Spoon Feeding Circles

Simplified Knowledge Management Classes Bangalore

My name is Subhashish Chattopadhyay. I have been teaching for IIT-JEE, Various International Exams (such as IMO [International Mathematics Olympiad], IPhO [International Physics Olympiad], IChO [International Chemistry Olympiad]), IGCSE (IB), CBSE, I.Sc, Indian State Board exams such as WB-Board, Karnataka PU-II etc since 1989. As I write this book in 2016, it is my 25 th year of teaching. I was a Visiting Professor to BARC Mankhurd, Chembur, Mumbai, Homi Bhabha Centre for Science Education (HBCSE) Physics Olympics camp BARC Campus.

I am Life Member of …

- **IAPT (Indian Association of Physics Teachers)**

- **IPA (Indian Physics Association)**

- **AMTI (Association of Mathematics Teachers of India)**

- **National Human Rights Association**

- **Men's Rights Movement (India and International)**

- **MGTOW Movement (India and International)**

And also of

IACT (Indian Association of Chemistry Teachers)

The selection for National Camp (for Official Science Olympiads - Physics, Chemistry, Biology, Astronomy) happens in the following steps ….

1) **NSEP** (National Standard Exam in Physics) and **NSEC** (National Standard Exam in Chemistry) held around 24 rth November. Approx 35,000 students appear for these exams every year. The exam fees is Rs 100 each. Since 1998 the IIT JEE toppers have been topping these exams and they get to know their rank / performance ahead of others.

2) **INPhO** (Indian National Physics Olympiad) and **INChO** (Indian National Chemistry Olympiad). Around 300 students in each subject are allowed to take these exams. Students coming from outside cities are paid fair from the Govt of India.

3) The Top 35 students of each subject are invited at HBCSE (Homi Bhabha Center for Science Education) Mankhurd, near Chembur, BARC, Mumbai. After a 2-3 weeks camp the top 5 are selected to represent India. The flight tickets and many other expenses are taken care by Govt of India.

Since last 50 years there has been no dearth of "Good Books". Those who are interested in studies have been always doing well. This e-Book does not intend to replace any standard text book. These topics are very old and already standardized.

There are 3 kinds of Text Books

- The thin Books - Good students who want more details are not happy with these. Average students who need more examples are not happy with these. Most students who want to "Cram" quickly and pass somehow find the thin books "good" as they have to read less !!

- The Thick Books - Most students do not like these, as they want to read as less as possible. Average students are "busy" with many other things and have no time to read all these.

- The Average sized Books - Good students do not get all details in any one book. Most bad students do not want to read books of "this much thickness" also !!

We know there can be no shoe that's fits in all.

Printed books are not e-Books! Can't be downloaded and kept in hard-disc for reading "later" ………

So if you read this book later, you will get all kinds of examples in a single place. This becomes a very good "Reference Material". I sincerely wish that all find this "very useful".

Students who do not practice lots of problems, do not do well. The rules of "doing well" had never changed …. Will never change !
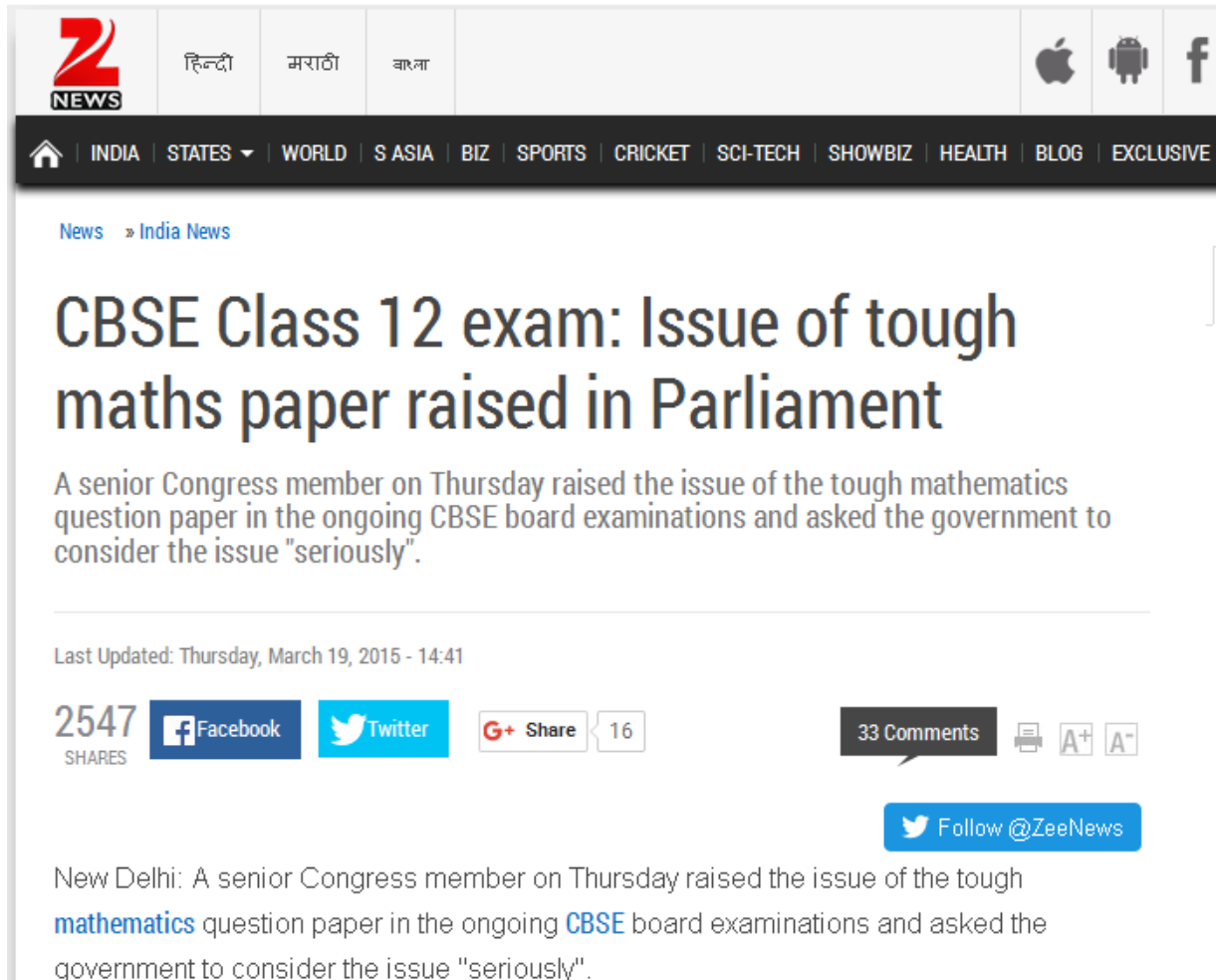
After 2016 CBSE Mathematics exam, lots of students complained that the paper was tough!

In 2015 also the same complain was there by many students

In March 2016, students of Karnataka PU-II also complained the same, regarding standard 12 (PU-II Mathematics Exam). Even though the Math Paper was identical to previous year, most students had not even solved the 2015 Question Paper.

Friday, March 25, 2016 - 13:28 | The **NEWS** Minute

HOME | NEWS | ANDHRA | KARNATAKA | KERALA | TAMIL NADU | TELANGANA | CULTURE | MEDIA | BLO

Exams

Online petition for lenient evaluation of K'taka II PU math paper gets over 8000 supporters

The campaign, which was launched on Monday, has garnered over 8000 supporters

TNM Staff | Wednesday, March 16, 2016 - 09:32

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Following a "very tough" math paper that left many II PU students in tears, Saket Ravindran a student launched an online campaign demanding lenient evaluation.

These complains are not new. In fact since last 40 years, (since my childhood), I always see this; every year the same setback, same complain!

In this e-Book I am trying to solve this problem. Those students who practice can learn.

No one can help those who are not studying, or practicing.
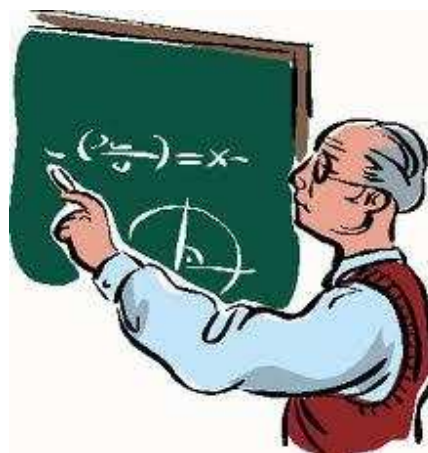
Learn more at http://skmclasses.weebly.com/iit-jee-home-tuitions-bangalore.html

Twitter – https://twitter.com/ZookeeperPhy

Facebook – https://www.facebook.com/IIT.JEE.by.Prof.Subhashish/

Blog – http://skmclasses.kinja.com
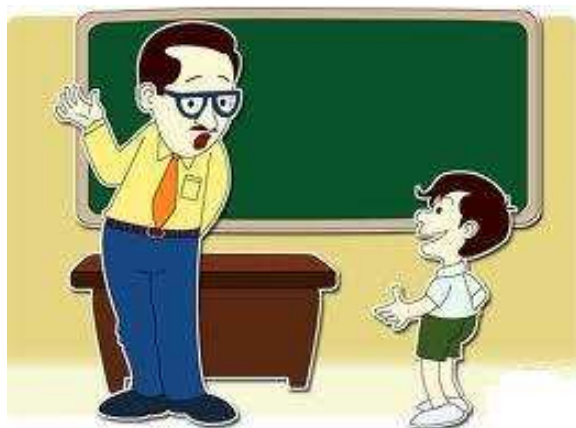
A very polite request :

I wish these e-Books are read only by Boys and Men. Girls and Women, better read something else; learn from somewhere else.

Preface

We all know that in the species "Homo Sapiens ", males are bigger than females. The reasons are explained in standard 10, or 11 (high school) Biology texts. **This shapes or size, influences all of our culture**. Before we recall / understand the reasons once again, let us see some random examples of the influence

Random - 1

If there is a Road rage, then who all fight ? (generally ?). Imagine two cars driven by adult drivers. Each car has a woman of similar age as that of the Man. The cars " touch " or " some issue happens". Who all comes out and fights ? Who all are most probable to drive the cars ?

(Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win)

Random - 2

Heavy metal music artists are all Men. Metallica, Black Sabbath, Motley Crue, Megadeth, Motorhead, AC/DC, Deep Purple, Slayer, Guns & Roses, Led Zeppelin, Aerosmith ….. the list can be in thousands. All these are grown-up Boys, known as Men.

(Men strive for perfection. Men are eager to excel. Men work hard. Men want to win.)

Random - 3

Apart from Marie Curie, only one more woman got Nobel Prize in Physics. (Maria Goeppert Mayer - 1963). So, … almost all are men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 4

The best Tabla Players are all Men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 5

History is all about, which Kings ruled. Kings, their men, and Soldiers went for wars. History is all about wars, fights, and killings by men.

Boys start fighting from school days. Girls do not fight like this

(Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.)

Random - 6

The highest award in Mathematics, the " Fields Medal " is around since decades. Till date only one woman could get that. (Maryam Mirzakhani - 2014). So, … almost all are men.

(Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women.)

Random - 7

Actor is a gender neutral word. Could the movie like " Top Gun " be made with Female actors ? The best pilots, astronauts, Fighters are all Men.

Random - 8

In my childhood had seen a movie named " The Tower in Inferno ". In the movie when the tall tower is in fire, women were being saved first, as only one lift was working….

Many decades later another movie is made. A box office hit. " The Titanic ". In this also …. As the ship is sinking women are being saved. **Men are disposable**. Men may get their turn later…

Movies are not training programs. Movies do not teach people what to do, or not to do. Movies only reflect the prevalent culture. Men are disposable, is the culture in the society. Knowingly, unknowingly, the culture is depicted in Movies, Theaters, Stories, Poems, Rituals, etc. I or you can't write a story, or make a movie in which after a minor car accident the Male passengers keep seating in the back seat, while the both the women drivers come out of the car and start fighting very bitterly on the road. There has been no story in this world, or no movie made, where after an accident or calamity, Men are being helped for safety first, and women are told to wait.

Random - 9

Artists generally follow the prevalent culture of the Society. In paintings, sculptures, stories, poems, movies, cartoon, Caricatures, knowingly / unknowingly, " the prevalent Reality " is depicted. The opposite will not go well with people. If deliberately " the opposite " is shown then it may only become a special art, considered as a special mockery.

Random - 10

Men go to "girl / woman's house" to marry / win, and bring her to his home. That is a sort of winning her. When a boy gets a " Girl-Friend ", generally he and his friends consider that as an achievement. The boy who " got / won " a girl-friend feels proud. His male friends feel, jealous, competitive and envious. Millions of stories have been written on these themes. Lakhs of movies show this. Boys / Men go for " bike race ", or say " Car Race ", where the winner " gets " the most beautiful girl of the college.

(Men want to excel. Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win.)

Prithviraj Chauhan ' went ` to " pickup " or " abduct " or " win " or " bring " his love. There was a Hindi movie (hit) song … " Pasand ho jaye, to ghar se utha laye ". It is not other way round. Girls do not go to Boy's house or man's house to marry. Nor the girls go in a gang to " pick-up " the boy / man and bring him to their home / place / den.

Random - 11

Rich people; often are very hard working. Successful business men, establish their business (empire), amass lot of wealth, with lot of difficulty. Lots of sacrifice, lots of hard work, gets into this. Rich people's wives had no contribution in this wealth creation. Women are smart, and successful upto the extent to choose the right/rich man to marry. So generally what happens in case of Divorces ? Search the net on " most costly divorces " and you will know. The women;(who had no contribution at all, in setting up the business / empire), often gets in Billions, or several Millions in divorce settlements.

Number 1

Rupert & Anna Murdoch -- $1.7 billion

One of the richest men in the world, **Rupert Murdoch** developed his worldwide media empire when he inherited his father's Australian newspaper in 1952. He married Anna Murdoch in the '60s and they remained together for 32 years, springing off three children.

They split amicably in 1998 but soon Rupert forced Anna off the board of News Corp and the gloves came off. The divorce was finalized in June 1999 when Rupert agreed to let his ex-wife leave with $1.7 billion worth of his assets, $110 million of it in cash. Seventeen days later, Rupert married Wendi Deng, one of his employees.

Ted Danson & Casey Coates -- $30 million

Ted Danson's claim to fame is undoubtedly his decade-long stint as Sam Malone on NBC's celebrated sitcom Cheers . While he did other TV shows and movies, he will always be known as the bartender of that place where everybody knows your name. He met his future first bride Casey, a designer, in 1976 while doing Erhard Seminars Training.

Ten years his senior, she suffered a paralyzing stroke while giving birth to their first child in 1979. In order to nurse her back to health, Danson took a break from acting for six months. But after two children and 15 years of marriage, the infatuation fell to pieces. Danson had started seeing Whoopi Goldberg while filming the comedy, Made in America and this precipitated the 1992 divorce. Casey got $30 million for her trouble.

See   https://zookeepersblog.wordpress.com/misandry-and-men-issues-a-short-summary-at-single-place/

See http://skmclasses.kinja.com/save-the-male-1761788732

It was Boys and Men, who brought the girls / women home. The Laws are biased, completely favoring women. The men are paying for their own mistakes.

See https://zookeepersblog.wordpress.com/biased-laws/

   (Man brings the Woman home. When she leaves, takes away her share of big fortune!)

Random - 12

A standardized test of Intelligence will never be possible. It never happened before, nor ever will happen in future; where the IQ test results will be acceptable by all. In the net there are thousands of charts which show that the intelligence scores of girls / women are lesser. Debates of Trillion words, does not improve performance of Girls.
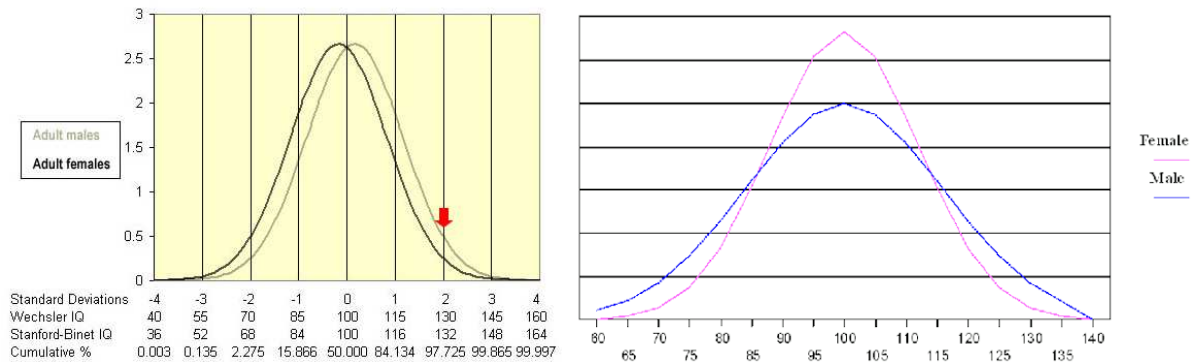
I am not wasting a single second debating or discussing with anyone, on this. I am simply accepting ALL the results. IQ is only one of the variables which is required for success in life. Thousands of books have been written on " Networking Skills ", EQ (Emotional Quotient), Drive, Dedication, Focus, " Tenacity towards the end goal " … etc. In each criteria, and in all together, women (in general) do far worse than men. Bangalore is known as " ….. capital of India ". [Fill in the blanks]. The blanks are generally filled as " Software Capital ", " IT Capital ", " Startup Capital ", etc. I am member in several startup eco-systems / groups. I have attended hundreds of meetings, regarding " technology startups ", or " idea startups ". These meetings have very few women. Starting up new companies are all " Men's Game " / " Men's business ". Only in Divorce settlements women will take their goodies, due to Biased laws. There is no dedication, towards wealth creation, by women.

Random - 13

Many men, as fathers, very unfortunately treat their daughters as " Princess ". Every " non-performing " woman / wife was " princess daughter " of some loving father. Pampering the girls, in name of " equal opportunity ", or " women empowerment ", have led to nothing.

See http://skmclasses.kinja.com/progressively-daughters-become-monsters-1764484338

See http://skmclasses.kinja.com/vivacious-vixens-1764483974

There can be thousands of more such random examples, where " Bigger Shape / size " of males have influenced our culture, our Society. **Let us recall the reasons**, that we already learned in standard 10 - 11, Biology text Books. In humans, women have a long gestation period, and also spends many years (almost a decade) to grow, nourish, and stabilize the child. (Million years of habit) Due to survival instinct Males want to inseminate. Boys and Men fight for the " facility (of womb + care) " the girl / woman may provide. Bigger size for males, has a winning advantage. Whoever wins, gets the " woman / facility ". The male who is of " Bigger Size ", has an advantage to win…. Leading to Natural selection over millions of years. In general " Bigger Males "; the " fighting instinct " in men; have led to wars, and solving tough problems (Mathematics, Physics, Technology, startups of new businesses, Wealth creation, Unreasonable attempts to make things [such as planes], Hard work ….)

So let us see the IIT-JEE results of girls. Statistics of several years show that there are around 17, (or less than 20) girls in top 1000 ranks, at all India level. Some people will yet not understand the performance, till it is said that … year after year we have around 980 boys in top 1000 ranks. Generally we see only 4 to 5 girls in top 500. In last 50 years not once any girl topped in IIT-JEE advanced. Forget about Single digit ranks, double digit ranks by girls have been extremely rare. It is all about " good boys ", " hard working ", " focused ", "**Bel-esprit** " **boys**.

In 2015, Only 2.6% of total candidates who qualified are girls (upto around 12,000 rank). while 20% of the Boys, amongst all candidates qualified. The Total number of students who appeared for the exam were around 1.4 million for IIT-JEE main. Subsequently 1.2 lakh (around 120 thousands) appeared for IIT-JEE advanced.

IIT-JEE results and analysis, of many years is given at https://zookeepersblog.wordpress.com/iit-jee-iseet-main-and-advanced-results/

In Bangalore it is rare to see a girl with rank better than 1000 in IIT-JEE advanced. We hardly see 6-7 boys with rank better than 1000. Hardly 2-3 boys get a rank better than 500.

See http://skmclasses.weebly.com/everybody-knows-so-you-should-also-know.html

Thousands of people are exposing the heinous crimes that Motherly Women are doing, or Female Teachers are committing. See https://www.facebook.com/WomenCriminals/

Some Random Examples must be known by all

It is extremely unfortunate that the " woman empowerment " has created. This is the kind of society and women we have now. I and many other sensible Men hate such women. Be away from such women, be aware of reality.

Mother Admits On Facebook to Sleeping with 15 Yr Old Son, They Have a Baby Together - Alwayzturntup

Sometimes it hard to believe w From Alwayzturntup

ALWAYZTURNTUP.ME

'Sex with my son is incredible - we're in love and we want a baby'

Ben Ford, who ditched his wife when he met his mother Kim West after 30 years, claims what the couple are doing 'isn't incest'

MIRROR.CO.UK

Woman sent to jail for the rest of her life after raping her four grandchildren is described as the 'most evil person' the judge has ever seen

Edwina Louis rape...

See More

Former Shelbyville ISD teacher who had sex with underage student gets 3 years in prison

After a two day break over the weekend, A Shelby County jury was back in the courtroom looking to conclude the trial of a former Shelbyville ISD teacher who had...

KLTV.COM | BY CALEB BEAMES

Woman sent to jail for raping her four grandchildren

A Ohio grandmother has been sentenced to four consecutive life terms after being found guilty of the rape of her own grandchildren. Edwina Louis, 53, will spend the rest of her life behind bars.

DAILYMAIL.CO.UK

http://www.thenativecanadian.com/.../eastern-ontario-teacher-...

The N.C. Chronicles.: Eastern Ontario teacher charged with 36 sexual offences

anti feminism, Child abuse, children's rights, Feminist hypocrisy,

THENATIVECANADIAN.COM | BY BLACKWOLF

Hyd woman kills newborn boy as she wanted daughter - Times of India

Having failed to bear a daughter for the third time, a shopkeeper's wife slit the throat of her 24day-old son with a shaving blade and left him to die in a street on Tuesday night.Purnima's first child was a stillborn boy, followed by another boy born five years ago.

TIMESOFINDIA.INDIATIMES.COM

Montgomery's son, Alan Vonn Webb, took the stand and was a key witness in her conviction.

"I want to see her placed somewhere she can never do that to children ...

See More

Woman sentenced to 40 years in prison for raping her children

A Murfreesboro mother found guilty of raping her own children learned her fate on Wednesday.

WAFF.COM | BY DENNIS FERRIER

gentler sex? Violence against men.'s photo.

Up to 64,000 women in UK are child-sex offenders

~ The Guardian

Women, the gentler sex? Violence against men.
April 8 at 1:38am ·

Like Page

In fact, the past decade has seen a dramatic increase in the number of incidents of women raping and sexually assaulting boys and men. On May 2014, Jezebel repo...

End violence against women . . . .

North Carolina Grandma Eats Her Daughter's New Born Baby After Smoking Bath Salts

Henderson, North Carolina— A North Carolina grandmother of 4 and recovering drug addict, is now in custody after she allegedly ate her daughter's newborn baby....

AZ-365.TOP

28-Year-Old Texas Teacher Accused of Sending Nude Picture to 14-Year-Old Former Student

BREITBART.COM

http://latest.com/.../attractive-girl-gang-lured-men-alleywa.../

http://www.wfmj.com/.../youngstown-woman-convicted-of-raping-...

Attractive Girl Gang Lured Men Into Alleyways Where Female Body Builder Would Attack Them

A Mexican street gang made up entirely of women has been accused of using their feminine wiles to lure men into alleyways and then beating them up and...

LATEST.COM

Youngstown woman convicted of raping a 1 year old is back in jail

A Youngstown woman who went to prison for raping a 1-year-old boy fifteen years ago is in trouble with the law again.

WFMJ.COM

End violence against women . . . .

Women are raping boys and young men

Rape advocacy has been maligned and twisted into a political agenda controlled by radicalized activists. Tim Patten takes a razor keen and well supported look into the manufactured rape culture and...

AVOICEFORMEN.COM | BY TIM PATTEN

Bronx Woman Convicted of Poisoning and Drowning Her Children

Lisette Bamenga researched methods on the Internet before she killed her son and daughter in 2012.

NYTIMES.COM | BY MARC SANTORA

A Russian-born newlywed slowly butchered her German husband — feeding strips of his flesh to their dog until he took his last breath. Svetlana Batukova, 46, was…

See More

Daily Mail
January 15, 2015 ·

Mother charged with rape and sodomy of her son's 12-year-old friend

She killed her husband and then fed him to her dog: police
A Russian-born newlywed butchered her German hubby — and fed strips of his flesh to her pooch, authorities said. Svetlana Batukova offed Horst Hans Henkels at their…
NYPOST.COM

Mom, 30, 'raped and had oral sex with her son's 12-year-old friend'
Nicole Marie Smith, 30, (pictured) of St Charles County, Missouri, has been jailed after she allegedly targeted the 12-year-old boy at her home.
DAILYM.AI

April 4 at 4:48am ·

This mother filmed herself raping her own son and then sold it to a man for $300. The courts just decide her fate. When you see what she got, you're going to be outraged.

Female prison officers commit 90pc of sex assaults on male teens in US juvenile detention centres
Lawsuit in Idaho highlights the prevalence of sexual victimization of juvenile offenders.
IBTIMES.CO.UK | BY NICOLE ROJAS

Mother Who Filmed Herself Raping Her 1-Year-Old Son Receives Shocking Sentence
"...then used the money to buy herself a laptop..."
AMERICANEWS.COM

In several countries or rather in several regions of the world, family system has collapsed, due to bad nature and naughty acts of women. Particularly in Britain, and America, almost 50% people are alone, lonely, separated, divorced or failed marriages. In 2013, 48% children were born out of wedlock. It was projected that by 2016, more than 51% children will be born, to unmarried mothers. In these developed countries " paternity fraud " by women, are close to 20%. You can see several articles in the net, and in wikipedia etc. This means 1 out of 5 children are calling a wrong man as dad. The lonely, alone " mothers " are frustrated. They see the children as burden. Love in the Society in general is lost, long time ago. The types of " Mothers " and " Women " we have now …………

By now if you have assumed that Indian women are not doing any crime then please become friends with MRA Guri https://www.facebook.com/profile.php?id=100004138754180

He has dedicated his life to expose Indian Criminals

Professor Subhashish Chattopadhyay

Spoon Feeding Series - Circles

Before we discuss Coordinate Geometry we must know the basics of the Graphs of Circles

Graph of $x^2 + y^2 = R^2$ will have the center at (0,0) and radius will be R

So graph of $x^2 + y^2 = 36$ is

| plot | $x^2 + y^2 = 36$ |
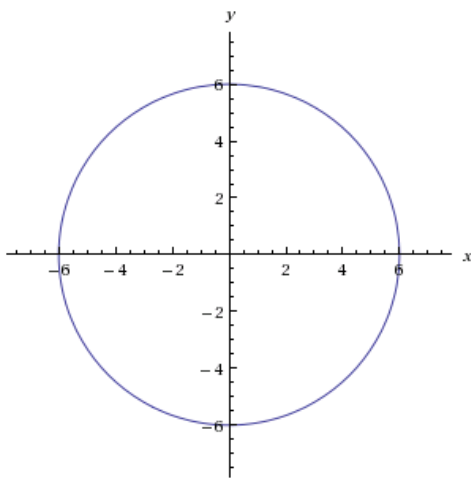
Graph of $(x - 3)^2 + (y - 4)^2 = 25$ is

| plot | $(x - 3)^2 + (y - 4)^2 = 25$ |
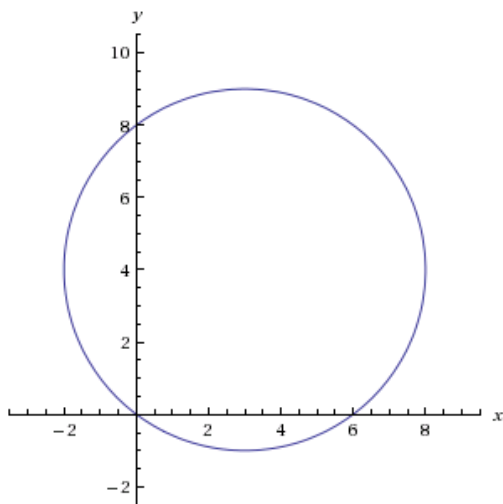
Center is at (3, 4)

The equation of the circle $x^2 + y^2 = a^2$ so $y = \sqrt{a^2 - x^2}$ center is (0,0)
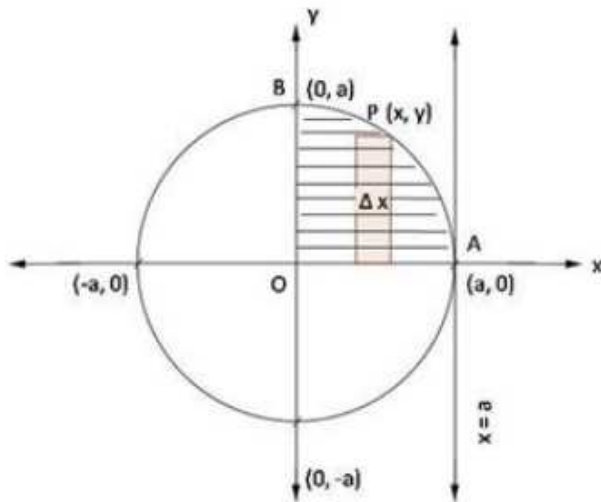
The general equation of circle is $(x - a)^2 + (y - b)^2 = r^2$ ....(A)
where (a, b) are centre and r is radius

∴ $(x + 2)^2 + (y - 3)^2 = 4^2$

⇒ $(x + 2)^2 + (y - 3)^2 = 16$

The general equation of circle is $(x - a)^2 + (y - b)^2 = r^2$ ....(A)
where (a, b) are centre and r is radius

From (A)

$(x - a)^2 + (y - b)^2 = \left(\sqrt{a^2 + b^2}\right)^2$

⇒ $x^2 - 2ax + a^2 + y^2 - 2bx + b^2 = a^2 + b^2$

⇒ $x^2 + y^2 - 2ax - 2by = 0$

The general equation of circle is $(x - a)^2 + (y - b)^2 = r^2$ ....(A)
where (a, b) are centre and r is radius

From (A)

$(x - 0)^2 + (y + 1)^2 = 1^2$

⇒ $x^2 + y^2 + 2y + 1 = 1$

⇒ $x^2 + y^2 + 2y = 0$

The general equation of circle is $(x - a)^2 + (y - b)^2 = r^2$ ....(A)
where (a, b) are centre and r is radius

From (A)

$$(x - a\cos\alpha)^2 + (y - a\sin\alpha)^2 = a^2$$
$$\Rightarrow \quad x^2 - 2a\cos\alpha x + y^2 - 2a\sin\alpha y + a^2(\cos^2\alpha + \sin^2\alpha) = a^2$$
$$\Rightarrow \quad x^2 + y^2 - 2a\cos\alpha x - 2a\sin\alpha y = 0$$

The general equation of circle is $(x - a)^2 + (y - b)^2 = r^2$ ....(A)
where (a, b) are centre and r is radius

From (A)

$$(x - a)^2 + (y - a)^2 = (\sqrt{2}a)^2$$
$$\Rightarrow \quad x^2 - 2ax + a^2 + y^2 - 2ay + a^2 = 2a^2$$
$$\Rightarrow \quad x^2 + y^2 - 2ax - 2ay = 0$$

A circle is a locus of a point whose distance from a fixed point (called centre) is always constant (called the radius).

