Spoon Feeding Complex or imaginary numbers

Simplified Knowledge Management Classes Bangalore

My name is Subhashish Chattopadhyay. I have been teaching for IIT-JEE, Various International Exams (such as IMO [International Mathematics Olympiad], IPhO [International Physics Olympiad], IChO [International Chemistry Olympiad]), IGCSE (IB), CBSE, I.Sc, Indian State Board exams such as WB-Board, Karnataka PU-II etc since 1989. As I write this book in 2016, it is my 25 th year of teaching. I was a Visiting Professor to BARC Mankhurd, Chembur, Mumbai, Homi Bhabha Centre for Science Education (HBCSE) Physics Olympics camp BARC Campus.

I am Life Member of …

- IAPT (Indian Association of Physics Teachers)

- IPA (Indian Physics Association)

- AMTI (Association of Mathematics Teachers of India)

- National Human Rights Association

- Men's Rights Movement (India and International)

- MGTOW Movement (India and International)

And also of

IACT (Indian Association of Chemistry Teachers)

The selection for National Camp (for Official Science Olympiads - Physics, Chemistry, Biology, Astronomy) happens in the following steps ….

1) **NSEP** (National Standard Exam in Physics) and **NSEC** (National Standard Exam in Chemistry) held around 24 rth November. Approx 35,000 students appear for these exams every year. The exam fees is Rs 100 each. Since 1998 the IIT JEE toppers have been topping these exams and they get to know their rank / performance ahead of others.

2) **INPhO** (Indian National Physics Olympiad) and **INChO** (Indian National Chemistry Olympiad). Around 300 students in each subject are allowed to take these exams. Students coming from outside cities are paid fair from the Govt of India.

3) The Top 35 students of each subject are invited at HBCSE (Homi Bhabha Center for Science Education) Mankhurd, near Chembur, BARC, Mumbai. After a 2-3 weeks camp the top 5 are selected to represent India. The flight tickets and many other expenses are taken care by Govt of India.

Since last 50 years there has been no dearth of "Good Books". Those who are interested in studies have been always doing well. This e-Book does not intend to replace any standard text book. These topics are very old and already standardized.

There are 3 kinds of Text Books

- The thin Books - Good students who want more details are not happy with these. Average students who need more examples are not happy with these. Most students who want to "Cram" quickly and pass somehow find the thin books "good" as they have to read less !!

- The Thick Books - Most students do not like these, as they want to read as less as possible. Average students are "busy" with many other things and have no time to read all these.

- The Average sized Books - Good students do not get all details in any one book. Most bad students do not want to read books of "this much thickness" also !!

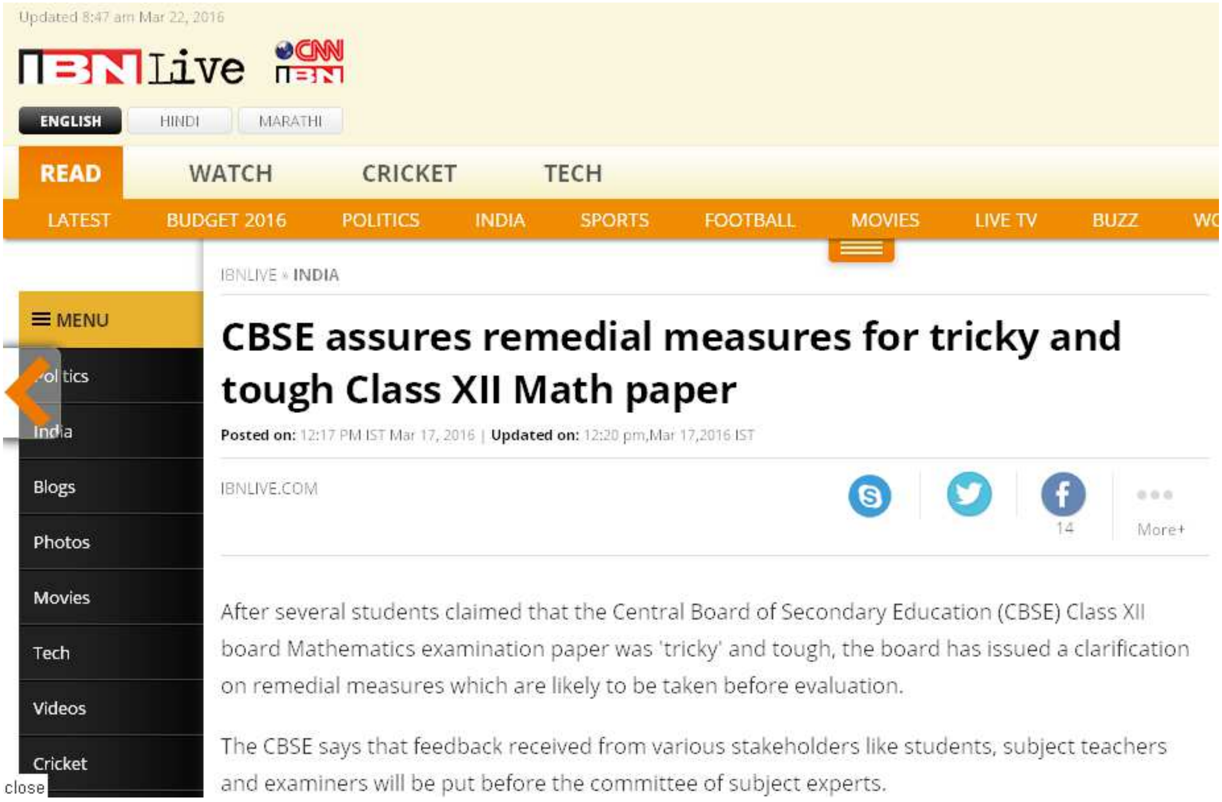
We know there can be no shoe that's fits in all.

Printed books are not e-Books! Can't be downloaded and kept in hard-disc for reading "later" ………

So if you read this book later, you will get all kinds of examples in a single place. This becomes a very good "Reference Material". I sincerely wish that all find this "very useful".

Students who do not practice lots of problems, do not do well. The rules of "doing well" had never changed …. Will never change !

After 2016 CBSE Mathematics exam, lots of students complained that the paper was tough!



Updated 8:47 am Mar 22, 2016

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IBNLIVE » INDIA

CBSE assures remedial measures for tricky and tough Class XII Math paper

Posted on: 12:17 PM IST Mar 17, 2016 | **Updated on:** 12:20 pm,Mar 17,2016 IST

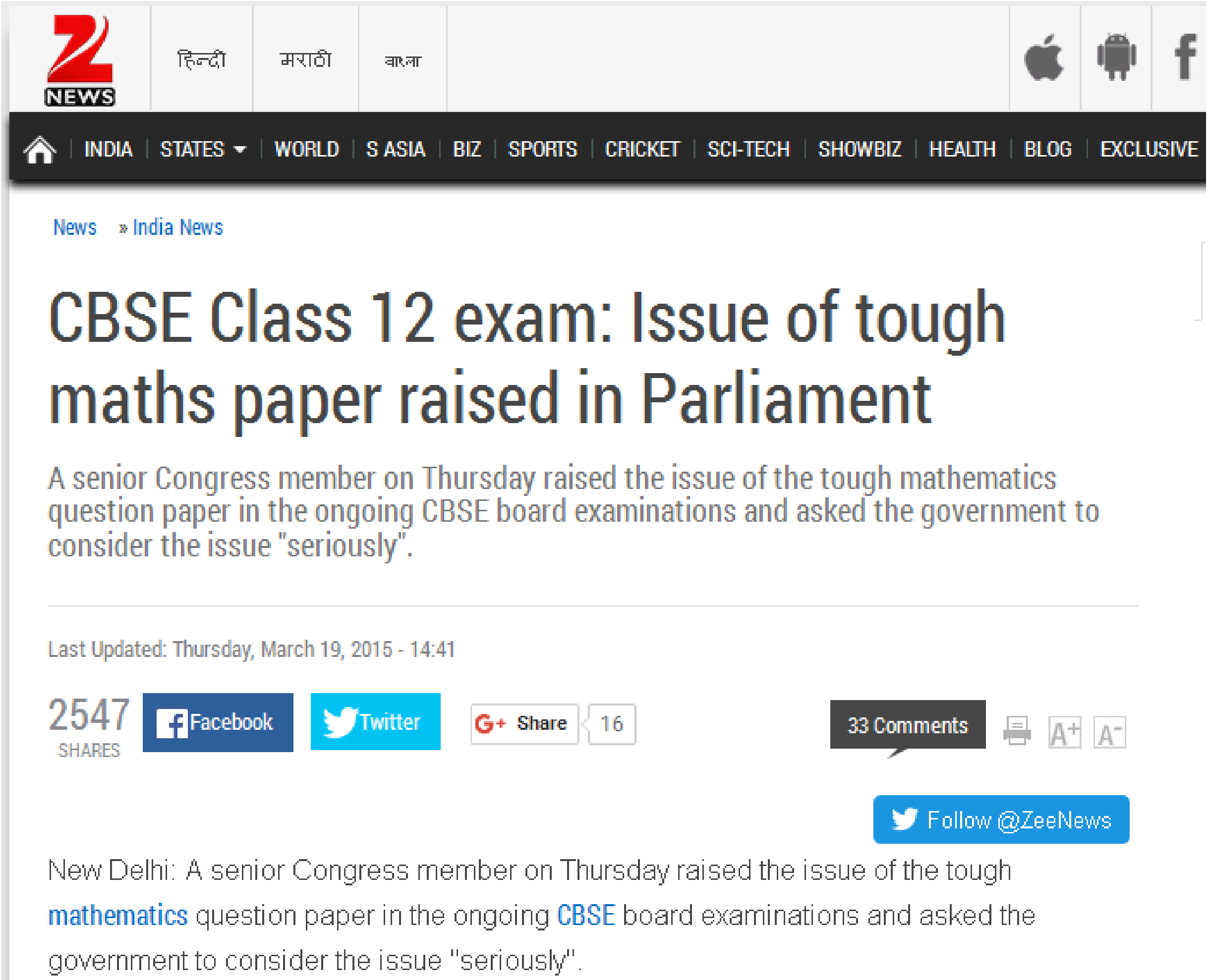
IBNLIVE.COM

14 More+

After several students claimed that the Central Board of Secondary Education (CBSE) Class XII board Mathematics examination paper was 'tricky' and tough, the board has issued a clarification on remedial measures which are likely to be taken before evaluation.

