# Spoon Feeding Trigonometry

**Simplified Knowledge Management Classes Bangalore**

My name is Subhashish Chattopadhyay. I have been teaching for IIT-JEE, Various International Exams ( such as IMO [ International Mathematics Olympiad ], IPhO [ International Physics Olympiad ], IChO [ International Chemistry Olympiad ] ), IGCSE ( IB ), CBSE, I.Sc, Indian State Board exams such as WB-Board, Karnataka PU-II etc since 1989. As I write this book in 2016, it is my 25 th year of teaching. I was a Visiting Professor to BARC Mankhurd, Chembur, Mumbai, Homi Bhabha Centre for Science Education ( HBCSE ) Physics Olympics camp BARC Campus.

**I am Life Member of …**

**- IAPT ( Indian Association of Physics Teachers )**

**- IPA ( Indian Physics Association )**

**- AMTI ( Association of Mathematics Teachers of India )**

**- National Human Rights Association**

**- Men's Rights Movement ( India and International )**

**- MGTOW Movement ( India and International )**

**And also of**

**IACT ( Indian Association of Chemistry Teachers )**

**The selection for National Camp ( for Official Science Olympiads - Physics, Chemistry, Biology, Astronomy ) happens in the following steps ….**

1 ) **NSEP** ( National Standard Exam in Physics ) and **NSEC** ( National Standard Exam in Chemistry ) held around 24 rth November. Approx 35,000 students appear for these exams every year. The exam fees is Rs 100 each. Since 1998 the IIT JEE toppers have been topping these exams and they get to know their rank / performance ahead of others.

2 ) **INPhO** ( Indian National Physics Olympiad ) and **INChO** ( Indian National Chemistry Olympiad ). Around 300 students in each subject are allowed to take these exams. Students coming from outside cities are paid fair from the Govt of India.

3 ) The Top 35 students of each subject are invited at HBCSE ( Homi Bhabha Center for Science Education ) Mankhurd, near Chembur, BARC, Mumbai. After a 2-3 weeks camp the top 5 are selected to represent India. The flight tickets and many other expenses are taken care by Govt of India.

Since last 50 years there has been no dearth of “Good Books“. Those who are interested in studies have been always doing well. This e-Book does not intend to replace any standard text book. These topics are very old and already standardized.

**There are 3 kinds of Text Books**

- The thin Books - Good students who want more details are not happy with these. Average students who need more examples are not happy with these. Most students who want to “Cram” quickly and pass somehow find the thin books “good” as they have to read less !!

**- The Thick Books - Most students do not like these, as they want to read as less as possible. Average students are “busy” with many other things and have no time to read all these.**

- The Average sized Books - Good students do not get all details in any one book. Most bad students do not want to read books of “this much thickness“ also !!

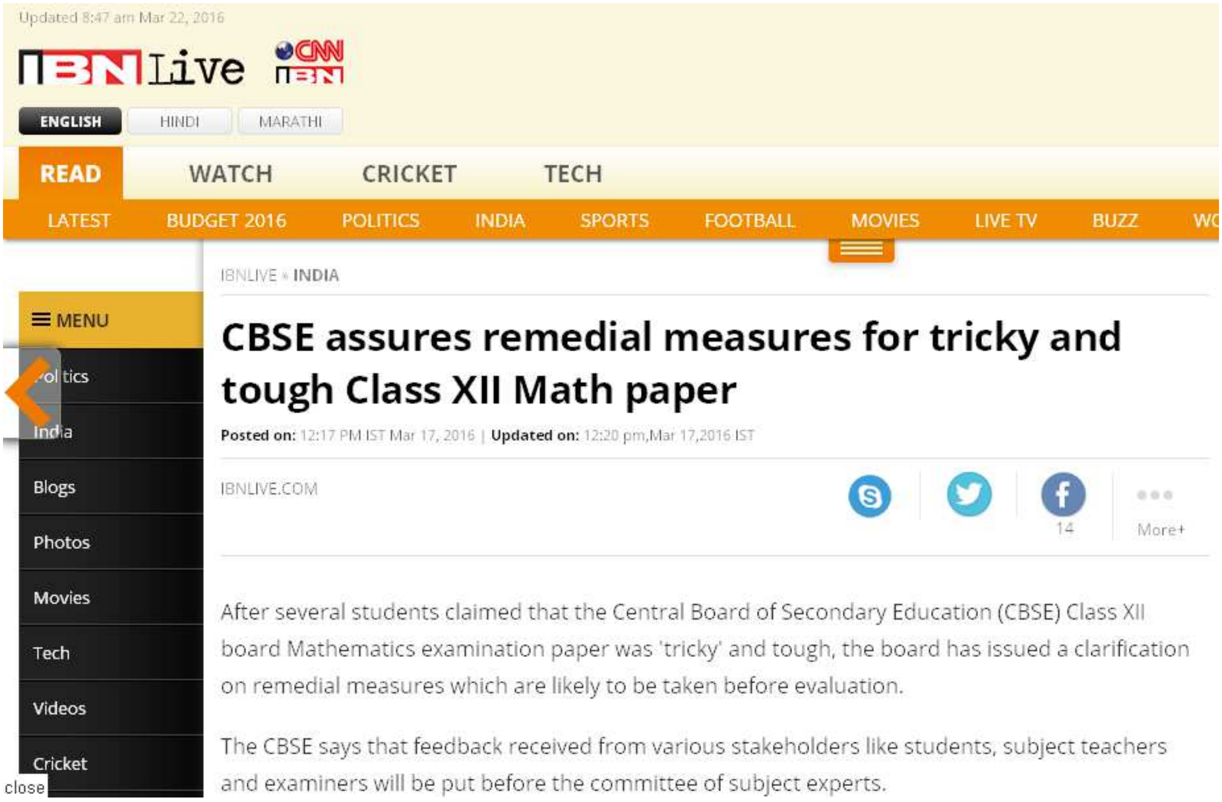
**We know there can be no shoe that’s fits in all.**