1. **Simplest or standard form** The equation of a circle whose centre is $(0, 0)$ and radius a is $x^2 + y^2 = a^2$

2. **Central form** The equation of a circle whose centre is (h, k) and radius a is $(x - h)^2 + (y - k)^2 = a^2$

3. **Zero or Point circle** The equation of a circle whose radius is zero is $x^2 + y^2 = 0$ or $(x - h)^2 + (y - k)^2 = 0$

4. The equation of a circle whose centre is (h, k) and the circle passes through the point (p, q) is $(x - h)^2 + (y - k)^2 = (p - h)^2 + (q - k)^2$

5. The equation of a circle whose centre is (h, k) and the circle touches x-axis ($k = a = $ radius and $h \neq k$) is $(x - h)^2 + (y - k)^2 = k^2$

6. The equation of a circle whose centre is (h, k) and the circle touches y-axis ($h = a = $ radius and $h \neq k$) is $(x - h)^2 + (y - k)^2 = h^2$

7. The equation of circle whose centre is (h, k) and the circle touches both the axis ($h = k = a$) is $(x - a)^2 + (y - a)^2 = a^2$

If a is constant then total number of circles touching both the axes will be 4
One each in each quadrant

8. The equation of circle which touches both the axis and line $x = 2c$ is $(x - c)^2 + (y \mp c)^2 = c^2$

9. The equation of circle which touches both the axis and line $x = 2c$ and $y = 2c$ is $(x - c)^2 + (y - c)^2 = c^2$

10. The equation of circle which passes through origin $(0, 0)$ and cuts the intercepts $2a$ and $2b$ on both the axes is $(x - a)^2 + (y - b)^2 = a^2 + b^2$ or $x^2 + y^2 - 2ax - 2by = 0$ centre (a, b), radius $= \sqrt{a^2 + b^2}$

This circle also passes through (0,0), 2a,0 and (0,2b)

11. The equation of circle which touches x-axis at a distance of a unit from the origin and cut the intercept b on y-axis is $(x - a)^2 +$

$$\left(y - \frac{\sqrt{4a^2 + b^2}}{2}\right)^2 = \left(\frac{\sqrt{4a^2 + b^2}}{2}\right)^2$$

centre $\left(a, \dfrac{\sqrt{4a^2 + b^2}}{2}\right)$, radius $= \dfrac{\sqrt{4a^2 + b^2}}{2}$

The circle above passes through (a,0)

12. The centre and radius of circle which touches y-axis at point $(0, a)$ and cuts the intercept b on x-axis is

centre $\left(\dfrac{\sqrt{4a^2 + b^2}}{2}, a \right)$, radius $= \dfrac{\sqrt{4a^2 + b^2}}{2}$

13. **Diameter form of a circle**
The equation of circle whose diameter ends coordinates are (x_1, y_1) and $B (x_2, y_2)$ is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

centre $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$

radius $= \dfrac{1}{2} \left[(x_1 - x_2)^2 + (y_1 - y_2)^2 \right]^{1/2}$

$y = mx$ be a chord of the circle $x^2 + y^2 = 2ax$ then, equation of the circle taking this chord as diameter is $x^2 + y^2 - \dfrac{2a}{1 + m^2} x - \dfrac{2am}{1 + m^2} y = 0$

14. **General equation of a circle**
$x^2 + y^2 + 2gx + 2fy + c = 0$
centre $(-g, -f)$, radius

$= \sqrt{g^2 + f^2 - c}$

if circle is real then $g^2 + f^2 = c$
if circle is point circle then $g^2 + f^2 = c$
if circle is imaginary then $g^2 + f^2 < c$

The circle is real, point circle, imaginary circle if radius of circle will be real, zero, and imaginary respectively.

Three geometrical conditions (parameter) are required for defining a circle.

Centre and radius of the circle $ax^2 + ay^2 + 2gx + 2fy + c = 0$ are as follows:

centre $\left(-\dfrac{g}{a}, -\dfrac{f}{a} \right)$,

radius $= \dfrac{\sqrt{g^2 + f^2 - ac}}{a}$

General equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a circle if $a = b$; $h = 0$ i.e.,

coefficient of x^2 = coefficient of y^2 and coefficient of $xy = 0$.

Before finding the centre and radius of general equation of circle, the coefficient of x^2 and y^2 is are made unity if they are not already.

General equation of circle whose centre is on x-axis is $x^2 + y^2 + 2gx + c = 0$

centre $(-g, 0)$; radius $= \sqrt{g^2 - c}$

General equation of circle whose centre is on X-axis and passes through origin is $x^2 + y^2 + 2gx = 0$ centre $(-g, 0)$, radius $= g$

General equation of circle touching X-axis at the origin and of radius f is $x^2 + y^2 + 2fy = 0$ centre $(0, -f)$, radius $= f$.

Maximum distance of point $P(x_1, y_1)$ from circle having centre c and radius $r = Cp + r$ and minimum distance $= Cp - r$.

Position of a Point with Respect to Circle

(i) **Position of Point $A(x_1, y_1)$ with respect to the circle $x^2 + y^2 = a^2$ is**

   (i) out side if $x_1^2 + y_1^2 - a^2 > 0$

   (ii) on the circle if $x_1^2 + y_1^2 - a^2 = 0$

   (iii) inside the circle if $x_1^2 + y_1^2 - a^2 < 0$

(ii) **Position of Point $A(x_1, y_1)$ with respect to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is**

   (i) outside the circle if $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c > 0$

   (ii) on the circle if $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = 0$

   (iii) inside the circle if $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c < 0$

The equation of the circle which passes through origin will have c = 0

The Least and Greatest Distance of a Point from a Circle

The diameter through point $P(x_1, y_1)$ cuts the circle at point A and B, then the point P from the circle is at the

(i) Least distance $= PA = PC - CA$

(ii) greatest distance $= PB = PC + CB$

Parametric Equation of a Circle

The number of variables is reduced by the parameteric equations and both co-ordiantes of a point are expressed in terms of one variable only.

(i) For the circle $x^2 + y^2 = a^2$, parametric equation are $x = a \cos \theta, y = a \sin \theta$, Hence for all values of θ, the point θ ($a \cos \theta$, $a \sin \theta$) lies on the circle $x^2 + y^2 = a^2$.

(ii) For the circle $(x - h)^2 + (y - k)^2 = a^2$ parametric equations are $x = h + a \cos \theta$; $y = k + a \sin \theta$

Hence for all values of θ, the point θ ($h + a \cos \theta$, $k + a \sin \theta$) is on the circle $(x - h)^2 + (y - k)^2 = a^2$

(iii) For the general equation of circle $x^2 + y^2 + 2gx + 2fy + c = 0$, Parametric equations are

$$x = -g + \sqrt{g^2 + f^2 - c} \, \cos \theta;$$

$$y = -f + \sqrt{g^2 + f^2 - c} \, \sin \theta.$$

Circle Passing Through Three Given Points

The equation of circle passing through three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is

$$\begin{vmatrix} x^2 + y^2 & x & y & 1 \\ x_1^2 + y_1^2 & x_1 & y_1 & 1 \\ x_2^2 + y_2^2 & x_2 & y_2 & 1 \\ x_3^2 + y_3^2 & x_3 & y_3 & 1 \end{vmatrix} = 0$$

Equation of tangent at point $P(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is

$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$$

Concentric circles The circle having same centre but different radius are called concentric circles.

1. $x^2 + y^2 = r_1^2$; $x^2 + y^2 = r_2^2$; $r_1 \neq r_2$
2. $(x - a)^2 + (y - b)^2 = r_1^2$; $(x - a)^2 + (y - b)^2 = r_2^2$
3. $x^2 + y^2 + 2gx + 2fy + c_1 = 0$
   $x^2 + y^2 + 2gx + 2fy + c_2 = 0$

Pair of Tangents and Chord of Contact

(i) The length of tangent drawn to the circle $x^2 + y^2 = a^2$ from an external point (x_1, y_1) is $= \sqrt{x_1^2 + y_1^2 - a^2}$
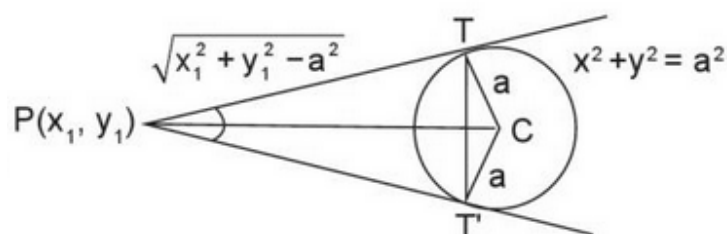
(ii) The length of tangent drawn to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ from external point (x_1, y_1) is $= \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$

$= \sqrt{S_1}$, where S_1 is known as **power of point** (x_1, y_1)

The equation of a pair of tangents drawn to the circle $x^2 + y^2 = a^2$ from an external point (x_1, y_1) is $SS_1 = T^2$ where $S = x^2 + y^2 - a^2$, $S_1 = x_1^2 + y_1^2 - a^2$ and $T = xx_1 + yy_1 - a^2$

The equation of a pair of tangents drawn to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ from external point (x_1, y_1) is $SS_1 = T^2$ where, $S = x^2 + y^2 + 2gx + 2fy + c$, $S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ and $T = xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c$

If the angle between a pairs of tangents drawn from an external point (x_1, y_1) to the circle $x^2 + y^2 = a^2$ be θ then $\tan \theta$

$$= \frac{2 \times a \times \sqrt{x_1^2 + y_1^2 - a^2}}{(x_1^2 + y_1^2 - a^2) - a^2}$$

$$= \frac{2 \times \text{radius} \times \text{length of tangent}}{(\text{length of tangent})^2 - (\text{radius})^2}$$

If two tangents intersect at right angle, then condition is $x_1^2 + y_1^2 = 2a^2$ (Angle between two circles is $= 90°$).

Let $S_1 = 0$, $S_2 = 0$ be equation of circles and $L = 0$ is equation of a line, then

(a) $S_1 + \lambda S_2 = 0$ is equation of circle passing through point of intersection of S_1 and S_2 ($\lambda \neq -1$)

(b) $S_1 + \lambda L = 0$ is equation of circle passing through point of intersection of line L and circle S_1.

Show that the circles $x^2 + y^2 - 14x - 10y + 58 = 0$ and $x^2 + y^2 - 2x + 6y - 26 = 0$ touch each other externally.

Answer

Given equation of the circles are

$$x^2 + y^2 - 14x - 10y + 58 = 0 \qquad \text{........ (1)}$$

and $\quad x^2 + y^2 - 2x + 6y - 26 = 0 \qquad \text{......... (2)}$

Centre and radius of the circle (1) are respectively $(-g, -f)$ or $(7, 5)$ and $\sqrt{g^2 + f^2 - c}$

$$= \sqrt{49 + 25 - 58} = \sqrt{16} = 4$$

Centre of the circle (2) is $(-g, -f)$ or $(1, -3)$ and

its radius $= \sqrt{g^2 + f^2 - c}$

$$= \sqrt{1 + 9 + 26} = \sqrt{36} = 6$$

Now, distance between the centre of these two circles

$$= \sqrt{(7-1)^2 + (5+3)^2} = \sqrt{36 + 64}$$

$$= \sqrt{100} = 10 = 4 + 6$$

$$= \text{Sum of their radius}$$

Hence, circles touch each other externally.

So review again

Equation of circle in various forms

(a)   The circle with centre as origin & radius 'r' has the equation; $x^2 + y^2 = r^2$.
(b)   The circle with centre (h, k) & radius 'r' has the equation; $(x - h)^2 + (y - k)^2 = r^2$.
(c)   The general equation of a circle is
$x^2 + y^2 + 2gx + 2fy + c = 0$

with centre as $(-g, -f)$ & radius $= \sqrt{g^2 + f^2 - c}$. If:
$g^2 + f^2 - c > 0 \Rightarrow$ real circle.
$g^2 + f^2 - c = 0 \Rightarrow$ point circle.
$g^2 + f^2 - c < 0 \Rightarrow$ imaginary circle, with real centre,   that is $(-g, -f)$

Every second degree equation in x & y, in which coefficient of x^2 is equal to coefficient of y^2 & the coefficient of xy is zero, always represents a circle.

The equation of circle with (x_1 , y_1) & (x_2 , y_2) as extremities of its diameter is:
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Note that this will be the circle of least radius passing through (x_1 , y_1) & (x_2 , y_2)

Question

Find the equation of the circle which passes through the point of intersection of the lines 3x – 2y – 1 = 0 and 4x + y – 27 = 0 and whose centre is (2, – 3).
Solution : Let P be the point of intersection of the lines AB and LM whose equations are respectively
3x – 2y – 1 = 0 ....(i)
and 4x + y – 27 = 0 ....(ii)
Solving (i) and (ii), we get x = 5, y = 7. So, coordinates of P are (5, 7). Let C(2, –3) be the centre of the circle. Since the circle passes through P, therefore

CP = radius = $\sqrt{(5-2)^2 + (7+3)^2}$ = $\sqrt{109}$

$(x - 2)^2 + (y + 3)^2 = 109$

Question

Find the centre & radius of the circle whose equation is $x^2 + y^2 - 4x + 6y + 12 = 0$
Solution : Comparing it with the general equation $x^2 + y^2 + 2gx + 2fy + c = 0$, we have
2g = – 4 so g = -2   2f = 6 so f = 3

& c = 12

Radius = $\sqrt{g^2 + f^2 - c}$ = 1

Question

Find the equation of the circle, the coordinates of the end points of whose diameter are (-1, 2) and (4, -3)

Solution : We know that the equation of the circle described on the line segment joining (x_1, y_1) and (x_2, y_2) as a diameter is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.

Here, $x_1 = -1$, $x_2 = 4$, $y_1 = 2$ and $y_2 = -3$.

So, the equation of the required circle is

$(x + 1)(x - 4) + (y - 2)(y + 3) = 0 \Rightarrow x^2 + y^2 - 3x + y - 10 = 0$

Intercepts made by a Circle on the Axes:

The intercepts made by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ on the co-ordinate axes are $2\sqrt{g^2 - c}$ & $2\sqrt{f^2 - c}$ respectively. If

$g^2 - c > 0 \quad \Rightarrow \quad$ circle cuts the x axis at two distinct points.

$g^2 = c \quad \Rightarrow \quad$ circle touches the x-axis.

$g^2 < c \quad \Rightarrow \quad$ circle lies completely above or below the x-axis.

Question

Find the equation to the circle touching the y-axis at a distance – 3 from the origin and intercepting a length 8 on the x-axis

Let the equation of the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$. Since it touches y-axis at $(0, -3)$ and $(0, -3)$ lies on the circle.

$\therefore \quad c = f^2 \quad ...(i) \qquad 9 - 6f + c = 0 \qquad ....(ii)$

From (i) and (ii), we get $9 - 6f + f^2 = 0 \Rightarrow (f - 3)^2 = 0 \Rightarrow f = 3$.

Putting f = 3 in (i) we obtain c = 9.

It is given that the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ intercepts length 8 on x-axis

$\therefore \quad 2\sqrt{g^2 - c} = 8 \Rightarrow 2\sqrt{g^2 - 9} = 8 \Rightarrow g^2 - 9 = 16 \Rightarrow g = \pm 5$

Hence, the required circle is $x^2 + y^2 \pm 10x + 6y + 9 = 0$.

Question

Find the equation of the circle which passes through the origin and cut off intercepts 3 and 4 from the positive parts of the axes respectively.

Solution We have

$OA = 3, OB = 4$

$\therefore \qquad OL = \dfrac{3}{2}$ and $CL = 2$

In $\triangle OLC,$
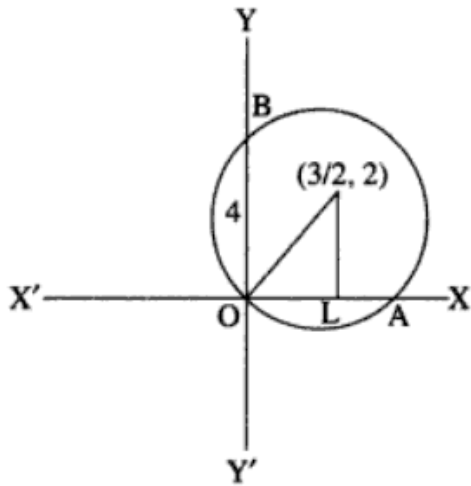
$$OC^2 = OL^2 + LC^2$$

$$\Rightarrow \qquad OC^2 = \left(\frac{3}{2}\right)^2 + 2^2$$

$$\Rightarrow \qquad OC = \frac{5}{2}$$

Thus, the required circle has its centre at $\left(\frac{3}{2}, 2\right)$ and radius $\frac{5}{2}$.

Hence its equation is

$$\left(x - \frac{3}{2}\right)^2 + (y-2)^2 = \left(\frac{5}{2}\right)^2$$

Question

Find the equation of the circle circumscribing the triangle formed by the lines $x + y = 6$, $2x + y = 4$ and $x + 2y = 5$.

Let the equation of sides AB, BC and CA of $\triangle ABC$ are respectively.

$$x + y = 6 \qquad\qquad ...(i)$$
$$2x + y = 4 \qquad\qquad ...(ii)$$
$$x + 2y = 5 \qquad\qquad ...(iii)$$

Solving (i) and (iii), (i) and (ii), (iii) and (ii), we get the coordinates A, B and C. The coordi-

nates of A, B and C are $(7, -1)$, $(-2, 8)$ and $(1, 2)$ respectively.

Let the equation of the circumference of $\triangle ABC$ be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \qquad ...(iv)$$

It passes through A $(7, -1)$, B $(-2, 8)$ and C $(1, 2)$ we get

$$50 + 14g - 2f + c = 0 \qquad ...(v)$$
$$68 - 4g + 16f + c = 0 \qquad ...(vi)$$
$$5 + 2g + 4f + c = 0 \qquad ...(vii)$$

Subtracting (v) from (vi), we get

$$18 - 18g + 18f = 0$$
$$\Rightarrow \qquad 1 - g + f = 0 \qquad ...(viii)$$

Subtracting (v) from (vii), we get

$$-45 - 12g + 6f = 0 \qquad ...(ix)$$

Solving (viii) and (ix) we get

$$g = -\frac{17}{2}, f = -\frac{19}{2}$$

Putting the values of g and f in (v), we get $c = 50$

Substituting the values of g, f and c in (iv), the equation of the required circumcircle is

$$x^2 + y^2 - 17x - 19y + 50 = 0$$

Question

Find the points of intersection of the line $2x + 3y = 18$ and the circle $x^2 + y^2 = 25$.

We have

$$2x + 3y = 18 \qquad ...(i)$$

and

$$x^2 + y^2 = 25 \qquad ...(ii)$$

Substituting $y = \dfrac{18 - 2x}{3}$ obtained from (i) in (ii), we get

$$x^2 + \left(\frac{18 - 2x}{3}\right)^2 = 25$$

$$\Rightarrow \qquad 13x^2 - 72x + 99 = 0$$

$$\Rightarrow \qquad (x - 3)(13x - 33) = 0$$

$$\Rightarrow \qquad x = 3 \text{ or } x = \frac{33}{13}$$

Substituting the values of x in (i), we get $y = 4$ or $y = \dfrac{56}{13}$ respectively. Hence, the points of intersection of the given line and the given circle are $(3, 4)$ and $\left(\dfrac{33}{13}, \dfrac{56}{13}\right)$

Parametric Equations of a Circle

The parametric equations of $(x - h)^2 + (y - k)^2 = r^2$ are: $x = h + r \cos \theta$; $y = k + r \sin \theta$; $-\pi < \theta \le \pi$ where (h, k) is the centre, r is the radius & θ is a parameter.

Find the parametric equations of the circle $x^2 + y^2 - 4x - 2y + 1 = 0$

We have : $x^2 + y^2 - 4x - 2y + 1 = 0 \quad \Rightarrow \quad (x^2 - 4x) + (y^2 - 2y) = -1$

$\Rightarrow \qquad (x - 2)^2 + (y - 1)^2 = 2^2$

So, the parametric equations of this circle are

$x = 2 + 2 \cos \theta$, $y = 1 + 2 \sin \theta$

Question

Find the equations of the following curves in Cartesian form. Also, find the centre and radius of the circle $x = a + c \cos \theta$, $y = b + c \sin \theta$

We have : $x = a + c \cos \theta$, $y = b + c \sin \theta$ $\Rightarrow$ $\cos \theta = \dfrac{x-a}{c}$, $\sin \theta = \dfrac{y-b}{c}$

$\Rightarrow$ $\left(\dfrac{x-a}{c}\right)^2 + \left(\dfrac{y-b}{c}\right)^2 = \cos^2\theta + \sin^2\theta$ $\Rightarrow$ $(x-a)^2 + (y-b)^2 = c^2$

Clearly, it is a circle with centre at (a, b) and radius c.

Question

Discuss the posi tion of the points (1, 2) and (6, 0) with respect to the circle
$X^2 + y^2 - 4x + 2y - 11 = 0$

Solution : We have $x^2 + y^2 - 4x + 2y - 11 = 0$ or S = 0, where S = $x^2 + y^2 - 4x + 2y - 11$.
For the point (1, 2), we have $S_1 = 1^2 + 2^2 - 4 \times 1 + 2 \times 2 - 11 < 0$
For the point (6, 0), we have $S_2 = 6^2 + 0^2 - 4 \times 6 + 2 \times 0 - 11 > 0$
Hence, the point (1, 2) lies inside the circle and the point (6, 0) lies outside the circle.

Line and Circle combined Problems

Let L = 0 be a line & S = 0 be a circle. If r is the radius of the circle & p is the length of the perpendicular from the centre on the line, then:
(i)   p > r   ⇔   the line does not meet the circle i. e. passes out side the circle.
(ii)   p = r   ⇔   the line touches the circle. (It is tangent to the circle)
(iii)   p < r   ⇔   the line is a secant of the circle.
(iv)   p = 0   ⇒   the line is a diameter of the circle.
Also, if y = mx + c is line and $x^2 + y^2 = a^2$ is circle then
(i) $c^2 > a^2 (1 + m^2)$ ⇔   the line is a secant of the circle.
(ii) $c^2 = a^2 (1 + m^2)$ ⇔   the line touches the circle. (It is tangent to the circle)
(iii) $c^2 < a^2 (1 + m^2)$ ⇔   the line does not meet the circle i. e. passes out side the circle.

Question

For what value of c will the line y = 2x + c be a tangent to the circle $x^2 + y^2 = 5$?
We have : y = 2x + c or 2x − y + c = 0 ......(i)   and $x^2 + y^2 = 5$ ....(ii)
If the line (i) touches the circle (ii), then
length of the ⊥ from the centre (0, 0) = radius of circle (ii)

$\Rightarrow$ $\left|\dfrac{2\times 0 - 0 + c}{\sqrt{2^2 + (-1)^2}}\right| = \sqrt{5}$ $\Rightarrow$ $\left|\dfrac{c}{\sqrt{5}}\right| = \sqrt{5}$

$\Rightarrow$ $\dfrac{c}{\sqrt{5}} = \pm \sqrt{5}$ $\Rightarrow$ c = ± 5

Hence, the line (i) touches the circle (ii) for c = ± 5

Tangent of Circle

(a) Slope form :
$y = mx + c$ is always a tangent to the circle $x^2 + y^2 = a^2$ if $c^2 = a^2(1 + m^2)$. Hence, equation

of tangent is $y = mx \pm a\sqrt{1+m^2}$ and the point of contact is $\left(-\dfrac{a^2 m}{c}, \dfrac{a^2}{c}\right)$.

(b) Point form :
   (i) The equation of the tangent to the circle $x^2 + y^2 = a^2$ at its point (x_1, y_1) is,
      $x x_1 + y y_1 = a^2$.
   (ii) The equation of the tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at its point
      (x, y_1) is: $xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$.

In general the equation of tangent to any second degree curve at point (x_1, y_1) on it can be obtained by

replacing x^2 by $x x_1$, y^2 by yy_1, x by $\dfrac{x+x_1}{2}$, y by $\dfrac{y+y_1}{2}$, xy by $\dfrac{x_1 y + x y_1}{2}$ and c remains as c.

(c) Parametric form :
   The equation of a tangent to circle $x^2 + y^2 = a^2$ at $(a\cos\alpha, a\sin\alpha)$ is
   $x\cos\alpha + y\sin\alpha = a$.

The point of intersection of the tangents at the points $P(\alpha)$ & $Q(\beta)$ is $\left(\dfrac{a\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}, \dfrac{a\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}\right)$

Question

Find the equation of the tangent to the circle $x^2 + y^2 - 30x + 6y + 109 = 0$ at $(4, -1)$
Equation of tangent is

$$4x + (-y) - 30\left(\frac{x+4}{2}\right) + 6\left(\frac{y + (-1)}{2}\right) + 109 = 0$$

or $4x - y - 15x - 60 + 3y - 3 + 109 = 0$ or $-11x + 2y + 46 = 0$
or $11x - 2y - 46 = 0$
Hence, the required equation of the tangent is $11x - 2y - 46 = 0$

Question

Find the equation of tangents to the circle $x2 + y2 - 6x + 4y - 12 = 0$ which are parallel to the line $4x + 3y + 5 = 0$

Given circle is $x2 + y2 - 6x + 4y - 12 = 0$ ....(i)
and given line is $4x + 3y + 5 = 0$ ....(ii)

Centre of circle (i) is (3, -2) and its radius is 5. Equation of any line
$4x + 3y + k = 0$ (iii) is parallel to the line (ii)
If line (iii) is tangent to circle, (i) then

$$\frac{|4(3) + 3(-2) + k|}{\sqrt{4^2 + 3^2}} = 5 \text{ or } |6 + k| = 25$$

Or $6 + k = \pm 25$ or $k = 19$ or -31

Hence equation of required tangents are $4x + 3y + 19 = 0$ and $4x + 3y - 31 = 0$

Normal

If a line is normal / orthogonal to a circle then it must pass through the centre of the circle. Using this fact normal to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) is; $y - y_1 = \dfrac{y_1 + f}{x_1 + g} (x - x_1)$.

Find the equation of the normal to the circle $x^2 + y^2 - 5x + 2y - 48 = 0$ at the point $(5, 6)$.
The equation of the tangent to the circle $x^2 + y^2 - 5x + 2y - 48 = 0$ at $(5, 6)$ is

$$5x + 6y - 5\left(\frac{x+5}{2}\right) + 2\left(\frac{x+6}{2}\right) - 48 = 0 \Rightarrow 10x + 12y - 5x - 25 + 2y + 12 - 96 = 0$$

$$\Rightarrow 5x + 14y - 109 = 0$$

$\therefore$ Slope of the tangent $= -\dfrac{5}{14}$ $\Rightarrow$ Slope of the normal $= \dfrac{14}{5}$

Hence, the equation of the normal at $(5, 6)$ is
$y - 6 = (14/5) (x - 5)$ $\Rightarrow$ $14x - 5y - 40 = 0$

Pair of Tangents from a Point

The equation of a pair of tangents drawn from the point A (x_1, y_1) to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is : $SS_1 = T^2$.

Where
$S \equiv x^2 + y^2 + 2gx + 2fy + c$; $S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$
$T \equiv xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c$.

Question

Find the equation of the pair of tangents drawn to the circle $x^2 + y^2 - 2x + 4y = 0$ from the point $(0, 1)$
Given circle is $S = x^2 + y^2 - 2x + 4y = 0$ ......(i)
Let $P \equiv (0, 1)$
For point P, $S_1 = 0^2 + 1^2 - 2.0 + 4.1 = 5$
Clearly P lies outside the circle
and $T \equiv x . 0 + y . 1 - (x + 0) + 2 (y + 1)$
i.e. $T \equiv -x + 3y + 2$.
Now equation of pair of tangents from P(0, 1) to circle (1) is $SS_1 = T^2$
or $5 (x^2 + y^2 - 2x + 4y) = (-x + 3y + 2)^2$
or $5x^2 + 5y^2 - 10x + 20y = x^2 + 9y^2 + 4 - 6xy - 4x + 12y$
or $4x^2 - 4y^2 - 6x + 8y + 6xy - 4 = 0$
or $2x^2 - 2y^2 + 3xy - 3x + 4y - 2 = 0$ ......(ii)
Note : Separate equation of pair of tangents : From (ii), $2x^2 + 3(y - 1) x - 2(2y^2 - 4y + 2) = 0$

$\therefore$ $x = \dfrac{3(y-1) \pm \sqrt{9(y-1)^2 + 8(2y^2 - 4y + 2)}}{4}$

or $4x - 3y + 3 = \pm \sqrt{25y^2 - 50y + 25} = \pm 5(y - 1)$

$\therefore$ Separate equations of tangents are $x - 2y + 2 = 0$ and $2x + y - 1 = 0$

Length of a Tangent and Power of a Point

The length of a tangent from an external point (x_1, y_1) to the circle

$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ is given by $L = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2f_1y + c} = \sqrt{S_1}$.