The CBSE says that feedback received from various stakeholders like students, subject teachers and examiners will be put before the committee of subject experts.

In 2015 also the same complain was there by many students



CBSE Class 12 exam: Issue of tough maths paper raised in Parliament

A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".

Last Updated: Thursday, March 19, 2015 - 14:41

New Delhi: A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".

In March 2016, students of Karnataka PU-II also complained the same, regarding standard 12 (PU-II Mathematics Exam). Even though the Math Paper was identical to previous year, most students had not even solved the 2015 Question Paper.


Friday, March 25, 2016 - 13:28

The NEWS Minute

HOME NEWS ANDHRA KARNATAKA KERALA TAMIL NADU TELANGANA CULTURE MEDIA BLOG

Exams

Online petition for lenient evaluation of K'taka II PU math paper gets over 8000 supporters

The campaign, which was launched on Monday, has garnered over 8000 supporters

TNM Staff | Wednesday, March 16, 2016 - 09:32

Follow @thenewsminute

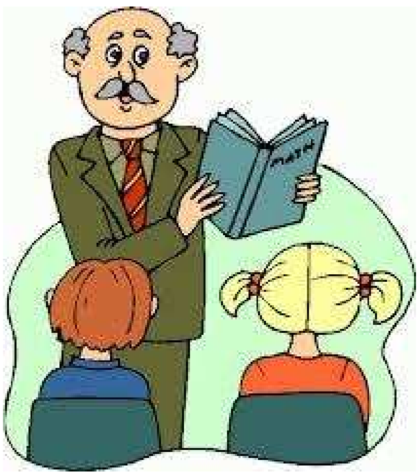
Share Tweet Email Share Reddit

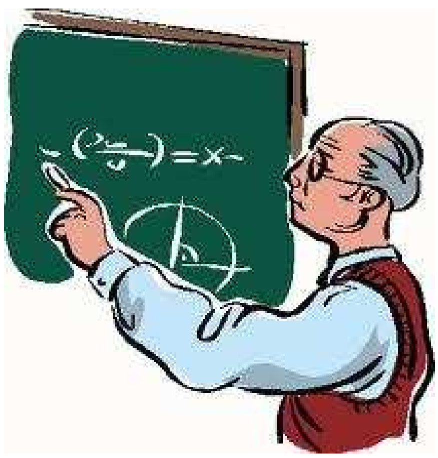
Following a "very tough" math paper that left many II PU students in tears, Saket Ravindran a student launched an online campaign demanding lenient evaluation.

These complains are not new. In fact since last 40 years, (since my childhood), I always see this; every year the same setback, same complain!

In this e-Book I am trying to solve this problem. Those students who practice can learn.

No one can help those who are not studying, or practicing.





Learn more at http://skmclasses.weebly.com/iit-jee-home-tuitions-bangalore.html

Twitter - https://twitter.com/ZookeeperPhy

Facebook - https://www.facebook.com/IIT.JEE.by.Prof.Subhashish/

Blog - http://skmclasses.kinja.com

A very polite request :

I wish these e-Books are read only by Boys and Men. Girls and Women, better read something else; learn from somewhere else.

Preface

We all know that in the species “Homo Sapiens “, males are bigger than females. The reasons are explained in standard 10, or 11 (high school) Biology texts. **This shapes or size, influences all of our culture**. Before we recall / understand the reasons once again, let us see some random examples of the influence

Random - 1

If there is a Road rage, then who all fight ? (generally ?). Imagine two cars driven by adult drivers. Each car has a woman of similar age as that of the Man. The cars “ touch “ or “ some issue happens”. Who all comes out and fights ? Who all are most probable to drive the cars ?

(Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win)

Random - 2

Heavy metal music artists are all Men. Metallica, Black Sabbath, Motley Crue, Megadeth, Motorhead, AC/DC, Deep Purple, Slayer, Guns & Roses, Led Zeppelin, Aerosmith ….. the list can be in thousands. All these are grown-up Boys, known as Men.

(Men strive for perfection. Men are eager to excel. Men work hard. Men want to win.)

Random - 3

Apart from Marie Curie, only one more woman got Nobel Prize in Physics. (Maria Goeppert Mayer - 1963). So, … almost all are men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 4

The best Tabla Players are all Men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 5

History is all about, which Kings ruled. Kings, their men, and Soldiers went for wars. History is all about wars, fights, and killings by men.

Boys start fighting from school days. Girls do not fight like this

(Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.)

Random - 6

The highest award in Mathematics, the " Fields Medal " is around since decades. Till date only one woman could get that. (Maryam Mirzakhani - 2014). So, … almost all are men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 7

Actor is a gender neutral word. Could the movie like " Top Gun " be made with Female actors ? The best pilots, astronauts, Fighters are all Men.

Random - 8

In my childhood had seen a movie named " The Tower in Inferno ". In the movie when the tall tower is in fire, women were being saved first, as only one lift was working….

Many decades later another movie is made. A box office hit. " The Titanic ". In this also …. As the ship is sinking women are being saved. **Men are disposable**. Men may get their turn later…

Movies are not training programs. Movies do not teach people what to do, or not to do. Movies only reflect the prevalent culture. Men are disposable, is the culture in the society. Knowingly, unknowingly, the culture is depicted in Movies, Theaters, Stories, Poems, Rituals, etc. I or you can't write a story, or make a movie in which after a minor car accident the Male passengers keep seating in the back seat, while the both the women drivers come out of the car and start fighting very bitterly on the road. There has been no story in this world, or no movie made, where after an accident or calamity, Men are being helped for safety first, and women are told to wait.

Random - 9

Artists generally follow the prevalent culture of the Society. In paintings, sculptures, stories, poems, movies, cartoon, Caricatures, knowingly / unknowingly, " the prevalent Reality " is depicted. The opposite will not go well with people. If deliberately " the opposite " is shown then it may only become a special art, considered as a special mockery.

Random - 10

Men go to "girl / woman's house" to marry / win, and bring her to his home. That is a sort of winning her. When a boy gets a " Girl-Friend ", generally he and his friends consider that as an achievement. The boy who " got / won " a girl-friend feels proud. His male friends feel, jealous, competitive and envious. Millions of stories have been written on these themes. Lakhs of movies show this. Boys / Men go for " bike race ", or say " Car Race ", where the winner " gets " the most beautiful girl of the college.




(Men want to excel. Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.)

Prithviraj Chauhan ' went ` to " pickup " or " abduct " or " win " or " bring " his love. There was a Hindi movie (hit) song … " Pasand ho jaye, to ghar se utha laye ". It is not other way round. Girls do not go to Boy's house or man's house to marry. Nor the girls go in a gang to " pick-up " the boy / man and bring him to their home / place / den.

Random - 11

Rich people; often are very hard working. Successful business men, establish their business (empire), amass lot of wealth, with lot of difficulty. Lots of sacrifice, lots of hard work, gets into this. Rich people's wives had no contribution in this wealth creation. Women are smart, and successful upto the extent to choose the right/rich man to marry. So generally what happens in case of Divorces ? Search the net on " most costly divorces " and you will know. The women;(who had no contribution at all, in setting up the business / empire), often gets in Billions, or several Millions in divorce settlements.

Number 1

Rupert & Anna Murdoch -- $1.7 billion

One of the richest men in the world, **Rupert Murdoch** developed his worldwide media empire when he inherited his father's Australian newspaper in 1952. He married Anna Murdoch in the '60s and they remained together for 32 years, springing off three children.

They split amicably in 1998 but soon Rupert forced Anna off the board of News Corp and the gloves came off. The divorce was finalized in June 1999 when Rupert agreed to let his ex-wife leave with $1.7 billion worth of his assets, $110 million of it in cash. Seventeen days later, Rupert married Wendi Deng, one of his employees.

Ted Danson & Casey Coates -- $30 million

Ted Danson's claim to fame is undoubtedly his decade-long stint as Sam Malone on NBC's celebrated sitcom Cheers . While he did other TV shows and movies, he will always be known as the bartender of that place where everybody knows your name. He met his future first bride Casey, a designer, in 1976 while doing Erhard Seminars Training.

Ten years his senior, she suffered a paralyzing stroke while giving birth to their first child in 1979. In order to nurse her back to health, Danson took a break from acting for six months. But after two children and 15 years of marriage, the infatuation fell to pieces. Danson had started seeing Whoopi Goldberg while filming the comedy, Made in America and this precipitated the 1992 divorce. Casey got $30 million for her trouble.

See https://zookeepersblog.wordpress.com/misandry-and-men-issues-a-short-summary-at-single-place/

See http://skmclasses.kinja.com/save-the-male-1761788732

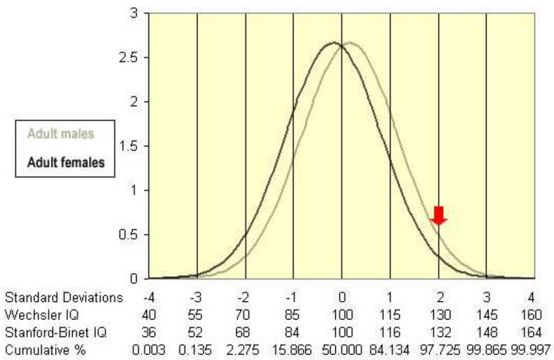
It was Boys and Men, who brought the girls / women home. The Laws are biased, completely favoring women. The men are paying for their own mistakes.

