Spoon Feeding Straight Lines

Simplified Knowledge Management Classes Bangalore

My name is Subhashish Chattopadhyay. I have been teaching for IIT-JEE, Various International Exams (such as IMO [International Mathematics Olympiad], IPhO [International Physics Olympiad], IChO [International Chemistry Olympiad]), IGCSE (IB), CBSE, I.Sc, Indian State Board exams such as WB-Board, Karnataka PU-II etc since 1989. As I write this book in 2016, it is my 25 th year of teaching. I was a Visiting Professor to BARC Mankhurd, Chembur, Mumbai, Homi Bhabha Centre for Science Education (HBCSE) Physics Olympics camp BARC Campus.

I am Life Member of …

- IAPT (Indian Association of Physics Teachers)

- IPA (Indian Physics Association)

- AMTI (Association of Mathematics Teachers of India)

- National Human Rights Association

- Men's Rights Movement (India and International)

- MGTOW Movement (India and International)

And also of

IACT (Indian Association of Chemistry Teachers)

The selection for National Camp (for Official Science Olympiads - Physics, Chemistry, Biology, Astronomy) happens in the following steps ….

1) **NSEP** (National Standard Exam in Physics) and **NSEC** (National Standard Exam in Chemistry) held around 24 rth November. Approx 35,000 students appear for these exams every year. The exam fees is Rs 100 each. Since 1998 the IIT JEE toppers have been topping these exams and they get to know their rank / performance ahead of others.

2) **INPhO** (Indian National Physics Olympiad) and **INChO** (Indian National Chemistry Olympiad). Around 300 students in each subject are allowed to take these exams. Students coming from outside cities are paid fair from the Govt of India.

3) The Top 35 students of each subject are invited at HBCSE (Homi Bhabha Center for Science Education) Mankhurd, near Chembur, BARC, Mumbai. After a 2-3 weeks camp the top 5 are selected to represent India. The flight tickets and many other expenses are taken care by Govt of India.

Since last 50 years there has been no dearth of "Good Books". Those who are interested in studies have been always doing well. This e-Book does not intend to replace any standard text book. These topics are very old and already standardized.

There are 3 kinds of Text Books

- The thin Books - Good students who want more details are not happy with these. Average students who need more examples are not happy with these. Most students who want to "Cram" quickly and pass somehow find the thin books "good" as they have to read less !!

- The Thick Books - Most students do not like these, as they want to read as less as possible. Average students are "busy" with many other things and have no time to read all these.

- The Average sized Books - Good students do not get all details in any one book. Most bad students do not want to read books of "this much thickness" also !!

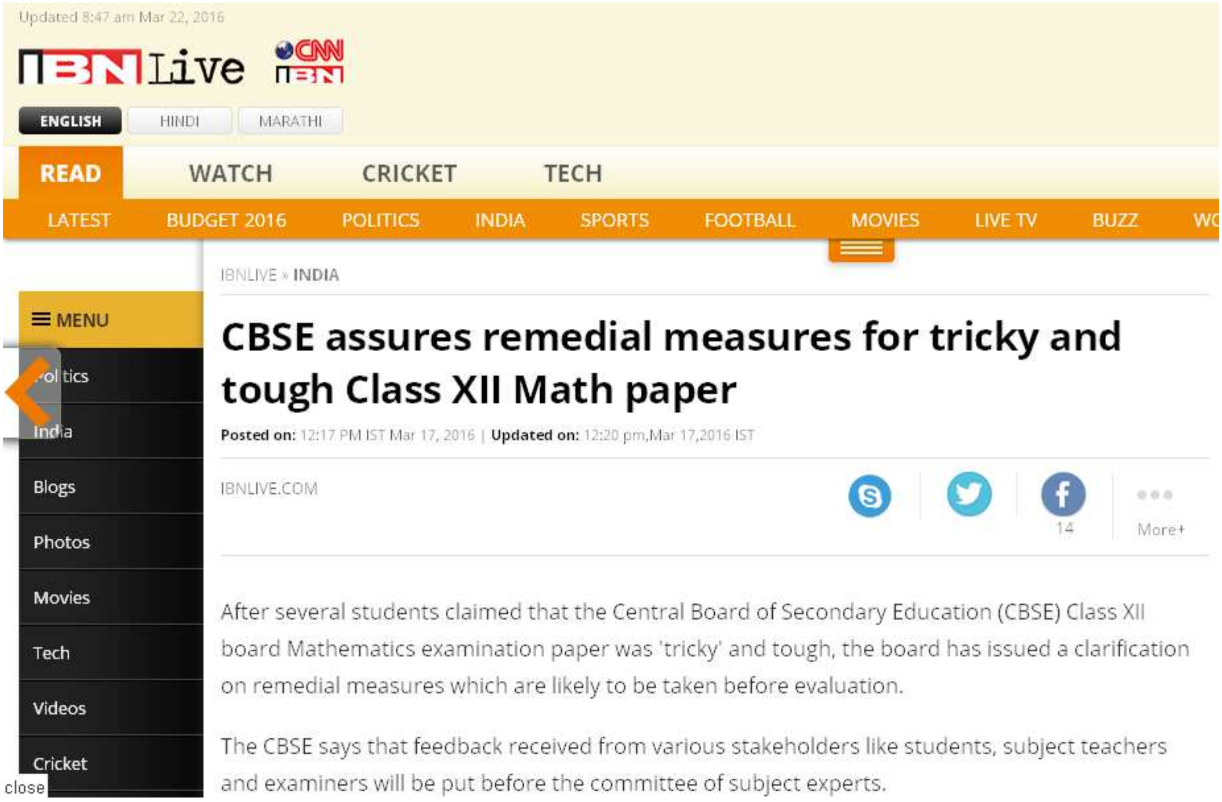
We know there can be no shoe that's fits in all.

Printed books are not e-Books! Can't be downloaded and kept in hard-disc for reading "later" ………

So if you read this book later, you will get all kinds of examples in a single place. This becomes a very good "Reference Material". I sincerely wish that all find this "very useful".

Students who do not practice lots of problems, do not do well. The rules of "doing well" had never changed …. Will never change !

After 2016 CBSE Mathematics exam, lots of students complained that the paper was tough!



Updated 8:47 am Mar 22, 2016

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IBNLIVE » INDIA

CBSE assures remedial measures for tricky and tough Class XII Math paper

Posted on: 12:17 PM IST Mar 17, 2016 | **Updated on:** 12:20 pm,Mar 17,2016 IST

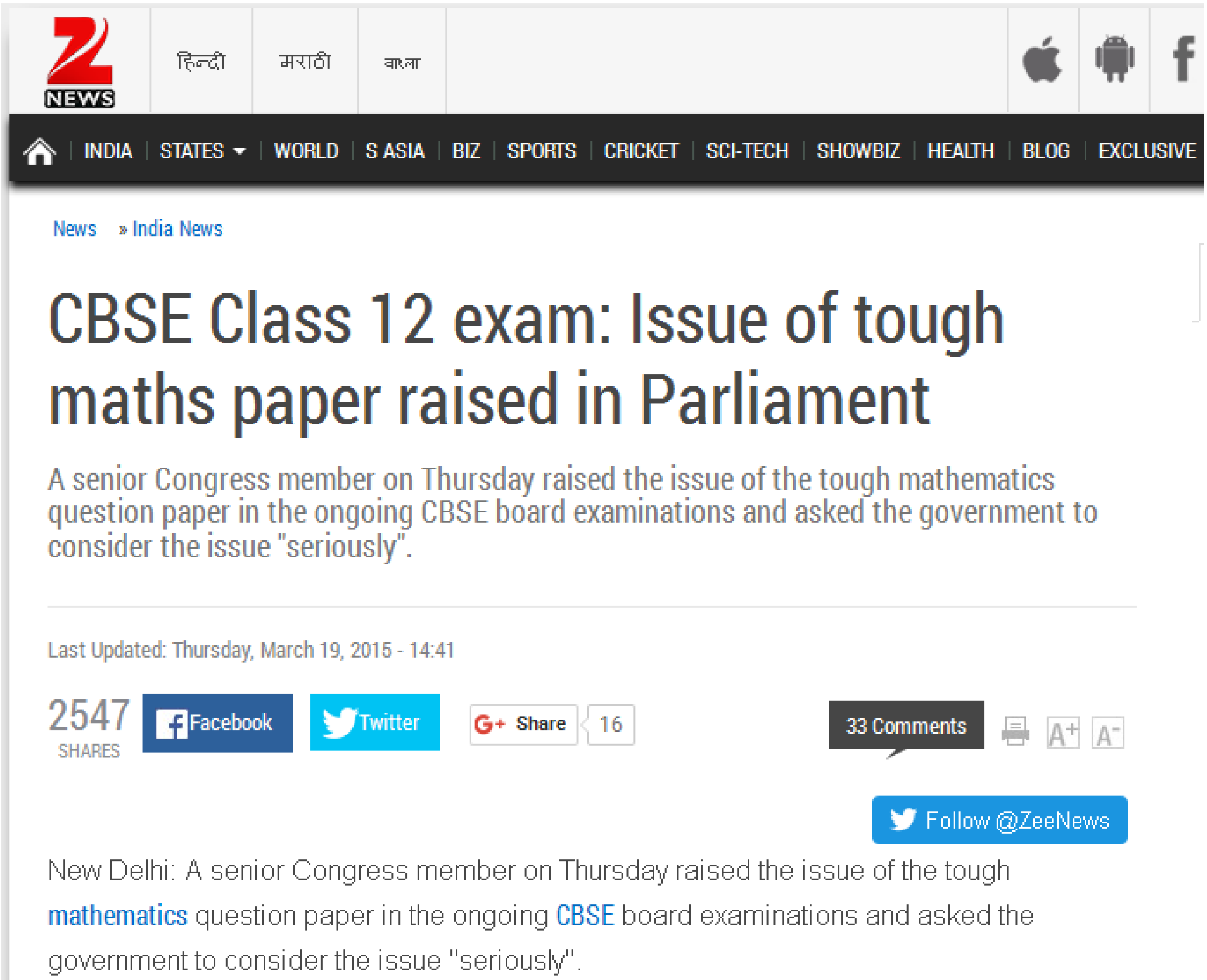
IBNLIVE.COM

14 More+

After several students claimed that the Central Board of Secondary Education (CBSE) Class XII board Mathematics examination paper was 'tricky' and tough, the board has issued a clarification on remedial measures which are likely to be taken before evaluation.

The CBSE says that feedback received from various stakeholders like students, subject teachers and examiners will be put before the committee of subject experts.

In 2015 also the same complain was there by many students



हिन्दी मराठी বাংলা

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News » India News

CBSE Class 12 exam: Issue of tough maths paper raised in Parliament

A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".

Last Updated: Thursday, March 19, 2015 - 14:41

2547 SHARES Facebook Twitter Share 16 33 Comments

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New Delhi: A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".

In March 2016, students of Karnataka PU-II also complained the same, regarding standard 12 (PU-II Mathematics Exam). Even though the Math Paper was identical to previous year, most students had not even solved the 2015 Question Paper.

Friday, March 25, 2016 - 13:28

The NEWS Minute

HOME NEWS ANDHRA KARNATAKA KERALA TAMIL NADU TELANGANA CULTURE MEDIA BLOG

Exams

Online petition for lenient evaluation of K'taka II PU math paper gets over 8000 supporters

The campaign, which was launched on Monday, has garnered over 8000 supporters

TNM Staff | Wednesday, March 16, 2016 - 09:32

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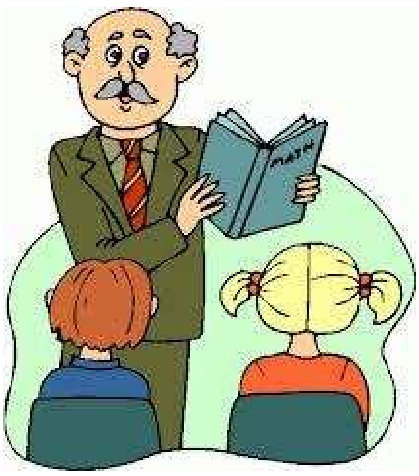
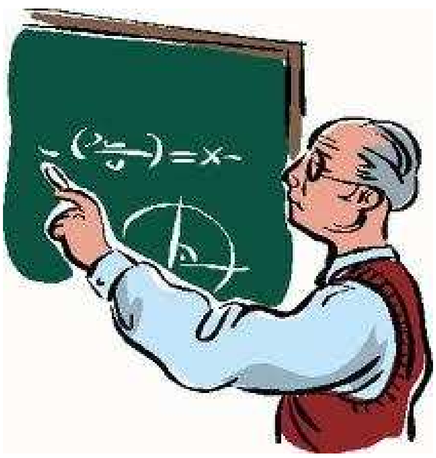
Following a "very tough" math paper that left many II PU students in tears, Saket Ravindran a student launched an online campaign demanding lenient evaluation.

These complains are not new. In fact since last 40 years, (since my childhood), I always see this; every year the same setback, same complain!

In this e-Book I am trying to solve this problem. Those students who practice can learn.

No one can help those who are not studying, or practicing.





Learn more at http://skmclasses.weebly.com/iit-jee-home-tuitions-bangalore.html

Twitter - https://twitter.com/ZookeeperPhy

Facebook - https://www.facebook.com/IIT.JEE.by.Prof.Subhashish/

Blog - http://skmclasses.kinja.com

A very polite request :

I wish these e-Books are read only by Boys and Men. Girls and Women, better read something else; learn from somewhere else.

Preface

We all know that in the species “Homo Sapiens “, males are bigger than females. The reasons are explained in standard 10, or 11 (high school) Biology texts. **This shapes or size, influences all of our culture**. Before we recall / understand the reasons once again, let us see some random examples of the influence

Random - 1

If there is a Road rage, then who all fight ? (generally ?). Imagine two cars driven by adult drivers. Each car has a woman of similar age as that of the Man. The cars “ touch “ or “ some issue happens”. Who all comes out and fights ? Who all are most probable to drive the cars ?

(Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win)

Random - 2

Heavy metal music artists are all Men. Metallica, Black Sabbath, Motley Crue, Megadeth, Motorhead, AC/DC, Deep Purple, Slayer, Guns & Roses, Led Zeppelin, Aerosmith ….. the list can be in thousands. All these are grown-up Boys, known as Men.