Square of length of the tangent from the point P is also called the power of point w.r.t. a circle.
Power of a point w.r.t. a circle remains constant.
Power of a point P is positive, negative or zero according as the point 'P' is outside, inside or on the circle respectively.

Question

Find the length of the tangent drawn from the point (5, 1) to the circle $x^2 + y^2 + 6x – 4y –3= 0$

Now length of the tangent from P(5, 1) to circle (i) = $\sqrt{(5^2 + 1^2 + 6 \times 5 - 4 (1) - 3)} = 7$

Director Circle

The locus of the point of intersection of two perpendicular tangents is called the director circle of the given circle. The director circle of a circle is the concentric circle having radius equal to 2 times the original circle.

Question

Find the equation of director circle of the circle $(x – 2)^2 + (y + 1)^2 = 2$

So Radius is $\sqrt{2}$ as 2 can be written as $(\sqrt{2})^2$

The centre is (2, -1)

radius of the director circle will be $\sqrt{2} \times \sqrt{2} = 2$

Thus the equation of the director circle will be $(x – 2)^2 + (y + 1)^2 = 2^2 = 4$

$x^2 + y^2 – 4x + 2y + 1 = 0$

Chord of Contact

If two tangents PT_1 & PT_2 are drawn from the point $P(x_1, y_1)$ to the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, then the equation of the chord of contact T_1T_2 is: $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$.
NOTE : Here R = radius; L = length of tangent.
(a)   Chord of contact exists only if the point 'P' is not inside.
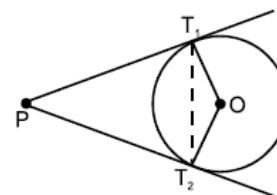
(b)   Length of chord of contact $T_1T_2 = \dfrac{2LR}{\sqrt{R^2 + L^2}}$.

(c)   Area of the triangle formed by the pair of the tangents & its chord of contact $= \dfrac{RL^3}{R^2 + L^2}$

(d)   Tangent of the angle between the pair of tangents from $(x_1, y_1) = \left(\dfrac{2RL}{L^2 - R^2} \right)$

(e)   Equation of the circle circumscribing the triangle PT_1T_2 is:
$(x - x_1)(x + g) + (y - y_1)(y + f) = 0$.

Question

Find the equation of the chord of contact of the tangents drawn from (1, 2) to the circle
$x^2 + y^2 - 2x + 4y + 7 = 0$
Given circle is $x^2 + y^2 - 2x + 4y + 7 = 0$ ....(i)
Let $P = (1, 2)$
For point P (1, 2), $x^2 + y^2 - 2x + 4y + 7 = 1 + 4 - 2 + 8 + 7 = 18 > 0$
Hence point P lies outside the circle
For point P (1, 2), $T = x \cdot 1 + y \cdot 2 - (x + 1) + 2(y + 2) + 7$
i.e. $T = 4y + 10$
Now equation of the chord of contact of point $P(1, 2)$ w.r.t. circle (i) will be
$4y + 10 = 0$ or $2y + 5 = 0$

Tangents are drawn to the circle $x^2 + y^2 = 12$ at the points where it is met by the circle
$x^2 + y^2 - 5x + 3y - 2 = 0$; find the point of intersection of these tangents.
Given circles are $S_1 \equiv x^2 + y^2 - 12 = 0$ .... (i)
and $S_2 = x^2 + y^2 - 5x + 3y - 2 = 0$ .... (ii)
Now equation of common chord of circle (i) and (ii) is
$S_1 - S_2 = 0$ i.e. $5x - 3y - 10 = 0$ .... (iii)
Let this line meet circle (i) [or (ii)] at A and B
Let the tangents to circle (i) at A and B meet at $P(\alpha, \beta)$, then AB will be the chord of contact of the tangents to the circle (i) from P, therefore equation of AB will be
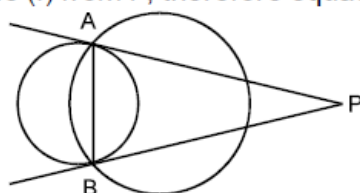
$x\alpha + y\beta - 12 = 0$ .... (iv)
Now lines (iii) and (iv) are same, therefore, equations (iii) and (iv) are identical

$\therefore$ $\dfrac{\alpha}{5} = \dfrac{\beta}{-3} = \dfrac{-12}{-10}$ $\therefore$ $\alpha = 6, \beta = -\dfrac{18}{5}$

Hence $P = \left(6, -\dfrac{18}{5} \right)$

Pole and Polar

(i) If through a point P in the plane of the circle, there be drawn any straight line to meet the circle in Q and R, the locus of the point of intersection of the tangents at Q & R is called the Polar of the point P; also P is called the Pole of the Polar.

(ii) The equation to the polar of a point P (x1, y1) w.r.t. the circle x2 + y2 = a2 is given by xx1 + yy1 = a2, & if the circle is general then the equation of the polar becomes xx1 + yy1 + g (x + x1) + f (y + y1) + c = 0 i.e. T = 0. Note that if the point (x1, y1) be on the circle then the tangent & polar will be represented by the same equation. Similarly if the point (x1, y1) be outside the circle then the chord of contact & polar will be represented by the same equation.

Pole of a given line Ax + By + C = 0 w.r.t. circle $x^2 + y^2 = a^2$ is $\left(-\dfrac{Aa^2}{C}, -\dfrac{Ba^2}{C} \right)$.

If the polar of a point P pass through a point Q then the polar of Q passes through P.

Two lines L_1 & L_2 are conjugate of each other if Pole of L_1 lies on L_2 & vice versa. Similarly two points P & Q are said to be conjugate of each other if the polar of P passes through Q & vice-versa.

Question

Find the equation of the polar of the point (2, −1) with respect to the circle
$x^2 + y^2 - 3x + 4y - 8 = 0$
Given circle is $x^2 + y^2 - 3x + 4y - 8 = 0$ ....(i)
Given point is (2, −1) let P = (2, −1). Now equation of the polar of point P with respect to circle (i)

$x.2 + y(-1) - 3\left(\dfrac{x+2}{2} \right) + 4\left(\dfrac{y-1}{2} \right) - 8 = 0$

or $\qquad 4x - 2y - 3x - 6 + 4y - 4 - 16 = 0 \qquad$ or $\qquad x + 2y - 26 = 0$

Question

Find the pole of the line 3x + 5y + 17 = 0 with respect to the circle $x^2 + y^2 + 4x + 6y + 9 = 0$
Given circle is $x^2 + y^2 + 4x + 6y + 9 = 0$ ....(i)
and given line is 3x + 5y + 17 = 0 ....(ii)
Let P(α, β) be the pole of line (ii) with respect to circle (i)
Now equation of polar of point P(α, β) with respect to circle (i) is
$\qquad x\alpha + y\beta + 2(x + \alpha) + 3(y + \beta) + 9 = 0$
or $\qquad (\alpha + 2)x + (\beta + 3)y + 2\alpha + 3\beta + 9 = 0$ ....(iii)
Now lines (ii) and (iii) are same, therefore,

$$\frac{\alpha + 2}{3} = \frac{\beta + 3}{5} = \frac{2\alpha + 3\beta + 9}{17}$$

(i)   (ii)   (iii)

From (i) and (ii), we get

$5\alpha + 10 = 3\beta + 9$ or $5\alpha - 3\beta = -1$ ....(iv)

From (i) and (iii), we get

$17\alpha + 34 = 6\alpha + 9\beta + 27$ or $11\alpha - 9\beta = -7$ ....(v)

Solving (iv) & (v), we get $\alpha = 1$, $\beta = 2$ Hence required pole is (1, 2).

Equation of the Chord with a given Middle Point

The equation of the chord of the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ in terms of its mid point M (x_1, y_1) is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ which is designated by $T = S_1$.

(i)   The shortest chord of a circle passing through a point 'M' inside the circle· is one chord whose middle point is M.

(ii)   The chord passing through a point 'M' inside the circle and which is at a maximum distance from the centre is a chord with middle point M.

Question

Find the equation of the chord of the circle $x^2 + y^2 + 6x + 8y - 11 = 0$, whose middle point is $(1, -1)$

Equation of given circle is $S \equiv x^2 + y^2 + 6x + 8y - 11 = 0$

Let $L \equiv (1, -1)$

For point L(1, -1), $S_1 = 1^2 + (-1)^2 + 6.1 + 8(-1) - 11 = -11$ and

$T \equiv x.1 + y(-1) + 3(x + 1) + 4(y - 1) - 11$ i.e. $T \equiv 4x + 3y - 12$

Now equation of the chord of circle (i) whose middle point is L(1, -1) is

$T = S_1$ or $4x + 3y - 12 = -11$ or $4x + 3y - 1 = 0$

Alternate method

The center is C (-3, -4) the point L is (1, -1)

Slope of CL is (-4 + 1) / (-3 -1) = ¾

Slope of the perpendicular line to CL is -4/3

So equation of the line is y + 1 = (-4/3) (x - 1)
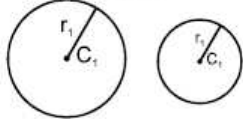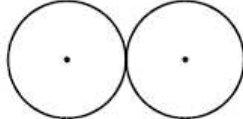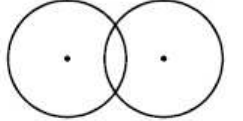
Or 4x + 3y – 1 = 0

Equation of the chord joining two points of circle

The equation of chord PQ to the circle $x^2 + y^2 = a^2$ joining two parametric points P(α) and Q(ß) on it is given by. Where P(α) is (aCos α, a Sin α) The equation of a straight line joining two point α & ß on the circle $x^2 + y^2 = a^2$ is

X Cos (α + ß)/2   + y Sin (α + ß)/2 = a Cos (α - ß)/2

Common Tangents to two Circles

Case		Number of Tangents	Condition
(i)		4 common tangents (2 direct and 2 transverse)	$r_1 + r_2 < c_1 c_2$.
(ii)		3 common tangents.	$r_1 + r_2 = c_1 c_2$.
(iii)		2 common tangents.	$\lvert r_1 - r_2 \rvert < c_1 c_2 < r_1 + r_2$
(iv)		1 common tangent.	$\lvert r_1 - r_2 \rvert = c_1 c_2$.
(v)		No common tangent.	$c_1 c_2 < \lvert r_1 - r_2 \rvert$.

Important Note

(i) The direct common tangents meet at a point which divides the line joining centre of circles externally in the ratio of their radii.
Transverse common tangents meet at a point which divides the line joining centre of circles internally in the ratio of their radii.

(ii) Length of an external (or direct) common tangent & internal (or transverse) common tangent to the two circles are given by: $L_{ext} = \sqrt{d^2 - (r_1 - r_2)^2}$ & $L_{int} = \sqrt{d^2 - (r_1 + r_2)^2}$,

where d = distance between the centres of the two circles and r_1, r_2 are the radii of the two circles. Note that length of internal common tangent is always less than the length of the external or direct common tangent.

Question

Examine if the two circles $x^2 + y^2 - 2x - 4y = 0$ and $x^2 + y^2 - 8y - 4 = 0$ touch each other externally or internally.
Given circles are $x^2 + y^2 - 2x - 4y = 0$ ....(i)
and $x^2 + y^2 - 8y - 4 = 0$ ....(ii)
Let A and B be the centres and r_1 and r_2 the radii of circles (i) and (ii) respectively, then
$A \equiv (1, 2)$, $B \equiv (0, 4)$, $r_1 = \sqrt{5}$, $r_2 = 2\sqrt{5}$

Now AB $= \sqrt{(1-0)^2 + (2-4)^2} = \sqrt{5}$ and $r_1 + r_2 = 3\sqrt{5}$, $\lvert r_1 - r_2 \rvert = \sqrt{5}$
Thus AB $= \lvert r_1 - r_2 \rvert$, hence the two circles touch each other internally.

Orthogonality Of Two Circles

Two circles S1= 0 & S2= 0 are said to be orthogonal or said to intersect orthogonally if the tangents at their point of intersection include a right angle. The condition for two circles to be orthogonal is:
$2 g_1 g_2 + 2 f_1 f_2 = c_1 + c_2$

(a) The centre of a variable circle orthogonal to two fixed circles lies on the radical axis of two circles.
(b) If two circles are orthogonal, then the polar of a point 'P' on first circle w.r.t. the second circle passes through the point Q which is the other end of the diameter through P. Hence locus of a point which moves such that its polars w.r.t. the circles S1 = 0, S2 = 0 & S3 = 0 are concurrent in a circle which is orthogonal to all the three circles.
(c) The centre of a circle which is orthogonal to three given circles is the radical centre provided the radical centre lies outside all the three circles.

Question

Obtain the equation of the circle orthogonal to both the circles $x2 + y2 + 3x – 5y+ 6 = 0$ … (i) and $4x^2 + 4y^2 – 28x +29 = 0$ …. (ii) and whose centre lies on the line $3x + 4y + 1 = 0$

Let the required circle be $x2 + y2 + 2gx + 2fy + c = 0$ ....(iii)
Since circle (iii) cuts circles (i) and (ii) orthogonally

$2g (3/2) + 2f (-5/2) = c + 6$
Or $3g – 5f = c + 6$ …. (iv)

Also $2g (-7/2) + 2f(0) = c + 29/4$ or $– 7g = c + 29/4$ …. (v)

or $40g – 20f = – 5.$

Or line equation is $3x + 4y = – 1$

we get $g = 0, f = ¼$

so $c = -29/4$

required circle equation is $4(x2 + y2) + 2y – 29 = 0$

Radical Axis and Radical Centre:

The radical axis of two circles is the locus of points whose powers w.r.t. the two circles are equal. The equation of radical axis of the two circles S1 = 0 & S2 = 0 is given by
S1 - S2 = 0 i.e. 2 (g1 - g2) x + 2 (f1 - f2) y + (c1 - c2) = 0.
The common point of intersection of the radical axes of three circles taken two at a time is called the radical centre of three circles. Note that the length of tangents from radical centre to the three circles are equal.
NOTE:
(a) If two circles intersect, then the radical axis is the common chord of the two circles.
(b) If two circles touch each other then the radical axis is the common tangent of the two circles at the common point of contact.
(c) Radical axis is always perpendicular to the line joining the centres of the two circles.

(d) Radical axis will pass through the mid point of the line joining the centres of the two circles
only if the two circles have equal radii.
(e) Radical axis bisects a common tangent between the two circles.
(f) A system of circles, every two which have the same radical axis, is called a coaxial system.
(g) Pairs of circles which do not have radical axis are concentric.

Question

Find the co-ordinates of the point from which the lengths of the tangents to the following three circles be equal.
$$3x^2 + 3y^2 + 4x - 6y - 1 = 0$$
$$2x^2 + 2y^2 - 3x - 2y - 4 = 0$$
$$2x^2 + 2y^2 - x + y - 1 = 0$$
Here we have to find the radical centre of the three circles. First reduce them to standard form in which coefficients of x^2 and y^2 be each unity. Subtracting in pairs the three radical axes are

$$\frac{17}{6}x - y + \frac{5}{3} = 0 \quad ; \quad -x - \frac{3}{2}y - \frac{3}{2} = 0$$

$$-\frac{11}{6}x + \frac{5}{2}y - \frac{1}{6} = 0.$$

solving any two, we get the point (-16/21, 31/63)
which satisfies the third also. This point is called the radical centre and by definition the length of the tangents from it to the three circles are equal.

Family of Circles

(a) The equation of the family of circles passing through the points of intersection of two circles S1 = 0 & S2 = 0 is : S1 + K S2 = 0
(K is not -1, provided the coefficient of x^2 & $y^2$2 in S1 & S2 are same)

(b) The equation of the family of circles passing through the point of intersection of a circle S = 0 & a line L = 0 is given by S + KL = 0.

(c) The equation of a family of circles passing through two given points (x1, y1) & (x2, y2) can be written in the form:

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + K \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0 \text{ where K is a parameter.}$$

(d)   The equation of a family of circles touching a fixed line $y - y_1 = m(x - x_1)$ at the fixed point (x_1, y_1) is $(x - x_1)^2 + (y - y_1)^2 + K[y - y_1 - m(x - x_1)] = 0$, where K is a parameter.
(e)   Family of circles circumscribing a triangle whose sides are given by $L_1 = 0$; $L_2 = 0$ and $L_3 = 0$ is given by; $L_1L_2 + \lambda L_2L_3 + \mu L_3L_1 = 0$ provided co-efficient of xy = 0 and co-efficient of x^2 = co-efficient of y^2.
(f)   Equation of circle circumscribing a quadrilateral whose side in order are represented by the lines $L_1 = 0$, $L_2 = 0$, $L_3 = 0$ & $L_4 = 0$ are $u L_1L_3 + \lambda L_2L_4 = 0$ where values of u & λ can be found out by using condition that co-efficient of x^2 = co-efficient of y^2 and co-efficient of xy = 0.

Question

Find the equations of the circles passing through the points of intersection of the circles $x^2 + y^2 - 2x - 4y - 4 = 0$ and $x^2 + y^2 - 10x - 12y + 40 = 0$ and whose radius is 4.

Any circle through the intersection of given circles is $S_1 + \lambda S_2 = 0$

or $\quad (x^2 + y^2 - 2x - 4y - 4) + l(x^2 + y^2 - 10x - 12y + 40) = 0$

or $\quad (x^2 + y^2) - 2\dfrac{(1+5\lambda)}{1+\lambda} x - 2\dfrac{(2+6\lambda)}{1+\lambda} y + \dfrac{40\lambda - 4}{1+\lambda} = 0$ ....(i)

$r = \sqrt{g^2 + f^2 - c} = 4$, given

$\therefore \quad 16 = \dfrac{(1+5\lambda)^2}{(1+\lambda)^2} + \dfrac{(2+6\lambda)^2}{(1+\lambda)^2} - \dfrac{40\lambda - 4}{1+\lambda}$

$16(1 + 2\lambda + \lambda^2) = 1 + 10\lambda + 25\lambda^2 + 4 + 24\lambda + 36\lambda^2 - 40\lambda^2 - 40\lambda + 4 + 4\lambda$

or $\quad 16 + 32\lambda + 16\lambda^2 = 21\lambda^2 - 2\lambda + 9 \quad$ or $\quad 5\lambda^2 - 34\lambda - 7 = 0$

$\therefore \quad (\lambda - 7)(5\lambda + 1) = 0 \qquad\qquad \therefore \quad \lambda = 7, -1/5$

Putting the values of λ in (i) the required circles are

$2x^2 + 2y^2 - 18x - 22y + 69 = 0 \qquad$ and $\qquad x^2 + y^2 - 2y - 15 = 0$

Question

Find the equations of circles which touche $2x - y + 3 = 0$ and pass through the points of intersection of the line $x + 2y - 1 = 0$ and the circle $x^2 + y^2 - 2x + 1 = 0$.

The required circle by $S + \lambda P = 0$ is

$x^2 + y^2 - 2x + 1 + \lambda(x + 2y - 1) = 0$

or $\quad x^2 + y^2 - x(2 - \lambda) + 2\lambda y + (1 - \lambda) = 0$

centre $(-g, -f)$ is $[(2 - \lambda)/2, -\lambda]$

$r = \sqrt{g^2 + f^2 - c}$

$= \sqrt{(2-\lambda)^2/4 + \lambda^2 - (1-\lambda)} \quad = \dfrac{1}{2}\sqrt{5\lambda^2} = (\lambda/2)\sqrt{5}$.

Since the circle touches the line $2x - y + 3 = 0$ therefore perpendicular from centre is equal to

radius $\dfrac{2.[(2-\lambda)/2] - (-\lambda) + 3}{\pm\sqrt{5}} = \dfrac{\lambda}{2}\sqrt{5}$. $\quad$ or $\quad 5 = \pm\dfrac{\lambda}{2}.5 \quad \therefore \quad \lambda = \pm 2$

Puttin the values of λ in (i) the required circles are

$x^2 + y^2 + 4y - 1 = 0$ and $\qquad x^2 + y^2 - 4x - 4y + 3 = 0$

Question

Find the equation of circle pasing through the points $A(1, 1)$ & $B(2, 2)$ and whose radiu is 1

Equation of AB is $x - y = 0 \qquad \therefore \qquad$ equation of circle is

$(x - 1)(x - 2) + (y - 1)(y - 2) + \lambda(x - y) = 0 \quad$ or $\quad x^2 + y^2 + (\lambda - 3)x - (\lambda + 3)y + 4 = 0$

radius $= \sqrt{\dfrac{(\lambda - 3)^2}{4} + \dfrac{(\lambda + 3)^2}{4} - 4}$

But radius = 1 (given) $\qquad \therefore \qquad \sqrt{\dfrac{(\lambda - 3)^2}{4} + \dfrac{(\lambda + 3)^2}{4} - 4} = 1$

or $\quad (\lambda - 3)^2 + (\lambda + 3)^2 - 16 = 4.\quad$ or $\quad 2\lambda^2 = 2 \qquad$ or $\qquad \lambda = \pm 1$

$\therefore \qquad$ equation of circle is

$x^2 + y^2 - 2x - 4y + 4 = 0 \quad \& \quad x^2 + y^2 - 4x - 2y + 4 = 0$

Question

Find the equation of the circle passing through the point (2, 1) and touching the line x + 2y – 1 = 0 at the point (3, – 1)

Equation of circle is
$(x – 3)^2 + (y + 1)^2 + λ (x + 2y – 1) = 0$
Since it passes through the point (2, 1)
1 + 4 + λ (2 + 2 – 1) = 0
λ = -5/3

So equation of circle is $(x – 3)^2 + (y + 1)^2 - (5/3) ((x + 2y – 1) = 0$

⇨ $3x^2 + 3y^2 – 23x – 4y + 35 = 0$

:-{D

Find the equation of the circle with centre (1, 1) and radius $\sqrt{2}$

Answer

The equation of a circle with centre (h, k) and radius r is given as

$(x – h)^2 + (y – k)^2 = r^2$

It is given that centre (h, k) = (1, 1) and radius (r) = $\sqrt{2}$.

Therefore, the equation of the circle is

$$\left(x-1\right)^2 +\left(y-1\right)^2 =\left(\sqrt{2}\right)^2$$
$$x^2 -2x+1+ y^2 -2y+1=2$$
$$x^2 + y^2 -2x-2y = 0$$

Question

Find the equation of a circle with centre (2, 2) and passes through the point (4, 5).

Answer

The centre of the circle is given as (h, k) = (2, 2).

Since the circle passes through point (4, 5), the radius (r) of the circle is the distance between the points (2, 2) and (4, 5).

$$\therefore r = \sqrt{\left(2-4\right)^2 +\left(2-5\right)^2} = \sqrt{\left(-2\right)^2 +\left(-3\right)^2} = \sqrt{4+9} = \sqrt{13}$$

Thus, the equation of the circle is

$$(x-h)^2+(y-k)^2=r^2$$

$$(x-2)^2+(y-2)^2=\left(\sqrt{13}\right)^2$$

$$x^2-4x+4+y^2-4y+4=13$$

$$x^2+y^2-4x-4y-5=0$$

Question

Does the point (−2.5, 3.5) lie inside, outside or on the circle $x^2 + y^2 = 25$?

Answer

The equation of the given circle is $x^2 + y^2 = 25$.

$x^2 + y^2 = 25$

$\Rightarrow (x - 0)^2 + (y - 0)^2 = 5^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 0$, $k = 0$, and $r = 5$.

∴ Centre = (0, 0) and radius = 5

Distance between point (−2.5, 3.5) and centre (0, 0)

$$=\sqrt{(-2.5-0)^2+(3.5-0)^2}$$

$$=\sqrt{6.25+12.25}$$

$$=\sqrt{18.5}$$

$$=4.3 \text{ (approx.)} < 5$$

Since the distance between point (−2.5, 3.5) and centre (0, 0) of the circle is less than the radius of the circle, point (−2.5, 3.5) lies inside the circle.

Equation of the circle passing through points A(x1,y1), B(x2,y2) and C(x3,y3) is

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ x_1^2+y_1^2 & x_1 & y_1 & 1 \\ x_2^2+y_2^2 & x_2 & y_2 & 1 \\ x_3^2+y_3^2 & x_3 & y_3 & 1 \end{vmatrix} = 0$$

Question

If equation $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ represent a circle then condition is

(a) $a = b$, $c = 0$ (b) $f = g$, $h = 0$

(c) $a = b$, $h = 0$ (d) $f = g$, $c = 0$.

                         (MNR 1979)

Ans. (c)

If quadratic equation represents circle, then

$$a = b \text{ and } h = 0$$

Question

The area of a circle whose centre is $(1, 2)$ and passes through $(4, 6)$ is

(a) 5π (b) 10π

(c) 25π (d) 15π. **(MNR 1982)**

Ans. (c)

Radius of circle $r =$

$$\sqrt{(4 - 1)^2 + (6 - 2)^2} = 5$$

Area of circle $\quad = \pi(r)^2 = \pi(5)^2 = 25\pi$.

Question

Length of tangent drawn from $(5, 1)$ to the circle $x^2 + y^2 + 6x - 4y - 3 = 0$ is

(a) 81   (b) 29

(c) 7   (d) 21.   **(MNR 1981)**

Ans. (c)

The length of tangent drawn from the point $p(5, 1)$ to circle $x^2 + y^2 + 6x - 4y - 3 = 0$ is

$$PT = \sqrt{25 + 1 + 30 - 4 - 3} = 7.$$

Question

Given two circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$, the equation of circle passing through intersection of circles and point $(1, 1)$ is

(a) $x^2 + y^2 - 6x + 4 = 0$
(b) $x^2 + y^2 - 3x + 1 = 0$
(c) $x^2 + y^2 - 4y + 2 = 0$
(d) $x^2 + y^2 - 3x - 1 = 0$.   (IIT 1980)

Ans. (b)

The equation of the circle passing through intersection of two circles is $x^2 + y^2 - 6x + 8 + \lambda(x^2 + y^2 - 6) = 0$ which passes through $(1, 1)$

$\therefore$ $1 + 1 - 6 + 8 + \lambda(1 + 1 - 6) = 0$

$\Rightarrow$ $4\lambda = 4 \Rightarrow \lambda = 1$

Required equation is $2(x^2 + y^2) - 6x + 2 = 0$

or $x^2 + y^2 - 3x + 1 = 0$

Question

A square is drawn in the circle $x^2 + y^2 - 2x + 4y + 3 = 0$ of which sides are parallel to coordinate axes, then vertices of the square are

(a) $\left(1 + \sqrt{2}, -2\right)$ (b) $\left(1 - \sqrt{2}, -2\right)$

(c) $\left(1, -2 + \sqrt{2}\right)$ (d) none of these.

     (IIT 1980)

Ans. (d)

The centre of the circle $x^2 + y^2 - 2x + 4y + 3 = 0$ is $(1, -2)$ and its radius $r = \sqrt{1 + 4 - 3} = \sqrt{2}$.

Question

The locus of mid point of the chord of the circle $x^2 + y^2 = 2x - 2y - 2 = 0$ which subtands an angle 120° at the centre, is

(a) $x^2 + y^2 - 2x - 2y + 1 = 0$

(b) $x^2 + y^2 + 2x + 2y + 1 = 0$

(c) $x^2 + y^2 + x + y + 1 = 0$

(d) $x^2 + y^2 - x - y - 1 = 0.$ (MNR 1994)

Ans. (a)

The centre of the circle $x^2 + y^2 - 2x - 2y - 2 = 0$ is (1, 1) and $r = 2$
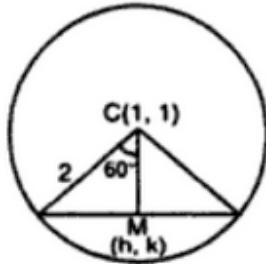
$$CM = 2 \cos 60° = 2 \cdot \frac{1}{2} = 1$$

$$\Rightarrow \quad (h - 1)^2 + (k - 1)^2 = 1$$

$$\Rightarrow \quad h^2 + k^2 - 2h - 2k + 1 = 0$$

∴ The locus is $x^2 + y^2 - 2x - 2y + 1 = 0$

Question

The locus of centre of the circle which touches the circle $x^2 + y^2 - 6x - 6y - 14 = 0$ externally and touches y-axis, is

(a) $x^2 - 6x - 10y + 14 = 0$

(b) $x^2 - 10x - 6y + 14 = 0$

(c) $y^2 - 6x - 10y + 14 = 0$

(d) $y^2 - 10r - 6x + 14 = 0.$ (IIT 1993)

Ans. (d)

Let $C_1(h, k)$ be the centre. Since circle touches
y-axis $\therefore$ radius $r_1 = h$
The centre of the circle $x^2 + y^2 - 6x - 6y + 14$
$= 0$ is $C_2(3, 3)$ and its
radius $r_2 = \sqrt{9 + 9 - 14} = 2$
Since desired circle touches the given circle
externally

$$C_1C_2 = r_1 + r_2$$

$\therefore \qquad \sqrt{(h-3)^2 + (k-3)^2} = h + 2$

$\Rightarrow \qquad (h-3)^2 + (k-3)^2 = (h+2)^2$

$\Rightarrow \quad h^2 + 9 - 6h + k^2 + 9 - 6k = h^2 + 4h + 4$

$\Rightarrow \qquad k^2 - 10h - 6k + 14 = 0$

$\therefore$ The locus is $y^2 - 10x - 6y + 14 = 0$

Question

The centre of the circle which passes through
$(0, 1)$ and touches the curve $y = x^2$ at $(2, 4)$, is

(a) $\left(\dfrac{-16}{5}, \dfrac{27}{10}\right)$ (b) $\left(\dfrac{-16}{7}, \dfrac{5}{10}\right)$

(c) $\left(\dfrac{-16}{5}, \dfrac{53}{10}\right)$ (d) none of these.