Printed books are not e-Books! Can’t be downloaded and kept in hard-disc for reading “later” ………

So if you read this book later, you will get all kinds of examples in a single place. This becomes a very good “Reference Material”. I sincerely wish that all find this “very useful”.

Students who do not practice lots of problems, do not do well. The rules of “doing well” had never changed …. Will never change !

After 2016 CBSE Mathematics exam, lots of students complained that the paper was tough!



Updated 8:47 am Mar 22, 2016

IBNLive CNN IBN

ENGLISH | HINDI | MARATHI

READ | WATCH | CRICKET | TECH

LATEST | BUDGET 2016 | POLITICS | INDIA | SPORTS | FOOTBALL | MOVIES | LIVE TV | BUZZ

MENU: Politics, India, Blogs, Photos, Movies, Tech, Videos, Cricket

IBNLIVE » INDIA

## CBSE assures remedial measures for tricky and tough Class XII Math paper

**Posted on:** 12:17 PM IST Mar 17, 2016 | **Updated on:** 12:20 pm,Mar 17,2016 IST

IBNLIVE.COM

14 More+

After several students claimed that the Central Board of Secondary Education (CBSE) Class XII board Mathematics examination paper was 'tricky' and tough, the board has issued a clarification on remedial measures which are likely to be taken before evaluation.

The CBSE says that feedback received from various stakeholders like students, subject teachers and examiners will be put before the committee of subject experts.

close

On 21 st May 2016 the CBSE standard 12 result was declared. I loved the headline

**INDIATODAY.IN** NEW DELHI, MAY 21, 2016 | UPDATED 16:40 IST

# CBSE Class 12 Results out: No leniency in Maths paper, high paper standard to be maintained in future

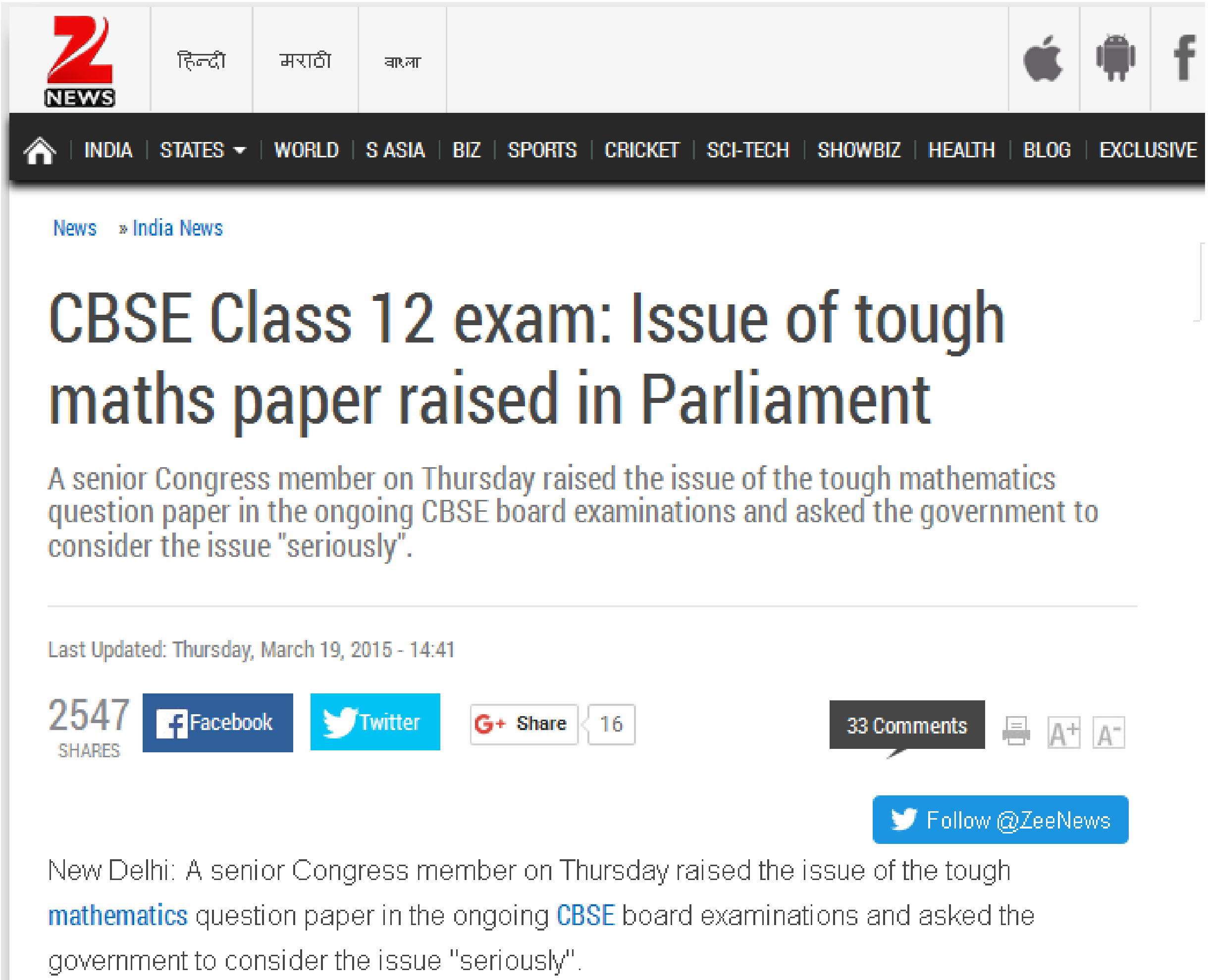
The CBSE Class 12 Mathematics board exam on March 14 reduced many students to tears as they found the paper quite lengthy and tough and many couldn't finish it on time. The results show an overall lowering of marks received in the Maths paper.