(Men strive for perfection. Men are eager to excel. Men work hard. Men want to win.)

Random - 3

Apart from Marie Curie, only one more woman got Nobel Prize in Physics. (Maria Goeppert Mayer - 1963). So, … almost all are men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 4

The best Tabla Players are all Men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 5

History is all about, which Kings ruled. Kings, their men, and Soldiers went for wars. History is all about wars, fights, and killings by men.

Boys start fighting from school days. Girls do not fight like this

(Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.)

Random - 6

The highest award in Mathematics, the " Fields Medal " is around since decades. Till date only one woman could get that. (Maryam Mirzakhani - 2014). So, … almost all are men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 7

Actor is a gender neutral word. Could the movie like " Top Gun " be made with Female actors ? The best pilots, astronauts, Fighters are all Men.

Random - 8

In my childhood had seen a movie named “ The Tower in Inferno “. In the movie when the tall tower is in fire, women were being saved first, as only one lift was working….


Many decades later another movie is made. A box office hit. “ The Titanic “. In this also …. As the ship is sinking women are being saved. **Men are disposable**. Men may get their turn later…

Movies are not training programs. Movies do not teach people what to do, or not to do. Movies only reflect the prevalent culture. Men are disposable, is the culture in the society. Knowingly, unknowingly, the culture is depicted in Movies, Theaters, Stories, Poems, Rituals, etc. I or you can’t write a story, or make a movie in which after a minor car accident the Male passengers keep seating in the back seat, while the both the women drivers come out of the car and start fighting very bitterly on the road. There has been no story in this world, or no movie made, where after an accident or calamity, Men are being helped for safety first, and women are told to wait.

Random - 9

Artists generally follow the prevalent culture of the Society. In paintings, sculptures, stories, poems, movies, cartoon, Caricatures, knowingly / unknowingly, “ the prevalent Reality “ is depicted. The opposite will not go well with people. If deliberately “ the opposite “ is shown then it may only become a special art, considered as a special mockery.

Random - 10

Men go to “girl / woman’s house” to marry / win, and bring her to his home. That is a sort of winning her. When a boy gets a “ Girl-Friend “, generally he and his friends consider that as an achievement. The boy who “ got / won “ a girl-friend feels proud. His male friends feel, jealous, competitive and envious. Millions of stories have been written on these themes. Lakhs of movies show this. Boys / Men go for “ bike race “, or say “ Car Race “, where the winner “ gets “ the most beautiful girl of the college.

(Men want to excel. Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.)

Prithviraj Chauhan ‘ went ` to “ pickup “ or “ abduct “ or “ win “ or “ bring “ his love. There was a Hindi movie (hit) song … “ Pasand ho jaye, to ghar se utha laye “. It is not other way round. Girls do not go to Boy’s house or man’s house to marry. Nor the girls go in a gang to “ pick-up “ the boy / man and bring him to their home / place / den.

Random - 11

Rich people; often are very hard working. Successful business men, establish their business (empire), amass lot of wealth, with lot of difficulty. Lots of sacrifice, lots of hard work, gets into this. Rich people's wives had no contribution in this wealth creation. Women are smart, and successful upto the extent to choose the right/rich man to marry. So generally what happens in case of Divorces ? Search the net on " most costly divorces " and you will know. The women;(who had no contribution at all, in setting up the business / empire), often gets in Billions, or several Millions in divorce settlements.

Number 1

Rupert & Anna Murdoch -- $1.7 billion

One of the richest men in the world, **Rupert Murdoch** developed his worldwide media empire when he inherited his father's Australian newspaper in 1952. He married Anna Murdoch in the '60s and they remained together for 32 years, springing off three children.

They split amicably in 1998 but soon Rupert forced Anna off the board of News Corp and the gloves came off. The divorce was finalized in June 1999 when Rupert agreed to let his ex-wife leave with $1.7 billion worth of his assets, $110 million of it in cash. Seventeen days later, Rupert married Wendi Deng, one of his employees.

Ted Danson & Casey Coates -- $30 million

Ted Danson's claim to fame is undoubtedly his decade-long stint as Sam Malone on NBC's celebrated sitcom Cheers . While he did other TV shows and movies, he will always be known as the bartender of that place where everybody knows your name. He met his future first bride Casey, a designer, in 1976 while doing Erhard Seminars Training.

Ten years his senior, she suffered a paralyzing stroke while giving birth to their first child in 1979. In order to nurse her back to health, Danson took a break from acting for six months. But after two children and 15 years of marriage, the infatuation fell to pieces. Danson had started seeing Whoopi Goldberg while filming the comedy, Made in America and this precipitated the 1992 divorce. Casey got $30 million for her trouble.

See https://zookeepersblog.wordpress.com/misandry-and-men-issues-a-short-summary-at-single-place/

See http://skmclasses.kinja.com/save-the-male-1761788732

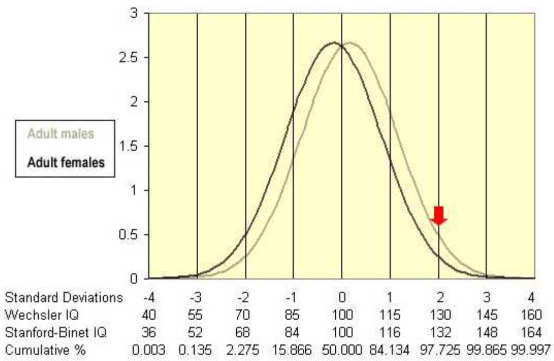
It was Boys and Men, who brought the girls / women home. The Laws are biased, completely favoring women. The men are paying for their own mistakes.

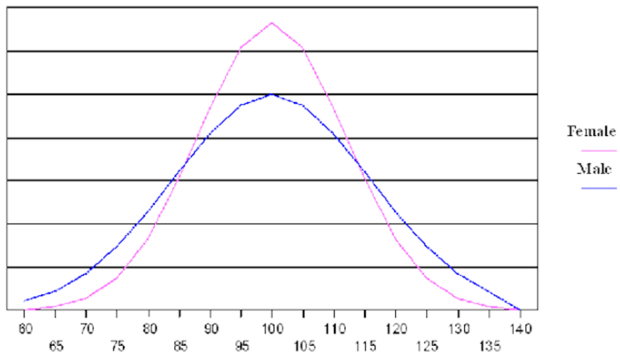
See https://zookeepersblog.wordpress.com/biased-laws/

(Man brings the Woman home. When she leaves, takes away her share of big fortune!)

Random - 12

A standardized test of Intelligence will never be possible. It never happened before, nor ever will happen in future; where the IQ test results will be acceptable by all. In the net there are thousands of charts which show that the intelligence scores of girls / women are lesser. Debates of Trillion words, does not improve performance of Girls.

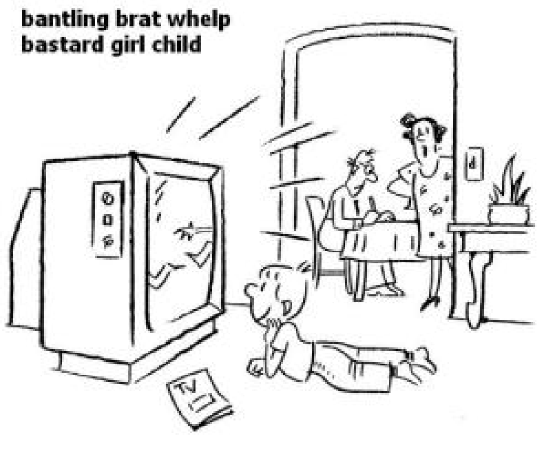




I am not wasting a single second debating or discussing with anyone, on this. I am simply accepting ALL the results. IQ is only one of the variables which is required for success in life. Thousands of books have been written on “ Networking Skills “, EQ (Emotional Quotient), Drive, Dedication, Focus, “ Tenacity towards the end goal “ … etc. In each criteria, and in all together, women (in general) do far worse than men. Bangalore is known as “ ….. capital of India “. [Fill in the blanks]. The blanks are generally filled as “ Software Capital “, “ IT Capital “, “ Startup Capital “, etc. I am member in several startup eco-systems / groups. I have attended hundreds of meetings, regarding “ technology startups “, or “ idea startups “. These meetings have very few women. Starting up new companies are all “ Men’s Game “ / “ Men’s business “. Only in Divorce settlements women will take their goodies, due to Biased laws. There is no dedication, towards wealth creation, by women.

Random - 13

Many men, as fathers, very unfortunately treat their daughters as “ Princess “. Every “ non-performing “ woman / wife was “ princess daughter “ of some loving father. Pampering the girls, in name of “ equal opportunity “, or “ women empowerment “, have led to nothing.



"Please turn it down - Daddy is trying to do your homework."

See http://skmclasses.kinja.com/progressively-daughters-become-monsters-1764484338

See http://skmclasses.kinja.com/vivacious-vixens-1764483974

There can be thousands of more such random examples, where " Bigger Shape / size " of males have influenced our culture, our Society. **Let us recall the reasons**, that we already learned in standard 10 - 11, Biology text Books. In humans, women have a long gestation period, and also spends many years (almost a decade) to grow, nourish, and stabilize the child. (Million years of habit) Due to survival instinct Males want to inseminate. Boys and Men fight for the " facility (of womb + care) " the girl / woman may provide. Bigger size for males, has a winning advantage. Whoever wins, gets the " woman / facility ". The male who is of " Bigger Size ", has an advantage to win…. Leading to Natural selection over millions of years. In general " Bigger Males "; the " fighting instinct " in men; have led to wars, and solving tough problems (Mathematics, Physics, Technology, startups of new businesses, Wealth creation, Unreasonable attempts to make things [such as planes], Hard work ….)

So let us see the IIT-JEE results of girls. Statistics of several years show that there are around 17, (or less than 20) girls in top 1000 ranks, at all India level. Some people will yet not understand the performance, till it is said that … year after year we have around 980 boys in top 1000 ranks. Generally we see only 4 to 5 girls in top 500. In last 50 years not once any girl topped in IIT-JEE advanced. Forget about Single digit ranks, double digit ranks by girls have been extremely rare. It is all about " good boys ", " hard working ", " focused ", "**Bel-esprit**" **boys**.

In 2015, Only 2.6% of total candidates who qualified are girls (upto around 12,000 rank). while 20% of the Boys, amongst all candidates qualified. The Total number of students who appeared for the exam were around 1.4 million for IIT-JEE main. Subsequently 1.2 lakh (around 120 thousands) appeared for IIT-JEE advanced.

IIT-JEE results and analysis, of many years is given at https://zookeepersblog.wordpress.com/iit-jee-iseet-main-and-advanced-results/

In Bangalore it is rare to see a girl with rank better than 1000 in IIT-JEE advanced. We hardly see 6-7 boys with rank better than 1000. Hardly 2-3 boys get a rank better than 500.

See http://skmclasses.weebly.com/everybody-knows-so-you-should-also-know.html

Thousands of people are exposing the heinous crimes that Motherly Women are doing, or Female Teachers are committing. See https://www.facebook.com/WomenCriminals/

Some Random Examples must be known by all

Mother Admits On Facebook to Sleeping with 15 Yr Old Son, They Have a Baby Together - Alwayzturntup

Sometimes it hard to believe w From Alwayzturntup

ALWAYZTURNTUP.ME

It is extremely unfortunate that the " woman empowerment " has created. This is the kind of society and women we have now. I and many other sensible Men hate such women. Be away from such women, be aware of reality.

'Sex with my son is incredible - we're in love and we want a baby'

Ben Ford, who ditched his wife when he met his mother Kim West after 30 years, claims what the couple are doing 'isn't incest'

MIRROR.CO.UK

Former Shelbyville ISD teacher who had sex with underage student gets 3 years in prison

After a two day break over the weekend, A Shelby County jury was back in the courtroom looking to conclude the trial of a former Shelbyville ISD teacher who had...

KLTV.COM | BY CALEB BEAMES

Woman sent to jail for the rest of her life after raping her four grandchildren is described as the 'most evil person' the judge has ever seen

Edwina Louis rape...

See More

Woman sent to jail for raping her four grandchildren

A Ohio grandmother has been sentenced to four consecutive life terms after being found guilty of the rape of her own grandchildren. Edwina Louis, 53, will spend the rest of her life behind bars.

DAILYMAIL.CO.UK

http://www.thenativecanadian.com/.../eastern-ontario-teacher-...

The N.C. Chronicles.: Eastern Ontario teacher charged with 36 sexual offences

anti feminism, Child abuse, children's rights, Feminist hypocrisy,

THENATIVECANADIAN.COM | BY BLACKWOLF

Hyd woman kills newborn boy as she wanted daughter - Times of India

Having failed to bear a daughter for the third time, a shopkeeper's wife slit the throat of her 24day-old son with a shaving blade and left him to die in a street on Tuesday night.Purnima's first child was a stillborn boy, followed by another boy born five years ago.

TIMESOFINDIA.INDIATIMES.COM

Montgomery's son, Alan Vonn Webb, took the stand and was a key witness in her conviction.

"I want to see her placed somewhere she can never do that to children ...

See More

Woman sentenced to 40 years in prison for raping her children

A Murfreesboro mother found guilty of raping her own children learned her fate on Wednesday.

WAFF.COM | BY DENNIS FERRIER

gentler sex? Violence against men.'s photo.

Women, the gentler sex? Violence against men. Like Page

April 8 at 1:38am ·

In fact, the past decade has seen a dramatic increase in the number of incidents of women raping and sexually assaulting boys and men. On May 2014, Jezebel repo...

End violence against women . . . .

North Carolina Grandma Eats Her Daughter's New Born Baby After Smoking Bath Salts

Henderson, North Carolina– A North Carolina grandmother of 4 and recovering drug addict, is now in custody after she allegedly ate her daughter's newborn baby....

AZ-365.TOP

28-Year-Old Texas Teacher Accused of Sending Nude Picture to 14-Year-Old Former Student

BREITBART.COM

http://latest.com/.../attractive-girl-gang-lured-men-alleywa.../

Attractive Girl Gang Lured Men Into Alleyways Where Female Body Builder Would Attack Them

A Mexican street gang made up entirely of women has been accused of using their feminine wiles to lure men into alleyways and then beating them up and...

LATEST.COM

http://www.wfmj.com/.../youngstown-woman-convicted-of-raping-...