(IIT 1983)

Ans. (c)

Let (h, k) be the centre of the circle

$\therefore \quad (h - 0)^2 + (k - 1)^2 = (h - 2)^2 + (k - 2)^2$

$\Rightarrow \qquad \qquad 4h + 9k = 19 \qquad \qquad ...(i)$

The equation to the tangent to parabola $y = x^2$ at $(2, 4)$ is

$$2 \cdot x = \frac{1}{2}(y + 4) \Rightarrow 4x - y - 4 = 0$$

which also touches circle and perpendicular to it will pass through centre.

$\therefore \qquad \qquad x + 4y = \lambda$

$\Rightarrow \qquad \qquad 2 + 16 = \lambda$

$\Rightarrow \qquad \qquad \lambda = 18$

Centre (h, k) lies on normal $x + 4y = 18$

$\qquad \qquad h + 4k = 18 \qquad \qquad ...(ii)$

Solving (i) and (ii) centre is $\left(-\dfrac{16}{5}, \dfrac{53}{10} \right)$

Question

AB is a diameter of a circle and C is any point on the circle, then

(a) The area of $\triangle$ ABC is maximum if it is an isosceles triangle

(b) The area of $\triangle$ ABC is minimum if it is an isosceles triangle

(c) The perimater of $\triangle$ABC is maximum if it is an isosceles triangle

(d) none of these.   (IIT 1983)

Ans. (a)

In semicircle the area of triangle is maximum
if its height is maximum
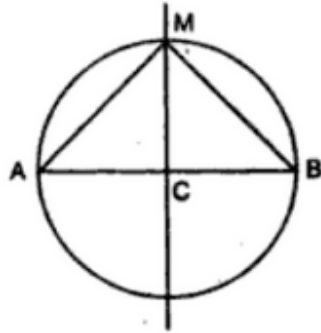
this is perpendicular bisector of AB.

∴ Triangle is an isosceles triangle.

Question

Circle $x^2 + y^2 + 4x - 7y + 12 = 0$ cuts intercept on y-axis, then intercept is equal to

(a) 1   (b) 3
(c) 4   (d) 7.

Ans. (a)

On y-axis $x = 0$

∴ $y^2 - 7y + 12 = 0$

⟹ $(y - 4)(y - 3) = 0$

⟹ $y = 3, 4$

∴   intercepts on y-axis = $|y_1 - y_2|$

                    = $|4 - 3| = 1$

Question

The equation of circle passing through the origin is $x^2 + y^2 - 6x + 2y = 0$, the equation of a diameter is

(a) $x + 3y = 0$ (b) $x + y = 0$
(c) $y = x$ (d) $3x + y = 0$.

Ans. (a)

The centré of the circle $x^2 + y^2 - 6x + 2y = 0$
is $(3, -1)$, diameter will pass through $(0, 0)$ and
$(3, -1)$

$$\therefore \qquad\qquad y = \frac{-1}{3}x$$

$$\Rightarrow \qquad\qquad 3y + x = 0.$$

Question

If the line $x + 2by + 7 = 0$ is a diameter of the
circle $x^2 + y^2 - 6x + 2y = 0$, $b =$
(a) 3   (b) -5
(c) -4 (d) 5. **(MP PET 1991)**

Ans. (d)

Centre of circle $(3, -1)$ lies on the diameter
$x + 2by + 7 = 0$

$$\therefore \qquad 3 - 2b + 7 = 0$$

$$\Rightarrow \qquad\qquad b = 5.$$

Question

The common chord of circles $x^2 + y^2 + 4x$
$+ 1 = 0$ and $x^2 + y^2 + 6x + 2y + 3 = 0$ is
(a) $x + y + 1 = 0$ (b) $x + y - 1 = 0$
(c) $2x + 2y + 5 = 0$ (d) $x + y - 3 = 0$.

                          (MP PET 1991)

Ans. (a)

Equation of common chord is $S_1 - S_2 = 0$

$$\Rightarrow \qquad 2x + 2y + 2 = 0$$

$$\Rightarrow \qquad\quad x + y + 1 = 0.$$

Question

The locus of mid point of a chord of the circle $x^2 + y^2 = 4$ which subtending angle $90°$ at the centre is

(a) $x + y = 2$ (b) $x^2 + y^2 = 1$

(c) $x^2 + y^2 = 2$ (d) $x + y = 1$.

(IIT 1984; MP PET 1990)

Ans. (c)

Let (h, k) be the mid point of the chord

$$h^2 + k^2 = (2\cos 45°)^2 = \left(2 \cdot \frac{1}{\sqrt{2}}\right)^2$$
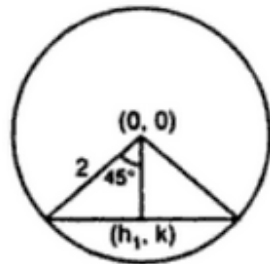
∴ locus of mid point is $x^2 + y^2 = 2$

Question

The pole of the line $9x + y - 28 = 0$ with respect to the circle $2x^2 + 2y^2 - 3x + 5y - 7 = 0$ is

(a) $(3, 1)$ (b) $1, 3)$

(c) $(3, -1)$,   (d) $(-3, 1)$.

(MNR 1984)

Ans. (c)

Let (x_1, y_1) be pole. Then the polar w.r.t. circle

$$x^2 + y^2 - \frac{3}{2}x + \frac{5}{2}y - \frac{7}{2} = 0 \text{ is}$$

$$xx_1 + yy_1 - \frac{3}{4}(x + x_1) + \frac{5}{4}(y + y_1) - \frac{7}{2} = 0$$

$$\Rightarrow x\left(x_1 - \frac{3}{4}\right) + y\left(y_1 + \frac{5}{4}\right) - \frac{3}{4}x_1$$

$$+ \frac{5}{4}y_1 - \frac{7}{2} = 0 \ ...(i)$$

which is identical to $9x + y - 28 = 0$ ...(ii)

$$\therefore \quad \frac{x_1 - 3/4}{9} = \frac{y_1 + 5/4}{1}$$

$$= \frac{-\frac{3}{4}x_1 + \frac{5}{4}y_1 - \frac{7}{2}}{-28} = \lambda$$

From (i) and (ii)

$$\frac{1}{9}\left(x_1 - \frac{3}{4}\right) = y_1 + \frac{5}{4}$$

$$\Rightarrow \qquad 4x_1 - 3 = (4y_1 + 5)9$$

$$\Rightarrow \qquad 4x_1 - 36y_1 = 48 \qquad ...(iii)$$

From (ii) and (iii)

$$28\left(y_1 + \frac{5}{4}\right) = \frac{3}{4}x_1 - \frac{5}{4}y_1 + \frac{7}{2}$$

$$\Rightarrow \qquad 28(4y_1 + 5) = 3x_1 - 5y_1 + 14$$

$$\Rightarrow \qquad 112y_1 + 140 = 3x_1 - 5y_1 + 14$$

$$\Rightarrow \qquad 3x_1 - 117y = 126 \qquad ...(iv)$$

solving (iii) and (iv), $x = 3, y = -1$

Question

The equation of circle which passes through (4, 5) and whose centre is (2, 2) is

(a) $x^2 + y^2 + 4x + 4y - 5 = 0$

(b) $x^2 + y^2 - 4x - 4y - 5 = 0$

(c) $x^2 + y^2 - 4x = 13$

(d) $x^2 + y^2 - 4x - 4y + 5 = 0$. (MNR 1985)

Ans. (b)

radius $= r = \sqrt{(4-2)^2 + (5-2)^2} = \sqrt{13}$

The equation of circle is

$$(x - 2)^2 + (y - 2)^2 = \left(\sqrt{13}\right)^2$$

$$\Rightarrow \quad x^2 + y^2 - 4x - 4y - 5 = 0$$

Question

The equation of tangents drawn from origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ is

(a) $x = 0$

(b) $y = 0$

(c) $(h^2 - r^2) x + 2rhy = 0$

(d) $(h^2 - r^2)x - 2rhy = 0$.

(ROORKEE 1989; IIT 1988)

Ans. (b), (d) A line drawn from origin is $y = mx$ which touches the circle whose centre is (r, h) and radius is r.

$$\therefore \quad \frac{mr - h}{\sqrt{1 + m^2}} = r$$

$$\Rightarrow m^2r^2 + h^2 - 2mrh = r^2(1 + m^2)$$

$$\Rightarrow 0m^2 + (h^2 - r^2) - 2rmh = 0$$

$$\Rightarrow \quad m = 0, \ m = \frac{h^2 - r^2}{2rh}$$

One tangent line is $y = 0$ which is x-axis and other tangent is

$$y = \frac{h^2 - r^2}{2rh} x \Rightarrow (h^2 - r^2)x - 2rhy = 0.$$

Question

The angle between tangents drawn from origin to the circle $(x - 7)^2 + (y + 1)^2 = 25$ is

(a) $\pi/3$ (b) $\pi/6$

(c) $\pi/2$ (d) 0. **(MNR 1990)**

Ans. (c)

Let $y = mx$ be the tangent. Perpendicular from centre $(7, -1)$ on the tangent = radius

$$\therefore \quad \frac{7m + 1}{\sqrt{1 + m^2}} = 5$$

$$\Rightarrow 49m^2 + 1 + 14m = 25 + 25m^2$$

$$\Rightarrow 24m^2 + 14m - 24 = 0. \text{ This gives two values}$$

of m

which are gradients of the tangents, m_1, m_2

$$\therefore m_1m_2 = \frac{-24}{24} = -1$$

$$\therefore \text{ Angle between tangents is } \frac{\pi}{2}.$$

Question

If two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect at two different points, then

(a) $2 < r < 8$ (b) $r = 2$

(c) $r < 2$ (d) $r > 2$. **(IIT 1989)**

Ans. (a)

If two circles intersect at two different points,

$$C_1C_2 < r_1 + r_2 \text{ and } r_1 - r_2 < C_1C_2$$

$\therefore \qquad 5 < r + 3 \implies 2 < r \qquad \text{...(i)}$

$\qquad r - 3 < 5 \implies r < 8 \qquad \text{...(ii)}$

By (i) and (ii) $2 < r < 8$.

Question

The number of common tangents to the circles $x^2 + y^2 + 2x + 8y - 23 = 0$ and $x^2 + y^2 - 4x - 10y + 19 = 0$ is

(a) 1   (b) 2

(c) 3   (d) 4.

Ans. (c)

$$C_1(- 1, - 4), \ r_1 = \sqrt{40}$$

$$C_2(2, 5), \ r_2 = \sqrt{10}$$

$$C_1C_2 = 3\sqrt{10}, \ r_1 + r_2 = 3\sqrt{10}$$

$\therefore$ There are three tangents

Question

Lines $2x - 3y = 5$ and $3x - 4y = 7$ are the equation of two diameters of a circle whose area is 154 sq. units, then the equation of circle is

(a) $x^2 + y^2 + 2x - 2y = 62$

(b) $x^2 + y^2 + 2x - 2y = 47$

(c) $x^2 + y^2 - 2x + 2y = 47$

(d) $x^2 - y^2 - 2x + 2y = 62$.   (IIT 1989)

Ans. (c)

On solving $2x - 3y = 5$ and $3x - 4y = 7$ we get centre $(1, -1)$ If r is the radius of circle

$$\pi r^2 = 154$$

$$\Rightarrow \quad r^2 = \frac{154 \times 7}{22} = 49$$

$$\Rightarrow \quad r = 7$$

∴ Equation of circle is

$$(x - 1)^2 + (y + 1)^2 = 7^2$$

$$\Rightarrow x^2 + y^2 - 2x + 2y = 47.$$

Question

If $(x, 3)$ and $(3, 5)$ are ends of a diameter of a circle and centre is $(2, y)$, the value of (x, y) is

(a) $(1, 4)$ (b) $(4, 1)$

(c) $(8, 2)$ (d) $(2, 8)$.

Ans. (a)

$(x, 3)$ and $(3, 5)$ are ends of the diameter. Then its mid point is the centre,

$$\therefore \qquad \frac{x + 3}{2} = 2 \Rightarrow x = 1$$

$$\frac{3 + 5}{2} = y \Rightarrow y = 4$$

$$\therefore \qquad (x, y) = (1, 4).$$

Question

The centre of a circle which passes through the points $(0, 0)$ and $(0, 1)$ and touches circle $x^2 + y^2 = 9$ is

(a) $\left(\frac{1}{2}, \frac{1}{2}\right)$

(b) $\left(\frac{1}{2}, \frac{3}{2}\right)$

(c) $\left(\sqrt{2}, \frac{1}{2}\right)$

(d) $\left(\frac{1}{2}, -\sqrt{2}\right)$.

(IIT 1992)

Ans. (c)

Equation of the circle passing through $(0, 0)$ is

$$x^2 + y^2 + 2gx + 2fy = 0 \quad \text{which also}$$

passes through $(0, 1)$

$$\therefore \quad 0 + 1 + 2g \cdot 0 + 2f \cdot 1 = 0 \Rightarrow f = -\frac{1}{2}$$

$$x^2 + y^2 + 2gx - y = 0$$

Its centre is $\left(-g, \dfrac{1}{2}\right)$, $r = \sqrt{g^2 + \dfrac{1}{4}}$

It touches the circle $x^2 + y^2 = 9$

$$C_1C_2 = r_1 - r_2$$

$$\sqrt{g^2 + \frac{1}{4}} = 3 - \sqrt{g^2 + \frac{1}{4}}$$

$$\Rightarrow \quad 2\sqrt{g^2 + \frac{1}{4}} = 3$$

$$\Rightarrow \quad 4\left(g^2 + \frac{1}{4}\right) = 9$$

$$\Rightarrow \quad 4g^2 + 1 = 9 \Rightarrow g^2 = 2$$

$$\Rightarrow \quad g = \sqrt{2}, -\sqrt{2}.$$

Question

The equation of circle which touches y-axis at $(0, 3)$ and cuts 8 intercept on x-axis, is

(a) $x^2 + y^2 + 10x - 6y + 9 = 0$

(b) $x^2 + y^2 + 6x - 10y + 9 = 0$

(c) $x^2 + y^2 + 8x + 4y + 2 = 0$

(d) $x^2 + y^2 - 10x - 6y + 9 = 0$.

Ans. (a), (d)

Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

Centre $(- g, - f)$, $r = \sqrt{g^2 + f^2 - c}$

It touches y-axis at $(0, 3)$, therefore

$$y^2 + 2fy + c = (y - 3)^2 = y^2 - 6y + 9$$

$\Rightarrow$ $\qquad\qquad f = - 3, c = 9$

and intercept on x-axis $= 2\sqrt{g^2 - c} = 8$

$\Rightarrow$ $\qquad\qquad g^2 - c = 16$

$\Rightarrow$ $\qquad\qquad g^2 = c + 16 = 9 + 16 = 25$

$\Rightarrow$ $\qquad\qquad g = \pm\ 5$

$\therefore$ Equation of circle is $x^2 + y^2 \pm 10x - 6y + 9 = 0$

Question

The equation of circle which passes through $(1, - 2)$ and $(3, - 4)$ and touches x-axis, is

(a) $x^2 + y^2 + 6x + 2y + 9 = 0$

(b) $x^2 + y^2 + 10x + 20y + 25 = 0$

(c) $x^2 + y^2 - 6x + 4y + 9 = 0$

(d) none of these.

Ans. (c)

Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

It touches x-axis

$$r = \sqrt{g^2 + f^2 - c} = f$$

$\Rightarrow$ $\qquad g^2 + f^2 - c = f^2$

$\qquad\qquad c = g^2$

and circle passes through $(1, - 2)$ and $(3, - 4)$

$1 + 4 + 2g - 4f + c = 0 \Rightarrow 2g - 4f + c = -5$

$9 + 16 + 6g - 8f + c = 0 \Rightarrow 6g - 8f + c = -25$

On subtracting $-4g + 4f = 20$

$-g + f = 5$

$2g - 4f + g^2 = -5$

$2g - 4(5 + g) + g^2 = -5$

$2g - 20 - 4g + g^2 = -5$

$\Rightarrow \qquad g^2 - 2g - 15 = 0$

$\Rightarrow \qquad (g - 5)(g + 3) = 0$

$\Rightarrow \qquad\qquad g = 5, -3.$

then $\qquad\qquad f = 10, 2, \quad c = 25, 9$

Equation of the circles are

$x^2 + y^2 + 10x - 6y + 25 = 0$

and $x^2 + y^2 - 6x + 4y + 9 = 0.$

Question

A locus of a point which moves in such a way that the sum of squares of distances of it from the sides of a square of side unit length is 9, is

(a) a st, line   (b) a circle

(c) a parabola   (d) an ellipse.

(IIT 1976)

Ans. (b)

Let (h, k) be a point.

According to the question,

$$k^2 + (1 - h)^2 + (1 - k)^2 + h^2 = 9$$
$$\Rightarrow k^2 + 1 + h^2 - 2h + 1 + k^2 - 2k + h^2 = 9$$
$$\Rightarrow \qquad\qquad 2(h^2 + k^2) - 2h - 2k = 7$$
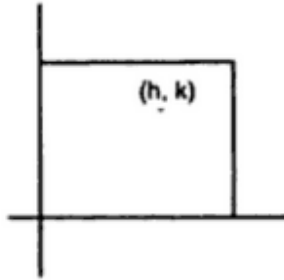
$\therefore$ locus is

$$2(x^2 + y^2) - 2x - 2y = 7$$
$$\Rightarrow x^2 + y^2 - x - y - \frac{7}{2} = 0 \text{ which is a circle.}$$

Question

If the circle $x^2 + y^2 = 25$ cuts the chord from the line $x - 2y = 2$, then the mid point of the chord is

(a) $\left(\dfrac{3}{5}, \dfrac{4}{5}\right)$ (b) $(-2, -2)$

(c) $\left(\dfrac{2}{5}, -\dfrac{4}{5}\right)$ (d) $\left(\dfrac{8}{3}, \dfrac{1}{3}\right)$.

Answer

Ans. (c)

The line perpendicular to $x - 2y - 2 = 0$ is $2x + y = \lambda$ which passes through $(0, 0)$

$\therefore$ $\lambda = 0$

$y + 2x = 0$. and $x - 2y - 2 = 0$

Solving them, $x = \dfrac{2}{5}$, $y = \dfrac{-4}{5}$.

Question

The equation of circle whose radius is 5 and thouches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at $(5, 5)$ externally, is

(a) $(x - 9)^2 + (y - 6)^2 = 5^2$
(b) $(x - 9)^2 + (y - 8)^2 = 5^2$
(c) $(x - 7)^2 + (y - 3)^2 = 5^2$
(d) $(x + 7)^2 + (y + 3)^2 = 5^2$

Ans. (b)

Let (h, k) be the centre. Since both circles touch externally each other and radii are 5. Point $(5, 5)$ is mid point of centre $(1, 2)$ and (h, k).

$$\frac{1 + h}{2} = 5, \quad \frac{2 + k}{2} = 5$$

$\Rightarrow$ $h = 9, k = 8$

$(x - 9)^2 + (y - 8)^2 = 5^2$.

Question

ABC is a right angled triangle with $\angle C$ right angle. If the coordinates of points A and B are $(-3, 4)$ and $(3, -4)$, the equation of circum circle of $\triangle$ ABC is

(a) $x^2 + y^2 - 6x + 8y = 0$

(b) $x^2 + y^2 = 25$

(c) $x^2 + y^2 - 3x + 4y + 5 = 0$

(d) $x^2 + y^2 - 3x = 25$.

Answer

. Ans. (b)

The mid point of AB is the centre (h, k) of desired circle

$$h = 0, \ k = 0$$

$$r = \frac{1}{2}\sqrt{36 + 64} = 5$$

$$x^2 + y^2 = 25.$$

Question

The equation of circle whose radius is 3 and touches the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ at $(-1, -1)$ internally, is

(a) $\left(x - \dfrac{4}{5}\right)^2 + \left(y + \dfrac{7}{5}\right)^2 = 3^2$

(b) $\left(x + \dfrac{4}{5}\right)^2 + \left(y - \dfrac{7}{5}\right)^2 = 3^2$

(c) $\left(x - \dfrac{4}{5}\right)^2 + \left(y - \dfrac{7}{5}\right)^2 = 3^2$

(d) $\left(x + \dfrac{4}{5}\right)^2 + \left(y + \dfrac{7}{5}\right)^2 = 3^2$.

Ans. (c)

Let (h, k) be the centre of the circle. If circles

touch each other,

$C_1(2, 3),\quad P(-1, -1)$

$PC_1 = 5,\ PC_2 = 3$

$C_1C_2 = 5 - 3 = 2$

C_2, divides PC_1 in the ratios

$2 : 3$, then coordinates of C_2 are,

$$x = \frac{3 \cdot 2 + 2 \cdot (-1)}{2 + 3},\ y = \frac{3 \cdot 3 + 2 \cdot (-1)}{2 + 3}$$

$$\Rightarrow \qquad x = \frac{4}{5},\ y = \frac{7}{5}$$

Required equation is $\left(x - \dfrac{4}{5}\right)^2 + \left(y - \dfrac{7}{5}\right)^2 = 3^2$.
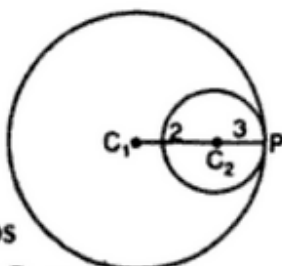
Question

The equation of circle which passes through intersection of $x^2 + y^2 - 6x + 2y + 4 = 0$ and, $x^2 + y^2 + 2x - 4y - 6 = 0$ and whose centre lies on $y = x$ is

(a) $3(x^2 + y^2) - 5x - 5y + 2 = 0$
(b) $7(x^2 + y^2) - 10x - 10y - 12 = 0$
(c) $x^2 + y^2 - 2x - 2y + 1 = 0$
(d) $x^2 + y^2 - 6x - 6y + 12 = 0.$

Ans. (b)

Equation of circle is

$x^2 + y^2 - 6x + 2y + 4 + \lambda(x^2 + y^2 + 2x$
$\qquad\qquad\qquad - 4y - 6) = 0$
$\Rightarrow \quad (1 + \lambda)x^2 + (1 + \lambda)y^2 - (6 - 2\lambda)x$
$\qquad\qquad + (2 - 4\lambda)y + 4 - 6\lambda = 0 \quad ...(i)$

Its centre $[3 - \lambda, -(1 - 2\lambda)]$ lies on the line $y = x$

$\therefore \qquad -(1 - 2\lambda) = 3 - \lambda$

$\Rightarrow \qquad -1 + 2\lambda = 3 - \lambda \Rightarrow \lambda = \dfrac{4}{3}$

Substituting the values of λ in (i), (b) is obtained.

Question

The equation of a circle of which the equation of normals is $x^2 + 2xy + 3x + 6y = 0$ and which contains the circle $x(x - 4) + y(y - 3) = 0$ is

(a) $x^2 + y^2 + 3x - 6y - 40 = 0$
(b) $x^2 + y^2 + 6x - 3y - 45 = 0$
(c) $x^2 + y^2 + 6x + 4y - 15 = 0$
(d) $x^2 + y^2 + 3x - 6y - 20 = 0.$

$\qquad\qquad\qquad$ **(ROORKEE 1990)**

Ans. (b)

Normals to the circle intersect at the centre of the circle

$$x^2 + 2xy + 3x + 6y = 0$$
$$\Rightarrow \quad x(x + 2y) + 3(x + 2y) = 0$$
$$\Rightarrow \quad (x + 3)(x + 2y) = 0$$

$\therefore$ Normals are $x = -3$, $y = -\dfrac{x}{2}$

Centre of the circle C_1 is $\left(-3, \dfrac{3}{2}\right)$ and let r be its radius. The centre of the circle $x^2 + y^2 - 4x$

$-3y = 0$ is $C_2\left(2, \dfrac{3}{2}\right)$

and $r_2 = \sqrt{4 + \dfrac{9}{4}} = \dfrac{5}{2}$

Since required circle contains it exactlly

$$C_1C_2 = r_1 - r_2$$
$$5 = r_1 - \dfrac{5}{2} \Rightarrow r_1 = \dfrac{15}{2}$$

$\therefore$ required equation is

$$(x + 3)^2 + \left(y - \dfrac{3}{2}\right)^2 = \left(\dfrac{15}{2}\right)^2$$
$$\Rightarrow x^2 + y^2 + 6x - 3y - 45 = 0.$$

Question

The locus of intersection point of two tangents to the circle $x = a\cos\theta$, $y = a\sin\theta$ such that difference between their paremetic angles is $\pi/2$ is

(a) a straight line   (b) a circle
(c) an ellipse   (d) a parabola.

Ans. (b)

The tangent to the circle $x^2 + y^2 = a^2$

at the point $x = a\cos\theta$, $y = a\sin\theta$ is

$x\cos\theta + y\sin\theta = a$ ...(i)

The tangent at the point

$x = a\cos\left(\theta + \dfrac{\pi}{2}\right)$, $y = a\sin\left(\dfrac{\pi}{2} + \theta\right)$ is

$-x\sin\theta + y\cos\theta = a$ ...(ii)

Squaring and adding (i) and (ii) we get

$x^2 + y^2 = 2a^2$

Question

The locus of intersection point of two tangents to the circle $x^2 + y^2 = a^2$ such that the difference between Parametric angles is $\pi/3$ is

(a) $x^2 + y^2 = a^2$ (b) $x^2 + y^2 = 4a^2$

(c) $x^2 + y^2 = \dfrac{4a^2}{3}$ (c) $x^2 + y^2 = 9$.

Ans. (c)

The tangent to the circle $x^2 + y^2 = a^2$ at $x = a\cos\theta$, $y = a\sin\theta$ is

$$x\cos\theta + y\sin\theta = a \qquad ...(i)$$

The tangent at the point

$$\left(a\cos\left(\theta + \frac{\pi}{3}\right) a\sin\left(\theta + \frac{\pi}{3}\right) \right) \text{ is}$$

$$x\cos\left(\theta + \frac{\pi}{3}\right) + y\sin\left(\theta + \frac{\pi}{3}\right) = a \qquad ...(ii)$$

$$\Rightarrow x\left[\frac{\cos\theta}{2} - \frac{\sqrt{3}}{2}\sin\theta \right]$$

$$+ y\left[\frac{1}{2}\sin\theta + \cos\theta \frac{\sqrt{3}}{2} \right] = a$$

$$\Rightarrow \frac{1}{2}(x\cos\theta + y\sin\theta) - \frac{\sqrt{3}}{2}(x\sin\theta - y\cos\theta)$$

$$= 9$$

$$\Rightarrow -\frac{\sqrt{3}}{2}(x\sin\theta - y\cos\theta) = a - \frac{a}{2} = \frac{a}{2}$$

$$\Rightarrow \qquad x\sin\theta - y\cos\theta = -\frac{a}{\sqrt{3}} \qquad ...(iii)$$

squaring and adding (i) and (iii)

$$x^2 + y^2 = a^2 + \frac{a^2}{3} = \frac{4a^2}{3}.$$

Question

From the inter section point of the circles $x^2 + y^2 = 12$ and $x^2 + y^2 - 5x + 3y - 2 = 0$ a tangent is drawn to the circle $x^2 + y^2 = 12$ then the coordinates of intersection point of tangents are

(a) (6, 12)   (b) (3, 4)
(c) (6, − 16/5)   (d) (1, 0).

Ans. (c)

Let the intersection point of tangents be (h, k)

the (chord of contact) w.r.t. the circle $x^2 + y^2 = 12$

is $xh + yk = 12$ ...(i)

and chord is $-5x + 3y = -10$ ...(ii)

Both represent the same line.

$$\frac{h}{-5} = \frac{k}{3} = -\frac{12}{10}$$

$\Rightarrow$ $h = 6, \ k = -\frac{18}{5}.$

Question

The equation of a circle which is ortho gonal to circles $x^2 + y^2 + 3x - 5y + 6 = 0$ and $4x^2 + 4y^2 - 28x + 29 = 0$ and whose centre lies on the line $3x + 4y + 1 = 0$ is

(a) $x^2 + y^2 + y/2 = \frac{29}{4}$

(b) $x^2 + y^2 + \frac{3}{2}x + \frac{5}{4} = 0$

(c) $x^2 + y^2 + \frac{7}{2}y + \frac{3}{2}x + 5 = 0$

(d) $x^2 + y^2 - \frac{y}{2} = \frac{29}{4}.$ **(DHANBAD 1989)**

Ans. (a)

Let the equation of the circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

centre $(-g, -f)$ and $r = \sqrt{g^2 + f^2 - c}$.

centre lies on the line $3x + 4y + 1 = 0$

$$\Rightarrow \quad -3g - 4f + 1 = 0$$

$$\Rightarrow \quad 3g + 4f - 1 = 0 \qquad \qquad ...(i)$$

Given circles are orthogonal,

$$2gg_1 + 2ff_1 = c_1 + c_2$$

$$3g - 5f = c + 6$$

$$7g = c + \frac{29}{4}$$

subtracting, $4g + 5f = +\dfrac{5}{4}$ \qquad ...(ii)

Solving (1) and (2) $g = 0$, $f = \dfrac{1}{4}$, $c = \dfrac{-29}{4}$

$\therefore$ Required equation is $x^2 + y^2 + \dfrac{7}{2}y$

$$- \frac{-29}{4} = 0$$

Question

The centre of the circle lies on the line $2x - 2y + 9 = 0$ and cuts the circle $x^2 + y^2 = 4$ orthogonally, then circle passes through two fixed points.

(a) $(1, 1)$, $(3, 3)$ (b) $\left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$, $(-4, 4)$

(c) $(0, 0)$, $(5, 5)$ (d) $(0, 0)$, $(-5, 5)$

Let the equation of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

Centre $(-g, -f)$ lies on the line $2x - 2y$
$$+ 9 = 0$$

$\therefore \quad -2g + 2f + 9 = 0 \qquad\qquad ...(i)$

$\Rightarrow \qquad\qquad 2g = 2f + 9$

It cuts circle $x^2 + y^2 = 4$ orthogonally.

