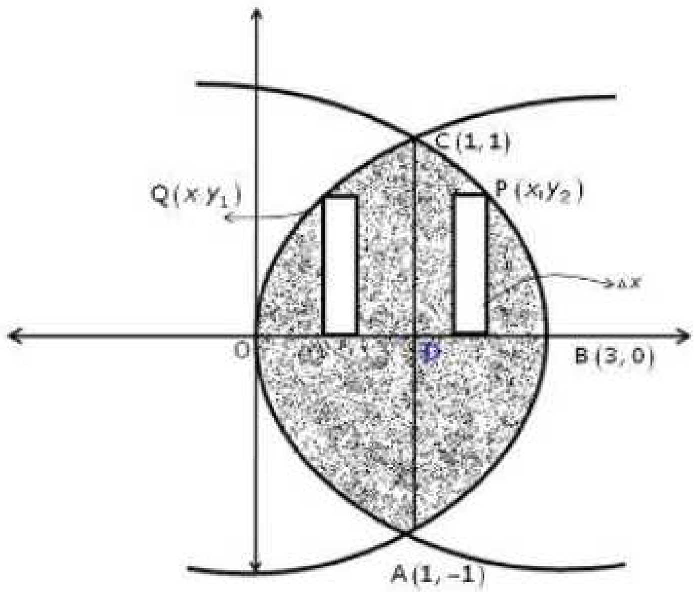
Equation (1) is a line passing through (2,2) and (0,0). Equation (2) is a parabola upward with vertex at (0,2). A rough sketch of curves is as under:-



-

Graphs of $x = y^2$ -- (1) and $x = 3 - 2y^2$ -- (2)

Equation (1) represents an upward parabola with vertex $(0,0)$ and axis $-y$. Equation (2) represents a parabola with vertex $(3,0)$ and axis as x-axis. They intersect at $(1,-1)$ and $(1,1)$. A rough sketch of the curves is as under:-



Graphs of $y = 4x - x^2$ -- (1) $y = x^2 - x$ -- (2)

Given curves are

$y = 4x - x^2$

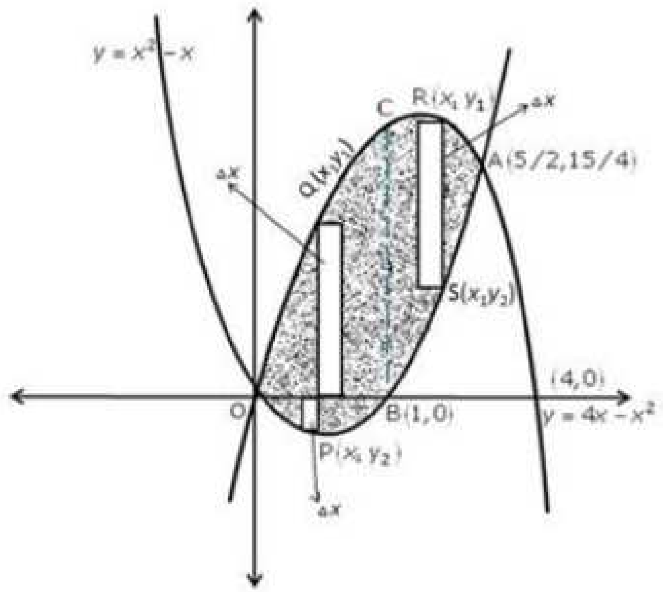
$\Rightarrow \quad -(y-4) = (x-2)^2 \qquad ---(1)$

and

$y = x^2 - x$

$\Rightarrow \quad \left(y + \frac{1}{4}\right)^2 = \left(x - \frac{1}{2}\right)^2 \qquad ---(2)$

Equation (1) represents a parabola downward with vertex at $(2,4)$ and meets axes at $(4,0),(0,0)$. Equation (2) represents a parabola upward whose vertex is $\left(\frac{1}{2},-\frac{1}{4}\right)$ and meets axes at $(1,0),(0,0)$ and $\left(\frac{5}{2},\frac{15}{4}\right)$. A rough sketch of the curves is as under:-



Area of the region above x-axis

-

Graphs of y = | x - 1 | -- (1) and y = 3 - | x | -- (2)