Youngstown woman convicted of raping a 1 year old is back in jail

A Youngstown woman who went to prison for raping a 1-year-old boy fifteen years ago is in trouble with the law again.

WFMJ.COM

Women are raping boys and young men

Rape advocacy has been maligned and twisted into a political agenda controlled by radicalized activists. Tim Patten takes a razor keen and well supported look into the manufactured rape culture and...

AVOICEFORMEN.COM | BY TIM PATTEN

End violence against women . . . .

Bronx Woman Convicted of Poisoning and Drowning Her Children

Lisette Bamenga researched methods on the Internet before she killed her son and daughter in 2012.

NYTIMES.COM | BY MARC SANTORA

A Russian-born newlywed slowly butchered her German husband — feeding strips of his flesh to their dog until he took his last breath. Svetlana Batukova, 46, was...

See More

She killed her husband and then fed him to her dog: police

A Russian-born newlywed butchered her German hubby — and fed strips of his flesh to her pooch, authorities said. Svetlana Batukova offed Horst Hans Henkels at their...

NYPOST.COM

Daily Mail
January 15, 2015 ·

Mother charged with rape and sodomy of her son's 12-year-old friend

Mom, 30, 'raped and had oral sex with her son's 12-year-old friend'

Nicole Marie Smith, 30, (pictured) of St Charles County, Missouri, has been jailed after she allegedly targeted the 12-year-old boy at her home.

DAILYM.AI

April 4 at 4:48am ·

Female prison officers commit 90pc of sex assaults on male teens in US juvenile detention centres

Lawsuit in Idaho highlights the prevalence of sexual victimization of juvenile offenders.

IBTIMES.CO.UK | BY NICOLE ROJAS

This mother filmed herself raping her own son and then sold it to a man for $300. The courts just decide her fate. When you see what she got, you're going to be outraged.

Mother Who Filmed Herself Raping Her 1-Year-Old Son Receives Shocking Sentence

"...then used the money to buy herself a laptop..."

AMERICANEWS.COM

In several countries or rather in several regions of the world, family system has collapsed, due to bad nature and naughty acts of women. Particularly in Britain, and America, almost 50% people are alone, lonely, separated, divorced or failed marriages. In 2013, 48% children were born out of wedlock. It was projected that by 2016, more than 51% children will be born, to unmarried mothers. In these developed countries " paternity fraud " by women, are close to 20%. You can see several articles in the net, and in wikipedia etc. This means 1 out of 5 children are calling a wrong man as dad. The lonely, alone “ mothers " are frustrated. They see the children as burden. Love in the Society in general is lost, long time ago. The types of “ Mothers “ and “ Women “ we have now …………


This is the type of women we have in this world. These kind of women were also someones daughter

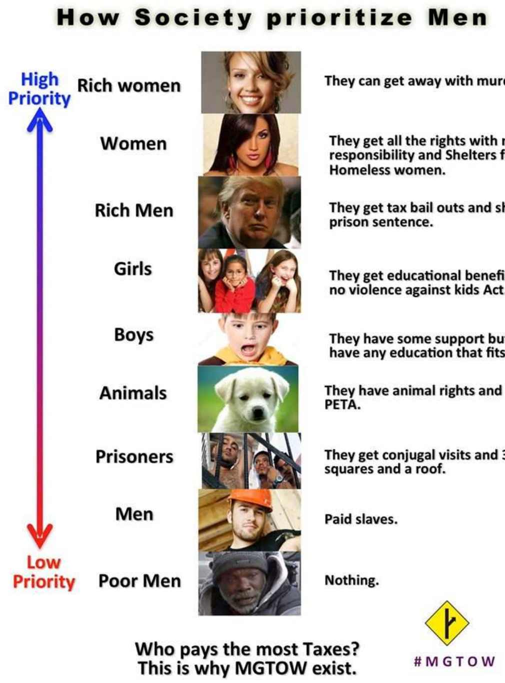
Mother Stabs Her Baby 90 Times With Scissors After He Bit Her While Breastfeeding Him!

Eight-month-old Xiao Bao was discovered by his uncle in a pool of blood Needed 100 stitches after the incident; he is now recovering in hospital Reports say his...

MOMMABUZZ.COM

By now if you have assumed that Indian women are not doing any crime then please become friends with MRA Guri https://www.facebook.com/profile.php?id=100004138754180

He has dedicated his life to expose Indian Criminals



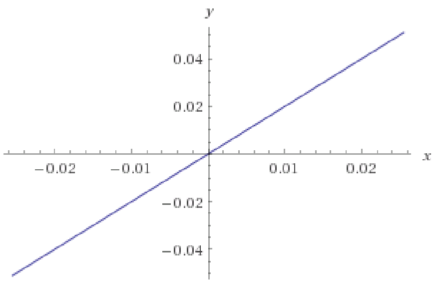
Professor Subhashish Chattopadhyay

Spoon Feeding Series - Straight Lines

Before we discuss Coordinate Geometry we must know the basics of the Graphs of Straight Lines

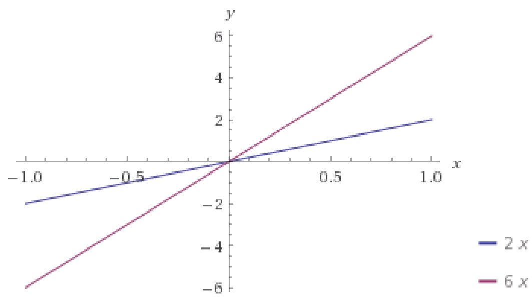
y = mx will be a straight line passing through the origin. Positive m will make the line move upwards as we move in positive x i.e. towards right.

plot	$y = 2x$



This is graph of y = 2 x Don't get foxed by the angle being almost 45^0 The scales in y-axis and x-axis are not same.

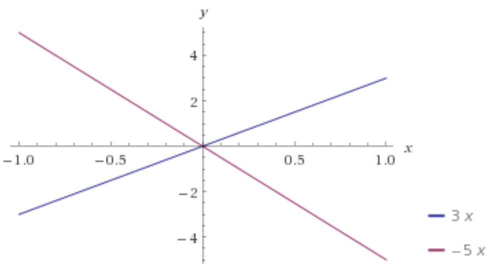
plot	$y = 2x$
	$y = 6x$



If we compare two graphs then it becomes more clear.

In this figure also scales of x-axis and y-axis are not same. But y = 6x has to be steeper than y = 2x

plot	$y = 3x$
	$y = -5x$

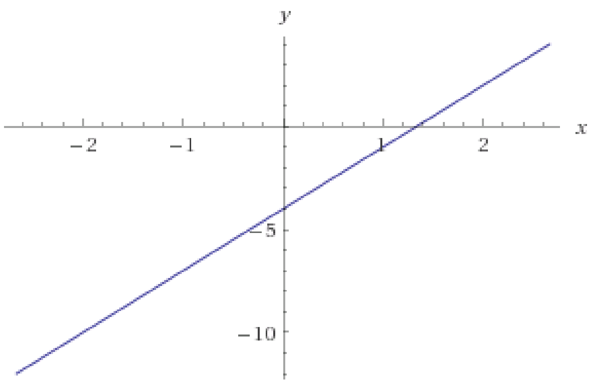


This is y = 3x and y = - 5 x graphs. For m = -5 the line moves down

For y = m x + c the c becomes the intercept in the y axis

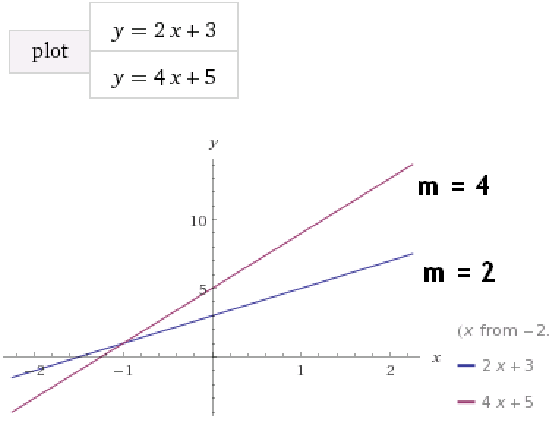
So y = 3x - 4 will look like

plot	$y = 3x - 4$



If c is a positive number then the intercept in y-axis will be on upper (positive) side.

Graphs of y = 2x + 3 and y = 4x + 5 will be



Again scales in x-axis and y-axis are different. But point made. See how the graphs pass through 3 and 5 respectively.

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Nature of Curves, Types of Graphs, Shapes are explained / discussed at

https://archive.org/details/AreaDefiniteIntegralNatureOfCurvesTypesOfGraphsShapesDiscussions

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Spoonfeeding

Write the equations for the x and y-axes.

Answer

The y-coordinate of every point on the x-axis is 0.

Therefore, the equation of the x-axis is $y = 0$.

The x-coordinate of every point on the y-axis is 0.

Therefore, the equation of the y-axis is $y = 0$.

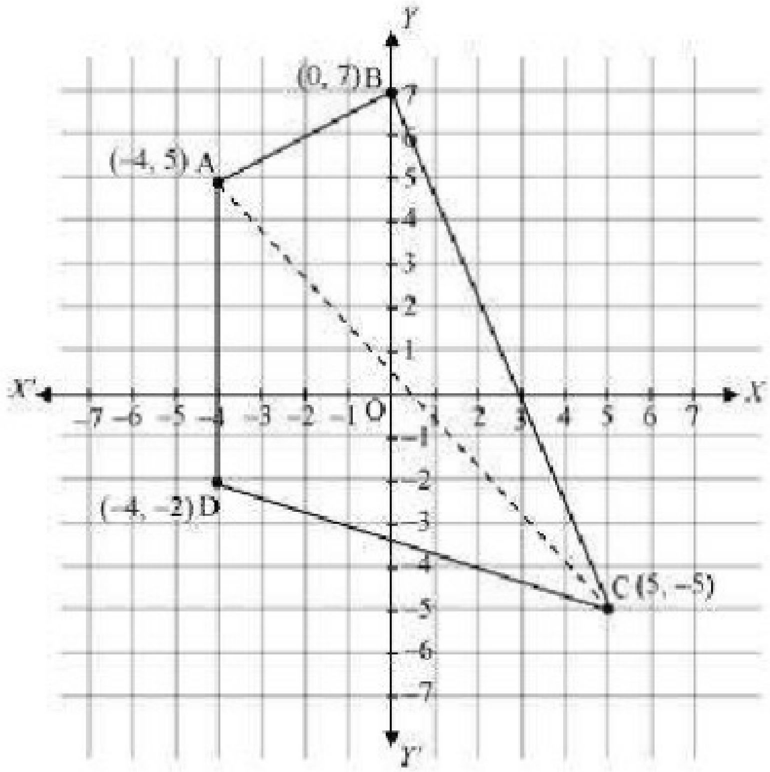
Question

Draw a quadrilateral in the Cartesian plane, whose vertices are (−4, 5), (0, 7), (5, −5) and (−4, −2). Also, find its area.

Answer

Let ABCD be the given quadrilateral with vertices A (−4, 5), B (0, 7), C (5, −5), and D (−4, −2).

Then, by plotting A, B, C, and D on the Cartesian plane and joining AB, BC, CD, and DA, the given quadrilateral can be drawn as



To find the area of quadrilateral ABCD, we draw one diagonal, say AC.

Accordingly, area (ABCD) = area (ΔABC) + area (ΔACD)

We know that the area of a triangle whose vertices are (x_1, y_1), (x_2, y_2), and (x_3, y_3) is

$$\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

Therefore, area of ΔABC

$$= \frac{1}{2}\left|-4(7+5)+0(-5-5)+5(5-7)\right| \text{ unit}^2$$

$$= \frac{1}{2}\left|-4(12)+5(-2)\right| \text{ unit}^2$$

$$= \frac{1}{2}\left|-48-10\right| \text{ unit}^2$$

$$= \frac{1}{2}\left|-58\right| \text{ unit}^2$$

$$= \frac{1}{2}\times 58 \text{ unit}^2$$

$$= 29 \text{ unit}^2$$

Area of ΔACD

$$= \frac{1}{2}\left|-4(-5+2)+5(-2-5)+(-4)(5+5)\right| \text{ unit}^2$$

$$= \frac{1}{2}\left|-4(-3)+5(-7)-4(10)\right| \text{ unit}^2$$

$$= \frac{1}{2}\left|12-35-40\right| \text{ unit}^2$$

$$= \frac{1}{2}\left|-63\right| \text{ unit}^2$$

$$= \frac{63}{2} \text{ unit}^2$$

Thus, area (ABCD) $= \left(29+\frac{63}{2}\right) \text{ unit}^2 = \frac{58+63}{2} \text{ unit}^2 = \frac{121}{2} \text{ unit}^2$

Question

The base of an equilateral triangle with side $2a$ lies along they y-axis such that the mid point of the base is at the origin. Find vertices of the triangle.

Answer

Let ABC be the given equilateral triangle with side $2a$.

Accordingly, AB = BC = CA = $2a$

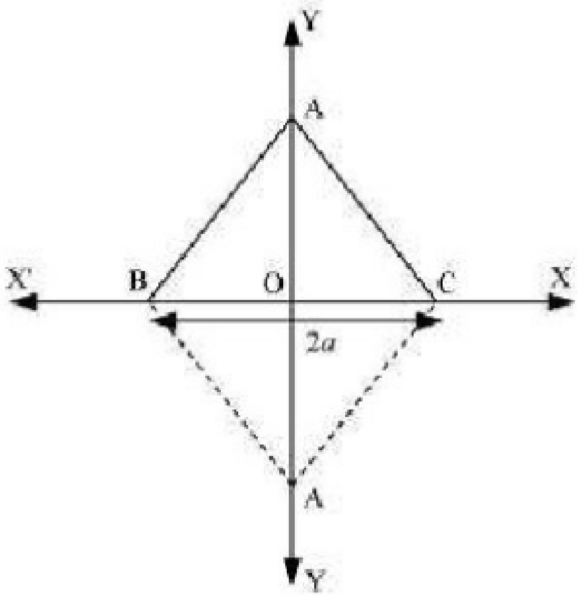
Assume that base BC lies along the y-axis such that the mid-point of BC is at the origin.

i.e., BO = OC = a, where O is the origin.

Now, it is clear that the coordinates of point C are $(0, a)$, while the coordinates of point B are $(0, -a)$.

It is known that the line joining a vertex of an equilateral triangle with the mid-point of its opposite side is perpendicular.