$2g. \; 0 + 2f. \; 0 = c - 4 \Rightarrow c = 4$

$x^2 + y^2 + (2f + 9)x + 2fy + 4 = 0$

$\Rightarrow \quad (x^2 + y^2 + 9x + 4) + f(2x + 2y) = 0$

Line $2x + 2y = 0$ cuts the circle $x^2 + y^2 + 9x + 4 = 0$ Solving both equation we get

$\left(-\dfrac{1}{2}, \dfrac{1}{2}\right), (-4, 4)$

Question

The coordinates of a point from which tangents drawn to three circles $x^2 + y^2 = 1$, $x^2 + y^2 + 8x + 15 = 0$, $x^2 + y^2 + 10y + 24 = 0$ are equal in length, are

(a) $(-2, 1)$

(b) $(1, -2)$

(c) $(2, 3)$

(d) $\left(-2, -\dfrac{5}{2}\right)$

(ROORKEE 1982)

Ans. (d)

Let P(h, k) be a point and let PT$_1$, PT$_2$, PT$_3$ be tangents to three given circles.

$$PT_1{}^2 = PT_2{}^2 = PT_3{}^2$$

$\Rightarrow \quad h^2 + k^2 - 1 = h^2 + k^2 + 8h + 15$
$$= h^2 + k^2 + 10k + 24 = 0$$
$$\text{(i)} \qquad \text{(ii)} \qquad\qquad \text{(iii)}$$

From (i) and (ii) $h = -2$

From (i) and (iii) $k = -5/2$

Question

The chord of contact drawn from a point on the circle $x^2 + y^2 = a^2$ to the circle $x^2 + y^2 = b^2$ touches the circle $x^2 + y^2 = c^2$ $a > b$, then a, b, c are in

(a) A. P. (b) G. P.

(c) H. P. (d) none of these.

(ROORKEE 1982)

Ans. (b)

Let (h, k) be on the circle $x^2 + y^2 = a^2$

$\therefore$ $\qquad h^2 + k^2 = a^2$ $\qquad$ ...(i)

The chord contact drawn from (h, k) to the circle $x^2 + y^2 = b^2$ is

$xh + yk = b^2$ which is tangent to the circle $x^2 + y^2 = c^2$

$\therefore$ $\dfrac{h.0 + k.0 - b^2}{\sqrt{h^2 + k^2}} = c$

$\Rightarrow$ $\qquad b^4 = c^2(h^2 + k^2)$

$\Rightarrow$ $\qquad b^4 = c^2 a^2 \Rightarrow b^2 = ac$

$\Rightarrow$ a, b, c are in G.P.

Question

the locus of the centre of a circle which always passes through two fixed points $(a, 0)$ and $(-a, 0)$ is

(a) $x = 1$ (b) $x + y = 6$

(c) $x + y = 2a$ (d) $x = 0$.

(IIT 1984)

And. (d)

Let the centre be (h, k). According to the question

$\qquad (h + a)^2 + k^2 = (h - a)^2 + k^2$

$\Rightarrow (x + a)^2 - (h - a)^2 = 0$

$\Rightarrow \qquad\qquad 4ah = 0$

$\Rightarrow \qquad$ the locus is $x = 0$

Question

Equation $x = a \cos\theta + b \sin\theta$ and $y = a \sin\theta - b \cos\theta$, $0 \le \theta \le 2\pi$ represent

(a) a parabola   (b) a straight line

(c) an ellipse   (d) a circle.

Ans. (d)

$\qquad x = a \cos\theta + b \sin\theta$

$\qquad y = a \sin\theta - b \cos\theta$

squaring and adding $x^2 + y^2 = a^2 + b^2$

Question

If $(x, 3)$ and $(3, 5)$ are ends of the diameter of a circle whose centre is $(2, y)$, values of is x and y are.

(a) $x = 1, y = 4$ (b) $x = 4, y = 1$

(c) $x = 8, y = 2$ (d) none of these.

(SCREENING 1986)

Ans. (a)

$$\frac{x + 3}{2} = 2, \quad \frac{3 + 5}{2} = y$$

$\Rightarrow \qquad\qquad x = 1, y = 4.$

Question

The centre of the circle is $(2, 3)$. If line $x + y = 1$ touches the circle, its equation is

(a) $x^2 + y^2 - 4x - 6y + 4 = 0$

(b) $x^2 + y^2 - 4x - 6y + 5 = 0$

(c) $x^2 + y^2 - 4x - 6y - 5 = 0$

(d) none of these.   **(RAJ. PET 1985, 89)**

Ans. (b)

The length of perpendicular from Centre $(2, 3)$ to the tangent $x + y - 1 = 0$ is equal to the radius

$$\therefore \quad \frac{2 + 3 - 1}{\sqrt{1 + 1}} = r$$

$$\Rightarrow \quad r = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$\therefore$ Required equation of circle is,

$$(x - 2)^2 + (y - 3)^2 = 8$$

or $\quad x^2 + y^2 - 4x - 6y + 5 = 0$

Question

The equation of a circum circle of the traingle whose sides âre $y + \sqrt{3}x = 6$, $y - \sqrt{3}x = 6$ and $y = 0$ is

(a) $x^2 + y^2 - 4y = 0$ (b) $x^2 + y^2 + 4x = 0$

(c) $x^2 + y^2 - 4y = 12$ (d) $x^2 + y^2 + 4x = 12$.

Ans. (c)

Solving equations we get the coordinates of vertices of the triangle

$(2\sqrt{3}, 0)$, $(-2\sqrt{3}, 0)$, $(0, 6)$

If the equation of the circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$, then

$12 + 0 + 4\sqrt{3}g + 0 + c = 0$ ...(i)

$12 + 0 - 4\sqrt{3}g + 0 + c = 0$ ...(ii)

$0 + 36 + 0 + 12f + c = 0$

                     $c = -12$ ...(iii)

Adding (i) and (ii)

From (iii) $f = -2$

From (i) $g = 0$

∴ The equation of the circle is

     $x^2 + y^2 - 4y - 12 = 0$.

Question

The number of common tangents to the circles

$x^2 + y^2 - 2x - 1 = 0$ and $x^2 + y^2 - 2y - 7 = 0$

is

(a) 1   (b) 2

(c) 3   (d) 4.

Ans. (a)

$C_1(1, 0)$, $C_2(0, 1)$, $r_1 = \sqrt{2}$, $r_2 = 2\sqrt{2}$

$$C_1C_2 = \sqrt{2}$$
$$C_1C_2 = r_2 - r_2$$
$$= 2\sqrt{2} - \sqrt{2} = \sqrt{2}$$

Circles touch internally. One tangent can be drawn.

Question

The four intersection points of lines $(2x - y + 1)$ $(x - 2y + 3) = 0$ with axes lie on a circle whose centre is

(a) $\left(\dfrac{3}{4}, \dfrac{5}{4}\right)$

(b) $\left(-\dfrac{7}{4}, \dfrac{5}{4}\right)$

(d) $(2, 3)$

(d) $\left(\dfrac{7}{5}, -\dfrac{5}{4}\right)$.

Ans. (b)

Equation of axes is $xy = 0$

The equation of circle is

$(2x - y + 1)(x - 2y + 3) + \lambda xy = 0$

If it is a circle, coefficient of xy is $= 0$

$\Rightarrow \qquad -5 + \lambda = 0 \Rightarrow \lambda = 5$

Equation of circle is

$\qquad 2(x^2 + y^2) + 7x - 5y + 3 = 0$

$\Rightarrow \ x^2 + y^2 + \dfrac{7}{2}x - \dfrac{5}{2}y + \dfrac{3}{2} = 0$

$\therefore \quad$ Centre is $\left(-\dfrac{7}{4}, \dfrac{5}{4}\right)$.

Question

The distances of the centres òf the circle $x^2 + y^2 - 2\lambda x = c^2$, where c is constant and λ is variable, from the origin are in G.P. then length of tangents drawn from the point on the circle $x^2 + y^2 = c^2$ to the circles are in

(a) A. P.   (b) G. P.

(c) H. P.   (d) none of these

Ans. (b)

Let the equation of three circles be $x^2 + y^2 - 2\lambda x = c^2$ whose centres are $(\lambda_1, 0)$, $(\lambda_2, 0)$, $(\lambda_3, 0)$ and $\lambda_2^2 = \lambda_1 \lambda_3$.

Then any point on $x^2 + y^2 = c^2$ is $x = c \cos\theta$, $y = c \sin\theta$, the length of tangents from it to the circles are

$$PT_1^2 = c^2 \cos^2\theta + c^2 \sin^2\theta - 2\lambda_1, c \cos\theta - c^2$$
$$\Rightarrow PT_1^2 = t_1^2 = c^2 - 2\lambda_1 c \cos\theta - c^2$$
$$\Rightarrow PT_1^2 = t_1^2 = -2\lambda_1 c \cos\theta$$

Similarly, $PT_2^2 = t_2^2 = -2\lambda_2 c \cos\theta$

$$PT_3^2 = -2\lambda_3 c \cos\theta$$
$$\Rightarrow (t_2^2)^2 = t_1^2 t_3^2$$
$$\therefore \quad t_1, t_2, t_3 \text{ are in G.P.}$$

Question

Two circles $x^2 + y^2 - 10x + 4y - 20 = 0$ and $x^2 + y^2 + 14x - 6y + 22 = 0$.
(a) touch externally   (b) intersect at two points
(c) do not intersect   (d) one contains other.

Ans. (a)

$$C_1 C_2 = r_1 + r_2 = 7 + 6 = 13$$

Question

The equation of a circle which touches the circle $x^2 + y^2 - 15x + 5y = 0$ at the point $(1, 2)$ and passes through the point $(0, 2)$, is
(a) $13(x^2 + y^2) - 13x - 61y + 70 = 0$
(b) $x^2 + y^2 + 2x = 0$
(c) $13(x^2 + y^2) - 13x - 61y + 9 = 0$
(d) $13(x^2 + y^2) + 13x + 61y + 9 = 0$.

Ans. (a)

The equation of tangent to the circle $x^2 + y^2 - 15x + 5y = 0$ at $(1,2)$ is

$$x + 2y - \frac{15}{2}(x + 1) + \frac{5}{2}(y + 2) = 0$$

$$\Rightarrow \qquad -13x + 9y - 5 = 0$$

$\therefore$ Required circle is $S + \lambda T = 0$ which passes through $(0, 2)$

$$\therefore \qquad \lambda = \frac{-14}{13}$$

Question

Which of the following lines is normal to the circle $(x - 1)^2 + (y - 2)^2 = 10$?

(a) $2x + y = 3$ (b) $x + 2y = 10$

(c) $x + y = 13$ (d) $x + y = 3.$

Ans. (d)

Centre $(1, 2)$ satisfies $x + y = 3$.

Question

The tangent to the circle $x^2 + y^2 = 5$ at $(1, -2)$ touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$ at the point

(a) $(-2, 1)$ (b) $(-3, 0)$

(c) $(-1, -1)$ (d) $(3, -1).$

Ans. (d)

The eqution of the tangent to the circle
$x^2 + y^2 = 5$ at $(1, -2)$ is

$$x \cdot 1 + y(-2) = 5$$

$$\Rightarrow \qquad x - 2y = 5 \qquad ...(i)$$

It also touches other circle, therefore the normal to the second circle is perpendicular to (1). and passes through the centre $(4, -3)$.

$$2x + y = \lambda$$

$$\Rightarrow \qquad 8 - 3 = \lambda \Rightarrow \lambda = 5$$

$$2x + y = 5 \qquad ...(ii)$$

Solving (i) and (ii) $x = 3, y = -1$.

Question

The circle $x^2 + y^2 - 5x - 13y - 14 = 0$ cuts intercepts on x-axis and y-axis, then intercepts are

(a) 9, 13   (b) 5, 13

(c) 9, 15   (d) 9, 13.

Ans. (c)

Intercept on x-axis $\quad = 2\sqrt{g^2 - c}$

$$= 2\sqrt{\frac{25}{4} + 14}$$

$$= 9$$

and intercept on y-axis $= 2\sqrt{f^2 - c}$

$$= 2\sqrt{\frac{169}{4} + 14}$$

$$= 15$$

Question

The equation of the circle which touches both axes and whose radius is a, is

(a) $x^2 + y^2 - 2ax - 2ay + a^2 = 0$
(b) $x^2 + y^2 - ax - ay = 0$
(c) $x^2 + y^2 + ax + ay = 0$
(d) $x^2 + y^2 - ax - ay = a^2$. **(MP PET 1984)**

Ans. (a)

If circle touches both axes and if (h, k) is the centre and a is the radius, then

$$h = k = a$$

∴ Equation of the circle is

$$(x - a)^2 + (y - a)^2 = a^2$$

$$\Rightarrow \quad x^2 + y^2 - 2ax - 2ay + a^2 = 0.$$

Question

The equation of the chord of the circle $x^2 + y^2 = a^2$ whose mid point is (x_1, y_1) is

(a) $xx_1 + yy_1 = x_1^2 + y_1^2$
(b) $xx_1 + yy_1 = a^2$
(d) $xx_1 - yy_1 = 0$
(d) $xy_1 + yx_1 = a^2$.
(IIT 1983; MP PET 1986)

Ans. (a)

If the mid point of any chord is (x_1, y_1), then its eqution is $T = S_1$

$$\Rightarrow \quad xx_1 + yy_1 - a^2 = x_1^2 + y_1^2 - a^2$$

$$\Rightarrow \quad xx_1 + yy_1 = x_1^2 + y_1^2.$$

Question

The centres of the circles $x^2 + y^2 = 1$, $x^2 + y^2 + 6x - 2y = 1$, and $x^2 + y^2 - 12x + 4y = 1$ are
(a) coincident (b) collinear
(c) non-collinear (d) none of these.

(MP PET 1986)

Ans. (b)

Centres are $C_1(0, 0)$, $C_2(-3, 1)$ and $C_3(6, -2)$

Slope of $C_1C_2 = -\dfrac{1}{3}$ = Slope of C_1C_3

∴ C_1, C_2, C_3 are collinear.

Question

If any circle passes through the point $(0, 0)$, $(a, 0)$ and $(0, b)$, its centre is

(a) (a, b) (b) $\left(\dfrac{a}{2}, \dfrac{b}{2}\right)$

(c) (b, a) (d) $\left(\dfrac{b}{2}, \dfrac{a}{2}\right)$.

(MNR 1975)

Ans. (b)

Required circle is $x^2 + y^2 - ax - by = 0$

whose centre is $\left(\dfrac{a}{2}, \dfrac{b}{2}\right)$.

Question

The line $lx + my + n = 0$ touches the circle $x^2 + y^2 = a^2$ if
(a) $n^2(l^2 + m^2) = a^2$ (b) $a^2(l^2 + m^2) = n^2$
(c) $n(l + m) = a$ (d) $a(l + m) = n$.

(MNR 1974; AMU 1981)

Ans. (b)

If the line $lx + my + n = 0$ touches the circle $x^2 + y^2 = a^2$ length of perpendicular from the centre to the tangnent = radius of the circle.

$$\frac{+n}{\sqrt{l^2 + m^2}} = a \Rightarrow a^2 (l^2 + m^2) = n^2.$$

Question

A pair of tangent lines are drawn from the origin to the circle $x^2 + y^2 + 20(x + y) + 20 = 0$. The equation of pair of tangents is

(a) $x^2 + y^2 + 10xy = 0$ (b) $x^2 + y^2 + 5xy = 0$

(c) $2x^2 + 2y^2 + 5xy = 0$ (d) $x^2 + y^2 - 5xy = 0$

(MP PET 1990)

Ans. (c)

Let the tangent drawn from origin be $y = mx$.

Centre of the circle is $(-10, -10)$ and $r =$

$$\sqrt{100 + 100 - 20} = 3\sqrt{20}$$

$$\therefore \quad \frac{-10m + 10}{\sqrt{1 + m^2}} = 3\sqrt{20}$$

$\Rightarrow \quad (100) (1 - m)^2 = 180 (1 + m^2)$

$\Rightarrow \quad 5 + 5m^2 - 10m = 9 + 9m^2$

$\Rightarrow \quad 4m^2 + 10m + 4 = 0$

$\Rightarrow \quad 2m^2 + 5m + 2 = 0$

(putting $m = \frac{y}{x}$)

$\Rightarrow \quad \frac{2y^2}{x^2} + 5\frac{y}{x} + 2 = 0$

$\Rightarrow \quad 2x^2 + 5xy + 2y^2 = 0.$

Question

Line $y = mx$ is a chord of a circle whose radius is a and x-axis is a diameter, the other and of chord is at origin the equation of the circle a assuming the chord the diameter of the circle, is

(a) $(1 + m^2)(x^2 + y^2) - 2ax = 0$

(b) $(1 + m^2)(x^2 + y^2) - 2a(x + my) = 0$

(c) $(1 + m^2)(x^2 + y^2) + 2a(x + my) = 0$

(d) $(1 + m^2)(x^2 + y^2) - 2a(x - my) = 0.$

(MP PET 1990)

Ans. (b)

A circle of radius a passes through the origin and diameter is x-axis

$\therefore$ Centre is $(a, 0)$.

Equation of circle is

$$(x - a)^2 + y^2 = a^2$$

$\Rightarrow \qquad x^2 + y^2 - 2ax = 0$

Required equation of the circle is

$x^2 + y^2 - 2ax + \lambda(y - mx) = 0$

$\Rightarrow x^2 + y^2 - 2ax - \lambda mx + \lambda y = 0$

Its centre is $\left(a + \dfrac{\lambda}{2}m, -\dfrac{\lambda}{2}\right)$ which lies on y

$= mx$

$\therefore \qquad \dfrac{-\lambda}{2} = m\left(a + \dfrac{\lambda m}{2}\right)$

$-\dfrac{\lambda}{2}(1 + m^2) = ma \Rightarrow \lambda = \dfrac{-2ma}{1 + m^2}$

$\therefore$ Required equation of the circle is

$x^2 + y^2 - 2ax + \dfrac{2ma}{1 + m^2}mx - \dfrac{2ma}{1 + m^2}y = 0$

$\Rightarrow (1 + m^2)(x^2 + y^2) - 2a(1 + m^2)x + 2m^2ax$
$\qquad\qquad\qquad\qquad\qquad\qquad - 2may = 0$

$\Rightarrow \qquad (1 + m^2)(x^2 + y^2) - 2a(x + my) = 0$

Question

Circle $x^2 + y^2 + 4x - 4y + 4 = 0$ touches

(a) x-axis \qquad\qquad (b) y-axis

(c) x-axis and y-axis (d) $y = x$.

(MP PET 1988)

Ans. (c)

The centre of the circle $x^2 + y^2 + 4x - 4y + 4 = 0$ is $(-2, 2)$ and $r = \sqrt{4 + 4 - 4} = 2$

$\therefore$ It touches x-axis and y-axis.

Question

The equation of the circle which touches axes and whose centre is (x_1, y_1) is

(a) $x^2 + y^2 + 2x_1(x + y) + x_1^2 = 0$

(b) $x^2 + y^2 - 2x_1(x + y) + x_1^2 = 0$

(c) $x^2 + y^2 = x_1^2 + y_1^2$

(d) $x^2 + y^2 + 2xx_1 + 2yy_1 = 0$.

Ans. (b)

If the circle touches both coordinate axes, then

$$x_1 = y_1 = \text{radius}$$

∴ Equation of the circle is

$$(x - x_1)^2 + (y - x_1)^2 = x_1^2$$
$$\Rightarrow x^2 + y^2 - 2x_1(x + y) + x_1^2 = 0$$

Question

A circle touches y-axis at $(0, 4)$ and cut intercept 6 units on x-axis, the radius of the circle is

(a) 3

(b) 4

(c) 5

(d) 6.

(MP PET 1992)

Ans. (c)

Let the equation of the circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

It touches y-axis at $(0, 4)$ then,

$$y^2 + 2fy + c \equiv (y - 4)^2 = y^2 - 8y + 16$$

$$\Rightarrow \qquad f = -4, c = 16$$

and $\qquad 2\sqrt{g^2 - c} = 6$

$$\Rightarrow \qquad g^2 - 16 = 9 \Rightarrow g = \pm 5$$

$\therefore$ Radius of the circle $r = \sqrt{g^2 + f^2 - c}$

$$= \sqrt{25 + 16 - 16} = 5.$$

Question

The equation of the circle which touches x-axis and whose centre is $(1, 2)$ is

(a) $x^2 + y^2 - 2x + 4y + 1 = 0$

(b) $x^2 + y^2 - 2x - 4y + 1 = 0$

(c) $x^2 + y^2 + 2x + 4y + 1 = 0$

(d) $x^2 + y^2 + 4x + 2y + 4 = 0.$

Ans. (b)

Here the circle touches x-axis, therefore $r = 2$

Required equation is $(x - 1)^2 + (y - 2)^2 = 2^2$

$$\Rightarrow \qquad x^2 + y^2 - 2x - 4y + 1 = 0$$

Question

If lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ touches a circle, then radius of the circle is

(a) 3/2 \qquad\qquad (b) 3/4

(c) 1/10 \qquad\qquad (d) 1/20.

(IIT 1984; RAJ. PET 1995)

Ans. (b)
Both tangent lines are parallel, the half of the distsnce between them is eqaul to the radius of circle. Point $(0, 1)$ lies on the equation (1). Length of perpendicular form it on other line

$$= 2r = \frac{4 - \frac{7}{2}}{\sqrt{9 + 16}}$$

$$= \left[\frac{-8 - 7}{\sqrt{36 + 64}} \right] = \frac{[-15]}{\sqrt{100}} = \frac{3}{2}$$

$$\Rightarrow \qquad r = \frac{3}{4}.$$

Question

The radius of the circle $x^2 + y^2 - 18x + 12y + k = 0$ is 11, then $k =$

(a) 4   (b) -4
(c) 5   (d) -5.

Ans. (b)

$$\sqrt{81 + 36 - k} = 11$$
$$\Rightarrow \qquad 117 - k = 121$$
$$\Rightarrow \qquad 117 - 121 = k = -4$$

Question

If the distance between centres of two circle is
$d =$ and their radii are r_1, r_2 and $d = r_1 + r_2$, then
(a) circles trouch externally
(b) circles touch internally
(c) circles intersect
(d) circles do not intersect.

Ans. (a)
It is clear that $C_1C_2 = r_1 + r_2$

Question

The locus of the centre of the circle which touches coordinate axes is

(a) $x^2 - y^2 = 0$

(b) $x^2 + y^2 = 0$

(c) $x^2 + y^2 = a$

(d) $x^2 - y^2 = a$, where a is constant.

Ans. (a)

If the centre (h, k) of the circle touching both axes then $|h| = |k| \Rightarrow h = \pm k$

$\therefore$ locus is

$\Rightarrow \quad (y - x)(y + x) \Rightarrow y^2 - x^2 = 0$.

Question

The line $(x - 2) + (y + 3) = 0$ cuts the circle $(x - 2)^2 + (y - 3)^2 = 11$

(a) at two points   (b) at one point

(c) at no point   (d) none of these.

(MNR 1975)

Ans. (c)

Centre of the circle is $(2, 3)$, $r = \sqrt{11}$

Line is $x + y + 1 = 0$

Length of perpendicular from centre to the line

$$p = \frac{2 + 3 + 1}{\sqrt{2}} = 3\sqrt{2}, r = \sqrt{11}$$

$p > r$ therefore line does not cut circle at any point.

Question

Circles $x^2 + y^2 - 2x - 4y = 0$ and $x^2 + y^2 - 8y - 4 = 0$

(a) touches internally (b) touches externally

(c) intersect   (d) deos not intersect.

Ans. (a)

$C_1(1, 2)$, $C_2(0, 4)$, $r_1 = \sqrt{5}$, $r_2 = \sqrt{20}$

$$C_1C_2 =$$

$$\sqrt{(1-0)^2 + (2-4)^2} = \sqrt{5}$$

$$r_2 - r_1 = 2\sqrt{5} - \sqrt{5} = \sqrt{5}$$

$$C_1C_2 = r_2 - r_1.$$

They touch internally.

Question

From the point (4, 3) tangents are drawn to the circle $x^2 + y^2 = 9$ the area of the triangle formed by the joining points of contact and tangents is,

(a) $\dfrac{24}{25}$

(b) $\dfrac{64}{25}$

(c) $\dfrac{192}{25}$

(d) $\dfrac{192}{5}$.

(IIT 1981, 87)

Ans. (c)

The equation of chord of contact AB of point

P (4, 3) is
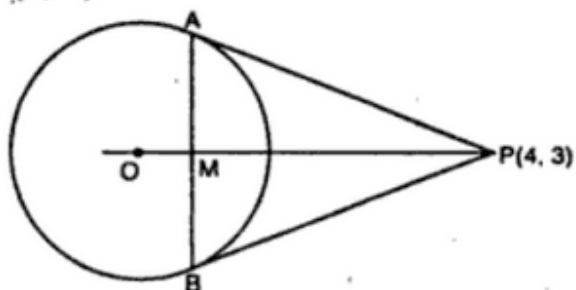
$$4x + 3y = 9$$

$$OM = \frac{9}{\sqrt{16+9}} = \frac{9}{5}$$

$$BM =$$

$$\sqrt{r^2 - OM^2} = \sqrt{9 - \frac{81}{25}} = \frac{12}{5}$$

$$AB = 2BM = \frac{24}{5}$$

$$OP = \sqrt{16 + 9} = 5$$

$$PM = OP - OM = 5 - \frac{9}{5} = \frac{16}{5}$$

Area of $\Delta PAB = \frac{1}{2} AM. OM$

$$= \frac{1}{2} \cdot \frac{24}{5} \times \frac{16}{5} = \frac{192}{25}$$

Question

Intersection point of $x^2 + y^2 = 25$ and $x^2 + y^2 - 8x + 7 = 0$ are

(a) (4, 3), (4, – 3)   (b) (– 4, – 3), (4, – 3)

(c) (4, 3), (– 4, – 3) (d) (3, 4), (– 3, – 4).

Ans. (a)

$$x^2 + y^2 = 25 \qquad \ldots(i)$$
$$x^2 + y^2 - 8x + 7 = 0 \qquad \ldots(ii)$$

Subtracting $\qquad 8x = 32 \Rightarrow x = 4$

$$y^2 = 25 - 16 = 9 \Rightarrow y = \pm 3$$

Intersection points are $(4, 3)$ and $(4, -3)$

Question

A point situated in the circle $x^2 + y^2 + 3x - 3y + 2 = 0$ is

(a) $(-1, 3)$ 　　　　　　(b) $(-2, 1)$

(c) $(2, 1)$ 　　　　　　(d) $(-3, 2)$.

Substitute and check that answer is (b)

Question

If circle $x^2 + y^2 = 3$ cuts a chord of length 2 units from the line $x - 2y = k$, then $k =$

(a) 0 　　　　　　(b) ± 1

(c) $\pm \sqrt{10}$ 　　　　　　(d) ± 3.

Ans. (c)

$$2 = 2\sqrt{r^2 - p^2}, \text{ where } p = \frac{k}{\sqrt{5}}$$

$$\Rightarrow \qquad 1 = \sqrt{3 - \frac{k^2}{5}}$$

$$\Rightarrow \qquad 5 = 15 - k^2$$

$$\Rightarrow \qquad k^2 = 10 \Rightarrow k = \pm \sqrt{10}$$

Question

If $y = mx + c$ is a tangent line to the circle $x^2 + y^2 = a^2$, the point of contact is

(a) $\left(\dfrac{-a^2}{c}, a^2 \right)$ (b) $\left(\dfrac{a^2}{c}, \dfrac{-a^2 m}{c} \right)$

(c) $\left(\dfrac{-a^2 m}{c}, \dfrac{a^2}{c} \right)$ (d) $\left(\dfrac{-a^2 c}{m}, \dfrac{a^2}{m} \right)$.

Ans. (c)

Let the point of contact be (h, k), then the tangent is

$$xh + ky = a^2 \qquad \text{...(i)}$$

and $\qquad mx - y = -c \qquad \text{...(ii)}$

Both represents the same line.

$\therefore \qquad \dfrac{h}{m} = \dfrac{k}{-1} = \dfrac{a^2}{-c}$

$\Rightarrow \qquad h = \dfrac{a^2 m}{c}, \ k = \dfrac{a^2}{c}.$

Question

If the line $lx + my = 1$ is a tangent line to the circle $x^2 + y^2 = a^2$, the locus of (l, m) is
(a) a straight line   (b) a circle
(c) an ellipse circle   (d) a parabola.

(MNR 1987)

Ans. (b)

$$\dfrac{-1^2}{\sqrt{l^2 + m^2}} = a \Rightarrow a^2(l^2 + m^2) = 1$$

$\therefore$ Locus is $x^2 + y^2 = \dfrac{1}{a^2}$

Question

The equation of the circle which touches axes and line $3x - 4y + 8 = 0$ and lies in 3rd quadrant is

(a) $x^2 + y^2 - 4x - 4y - 4 = 0$
(b) $x^2 + y^2 - 4x + 4y + 4 = 0$
(c) $x^2 + y^2 + 4x + 4y + 4 = 0$
(d) $x^2 + y^2 - 4x + 4y - 4 = 0$.

Ans. (c)
The circle touches coordinate axes. The relation between coordinates $(-h, -k)$ and radius is $|h| = |k| = r$. Length of perpendicular from the centre to $3x - 4y + 8 = 0$ is

$$\frac{-3h + 4h + 8}{\sqrt{9 + 16}} = h$$

$$\Rightarrow \qquad h + 8 = 5h$$
$$\Rightarrow \qquad h = 2$$

$\therefore$ Centre of the circle is $(-2, -2)$, $r = 2$

$$(x + 2)^2 + (y + 2)^2 = 2^2$$

$$x^2 + y^2 + 4x + 4y + h = 0.$$

Question

If one end of the diameter of the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ is $(3, 4)$, other end is

(a) $(0, 0)$ (b) $(1, 1)$
(c) $(1, 2)$ (d) $(2, 1)$.