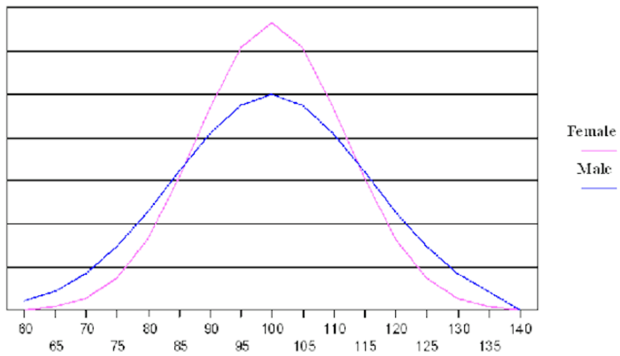
See https://zookeepersblog.wordpress.com/biased-laws/

(Man brings the Woman home. When she leaves, takes away her share of big fortune!)

Random - 12

A standardized test of Intelligence will never be possible. It never happened before, nor ever will happen in future; where the IQ test results will be acceptable by all. In the net there are thousands of charts which show that the intelligence scores of girls / women are lesser. Debates of Trillion words, does not improve performance of Girls.





I am not wasting a single second debating or discussing with anyone, on this. I am simply accepting ALL the results. IQ is only one of the variables which is required for success in life. Thousands of books have been written on " Networking Skills ", EQ (Emotional Quotient), Drive, Dedication, Focus, " Tenacity towards the end goal " … etc. In each criteria, and in all together, women (in general) do far worse than men. Bangalore is known as " ….. capital of India ". [Fill in the blanks]. The blanks are generally filled as " Software Capital ", " IT Capital ", " Startup Capital ", etc. I am member in several startup eco-systems / groups. I have attended hundreds of meetings, regarding " technology startups ", or " idea startups ". These meetings have very few women. Starting up new companies are all " Men's Game " / " Men's business ". Only in Divorce settlements women will take their goodies, due to Biased laws. There is no dedication, towards wealth creation, by women.

Random - 13

Many men, as fathers, very unfortunately treat their daughters as " Princess ". Every " non-performing " woman / wife was " princess daughter " of some loving father. Pampering the girls, in name of " equal opportunity ", or " women empowerment ", have led to nothing.

See http://skmclasses.kinja.com/progressively-daughters-become-monsters-1764484338

See http://skmclasses.kinja.com/vivacious-vixens-1764483974

There can be thousands of more such random examples, where " Bigger Shape / size " of males have influenced our culture, our Society. **Let us recall the reasons**, that we already learned in standard 10 - 11, Biology text Books. In humans, women have a long gestation period, and also spends many years (almost a decade) to grow, nourish, and stabilize the child. (Million years of habit) Due to survival instinct Males want to inseminate. Boys and Men fight for the " facility (of womb + care) " the girl / woman may provide. Bigger size for males, has a winning advantage. Whoever wins, gets the " woman / facility ". The male who is of " Bigger Size ", has an advantage to win.... Leading to Natural selection over millions of years. In general " Bigger Males "; the " fighting instinct " in men; have led to wars, and solving tough problems (Mathematics, Physics, Technology, startups of new businesses, Wealth creation, Unreasonable attempts to make things [such as planes], Hard work ....)

So let us see the IIT-JEE results of girls. Statistics of several years show that there are around 17, (or less than 20) girls in top 1000 ranks, at all India level. Some people will yet not understand the performance, till it is said that ... year after year we have around 980 boys in top 1000 ranks. Generally we see only 4 to 5 girls in top 500. In last 50 years not once any girl topped in IIT-JEE advanced. Forget about Single digit ranks, double digit ranks by girls have been extremely rare. It is all about " good boys ", " hard working ", " focused ", "**Bel-esprit** " **boys**.

In 2015, Only 2.6% of total candidates who qualified are girls (upto around 12,000 rank). while 20% of the Boys, amongst all candidates qualified. The Total number of students who appeared for the exam were around 1.4 million for IIT-JEE main. Subsequently 1.2 lakh (around 120 thousands) appeared for IIT-JEE advanced.

IIT-JEE results and analysis, of many years is given at https://zookeepersblog.wordpress.com/iit-jee-iseet-main-and-advanced-results/

In Bangalore it is rare to see a girl with rank better than 1000 in IIT-JEE advanced. We hardly see 6-7 boys with rank better than 1000. Hardly 2-3 boys get a rank better than 500.

See http://skmclasses.weebly.com/everybody-knows-so-you-should-also-know.html

Thousands of people are exposing the heinous crimes that Motherly Women are doing, or Female Teachers are committing. See https://www.facebook.com/WomenCriminals/

Some Random Examples must be known by all

Mother Admits On Facebook to Sleeping with 15 Yr Old Son, They Have a Baby Together - Alwayzturntup

Sometimes it hard to believe w From Alwayzturntup

ALWAYZTURNTUP.ME

It is extremely unfortunate that the " woman empowerment " has created. This is the kind of society and women we have now. I and many other sensible Men hate such women. Be away from such women, be aware of reality.

'Sex with my son is incredible - we're in love and we want a baby'

Ben Ford, who ditched his wife when he met his mother Kim West after 30 years, claims what the couple are doing 'isn't incest'

MIRROR.CO.UK

Former Shelbyville ISD teacher who had sex with underage student gets 3 years in prison

After a two day break over the weekend, A Shelby County jury was back in the courtroom looking to conclude the trial of a former Shelbyville ISD teacher who had...

KLTV.COM | BY CALEB BEAMES

Woman sent to jail for the rest of her life after raping her four grandchildren is described as the 'most evil person' the judge has ever seen

Edwina Louis rape...

See More

Woman sent to jail for raping her four grandchildren

A Ohio grandmother has been sentenced to four consecutive life terms after being found guilty of the rape of her own grandchildren. Edwina Louis, 53, will spend the rest of her life behind bars.

DAILYMAIL.CO.UK

http://www.thenativecanadian.com/.../eastern-ontario-teacher-...

The N.C. Chronicles.: Eastern Ontario teacher charged with 36 sexual offences

anti feminism, Child abuse, children's rights, Feminist hypocrisy,

THENATIVECANADIAN.COM | BY BLACKWOLF

Hyd woman kills newborn boy as she wanted daughter - Times of India

Having failed to bear a daughter for the third time, a shopkeeper's wife slit the throat of her 24day-old son with a shaving blade and left him to die in a street on Tuesday night.Purnima's first child was a stillborn boy, followed by another boy born five years ago.

TIMESOFINDIA.INDIATIMES.COM

Montgomery's son, Alan Vonn Webb, took the stand and was a key witness in her conviction.

"I want to see her placed somewhere she can never do that to children ...

See More

Woman sentenced to 40 years in prison for raping her children

A Murfreesboro mother found guilty of raping her own children learned her fate on Wednesday.

WAFF.COM | BY DENNIS FERRIER

gentler sex? Violence against men.'s photo.

Women, the gentler sex? Violence against men. Like Page

April 8 at 1:38am

In fact, the past decade has seen a dramatic increase in the number of incidents of women raping and sexually assaulting boys and men. On May 2014, Jezebel repo...

End violence against women . . . .

North Carolina Grandma Eats Her Daughter's New Born Baby After Smoking Bath Salts

Henderson, North Carolina– A North Carolina grandmother of 4 and recovering drug addict, is now in custody after she allegedly ate her daughter's newborn baby....

AZ-365.TOP

28-Year-Old Texas Teacher Accused of Sending Nude Picture to 14-Year-Old Former Student

BREITBART.COM

http://latest.com/.../attractive-girl-gang-lured-men-alleywa.../

Attractive Girl Gang Lured Men Into Alleyways Where Female Body Builder Would Attack Them

A Mexican street gang made up entirely of women has been accused of using their feminine wiles to lure men into alleyways and then beating them up and...

LATEST.COM

http://www.wfmj.com/.../youngstown-woman-convicted-of-raping-...

Youngstown woman convicted of raping a 1 year old is back in jail

A Youngstown woman who went to prison for raping a 1-year-old boy fifteen years ago is in trouble with the law again.

WFMJ.COM

End violence against women . . . .

Women are raping boys and young men

Rape advocacy has been maligned and twisted into a political agenda controlled by radicalized activists. Tim Patten takes a razor keen and well supported look into the manufactured rape culture and...

AVOICEFORMEN.COM | BY TIM PATTEN

Bronx Woman Convicted of Poisoning and Drowning Her Children

Lisette Bamenga researched methods on the Internet before she killed her son and daughter in 2012.

NYTIMES.COM | BY MARC SANTORA

A Russian-born newlywed slowly butchered her German husband — feeding strips of his flesh to their dog until he took his last breath. Svetlana Batukova, 46, was...

See More

She killed her husband and then fed him to her dog: police

A Russian-born newlywed butchered her German hubby — and fed strips of his flesh to her pooch, authorities said. Svetlana Batukova offed Horst Hans Henkels at their...

NYPOST.COM

Daily Mail
January 15, 2015

Mother charged with rape and sodomy of her son's 12-year-old friend

Mom, 30, 'raped and had oral sex with her son's 12-year-old friend'

Nicole Marie Smith, 30, (pictured) of St Charles County, Missouri, has been jailed after she allegedly targeted the 12-year-old boy at her home.

DAILYM.AI

April 4 at 4:48am

Female prison officers commit 90pc of sex assaults on male teens in US juvenile detention centres

Lawsuit in Idaho highlights the prevalence of sexual victimization of juvenile offenders.

IBTIMES.CO.UK | BY NICOLE ROJAS

This mother filmed herself raping her own son and then sold it to a man for $300. The courts just decide her fate. When you see what she got, you're going to be outraged.