**RELATED STORIES**

- CBSE Board result 2016 declared! Thiruvanathpuram obtains the highest part percentage, check how your region scored
- Meet CBSE topper Sukriti Gupta: Check her percentage here!
- CBSE Class 12 Boards 2016: Results announced ahead of time!
- CBSE results declared at www.cbse.nic.in: Steps to check online
- Exclusive! CBSE declares Class 12 Results at www.cbseresults.nic.in and cbse.nic.in

The CBSE (Central Board of Secondary Education) Class 12 Board exam results have been announced today, i.e on May 21, around 10:30 am ahead of time. Students may check their scores at the official website, *www.cbseresults.nic.in*. (**Read: CBSE Class 12 Boards 2016: Results announced ahead of time! Check your score at cbseresults.nic.in**)

In 2015 also the same complain was there by many students



हिन्दी | मराठी | বাংলা

INDIA | STATES | WORLD | S ASIA | BIZ | SPORTS | CRICKET | SCI-TECH | SHOWBIZ | HEALTH | BLOG | EXCLUSIVE

News » India News

# CBSE Class 12 exam: Issue of tough maths paper raised in Parliament

A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".

Last Updated: Thursday, March 19, 2015 - 14:41

2547 SHARES Facebook Twitter Share 16 33 Comments

Follow @ZeeNews

New Delhi: A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".

So we see that by raising frivolous requests, even upto parliament, actually does not help. Many times requests from several quarters have been put to CBSE, or Parliament etc for easy Math Paper. These kinds of requests actually can-not be entertained, never will be.

In March 2016, students of Karnataka PU-II also complained the same, regarding standard 12 ( PU-II Mathematics Exam ). Even though the Math Paper was identical to previous year, most students had not even solved the 2015 Question Paper.

Friday, March 25, 2016 - 13:28

The NEWS Minute

HOME NEWS ANDHRA KARNATAKA KERALA TAMIL NADU TELANGANA CULTURE MEDIA BLOG

Exams

# Online petition for lenient evaluation of K'taka II PU math paper gets over 8000 supporters

The campaign, which was launched on Monday, has garnered over 8000 supporters

TNM Staff | Wednesday, March 16, 2016 - 09:32

Follow @thenewsminute

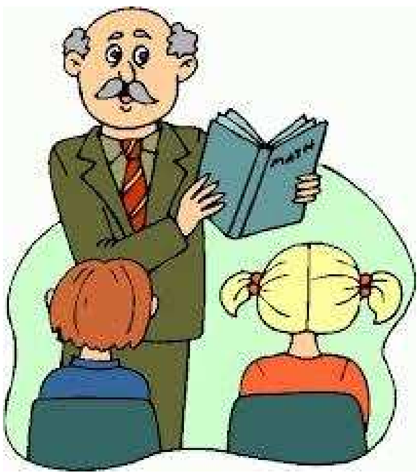
Share Tweet Email Share Reddit

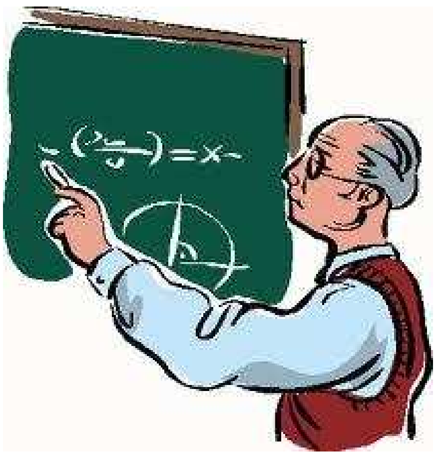
Following a “very tough” math paper that left many II PU students in tears, Saket Ravindran a student launched an online campaign demanding lenient evaluation.

These complains are not new. In fact since last 40 years, ( since my childhood ), I always see this; every year the same setback, same complain!