(B. I. T. RANCHI 1991; MP PET 1986)

Ans. (c)
Centre is $(2, 3)$. Let (h, k) be other end.

$$\therefore \qquad \frac{3 + h}{2} = 2, \quad \frac{4 + k}{2} = 3$$

$$\Rightarrow \qquad h = 1, k = 2.$$

Question

If OA and OB are tangents drawn from O to circle $x^2 + y^2 - 6x - 8y + 21 = 0$, then AB =

(a) 11

(b) $\frac{4}{5}\sqrt{21}$

(c) $\sqrt{\frac{17}{3}}$

(d) none of these.

Ans. (b)

If OA and OB are tangent lines, then chord of contact of the circle is

$x.0 + y.0 - 3(x + 0) - 4(y + 0) + 21 = 0$

$\Rightarrow -3x - 4y + 21 = 0$

$\Rightarrow 3x + 4y - 21 = 0$

Length of perpendicular from (3, 4) to the line is

$$p = \frac{9 + 16 - 21}{\sqrt{9 + 16}} = \frac{4}{5}, r = 2$$

$$AB = 2\sqrt{r^2 - p^2}$$

$$= 2\sqrt{4 - \frac{16}{25}} = \frac{2 \times 2\sqrt{21}}{5}$$

Question

If circles $x^2 + y^2 + px + qy - 5 = 0$ and $x^2 + y^2 + 5x + py + 7 = 0$ cut orthogonally, then $p =$

(a) 1

(b) $\frac{1}{2}$

(c) $\frac{1}{4}$

(d) $\frac{3}{4}$.

Ans. (b)

Here $C_1\left(\dfrac{-p}{2}, \dfrac{-3}{2}\right)$, $C_2\left(\dfrac{-5}{2}, \dfrac{-p}{2}\right)$

If circles cut orthogonally,

$$2gg_1 + 2ff_1 = c_1 + c_2$$

$$\Rightarrow \quad 2\left(\dfrac{p}{2}\right)\left(\dfrac{5}{2}\right) + 2\left(\dfrac{3}{2}\right)\left(\dfrac{p}{2}\right) = -5 + 7$$

$$\Rightarrow \qquad\qquad 5p + 3p = 4 \Rightarrow p = \dfrac{1}{2}$$

Question

If circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$ touch each other, then

(a) $\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{1}{c}$

(b) $\dfrac{1}{a^2} + \dfrac{1}{b^2} = \dfrac{1}{c^2}$

(c) $\dfrac{1}{a} + \dfrac{1}{b} = c^2$

(d) $\dfrac{1}{a^2} + \dfrac{1}{b^2} = \dfrac{1}{c}$

$$\text{(MNR 1987)}$$

Ans. (a)

$C_1(- a, 0)$, $C_2(0, - b)$. $r_1 = \sqrt{a^2 - c}$,

$$r_2 = \sqrt{b^2 - c}$$

Commom tangent is $ax - by = 0$

Perpendicular from $- (a, 0)$ to $ax - by = 0$

$$= \sqrt{a^2 - c}$$

$$\Rightarrow \qquad \frac{- a^2}{\sqrt{a^2 + b^2}} = \sqrt{a^2 - c}$$

$$\Rightarrow \quad a^4 = (a^2 - c)(a^2 + b^2) = a^4 + a^2 b^2$$
$$- c(a^2 + b^2)$$

$$\Rightarrow \qquad c(a^2 + b^2) = a^2 b^2$$

$$\Rightarrow \qquad \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}.$$

Question

The equation of the circle whose centre is $(3, - 1)$ and cut the chord of length of 6 units from the line $2x - 5y + 18 = 0$ is

(a) $(x - 3)^2 + (y + 1)^2 = 38$

(b) $(x + 3)^2 (y - 1)^2 = 38$

(c) $(x - 3)^2 + (y + 1)^2 = \sqrt{38}$

(d) $(x - 3)^2 + (y - 1)^2 = 38$.

(ROORKEE 1977)

Ans. (a)

Length of perpendicular from centre is

$$p = \frac{6 + 5 + 18}{\sqrt{4 + 25}}$$

$$= \frac{29}{\sqrt{29}} = \sqrt{29}$$

Lengtht of chord $l = 2\sqrt{r^2 - p^2}$

$\Rightarrow \qquad\qquad l^2 = 4(r^2 - 29)$

$\Rightarrow \qquad\qquad 36 = 4(r^2 - 29)$

$\Rightarrow \qquad\qquad 9 + 29 = r^2$

$\Rightarrow \qquad\qquad r^2 = 38.$

Required equation is $(x - 3)^2 + (y + 1)^2 = 38$

Question

The angle between the tangents to the circle $x^2 + y^2 = 169$ at the points (5, 12) and (12, − 5) is

(a) $\pi/6$ (b) $\pi/4$

(c) $\pi/3$ (d) $\pi/2$.

Ans. (d)

The tangents at points (5, 12) and (12 − 5) are

$5x + 12y = 169$ ...(i) $\Rightarrow$ $m_1 = -5/12$

$12x - 5y = 169$ ...(ii) $m_2 = 12/5$

Here $m_1 m_2 = -1$

$\therefore$ These tangents are perpendicular.

Question

Which of the following is the diameter of the circle $x^2 + y^2 - 6x - 8y - 9 = 0$?

(a) $3x - 4y = 0$ (b) $4x - 3y = 9$

(c) $x + y = 7$ (d) $x - y = 1$.

Ans. (c)

Centre $(3, 4)$, satisfies $x + y = 7$

Question

Circles $x^2 + y^2 - 6x - 2y + 1 = 0$ and $x^2 + y^2 + 2x - 8y + 13 = 0$

(a) do not touch each other

(b) do not intersect

(c) intersect at two points

(d) touch each other.   **(MP PET 1969)**

Ans. (d)

$C_1(3, 1)$, $C_2(-1, 4)$, $r_1 = 3$, $r_2 = 2$

$C_1C_2 = 5$, $C_1C_2 = r_1 + r_2$

Question

The area of the triangle formed by tangents drawn from the point (h, k) to the circle $x^2 + y^2 = a^2$ and a line joining points of contact is

(a) $\dfrac{a(h^2 + k^2 - a^2)^{3/2}}{h^2 + k^2}$ (b) $\dfrac{a(h^2 + k^2 - a^2)^{1/2}}{h^2 + k^2}$

(c) $\dfrac{(h^2 + k^2 - a^2)^{3/2}}{h^2 + k^2}$ (d) $\dfrac{(h^2 + k^2 - a^2)^{1/2}}{h^2 + k^2}$

      (MNR 1980)

Ans. (a)

The chord of contact of point (h, k) is

$$xh + yk = a^2 \qquad ...(i)$$

Perpendicular from (h, k) on chord

$$p = \frac{h^2 + k^2 - a^2}{\sqrt{h^2 + k^2}}$$

Perpendicular from centre on chord

$$d = \frac{-a^2}{\sqrt{h^2 + k^2}}$$

Length of chord $l = 2\sqrt{r^2 - d^2}$

$$= 2\sqrt{a^2 - \frac{a^4}{h^2 + k^2}}$$

Question

The equation of the circle whose centre is at the point $(1, -2)$ and passes through the centre of the circle $x^2 + y^2 + 2y - 3 = 0$ is

(a) $x^2 + y^2 - 2x + 4y + 3 = 0$
(b) $x^2 + y^2 - 2x + 4y - 3 = 0$
(c) $x^2 + y^2 + 2x - 4y - 3 = 0$
(d) $x^2 + y^2 - 2x - 4y - 3 = 0$.

Ans (a)

The centre of the given circle is $(0, -1)$, centre of desired circle is $(1, -2)$ and

$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

∴ Required equation is $(x - 1)^2 + (y + 2)^2 = 2$

Question

Line $(x - a) \cos\alpha + (y - b) \sin\alpha = r$ touches the circle $(x - a)^2 + (y - b)^2 = r^2$

(a) If $\alpha = 30°$ (b) if $\alpha = 60°$

(c) for all α (d) none of these.

Ans. (c)

Length of perpendicular from centre (a, b) to the tangent line is

$$\frac{[(a - a) \cos\alpha + (b - b) \sin\alpha - r]}{\sqrt{\cos^2\alpha + \sin^2\alpha}} = r$$

$\Rightarrow$ $|-r| = r$

$\therefore$ It is tangent line for all value of α.

Question

If the circle $x^2 + y^2 + 2gx + 2fy = 0$ cuts intercepts on axes 10 and 24 respectively then radius of the circle is.

(a) 17   (b) 9

(c) 13   (d) 5.

Ans. (c)

Here $c = 0$

$$2\sqrt{g^2 - c} = 10, \; 2\sqrt{f^2 - c} = 24$$

$\Rightarrow$ $g^2 = 25, \; f^2 = 144$

Radius of circle $= \sqrt{g^2 + f^2 - c}$

$$= \sqrt{25 + 144} = 13$$

Question

The locus of the mid point of the chord of the circle $x^2 + y^2 = a^2$ and parallel to line $y = 2x$. is

(a) a circle of radius a

(b) a straight line with slope $-1/2$

(c) a circle with centre $(0, 0)$

(d) a straight line with slope -2.

Ans. (b)

The line parallel to $y = 2x$ is $2x - y + \lambda = 0$ which is perpendicular to the line joining centre and mid point of the chord.

$\therefore$ Locus is a straight line whose slope is $-\dfrac{1}{2}$

Question

A circle touches x-axis and cuts a chord of length $2l$ on y-axis, the locus of the centre of the circle is

(a) a straight line   (b) a circle

(c) an ellipse circle   (d) a parabola.

Ans. (d)

Let (h, k) be the centre of the circle. It touches x-axis. then radius $= k$

$$2l = 2\sqrt{k^2 - c}$$

$\Rightarrow \qquad\qquad l^2 = k^2 - c \Rightarrow c = k^2 - l^2$

$\therefore$ radius $\qquad k = \sqrt{h^2 + k^2 - c}$

$\Rightarrow \qquad\qquad k^2 = h^2 + l^2$

$\Rightarrow \qquad k^2 - h^2 = l^2$

$\therefore$ Locus is $y^2 - x^2 = l^2$ which is a hyperbola.

Question

The equation of circle touching both axes and passing through the points $(1, 2)$ is

(a) $x^2 + y^2 - 2x - 2y + 1 = 0$,

$x^2 + y^2 - 10x - 10y + 25 = 0$

(b) $x^2 + y^2 - 2x - 2y - 1 = 0$,

$x^2 + y^2 - 10x - 10y - 25 = 0$

(c) $x^2 + y^2 + 2x + 2y + 1 = 0$,

$x^2 + y^2 + 10x + 10y + 25 = 0$

(d) none of these.

Ans. (a)

If the centre of the circle touching both axes is (h, h) and radius $r = h$

$\therefore$ $(h - 1)^2 + (h - 2)^2 = h^2$

$\Rightarrow$ $h^2 - 6h + 5 = 0$

$\Rightarrow$ $h = 1, 5$

centre $C_1(1, 1), r = 1$, and $C_2(5, 5), r_2 = 5$

$(x - 1)^2 + (y - 1)^2 = 1^2$ and $(x - 5)^2 + (y - 5)^2 = 5^2$.

Question

The two tangents drawn from the origin to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ are mutually perpendicular if

(a) $g^2 + f^2 = 2c$ (b) $g = f = c^2$

(c) $g = f = c$ (d) none of these

Ans. (a)

Let $y = mx$ be the tangent drawn from $(0, 0)$ then,

$$\frac{-mg + f}{\sqrt{m^2 + 1}} = \sqrt{g^2 + f^2 - c}$$

$$\Rightarrow \quad m^2g^2 + f^2 - 2fmg = (1 + m^2)(g^2 + f^2 - c)$$

$$\Rightarrow m^2(f^2 - c) + 2fmg + g^2 - c = 0$$

$$m_1 m_2 = \frac{g^2 - c}{f^2 - c} = -1$$

$$\Rightarrow \quad g^2 - c = -f^2 + c$$

$$\Rightarrow \quad f^2 + g^2 = 2c.$$

Question

The equation of the circle passing through (2, 1) and touching y-axis at the origin is

(a) $x^2 + y^2 - 5x = 0$ (b) $2x^2 + 2y^2 - 5x = 0$

(c) $x^2 + y^2 + 5x = 0$ (d) $2x^2 + 2y^2 + 5x = 0$.

Ans. (b)

This circle touches y-axis at $(0, 0)$, then its centre is on x-axis at $(h, 0)$ and equation of circle is $x^2 + y^2 - 2hx = 0$ which passes through (2, 1).

$$\therefore \quad 4 + 1 - 2h.2 = 0$$

$$\Rightarrow \quad h = \frac{5}{4}$$

$$\therefore \quad 2x^2 + 2y^2 - 5x = 0.$$

Question

The least and the greatest distances of the point $(10, 7)$ from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ are 5 and 15. Is this statement true or false?

Answer

True. C is $(2, 1)$, P $(10, 7)$ and $r = 5$

$\therefore \quad PC = 10$
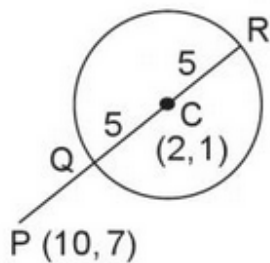
If the diameter through P cuts the circle in Q and R, then PQ is least and PR is greatest.

$\therefore \quad PQ = PC - r = 10 - 5 = 5$ and
$\quad\quad PR = PC + r = 10 + 5 = 15$

Question

Find the equations of a circles which pass through the points (7, 10) and (–7, –4) and have radius equal to 10 units.

Answer

Let the centre be (h, k), then distances of this point from the given points are equal, each equal to the radius 10, i.e.,

$$(h - 7)^2 + (k - 10)^2 = 10^2 \quad \text{............ (1)}$$

and $(h + 7)^2 + (k + 4)^2 = 10^2 \quad \text{............ (2)}$

Subtracting (ii) from (i), we get

$$-28h - 20k - 8k + 100 - 16 = 0$$

or $h + k = 3 \quad \text{.......... (3)}$

Substituting the value of $k = 3 - h$ from (3) in (1), we obtain $(h - 7)^2 + (3 - h - 10)^2 = 10^2$

$\Rightarrow \quad (h - 7)^2 + (-7 - h)^2 = 10^2$

$\Rightarrow \quad 2h^2 + 98 = 100$

$\Rightarrow \quad 2h^2 = 2$

$\Rightarrow \quad h = \pm 1$

When $h = 1$, then from (3), $k = 3 - h = 3 - 1 = 2$

Hence, the centre of the circle is (1, 2) and its equation is $(x - 1)^2 + (y - 2)^2 = 10^2$.

When $h = -1$, then from (3), $k = 3 - h = 3 - (-1) \Rightarrow k = 4$.

Hence, the centre of the circle is (–1, 4) and its equation is $(x - (-1))^2 + (y - 4)^2 = 10^2$.

Question

If the distance of a point P from $(6, 0)$ is twice its distance from the point $(1, 3)$ prove that the locus of P is a circle. Find its centre and radius.

Answer

Let the given points be $A\,(6, 0)$ and $B(1, 3)$.
Let, $P(x, y)$ be any point such that $|PA| = 2\,|PB|$

$\Rightarrow \quad PA^2 = 4PB^2$

$\Rightarrow \quad (x - 6)^2 + (y - 0)^2 = 4\{(x - 1)^2 + (y - 3)^2\}$

$\Rightarrow \quad 3x^2 + 3y^2 + 4x - 24y + 4 = 0$

$\Rightarrow \quad x^2 + y^2 + \dfrac{4}{3}x - 8y + \dfrac{4}{3} = 0$, which is of the

form

$x^2 + y^2 + 2gx + 2fy + c = 0$, with

$2g = \dfrac{4}{3}, 2f = -8, c = \dfrac{4}{3}$

i.e., $\quad g = \dfrac{2}{3}, f = -4, c = \dfrac{4}{3}$

Since, $g^2 + f^2 - c = \dfrac{4}{9} + 16 - \dfrac{4}{3} = \dfrac{136}{9} > 0$

therefore, locus of P is a circle with centre

$(-g, -f) = \left(-\dfrac{2}{3}, 4\right)$ and radius $= \sqrt{\dfrac{136}{9}} = \dfrac{2}{3}\sqrt{34}$

$= \sqrt{\dfrac{136}{9}} = \dfrac{2}{3}\sqrt{34}$

Question (A NCERT Problem)

Find the equation of the circle passing through the points (4, 1) and (6, 5) and whose centre is on the line $4x + y = 16$.

Answer

Let the equation of the circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \qquad \dots \dots (i)$$

As the points (4, 1) and (6, 5) lie on (i), therefore, $4^2 + 1^2 + 8g + 2f + c = 0$

$$\Rightarrow \quad 8g + 2f + c = -17 \qquad \dots \dots (ii)$$

and $6^2 + 5^2 + 12g + 10f + c = 0$

$$\Rightarrow \quad 12g + 10f + c = -61 \qquad \dots \dots (iii)$$

Subtracting (iii) from (ii), we get

$$-4g - 8f = -17 + 61$$

$$\Rightarrow \quad 4g + 8f = -44$$

$\Rightarrow \quad g + 2f = -11$ .... (iv)

Also, the centre $(-g, -f)$ of (i) lies on the line $4x + y = 16$

$\Rightarrow \quad 4(-g) - f = 16$

$\Rightarrow \quad 4g + f = -16$ .... (v)

Multiplying (v) by 2 and subtracting it from (iv), we obtain $g + 2f - 2(4g + f) = -11 - 2(-16)$

$\Rightarrow \quad -7g = 21$

$\Rightarrow \quad g = -3$

Substituting this value of g in (iv), we get $-3 + 2f = -11$

$\Rightarrow \quad 2f = -11 + 3$

$\Rightarrow \quad f = -\dfrac{8}{2} = -4$

Substituting $g = -3, f = -4$ in (ii), we obtain
$c = -17 - 8g - 2f = -17 - 8(-3) - 2(-4) = 15$

Hence, the required equation of the circle in reference is $x^2 + y^2 + 2(-3)x + 2(-4)y + 15 = 0$ (from (i))

or $\quad x^2 + y^2 - 6x - 8y + 15 = 0$

Question

The equation of the circle which touches both axes and whose centre is (x_1, y_1) is

[PET-88]

(a) $x^2 + y^2 + 2x_1(x + y) + x_1^2 = 0$
(b) $x^2 + y^2 - 2x_1(x + y) + x_1^2 = 0$
(c) $x^2 + y^2 = x_1^2 + y_1^2$
(d) $x^2 + y^2 + 2xx_1 + 2yy_1 = 0$

(b) The equation will be $x^2 + y^2 - 2x_1(x + y) + x^2_1 = 0$.

Question

The equation of the cirlce which touches x-axis and whose centre is $(1, 2)$ is

[PET-84]

(a) $x^2 + y^2 - 2x + 4y + 1 = 0$
(b) $x^2 + y^2 - 2x - 4y + 1 = 0$
(c) $x^2 + y^2 + 2x + 4y + 1 = 0$
(d) $x^2 + y^2 + 4x + 2y + 4 = 0$

(b) Center $(1, 2)$ and since circle touches x-axis, therefore, radius is equal to 2
Hence, the equation is $(x - 1)^2 + (y - 2)^2 = 2^2$
$\Rightarrow \quad x^2 + y^2 - 2x - 4y + 1 = 0$.

\

Question

If the radius of the circle $x^2 + y^2 - 18x + 12y + k = 0$ be 11, then k is equal to

[PET-87]

(a) 347   (b) 4
(c) -4 (d) 49

(c) $(\text{radius})^2 = g^2 + f^2 - c$ or $121 = 81 + 36 - k$
$\Rightarrow \quad k = -4$.

Question

The equation of a circle which touches both axes and the line $3x - 4y + 8 = 0$ and lies in the third quadrant is

[PET-86]

(a) $x^2 + y^2 - 4x + 4y - 4 = 0$
(b) $x^2 + y^2 - 4x + 4y + 4 = 0$
(c) $x^2 + y^2 + 4x + 4y + 4 = 0$
(d) $x^2 + y^2 - 4x - 4y - 4 = 0$

Answer

(c) The equation of a circle in third quadrant touching the **coordinate** axis with center $(-a, -a)$ and radius a is $x^2 + y^2 + 2ax + 2ay + a^2 = 0$ and we know $\left| \dfrac{3(-a) - 4(-a) + 8}{\sqrt{9 + 16}} \right| = a$

$\Rightarrow \quad a = 2.$

Hence the required equation is $x^2 + y^2 + 4x + 4y + 4 = 0.$

Question

If one end of a diameter of the cirlce $x^2 + y^2 - 4x - 6y + 11 = 0$ be $(3, 4)$ then the other end is

[PET-86; RANCHI BIT-91]

(a) $(0, 0)$ (b) $(1, 1)$
(c) $(1, 2)$ (d) $(2, 1)$

Answer

(c) Center is $(2, 3)$, one end is $(3, 4)$. P_2 divides the joins of P, and O in ratio of $2 : 1.$

Hence P_2 is $\left(\dfrac{4-3}{2-1}, \dfrac{6-4}{2-1} \right) \equiv (1, 2)$

Question

The equation of the circle having centre $(1, -2)$ and passing through the point of intersection of lines $3x + y = 14$, $2x + 5y = 18$ is

[PET-90]

(a) $x^2 + y^2 - 2x + 4y - 20 = 0$
(b) $x^2 + y^2 - 2x - 4y - 20 = 0$
(c) $x^2 + y^2 + 2x - 4y - 20 = 0$
(d) $x^2 + y^2 + 2x + 4y - 20 = 0$

Answer

(a) The point of intersection of $3x + y - 14 = 0$ and $2x + 5y - 18 = 0$

are $\quad x = \dfrac{-18 + 70}{15 - 2}, \quad y = \dfrac{-28 + 54}{13}$

$\Rightarrow \quad x = 4, y = 2$

i.e., point is $(4, 2)$

Therefore radius is $\sqrt{(9) + (16)} = 5$ and equation is $x^2 + y^2 - 2x + 4y - 20 = 0$.

Question

Equation of the circle which touches the lines $x = 0, y = 0$ and $3x + 4y = 4$ is

[PET-91]

(a) $x^2 - 4x + y^2 + 4y + 4 = 0$
(b) $x^2 - 4x + y^2 - 4y + 4 = 0$
(c) $x^2 + 4x + y^2 + 4y + 4 = 0$
(d) $x^2 + 4x + y^2 - 4y + 4 = 0$

Answer

(b) Let center of circle be (h, k). Since it touches both axis therefore $h = k = a$. Hence, equation can be $(x - a)^2 + (y - a)^2 = a^2$. But it also touches the line $3x + 4y = 4$.

Therefore, $\dfrac{3a + 4a - 4}{5} = a \Rightarrow a = 2$. Hence the required equation of circle is $x^2 + y^2 - 4x - 4y + 4 = 0$.

Question

The centre and radius of the circle $2x^2 + 2y^2 - x = 0$ are *[PET-84, 87]*
(a) $(1/4, 0)$ and $1/4$
(b) $(-1/2, 0)$ and $1/2$
(c) $(1/2, 0)$ and $1/2$
(d) $(0, -1/4)$ and $1/4$

Answer

(a) centre $(-g, -f) = \left(\dfrac{1}{4}, 0\right)$

and $R = \sqrt{\dfrac{1}{16} + 0 - 0} = \dfrac{1}{4}$

Question

The equation of the circle which touches x-axis at $(3, 0)$ and passes through $(1, 4)$ is given by *[PET-93]*
(a) $x^2 + y^2 - 6x - 5y + 9 = 0$
(b) $x^2 + y^2 + 6x + 5y - 9 = 0$
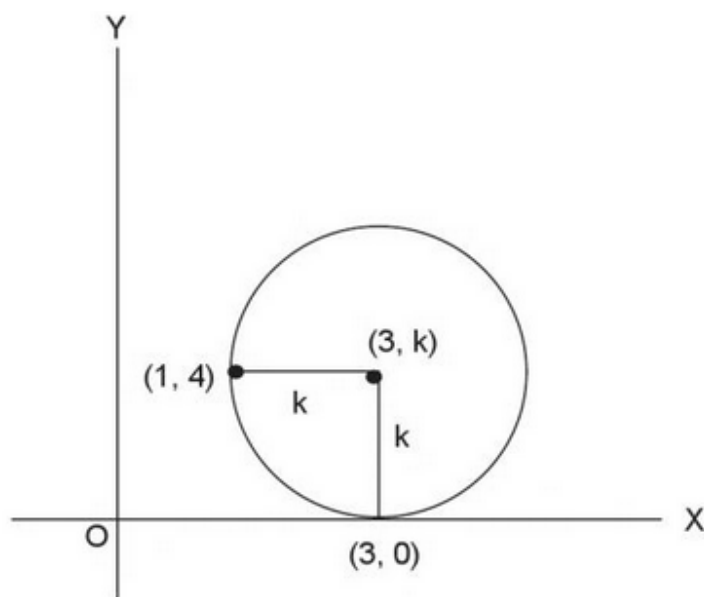(c) $x^2 + y^2 - 6x + 5y - 9 = 0$
(d) $x^2 + y^2 + 6x - 5y + 9 = 0$

Answer

(a) $k^2 = (3 - 1)^2 + (k - 4)^2$

$\Rightarrow \quad k^2 = 4 + (k - 4)^2$

$\Rightarrow \quad k = \dfrac{5}{2}$.

Hence required equation of circle is

$$(x - 3)^2 + \left(y - \frac{5}{2}\right)^2 = \left(\frac{5}{2}\right)^2$$

$\Rightarrow \quad x^2 + y^2 - 6x - 5y + 9 = 0$

Question

The equation of circle whose diameter is the line joining the points $(-4, 3)$ and $(12, -1)$ is
[IIT-71; AMU-79; MPPET-84; Roorkee-69]
(a) $x^2 + y^2 + 8x + 2y + 51 = 0$
(b) $x^2 + y^2 + 8x - 2y - 51 = 0$
(c) $x^2 + y^2 + 8x + 2y - 51 = 0$
(d) $x^2 + y^2 - 8x - 2y - 51 = 0$

Answer

(d) Required equation is $(x - x_1)(x - x_2) +$
$(y - y_1)(y - y_2) = 0$
$(x + 4)(x - 12) + (y - 3)(y + 1) = 0$
$= x^2 + y^2 - 8x - 2y - 51 = 0$

Question

The area of a circle whose centre is (h, k) and radius a is $\qquad$ **[PET-94]**

(a) $\pi(h^2 + k^2 - a^2)$ (b) $\pi a^2 hk$

(c) πa^2 (d) None of these

Answer

(c) Since area $= \pi r^2$, where $r = a$

$\Rightarrow$ area $= \pi a^2$.

Question

If the equation $\dfrac{K(x+1)^2}{3} + \dfrac{(y+2)^2}{4} = 1$ repre-

sents a circle, then K is equal to

[PET-94]

(a) 3/4   (b) 1

(c) 4/3   (d) 12

Answer

(a) It represent a circle, if $a = b$

$\Rightarrow \quad \dfrac{3}{k} = 4 \Rightarrow k = \dfrac{3}{4}$.

Question

If (α, β) is the centre of a circle passing through the origin, then its equation is

[MPPET-1999]

(a) $x^2 + y^2 - \alpha x - \beta y = 0$
(b) $x^2 + y^2 + 2\alpha x + \beta y = 0$
(c) $x^2 + y^2 - 2\alpha x - 2\beta y = 0$
(d) $x^2 + y^2 + \alpha x + \beta y = 0$

Answer

(c) Radius = Distance from origin = $\sqrt{\alpha^2 + \beta^2}$

$\therefore \quad (x - \alpha)^2 + (y - \beta)^2 = \alpha^2 + \beta^2$

$\Rightarrow \quad x^2 + y^2 - 2\alpha x - 2\beta y = 0$

Question

If the area of the circle $4x^2 + 4y^2 - 8x + 16y + k = 0$ is 9π sq. units, then the value of k is   *[MPPET-2005]*

(a) 4   (b) 16
(c) −16   (d) ±16

Answer

(c) The equation of the circle is

$$x^2 + y^2 - 2x + 4y + \frac{k}{4} = 0$$

$$\therefore \quad \text{Radius of circle} = \sqrt{1 + 4 - \frac{k}{4}}$$

$$\Rightarrow \quad \pi\left(5 - \frac{k}{4}\right) = 9\pi$$

$$\Rightarrow \quad 5 - 9 = \frac{k}{4}$$

$$\Rightarrow \quad k = -16$$

Question

Circle $x^2 + y^2 - 8x + 4y + 4 = 0$ touches

[CET(Karnataka)-99, 04]

(a) y-axis   (b) both the axes

(c) None of the axes   (d) x-axis

Answer

$g = -4, f = 2, c = 4$

$\Rightarrow \quad f^2 = c$

$\Rightarrow \quad$ circle touches y-axis.

Question

The minimum distance of the point $(2, -7)$ from the circle $x^2 + y^2 - 14x - 10y - 151 = 0$ is

[NDA-2004]

(a) 2   (b) 3

(c) 5   (d) 7

Answer

Centre of the circle = (7, 5), radius = 15
Distance of the given point from centre = 13
$\therefore$ required distance = 15 – 13 = 2

Question

For which value of a in equation $x^2 + y^2 + (a^2 - 4)xy + 2x + 2y + a = 0$ represents circle?