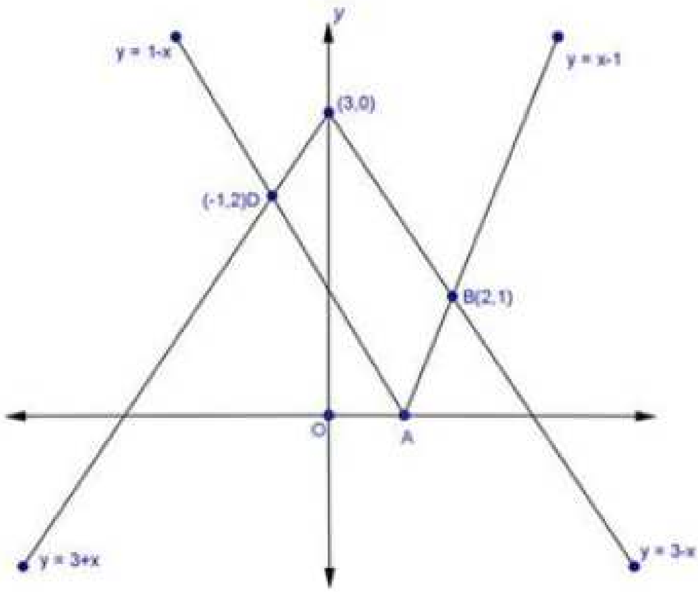
$$y = |x-1|$$

$$\Rightarrow \quad y = \begin{cases} 1-x, \text{ if } x < 1 & ---(1) \\ x-1, \text{ if } x \ge 1 & ---(2) \end{cases}$$

and $\quad y = 3 - |x|$

$$\Rightarrow \quad y = \begin{cases} 3+x, \text{ if } x < 0 & ---(3) \\ 3-x, \text{ if } x \ge 0 & ---(4) \end{cases}$$

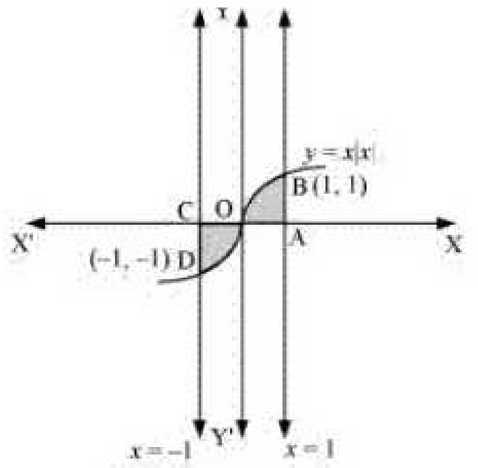
Drawing the rough sketch of lines (1), (2), (3) and (4) as under:-



Shaded region is the required area

-

Graphs of y = x|x| - (1) and x = -1 and x = 1



$$\text{Required area} = \int_{-1}^{1} y\,dx$$

$$= \int_{-1}^{1} x|x|\,dx$$

$$= \int_{-1}^{0} x^2 dx + \int_{0}^{1} x^2 dx$$

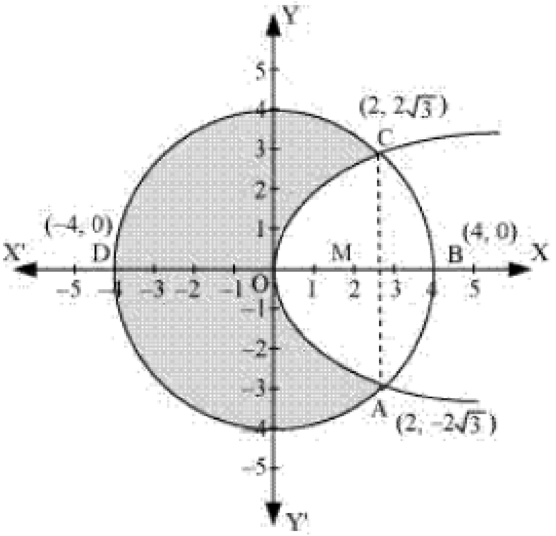
$$= \left[\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{1}$$

$$= -\left(-\frac{1}{3}\right) + \frac{1}{3}$$

$$= \frac{2}{3} \text{ sq. units}$$

-

Graphs of $x^2 + y^2 = 16$ -- (1) and $y^2 = 6x$ -- (2)



Area bounded by the circle and parabola

$$= 2\left[\text{Area}(\text{OADO}) + \text{Area}(\text{ADBA})\right]$$

$$= 2\left[\int_0^2 \sqrt{16x}\,dx + \int_2^4 \sqrt{16 - x^2}\,dx\right]$$

-

Function

A function is a relation for which there is only one value of y corresponding to any value of x. We sometimes write y = f (x), which is notation meaning 'y is a function of x'.