Hence, vertex A lies on the y-axis.



On applying Pythagoras theorem to ΔAOC, we obtain

$(AC)^2 = (OA)^2 + (OC)^2$

$\Rightarrow (2a)^2 = (OA)^2 + a^2$

$\Rightarrow 4a^2 - a^2 = (OA)^2$

$\Rightarrow (OA)^2 = 3a^2$

$\Rightarrow$ OA = $\sqrt{3}a$

$\therefore$Coordinates of point A = $\left(\pm\sqrt{3}a, 0\right)$

Thus, the vertices of the given equilateral triangle are (0, *a*), (0, −*a*), and $\left(\sqrt{3}a,\ 0\right)$ or (0, *a*), (0, −*a*), and $\left(-\sqrt{3}a,\ 0\right)$.

Question

Find the distance between $P\left(x_1,\ y_1\right)$ and $Q\left(x_2,\ y_2\right)$ when: (i) PQ is parallel to the *y*-axis, (ii) PQ is parallel to the *x*-axis.

Answer

The given points are $P\left(x_1,\ y_1\right)$ and $Q\left(x_2,\ y_2\right)$.

(i) When PQ is parallel to the *y*-axis, $x_1 = x_2$.

In this case, distance between P and Q $= \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$

$= \sqrt{\left(y_2 - y_1\right)^2}$

$= \left|y_2 - y_1\right|$

(ii) When PQ is parallel to the *x*-axis, $y_1 = y_2$.

In this case, distance between P and Q $= \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$

$= \sqrt{\left(x_2 - x_1\right)^2}$

$= \left|x_2 - x_1\right|$

Question

Find a point on the x-axis, which is equidistant from the points (7, 6) and (3, 4).

Answer

Let (a, 0) be the point on the x axis that is equidistant from the points (7, 6) and (3, 4).

Accordingly, $\sqrt{(7-a)^2+(6-0)^2} = \sqrt{(3-a)^2+(4-0)^2}$

$\Rightarrow \sqrt{49+a^2-14a+36} = \sqrt{9+a^2-6a+16}$

$\Rightarrow \sqrt{a^2-14a+85} = \sqrt{a^2-6a+25}$

On squaring both sides, we obtain

$a^2 - 14a + 85 = a^2 - 6a + 25$

$\Rightarrow -14a + 6a = 25 - 85$

$\Rightarrow -8a = -60$

$\Rightarrow a = \frac{60}{8} = \frac{15}{2}$

Thus, the required point on the x-axis is $\left(\frac{15}{2}, 0\right)$.

Question

Find the slope of a line, which passes through the origin, and the mid-point of the line segment joining the points P (0, −4) and B (8, 0).

Answer

The coordinates of the mid-point of the line segment joining the points P (0, −4) and B (8, 0) are $\left(\frac{0+8}{2}, \frac{-4+0}{2}\right) = (4, -2)$

It is known that the slope (m) of a non-vertical line passing through the points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2-y_1}{x_2-x_1}, x_2 \neq x_1$.

Therefore, the slope of the line passing through (0, 0) and (4, −2) is

$\frac{-2-0}{4-0} = \frac{-2}{4} = -\frac{1}{2}$.

So the slope of the line is -1/2

Question

Without using the Pythagoras theorem, show that the points (4, 4), (3, 5) and (−1, −1) are the vertices of a right angled triangle.

Answer

The vertices of the given triangle are A (4, 4), B (3, 5), and C (−1, −1).

It is known that the slope (m) of a non-vertical line passing through the points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}, x_2 \neq x_1$.

∴Slope of AB (m_1) $= \frac{5-4}{3-4} = -1$

Slope of BC (m_2) $= \frac{-1-5}{-1-3} = \frac{-6}{-4} = \frac{3}{2}$

Slope of CA (m_3) $= \frac{4+1}{4+1} = \frac{5}{5} = 1$

It is observed that $m_1 m_3 = -1$

This shows that line segments AB and CA are perpendicular to each other

i.e., the given triangle is right-angled at A (4, 4).

Thus, the points (4, 4), (3, 5), and (−1, −1) are the vertices of a right-angled triangle.

Question

Find the equation of the line which passes through the point (−4, 3) with slope $\frac{1}{2}$.

Answer

We know that the equation of the line passing through point (x_0, y_0), whose slope is m, is $(y - y_0) = m(x - x_0)$.

Thus, the equation of the line passing through point (−4, 3), whose slope is $\frac{1}{2}$, is

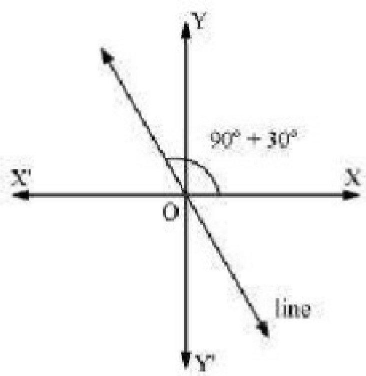
$$(y-3) = \frac{1}{2}(x+4)$$
$$2(y-3) = x+4$$
$$2y-6 = x+4$$
$$\text{i.e., } x-2y+10=0$$

Question

Find the slope of the line, which makes an angle of 30° with the positive direction of y-axis measured anticlockwise.

Answer

If a line makes an angle of 30° with the positive direction of the y-axis measured anticlockwise, then the angle made by the line with the positive direction of the x-axis measured anticlockwise is 90° + 30° = 120°.



Thus, the slope of the given line is tan 120° = tan (180° − 60°) = −tan 60° $= -\sqrt{3}$

Question

Find the value of x for which the points (x, −1), (2, 1) and (4, 5) are collinear.

Answer

If points A (x, −1), B (2, 1), and C (4, 5) are collinear, then

Slope of AB = Slope of BC

$$\Rightarrow \frac{1-(-1)}{2-x} = \frac{5-1}{4-2}$$

$$\Rightarrow \frac{1+1}{2-x} = \frac{4}{2}$$

$$\Rightarrow \frac{2}{2-x} = 2$$

$$\Rightarrow 2 = 4 - 2x$$

$$\Rightarrow 2x = 2$$

$$\Rightarrow x = 1$$

Thus, the required value of x is 1.

Question

Find the equation of the line which passes though (0, 0) with slope m.

Answer

We know that the equation of the line passing through point (x_0, y_0), whose slope is m, is $(y - y_0) = m(x - x_0)$.

Thus, the equation of the line passing through point (0, 0), whose slope is m, is

$(y - 0) = m(x - 0)$

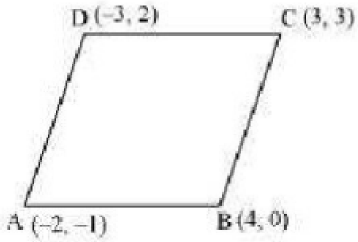
i.e., $y = mx$

Question

Without using distance formula, show that points (−2, −1), (4, 0), (3, 3) and (−3, 2) are vertices of a parallelogram.

Answer

Let points (−2, −1), (4, 0), (3, 3), and (−3, 2) be respectively denoted by A, B, C, and D.



Slope of AB $= \frac{0+1}{4+2} = \frac{1}{6}$

Slope of CD $= \frac{2-3}{-3-3} = \frac{-1}{-6} = \frac{1}{6}$

⇒ Slope of AB = Slope of CD

⇒ AB and CD are parallel to each other.

Now, slope of BC $= \frac{3-0}{3-4} = \frac{3}{-1} = -3$

Slope of AD $= \frac{2+1}{-3+2} = \frac{3}{-1} = -3$

⇒ Slope of BC = Slope of AD

⇒ BC and AD are parallel to each other.

Therefore, both pairs of opposite sides of quadrilateral ABCD are parallel. Hence, ABCD is a parallelogram.

Thus, points (−2, −1), (4, 0), (3, 3), and (−3, 2) are the vertices of a parallelogram.

Question

Find the angle between the x-axis and the line joining the points (3, −1) and (4, −2).

Answer

The slope of the line joining the points (3, −1) and (4, −2) is $m = \frac{-2-(-1)}{4-3} = -2+1 = -1$

Now, the inclination (θ) of the line joining the points (3, −1) and (4, − 2) is given by

$\tan\theta = -1$

$\Rightarrow \theta = (90° + 45°) = 135°$

Thus, the angle between the x-axis and the line joining the points (3, −1) and (4, −2) is 135°.

Question

The slope of a line is double of the slope of another line. If tangent of the angle between them is $\frac{1}{3}$, find the slopes of he lines.

Answer

Let m_1 and m be the slopes of the two given lines such that $m_1 = 2m$.

We know that if θisthe angle between the lines l_1 and l_2 with slopes m_1 and m_2, then

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|.$$

It is given that the tangent of the angle between the two lines is $\frac{1}{3}$.

$$\therefore \frac{1}{3} = \left|\frac{m-2m}{1+(2m)\cdot m}\right|$$

$$\Rightarrow \frac{1}{3} = \left|\frac{-m}{1+2m^2}\right|$$

$$\Rightarrow \frac{1}{3} = \left|\frac{-m}{1+2m^2}\right|$$

$$\Rightarrow \frac{1}{3} = \frac{-m}{1+2m^2} \text{ or } \frac{1}{3} = -\left(\frac{-m}{1+2m^2}\right) = \frac{m}{1+2m^2}$$

Case I

$$\Rightarrow \frac{1}{3} = \frac{-m}{1+2m^2}$$

$$\Rightarrow 1+2m^2 = -3m$$

$$\Rightarrow 2m^2 + 3m + 1 = 0$$

$$\Rightarrow 2m^2 + 2m + m + 1 = 0$$

$$\Rightarrow 2m(m+1) + 1(m+1) = 0$$

$$\Rightarrow (m+1)(2m+1) = 0$$

$$\Rightarrow m = -1 \text{ or } m = -\frac{1}{2}$$

If $m = -1$, then the slopes of the lines are -1 and -2.

If $m = -\frac{1}{2}$, then the slopes of the lines are $-\frac{1}{2}$ and -1.

Case II

$$\frac{1}{3} = \frac{m}{1+2m^2}$$
$$\Rightarrow 2m^2 + 1 = 3m$$
$$\Rightarrow 2m^2 - 3m + 1 = 0$$
$$\Rightarrow 2m^2 - 2m - m + 1 = 0$$
$$\Rightarrow 2m(m-1) - 1(m-1) = 0$$
$$\Rightarrow (m-1)(2m-1) = 0$$
$$\Rightarrow m = 1 \text{ or } m = \frac{1}{2}$$

If $m = 1$, then the slopes of the lines are 1 and 2.

If $m = \frac{1}{2}$, then the slopes of the lines are $\frac{1}{2}$ and 1.

Hence, the slopes of the lines are -1 and -2 or $-\frac{1}{2}$ and -1 or 1 and 2 or $\frac{1}{2}$ and 1

Question

A line passes through (x_1, y_1) and (h, k). If slope of the line is m, show that $k - y_1 = m(h - x_1)$.

Answer

The slope of the line passing through (x_1, y_1) and (h, k) is $\frac{k - y_1}{h - x_1}$.

It is given that the slope of the line is m.

$$\therefore \frac{k - y_1}{h - x_1} = m$$

$$\Rightarrow k - y_1 = m(h - x_1)$$

Hence, $k - y_1 = m(h - x_1)$

Question

If three point $(h, 0)$, (a, b) and $(0, k)$ lie on a line, show that $\frac{a}{h}+\frac{b}{k}=1$

Answer

If the points A $(h, 0)$, B (a, b), and C $(0, k)$ lie on a line, then

Slope of AB = Slope of BC

$$\frac{b-0}{a-h}=\frac{k-b}{0-a}$$

$$\Rightarrow \frac{b}{a-h}=\frac{k-b}{-a}$$

$$\Rightarrow -ab=(k-b)(a-h)$$

$$\Rightarrow -ab=ka-kh-ab+bh$$

$$\Rightarrow ka+bh=kh$$

On dividing both sides by kh, we obtain

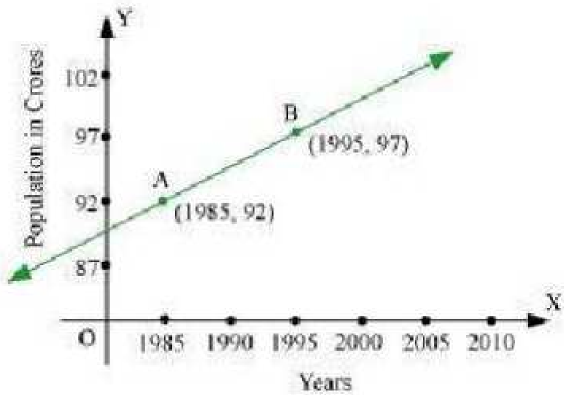
$$\frac{ka}{kh}+\frac{bh}{kh}=\frac{kh}{kh}$$

$$\Rightarrow \frac{a}{h}+\frac{b}{k}=1$$

Hence, $\frac{a}{h}+\frac{b}{k}=1$

Question

Consider the given population and year graph. Find the slope of the line AB and using it, find what will be the population in the year 2010?



Answer

Since line AB passes through points A (1985, 92) and B (1995, 97), its slope is

$$\frac{97-92}{1995-1985}=\frac{5}{10}=\frac{1}{2}$$

Let y be the population in the year 2010. Then, according to the given graph, line AB must pass through point C (2010, y).

∴Slope of AB = Slope of BC

$$\Rightarrow \frac{1}{2}=\frac{y-97}{2010-1995}$$

$$\Rightarrow \frac{1}{2}=\frac{y-97}{15}$$

$$\Rightarrow \frac{15}{2}=y-97$$

$$\Rightarrow y-97=7.5$$

$$\Rightarrow y=7.5+97=104.5$$

Thus, the slope of line AB is $\frac{1}{2}$, while in the year 2010, the population will be 104.5 crores.

Question

Find the equation of the line which passes though $\left(2,\ 2\sqrt{3}\right)$ and is inclined with the x-axis at an angle of 75°.

Answer

The slope of the line that inclines with the x-axis at an angle of 75° is

m = tan 75°

$$\Rightarrow m = \tan\left(45° + 30°\right) = \frac{\tan 45° + \tan 30°}{1 - \tan 45° \cdot \tan 30°} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

We know that the equation of the line passing through point (x_0, y_0), whose slope is m, is $(y - y_0) = m(x - x_0)$.