Mother Who Filmed Herself Raping Her 1-Year-Old Son Receives Shocking Sentence

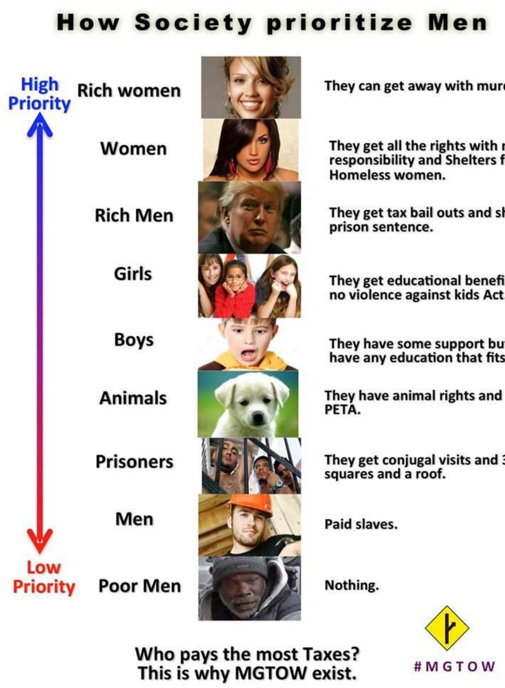
"...then used the money to buy herself a laptop..."

AMERICANEWS.COM

In several countries or rather in several regions of the world, family system has collapsed, due to bad nature and naughty acts of women. Particularly in Britain, and America, almost 50% people are alone, lonely, separated, divorced or failed marriages. In 2013, 48% children were born out of wedlock. It was projected that by 2016, more than 51% children will be born, to unmarried mothers. In these developed countries " paternity fraud " by women, are close to 20%. You can see several articles in the net, and in wikipedia etc. This means 1 out of 5 children are calling a wrong man as dad. The lonely, alone “ mothers " are frustrated. They see the children as burden. Love in the Society in general is lost, long time ago. The types of “ Mothers “ and “ Women “ we have now …………

By now if you have assumed that Indian women are not doing any crime then please become friends with MRA Guri https://www.facebook.com/profile.php?id=100004138754180

He has dedicated his life to expose Indian Criminals



Professor Subhashish Chattopadhyay

Spoon Feeding Series - Complex or Imaginary Numbers

Question

Express the given complex number in the form $a + ib$: $(5i)\left(-\frac{3}{5}i\right)$

Answer

$$(5i)\left(\frac{-3}{5}i\right) = -5\times\frac{3}{5}\times i\times i$$
$$= -3i^2$$
$$= -3(-1) \qquad \left[i^2 = -1\right]$$
$$= 3$$

Question

Express the given complex number in the form $a + ib$: $i^9 + i^{19}$

Answer

$$i^9 + i^{19} = i^{4\times2+1} + i^{4\times4+3}$$
$$= \left(i^4\right)^2\cdot i + \left(i^4\right)^4\cdot i^3$$
$$= 1\times i + 1\times(-i) \qquad \left[i^4 = 1,\ i^3 = -i\right]$$
$$= i + (-i)$$
$$= 0$$

Question

Express the given complex number in the form $a + ib$: i^{-39}

Answer

$$i^{-39} = i^{-4\times 9-3} = \left(i^4\right)^{-9} \cdot i^{-3}$$

$$= (1)^{-9} \cdot i^{-3} \qquad \left[i^4 = 1\right]$$

$$= \frac{1}{i^3} = \frac{1}{-i} \qquad \left[i^3 = -i\right]$$

$$= \frac{-1}{i} \times \frac{i}{i}$$

$$= \frac{-i}{i^2} = \frac{-i}{-1} = i \qquad \left[i^2 = -1\right]$$

Question

Express the given complex number in the form $a + ib$: $3(7 + i7) + i(7 + i7)$

Answer

$$3(7+i7)+i(7+i7) = 21+21i+7i+7i^2$$

$$= 21+28i+7\times(-1) \qquad \left[\because i^2 = -1\right]$$

$$= 14+28i$$

Question

Express the given complex number in the form $a + ib$: $(1 - i) - (-1 + i6)$

Answer

$$(1-i)-(-1+i6) = 1-i+1-6i$$

$$= 2-7i$$

Question

Express the given complex number in the form $a + ib$: $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$

Answer

$$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$$
$$=\frac{1}{5}+\frac{2}{5}i-4-\frac{5}{2}i$$
$$=\left(\frac{1}{5}-4\right)+i\left(\frac{2}{5}-\frac{5}{2}\right)$$
$$=\frac{-19}{5}+i\left(\frac{-21}{10}\right)$$
$$=\frac{-19}{5}-\frac{21}{10}i$$

Question

Express the given complex number in the form $a + ib$: $\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

Answer

$$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(\frac{-4}{3}+i\right)$$
$$=\frac{1}{3}+\frac{7}{3}i+4+\frac{1}{3}i+\frac{4}{3}-i$$
$$=\left(\frac{1}{3}+4+\frac{4}{3}\right)+i\left(\frac{7}{3}+\frac{1}{3}-1\right)$$
$$=\frac{17}{3}+i\frac{5}{3}$$

Question

Express the given complex number in the form $a + ib$: $(1 - i)^4$

Answer

$$(1-i)^4 = \left[(1-i)^2\right]^2$$
$$= \left[1^2 + i^2 - 2i\right]^2$$
$$= [1-1-2i]^2$$
$$= (-2i)^2$$
$$= (-2i)\times(-2i)$$
$$= 4i^2 = -4 \qquad \left[i^2 = -1\right]$$

Question

Express the given complex number in the form $a + ib$: $\left(\frac{1}{3}+3i\right)^3$

Answer

$$\left(\frac{1}{3}+3i\right)^3 = \left(\frac{1}{3}\right)^3 + (3i)^3 + 3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3}+3i\right)$$
$$= \frac{1}{27} + 27i^3 + 3i\left(\frac{1}{3}+3i\right)$$
$$= \frac{1}{27} + 27(-i) + i + 9i^2 \qquad \left[i^3 = -i\right]$$
$$= \frac{1}{27} - 27i + i - 9 \qquad \left[i^2 = -1\right]$$
$$= \left(\frac{1}{27} - 9\right) + i(-27+1)$$
$$= \frac{-242}{27} - 26i$$

Question

Express the given complex number in the form $a + ib$: $\left(-2-\frac{1}{3}i\right)^3$

Answer

$$\left(-2-\frac{1}{3}i\right)^3 = (-1)^3\left(2+\frac{1}{3}i\right)^3$$

$$= -\left[2^3+\left(\frac{i}{3}\right)^3+3(2)\left(\frac{i}{3}\right)\left(2+\frac{i}{3}\right)\right]$$

$$= -\left[8+\frac{i^3}{27}+2i\left(2+\frac{i}{3}\right)\right]$$

$$= -\left[8-\frac{i}{27}+4i+\frac{2i^2}{3}\right] \qquad \left[i^3=-i\right]$$

$$= -\left[8-\frac{i}{27}+4i-\frac{2}{3}\right] \qquad \left[i^2=-1\right]$$

$$= -\left[\frac{22}{3}+\frac{107i}{27}\right]$$

$$= -\frac{22}{3}-\frac{107}{27}i$$

Question

Find the multiplicative inverse of the complex number 4 − 3*i*

Answer

Let $z = 4 - 3i$

Then, $\bar{z} = 4 + 3i$ and $|z|^2 = 4^2 + (-3)^2 = 16 + 9 = 25$

Therefore, the multiplicative inverse of 4 − 3*i* is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{4+3i}{25} = \frac{4}{25} + \frac{3}{25}i$$

Question

Find the multiplicative inverse of the complex number $\sqrt{5} + 3i$

Answer

Let $z = \sqrt{5} + 3i$

Then, $\bar{z} = \sqrt{5} - 3i$ and $|z|^2 = (\sqrt{5})^2 + 3^2 = 5 + 9 = 14$

Therefore, the multiplicative inverse of $\sqrt{5} + 3i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\sqrt{5} - 3i}{14} = \frac{\sqrt{5}}{14} - \frac{3i}{14}$$

Question

Find the multiplicative inverse of the complex number $-i$

Answer

Let $z = -i$

Then, $\bar{z} = i$ and $|z|^2 = 1^2 = 1$

Therefore, the multiplicative inverse of $-i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{i}{1} = i$$

Question

Express the following expression in the form of $a + ib$.

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$$

Answer

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$$

$$= \frac{(3)^2-(i\sqrt{5})^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i} \qquad [(a+b)(a-b)=a^2-b^2]$$

$$= \frac{9-5i^2}{2\sqrt{2}i}$$

$$= \frac{9-5(-1)}{2\sqrt{2}i} \qquad [i^2=-1]$$

$$= \frac{9+5}{2\sqrt{2}i} \times \frac{i}{i}$$

$$= \frac{14i}{2\sqrt{2}i^2}$$

$$= \frac{14i}{2\sqrt{2}(-1)}$$

$$= \frac{-7i}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{-7\sqrt{2}i}{2}$$

Question

Find the modulus and the argument of the complex number $z = -1 - i\sqrt{3}$

Answer

$z = -1 - i\sqrt{3}$

Let $r\cos\theta = -1$ and $r\sin\theta = -\sqrt{3}$

On squaring and adding, we obtain

$$(r\cos\theta)^2 + (r\sin\theta)^2 = (-1)^2 + (-\sqrt{3})^2$$

$$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 1 + 3$$

$$\Rightarrow r^2 = 4 \qquad [\cos^2\theta + \sin^2\theta = 1]$$

$$\Rightarrow r = \sqrt{4} = 2 \qquad [\text{Conventionally, } r > 0]$$

$$\therefore \text{Modulus} = 2$$

$$\therefore 2\cos\theta = -1 \text{ and } 2\sin\theta = -\sqrt{3}$$

$$\Rightarrow \cos\theta = \frac{-1}{2} \text{ and } \sin\theta = \frac{-\sqrt{3}}{2}$$

Since both the values of sin θ and cos θ are negative and sinθ and cosθ are negative in III quadrant,

$$\text{Argument} = -\left(\pi - \frac{\pi}{3}\right) = \frac{-2\pi}{3}$$

Thus, the modulus and argument of the complex number $-1-\sqrt{3}\,\mathrm{i}$ are 2 and $\frac{-2\pi}{3}$ respectively.