[Gujarat CET-2007]

(a) 3   (b) 2
(c) 4   (d) None of these

Answer

(b) General equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$
Equation $x^2 + y^2 + (a^2 - 4) xy + 2x + 2y + a = 0$
represents a circle if coefficient of $xy = 0$
$\Rightarrow$ $a^2 - 4 = 0$
$\Rightarrow$ $a^2 = 4$ $\therefore a = \pm 2$

Question

A circle touches the y-axis at the point (0, 4) and cuts the x-axis in a chord of length 6 units. The radius of the circle is

[PET-92]

(a) 3   (b) 4
(c) 5   (d) 6

Answer

(c) $r = \dfrac{\sqrt{4a^2 + b^2}}{2} = \dfrac{\sqrt{64 + 36}}{2} = 5$

Here $a = 4$, $b = 6$

Question

The equation of a diameter passing through origin of circle $x^2 + y^2 - 6x + 2y = 0$

(a) $x + 3y = 0$ (b) $x - 3y = 0$

(c) $3x + y = 0$ (d) $3x - y = 0$

Answer

(a) Center $(3, -1)$. Line through it and origin is $x + 3y = 0$

Question

Radius of the circle $x^2 + y^2 + 2x \cos\theta + 2y \sin\theta - 8 = 0$, is   *[MNR-74]*

(a) 1 (b) 3

(c) $2\sqrt{3}$ (d) $\sqrt{10}$

Answer

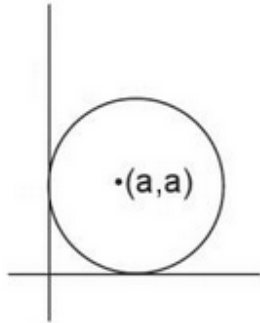(b) radius $= \sqrt{\cos^2\theta + \sin^2\theta - (-8)} = 3$

Question

The equation of the circle which touches both the axes and whose radius is a, is

[PET-1984]

(a) $x^2 + y^2 - 2ax - 2ay + a^2 = 0$

(b) $x^2 + y^2 + ax + ay - a^2 = 0$

Answer

(a) If circle touches both axis (Let it be in 1st quadrant)

Centre (a, a) radius $= a$

$\therefore$ equation is $(x - a)^2 + (y - a)^2 = a^2$

$x^2 + y^2 - 2ax - 2ay + a^2 = 0$

$\therefore$ In all quadrant equation can be $x^2 + y^2 \pm 2ax \pm 2ay + a^2 = 0$

Question

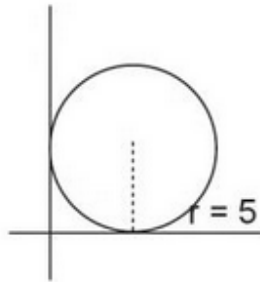The equation of the circle in the first quadrant which touches each axis at a distance 5 from the origin is

(a) $x^2 + y^2 + 5x + 5y + 25 = 0$
(b) $x^2 + y^2 - 10x - 10y + 25 = 0$
(c) $x^2 + y^2 - 5x - 5y + 25 = 0$
(d) $x^2 + y^2 - 12y + 27 = 0$

Answer

(b) Here centre is (r, r)

Where radius $= r$

Distance of point of contact with axis from origin $= 5 =$ radius

Equation is $(x - 5)^2 + (y - 5) = 5^2$

$x^2 + y^2 - 10x - 10y + 25 = 0$

Question

Equation of a circle passing through origin is $x^2 + y^2 - 6x + 2y = 0$. What is the equation of one of its diameters?

[NDA-2008]

(a) $x + 3y = 0$ (b) $x + y = 0$

(c) $x = y$ (d) $3x + y = 0$

Answer

(a) Centre of circle is

Diameter always passes through centre $(3, -1)$

Satisfy each option to get (a)

Question

If the coordinates of one end of the diameter of the circle $x^2 + y^2 - 8x - 4y + c = 0$ are $(-3, 2)$, then the coordiantes of other end are   *[Roorkee-95]*

(a) $(5, 3)$ (b) $(6, 2)$
(c) $(1, -8)$ (d) $(11, 2)$

Answer

(d) Centre $(4, 2)$
Let other end be (α, β)
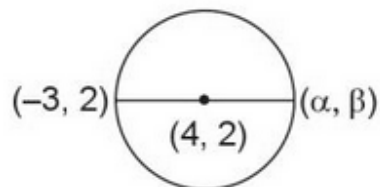
$\therefore \quad \left(\dfrac{\alpha-3}{2}, \dfrac{\beta+2}{2}\right) \equiv (4, 2) \quad (\alpha, \beta) \equiv (11, 2)$

Question

If area of the circle $4x^2 + 4y^2 - 8x + 16y + k = 0$ is 9π, then k is equal to
(a) -16 (b) 16
(c) ± 16 (d) 4

Answer

(a) Area $= 9\pi$

$\pi r^2 = 9\pi \Rightarrow r = 3$

Equation of circle is $yx^2 + 4y^2 - 8x + 16y + k = 0$

or $\quad x^2 + y^2 - 2x + 4y + \dfrac{k}{4} = 0$

$\therefore \quad \sqrt{g^2 + f^2 - c} = 3$

$\Rightarrow \quad \sqrt{(-1)^2 + 2^2 - \dfrac{k}{4}} = 3$

Question
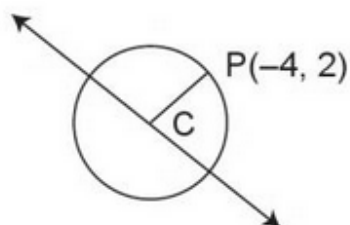
Find the equation of circle of lowest size which passes through the point $(-4, 2)$ and whose centre lies on the line $3x + 2y = 5$.

[MP-1997]

Answer

The length of perpendicular from the point $(-4, 2)$ to the line $3x + 2y = 5$ must be equal to radius of circle of lowest size.

$\therefore \quad$ Radius of circle $= \dfrac{-3(-4) - 2(2) + 5}{\sqrt{(-3)^2 + (-2)^2}}$

$$= \frac{12-4+5}{\sqrt{9+4}} = \frac{13}{\sqrt{13}} = \sqrt{13}$$

Given line is $3x + 2y = 5$ .... (1)

Equation of any line perpendicular to line (1) is

$2x - 3y + \lambda = 0$

It passes through $(-4, 2)$

$\therefore$ $2(-4) - 3(2) + \lambda = 0$

$\Rightarrow$ $\lambda = 14$

$\therefore$ Equation of perpendicular line is

$2x - 3y + 14 = 0$ .... (2)

Solving equations (1) and (2), we get $x = -1$ and $y = 4$.

Hence, centre of circle is $(-1, 4)$.

Therefore, required equation is

$(x + 1)^2 + (y - 4)^2 = (\sqrt{13})^2$

$\Rightarrow$ $x^2 + 2x + 1 + y^2 + 16 - 8y = 13$

$\Rightarrow$ $x^2 + y^2 + 2x - 8y + 4 = 0$

Question

Find the co-ordinates of the points of intersection of the line $5x - y + 2 = 0$ and the circle $x^2 + y^2 - 13x - 4y - 9 = 0$ and also find the length of the intercepted chord.

Answer

Equation of line is $5x - y + 2 = 0$ .... (1)

Equation of circle is $x^2 + y^2 - 13x - 4y - 9 = 0$ .... (2)

Putting the value of y in equation (2) from equation (1),

we get $x^2 + (5x + 2)^2 - 13x - 4(5x + 2) - 9 = 0$

$\Rightarrow \quad 26x^2 - 13x - 13 = 0$

$\Rightarrow \quad 2x^2 - x - 1 = 0$

$\Rightarrow \quad (x - 1)(2x + 1) = 0$

$\therefore \quad x = 1$ or $x = -\dfrac{1}{2}$

When $x = 1$, $y = 5(1) + 2 = 7$

When $x = -\dfrac{1}{2}$,

$$y = 5\left(-\frac{1}{2}\right) + 2 = -\frac{1}{2}$$

Coordinates of points of intersection are

$(1, 7)$ and $\left(-\dfrac{1}{2}, -\dfrac{1}{2}\right)$

Question

Find the cartesian equation of the curve whose parametric equations are

$$x = \frac{2at}{1+t^2}, y = \frac{a(1-t^2)}{1+t^2}, t \in R, a > 0$$

Answer

Given equations are $x = \dfrac{2at}{1+t^2}$ .... (1)

$$y = \frac{a(1-t^2)}{1+t^2}$$ .... (2)

$\Rightarrow$ put $t = \tan\theta$ to get
$x = a\sin 2\theta, y = a\cos 2\theta$

Squaring and adding these equations, we get

$x^2 + y^2 = a^2(\sin^2 2\theta + \cos^2 2\theta)$
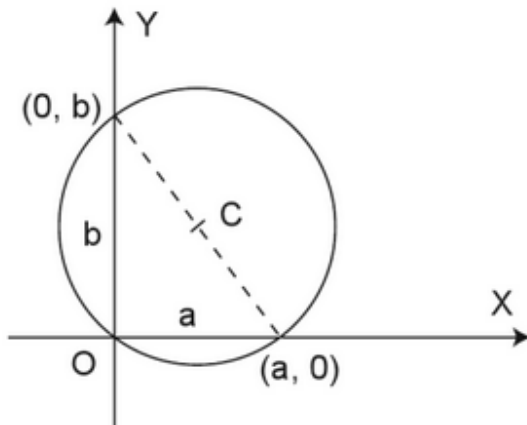
$\Rightarrow \quad x^2 + y^2 = a^2$

Question

Find the equation of the circle passing through $(0, 0)$ and making intercepts a and b on the **coordinate** axes.

[NCERT]

Answer

As the circle passes through the origin and makes intercepts a and b on x-axis and y-axis respectively, therefore, the points $A(a, 0)$ and $B(0, b)$ also lie on the circle in reference.

As $\angle AOB = 90°$, therefore, $[AB]$ is a diameter of the circle. Using diameter form, the equation of the circle is $(x - a)(x - 0) + (y - 0)(y - b) = 0 \mid$ Diameter form

or $\quad x^2 + y^2 - ax - by = 0$

Question

Find the co-ordinates of the points of intersection of the line $2x + y - 1 = 0$ and the circle $x^2 + y^2 + 6x - 4y + 8 = 0$ and show that given line is a tangent to the circle.

Answer

Equation of line is $2x + y - 1 = 0$

$\Rightarrow \quad y = 1 - 2x$ .... (1)

Equation of circle is $x^2 + y^2 + 6x - 4y + 8 = 0$

.... (2)

Putting the value of y in equation (2) from equation (1), we get

$\quad x^2 + (1 - 2x)^2 + 6x - 4(1 - 2x) + 8 = 0$

$\Rightarrow \quad x^2 + 1 - 4x + 4x^2 + 6x - 4 + 8x + 8 = 0$

$\Rightarrow \quad 5x^2 + 10x + 5 = 0$

$\Rightarrow \quad x^2 + 2x + 1 = 0$

$\Rightarrow \quad (x + 1)^2 = 0$

$\Rightarrow \quad (x + 1) \cdot (x + 1) = 0$

$\therefore \quad x = -1, -1$

$\therefore \quad y = 1 - 2(-1) = 1 + 2 = 3$

$\therefore \quad$ Points of intersection are $(-1, 3)$ and $(-1, 3)$

Hence points of intersection are coincident i.e., one point. Therefore by definition of tangency, given line $2x + y - 1 = 0$ is tangent to the circle $x^2 + y^2 + 6x - 4y + 8 = 0$.

Question

Find the value of p if the line $x \cos \alpha + y \sin \alpha = p$ touches the circle $x^2 + y^2 - 12x = 0$.

Answer

Given circle is, $x^2 + y^2 - 12x = 0$
Here, $2g = -12$, $2f = 0$ and $c = 0$
$\Rightarrow \quad g = -6, f = 0$ and $c = 0$
$\therefore \quad$ Centre of the circle $= (-g, -f) = (6, 0)$ and

$\quad$ Radius of the circle $= \sqrt{g^2 + f^2 - c}$

$$= \sqrt{36 + 0 - 0} = 6$$

Since the line $x \cos \alpha + y \sin \alpha = p$ touches the circle, therefore length of the perpendicular drawn from centre $(6, 0)$ on this line is equal to radius 6.

$\therefore \quad \dfrac{6\cos\alpha + 0 - p}{\sqrt{\cos^2\alpha + \sin^2\alpha}} = \pm 6$

$\Rightarrow \quad 6 \cos \alpha - p = \pm 6$
$\Rightarrow \quad p = 6 \cos \alpha \pm 6$
$\Rightarrow \quad \quad = 6 (\cos \alpha - 1), 6 (\cos \alpha + 1)$
$\Rightarrow \quad p = 6 (\cos \alpha + 1),$

$\quad\quad\quad\quad (\because p$ is always positive$)$

$$= 6 \left(2\cos^2 \frac{\alpha}{2} - 1 + 1 \right)$$

$\Rightarrow \quad p = 12 \cos^2 \dfrac{\alpha}{2}$

Question

Point $(3, -1)$ is centre of a circle. This circle cuts off a chord of length 6 units on the line $2x + 5y + 18 = 0$. Find the equation of the circle.
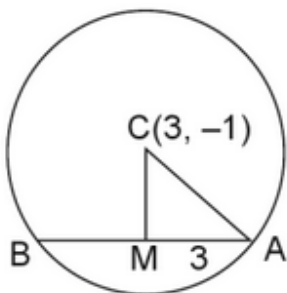
Answer

Given, centre of the circle is $(3, -1)$ and equation of the chord AB is $2x + 5y + 18 = 0$
.... (1)

Draw a perpendicular CM on the line AB,

then $MA = \dfrac{1}{2} AB = \dfrac{1}{2} \times 6 = 3$

Now, $CM =$ Length of the perpendicular drawn from $C(3, -1)$ on the line (1).

$$= \frac{2 \times 3 + 5 \times (-1) + 18}{\sqrt{4 + 25}} = \frac{6 - 5 + 18}{\sqrt{29}} = \frac{19}{\sqrt{29}}$$

$\therefore$ Radius of the circle. $CA = \sqrt{CM^2 + MA^2}$

$$= \sqrt{\left(\frac{19}{\sqrt{29}}\right)^2 + 3^2} = \sqrt{\frac{361}{29} + 9}$$

$$= \sqrt{\frac{361 + 261}{29}}$$

$$= \sqrt{\frac{622}{29}}$$

$\therefore$ Equation of the circle is $(x - 3)^2 + (y + 1)^2$

$$= \frac{622}{29}$$

$\Rightarrow \quad 29(x^2 - 6x + 9 + y^2 + 2y + 1) = 622$

$\Rightarrow \quad 29x^2 + 29y^2 - 174x + 58y + 290 - 622 = 0$

$\Rightarrow \quad 29x^2 + 29y^2 - 174x + 58y - 332 = 0$

Question

Find the tangents to the circle $x^2 + y^2 = a^2$ which make the triangle with axes of area a^2.

Answer

Any tangent to the circle $x^2 + y^2 = a^2$ be

$$y = mx \pm a\sqrt{1+m^2} \qquad \ldots\ldots (1)$$

This line cuts the X-axis at A. when $y = 0$

$$\therefore \quad 0 = mx \pm a\sqrt{1+m^2}$$

$$\Rightarrow \quad x = \mp \frac{a\sqrt{1+m^2}}{m}$$

Again, tangent (1) cuts the Y-axis at B, where $x = 0$

$$\therefore \quad y = \pm a\sqrt{1+m^2}$$

$$\Rightarrow \quad OB = \pm a\sqrt{1+m^2}$$

$\therefore$ Area of the triangle formed by axes and tangent

$$= \frac{1}{2}OA.OB = \frac{1}{2}\left(\mp\frac{a\sqrt{1+m^2}}{m}\right).\left(\pm a\sqrt{1+m^2}\right)$$

$$= \mp\frac{1}{2}a^2.\frac{1+m^2}{m}$$

According to question, $\pm\dfrac{a^2(1+m^2)}{2m} = a^2$

$$\Rightarrow \quad 1 + m^2 = \pm 2m$$

$$\Rightarrow \quad 1 + m^2 \mp 2m = 0$$

$\Rightarrow \quad (1 \mp m)^2 = 0$

$\Rightarrow \quad m = \pm 1$

Putting this value of m in equation (1),

$y = \pm x \pm a \sqrt{1+1}$

$\Rightarrow \quad y = \pm x \pm a \sqrt{2}$

Which are the equation of tangents.

Question

If the line $3x + 4y - 1 = 0$ touches the circle $(x - 1)^2 + (y - 2)^2 = r^2$, then find the value of r.

Answer

Given line is $3x + 4y - 1 = 0$ .... (1)

Given circle is $(x - 1)^2 + (y - 2)^2 = r^2$ ...... (2)

Its centre is $(1, 2)$ and radius is 2.

If line (1) touches the circle (2), then length of the perpendicular drawn from centre $(1, 2)$ to the line (1) is equal to radius r.

i.e., $r = \dfrac{3 \times 1 + 4 \times 2 - 1}{\sqrt{9+16}} = \dfrac{3 + 8 - 1}{\sqrt{25}} = \dfrac{10}{5} = 2$

Question

Under which one of the following conditions does the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ meet the x-axis in two points on opposite sides of the origin?   *[NDA-2007]*

(a) $c > 0$ (b) $c < 0$

(c) $c = 0$ (d) $c \leq 0$

Answer

(b) The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ meets x-axis ($y = 0$) in two points on opposite sides of origin.

If mean $x^2 + 2gx + c = 0$

$\Rightarrow \quad x = \dfrac{-2g \pm \sqrt{4g^2 - 4c}}{2}$

$\Rightarrow \quad x = -g \pm \sqrt{g^2 - c}$

Circle meets the x-axis in two points on opposite side of origin.

Hence, $-g + \sqrt{g^2 - c} > 0$

$\qquad -g - \sqrt{g^2 - c} < 0$

$\Rightarrow \quad \sqrt{g^2 - c} > g \Rightarrow g^2 - c > g^2$

$\Rightarrow \quad -c > 0 \therefore c < 0$

Question

If points $(0, 0)$; $(1, 0)$; $(0, 2)$ and $(0, a)$ are concyclic, then a is equal to

[PET (Raj.)-97]

(a) $0, 1$ (b) $0, 2$

(c) $-1, 2$ (d) None of these

Answer

(b) Obviously $(0, 0)$; $(1, 0)$ and $(0, 2)$ are vertices of a right angled triangle, so equation of the circle passing through these points is

$$(x - 1)x + y(y - 2) = 0$$

$$\Rightarrow \quad x^2 + y^2 - x - 2y = 0$$

Since this also passes through $(0, a)$,

so $\quad a^2 - 2a = 0 \Rightarrow a = 0, 2$

Question

The side of an equilateral triangle drawn in the circle $x^2 + y^2 = a^2$ is

[UPSEAT-99]

(a) $\sqrt{2}a$

(b) $\sqrt{3}a$

(c) $\sqrt{3/2}a$

(d) None of these

Answer

(b) Using cosine formula in $\triangle AOB$

$$\cos 120° = \frac{a^2 + a^2 - AB^2}{2a.a}$$

$$\Rightarrow AB = \sqrt{3}a$$

Question

Let L_1 be a **straight** line passing through the origin and L_2 be the **straight** line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, then which of the following equations can represent L_1

[IIT-1999]

(a) $x + y = 0$
(b) $x - y = 0$
(c) $x + 7y = 0$
(d) $x - 7y = 0$

Answer

(b, c) Let the equation of line passing through origin be $y = mx$. Therefore
We know that chord of equal length are equidistant from centre.

$\therefore$ Distance of both chord are equal centre

$$\frac{\dfrac{1-3m}{1+m^2}}{\sqrt{\left(\dfrac{1-3m}{1+m^2}\right)^2 + m^2 \left(\dfrac{1-3m}{1+m^2}\right)^2}}$$

Only solving we get

$\Rightarrow$ $\quad 7m^2 - 6m - 1 = 0$

$\Rightarrow$ $\quad (7m + 1)(m - 1) = 0$

$\Rightarrow$ $\quad m = 1, -\dfrac{1}{7}$

$\therefore$ $\quad$ Lines are $y = x, y = -\dfrac{1}{7}x$

or $\quad x - y = 0,$

$\qquad x + 7y = 0.$

Question

The equation of the chord of the circle $x^2 + y^2 = a^2$ having (x_1, y_1) as its mid-points is

[IIT-83; PET-86; CET-03]

(a) $xy_1 + yx_1 = a^2$

(b) $x_1 + y_1 = a$

(c) $xx_1 + yy_1 = x_1^2 + y_1^2$

(d) $xx_1 + yy_1 = a^2$

Answer

(c) $T = S_1$ is the equation of desired chord,
hence $xx_1 + yy_1 - a^2 = x_1^2 + y_1^2 - a^2$

$\Rightarrow$ $\quad xx_1 + yy_1 = x_1^2 + y_1^2$

Question

ABCD is a square the length, of whose side is *a*. Taking *AB* and *AD* as the **coordinate** axes, the equation of the circle passing through the vertices of the square is

(a) $x^2 + y^2 + ax + ay = 0$
(b) $x^2 + y^2 - ax - ay = 0$
(c) $x^2 + y^2 + 2ax + 2ay = 0$
(d) $x^2 + y^2 - 2ax - 2ay = 0$

Answer

(b) According to the figure $A(0, 0)$, $B(a, 0)$ and $D(0, a)$ and centre is $\left(\dfrac{a}{2}, \dfrac{a}{2}\right)$. Therefore, the equation of circle is $\left(x - \dfrac{a}{2}\right)^2 + \left(y - \dfrac{a}{2}\right)^2 = \dfrac{a^2}{2}$

$\Rightarrow \quad x^2 + y^2 - ax - ay = 0$.

Question

The locus of the middle points of those chords of the circle $x^2 + y^2 = 4$ which subtend a right angle at the origin is

[IIT-84; PET-97]

(a) $x^2 + y^2 - 2x - 2y = 0$
(b) $x^2 + y^2 = 4$
(c) $x^2 + y^2 = 2$
(d) $(x - 1)^2 + (y - 1)^2 = 5$

Answer

(c) Let the mid-point of chord is (h, k). Also radius of circle is 2. Therefore, $\dfrac{oc}{ob} = \cos 45°$

$\Rightarrow \quad \dfrac{\sqrt{h^2 + k^2}}{2} = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad h^2 + k^2 = 2$

Hence locus is $x^2 + y^2 = 2$

Question

$y = mx$ is a chord of a circle of radius a and the diameter of the circle lies among x-axis and one end of this chord is origin. The equation of the circle described on this chord as diameter is

$\hspace{6cm}$ *[PET-90]*

(a) $(1 + m^2)(x^2 + y^2) - 2ax = 0$

(b) $(1 + m^2)(x^2 + y^2) - 2a(x + my) = 0$

(c) $(1 + m^2)(x^2 + y^2) + 2a(x + my) = 0$

(d) $(1 + m^2)(x^2 + y^2) - 2a(x - my) = 0$

Answer

(b) Here the equation of circle is $(x - a)^2 + (y - 0)^2 = a^2$

$\Rightarrow \quad x^2 + y^2 - 2ax = 0$

Now the point of intersection of circle and chord i.e., O and B are $0(0, 0)$ and

$B\left(\dfrac{2a}{1+m^2}, \dfrac{2am}{1+m^2}\right)$

Hence the equation of circle are chord OB's diameter is $(x^2 + y^2)(1 + m)^2 - 2a(x + my) = 0$.

Question

The **straight** line $x - y - 3 = 0$ touches the circle $x^2 + y^2 - 4x + 6y + 11 = 0$ at the point whose co-ordinates are

$\qquad\qquad\qquad\qquad$ *[PET-93]*

(a) $(1, -2)$ $\qquad\qquad$ (b) $(1, 2)$

(c) $(-1, 2)$ $\qquad\qquad$ (d) $(-1, -2)$

Answer

(a) The point must satisfy the line and the circle simultaneously. Obviously the point of contact is $(1, -2)$.

Question

At which point the line $y = x + \sqrt{2}$ a touches to the circle $x^2 + y^2 = a^2$ or Line $y = x + a\sqrt{2}$ is a tangent to the circle $x^2 + y^2 = a^2$ at

(a) $\left(\dfrac{a}{\sqrt{2}}, \dfrac{a}{\sqrt{2}} \right)$

(b) $\left(-\dfrac{a}{\sqrt{2}}, -\dfrac{a}{\sqrt{2}} \right)$

(c) $\left(\dfrac{a}{\sqrt{2}}, -\dfrac{a}{\sqrt{2}} \right)$

(d) $\left(-\dfrac{a}{\sqrt{2}}, \dfrac{a}{\sqrt{2}} \right)$

Answer

(d) Suppose that point be (h, k). Tangent at (h, k) is $hx + ky = a^2 \equiv x - y = -\sqrt{2}a$

or $\quad \dfrac{h}{1} = \dfrac{k}{-1} = \dfrac{a^2}{-\sqrt{2}a}$

or $\quad h = -\dfrac{a}{\sqrt{2}}, k = \dfrac{a}{\sqrt{2}}$.

Question

The centre of the circle $x = -1 + 2\cos\theta, y = 3 + 2\sin\theta$ is $\qquad$ [PET-1995]

(a) $(1, -3)$ (b) $(-1, 3)$

(c) $(1, 3)$ (d) None of these

Answer

(b) Given that $\dfrac{x+1}{2} = \cos\theta$.

Also $\dfrac{y-3}{2} = \sin\theta$

$\Rightarrow \quad \left(\dfrac{x+1}{2}\right)^2 + \left(\dfrac{y-3}{2}\right)^2 = 1$

$\Rightarrow \quad (x+1)^2 + (y-3)^2 = 4.$

Question

$x = 7$ touches the circle $x^2 + y^2 - 4x - 6y - 12 = 0$, then the coordinates of the point of contact are

(a) $(7, 3)$ (b) $(7, 4)$

(c) $(7, 8)$ (d) $(7, 2)$

Answer

(a) Putting $x = 7$, we get $y^2 - 6y + 9 = 0$

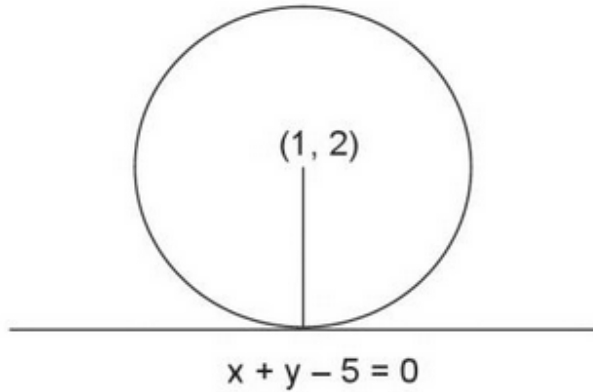$\Rightarrow \quad y = 3$. Hence the point of contact is $(7, 3)$.

Question

The equation of circle with centre $(1, 2)$ and tangent $x + y - 5 = 0$ is   *[PET-01]*

(a) $x^2 + y^2 + 2x - 4y + 6 = 0$

(b) $x^2 + y^2 - 2x - 4y + 3 = 0$

(c) $x^2 + y^2 - 2x + 4y + 8 = 0$

(d) $x^2 + y^2 - 2x - 4y + 86 = 0$

Answer

(b) $\because$ Radius of circle = Perpendicular distance of tangent from the centre of circle.

$$x + y - 5 = 0$$

$\Rightarrow \quad r = \dfrac{1 + 2 - 5}{\sqrt{1+1}} = \sqrt{2}$

Hence the equation of required circle is

$(x - 1)^2 + (y - 2)^2 = (\sqrt{2})^2$

$\Rightarrow \quad x^2 + y^2 - 2x - 4y + 3 = 0$

OR

Verification Method: Centre of circles in options are as follows

(a) $(-1, 2)$ (b) $(1, 2)$

(c) $(1, -2)$ (d) $(-1, -2)$

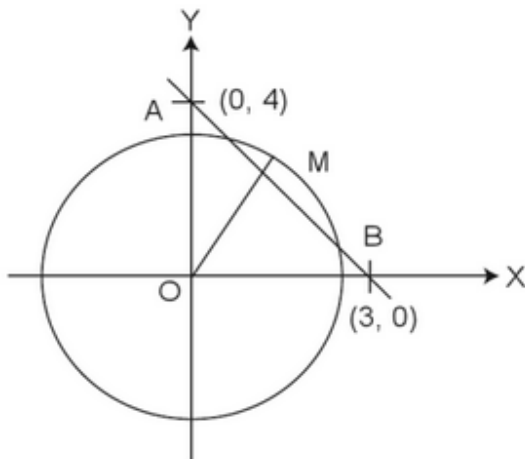So (b) can be the only answer

Question

The length of the chord joining the points in which the **straight** line $\dfrac{x}{3} + \dfrac{y}{4} = 1$ cuts the circle $x^2 + y^2 = \dfrac{169}{25}$ is

[Orissa JEE-2003]

(a) 1
(b) 2
(c) 4
(d) 8

Answer

(b) OM = length of the perpendicular to the line $4x + 3y = 12$ from $(0, 0) = 12/5$

Radius of the circle is $\dfrac{13}{5}$.

Required length $= 2 \sqrt{\dfrac{169}{25} - \dfrac{144}{25}} = 2$

Question

If the line $3x - 2y = k$ meets the circle $x^2 + y^2 = 4r^2$ only at one point, then k is equal to

[CET (Karnataka)-2003]

(a) $52\, r^2$

(b) $20\, r^2$

(c) $(52/9)\, r^2$

(d) $(20/9)\, r^2$

Answer

Line is a tangent to the circle, so
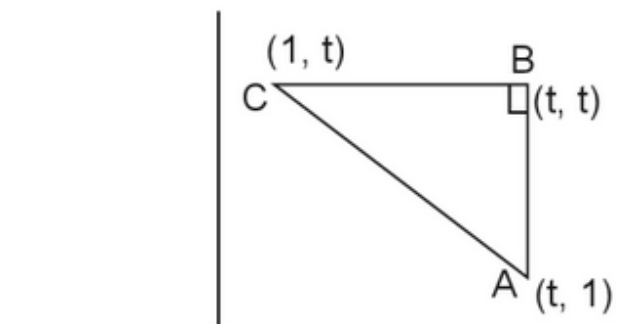
$$p = a \Rightarrow \frac{k}{\sqrt{13}} = 2r$$

Question

Circle passing through the points $(t, 1)$, $(1, t)$ and (t, t) for all values of t passes through the point

[Orissa JEE-2007]

(a) $(0, 0)$

(b) $(1, 1)$

(c) $(1, -1)$

(d) $(-1, 1)$

Answer

3 points form right angled triangle
AC is diameter of circumcircle
equation of circle is

$$(x - t)(x - 1) + (y - 1)(y - t) = 0$$

which clearly pairs through $(1, 1)$

Question

If the equation of the tangent to the circle $x^2 + y^2 - 2x + 6y - 6 = 0$ parallel to $3x - 4y + 7 = 0$ is $3x - 4y + k = 0$, then the values of k are

[Kerala (Engg.)-05]

(a) $5, -35$ (b) $-5, 35$

(c) $7, -32$ (d) $-7, 32$

Answer

(a) Equation of circle is, $x^2 + y^2 - 2x + 6y - 6 = 0$

$$(x - 1)^2 + (y + 3)^2 = (4)^2$$

Radius of circle $= 4$

And centre of circle $= (1, -3)$

Equation of tangent $3x - 4y + k = 0$

$\therefore \quad \dfrac{3 \times 1 - 4 \times (-3) + k = 0}{\sqrt{(3)^2 + (-4)^2}} = \pm 4$.