Thus, if a line passes though $\left(2,\ 2\sqrt{3}\right)$ and inclines with the x-axis at an angle of 75°,

then the equation of the line is given as

$$\left(y - 2\sqrt{3}\right) = \frac{\sqrt{3}+1}{\sqrt{3}-1}(x-2)$$

$$\left(y - 2\sqrt{3}\right)\left(\sqrt{3}-1\right) = \left(\sqrt{3}+1\right)(x-2)$$

$$y\left(\sqrt{3}-1\right) - 2\sqrt{3}\left(\sqrt{3}-1\right) = x\left(\sqrt{3}+1\right) - 2\left(\sqrt{3}+1\right)$$

$$\left(\sqrt{3}+1\right)x - \left(\sqrt{3}-1\right)y = 2\sqrt{3} + 2 - 6 + 2\sqrt{3}$$

$$\left(\sqrt{3}+1\right)x - \left(\sqrt{3}-1\right)y = 4\sqrt{3} - 4$$

$$\text{i.e., } \left(\sqrt{3}+1\right)x - \left(\sqrt{3}-1\right)y = 4\left(\sqrt{3}-1\right)$$

Question

Find the equation of the line which intersects the x-axis at a distance of 3 units to the left of origin with slope −2.

Answer

It is known that if a line with slope m makes x-intercept d, then the equation of the line is given as

$y = m(x - d)$

For the line intersecting the x-axis at a distance of 3 units to the left of the origin, $d = -3$.

The slope of the line is given as $m = -2$

Thus, the required equation of the given line is

$y = -2\,[x - (-3)]$

$y = -2x - 6$

i.e., $2x + y + 6 = 0$

Question

Find the equation of the line which intersects the y-axis at a distance of 2 units above the origin and makes an angle of 30° with the positive direction of the x-axis.

Answer

It is known that if a line with slope m makes y-intercept c, then the equation of the line is given as

$y = mx + c$

Here, $c = 2$ and $m = \tan 30° = \frac{1}{\sqrt{3}}$.

Thus, the required equation of the given line is

$$y = \frac{1}{\sqrt{3}}x + 2$$

$$y = \frac{x + 2\sqrt{3}}{\sqrt{3}}$$

$$\sqrt{3}y = x + 2\sqrt{3}$$

i.e., $x - \sqrt{3}y + 2\sqrt{3} = 0$

Question

Find the equation of the line that passes through the points (-1, 1) and (2,-4)

Answer

It is known that the equation of the line passing through points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Therefore, the equation of the line passing through the points (−1, 1) and (2, −4) is

$$(y-1) = \frac{-4-1}{2+1}(x+1)$$

$$(y-1) = \frac{-5}{3}(x+1)$$

$$3(y-1) = -5(x+1)$$

$$3y - 3 = -5x - 5$$

$$\text{i.e., } 5x + 3y + 2 = 0$$

Question

Find the equation of the line which is at a perpendicular distance of 5 units from the origin and the angle made by the perpendicular with the positive x-axis is 30°

Answer

If p is the length of the normal from the origin to a line and ω is the angle made by the normal with the positive direction of the x-axis, then the equation of the line is given by $x\cos\omega + y\sin\omega = p$.

Here, $p = 5$ units and $\omega = 30°$

Thus, the required equation of the given line is

$x \cos 30° + y \sin 30° = 5$

$$x\frac{\sqrt{3}}{2} + y\cdot\frac{1}{2} = 5$$

$$\text{i.e., } \sqrt{3}x + y = 10$$

Question

A line perpendicular to the line segment joining the points (1, 0) and (2, 3) divides it in the ratio 1:n. Find the equation of the line.

Answer

According to the section formula, the coordinates of the point that divides the line segment joining the points (1, 0) and (2, 3) in the ratio 1: n is given by

$$\left(\frac{n(1)+1(2)}{1+n},\frac{n(0)+1(3)}{1+n}\right)=\left(\frac{n+2}{n+1},\frac{3}{n+1}\right)$$

The slope of the line joining the points (1, 0) and (2, 3) is

$$m=\frac{3-0}{2-1}=3$$

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.

Therefore, slope of the line that is perpendicular to the line joining the points (1, 0) and (2, 3) $=-\frac{1}{m}=-\frac{1}{3}$

Now, the equation of the line passing through $\left(\frac{n+2}{n+1},\frac{3}{n+1}\right)$ and whose slope is $-\frac{1}{3}$ is given by

$$\left(y-\frac{3}{n+1}\right)=\frac{-1}{3}\left(x-\frac{n+2}{n+1}\right)$$
$$\Rightarrow 3\left[(n+1)y-3\right]=-\left[x(n+1)-(n+2)\right]$$
$$\Rightarrow 3(n+1)y-9=-(n+1)x+n+2$$
$$\Rightarrow (1+n)x+3(1+n)y=n+11$$

Question

Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point (2, 3).

Answer

The equation of a line in the intercept form is

$$\frac{x}{a}+\frac{y}{b}=1 \qquad \text{... (i)}$$

Here, a and b are the intercepts on x and y axes respectively.

It is given that the line cuts off equal intercepts on both the axes. This means that $a = b$.

Accordingly, equation (i) reduces to

$$\frac{x}{a}+\frac{y}{a}=1$$

$$\Rightarrow x+y=a \qquad \text{... (ii)}$$

Since the given line passes through point (2, 3), equation (ii) reduces to

$2 + 3 = a \Rightarrow a = 5$

On substituting the value of a in equation (ii), we obtain

$x + y = 5$, which is the required equation of the line

Question

The perpendicular from the origin to a line meets it at the point (– 2, 9), find the equation of the line.

Answer

The slope of the line joining the origin (0, 0) and point (−2, 9) is $m_1=\frac{9-0}{-2-0}=-\frac{9}{2}$

Accordingly, the slope of the line perpendicular to the line joining the origin and point (−2, 9) is

$$m_2=-\frac{1}{m_1}=-\frac{1}{\left(-\frac{9}{2}\right)}=\frac{2}{9}$$

Now, the equation of the line passing through point (−2, 9) and having a slope m_2 is

$$(y-9)=\frac{2}{9}(x+2)$$
$$9y-81=2x+4$$
$$\text{i.e., } 2x-9y+85=0$$

Question

The length L (in centimetre) of a copper rod is a linear function of its Celsius temperature C. In an experiment, if L = 124.942 when C = 20 and L = 125.134 when C = 110, express L in terms of C.

Answer

It is given that when C = 20, the value of L is 124.942, whereas when C = 110, the value of L is 125.134.

Accordingly, points (20, 124.942) and (110, 125.134) satisfy the linear relation between L and C.

Now, assuming C along the x-axis and L along the y-axis, we have two points i.e., (20, 124.942) and (110, 125.134) in the XY plane.

Therefore, the linear relation between L and C is the equation of the line passing through points (20, 124.942) and (110, 125.134).

$$(L - 124.942) = \frac{125.134-124.942}{110-20}(C-20)$$

$$L-124.942=\frac{0.192}{90}(C-20)$$

$$\text{i.e., } L=\frac{0.192}{90}(C-20)+124.942, \text{ which is the required linear relation}$$

Question

The owner of a milk store finds that, he can sell 980 litres of milk each week at Rs 14/litre and 1220 litres of milk each week at Rs 16/litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at Rs 17/litre?

Answer

The relationship between selling price and demand is linear.

Assuming selling price per litre along the x-axis and demand along the y-axis, we have two points i.e., (14, 980) and (16, 1220) in the XY plane that satisfy the linear relationship between selling price and demand.

Therefore, the linear relationship between selling price per litre and demand is the equation of the line passing through points (14, 980) and (16, 1220).

$$y-980=\frac{1220-980}{16-14}(x-14)$$

$$y-980=\frac{240}{2}(x-14)$$

$$y-980=120(x-14)$$

$$\text{i.e., } y=120(x-14)+980$$

When x = Rs 17/litre,

$$y=120(17-14)+980$$

$$\Rightarrow y=120\times 3+980=360+980=1340$$

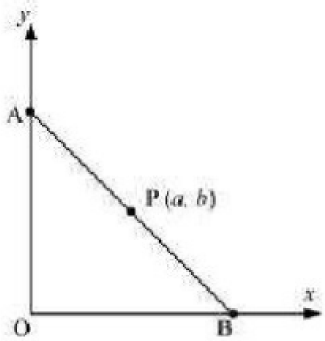
Thus, the owner of the milk store could sell 1340 litres of milk weekly at Rs 17/litre.

Question

P (a, b) is the mid-point of a line segment between axes. Show that equation of the line is $\frac{x}{a}+\frac{y}{b}=2$

Answer

Let AB be the line segment between the axes and let P (a, b) be its mid-point.



Let the coordinates of A and B be (0, y) and (x, 0) respectively.

Since P (a, b) is the mid-point of AB,

$$\left(\frac{0+x}{2}, \frac{y+0}{2}\right) = (a, b)$$

$$\Rightarrow \left(\frac{x}{2}, \frac{y}{2}\right) = (a, b)$$

$$\Rightarrow \frac{x}{2} = a \text{ and } \frac{y}{2} = b$$

$$\therefore x = 2a \text{ and } y = 2b$$

Thus, the respective coordinates of A and B are (0, 2*b*) and (2*a*, 0).

The equation of the line passing through points (0, 2*b*) and (2*a*, 0) is

$$(y - 2b) = \frac{(0 - 2b)}{(2a - 0)}(x - 0)$$

$$y - 2b = \frac{-2b}{2a}(x)$$

$$a(y - 2b) = -bx$$

$$ay - 2ab = -bx$$

$$\textbf{i.e.,}\ bx + ay = 2ab$$

On dividing both sides by *ab*, we obtain

$$\frac{bx}{ab} + \frac{ay}{ab} = \frac{2ab}{ab}$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = 2$$

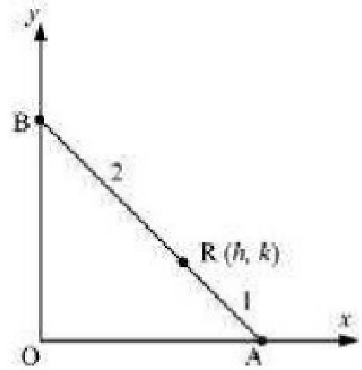
Thus, the equation of the line is $\frac{x}{a} + \frac{y}{b} = 2$.

Question

Point R (h, k) divides a line segment between the axes in the ratio 1:2. Find equation of the line.

Answer

Let AB be the line segment between the axes such that point R (h, k) divides AB in the ratio 1: 2.



Let the respective coordinates of A and B be (x, 0) and (0, y).

Since point R (h, k) divides AB in the ratio 1: 2, according to the section formula,

$$(h,k)=\left(\frac{1\times 0+2\times x}{1+2},\frac{1\times y+2\times 0}{1+2}\right)$$

$$\Rightarrow (h,k)=\left(\frac{2x}{3},\frac{y}{3}\right)$$

$$\Rightarrow h=\frac{2x}{3} \text{ and } k=\frac{y}{3}$$

$$\Rightarrow x = \frac{3h}{2} \text{ and } y = 3k$$

Therefore, the respective coordinates of A and B are $\left(\frac{3h}{2}, 0\right)$ and $(0, 3k)$.

Now, the equation of line AB passing through points $\left(\frac{3h}{2}, 0\right)$ and $(0, 3k)$ is

$$(y-0) = \frac{3k-0}{0-\frac{3h}{2}}\left(x-\frac{3h}{2}\right)$$

$$y = -\frac{2k}{h}\left(x-\frac{3h}{2}\right)$$

$$hy = -2kx + 3hk$$

$$\text{i.e., } 2kx + hy = 3hk$$

Thus, the required equation of the line is $2kx + hy = 3hk$

Question

By using the concept of equation of a line, prove that the three points (3, 0), (−2, −2) and (8, 2) are collinear.

Answer

In order to show that points (3, 0), (−2, −2), and (8, 2) are collinear, it suffices to show that the line passing through points (3, 0) and (−2, −2) also passes through point (8, 2).

The equation of the line passing through points (3, 0) and (−2, −2) is

$$(y-0)=\frac{(-2-0)}{(-2-3)}(x-3)$$

$$y=\frac{-2}{-5}(x-3)$$

$$5y=2x-6$$

i.e., $2x-5y=6$

It is observed that at x = 8 and y = 2,

L.H.S. = 2 × 8 − 5 × 2 = 16 − 10 = 6 = R.H.S.

Therefore, the line passing through points (3, 0) and (−2, −2) also passes through point (8, 2). Hence, points (3, 0), (−2, −2), and (8, 2) are collinear.

Question

Reduce the following equations into slope-intercept form and find their slopes and the y-intercepts.

(i) $x + 7y = 0$ (ii) $6x + 3y - 5 = 0$ (iii) $y = 0$

Answer

(i) The given equation is $x + 7y = 0$.

It can be written as

$$y = -\frac{1}{7}x + 0 \qquad \ldots(1)$$

This equation is of the form $y = mx + c$, where $m = -\frac{1}{7}$ and $c = 0$.

Therefore, equation (1) is in the slope-intercept form, where the slope and the y-intercept are $-\frac{1}{7}$ and 0 respectively.

(ii) The given equation is $6x + 3y - 5 = 0$.

It can be written as

$$y = \frac{1}{3}(-6x + 5)$$

$$y = -2x + \frac{5}{3} \qquad \ldots(2)$$

This equation is of the form $y = mx + c$, where $m = -2$ and $c = \frac{5}{3}$.

Therefore, equation (2) is in the slope-intercept form, where the slope and the y-intercept are -2 and $\frac{5}{3}$ respectively.

Question

Reduce the following equations into normal form. Find their perpendicular distances from the origin and angle between perpendicular and the positive x-axis.

(i) $x-\sqrt{3}y+8=0$ (ii) $y - 2 = 0$ (iii) $x - y = 4$

Answer

(i) The given equation is $x-\sqrt{3}y+8=0$.

It can be reduced as:

$$x-\sqrt{3}y=-8$$
$$\Rightarrow -x+\sqrt{3}y=8$$

On dividing both sides by $\sqrt{(-1)^2+\left(\sqrt{3}\right)^2}=\sqrt{4}=2$, we obtain

$$-\frac{x}{2}+\frac{\sqrt{3}}{2}y=\frac{8}{2}$$
$$\Rightarrow\left(-\frac{1}{2}\right)x+\left(\frac{\sqrt{3}}{2}\right)y=4$$
$$\Rightarrow x\cos 120^\circ+y\sin 120^\circ=4 \qquad ...(1)$$

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of line

$x \cos \omega + y \sin \omega = p$, we obtain $\omega = 120^\circ$ and $p = 4$.