In this e-Book I am trying to solve this problem. Those students who practice can learn.

**No one can help those who are not studying, or practicing.**





Learn more at http://skmclasses.weebly.com/iit-jee-home-tuitions-bangalore.html

Twitter - https://twitter.com/ZookeeperPhy

Facebook - https://www.facebook.com/IIT.JEE.by.Prof.Subhashish/

Blog - http://skmclasses.kinja.com


### A very polite request :

I wish these e-Books are read only by Boys and Men. Girls and Women, better read something else; learn from somewhere else.

## Preface

We all know that in the species “Homo Sapiens “, males are bigger than females. The reasons are explained in standard 10, or 11 ( high school ) Biology texts. **This shapes or size, influences all of our culture**. Before we recall / understand the reasons once again, let us see some random examples of the influence

Random - 1

If there is a Road rage, then who all fight ? ( generally ? ). Imagine two cars driven by adult drivers. Each car has a woman of similar age as that of the Man. The cars “ touch “ or “ some issue happens”. Who all comes out and fights ? Who all are most probable to drive the cars ?

( Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win )

Random - 2

Heavy metal music artists are all Men. Metallica, Black Sabbath, Motley Crue, Megadeth, Motorhead, AC/DC, Deep Purple, Slayer, Guns & Roses, Led Zeppelin, Aerosmith ….. the list can be in thousands. All these are grown-up Boys, known as Men.

( Men strive for perfection. Men are eager to excel. Men work hard. Men want to win. )

Random - 3

Apart from Marie Curie, only one more woman got Nobel Prize in Physics. ( Maria Goeppert Mayer - 1963 ). So, … almost all are men.

( Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women. )

Random - 4

The best Tabla Players are all Men.


( Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women. )

Random - 5

History is all about, which Kings ruled. Kings, their men, and Soldiers went for wars. History is all about wars, fights, and killings by men.

**Boys start fighting from school days. Girls do not fight like this**

( Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win. )

Random - 6

The highest award in Mathematics, the “ Fields Medal “ is around since decades. Till date only one woman could get that. ( Maryam Mirzakhani - 2014 ). So, … almost all are men.

( Men want to excel. Men strive for perfection. Men want to win. Men work hard. Men do better than women. )

Random - 7

Actor is a gender neutral word. Could the movie like “ Top Gun “ be made with Female actors ? The best pilots, astronauts, Fighters are all Men.

Random - 8

In my childhood had seen a movie named “ The Tower in Inferno “. In the movie when the tall tower is in fire, women were being saved first, as only one lift was working….


Many decades later another movie is made. A box office hit. “ The Titanic “. In this also …. As the ship is sinking women are being saved. **Men are disposable**. Men may get their turn later…

Movies are not training programs. Movies do not teach people what to do, or not to do. Movies only reflect the prevalent culture. Men are disposable, is the culture in the society. Knowingly, unknowingly, the culture is depicted in Movies, Theaters, Stories, Poems, Rituals, etc. I or you can’t write a story, or make a movie in which after a minor car accident the Male passengers keep seating in the back seat, while the both the women drivers come out of the car and start fighting very bitterly on the road. There has been no story in this world, or no movie made, where after an accident or calamity, Men are being helped for safety first, and women are told to wait.

Random - 9

Artists generally follow the prevalent culture of the Society. In paintings, sculptures, stories, poems, movies, cartoon, Caricatures, knowingly / unknowingly, “ the prevalent Reality “ is depicted. The opposite will not go well with people. If deliberately “ the opposite “ is shown then it may only become a special art, considered as a special mockery.



Random - 10

Men go to “girl / woman’s house” to marry / win, and bring her to his home. That is a sort of winning her. When a boy gets a “ Girl-Friend “, generally he and his friends consider that as an achievement. The boy who “ got / won “ a girl-friend feels proud. His male friends feel, jealous, competitive and envious. Millions of stories have been written on these themes. Lakhs of movies show this. Boys / Men go for “ bike race “, or say “ Car Race “, where the winner “ gets “ the most beautiful girl of the college.

( Men want to excel. Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win. )

Prithviraj Chauhan ‘ went ` to “ pickup “ or “ abduct “ or “ win “ or “ bring “ his love. There was a Hindi movie ( hit ) song … “ Pasand ho jaye, to ghar se utha laye “. It is not other way round. Girls do not go to Boy’s house or man’s house to marry. Nor the girls go in a gang to “ pick-up “ the boy / man and bring him to their home / place / den.

Random - 11

We have the word "ice cold". While, when it snows heavily, the cleaning of the roads is done by Men. Ice avalanche is cleared by Guns, by Men.

Can women do this please ?