Question

Find the modulus and the argument of the complex number $z = -\sqrt{3} + i$

Answer

$z = -\sqrt{3} + i$

Let $r\cos\theta = -\sqrt{3}$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = \left(-\sqrt{3}\right)^2 + 1^2$$

$$\Rightarrow r^2 = 3 + 1 = 4 \qquad \left[\cos^2\theta + \sin^2\theta = 1\right]$$

$$\Rightarrow r = \sqrt{4} = 2 \qquad [\text{Conventionally, } r > 0]$$

$$\therefore \text{Modulus } = 2$$

$$\therefore \ 2\cos\theta = -\sqrt{3} \text{ and } 2\sin\theta = 1$$

$$\Rightarrow \cos\theta = \frac{-\sqrt{3}}{2} \text{ and } \sin\theta = \frac{1}{2}$$

$$\therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \qquad [\text{As } \theta \text{ lies in the II quadrant}]$$

Thus, the modulus and argument of the complex number $-\sqrt{3} + i$ are 2 and $\frac{5\pi}{6}$ respectively.

Question

Convert the given complex number in polar form: 1 – *i*

Answer

1 – *i*

Let *r* cos θ = 1 and *r* sin θ = –1

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = 1^2 + (-1)^2$$
$$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 1 + 1$$
$$\Rightarrow r^2 = 2$$
$$\Rightarrow r = \sqrt{2} \qquad [\text{Conventionally, } r > 0]$$
$$\therefore \sqrt{2}\cos\theta = 1 \text{ and } \sqrt{2}\sin\theta = -1$$
$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \text{ and } \sin\theta = -\frac{1}{\sqrt{2}}$$
$$\therefore \theta = -\frac{\pi}{4} \qquad [\text{As } \theta \text{ lies in the IV quadrant}]$$
$$\therefore 1 - i = r\cos\theta + i r\sin\theta = \sqrt{2}\cos\left(-\frac{\pi}{4}\right) + i\sqrt{2}\sin\left(-\frac{\pi}{4}\right) = \sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$$

This is the required polar form.

Question

Convert the given complex number in polar form: $-1 + i$

Answer

$-1 + i$

Let $r\cos\theta = -1$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + 1^2$$

$$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 1 + 1$$

$$\Rightarrow r^2 = 2$$

$$\Rightarrow r = \sqrt{2} \qquad [\text{Conventionally, } r > 0]$$

$$\therefore \sqrt{2}\cos\theta = -1 \text{ and } \sqrt{2}\sin\theta = 1$$

$$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}} \text{ and } \sin\theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in the II quadrant}]$$

It can be written,

$$\therefore -1 + i = r\cos\theta + i r\sin\theta = \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

This is the required polar form.

Question

Convert the given complex number in polar form: $-1-i$

Answer

$-1-i$

Let $r\cos\theta = -1$ and $r\sin\theta = -1$

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + (-1)^2$$

$$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 1+1$$

$$\Rightarrow r^2 = 2$$

$$\Rightarrow r = \sqrt{2} \qquad [\text{Conventionally, } r>0]$$

$$\therefore \sqrt{2}\cos\theta = -1 \text{ and } \sqrt{2}\sin\theta = -1$$

$$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}} \text{ and } \sin\theta = -\frac{1}{\sqrt{2}}$$

$$\therefore \theta = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in the III quadrant}]$$

$$\therefore -1-i = r\cos\theta + ir\sin\theta = \sqrt{2}\cos\frac{-3\pi}{4} + i\sqrt{2}\sin\frac{-3\pi}{4} = \sqrt{2}\left(\cos\frac{-3\pi}{4} + i\sin\frac{-3\pi}{4}\right)$$ This is the required polar form.

Question

Convert the given complex number in polar form: -3

Answer

-3

Let $r\cos\theta = -3$ and $r\sin\theta = 0$

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = (-3)^2$$
$$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 9$$
$$\Rightarrow r^2 = 9$$
$$\Rightarrow r = \sqrt{9} = 3 \qquad [\text{Conventionally, } r > 0]$$
$$\therefore 3\cos\theta = -3 \text{ and } 3\sin\theta = 0$$
$$\Rightarrow \cos\theta = -1 \text{ and } \sin\theta = 0$$
$$\therefore \theta = \pi$$
$$\therefore -3 = r\cos\theta + ir\sin\theta = 3\cos\pi + i3\sin\pi = 3(\cos\pi + i\sin\pi)$$

This is the required polar form.

Question

Convert the given complex number in polar form: $\sqrt{3} + i$

Answer

$\sqrt{3} + i$

Let $r\cos\theta = \sqrt{3}$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = \left(\sqrt{3}\right)^2 + 1^2$$
$$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 3 + 1$$
$$\Rightarrow r^2 = 4$$
$$\Rightarrow r = \sqrt{4} = 2 \qquad [\text{Conventionally, } r > 0]$$
$$\therefore 2\cos\theta = \sqrt{3} \text{ and } 2\sin\theta = 1$$
$$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \text{ and } \sin\theta = \frac{1}{2}$$
$$\therefore \theta = \frac{\pi}{6} \qquad [\text{As } \theta \text{ lies in the I quadrant}]$$

$$\therefore \sqrt{3}+i = r\cos\theta + i\,r\sin\theta = 2\cos\frac{\pi}{6} + i\,2\sin\frac{\pi}{6} = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

This is the required polar form

Question

Convert the given complex number in polar form: i

Answer

i

Let $r\cos\theta = 0$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = 0^2 + 1^2$$

$$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 1$$

$$\Rightarrow r^2 = 1$$

$$\Rightarrow r = \sqrt{1} = 1 \qquad [\text{Conventionally, } r > 0]$$

$$\therefore \cos\theta = 0 \text{ and } \sin\theta = 1$$

$$\therefore \theta = \frac{\pi}{2}$$

$$\therefore i = r\cos\theta + i\,r\sin\theta = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$$

This is the required polar form.

Question

Evaluate: $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$

Answer

$$\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$$
$$=\left[i^{4\times4+2}+\frac{1}{i^{4\times6+1}}\right]^3$$
$$=\left[\left(i^4\right)^4\cdot i^2+\frac{1}{\left(i^4\right)^6\cdot i}\right]^3$$
$$=\left[i^2+\frac{1}{i}\right]^3 \qquad \left[i^4=1\right]$$
$$=\left[-1+\frac{1}{i}\times\frac{i}{i}\right]^3 \qquad \left[i^2=-1\right]$$
$$=\left[-1+\frac{i}{i^2}\right]^3$$
$$=\left[-1-i\right]^3$$
$$=(-1)^3\left[1+i\right]^3$$
$$=-\left[1^3+i^3+3\cdot1\cdot i(1+i)\right]$$
$$=-\left[1+i^3+3i+3i^2\right]$$
$$=-\left[1-i+3i-3\right]$$
$$=-\left[-2+2i\right]$$
$$=2-2i$$

Question

For any two complex numbers z_1 and z_2, prove that

$\text{Re}(z_1z_2) = \text{Re}\, z_1 \text{Re}\, z_2 - \text{Im}\, z_1 \text{Im}\, z_2$

Answer

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$$\therefore z_1z_2 = (x_1 + iy_1)(x_2 + iy_2)$$
$$= x_1(x_2 + iy_2) + iy_1(x_2 + iy_2)$$
$$= x_1x_2 + ix_1y_2 + iy_1x_2 + i^2y_1y_2$$
$$= x_1x_2 + ix_1y_2 + iy_1x_2 - y_1y_2 \qquad [i^2 = -1]$$
$$= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)$$
$$\Rightarrow \text{Re}(z_1z_2) = x_1x_2 - y_1y_2$$
$$\Rightarrow \text{Re}(z_1z_2) = \text{Re}\, z_1 \text{Re}\, z_2 - \text{Im}\, z_1 \text{Im}\, z_2$$

Hence, proved.

Question

Reduce $\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ to the standard form.

Answer

$$\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)=\left[\frac{(1+i)-2(1-4i)}{(1-4i)(1+i)}\right]\left[\frac{3-4i}{5+i}\right]$$

$$=\left[\frac{1+i-2+8i}{1+i-4i-4i^2}\right]\left[\frac{3-4i}{5+i}\right]=\left[\frac{-1+9i}{5-3i}\right]\left[\frac{3-4i}{5+i}\right]$$

$$=\left[\frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}\right]=\frac{33+31i}{28-10i}=\frac{33+31i}{2(14-5i)}$$

$$=\frac{(33+31i)}{2(14-5i)}\times\frac{(14+5i)}{(14+5i)} \qquad \left[\textbf{On multiplying} \text{ numerator and denominator by } (14+5i)\right]$$

$$=\frac{462+165i+434i+155i^2}{2\left[(14)^2-(5i)^2\right]}=\frac{307+599i}{2(196-25i^2)}$$

$$=\frac{307+599i}{2(221)}=\frac{307+599i}{442}=\frac{307}{442}+\frac{599i}{442}$$

This is the required standard form

Question

If $x - iy = \sqrt{\frac{a-ib}{c-id}}$ prove that $\left(x^2+y^2\right)^2 = \frac{a^2+b^2}{c^2+d^2}$.