Thus, the perpendicular distance of the line from the origin is 4, while the angle between the perpendicular and the positive x-axis is 120°.

(ii) The given equation is $y - 2 = 0$.

It can be reduced as $0.x + 1.y = 2$

On dividing both sides by $\sqrt{0^2+1^2}=1$, we obtain $0.x + 1.y = 2$

$\Rightarrow x \cos 90^\circ + y \sin 90^\circ = 2$ ... (1)

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of line

$x \cos \omega + y \sin \omega = p$, we obtain $\omega = 90°$ and $p = 2$.

Thus, the perpendicular distance of the line from the origin is 2, while the angle between the perpendicular and the positive x-axis is 90°.

(iii) The given equation is $x - y = 4$.

It can be reduced as $1.x + (-1)\,y = 4$

On dividing both sides by $\sqrt{1^2+(-1)^2}=\sqrt{2}$, we obtain

$$\frac{1}{\sqrt{2}}x+\left(-\frac{1}{\sqrt{2}}\right)y=\frac{4}{\sqrt{2}}$$

$$\Rightarrow x\cos\left(2\pi-\frac{\pi}{4}\right)+y\sin\left(2\pi-\frac{\pi}{4}\right)=2\sqrt{2}$$

$$\Rightarrow x\cos 315°+y\sin 315°=2\sqrt{2} \qquad ...(1)$$

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of line

$x \cos \omega + y \sin \omega = p$, we obtain $\omega = 315°$ and $p=2\sqrt{2}$.

Thus, the perpendicular distance of the line from the origin is $2\sqrt{2}$, while the angle between the perpendicular and the positive x-axis is 315°.

Question

Find the distance of the point (−1, 1) from the line $12(x + 6) = 5(y - 2)$.

Answer

The given equation of the line is $12(x + 6) = 5(y - 2)$.

$\Rightarrow 12x + 72 = 5y - 10$

$\Rightarrow 12x - 5y + 82 = 0$ ... (1)

On comparing equation (1) with general equation of line $Ax + By + C = 0$, we obtain $A = 12$, $B = -5$, and $C = 82$.

It is known that the perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$.

The given point is $(x_1, y_1) = (-1, 1)$.

Therefore, the distance of point (−1, 1) from the given line

$$= \frac{|12(-1) + (-5)(1) + 82|}{\sqrt{(12)^2 + (-5)^2}} \text{ units} = \frac{|-12 - 5 + 82|}{\sqrt{169}} \text{ units} = \frac{|65|}{13} \text{ units} = 5 \text{ units}$$

Question

Find the points on the x-axis, whose distances from the line $\frac{x}{3} + \frac{y}{4} = 1$ are 4 units.

Answer

The given equation of line is

$$\frac{x}{3} + \frac{y}{4} = 1$$

$$\text{or, } 4x + 3y - 12 = 0 \qquad \text{...(1)}$$

On comparing equation (1) with general equation of line $Ax + By + C = 0$, we obtain $A = 4$, $B = 3$, and $C = -12$.

Let $(a, 0)$ be the point on the x-axis whose distance from the given line is 4 units.

It is known that the perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$.

Therefore,

$$4 = \frac{|4a + 3\times 0 - 12|}{\sqrt{4^2 + 3^2}}$$

$$\Rightarrow 4 = \frac{|4a - 12|}{5}$$

$$\Rightarrow |4a - 12| = 20$$

$$\Rightarrow \pm(4a - 12) = 20$$

$$\Rightarrow (4a - 12) = 20 \text{ or } -(4a - 12) = 20$$

$$\Rightarrow 4a = 20 + 12 \text{ or } 4a = -20 + 12$$

$$\Rightarrow a = 8 \text{ or } -2$$

Thus, the required points on the x-axis are (−2, 0) and (8, 0).

Question

Find the distance between parallel lines

(i) $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$

(ii) $l(x + y) + p = 0$ and $l(x + y) - r = 0$

Answer

It is known that the distance (d) between parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by $d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$.

(i) The given parallel lines are $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$.

Here, $A = 15$, $B = 8$, $C_1 = -34$, and $C_2 = 31$.

Therefore, the distance between the parallel lines is

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} = \frac{|-34 - 31|}{\sqrt{(15)^2 + (8)^2}} \text{ units} = \frac{|-65|}{17} \text{ units} = \frac{65}{17} \text{ units}$$

(ii) The given parallel lines are $l(x + y) + p = 0$ and $l(x + y) - r = 0$.

$lx + ly + p = 0$ and $lx + ly - r = 0$

Here, $A = l$, $B = l$, $C_1 = p$, and $C_2 = -r$.

Therefore, the distance between the parallel lines is

$$d=\frac{|C_1-C_2|}{\sqrt{A^2+B^2}}=\frac{|p+r|}{\sqrt{l^2+l^2}}\text{ units}=\frac{|p+r|}{\sqrt{2l^2}}\text{ units}=\frac{|p+r|}{l\sqrt{2}}\text{ units}=\frac{1}{\sqrt{2}}\left|\frac{p+r}{l}\right|\text{ units}$$

Question

Find equation of the line parallel to the line $3x - 4y + 2 = 0$ and passing through the point $(-2, 3)$.

Answer

The equation of the given line is

$$3x-4y+2=0$$

$$\text{or } y=\frac{3x}{4}+\frac{2}{4}$$

$$\text{or } y=\frac{3}{4}x+\frac{1}{2}$$, which is of the form $y = mx + c$

$\therefore$ Slope of the given line $=\frac{3}{4}$

It is known that parallel lines have the same slope.

$\therefore$ Slope of the other line $= m=\frac{3}{4}$

Now, the equation of the line that has a slope of $\frac{3}{4}$ and passes through the point $(-2, 3)$ is

$$(y-3)=\frac{3}{4}\{x-(-2)\}$$

$$4y-12=3x+6$$

$$\text{i.e., } 3x-4y+18=0$$

Question

Find equation of the line perpendicular to the line $x - 7y + 5 = 0$ and having x intercept 3.

Answer

The given equation of line is $x-7y+5=0$.

Or, $y=\frac{1}{7}x+\frac{5}{7}$, which is of the form $y = mx + c$

$\therefore$Slope of the given line $=\frac{1}{7}$

The slope of the line perpendicular to the line having a slope of $\frac{1}{7}$ is $m=-\frac{1}{\left(\frac{1}{7}\right)}=-7$

The equation of the line with slope −7 and x-intercept 3 is given by

$y = m(x - d)$

$\Rightarrow y = -7(x - 3)$

$\Rightarrow y = -7x + 21$

$\Rightarrow 7x + y = 21$

Question

Two lines passing through the point (2, 3) intersects each other at an angle of 60°. If slope of one line is 2, find equation of the other line.

Answer

It is given that the slope of the first line, $m_1 = 2$.

Let the slope of the other line be m_2.

The angle between the two lines is 60°.

$$\therefore \tan 60° = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\Rightarrow \sqrt{3} = \left|\frac{2 - m_2}{1 + 2m_2}\right|$$

$$\Rightarrow \sqrt{3} = \pm\left(\frac{2 - m_2}{1 + 2m_2}\right)$$

$$\Rightarrow \sqrt{3} = \frac{2 - m_2}{1 + 2m_2} \text{ or } \sqrt{3} = -\left(\frac{2 - m_2}{1 + 2m_2}\right)$$

$$\Rightarrow \sqrt{3}\left(1 + 2m_2\right) = 2 - m_2 \text{ or } \sqrt{3}\left(1 + 2m_2\right) = -\left(2 - m_2\right)$$

$$\Rightarrow \sqrt{3} + 2\sqrt{3}m_2 + m_2 = 2 \text{ or } \sqrt{3} + 2\sqrt{3}m_2 - m_2 = -2$$

$$\Rightarrow \sqrt{3} + \left(2\sqrt{3} + 1\right)m_2 = 2 \text{ or } \sqrt{3} + \left(2\sqrt{3} - 1\right)m_2 = -2$$

$$\Rightarrow m_2 = \frac{2 - \sqrt{3}}{\left(2\sqrt{3} + 1\right)} \text{ or } m_2 = \frac{-\left(2 + \sqrt{3}\right)}{\left(2\sqrt{3} - 1\right)}$$

Case I : $m_2 = \left(\frac{2 - \sqrt{3}}{2\sqrt{3} + 1}\right)$

The equation of the line passing through point (2, 3) and having a slope of $\frac{(2-\sqrt{3})}{(2\sqrt{3}+1)}$ is

$$(y-3)=\frac{2-\sqrt{3}}{2\sqrt{3}+1}(x-2)$$
$$(2\sqrt{3}+1)y-3(2\sqrt{3}+1)=(2-\sqrt{3})x-2(2-\sqrt{3})$$
$$(\sqrt{3}-2)x+(2\sqrt{3}+1)y=-4+2\sqrt{3}+6\sqrt{3}+3$$
$$(\sqrt{3}-2)x+(2\sqrt{3}+1)y=-1+8\sqrt{3}$$

Question

The perpendicular from the origin to the line $y = mx + c$ meets it at the point (−1, 2). Find the values of m and c.

Answer

The given equation of line is $y = mx + c$.

It is given that the perpendicular from the origin meets the given line at (−1, 2).

Therefore, the line joining the points (0, 0) and (−1, 2) is perpendicular to the given line.

∴Slope of the line joining (0, 0) and (−1, 2) $=\frac{2}{-1}=-2$

The slope of the given line is m.

$$\therefore m\times -2=-1 \qquad [\text{The two lines are perpendicular}]$$
$$\Rightarrow m=\frac{1}{2}$$

Since point (−1, 2) lies on the given line, it satisfies the equation $y = mx + c$.

$$\therefore 2=m(-1)+c$$
$$\Rightarrow 2=\frac{1}{2}(-1)+c$$
$$\Rightarrow c=2+\frac{1}{2}=\frac{5}{2}$$

Thus, the respective values of m and c are $\frac{1}{2}$ and $\frac{5}{2}$

Question

If p and q are the lengths of perpendiculars from the origin to the lines $x\cos\theta - y\sin\theta = k\cos 2\theta$ and $x\sec\theta + y\,\text{cosec}\,\theta = k$, respectively, prove that $p^2 + 4q^2 = k^2$

Answer

The equations of given lines are

$x\cos\theta - y\sin\theta = k\cos 2\theta$ ... (1)

$x\sec\theta + y\,\text{cosec}\,\theta = k$ ... (2)

The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

On comparing equation (1) to the general equation of line i.e., $Ax + By + C = 0$, we obtain $A = \cos\theta$, $B = -\sin\theta$, and $C = -k\cos 2\theta$.

It is given that p is the length of the perpendicular from (0, 0) to line (1).

$$\therefore p = \frac{|A(0) + B(0) + C|}{\sqrt{A^2 + B^2}} = \frac{|C|}{\sqrt{A^2 + B^2}} = \frac{|-k\cos 2\theta|}{\sqrt{\cos^2\theta + \sin^2\theta}} = |-k\cos 2\theta| \quad ...(3)$$

On comparing equation (2) to the general equation of line i.e., $Ax + By + C = 0$, we obtain $A = \sec\theta$, $B = \text{cosec}\,\theta$, and $C = -k$.

It is given that q is the length of the perpendicular from (0, 0) to line (2).

$$\therefore q = \frac{|A(0) + B(0) + C|}{\sqrt{A^2 + B^2}} = \frac{|C|}{\sqrt{A^2 + B^2}} = \frac{|-k|}{\sqrt{\sec^2\theta + \text{cosec}^2\theta}} \quad ...(4)$$

From (3) and (4), we have

$$p^2 + 4q^2 = \left(\left|-k\cos 2\theta\right|\right)^2 + 4\left(\frac{|-k|}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}}\right)^2$$

$$= k^2\cos^2 2\theta + \frac{4k^2}{\left(\sec^2\theta + \operatorname{cosec}^2\theta\right)}$$

$$= k^2\cos^2 2\theta + \frac{4k^2}{\left(\dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}\right)}$$

$$= k^2\cos^2 2\theta + \frac{4k^2}{\left(\dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta}\right)}$$

$$= k^2\cos^2 2\theta + \frac{4k^2}{\left(\dfrac{1}{\sin^2\theta\cos^2\theta}\right)}$$

$$= k^2\cos^2 2\theta + 4k^2\sin^2\theta\cos^2\theta$$

$$= k^2\cos^2 2\theta + k^2\left(2\sin\theta\cos\theta\right)^2$$

$$= k^2\cos^2 2\theta + k^2\sin^2 2\theta$$

$$= k^2\left(\cos^2 2\theta + \sin^2 2\theta\right)$$

$$= k^2$$

Hence, we proved that $p^2 + 4q^2 = k^2$.

Question

If p is the length of perpendicular from the origin to the line whose intercepts on the axes are a and b, then show that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$.

Answer

It is known that the equation of a line whose intercepts on the axes are a and b is

$$\frac{x}{a}+\frac{y}{b}=1$$
$$\text{or } bx+ay=ab$$
$$\text{or } bx+ay-ab=0 \qquad \ldots(1)$$

The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by

$$d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}.$$

On comparing equation (1) to the general equation of line $Ax + By + C = 0$, we obtain $A = b$, $B = a$, and $C = -ab$.

Therefore, if p is the length of the perpendicular from point $(x_1, y_1) = (0, 0)$ to line (1), we obtain

$$p=\frac{|A(0)+B(0)-ab|}{\sqrt{b^2+a^2}}$$
$$\Rightarrow p=\frac{|-ab|}{\sqrt{a^2+b^2}}$$

On squaring both sides, we obtain

$$p^2=\frac{(-ab)^2}{a^2+b^2}$$
$$\Rightarrow p^2\left(a^2+b^2\right)=a^2b^2$$
$$\Rightarrow \frac{a^2+b^2}{a^2b^2}=\frac{1}{p^2}$$
$$\Rightarrow \frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

Question

Find the values of θ and p, if the equation $x\cos\theta + y\sin\theta = p$ is the normal form of the line $\sqrt{3}x + y + 2 = 0$.