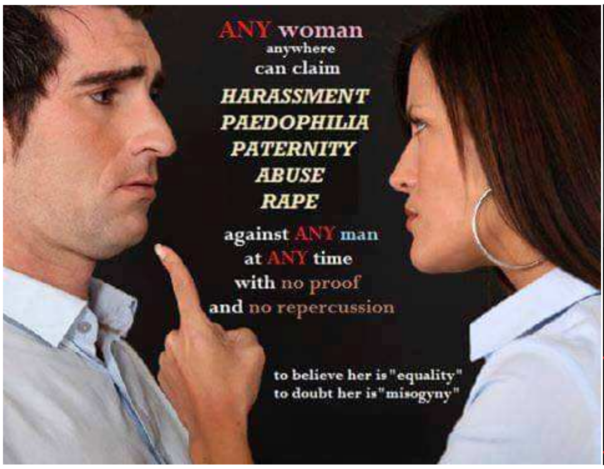
Random - 12

There are many remote mines in this world which are connected by rails through Hilly regions. These railroads move through steep ups and downs. Optimum speed of the train has to be maintained so that the brakes do not burn out, but the next climbing can be done. Sudden braking is not possible as the load of the wagons will derail the train, and will mean huge loss and deaths. The Drivers are Men who risk their lives in every journey.

Random - 13

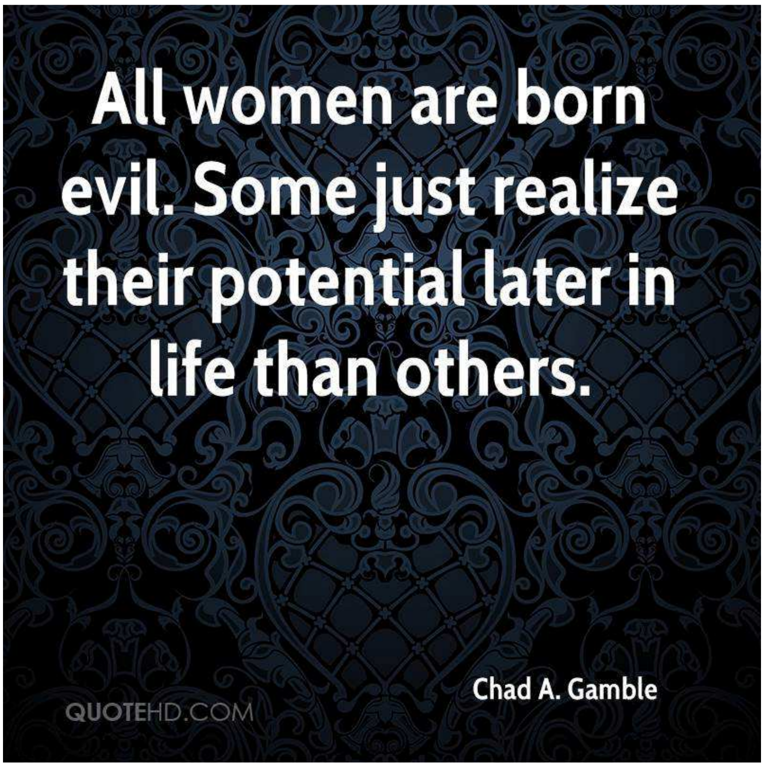
Almost all of us are very biased. Instead of I asking some questions, see the following images

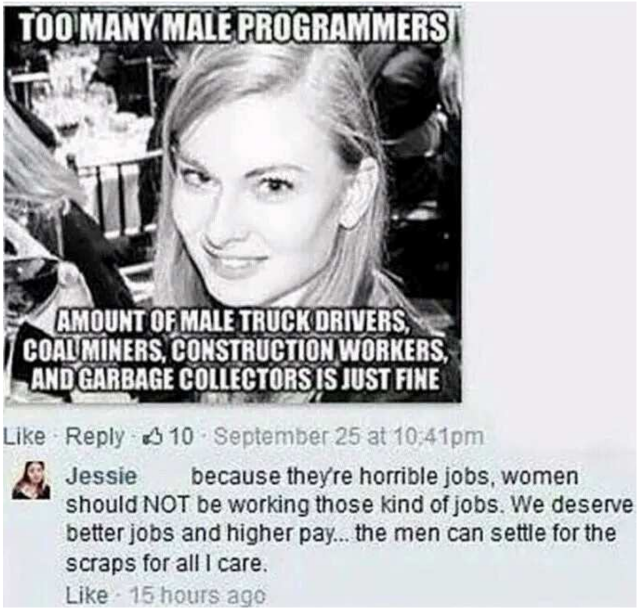






GARBAGE COLLECTOR
ALMOST 100% MEN
NO ONE HOWLS FOR EQUALITY
SEWER WORKER
ALMOST 100% MEN
NO ONE HOWLS FOR EQUALITY
MINER
ALMOST 100% MEN
NO ONE HOWLS FOR EQUALITY
OFFICE WORKER IN
AIR-CONDITIONED ROOM
OMG! EQUALITY, EMANCIPATION,
WOMEN ON THE LABOUR
MARKET, DISCRIMINATION!!!


All women are born evil. Some just realize their potential later in life than others.
Chad A. Gamble
QUOTEHD.COM




**Proof that girls are evil**

First we state that girls require time and money.