Answer

$$x - iy = \sqrt{\frac{a-ib}{c-id}}$$

$$= \sqrt{\frac{a-ib}{c-id} \times \frac{c+id}{c+id}} \left[\text{On multiplying numerator and denominator by } (c+id)\right]$$

$$= \sqrt{\frac{(ac+bd)+i(ad-bc)}{c^2+d^2}}$$

$$\therefore (x-iy)^2 = \frac{(ac+bd)+i(ad-bc)}{c^2+d^2}$$

$$\Rightarrow x^2 - y^2 - 2ixy = \frac{(ac+bd)+i(ad-bc)}{c^2+d^2}$$

On comparing real and imaginary parts, we obtain

$$x^2 - y^2 = \frac{ac+bd}{c^2+d^2}, \quad -2xy = \frac{ad-bc}{c^2+d^2} \qquad (1)$$

$$\left(x^2+y^2\right)^2=\left(x^2-y^2\right)^2+4x^2y^2$$

$$=\left(\frac{ac+bd}{c^2+d^2}\right)^2+\left(\frac{ad-bc}{c^2+d^2}\right)^2 \qquad [\text{Using } (1)]$$

$$=\frac{a^2c^2+b^2d^2+2acbd+a^2d^2+b^2c^2-2adbc}{\left(c^2+d^2\right)^2}$$

$$=\frac{a^2c^2+b^2d^2+a^2d^2+b^2c^2}{\left(c^2+d^2\right)^2}$$

$$=\frac{a^2\left(c^2+d^2\right)+b^2\left(c^2+d^2\right)}{\left(c^2+d^2\right)^2}$$

$$=\frac{\left(c^2+d^2\right)\left(a^2+b^2\right)}{\left(c^2+d^2\right)^2}$$

$$=\frac{a^2+b^2}{c^2+d^2}$$

Hence, proved.

Question

Convert the following in the polar form:

(i) $\frac{1+7i}{(2-i)^2}$, (ii) $\frac{1+3i}{1-2i}$

Answer

(i) Here, $z = \frac{1+7i}{(2-i)^2}$

$$= \frac{1+7i}{(2-i)^2} = \frac{1+7i}{4+i^2-4i} = \frac{1+7i}{4-1-4i}$$

$$= \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i} = \frac{3+4i+21i+28i^2}{3^2+4^2}$$

$$= \frac{3+4i+21i-28}{3^2+4^2} = \frac{-25+25i}{25}$$

$$= -1+i$$

Let $r\cos\theta = -1$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$r^2(\cos^2\theta + \sin^2\theta) = 1 + 1$

$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 2$

$\Rightarrow r^2 = 2$ $\quad [\cos^2\theta + \sin^2\theta = 1]$

$\Rightarrow r = \sqrt{2}$ $\quad$ [Conventionally, $r > 0$]

$\therefore \sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ $\quad$ [As θ lies in II quadrant]

$\therefore z = r\cos\theta + i\,r\sin\theta$

$= \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

This is the required polar form.

Question

(ii) Here, $z = \frac{1+3i}{1-2i}$

$= \frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i}$

$= \frac{1+2i+3i-6}{1+4}$

$= \frac{-5+5i}{5} = -1+i$

Let $r\cos\theta = -1$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$r^2(\cos^2\theta + \sin^2\theta) = 1 + 1$

$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 2$

$\Rightarrow r^2 = 2$ $\quad [\cos^2\theta + \sin^2\theta = 1]$

$\Rightarrow r = \sqrt{2}$ [Conventionally, $r > 0$]

$\therefore \sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ [As θ lies in II quadrant]

$\therefore z = r\cos\theta + i\,r\sin\theta$

$= \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

This is the required polar form.

Question

If $z_1 = 2 - i,\ z_2 = 1 + i$, find $\left|\dfrac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right|$

Answer

$$z_1 = 2 - i,\ z_2 = 1 + i$$

$$\therefore \left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right| = \left|\frac{(2 - i) + (1 + i) + 1}{(2 - i) - (1 + i) + 1}\right|$$

$$= \left|\frac{4}{2 - 2i}\right| = \left|\frac{4}{2(1 - i)}\right|$$

$$= \left|\frac{2}{1 - i} \times \frac{1 + i}{1 + i}\right| = \left|\frac{2(1 + i)}{1^2 - i^2}\right|$$

$$= \left|\frac{2(1 + i)}{1 + 1}\right| \qquad \left[i^2 = -1\right]$$

$$= \left|\frac{2(1 + i)}{2}\right|$$

$$= |1 + i| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Question

If $a + ib = \dfrac{(x+i)^2}{2x^2+1}$, prove that $a^2 + b^2 = \dfrac{(x^2+1)^2}{(2x+1)^2}$

Answer

$$a + ib = \frac{(x+i)^2}{2x^2+1}$$

$$= \frac{x^2 + i^2 + 2xi}{2x^2+1}$$

$$= \frac{x^2 - 1 + i2x}{2x^2+1}$$

$$= \frac{x^2-1}{2x^2+1} + i\left(\frac{2x}{2x^2+1}\right)$$

On comparing real and imaginary parts, we obtain

$$a = \frac{x^2-1}{2x^2+1} \text{ and } b = \frac{2x}{2x^2+1}$$

$$\therefore a^2 + b^2 = \left(\frac{x^2 - 1}{2x^2 + 1}\right)^2 + \left(\frac{2x}{2x^2 + 1}\right)^2$$

$$= \frac{x^4 + 1 - 2x^2 + 4x^2}{(2x + 1)^2}$$

$$= \frac{x^4 + 1 + 2x^2}{(2x^2 + 1)^2}$$

$$= \frac{(x^2 + 1)^2}{(2x^2 + 1)^2}$$

$$\therefore a^2 + b^2 = \frac{(x^2 + 1)^2}{(2x^2 + 1)^2}$$

Hence, proved.

Question

Let $z_1 = 2 - i,\ z_2 = -2 + i$. Find

(i) $\text{Re}\left(\frac{z_1 z_2}{\overline{z}_1}\right)$, (ii) $\text{Im}\left(\frac{1}{z_1 \overline{z}_1}\right)$

Answer

$z_1 = 2 - i,\ z_2 = -2 + i$

(i) $z_1 z_2 = (2-i)(-2+i) = -4 + 2i + 2i - i^2 = -4 + 4i - (-1) = -3 + 4i$

$\overline{z}_1 = 2 + i$

$\therefore \frac{z_1 z_2}{\overline{z}_1} = \frac{-3+4i}{2+i}$

On multiplying numerator and denominator by (2 − *i*), we obtain

$$\frac{z_1 z_2}{\overline{z}_1} = \frac{(-3+4i)(2-i)}{(2+i)(2-i)} = \frac{-6+3i+8i-4i^2}{2^2+1^2} = \frac{-6+11i-4(-1)}{2^2+1^2}$$

$$= \frac{-2+11i}{5} = \frac{-2}{5} + \frac{11}{5}i$$

On comparing real parts, we obtain

$$\text{Re}\left(\frac{z_1 z_2}{\overline{z}_1}\right) = \frac{-2}{5}$$

(ii) $\frac{1}{z_1 \overline{z}_1} = \frac{1}{(2-i)(2+i)} = \frac{1}{(2)^2 + (1)^2} = \frac{1}{5}$

On comparing imaginary parts, we obtain

$$\text{Im}\left(\frac{1}{z_1 \overline{z}_1}\right) = 0$$

Find the modulus and argument of the complex number $\frac{1+2i}{1-3i}$

Answer

Let $z=\frac{1+2i}{1-3i}$, then

$$z=\frac{1+2i}{1-3i}\times\frac{1+3i}{1+3i}=\frac{1+3i+2i+6i^2}{1^2+3^2}=\frac{1+5i+6(-1)}{1+9}$$
$$=\frac{-5+5i}{10}=\frac{-5}{10}+\frac{5i}{10}=\frac{-1}{2}+\frac{1}{2}i$$

Let $z=r\cos\theta+ir\sin\theta$

i.e., $r\cos\theta=\frac{-1}{2}$ and $r\sin\theta=\frac{1}{2}$

On squaring and adding, we obtain

$$r^2\left(\cos^2\theta+\sin^2\theta\right)=\left(\frac{-1}{2}\right)^2+\left(\frac{1}{2}\right)^2$$

$$\Rightarrow r^2=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$$

$$\Rightarrow r=\frac{1}{\sqrt{2}} \qquad \left[\text{Conventionally, } r>0\right]$$

$$\therefore \frac{1}{\sqrt{2}}\cos\theta=\frac{-1}{2} \text{ and } \frac{1}{\sqrt{2}}\sin\theta=\frac{1}{2}$$

$$\Rightarrow \cos\theta=\frac{-1}{\sqrt{2}} \text{ and } \sin\theta=\frac{1}{\sqrt{2}}$$

$$\therefore \theta=\pi-\frac{\pi}{4}=\frac{3\pi}{4} \qquad \left[\text{As } \theta \text{ lies in the II quadrant}\right]$$

Therefore, the modulus and argument of the given complex number are $\frac{1}{\sqrt{2}}$ and $\frac{3\pi}{4}$ respectively.

Question

Find the real numbers x and y if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$.

Answer

Let $z = (x - iy)(3 + 5i)$

$$z = 3x + 5xi - 3yi - 5yi^2 = 3x + 5xi - 3yi + 5y = (3x + 5y) + i(5x - 3y)$$

$$\therefore \bar{z} = (3x + 5y) - i(5x - 3y)$$

It is given that, $\bar{z} = -6 - 24i$

$$\therefore (3x + 5y) - i(5x - 3y) = -6 - 24i$$

Equating real and imaginary parts, we obtain

$$3x + 5y = -6 \qquad \text{... (i)}$$

$$5x - 3y = 24 \qquad \text{... (ii)}$$

Multiplying equation (i) by 3 and equation (ii) by 5 and then adding them, we obtain

$$\begin{array}{r} 9x + 15y = -18 \\ 25x - 15y = 120 \\ \hline 34x \quad = 102 \end{array}$$

$$\therefore x = \frac{102}{34} = 3$$

Putting the value of x in equation (i), we obtain

$$3(3) + 5y = -6$$

$$\Rightarrow 5y = -6 - 9 = -15$$

$$\Rightarrow y = -3$$

Thus, the values of x and y are 3 and −3 respectively.