Answer

The equation of the given line is $\sqrt{3}x + y + 2 = 0$.

This equation can be reduced as

$$\sqrt{3}x + y + 2 = 0$$
$$\Rightarrow -\sqrt{3}x - y = 2$$

On dividing both sides by $\sqrt{\left(-\sqrt{3}\right)^2 + (-1)^2} = 2$, we obtain

$$-\frac{\sqrt{3}}{2}x - \frac{1}{2}y = \frac{2}{2}$$
$$\Rightarrow \left(-\frac{\sqrt{3}}{2}\right)x + \left(-\frac{1}{2}\right)y = 1 \qquad ...(1)$$

On comparing equation (1) to $x\cos\theta + y\sin\theta = p$, we obtain

$$\cos\theta = -\frac{\sqrt{3}}{2},\ \sin\theta = -\frac{1}{2}, \text{ and } p = 1$$

Since the values of sin θ and cos θ are negative, $\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

Thus, the respective values of θ and p are $\frac{7\pi}{6}$ and 1

Question

Find the equations of the lines, which cut-off intercepts on the axes whose sum and product are 1 and −6, respectively.

Answer

Let the intercepts cut by the given lines on the axes be a and b.

It is given that

$a + b = 1$ ... (1)

$ab = -6$ ... (2)

On solving equations (1) and (2), we obtain

$a = 3$ and $b = -2$ or $a = -2$ and $b = 3$

It is known that the equation of the line whose intercepts on the axes are a and b is

$$\frac{x}{a} + \frac{y}{b} = 1 \text{ or } bx + ay - ab = 0$$

Case I: $a = 3$ and $b = -2$

In this case, the equation of the line is $-2x + 3y + 6 = 0$, i.e., $2x - 3y = 6$.

Case II: $a = -2$ and $b = 3$

In this case, the equation of the line is $3x - 2y + 6 = 0$, i.e., $-3x + 2y = 6$.

Thus, the required equation of the lines are $2x - 3y = 6$ and $-3x + 2y = 6$.

Question

What are the points on the y-axis whose distance from the line $\frac{x}{3}+\frac{y}{4}=1$ is 4 units.

Answer

Let (0, b) be the point on the y-axis whose distance from line $\frac{x}{3}+\frac{y}{4}=1$ is 4 units.

The given line can be written as $4x + 3y - 12 = 0$ ... (1)

On comparing equation (1) to the general equation of line $Ax + By + C = 0$, we obtain $A = 4$, $B = 3$, and $C = -12$.

It is known that the perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$.

Therefore, if (0, b) is the point on the y-axis whose distance from line $\frac{x}{3}+\frac{y}{4}=1$ is 4 units, then:

$$4=\frac{|4(0)+3(b)-12|}{\sqrt{4^2+3^2}}$$

$$\Rightarrow 4=\frac{|3b-12|}{5}$$

$$\Rightarrow 20=|3b-12|$$

$$\Rightarrow 20=\pm(3b-12)$$

$$\Rightarrow 20=(3b-12) \text{ or } 20=-(3b-12)$$

$$\Rightarrow 3b=20+12 \text{ or } 3b=-20+12$$

$$\Rightarrow b=\frac{32}{3} \text{ or } b=-\frac{8}{3}$$

Thus, the required points are $\left(0,\frac{32}{3}\right)$ and $\left(0,-\frac{8}{3}\right)$.

Question

Find the perpendicular distance from the origin to the line joining the points $(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$.

Answer

The equation of the line joining the points $(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$ is given by

$$y - \sin\theta = \frac{\sin\phi - \sin\theta}{\cos\phi - \cos\theta}(x - \cos\theta)$$

$$y(\cos\phi - \cos\theta) - \sin\theta(\cos\phi - \cos\theta) = x(\sin\phi - \sin\theta) - \cos\theta(\sin\phi - \sin\theta)$$

$$x(\sin\theta - \sin\phi) + y(\cos\phi - \cos\theta) + \cos\theta\sin\phi - \cos\theta\sin\theta - \sin\theta\cos\phi + \sin\theta\cos\theta = 0$$

$$x(\sin\theta - \sin\phi) + y(\cos\phi - \cos\theta) + \sin(\phi - \theta) = 0$$

$$Ax + By + C = 0, \text{ where } A = \sin\theta - \sin\phi,\ B = \cos\phi - \cos\theta, \text{ and } C = \sin(\phi - \theta)$$

It is known that the perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$.

Therefore, the perpendicular distance (d) of the given line from point $(x_1, y_1) = (0, 0)$ is

$$d = \frac{|(\sin\theta - \sin\phi)(0) + (\cos\phi - \cos\theta)(0) + \sin(\phi - \theta)|}{\sqrt{(\sin\theta - \sin\phi)^2 + (\cos\phi - \cos\theta)^2}}$$

$$= \frac{|\sin(\phi - \theta)|}{\sqrt{\sin^2\theta + \sin^2\phi - 2\sin\theta\sin\phi + \cos^2\phi + \cos^2\theta - 2\cos\phi\cos\theta}}$$

$$= \frac{|\sin(\phi-\theta)|}{\sqrt{(\sin^2\theta+\cos^2\theta)+(\sin^2\phi+\cos^2\phi)-2(\sin\theta\sin\phi+\cos\theta\cos\phi)}}$$

$$= \frac{|\sin(\phi-\theta)|}{\sqrt{1+1-2(\cos(\phi-\theta))}}$$

$$= \frac{|\sin(\phi-\theta)|}{\sqrt{2(1-\cos(\phi-\theta))}}$$

$$= \frac{|\sin(\phi-\theta)|}{\sqrt{2\left(2\sin^2\left(\frac{\phi-\theta}{2}\right)\right)}}$$

$$= \frac{|\sin(\phi-\theta)|}{\left|2\sin\left(\frac{\phi-\theta}{2}\right)\right|}$$

Question

Find the equation of the line parallel to y-axis and drawn through the point of intersection of the lines $x - 7y + 5 = 0$ and $3x + y = 0$.

Answer

The equation of any line parallel to the y-axis is of the form

$x = a$ ... (1)

The two given lines are

$x - 7y + 5 = 0$ ... (2)

$3x + y = 0$ ... (3)

On solving equations (2) and (3), we obtain $x = -\frac{5}{22}$ and $y = \frac{15}{22}$.

Therefore, $\left(-\frac{5}{22}, \frac{15}{22}\right)$ is the point of intersection of lines (2) and (3).

Since line $x = a$ passes through point $\left(-\frac{5}{22}, \frac{15}{22}\right)$, $a = -\frac{5}{22}$.

Thus, the required equation of the line is $x = -\frac{5}{22}$

Question

Find the equation of a line drawn perpendicular to the line $\frac{x}{4}+\frac{y}{6}=1$ through the point, where it meets the *y*-axis.

Answer

The equation of the given line is $\frac{x}{4}+\frac{y}{6}=1$.

This equation can also be written as $3x + 2y - 12 = 0$

$y=\frac{-3}{2}x+6$, which is of the form $y = mx + c$

∴Slope of the given line $=-\frac{3}{2}$

∴Slope of line perpendicular to the given line $=-\frac{1}{\left(-\frac{3}{2}\right)}=\frac{2}{3}$

Let the given line intersect the *y*-axis at (0, *y*).

On substituting *x* with 0 in the equation of the given line, we obtain $\frac{y}{6}=1 \Rightarrow y=6$

∴The given line intersects the *y*-axis at (0, 6).

The equation of the line that has a slope of $\frac{2}{3}$ and passes through point (0, 6) is

$$(y-6)=\frac{2}{3}(x-0)$$
$$3y-18=2x$$
$$2x-3y+18=0$$

Thus, the required equation of the line is $2x-3y+18=0$.

Question

Find the area of the triangle formed by the lines $y - x = 0$, $x + y = 0$ and $x - k = 0$.

Answer

The equations of the given lines are

$y - x = 0$ ... (1)

$x + y = 0$ ... (2)

$x - k = 0$ ... (3)

The point of intersection of lines (1) and (2) is given by

$x = 0$ and $y = 0$

The point of intersection of lines (2) and (3) is given by

$x = k$ and $y = -k$

The point of intersection of lines (3) and (1) is given by

$x = k$ and $y = k$

Thus, the vertices of the triangle formed by the three given lines are (0, 0), $(k, -k)$, and (k, k).

We know that the area of a triangle whose vertices are (x_1, y_1), (x_2, y_2), and (x_3, y_3) is

$$\frac{1}{2}\left|x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right|$$

Question

Find the equation of the lines through the point (3, 2) which make an angle of 45° with the line $x - 2y = 3$.

Answer

Let the slope of the required line be m_1.

The given line can be written as $y = \frac{1}{2}x - \frac{3}{2}$, which is of the form $y = mx + c$

∴Slope of the given line = $m_2 = \frac{1}{2}$

It is given that the angle between the required line and line $x - 2y = 3$ is 45°.

We know that if θ isthe acute angle between lines l_1 and l_2 with slopes m_1 and m_2 respectively, then $\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$.

$$\therefore \tan 45^\circ = \frac{|m_1 - m_2|}{1 + m_1 m_2}$$

$$\Rightarrow 1 = \left| \frac{\frac{1}{2} - m_1}{1 + \frac{m_1}{2}} \right|$$

$$\Rightarrow 1 = \left| \frac{\left(\frac{1 - 2m_1}{2}\right)}{\frac{2 + m_1}{2}} \right|$$

$$\Rightarrow 1 = \left| \frac{1 - 2m_1}{2 + m_1} \right|$$

$$\Rightarrow 1 = \pm \left(\frac{1 - 2m_1}{2 + m_1} \right)$$

$$\Rightarrow 1 = \frac{1 - 2m_1}{2 + m_1} \text{ or } 1 = -\left(\frac{1 - 2m_1}{2 + m_1} \right)$$

$$\Rightarrow 2 + m_1 = 1 - 2m_1 \text{ or } 2 + m_1 = -1 + 2m_1$$

$$\Rightarrow m_1 = -\frac{1}{3} \text{ or } m_1 = 3$$

Case I: $m_1 = 3$

The equation of the line passing through (3, 2) and having a slope of 3 is:

$y - 2 = 3(x - 3)$

$y - 2 = 3x - 9$

$3x - y = 7$

Case II: $m_1 = -\frac{1}{3}$

The equation of the line passing through (3, 2) and having a slope of $-\frac{1}{3}$ is:

$$y-2=-\frac{1}{3}(x-3)$$
$$3y-6=-x+3$$
$$x+3y=9$$

Thus, the equations of the lines are $3x - y = 7$ and $x + 3y = 9$.

Question

Find the equation of the line passing through the point of intersection of the lines $4x + 7y - 3 = 0$ and $2x - 3y + 1 = 0$ that has equal intercepts on the axes.

Answer

Let the equation of the line having equal intercepts on the axes be

$$\frac{x}{a}+\frac{y}{a}=1$$
$$\text{Or } x+y=a \qquad ...(1)$$

On solving equations $4x + 7y - 3 = 0$ and $2x - 3y + 1 = 0$, we obtain $x=\frac{1}{13}$ and $y=\frac{5}{13}$.

$\therefore\left(\frac{1}{13},\frac{5}{13}\right)$ is the point of intersection of the two given lines.

Since equation (1) passes through point $\left(\frac{1}{13},\frac{5}{13}\right)$,

$$\frac{1}{13}+\frac{5}{13}=a$$
$$\Rightarrow a=\frac{6}{13}$$

$\therefore$ Equation (1) becomes $x+y=\frac{6}{13}$, **i.e.,** $13x+13y=6$

Thus, the required equation of the line is $13x+13y=6$

Question

Show that the equation of the line passing through the origin and making an angle θ with the line $y=mx+c$ is $\frac{y}{x}=\frac{m\pm\tan\theta}{1\mp m\tan\theta}$.

Answer

Let the equation of the line passing through the origin be $y = m_1x$.

If this line makes an angle of θ with line $y = mx + c$, then angle θ is given by

$$\therefore \tan\theta=\left|\frac{m_1-m}{1+m_1m}\right|$$
$$\Rightarrow \tan\theta=\left|\frac{\frac{y}{x}-m}{1+\frac{y}{x}m}\right|$$
$$\Rightarrow \tan\theta=\pm\left(\frac{\frac{y}{x}-m}{1+\frac{y}{x}m}\right)$$

$$\Rightarrow \tan\theta = \frac{\frac{y}{x} - m}{1 + \frac{y}{x} m} \quad \text{or} \quad \tan\theta = -\left(\frac{\frac{y}{x} - m}{1 + \frac{y}{x} m}\right)$$

Case I:

$$\tan\theta = \frac{\frac{y}{x} - m}{1 + \frac{y}{x} m}$$

$$\tan\theta = \frac{\frac{y}{x} - m}{1 + \frac{y}{x} m}$$

$$\Rightarrow \tan\theta + \frac{y}{x} m \tan\theta = \frac{y}{x} - m$$

$$\Rightarrow m + \tan\theta = \frac{y}{x}(1 - m\tan\theta)$$

$$\Rightarrow \frac{y}{x} = \frac{m + \tan\theta}{1 - m\tan\theta}$$

Question

Find the distance of the line $4x + 7y + 5 = 0$ from the point (1, 2) along the line $2x - y = 0$.

Answer

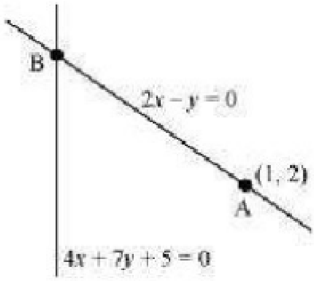
The given lines are

$2x - y = 0$ … (1)

$4x + 7y + 5 = 0$ … (2)

A (1, 2) is a point on line (1).

Let B be the point of intersection of lines (1) and (2).



On solving equations (1) and (2), we obtain $x = \frac{-5}{18}$ and $y = \frac{-5}{9}$.