Hence $k = 5, -35$

Question

If the **straight** line $y = mx$ is outside the circle $x^2 + y^2 - 20y + 90 = 0$, then

[Roorkee-1999]

(a) $m > 3$ (b) $m < 3$

(c) $|m| > 3$ (d) $|m| < 3$

Answer

(d) If the **straight** line $y = mx$ is outside the given circle then distance from centre of circle

> radius of circle $\dfrac{10}{\sqrt{1+m^2}} > \sqrt{10}$

$\Rightarrow \quad (1 + m^2) < 10 \Rightarrow m^2 < 9$

$\Rightarrow \quad |m| < 3.$

Question

The number of distinct tangents that can be drawn from the origin to the circle $x^2 + y^2 = 2(x + y)$ is

[ICS-90]

(a) 0   (b) 1

(c) 2   (d) 3

Answer

(b) Origin lies on the circle, so number of distinct tangent is 1

Question

Consider a family of circles which are passing through the point $(-1, 1)$ and are tangent to x-axis. If (h, k) are the co-ordinates of the centre of the circles, then the set of values of k is given by the interval

[AIEEE-2007]

(a $0 < k < \dfrac{1}{2}$ (b) $k \geq \dfrac{1}{2}$

(c) $-\dfrac{1}{2} \leq k \leq \dfrac{1}{2}$ (d) $k \leq \dfrac{1}{2}$

Answer

(b) $k^2 = (h + 1)^2 + (k - 1)^2$

$\Rightarrow \quad 2k = h^2 + 2h + 2$

$\Rightarrow \quad 2k = (h + 1)^2 + 1$

$\Rightarrow \quad 2k \geq 1$

$\Rightarrow \quad k \geq 1/2$.

Question

The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if

[AIEEE-2010]

(a) $-35 < m < 15$
(b) $15 < m < 65$
(c) $35 < m < 85$
(d) $-85 < m < -35$

Answer

(a) $r = \sqrt{4 + 16 + 5} = 5$

$\left| \dfrac{6 - 16 - m}{5} \right| = 5$

$\Rightarrow \quad -25 < m + 10 < 25$

$\Rightarrow \quad -35 < m < 15$

Hence correct option is (a)

Question

Lines $x = 4$, $y = 2$, $x = -2$ and $y = -3$ represent sides of rectangle. Find the equation of a circle whose one diameter is diagonal of this rectangle.
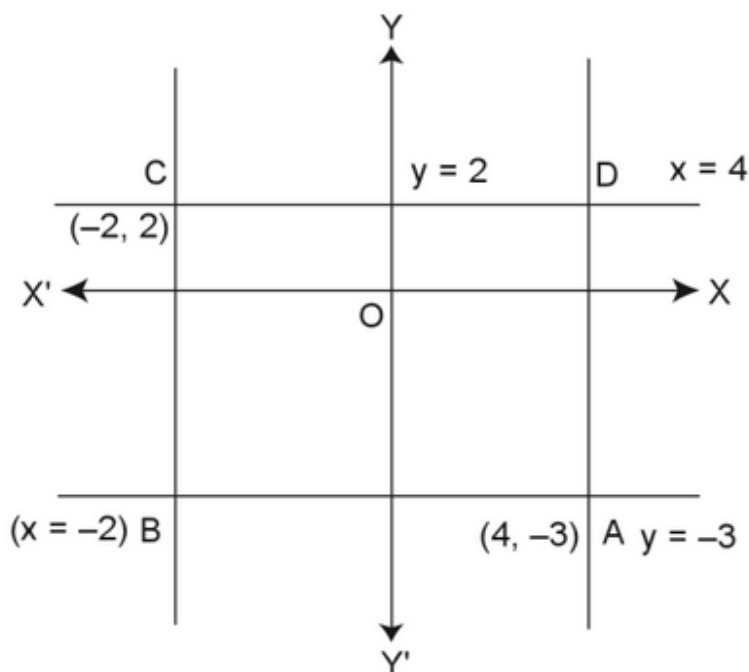
Answer

Let equation of the sides AB, BC, CD and DA of the rectangle are respectively

$$y = -3 \quad \text{..... (1)}, \quad x = -2 \quad \text{......... (2)},$$
$$y = 2 \quad \text{......(3) and } y = 4 \quad \text{............ (4)}$$

Intersection of line (1) and (4) is $A(4, -3)$

Intersection of line (2) and (3) is $C(-2, 2)$

∴ Ends of the diagonal AC are $A(4, -3)$ and $C(-2, 2)$

∴ Equation of the circle whose diameter is AC be

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$
$$\Rightarrow \quad (x - 4)(x + 2) + (y + 3)(y - 2) = 0$$
$$\Rightarrow \quad x^2 - 2x + y^2 + y - 8 - 6 = 0$$
$$\Rightarrow \quad x^2 + y^2 - 2x + y - 14 = 0$$

Again, co-ordinates of points B and D are respectively $(-2, -3,)$ and $(4, 2)$.

∴ Equation of the cirlce whose diameter is BD be

$$(x + 2)(x - 4) + (y + 3)(y - 2) = 0$$
$$\Rightarrow \quad x^2 - 2x - 8 + y^2 + y - 6 = 0 \Rightarrow x^2 + y^2 - 2x + y - 14 = 0$$

Therefore, equation of the circle whose two diameters are diagonal of rectangle be
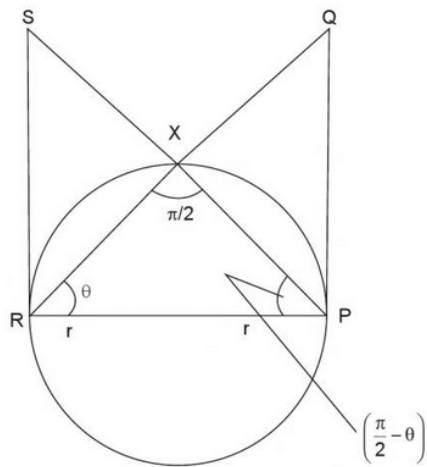$$x^2 + y^2 - 2x + y - 14 = 0$$

Question

Let PQ and RS be tangents at the extremeties of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle, then $2r$ equals   [IIT-01]

(a) $\sqrt{PQ.RS}$

(b) $\dfrac{PQ + RS}{2}$

(c) $\dfrac{2PQ.RS}{PQ + RS}$

(d) $\sqrt{\dfrac{PQ^2 + RS^2}{2}}$

Answer

(a) $\tan\theta = \dfrac{PQ}{PR} = \dfrac{PQ}{2r}$ .....(i),

Also $\tan\theta\left(\dfrac{\pi}{2} - \theta\right) = \dfrac{RS}{2r}$ ......(ii)

i.e., $\cot\theta = \dfrac{RS}{2r}$. Multiplying (i) and (ii), side-wise we find

$\therefore$ $\therefore \tan\theta, \cot\theta = \dfrac{PQ.RS}{4r^2}$ i.e.,

$\Rightarrow$ $4r^2 = PQ.RS \Rightarrow 2r = \sqrt{(PQ)(RS)}$.

Question

If OA and OB be the tangents to the circle $x^2 + y^2 - 6x - 8y + 21 = 0$ drawn from the origin O, then AB is equal to

(a) 11

(b) $\dfrac{4}{5}\sqrt{21}$

(c) $\sqrt{\dfrac{17}{3}}$

(d) None of these

Answer

(b) Here the equation of AB (chord of contact) is

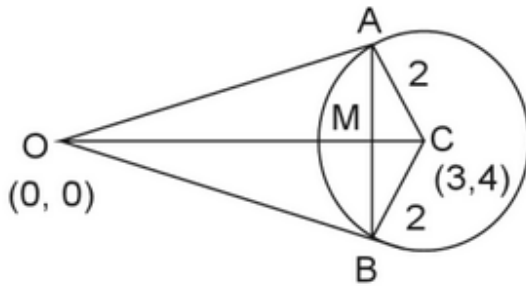$$0 + 0 - 3(x + 0) - 4(y + 0) + 21 = 0$$

$\Rightarrow \quad 3x + 4y - 21 = 0 \qquad \qquad \text{............ (i)}$

CM = perpendicular distance from $(3, 4)$ to line (i) is

$$\frac{3 \times 3 + 4 \times 4 - 21}{\sqrt{9 + 16}} = \frac{4}{5},$$

$$AM = \sqrt{AC^2 - CM^2}$$

$$= \sqrt{4 - \frac{16}{25}} = \frac{2}{5}\sqrt{21}$$

$$\therefore \quad AB = 2AM = \frac{4}{5}\sqrt{21} \ .$$

Question

The equation of the circle which passes through the intersection of $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$ and whose centre lies on $13x + 30y = 0$ is   *[DCE-2001]*

(a) $x^2 + y^2 + 30x - 13y - 25 = 0$
(b) $4x^2 + 4y^2 + 30x - 13y - 25 = 0$
(c) $2x^2 + 2y^2 + 30x - 13y - 25 = 0$
(d) $x^2 + y^2 + 30x - 13y + 25 = 0$

Answer

(b) The equation of required circle is $S_1 + \lambda S_2 = 0$

$\Rightarrow \quad x^2(1 + \lambda) + y^2(1 + \lambda) + x(2 + 13\lambda) - y \left(\frac{7}{2} + 3\lambda\right) - \frac{25}{2} = 0$

$$\text{Centre} = \left(\frac{-(2+13\lambda)}{2}, \frac{\frac{7}{2}+3\lambda}{2} \right)$$

$\because$ Centre lies on $13x + 30y = 0$

$$\Rightarrow \quad -13\left(\frac{2+13\lambda}{2} \right) + 30\left(\frac{\frac{7}{2}+3\lambda}{2} \right) = 0$$

$\Rightarrow \quad \lambda = 1$

Hence the equation of required circle is

$$4x^2 + 4y^2 + 30x - 13y - 25 = 0$$

Question

If P is a point such that the ratio of the squares of the lengths, of the tangents from P to the circles $x^2 + y^2 + 2x - 4y - 20 = 0$ and $x^2 + y^2 - 4x + 2y - 44 = 0$ is 2 : 3, then the locus of P is a circle with centre

[EAMCET-2003]

(a) (7, –8)   (b) (–7, 8)
(c) (7, 8)   (d) (–7, –8)

Answer

(b) $\dfrac{x^2 + y^2 + 2x - 4y - 20}{x^2 + y^2 - 4x + 2y - 44} = \dfrac{2}{3}$

$\Rightarrow \quad x^2 + y^2 + 14x - 16y + 28 = 0$

$\therefore$ Centre = (–7, 8)

Question

A circle C_1 of radius 2 touches both x-axis and y-axis. Another circle C_2 whose radius is greater than 2 touches circle C_1 and both the axes. Then the radius of circle C_2 is

[AMU-2005]

(a) $6 - 4\sqrt{2}$

(b) $6 - 4\sqrt{2}$

(c) $6 - 4\sqrt{3}$

(d) $6 + 4\sqrt{3}$

Answer

(b) First circle touches both axes and radius is 2 unit.

Hence centre of circle is $(2, 2)$.

Let radius of other circle be a and this circle also touches both the axes.

Hence centre of circle is (a, a). This circle touches first circle

Hence, $\sqrt{(a-2)^2 + (a-2)^2} = a + 2$ squaring both the sides,

$\Rightarrow \quad a^2 - 12a + 4 = 0,$

$$a = \frac{12 \pm \sqrt{(12)^2 - 4 \times 4 \times 1}}{2}$$

$$= \frac{12 \pm \sqrt{128}}{2} = 6 \pm 4\sqrt{2}$$

But $a > 2$, Hence $a = 6 - 4\sqrt{2}$ is neglected.

Hence, $a = 6 + 4\sqrt{2}$.

Question

The locus of the centre of a circle which touches externally the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y-axis, is given by the equation   *[IIT-1993; DCE-2000]*
(a) $x^2 - 6x - 10y + 14 = 0$
(b) $x^2 - 10x - 6y + 14 = 0$
(c) $y^2 - 6x - 10y + 14 = 0$
(d) $y^2 - 10x - 6y + 14 = 0$

Answer

(d)   Let the centre be (h, k), then radius $= h$
Also $CC_1 = R_1 + R_2$

or $\sqrt{(h-3)^2 + (k-3)^2} = h + \sqrt{9+9-14}$

$\Rightarrow$ $(h-3)^2 + (k-3)^2 = h^2 + 4 + 4h$

$\Rightarrow$ $k^2 - 10h - 6k + 14 = 0$

or $y^2 - 10x - 6y + 14 = 0$

Question

If $a > 2b > 0$ then the positive value of m for which $y = mx - b\sqrt{1+m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x-a)^2 + y^2 = b^2$, is
                    [IIT (Screening)-2002]

(a) $\dfrac{2b}{\sqrt{a^2 - 4b^2}}$ (b) $\dfrac{\sqrt{a^2 - 4b^2}}{2b}$

(c) $\dfrac{2a}{a - 2b}$ (d) $\dfrac{b}{a - 2b}$

Answer

(a) Any tangent to $x^2 + y^2 = b^2$ is

$$y = mx - b\sqrt{1+m^2}.$$ It touches $(x - a)^2 + y^2 = b^2$

if $\dfrac{ma - b\sqrt{1+m^2}}{\sqrt{m^2+1}} = b$ or $ma = 2b\sqrt{1+m^2}$

or $m^2 a^2 = 4b^2 + 4b^2 m^2$, $\therefore m = \pm \dfrac{2b}{\sqrt{a^2 - 4b^2}}$

A circle is inscribed in an equilateral triangle of side a, the area of any square inscribed in the circle is **[IIT-1994]**
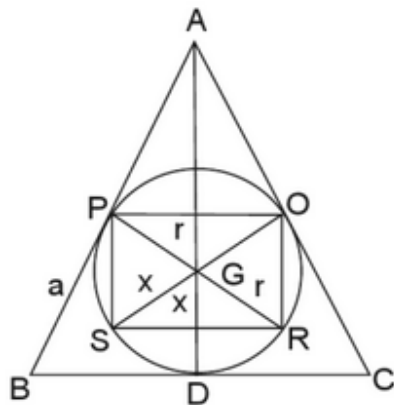
(a) $\dfrac{a^2}{3}$ (b) $\dfrac{2a^2}{3}$

(c) $\dfrac{a^2}{6}$ (d) $\dfrac{a^2}{12}$

Answer

(c) If p be the altitude, then $p = a \sin 60° = \dfrac{a}{2}\sqrt{3}$

Since the triangle is equilateral, therefore centroid, orthocentre, circumcentre and incentre all coincide.

Hence, radius of the inscribed circle $= \dfrac{1}{3}p$

$= \dfrac{a}{2\sqrt{3}} = r$ or diameter $= 2r = \dfrac{a}{\sqrt{3}}$

Now if x be the side of the square inscribed, then angle in a semicircle being a right angle,

hence

$$x^2 + x^2 = d^2 = 4r^2$$

$$\Rightarrow \quad 2x^2 = \dfrac{a^2}{3} \quad \therefore \text{Area} = x^2 = \dfrac{a^2}{6}$$

Question

Points E and F are given by

[IIT-JEE-2008]

(a) $\left(\dfrac{\sqrt{3}}{2}, \dfrac{3}{2}\right), \left(\sqrt{3}, 0\right)$

(b) $\left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right), \left(\sqrt{3}, 0\right)$

(c) $\left(\dfrac{\sqrt{3}}{2}, \dfrac{3}{2}\right), \left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$

(d) $\left(\dfrac{3}{2}, \dfrac{\sqrt{3}}{2}\right), \left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$

Answer

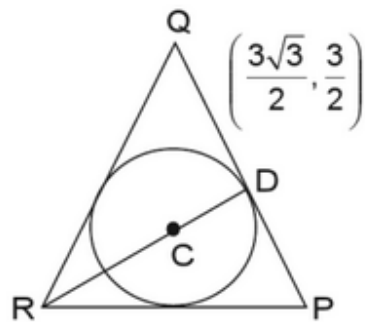(a) C divides RD in the ratio $2 : 1$

As $C \equiv (\sqrt{3}, 1)$ and $D \equiv \left(\dfrac{3\sqrt{3}}{2}, \dfrac{3}{2}\right)$

We have

$$R \equiv \left(\frac{3\sqrt{3} - 2.\frac{3\sqrt{3}}{2}}{3-2}, \frac{3.1 - 2.\frac{3}{2}}{3-2} \right) \equiv (0, 0)$$

The points P and Q are at a distance of $\sqrt{3}$ from D. The equation to PQ in parametric form is

$$\frac{x - \frac{3\sqrt{3}}{2}}{-\frac{1}{2}} = \frac{y - \frac{3}{2}}{\frac{\sqrt{3}}{2}} = \pm\sqrt{3}$$

Which give two points $Q(\sqrt{3}, \ 3)$ and $P(2\sqrt{3}, 0)$

E is the mid-point of $QR \equiv \left(\frac{\sqrt{3}}{2}, \frac{3}{2} \right)$

F is the mid-point of $PR \equiv (\sqrt{3}, 0)$

Question

Equations of the sides QR, RP are

[IIT JEE-2008]

(a) $y = \dfrac{2}{\sqrt{3}}x + 1$, $y = -\dfrac{2}{\sqrt{3}}x - 1$

(b) $y = \dfrac{1}{\sqrt{3}}x$, $y = 0$

(c) $y = \dfrac{\sqrt{3}}{2}x + 1$, $y = -\dfrac{\sqrt{3}}{2}x - 1$

(d) $y = \sqrt{3}x$, $y = 0$

Answer

(d) The equation to RP is simply $y = 0$

The equation to QR is $y - 3$

$$= \sqrt{3}(r - \sqrt{3}) \text{ i.e., } y = \sqrt{3}x$$

Question

The number of tangents drawn from the points (0, 0) to the circle $x^2 + y^2 + 2x + 6y - 15 = 0$ is

(a) one   (b) two

(c) Infinity   (d) none of these.

(MP PET 1992)

Answer

Ans. (d)

Centre of circle is $(-1, -3)$,

$$r = \sqrt{1 + 4 + 15} = \sqrt{20}$$

Distances between $(0, 0)$ and $(-1, -3)$

$$d = \sqrt{1 + 9} = \sqrt{10}$$

$\because$ $d < r$, point $(0, 0)$ lies within the circle.

Question

If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ touches x-axis, then

(a) $g = f$ (b) $g^2 = c$

(c) $f^2 = c$ (d) $g^2 + f^2 = c$.

Answer

Ans. (b)

Since circle touches x-axis, then at $y = 0$ the equation in x is *a* perfect square, *i.e.*, the rods of $x^2 + 2gx + c = 0$ are equal.

$\therefore$ $2\sqrt{g^2 - c} = 0 \Rightarrow g^2 = c$

:-{D

To recall standard integrals

$f(x)$	$\int f(x)dx$	$f(x)$	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1}$ $(n \neq -1)$	$[g(x)]^n g'(x)$	$\frac{[g(x)]^{n+1}}{n+1}$ $(n \neq -1)$
$\frac{1}{x}$	$\ln|x|$	$\frac{g'(x)}{g(x)}$	$\ln|g(x)|$
e^x	e^x	a^x	$\frac{a^x}{\ln a}$ $(a > 0)$
$\sin x$	$-\cos x$	$\sinh x$	$\cosh x$
$\cos x$	$\sin x$	$\cosh x$	$\sinh x$
$\tan x$	$-\ln|\cos x|$	$\tanh x$	$\ln \cosh x$
$\operatorname{cosec} x$	$\ln|\tan \frac{x}{2}|$	$\operatorname{cosech} x$	$\ln|\tanh \frac{x}{2}|$
$\sec x$	$\ln|\sec x + \tan x|$	$\operatorname{sech} x$	$2\tan^{-1} e^x$
$\sec^2 x$	$\tan x$	$\operatorname{sech}^2 x$	$\tanh x$
$\cot x$	$\ln|\sin x|$	$\coth x$	$\ln|\sinh x|$
$\sin^2 x$	$\frac{x}{2} - \frac{\sin 2x}{4}$	$\sinh^2 x$	$\frac{\sinh 2x}{4} - \frac{x}{2}$
$\cos^2 x$	$\frac{x}{2} + \frac{\sin 2x}{4}$	$\cosh^2 x$	$\frac{\sinh 2x}{4} + \frac{x}{2}$

$f(x)$	$\int f(x)\,dx$	$f(x)$	$\int f(x)\,dx$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a}$ $(a>0)$	$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|$ $(0<|x|<a)$
		$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$ $(|x|>a>0)$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\frac{x}{a}$ $(-a<x<a)$	$\frac{1}{\sqrt{a^2+x^2}}$	$\ln\left|\frac{x+\sqrt{a^2+x^2}}{a}\right|$ $(a>0)$
		$\frac{1}{\sqrt{x^2-a^2}}$	$\ln\left|\frac{x+\sqrt{x^2-a^2}}{a}\right|$ $(x>a>0)$
$\sqrt{a^2-x^2}$	$\frac{a^2}{2}\left[\sin^{-1}\left(\frac{x}{a}\right) + \frac{x\sqrt{a^2-x^2}}{a^2}\right]$	$\sqrt{a^2+x^2}$	$\frac{a^2}{2}\left[\sinh^{-1}\left(\frac{x}{a}\right) + \frac{x\sqrt{a^2+x^2}}{a^2}\right]$
		$\sqrt{x^2-a^2}$	$\frac{a^2}{2}\left[-\cosh^{-1}\left(\frac{x}{a}\right) + \frac{x\sqrt{x^2-a^2}}{a^2}\right]$

Some series Expansions -

$$\frac{\pi}{2} = \left(\frac{2}{1}\frac{2}{3}\right)\left(\frac{4}{3}\frac{4}{5}\right)\left(\frac{6}{5}\frac{6}{7}\right)\left(\frac{8}{7}\frac{8}{9}\right)\cdots$$

$$\pi = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \frac{4}{13} - \cdots$$

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$$

$$\pi = \sqrt{12}\left(1 - \frac{1}{3\cdot 3} + \frac{1}{5\cdot 3^2} - \frac{1}{7\cdot 3^3} + \cdots\right)$$

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\int_0^{\pi/2} \log\sin x\, dx = -\frac{\pi}{2}\log 2 = \frac{\pi}{2}\log\frac{1}{2}$$

Solve a series problem

If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$ upto $\infty = \frac{\pi^2}{6}$, then value of

$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ up to ∞ is

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{6}$ (c) $\frac{\pi^2}{8}$ (d) $\frac{\pi^2}{12}$

Ans. (c)

Solution We have $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ upto ∞

$$= \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} \cdots \text{upto } \infty$$

$$- \frac{1}{2^2}\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right]$$

$$= \frac{\pi^2}{6} - \frac{1}{4}\left(\frac{\pi^2}{6}\right) = \frac{\pi^2}{8}$$

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \cdots \infty = \frac{\pi^2}{12}$$

$$\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \cdots \infty = \frac{\pi^2}{24}$$

$$\frac{\sin \sqrt{x}}{\sqrt{x}} = 1 - \frac{x}{3!} + \frac{x^2}{5!} - \frac{x^3}{7!} + \frac{x^4}{9!} - \frac{x^5}{11!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = \sum_{k=0}^{n} \frac{(-1)^k x^{2k}}{(2k)!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots = \sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots = \sum_{k=0}^{n} \frac{x^{2k}}{(2k)!}$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots = \sum_{k=0}^{n} \frac{x^{2k+1}}{(2k+1)!}$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad (-1 \leq x < 1)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} \cdots +$$

$$\frac{2^{2n}\left(2^{2n}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \qquad |x| < \frac{\pi}{2}$$

$$\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \cdots + \frac{E_n x^{2n}}{(2n)!} + \cdots \quad |x| < \frac{\pi}{2}$$

$$\csc x = \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^5}{15120} + \cdots +$$

$$\frac{2\left(2^{2n-1}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \qquad 0 < |x| < \pi$$

$$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - - \cdots - \frac{2^{2n}B_n x^{2n-1}}{(2n)!} - \cdots \quad 0 < |x| < \pi$$

$$\tan x = x + \frac{x^3}{3} + \frac{2\,x^5}{15} + \cdots$$

$$\sec x = 1 + \frac{x^2}{\underline{2}} + \frac{5\,x^4}{\underline{4}} + \cdots$$

$$\log (\cos x) = -\frac{x^2}{\underline{2}} - \frac{2\,x^4}{\underline{4}} - \cdots$$

$$\log (1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \cdots$$

$$\sin^{-1} x = x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \cdots \quad |x| < 1$$

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$= \frac{\pi}{2} - \left(x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \cdots \right) \quad |x| < 1$$

$$\tan^{-1} x = \begin{cases} x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \cdots & |x| < 1 \\[2mm] \pm\dfrac{\pi}{2} - \dfrac{1}{x} + \dfrac{1}{3x^3} - \dfrac{1}{5x^5} + \cdots & \begin{cases} + \text{ if } x \geq 1 \\ - \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$$

$$= \frac{\pi}{2} - \left(\frac{1}{x} + \frac{1}{2\cdot 3x^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5x^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \cdots \right) \quad |x| > 1$$

$$\csc^{-1} x = \sin^{-1}(1/x)$$

$$= \frac{1}{x} + \frac{1}{2\cdot 3x^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5x^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \cdots \quad |x| > 1$$

$$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$$

$$= \begin{cases} \dfrac{\pi}{2} - \left(x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \cdots \right) & |x| < 1 \\[2mm] p\pi + \dfrac{1}{x} - \dfrac{1}{3x^3} + \dfrac{1}{5x^5} + \cdots & \begin{cases} p = 0 \text{ if } x \geq 1 \\ p = 1 \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\ln x = 2\left[\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \dots \right]$$

$$= 2\sum_{n=1}^{\infty} \frac{1}{2n-1}\left(\frac{x-1}{x+1}\right)^{2n-1} \quad (x > 0)$$

$$\ln x = \frac{x-1}{x} + \frac{1}{2}\left(\frac{x-1}{x}\right)^2 + \frac{1}{3}\left(\frac{x-1}{x}\right)^3 + \dots$$

$$= \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{x-1}{x}\right)^n \quad (x > \frac{1}{2})$$

$$\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \dots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1}\frac{1}{n}(x-1)^n \quad (0 < x \le 2)$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \dots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1}\frac{1}{n}x^n \quad (|x| < 1)$$

$$\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots \infty \, (-1 \le x < 1)$$

$$\log_e(1+x) - \log_e(1-x) =$$

$$\log_e \frac{1+x}{1-x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \infty\right) (-1 < x < 1)$$

$$\log_e\left(1 + \frac{1}{n}\right) = \log_e\frac{n+1}{n} = 2 \left[\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \dots \infty\right]$$

$$\log_e(1+x) + \log_e(1-x) = \log_e(1-x^2) = -2\left(\frac{x^2}{2} + \frac{x^4}{4} + \dots \infty\right)(-1 < x < 1)$$

$$\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots = \frac{1}{1\cdot2} + \frac{1}{3\cdot4} + \frac{1}{5\cdot6} + \dots$$

Important Results

(i) (a) $\displaystyle\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x}\, dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cos^n x}{\sin^n x + \cos^n x}\, dx$

(b) $\displaystyle\int_0^{\pi/2} \frac{\tan^n x}{1 + \tan^n x}\, dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{dx}{1 + \tan^n x}$

(c) $\displaystyle\int_0^{\pi/2} \frac{dx}{1 + \cot^n x} = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{1 + \cot^n x}\, dx$

(d) $\displaystyle\int_0^{\pi/2} \frac{\tan^n x}{\tan^n x + \cot^n x}\, dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{\tan^n x + \cot^n x}\, dx$

(e) $\displaystyle\int_0^{\pi/2} \frac{\sec^n x}{\sec^n x + \csc^n x}\, dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\csc^n x}{\sec^n x + \csc^n x}\, dx$ where, $n \in R$

(ii) $\displaystyle\int_0^{\pi/2} \frac{a^{\sin^n x}}{a^{\sin^n x} + a^{\cos^n x}}\, dx = \int_0^{\pi/2} \frac{a^{\cos^n x}}{a^{\sin^n x} + a^{\cos^n x}}\, dx = \frac{\pi}{4}$

(iii) (a) $\displaystyle\int_0^{\pi/2} \log \sin x \, dx = \int_0^{\pi/2} \log \cos x \, dx = -\frac{\pi}{2} \log 2$

(b) $\displaystyle\int_0^{\pi/2} \log \tan x \, dx = \int_0^{\pi/2} \log \cot x \, dx = 0$

(c) $\displaystyle\int_0^{\pi/2} \log \sec x \, dx = \int_0^{\pi/2} \log \csc x \, dx = \frac{\pi}{2} \log 2$

(iv) (a) $\displaystyle\int_0^{\infty} e^{-ax} \sin bx \, dx = \frac{b}{a^2 + b^2}$

(b) $\displaystyle\int_0^{\infty} e^{-ax} \cos bx \, dx = \frac{a}{a^2 + b^2}$

(c) $\displaystyle\int_0^{\infty} e^{-ax} x^n \, dx = \frac{n!}{a^n + 1}$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left(x + \sqrt{x^2 - a^2}\right) + C$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right) + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right) + C$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{a^2 + x^2}\, dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right) + C$$

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Good Luck to you for your Preparations, References, and Exams

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