Some very common mathematical constructions are not functions. For example, consider the relation $x^2 + y^2 = 4$ because multiple values can satisfy the equation. Ay put y = 0, then for x = 2 and x = -2 both the expression is 4

There is a simple test to check if a relation is a function, by looking at its graph. This test is called the
vertical line test. If it is possible to draw any vertical line (a line of constant x) which crosses the graph of the relation more than once, then the relation is not a function. If more than one intersection point exists, then the intersections correspond to multiple values of y for a single value of x.

An inverse function is a function which "does the reverse" of a given function. More formally, if f is a
function with domain X, then f^{-1} is its inverse function if and only if for every x ε X we have

$$f^{-1}(f(x)) = \mathbf{x}$$

A simple way to think about this is that a function, say y = f (x), gives you a y-value if you substitute an
x-value into f (x). The inverse function tells you tells you which x-value was used to get a particular y-valuewhen you substitue the y-value into f^{-1} (x).

If f(x) = 3x + 2 then find f^{-1} (x)

Solution :
Put y = 3x + 2 and solve for x => y - 2 = 3x or x = (y - 2)/3
Now exchange x and y
So f^{-1}(x) = (x - 2)/3

-

We say that a function g(x) is periodic if there is a positive or negative number T for which g(x + T) = g(x) for all x We call T a period of g(x)

A periodic function has many periods

Since the graph of g repeats after x increases by T, it also repeats after x increases by 2T, or −3T, or any integer multiple (positive or negative) of T. This means that a periodic function always has many periods. (That's why the definition refers to "a period" rather than "the period.")

The period of a periodic function is its smallest positive period. It is the size of a single cycle.

If the function g(x) is periodic, then its frequency is the number of cycles per unit x.

In general, if f is the frequency of a periodic function g(x) and T is its period, then we have

$$f = \frac{1}{T} \quad \textbf{and} \quad T = \frac{1}{f}$$

if the period is measured in seconds, then the frequency is measured in cycles per second.

The term Hertz is a special unit used to measure time frequencies; it equals one cycle per second. Hertz is abbreviated Hz; thus a kilohertz (kHz) and a megahertz (MHz) are 1,000 and 1,000,000 cycles per second, respectively. This unit is commonly used to describe sound, light, radio, and television waves.

Circular functions. While there are innumerable examples of periodic functions, two in particular are considered basic: the sine and the cosine. They are called circular functions because they are defined by means of a circle. To be specific, take the circle of radius 1 centered at the origin in the x, y-plane. Given any real number t, measure a distance of t units around the circumference of the circle.

Start on the positive x-axis, and measure counterclockwise if t is positive, clockwise if t is negative. The coordinates of the point you reach this way are, by definition, the cosine and the sine functions of t, respectively:
$x = \cos(t)$
$y = \sin(t)$

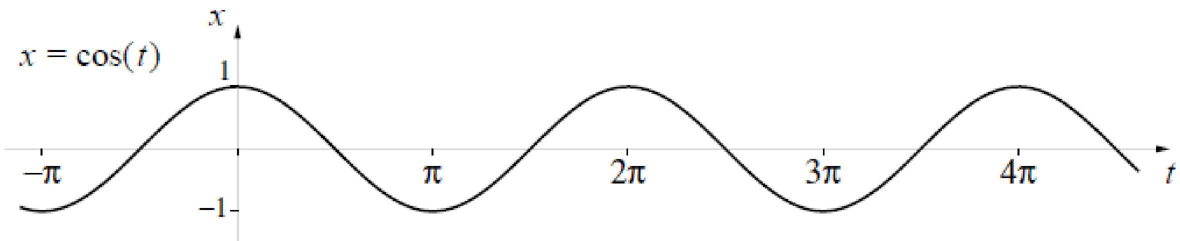
The whole circumference of the circle measures 2π units. Therefore, if we add 2π units to the t units we have already measured, we will arrive back at the same point on the circle. That is, we get to the same point on the circle by measuring either t or $t + 2\pi$ units around the circumference. We can describe the coordinates of this point two ways:

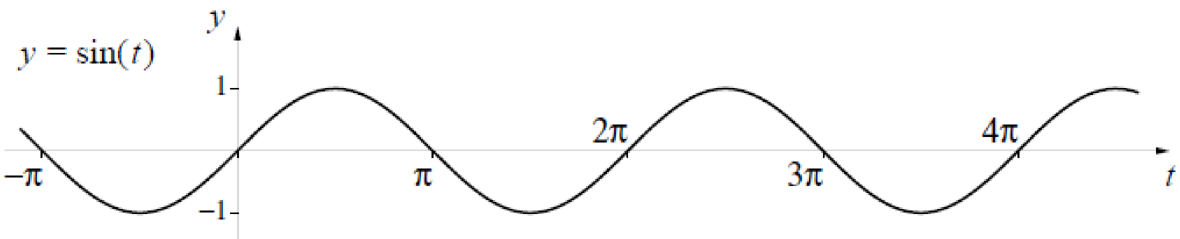
$(\cos(t), \sin(t))$ or $(\cos(t + 2\pi), \sin(t + 2\pi))$

Thus

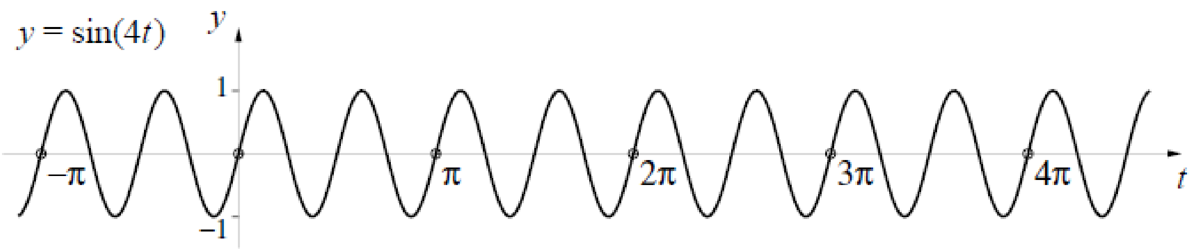
$\cos(t + 2\pi) = \cos(t)$ $\sin(t + 2\pi) = \sin(t)$,
so cos(t) and sin(t) are both periodic, and they have the same period, 2π.

Here are their graphs. By reading their slopes we can see $(\sin t)' = \cos t$ and $(\cos t)' = -\sin t$





While graph of $y = \sin(4t)$ will be



Their scales are identical, so it is clear that the frequency of sin(4t) is four times the frequency of sin(t). The general pattern is described in the following table.

function		period	frequency
$\sin(t)$	$\cos(t)$	2π	$1/2\pi$
$\sin(4t)$	$\cos(4t)$	$2\pi/4$	$4/2\pi$
$\sin(bt)$	$\cos(bt)$	$2\pi/b$	$b/2\pi$

Notice that it is the frequency—not the period—that is increased by a factor of b when we multiply the input variable by b.

Constructing a circular function with a given frequency

By using the information in the table, we can circular functions with any period or frequency whatsoever. For instance, suppose we wanted a cosine function x = cos(bt) with a frequency of 5 cycles per unit t. This means

$$5 = \text{frequency} = \frac{b}{2\pi}$$

which implies that we should set b = 10π and x = cos(10πt)

-

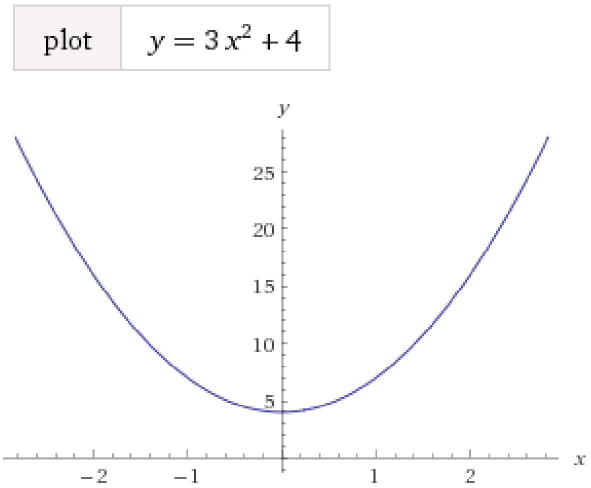
Domain and Range of functions (graphs)

In simple words - Given a function, the values of x that are allowed to be supplied to the function, so that all terms remain real, is the domain. So Domain is the values for x as it varies.