Question

Find the modulus of $\frac{1+i}{1-i}-\frac{1-i}{1+i}$.

Answer

$$\frac{1+i}{1-i}-\frac{1-i}{1+i}=\frac{(1+i)^2-(1-i)^2}{(1-i)(1+i)}$$

$$=\frac{1+i^2+2i-1-i^2+2i}{1^2+1^2}$$

$$=\frac{4i}{2}=2i$$

$$\therefore\left|\frac{1+i}{1-i}-\frac{1-i}{1+i}\right|=|2i|=\sqrt{2^2}=2$$

Question

If $(x + iy)^3 = u + iv$, then show that $\frac{u}{x} + \frac{v}{y} = 4\left(x^2 - y^2\right)$

Answer

$$(x + iy)^3 = u + iv$$

$$\Rightarrow x^3 + (iy)^3 + 3 \cdot x \cdot iy(x + iy) = u + iv$$

$$\Rightarrow x^3 + i^3y^3 + 3x^2yi + 3xy^2i^2 = u + iv$$

$$\Rightarrow x^3 - iy^3 + 3x^2yi - 3xy^2 = u + iv$$

$$\Rightarrow \left(x^3 - 3xy^2\right) + i\left(3x^2y - y^3\right) = u + iv$$

On equating real and imaginary parts, we obtain

$$u = x^3 - 3xy^2,\ v = 3x^2y - y^3$$

$$\therefore \frac{u}{x} + \frac{v}{y} = \frac{x^3 - 3xy^2}{x} + \frac{3x^2y - y^3}{y}$$

$$= \frac{x\left(x^2 - 3y^2\right)}{x} + \frac{y\left(3x^2 - y^2\right)}{y}$$

$$= x^2 - 3y^2 + 3x^2 - y^2$$

$$= 4x^2 - 4y^2$$

$$= 4\left(x^2 - y^2\right)$$

$$\therefore \frac{u}{x} + \frac{v}{y} = 4\left(x^2 - y^2\right)$$

Hence, proved.

Question

If α and β are different complex numbers with $|\beta| = 1$, then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$.

Answer

Let α = *a* + *ib* and β = *x* + *iy*

It is given that, $|\beta| = 1$

$$\therefore \sqrt{x^2+y^2} = 1$$
$$\Rightarrow x^2+y^2 = 1 \qquad \text{... (i)}$$

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = \left|\frac{(x+iy)-(a+ib)}{1-(a-ib)(x+iy)}\right|$$

$$= \left|\frac{(x-a)+i(y-b)}{1-(ax+aiy-ibx+by)}\right|$$

$$= \left|\frac{(x-a)+i(y-b)}{(1-ax-by)+i(bx-ay)}\right|$$

$$= \frac{|(x-a)+i(y-b)|}{|(1-ax-by)+i(bx-ay)|} \qquad \left[\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$$

$$= \frac{\sqrt{(x-a)^2+(y-b)^2}}{\sqrt{(1-ax-by)^2+(bx-ay)^2}}$$

$$= \frac{\sqrt{x^2+a^2-2ax+y^2+b^2-2by}}{\sqrt{1+a^2x^2+b^2y^2-2ax+2abxy-2by+b^2x^2+a^2y^2-2abxy}}$$

$$= \frac{\sqrt{\left(x^2+y^2\right)+a^2+b^2-2ax-2by}}{\sqrt{1+a^2\left(x^2+y^2\right)+b^2\left(y^2+x^2\right)-2ax-2by}}$$

$$= \frac{\sqrt{1+a^2+b^2-2ax-2by}}{\sqrt{1+a^2+b^2-2ax-2by}} \qquad \left[\text{Using } (1)\right]$$

$$= 1$$

$$\therefore \left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = 1$$

The above method is very long and stupid. There is a very short way of doing this.

Understand the fundamentals…. | Z | = 1 => Z (Ź) = 1

There is a typing issue here the Z bar symbol is not there. A bar symbol is available Ā

So writing Z bar as Ź

In the problem it is given | ß | = 1 so ß (ß bar) = 1

The denominator 1 - ß (α bar) can be written as ß (ß bar) - ß (α bar)

Take ß common so ß(ß bar - α bar)

Now | ß - α | = | ß bar - α bar | You should know that | Z ± Ā | = | Ź ± A |

Or say | A - Z | = | Ā - Ź | and so on

So our problem simplifies as 1 / | ß | which is 1

:-{D

Question

Find the number of non-zero integral solutions of the equation $|1-i|^x = 2^x$

Answer

$$|1-i|^x = 2^x$$
$$\Rightarrow \left(\sqrt{1^2+(-1)^2}\right)^x = 2^x$$
$$\Rightarrow \left(\sqrt{2}\right)^x = 2^x$$
$$\Rightarrow 2^{\frac{x}{2}} = 2^x$$
$$\Rightarrow \frac{x}{2} = x$$
$$\Rightarrow x = 2x$$
$$\Rightarrow 2x - x = 0$$
$$\Rightarrow x = 0$$

Thus, 0 is the only integral solution of the given equation. Therefore, the number of non-zero integral solutions of the given equation is 0.

Question

If $(a + ib)(c + id)(e + if)(g + ih) = A + iB$, then show that $(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$.

Answer

$$(a+ib)(c+id)(e+if)(g+ih) = A + iB$$
$$\therefore |(a+ib)(c+id)(e+if)(g+ih)| = |A+iB|$$
$$\Rightarrow |(a+ib)| \times |(c+id)| \times |(e+if)| \times |(g+ih)| = |A+iB| \qquad [|z_1 z_2| = |z_1||z_2|]$$
$$\Rightarrow \sqrt{a^2+b^2} \times \sqrt{c^2+d^2} \times \sqrt{e^2+f^2} \times \sqrt{g^2+h^2} = \sqrt{A^2+B^2}$$

On squaring both sides, we obtain

$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$

Hence, proved.

Question

If $\left(\frac{1+i}{1-i}\right)^m = 1$, then find the least positive integral value of m

Answer

$$\left(\frac{1+i}{1-i}\right)^m = 1$$

$$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^m = 1$$

$$\Rightarrow \left(\frac{(1+i)^2}{1^2+1^2}\right)^m = 1$$

$$\Rightarrow \left(\frac{1^2+i^2+2i}{2}\right)^m = 1$$

$$\Rightarrow \left(\frac{1-1+2i}{2}\right)^m = 1$$

$$\Rightarrow \left(\frac{2i}{2}\right)^m = 1$$

$$\Rightarrow i^m = 1$$

$\therefore m = 4k$, where k is some integer.

Therefore, the least positive integer is 1

Thus the least positive integral value of m is 4

Question

$$\begin{aligned} i^{49}+i^{68}+i^{89}+i^{110} &= i^{4\times 12}\times +i^{1}+i^{4\times 17}+i^{4\times 22}\times i^{1}+i^{4\times 27}\times i^{2} \\ &= 1\times i+1+1\times i+1\times i^{2} \\ &= i+1+i-1 \\ &= 2i \end{aligned}$$

$$\therefore\ i^{49}+i^{68}+i^{89}+i^{110} = 2i$$

$$\begin{aligned} i^{30}+i^{80}+i^{120} &= i^{4\times 7}\times i^{2}+i^{4\times 20}+i^{4\times 30} \\ &= 1\times i^{2}+1+1 \\ &= -1+1+1 \\ &= 1 \end{aligned}$$

$$\therefore\ i^{30}+i^{80}+i^{120} = 1$$

$$\begin{aligned} i+i^{2}+i^{3}+i^{4} &= 1+(-1)+(-i)+1 \\ &= 0 \end{aligned}$$

$$\therefore\ i+i^{2}+i^{3}+i^{4} = 0$$

$$\begin{aligned} i^5 + i^{10} + i^{15} &= i^{4\times1} \times i^1 + i^{4\times2} \times i^2 + i^{4\times3} \times i^3 \\ &= 1 \times i + 1 \times i^2 + 1 \times i^3 \\ &= i - 1 - i \\ &= -1 \end{aligned}$$

$$\therefore\ i^5 + i^{10} + i^{15} = -1$$

$$\begin{aligned} \frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} &= \frac{i^{4\times148} + i^{147} \times i^2 + i^{4\times147} + i^{4\times146} \times i^2 + i^{4\times146}}{i^{4\times145} \times i^2 + i^{4\times145} + i^{4\times144} \times i^2 + i^{4\times144} + i^{4\times143} \times i^2} \\ &= \frac{1 + 1 \times i^2 + 1 + 1 \times i^2 + 1}{1 \times i^2 + 1 + 1 \times i^2 + 1 + 1 \times i^2} \\ &= \frac{1 - 1 + 1 - 1 + 1}{-1 + 1 - 1 + 1 - 1} \\ &= \frac{1}{-1} \\ &= -1 \end{aligned}$$

$$\therefore\ \frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} = -1$$

$$\begin{aligned} &1 + i^2 + i^4 + i^6 + i^8 + \ldots + i^{20} \\ &= 1 + i^2 + i^4 + i^{4\times1} \times i^2 + i^{4\times2} + i^{4\times2} \times i^2 + i^{4\times3} + i^{4\times3} \times i^2 + i^{4\times4} + i^{4\times4} \times i^2 + i^{4\times5} \\ &= 1 - 1 + 1 + 1 \times i^2 + 1 + 1 \times i^2 + 1 + 1 \times i^2 + 1 + 1 \times i^2 + 1 \\ &= 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 \\ &= 1 \end{aligned}$$