GIRLS = TIME × MONEY

And as we all know "time is money"

TIME = MONEY

Therefore:

GIRLS = MONEY × MONEY = $(\text{MONEY})^2$

And because "money is the root of all evil":

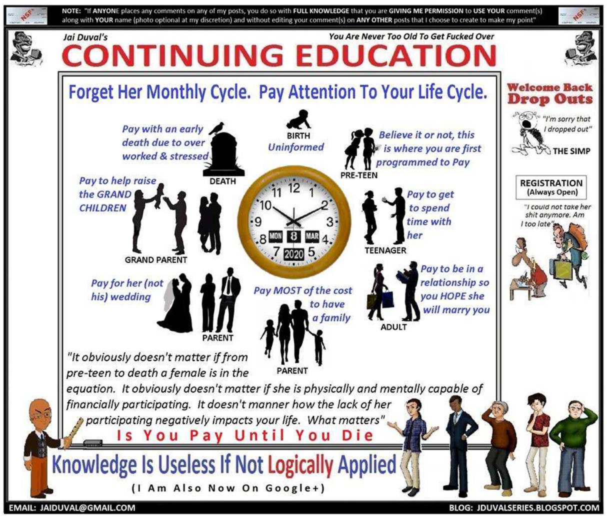
MONEY = $\sqrt{\text{EVIL}}$

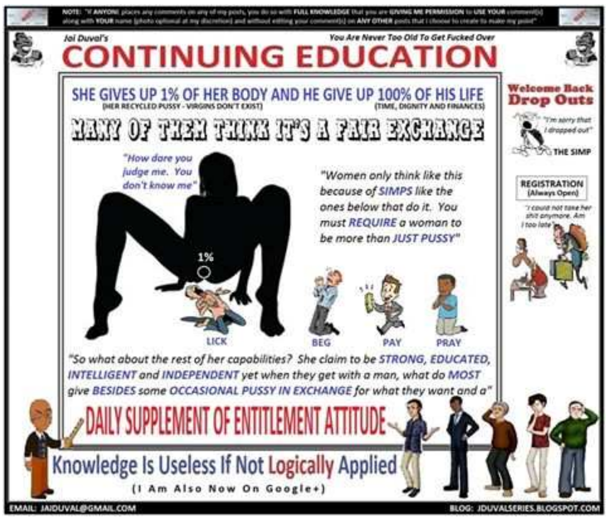
Therefore:

GIRLS = $(\sqrt{\text{EVIL}})^2$

We are forced to conclude that:

GIRLS = EVIL



Random - 14

Rich people; often are very hard working. Successful business men, establish their business ( empire ), amass lot of wealth, with lot of difficulty. Lots of sacrifice, lots of hard work, gets into this. Rich people's wives had no contribution in this wealth creation. Women are smart, and successful upto the extent to choose the right/rich man to marry. So generally what happens in case of Divorces ? Search the net on " most costly divorces " and you will know. The women;( who had no contribution at all, in setting up the business / empire ), often gets in Billions, or several Millions in divorce settlements.

Number 1

Rupert & Anna Murdoch -- $1.7 billion

One of the richest men in the world, **Rupert Murdoch** developed his worldwide media empire when he inherited his father's Australian newspaper in 1952. He married Anna Murdoch in the '60s and they remained together for 32 years, springing off three children.

They split amicably in 1998 but soon Rupert forced Anna off the board of News Corp and the gloves came off. The divorce was finalized in June 1999 when Rupert agreed to let his ex-wife leave with $1.7 billion worth of his assets, $110 million of it in cash. Seventeen days later, Rupert married Wendi Deng, one of his employees.

Ted Danson & Casey Coates -- $30 million

Ted Danson's claim to fame is undoubtedly his decade-long stint as Sam Malone on NBC's celebrated sitcom Cheers . While he did other TV shows and movies, he will always be known as the bartender of that place where everybody knows your name. He met his future first bride Casey, a designer, in 1976 while doing Erhard Seminars Training.

Ten years his senior, she suffered a paralyzing stroke while giving birth to their first child in 1979. In order to nurse her back to health, Danson took a break from acting for six months. But after two children and 15 years of marriage, the infatuation fell to pieces. Danson had started seeing Whoopi Goldberg while filming the comedy, Made in America and this precipitated the 1992 divorce. Casey got $30 million for her trouble.

See https://zookeepersblog.wordpress.com/misandry-and-men-issues-a-short-summary-at-single-place/

See http://skmclasses.kinja.com/save-the-male-1761788732

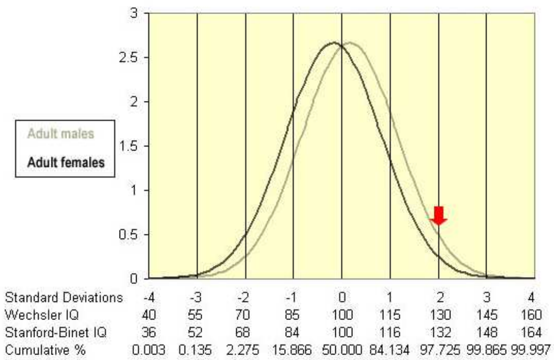
It was Boys and Men, who brought the girls / women home. The Laws are biased, completely favoring women. The men are paying for their own mistakes.