Range are the values y = f(x) takes, keeping the function as Real. So Range is the way y varies.

Consider the parabola $y = f(x) = 3x^2 + 4$

The graph will be



We see that y is minimum 5 or more. So Range is [5, ∞) while Domain is (-∞ , ∞) or] -∞ , ∞ [

Note : the round brackets (or) means open bracket. Meaning close to that value but not equal to.

∞ is always written with open bracket ") " as we can have a very high value, but exactly infinity is undefined. Other way of writing the same thing is $5 \le y < \infty$

Thus the ≤ sign is giving the symbol " ["

In some statement if we get $6 < y < 9$ then same thing will be written as (6, 9)

But $1 \le y \le 8$ will be [1, 8]

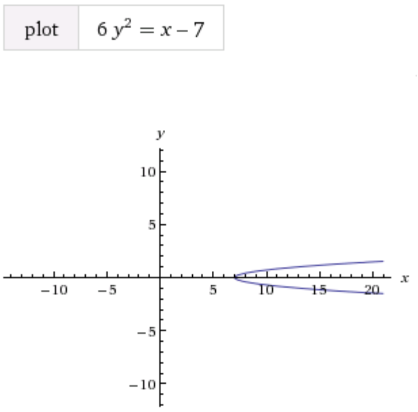
For the above problem for all values (positive, negative, zero) of x the graph exists. So $-\infty < x < \infty$ watch the ≤ sign is not being used for infinity.

Now some Mathematicians take pleasure in writing the same thing as] -∞ , ∞ [as this is easier to print.

-

Spoon Feeding : Find Domain and Range for $x = 6y^2 + 7$ and write the answer in all methods

Graph will be



So Domain is : $7 \le x$ or $7 \le x < \infty$ or $[7, \infty)$ or $[7, \infty[$

And Range is : $]-\infty, \infty[$ or $(-\infty, \infty)$ or $-\infty < y < \infty$

-

Spoon Feeding : One of the most favorite questions by Math teachers, of standard 11 is to ask Domain and Range of $y = f(x) = \sqrt{\frac{9 - x}{x - 1}}$

Let me try to solve this without drawing the graph. If the student can guess the graph, of-course it becomes very easy to solve.

If x becomes more than 9 then Numerator(N) becomes negative while Denominator(D) remains positive. As imaginary value of y is not allowed, x has to be less than or equal to 9 so $x \le 9$

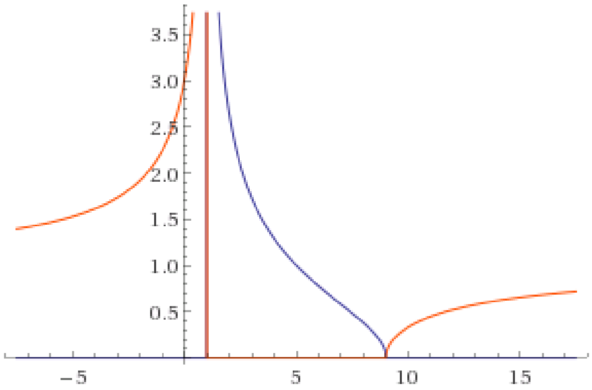
N can be zero, but D is not allowed to be zero as dividing by 0 is not defined. So $x = 1$ is not allowed. If $x < 1$ then the N remain +ve but D becomes -ve. As y cannot be imaginary, $1 < x$, meaning x has to be greater than 1. But x can be arbitrarily close to 1 say $1 + \delta$ where δ is very small positive number. In that case D becomes δ while $N = 9 - (1 + \delta) = 8 - \delta$

$(8 - \delta) / \delta$ tends to ∞ So Range for y will go upwards to infinity. But what will be the least value ?

We already saw that N can be zero (0) but not negative. So Range will be from 0 to infinity.

Now let us see the graph

plot	$y = \sqrt{\dfrac{9 - x}{x - 1}}$



Ignore the red lines of imaginary solutions.

Let us write the Solutions

Domain : $1 < x \leq 9$ or (1, 9] or] 1, 9]   Happy ?