To recall standard integrals

$f(x)$	$\int f(x)dx$	$f(x)$	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1}$ $(n \neq -1)$	$[g(x)]^n g'(x)$	$\frac{[g(x)]^{n+1}}{n+1}$ $(n \neq -1)$
$\frac{1}{x}$	$\ln\|x\|$	$\frac{g'(x)}{g(x)}$	$\ln\|g(x)\|$
e^x	e^x	a^x	$\frac{a^x}{\ln a}$ $(a > 0)$
$\sin x$	$-\cos x$	$\sinh x$	$\cosh x$
$\cos x$	$\sin x$	$\cosh x$	$\sinh x$
$\tan x$	$-\ln\|\cos x\|$	$\tanh x$	$\ln \cosh x$
$\operatorname{cosec} x$	$\ln\left\|\tan \frac{x}{2}\right\|$	$\operatorname{cosech} x$	$\ln\left\|\tanh \frac{x}{2}\right\|$
$\sec x$	$\ln\|\sec x + \tan x\|$	$\operatorname{sech} x$	$2\tan^{-1} e^x$
$\sec^2 x$	$\tan x$	$\operatorname{sech}^2 x$	$\tanh x$
$\cot x$	$\ln\|\sin x\|$	$\coth x$	$\ln\|\sinh x\|$
$\sin^2 x$	$\frac{x}{2} - \frac{\sin 2x}{4}$	$\sinh^2 x$	$\frac{\sinh 2x}{4} - \frac{x}{2}$
$\cos^2 x$	$\frac{x}{2} + \frac{\sin 2x}{4}$	$\cosh^2 x$	$\frac{\sinh 2x}{4} + \frac{x}{2}$

$f(x)$	$\int f(x)\,dx$	$f(x)$	$\int f(x)\,dx$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a}$ $(a > 0)$	$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\|$ $(0<\|x\|<a)$
		$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\|$ $(\|x\| > a>0)$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\frac{x}{a}$ $(-a < x < a)$	$\frac{1}{\sqrt{a^2+x^2}}$	$\ln\left\|\frac{x+\sqrt{a^2+x^2}}{a}\right\|$ $(a > 0)$
		$\frac{1}{\sqrt{x^2-a^2}}$	$\ln\left\|\frac{x+\sqrt{x^2-a^2}}{a}\right\|$ $(x>a>0)$
$\sqrt{a^2-x^2}$	$\frac{a^2}{2}\left[\sin^{-1}\left(\frac{x}{a}\right) + \frac{x\sqrt{a^2-x^2}}{a^2}\right]$	$\sqrt{a^2+x^2}$	$\frac{a^2}{2}\left[\sinh^{-1}\left(\frac{x}{a}\right) + \frac{x\sqrt{a^2+x^2}}{a^2}\right]$
		$\sqrt{x^2-a^2}$	$\frac{a^2}{2}\left[-\cosh^{-1}\left(\frac{x}{a}\right) + \frac{x\sqrt{x^2-a^2}}{a^2}\right]$

Some series Expansions -

$$\frac{\pi}{2} = \left(\frac{2}{1}\frac{2}{3}\right)\left(\frac{4}{3}\frac{4}{5}\right)\left(\frac{6}{5}\frac{6}{7}\right)\left(\frac{8}{7}\frac{8}{9}\right)\ldots$$

$$\pi = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \frac{4}{13} - \cdots$$

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots.$$

$$\pi = \sqrt{12}\left(1 - \frac{1}{3\cdot 3} + \frac{1}{5\cdot 3^2} - \frac{1}{7\cdot 3^3} + \cdots\right)$$

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots = \sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\int_0^{\pi/2} \log \sin x \, dx = -\frac{\pi}{2}\log 2 = \frac{\pi}{2}\log\frac{1}{2}$$

Solve a series problem

If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$ upto $\infty = \frac{\pi^2}{6}$, then value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ up to ∞ is

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{6}$ (c) $\frac{\pi^2}{8}$ (d) $\frac{\pi^2}{12}$

Ans. (c)

Solution We have $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ upto ∞

$$= \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} \cdots \text{ upto } \infty$$

$$- \frac{1}{2^2}\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right]$$

$$= \frac{\pi^2}{6} - \frac{1}{4}\left(\frac{\pi^2}{6}\right) = \frac{\pi^2}{8}$$

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \ldots \infty = \frac{\pi^2}{12}$$

$$\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \ldots \infty = \frac{\pi^2}{24}$$

$$\frac{\sin\sqrt{x}}{\sqrt{x}} = 1 - \frac{x}{3!} + \frac{x^2}{5!} - \frac{x^3}{7!} + \frac{x^4}{9!} - \frac{x^5}{11!} + \ldots.$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = \sum_{k=0}^{n} \frac{(-1)^k x^{2k}}{(2k)!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots = \sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots = \sum_{k=0}^{n} \frac{x^{2k}}{(2k)!}$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots = \sum_{k=0}^{n} \frac{x^{2k+1}}{(2k+1)!}$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad (-1 \le x < 1)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} \cdots + \frac{2^{2n}\left(2^{2n}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \qquad |x| < \frac{\pi}{2}$$

$$\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \cdots + \frac{E_n x^{2n}}{(2n)!} + \cdots \quad |x| < \frac{\pi}{2}$$

$$\csc x = \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^5}{15120} + \cdots + \frac{2\left(2^{2n-1}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \qquad 0 < |x| < \pi$$

$$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - - \cdots - \frac{2^{2n} B_n x^{2n-1}}{(2n)!} - \cdots \quad 0 < |x| < \pi$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$$

$$\sec x = 1 + \frac{x^2}{\lfloor 2} + \frac{5x^4}{\lfloor 4} + \cdots$$

$$\log(\cos x) = -\frac{x^2}{\lfloor 2} - \frac{2x^4}{\lfloor 4} - \cdots$$

$$\log(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \cdots$$

$$\sin^{-1} x = x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \cdots \quad |x| < 1$$

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$= \frac{\pi}{2} - \left(x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \cdots\right) \quad |x| < 1$$

$$\tan^{-1} x = \begin{cases} x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots & |x| < 1 \\ \pm\frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \cdots & \begin{cases} + \text{ if } x \geq 1 \\ - \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$$

$$= \frac{\pi}{2} - \left(\frac{1}{x} + \frac{1}{2\cdot 3x^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5x^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \cdots\right) \quad |x| > 1$$

$$\csc^{-1} x = \sin^{-1}(1/x)$$

$$= \frac{1}{x} + \frac{1}{2\cdot 3x^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5x^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \cdots \quad |x| > 1$$

$$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$$

$$= \begin{cases} \frac{\pi}{2} - \left(x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots\right) & |x| < 1 \\ p\pi + \frac{1}{x} - \frac{1}{3x^3} + \frac{1}{5x^5} + \cdots & \begin{cases} p = 0 \text{ if } x \geq 1 \\ p = 1 \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\ln x = 2\left[\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \ldots\right]$$

$$= 2\sum_{n=1}^{\infty} \frac{1}{2n-1}\left(\frac{x-1}{x+1}\right)^{2n-1} \quad (x > 0)$$

$$\ln x = \frac{x-1}{x} + \frac{1}{2}\left(\frac{x-1}{x}\right)^2 + \frac{1}{3}\left(\frac{x-1}{x}\right)^3 + \ldots$$

$$= \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{x-1}{x}\right)^n \quad (x > \frac{1}{2})$$

$$\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}(x-1)^n \quad (0 < x \le 2)$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}x^n \quad (|x| < 1)$$

$$\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots\infty \, (-1 \le x < 1)$$

$$\log_e(1+x) - \log_e(1-x) =$$

$$\log_e \frac{1+x}{1-x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\infty\right)(-1 < x < 1)$$

$$\log_e\left(1+\frac{1}{n}\right) = \log_e \frac{n+1}{n} = 2 \left[\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \ldots\infty\right]$$

$$\log_e(1+x) + \log_e(1-x) = \log_e(1-x^2) = -2\left(\frac{x^2}{2} + \frac{x^4}{4} + \ldots\infty\right)(-1 < x < 1)$$

$$\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots = \frac{1}{1\cdot 2} + \frac{1}{3.4} + \frac{1}{5.6} + \ldots$$

Important Results

(i) (a) $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cos^n x}{\sin^n x + \cos^n x} dx$

(b) $\int_0^{\pi/2} \frac{\tan^n x}{1 + \tan^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{dx}{1 + \tan^n x}$

(c) $\int_0^{\pi/2} \frac{dx}{1 + \cot^n x} = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{1 + \cot^n x} dx$

(d) $\int_0^{\pi/2} \frac{\tan^n x}{\tan^n x + \cot^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{\tan^n x + \cot^n x} dx$

(e) $\int_0^{\pi/2} \frac{\sec^n x}{\sec^n x + \text{cosec}^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\text{cosec}^n x}{\sec^n x + \text{cosec}^n x} dx$ where, $n \in R$

(ii) $\int_0^{\pi/2} \frac{a^{\sin^n x}}{a^{\sin^n x} + a^{\cos^n x}} dx = \int_0^{\pi/2} \frac{a^{\cos^n x}}{a^{\sin^n x} + a^{\cos^n x}} dx = \frac{\pi}{4}$

(iii) (a) $\int_0^{\pi/2} \log \sin x \, dx = \int_0^{\pi/2} \log \cos x \, dx = -\frac{\pi}{2} \log 2$

(b) $\int_0^{\pi/2} \log \tan x \, dx = \int_0^{\pi/2} \log \cot x \, dx = 0$

(c) $\int_0^{\pi/2} \log \sec x \, dx = \int_0^{\pi/2} \log \text{cosec} x \, dx = \frac{\pi}{2} \log 2$

(iv) (a) $\int_0^{\infty} e^{-ax} \sin bx \, dx = \frac{b}{a^2 + b^2}$

(b) $\int_0^{\infty} e^{-ax} \cos bx \, dx = \frac{a}{a^2 + b^2}$

(c) $\int_0^{\infty} e^{-ax} x^n \, dx = \frac{n!}{a^n + 1}$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left(x + \sqrt{x^2 - a^2}\right) + C$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left(\frac{x - a}{x + a}\right) + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left(\frac{a + x}{a - x}\right) + C$$

$$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{a^2 + x^2}\,dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right) + C$$

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Good Luck to you for your Preparations, References, and Exams

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