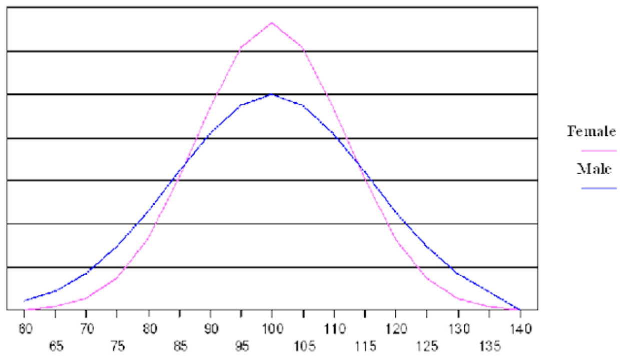
See https://zookeepersblog.wordpress.com/biased-laws/

( Man brings the Woman home. When she leaves, takes away her share of big fortune! )

Random - 15

A standardized test of Intelligence will never be possible. It never happened before, nor ever will happen in future; where the IQ test results will be acceptable by all. In the net there are thousands of charts which show that the intelligence scores of girls / women are lesser. Debates of Trillion words, does not improve performance of Girls.




I am not wasting a single second debating or discussing with anyone, on this. I am simply accepting ALL the results. IQ is only one of the variables which is required for success in life. Thousands of books have been written on " Networking Skills ", EQ ( Emotional Quotient ), Drive, Dedication, Focus, " Tenacity towards the end goal " … etc. In each criteria, and in all together, women ( in general ) do far worse than men. Bangalore is known as " ….. capital of India ". [ Fill in the blanks ]. The blanks are generally filled as " Software Capital ", " IT Capital ", " Startup Capital ", etc. I am member in several startup eco-systems / groups. I have attended hundreds of meetings, regarding " technology startups ", or " idea startups ". These meetings have very few women. Starting up new companies are all " Men's Game " / " Men's business ". Only in Divorce settlements women will take their goodies, due to Biased laws. There is no dedication, towards wealth creation, by women.

Random - 16

Many men, as fathers, very unfortunately treat their daughters as " Princess ". Every " non-performing " woman / wife was " princess daughter " of some loving father. Pampering the girls, in name of " equal opportunity ", or " women empowerment ", have led to nothing.


See http://skmclasses.kinja.com/progressively-daughters-become-monsters-1764484338

See http://skmclasses.kinja.com/vivacious-vixens-1764483974

There can be thousands of more such random examples, where " Bigger Shape / size " of males have influenced our culture, our Society. **Let us recall the reasons**, that we already learned in standard 10 - 11, Biology text Books. In humans, women have a long gestation period, and also spends many years ( almost a decade ) to grow, nourish, and stabilize the child. ( Million years of habit ) Due to survival instinct Males want to inseminate. Boys and Men fight for the " facility ( of womb + care ) " the girl / woman may provide. Bigger size for males, has a winning advantage. Whoever wins, gets the " woman / facility ". The male who is of " Bigger Size ", has an advantage to win…. Leading to Natural selection over millions of years. In general " Bigger Males "; the " fighting instinct " in men; have led to wars, and solving tough problems ( Mathematics, Physics, Technology, startups of new businesses, Wealth creation, Unreasonable attempts to make things [ such as planes ], Hard work …. )

**So let us see the IIT-JEE results of girls.** Statistics of several years show that there are around 17, ( or less than 20 ) girls in top 1000 ranks, at all India level. Some people will yet not understand the performance, till it is said that … year after year we have around 980 boys in top 1000 ranks. Generally we see only 4 to 5 girls in top 500. In last 50 years not once any girl topped in IIT-JEE advanced. Forget about Single digit ranks, double digit ranks by girls have been extremely rare. It is all about " good boys ", " hard working ", " focused ", "**Bel-esprit** " **boys**.

**In 2015, Only 2.6% of total candidates who qualified are girls ( upto around 12,000 rank ). while 20% of the Boys, amongst all candidates qualified. The Total number of students who appeared for the exam were around 1.4 million for IIT-JEE main. Subsequently 1.2 lakh ( around 120 thousands ) appeared for IIT-JEE advanced.**

IIT-JEE results and analysis, of many years is given at https://zookeepersblog.wordpress.com/iit-jee-iseet-main-and-advanced-results/

In Bangalore it is rare to see a girl with rank better than 1000 in IIT-JEE advanced. We hardly see 6-7 boys with rank better than 1000. Hardly 2-3 boys get a rank better than 500.

See http://skmclasses.weebly.com/everybody-knows-so-you-should-also-know.html

Thousands of people are exposing the heinous crimes that Motherly Women are doing, or Female Teachers are committing. See https://www.facebook.com/WomenCriminals/

Some Random Examples must be known by all


Mother Admits On Facebook to Sleeping with 15 Yr Old Son, They Have a Baby Together - Alwayzturntup

Sometimes it hard to believe w From Alwayzturntup

ALWAYZTURNTUP.ME

It is extremely unfortunate that the " woman empowerment " has created. This is the kind of society and women we have now. I and many other sensible Men hate such women. Be away from such women, be aware of reality.


'Sex with my son is incredible - we're in love and we want a baby'

Ben Ford, who ditched his wife when he met his mother Kim West after 30 years, claims what the couple are doing 'isn't incest'

MIRROR.CO.UK

Former Shelbyville ISD teacher who had sex with underage student gets 3 years in prison

After a two day break over the weekend, A Shelby County jury was back in the courtroom looking to conclude the trial of a former Shelbyville ISD teacher who had...