Range : $0 \leq y < \infty$ or [0, ∞) or [0, ∞ [☺

Spoon Feeding : Find Domain and Range of y = f(x) = $\sqrt{\dfrac{8 - x}{1 - x}}$

If x is in between 1 and 8, say 6 then N is +ve while D is -ve, which is not allowed.

If x is greater than 8 (8 < x) then both N and D are negative so fine for us, as y the function is real. If x is less than 1 say 0 or -100 then also y is real. As both N, and D is positive.

We need to analyze the function when x is very close to 1 i.e. x = 1 - δ where δ is very small positive number. D becomes 1 - (1 - δ) = δ

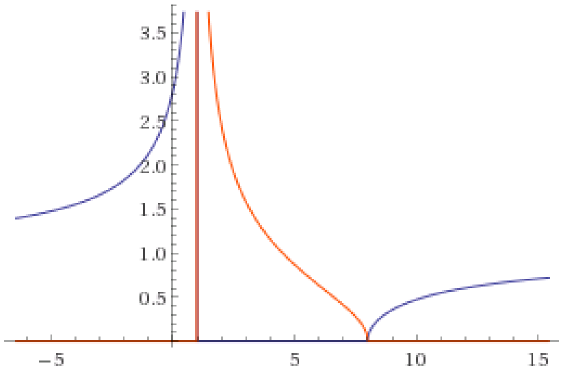
Positive D divided by δ will tend to infinity.

Now we can write the Domain : $8 \leq x$ or $x < 1$

While Range : $0 \leq y < \infty$ or [0, ∞) or [0, ∞ [☺

Let us confirm by drawing the graph. (Though in the exam you have to just guess the graph)

plot	$y = \sqrt{\dfrac{8-x}{1-x}}$



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To recall standard integrals

$f(x)$	$\int f(x)dx$	$f(x)$	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1} \quad (n \neq -1)$	$[g(x)]^n g'(x)$	$\frac{[g(x)]^{n+1}}{n+1} \quad (n \neq -1)$
$\frac{1}{x}$	$\ln\|x\|$	$\frac{g'(x)}{g(x)}$	$\ln\|g(x)\|$
e^x	e^x	a^x	$\frac{a^x}{\ln a} \quad (a > 0)$
$\sin x$	$-\cos x$	$\sinh x$	$\cosh x$
$\cos x$	$\sin x$	$\cosh x$	$\sinh x$
$\tan x$	$-\ln\|\cos x\|$	$\tanh x$	$\ln\cosh x$
$\text{cosec}\, x$	$\ln\left\|\tan\frac{x}{2}\right\|$	$\text{cosech}\, x$	$\ln\left\|\tanh\frac{x}{2}\right\|$
$\sec x$	$\ln\|\sec x + \tan x\|$	$\text{sech}\, x$	$2\tan^{-1} e^x$
$\sec^2 x$	$\tan x$	$\text{sech}^2 x$	$\tanh x$
$\cot x$	$\ln\|\sin x\|$	$\coth x$	$\ln\|\sinh x\|$
$\sin^2 x$	$\frac{x}{2} - \frac{\sin 2x}{4}$	$\sinh^2 x$	$\frac{\sinh 2x}{4} - \frac{x}{2}$
$\cos^2 x$	$\frac{x}{2} + \frac{\sin 2x}{4}$	$\cosh^2 x$	$\frac{\sinh 2x}{4} + \frac{x}{2}$

$f(x)$	$\int f(x)\,dx$	$f(x)$	$\int f(x)\,dx$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a}$	$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert$ $(0<\lvert x\rvert<a)$
	$(a>0)$	$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert$ $(\lvert x\rvert>a>0)$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\frac{x}{a}$	$\frac{1}{\sqrt{a^2+x^2}}$	$\ln\left\lvert\frac{x+\sqrt{a^2+x^2}}{a}\right\rvert$ $(a>0)$
	$(-a<x<a)$	$\frac{1}{\sqrt{x^2-a^2}}$	$\ln\left\lvert\frac{x+\sqrt{x^2-a^2}}{a}\right\rvert$ $(x>a>0)$
$\sqrt{a^2-x^2}$	$\frac{a^2}{2}\left[\sin^{-1}\left(\frac{x}{a}\right)\right.$	$\sqrt{a^2+x^2}$	$\frac{a^2}{2}\left[\sinh^{-1}\left(\frac{x}{a}\right)+\frac{x\sqrt{a^2+x^2}}{a^2}\right]$
	$\left.+\frac{x\sqrt{a^2-x^2}}{a^2}\right]$	$\sqrt{x^2-a^2}$	$\frac{a^2}{2}\left[-\cosh^{-1}\left(\frac{x}{a}\right)+\frac{x\sqrt{x^2-a^2}}{a^2}\right]$