∴Coordinates of point B are $\left(\frac{-5}{18}, \frac{-5}{9}\right)$

By using distance formula, the distance between points A and B can be obtained as

$$AB = \sqrt{\left(1+\frac{5}{18}\right)^2 + \left(2+\frac{5}{9}\right)^2} \text{ units}$$

$$= \sqrt{\left(\frac{23}{18}\right)^2 + \left(\frac{23}{9}\right)^2} \text{ units}$$

$$= \sqrt{\left(\frac{23}{2\times 9}\right)^2 + \left(\frac{23}{9}\right)^2} \text{ units}$$

$$= \sqrt{\left(\frac{23}{9}\right)^2 \left(\frac{1}{2}\right)^2 + \left(\frac{23}{9}\right)^2} \text{ units}$$

$$= \sqrt{\left(\frac{23}{9}\right)^2 \left(\frac{1}{4}+1\right)} \text{ units}$$

$$= \frac{23}{9}\sqrt{\frac{5}{4}} \text{ units}$$

$$= \frac{23}{9}\times\frac{\sqrt{5}}{2} \text{ units}$$

$$= \frac{23\sqrt{5}}{18} \text{ units}$$

Thus, the required distance is $\frac{23\sqrt{5}}{18}$ **units**

Question

Find the direction in which a straight line must be drawn through the point (−1, 2) so that its point of intersection with the line $x + y = 4$ may be at a distance of 3 units from this point.

Answer

Let $y = mx + c$ be the line through point (−1, 2).

Accordingly, $2 = m(-1) + c$.

$\Rightarrow 2 = -m + c$

$\Rightarrow c = m + 2$

$\therefore y = mx + m + 2 \ldots (1)$

The given line is

$x + y = 4 \ldots (2)$

On solving equations (1) and (2), we obtain

$$x = \frac{2-m}{m+1} \text{ and } y = \frac{5m+2}{m+1}$$

$\therefore \left(\frac{2-m}{m+1}, \frac{5m+2}{m+1}\right)$ is the point of intersection of lines (1) and (2).

Since this point is at a distance of 3 units from point (− 1, 2), according to distance formula,

$$\sqrt{\left(\frac{2-m}{m+1}+1\right)^2 + \left(\frac{5m+2}{m+1}-2\right)^2} = 3$$

$$\Rightarrow \left(\frac{2-m+m+1}{m+1}\right)^2 + \left(\frac{5m+2-2m-2}{m+1}\right)^2 = 3^2$$

$$\Rightarrow \frac{9}{(m+1)^2} + \frac{9m^2}{(m+1)^2} = 9$$

$$\Rightarrow \frac{9}{(m+1)^2} + \frac{9m^2}{(m+1)^2} = 9$$

$$\Rightarrow \frac{1+m^2}{(m+1)^2} = 1$$

$$\Rightarrow 1 + m^2 = m^2 + 1 + 2m$$

$$\Rightarrow 2m = 0$$

$$\Rightarrow m = 0$$

Thus, the slope of the required line must be zero i.e., the line must be parallel to the x-axis.

Question

Find equation of the line which is equidistant from parallel lines $9x + 6y - 7 = 0$ and $3x + 2y + 6 = 0$.

Answer

The equations of the given lines are

$9x + 6y - 7 = 0$ ... (1)

$3x + 2y + 6 = 0$ ... (2)

Let P (h, k) be the arbitrary point that is equidistant from lines (1) and (2). The perpendicular distance of P (h, k) from line (1) is given by

$$d_1 = \frac{|9h+6k-7|}{(9)^2+(6)^2} = \frac{|9h+6k-7|}{\sqrt{117}} = \frac{|9h+6k-7|}{3\sqrt{13}}$$

The perpendicular distance of P (h, k) from line (2) is given by

$$d_2 = \frac{|3h+2k+6|}{\sqrt{(3)^2+(2)^2}} = \frac{|3h+2k+6|}{\sqrt{13}}$$

Since P (h, k) is equidistant from lines (1) and (2), $d_1 = d_2$

:-{D

To recall standard integrals

$f(x)$	$\int f(x)dx$	$f(x)$	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1} \quad (n \neq -1)$	$[g(x)]^n g'(x)$	$\frac{[g(x)]^{n+1}}{n+1} \quad (n \neq -1)$
$\frac{1}{x}$	$\ln\lvert x\rvert$	$\frac{g'(x)}{g(x)}$	$\ln\lvert g(x)\rvert$
e^x	e^x	a^x	$\frac{a^x}{\ln a} \quad (a > 0)$
$\sin x$	$-\cos x$	$\sinh x$	$\cosh x$
$\cos x$	$\sin x$	$\cosh x$	$\sinh x$
$\tan x$	$-\ln\lvert\cos x\rvert$	$\tanh x$	$\ln \cosh x$
$\mathrm{cosec}\, x$	$\ln\left\lvert\tan\frac{x}{2}\right\rvert$	$\mathrm{cosech}\, x$	$\ln\left\lvert\tanh\frac{x}{2}\right\rvert$
$\sec x$	$\ln\lvert\sec x + \tan x\rvert$	$\mathrm{sech}\, x$	$2\tan^{-1} e^x$
$\sec^2 x$	$\tan x$	$\mathrm{sech}^2 x$	$\tanh x$
$\cot x$	$\ln\lvert\sin x\rvert$	$\coth x$	$\ln\lvert\sinh x\rvert$
$\sin^2 x$	$\frac{x}{2} - \frac{\sin 2x}{4}$	$\sinh^2 x$	$\frac{\sinh 2x}{4} - \frac{x}{2}$
$\cos^2 x$	$\frac{x}{2} + \frac{\sin 2x}{4}$	$\cosh^2 x$	$\frac{\sinh 2x}{4} + \frac{x}{2}$

$f(x)$	$\int f(x)\,dx$	$f(x)$	$\int f(x)\,dx$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a}$	$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert \quad (0<\lvert x\rvert<a)$
	$(a > 0)$	$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert \quad (\lvert x\rvert > a>0)$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\frac{x}{a}$	$\frac{1}{\sqrt{a^2+x^2}}$	$\ln\left\lvert\frac{x+\sqrt{a^2+x^2}}{a}\right\rvert \quad (a > 0)$
	$(-a < x < a)$	$\frac{1}{\sqrt{x^2-a^2}}$	$\ln\left\lvert\frac{x+\sqrt{x^2-a^2}}{a}\right\rvert \quad (x>a>0)$
$\sqrt{a^2-x^2}$	$\frac{a^2}{2}\left[\sin^{-1}\left(\frac{x}{a}\right) + \frac{x\sqrt{a^2-x^2}}{a^2}\right]$	$\sqrt{a^2+x^2}$	$\frac{a^2}{2}\left[\sinh^{-1}\left(\frac{x}{a}\right) + \frac{x\sqrt{a^2+x^2}}{a^2}\right]$
		$\sqrt{x^2-a^2}$	$\frac{a^2}{2}\left[-\cosh^{-1}\left(\frac{x}{a}\right) + \frac{x\sqrt{x^2-a^2}}{a^2}\right]$

Some series Expansions -

$$\frac{\pi}{2} = \left(\frac{2}{1}\frac{2}{3}\right)\left(\frac{4}{3}\frac{4}{5}\right)\left(\frac{6}{5}\frac{6}{7}\right)\left(\frac{8}{7}\frac{8}{9}\right)\ldots$$

$$\pi = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \frac{4}{13} - \cdots$$

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots\cdot$$

$$\pi = \sqrt{12}\left(1 - \frac{1}{3\cdot 3} + \frac{1}{5\cdot 3^2} - \frac{1}{7\cdot 3^3} + \cdots\right)$$

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots = \sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\int_0^{\pi/2} \log \sin x \, dx = -\frac{\pi}{2}\log 2 = \frac{\pi}{2}\log\frac{1}{2}$$

Solve a series problem

If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$ upto $\infty = \frac{\pi^2}{6}$, then value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ up to ∞ is

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{6}$ (c) $\frac{\pi^2}{8}$ (d) $\frac{\pi^2}{12}$

Ans. (c)

Solution We have $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ upto ∞

$$= \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} \cdots \text{ upto } \infty$$

$$- \frac{1}{2^2}\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right]$$

$$= \frac{\pi^2}{6} - \frac{1}{4}\left(\frac{\pi^2}{6}\right) = \frac{\pi^2}{8}$$

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \ldots \infty = \frac{\pi^2}{12}$$

$$\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \ldots \infty = \frac{\pi^2}{24}$$

$$\frac{\sin\sqrt{x}}{\sqrt{x}} = 1 - \frac{x}{3!} + \frac{x^2}{5!} - \frac{x^3}{7!} + \frac{x^4}{9!} - \frac{x^5}{11!} + \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = \sum_{k=0}^{n} \frac{(-1)^k x^{2k}}{(2k)!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots = \sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots = \sum_{k=0}^{n} \frac{x^{2k}}{(2k)!}$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots = \sum_{k=0}^{n} \frac{x^{2k+1}}{(2k+1)!}$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad (-1 \le x < 1)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} \cdots + \frac{2^{2n}\left(2^{2n}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \qquad |x| < \frac{\pi}{2}$$

$$\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \cdots + \frac{E_n x^{2n}}{(2n)!} + \cdots \quad |x| < \frac{\pi}{2}$$

$$\csc x = \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^5}{15120} + \cdots + \frac{2\left(2^{2n-1}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \qquad 0 < |x| < \pi$$

$$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - - \cdots - \frac{2^{2n} B_n x^{2n-1}}{(2n)!} - \cdots \quad 0 < |x| < \pi$$

$$\tan x = x + \frac{x^3}{3} + \frac{2\,x^5}{15} + \cdots$$

$$\sec x = 1 + \frac{x^2}{\underline{|2}} + \frac{5\,x^4}{\underline{|4}} + \cdots$$

$$\log\,(\cos x) = -\frac{x^2}{\underline{|2}} - \frac{2\,x^4}{\underline{|4}} - \cdots$$

$$\log\,(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \cdots$$

$$\sin^{-1} x = x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \cdots \quad |x| < 1$$

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$= \frac{\pi}{2} - \left(x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \cdots\right) \quad |x| < 1$$

$$\tan^{-1} x = \begin{cases} x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \cdots & |x| < 1 \\ \pm\dfrac{\pi}{2} - \dfrac{1}{x} + \dfrac{1}{3x^3} - \dfrac{1}{5x^5} + \cdots & \begin{cases} + \text{ if } x \geq 1 \\ - \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$$

$$= \frac{\pi}{2} - \left(\frac{1}{x} + \frac{1}{2\cdot 3x^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5x^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \cdots\right) \quad |x| > 1$$

$$\csc^{-1} x = \sin^{-1}(1/x)$$

$$= \frac{1}{x} + \frac{1}{2\cdot 3x^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5x^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \cdots \quad |x| > 1$$

$$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$$

$$= \begin{cases} \dfrac{\pi}{2} - \left(x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \cdots\right) & |x| < 1 \\ p\pi + \dfrac{1}{x} - \dfrac{1}{3x^3} + \dfrac{1}{5x^5} + \cdots & \begin{cases} p = 0 \text{ if } x \geq 1 \\ p = 1 \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\ln x = 2\left[\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \ldots\right]$$

$$= 2\sum_{n=1}^{\infty} \frac{1}{2n-1}\left(\frac{x-1}{x+1}\right)^{2n-1} \quad (x > 0)$$

$$\ln x = \frac{x-1}{x} + \frac{1}{2}\left(\frac{x-1}{x}\right)^2 + \frac{1}{3}\left(\frac{x-1}{x}\right)^3 + \ldots$$

$$= \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{x-1}{x}\right)^n \quad (x > \frac{1}{2})$$

$$\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}(x-1)^n \quad (0 < x \le 2)$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}x^n \quad (|x| < 1)$$

$$\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots\infty (-1 \le x < 1)$$

$$\log_e(1+x) - \log_e(1-x) =$$

$$\log_e \frac{1+x}{1-x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\infty\right)(-1 < x < 1)$$

$$\log_e\left(1+\frac{1}{n}\right) = \log_e \frac{n+1}{n} = 2 \quad \left[\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \ldots\infty\right]$$

$$\log_e (1 + x) + \log_e (1 - x) = \log_e(1-x^2) = -2\left(\frac{x^2}{2} + \frac{x^4}{4} + \ldots\infty\right)(-1 < x < 1)$$

$$\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots \quad = \frac{1}{1 \cdot 2} + \frac{1}{3.4} + \frac{1}{5.6} + \ldots$$

Important Results

(i) (a) $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cos^n x}{\sin^n x + \cos^n x} dx$

(b) $\int_0^{\pi/2} \frac{\tan^n x}{1 + \tan^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{dx}{1 + \tan^n x}$

(c) $\int_0^{\pi/2} \frac{dx}{1 + \cot^n x} = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{1 + \cot^n x} dx$

(d) $\int_0^{\pi/2} \frac{\tan^n x}{\tan^n x + \cot^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{\tan^n x + \cot^n x} dx$

(e) $\int_0^{\pi/2} \frac{\sec^n x}{\sec^n x + \operatorname{cosec}^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\operatorname{cosec}^n x}{\sec^n x + \operatorname{cosec}^n x} dx$ where, $n \in R$

(ii) $\int_0^{\pi/2} \frac{a^{\sin^n x}}{a^{\sin^n x} + a^{\cos^n x}} dx = \int_0^{\pi/2} \frac{a^{\cos^n x}}{a^{\sin^n x} + a^{\cos^n x}} dx = \frac{\pi}{4}$

(iii) (a) $\int_0^{\pi/2} \log \sin x \, dx = \int_0^{\pi/2} \log \cos x \, dx = -\frac{\pi}{2} \log 2$

(b) $\int_0^{\pi/2} \log \tan x \, dx = \int_0^{\pi/2} \log \cot x \, dx = 0$

(c) $\int_0^{\pi/2} \log \sec x \, dx = \int_0^{\pi/2} \log \operatorname{cosec} x \, dx = \frac{\pi}{2} \log 2$

(iv) (a) $\int_0^{\infty} e^{-ax} \sin bx \, dx = \frac{b}{a^2 + b^2}$

(b) $\int_0^{\infty} e^{-ax} \cos bx \, dx = \frac{a}{a^2 + b^2}$

(c) $\int_0^{\infty} e^{-ax} x^n \, dx = \frac{n!}{a^n + 1}$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left(x + \sqrt{x^2 - a^2}\right) + C$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left(\frac{x - a}{x + a}\right) + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left(\frac{a + x}{a - x}\right) + C$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{a^2 + x^2}\, dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \sinh^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \cosh^{-1}\left(\frac{x}{a}\right) + C$$

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Good Luck to you for your Preparations, References, and Exams

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