Some series Expansions -

$$\frac{\pi}{2}=\left(\frac{2}{1}\frac{2}{3}\right)\left(\frac{4}{3}\frac{4}{5}\right)\left(\frac{6}{5}\frac{6}{7}\right)\left(\frac{8}{7}\frac{8}{9}\right)\dots$$

$$\pi=\frac{4}{1}-\frac{4}{3}+\frac{4}{5}-\frac{4}{7}+\frac{4}{9}-\frac{4}{11}+\frac{4}{13}-\cdots$$

$$\frac{\pi}{4}=\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\dots$$

$$\pi=\sqrt{12}\left(1-\frac{1}{3\cdot 3}+\frac{1}{5\cdot 3^2}-\frac{1}{7\cdot 3^3}+\cdots\right)$$

$$\frac{\pi^2}{6}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\dots=\sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\int_0^{\pi/2}\log\sin x\,dx=-\frac{\pi}{2}\log 2=\frac{\pi}{2}\log\frac{1}{2}$$

Solve a series problem

If $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots$ upto $\infty = \frac{\pi^2}{6}$, then value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots$ up to ∞ is

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{6}$ (c) $\frac{\pi^2}{8}$ (d) $\frac{\pi^2}{12}$

Ans. (c)

Solution We have $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots$ upto ∞

$$= \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}\cdots \text{ upto } \infty$$

$$-\frac{1}{2^2}\left[1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots\right]$$

$$= \frac{\pi^2}{6}-\frac{1}{4}\left(\frac{\pi^2}{6}\right) = \frac{\pi^2}{8}$$

$$1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\frac{1}{5^2}-\frac{1}{6^2}+\ldots\infty = \frac{\pi^2}{12}$$

$$\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+\ldots\infty = \frac{\pi^2}{24}$$

$$\frac{\sin\sqrt{x}}{\sqrt{x}} = 1-\frac{x}{3!}+\frac{x^2}{5!}-\frac{x^3}{7!}+\frac{x^4}{9!}-\frac{x^5}{11!}+\ldots$$

$$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots = \sum_{k=0}^{n}\frac{(-1)^k x^{2k}}{(2k)!}$$

$$\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots = \sum_{k=0}^{n}\frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$\cosh x = 1+\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots = \sum_{k=0}^{n}\frac{x^{2k}}{(2k)!}$$

$$\sinh x = x+\frac{x^3}{3!}+\frac{x^5}{5!}+\cdots = \sum_{k=0}^{n}\frac{x^{2k+1}}{(2k+1)!}$$

$$\tan^{-1} x = x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots \qquad (-1 \le x < 1)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} \cdots + \frac{2^{2n}\left(2^{2n}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \qquad |x| < \frac{\pi}{2}$$

$$\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \cdots + \frac{E_n x^{2n}}{(2n)!} + \cdots \quad |x| < \frac{\pi}{2}$$

$$\csc x = \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^5}{15120} + \cdots + \frac{2\left(2^{2n-1}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \qquad 0 < |x| < \pi$$

$$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - -\cdots - \frac{2^{2n} B_n x^{2n-1}}{(2n)!} - \cdots \quad 0 < |x| < \pi$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$$

$$\sec x = 1 + \frac{x^2}{\lfloor 2} + \frac{5x^4}{\lfloor 4} + \cdots$$

$$\log(\cos x) = -\frac{x^2}{\lfloor 2} - \frac{2x^4}{\lfloor 4} - \cdots$$

$$\log(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \cdots$$

$$\sin^{-1} x = x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \cdots \quad |x| < 1$$

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$= \frac{\pi}{2} - \left(x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \cdots\right) \quad |x| < 1$$

$$\tan^{-1} x = \begin{cases} x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \cdots & |x| < 1 \\ \pm\dfrac{\pi}{2} - \dfrac{1}{x} + \dfrac{1}{3x^3} - \dfrac{1}{5x^5} + \cdots & \begin{cases} + \text{ if } x \geq 1 \\ - \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$$

$$= \frac{\pi}{2} - \left(\frac{1}{x} + \frac{1}{2\cdot 3x^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5x^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \cdots\right) \quad |x| > 1$$

$$\csc^{-1} x = \sin^{-1}(1/x)$$

$$= \frac{1}{x} + \frac{1}{2\cdot 3x^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5x^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \cdots \quad |x| > 1$$

$$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$$

$$= \begin{cases} \dfrac{\pi}{2} - \left(x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \cdots\right) & |x| < 1 \\ p\pi + \dfrac{1}{x} - \dfrac{1}{3x^3} + \dfrac{1}{5x^5} + \cdots & \begin{cases} p = 0 \text{ if } x \geq 1 \\ p = 1 \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\ln x = 2\left[\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \ldots\right]$$

$$= 2\sum_{n=1}^{\infty} \frac{1}{2n-1}\left(\frac{x-1}{x+1}\right)^{2n-1} \quad (x > 0)$$

$$\ln x = \frac{x-1}{x} + \frac{1}{2}\left(\frac{x-1}{x}\right)^2 + \frac{1}{3}\left(\frac{x-1}{x}\right)^3 + \ldots$$

$$= \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{x-1}{x}\right)^n \quad (x > \frac{1}{2})$$

$$\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}(x-1)^n \quad (0 < x \le 2)$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} x^n \quad (|x| < 1)$$

$$\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots\infty \quad (-1 \le x < 1)$$

$$\log_e(1+x) - \log_e(1-x) =$$

$$\log_e \frac{1+x}{1-x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\infty\right) \quad (-1 < x < 1)$$

$$\log_e\left(1 + \frac{1}{n}\right) = \log_e \frac{n+1}{n} = 2 \left[\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \ldots\infty\right]$$

$$\log_e (1 + x) + \log_e (1 - x) = \log_e(1 - x^2) = -2\left(\frac{x^2}{2} + \frac{x^4}{4} + \ldots\infty\right) \quad (-1 < x < 1)$$

$$\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots = \frac{1}{1\cdot 2} + \frac{1}{3.4} + \frac{1}{5.6} + \ldots$$

Important Results

(i) (a) $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cos^n x}{\sin^n x + \cos^n x} dx$

(b) $\int_0^{\pi/2} \frac{\tan^n x}{1 + \tan^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{dx}{1 + \tan^n x}$

(c) $\int_0^{\pi/2} \frac{dx}{1 + \cot^n x} = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{1 + \cot^n x} dx$

(d) $\int_0^{\pi/2} \frac{\tan^n x}{\tan^n x + \cot^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{\tan^n x + \cot^n x} dx$

(e) $\int_0^{\pi/2} \frac{\sec^n x}{\sec^n x + \operatorname{cosec}^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\operatorname{cosec}^n x}{\sec^n x + \operatorname{cosec}^n x} dx$ where, $n \in R$

(ii) $\int_0^{\pi/2} \frac{a^{\sin^n x}}{a^{\sin^n x} + a^{\cos^n x}} dx = \int_0^{\pi/2} \frac{a^{\cos^n x}}{a^{\sin^n x} + a^{\cos^n x}} dx = \frac{\pi}{4}$

(iii) (a) $\int_0^{\pi/2} \log \sin x \, dx = \int_0^{\pi/2} \log \cos x \, dx = -\frac{\pi}{2} \log 2$

(b) $\int_0^{\pi/2} \log \tan x \, dx = \int_0^{\pi/2} \log \cot x \, dx = 0$

(c) $\int_0^{\pi/2} \log \sec x \, dx = \int_0^{\pi/2} \log \operatorname{cosec} x \, dx = \frac{\pi}{2} \log 2$

(iv) (a) $\int_0^{\infty} e^{-ax} \sin bx \, dx = \frac{b}{a^2 + b^2}$

(b) $\int_0^{\infty} e^{-ax} \cos bx \, dx = \frac{a}{a^2 + b^2}$

(c) $\int_0^{\infty} e^{-ax} x^n \, dx = \frac{n!}{a^n + 1}$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left(x + \sqrt{x^2 - a^2}\right) + C$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right) + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right) + C$$

$$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{a^2 + x^2}\,dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right) + C$$

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Good Luck to you for your Preparations, References, and Exams

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