KLTV.COM | BY CALEB BEAMES

Woman sent to jail for the rest of her life after raping her four grandchildren is described as the 'most evil person' the judge has ever seen

Edwina Louis rape...

See More

Woman sent to jail for raping her four grandchildren

A Ohio grandmother has been sentenced to four consecutive life terms after being found guilty of the rape of her own grandchildren. Edwina Louis, 53, will spend the rest of her life behind bars.

DAILYMAIL.CO.UK

http://www.thenativecanadian.com/…/eastern-ontario-teacher-…


The N.C. Chronicles.: Eastern Ontario teacher charged with 36 sexual offences

anti feminism, Child abuse, children's rights, Feminist hypocrisy,

THENATIVECANADIAN.COM | BY BLACKWOLF

Hyd woman kills newborn boy as she wanted daughter - Times of India

Having failed to bear a daughter for the third time, a shopkeeper's wife slit the throat of her 24day-old son with a shaving blade and left him to die in a street on Tuesday night.Purnima's first child was a stillborn boy, followed by another boy born five years ago.

TIMESOFINDIA.INDIATIMES.COM

Montgomery's son, Alan Vonn Webb, took the stand and was a key witness in her conviction.

"I want to see her placed somewhere she can never do that to children ...

See More

Woman sentenced to 40 years in prison for raping her children

A Murfreesboro mother found guilty of raping her own children learned her fate on Wednesday.

WAFF.COM | BY DENNIS FERRIER

gentler sex? Violence against men.'s photo.


Women, the gentler sex? Violence against men. Like Page

April 8 at 1:38am

In fact, the past decade has seen a dramatic increase in the number of incidents of women raping and sexually assaulting boys and men. On May 2014, Jezebel repo...

In Facebook, and internet + whatsapp etc we have unending number of posts describing frustration of men / husbands on naughty unreasonable women. Most women are very illogical, punic, perfidious, treacherous, naughty, gamey bitches.

We also see zillions of Jokes which basically describe how unreasonable women / girls are. How stupid they are, making life of Boys / Men / Husband a hell.

While each of these girls was someones daughter. Millions of foolish Dads are into Fathers rights movement, who want their daughter back for pampering.

Most girls are being cockered coddled cosseted mollycoddled featherbedded spoilt into brats.

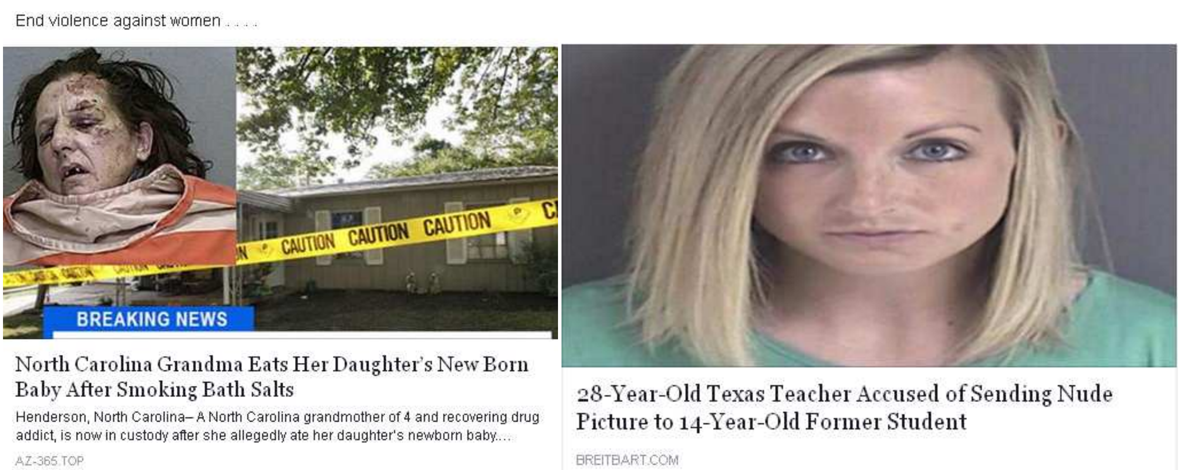
Foolish fathers are breeding Monsters who are filing false rape cases. Enacting Biased Laws. Filing False domestic violence cases. Filing false sexual assault cases. Asking for alimony, and taking custody of the Daughter, not allowing the " monster " to meet dad. The cycle goes on and on and on.

Foolish men keep pampering future demons who make other Men's life a hell. ( Now read this again from beginning ). Every day we see the same posts of frustration.




https://nicewemen.wordpress.com/

Each women as described below was someone's Pampered Princess …



http://latest.com/.../attractive-girl-gang-lured-men-alleywa.../

Attractive Girl Gang Lured Men Into Alleyways Where Female Body Builder Would Attack Them

A Mexican street gang made up entirely of women has been accused of using their feminine wiles to lure men into alleyways and then beating them up and...

LATEST.COM

http://www.wfmj.com/.../youngstown-woman-convicted-of-raping-...

Youngstown woman convicted of raping a 1 year old is back in jail

A Youngstown woman who went to prison for raping a 1-year-old boy fifteen years ago is in trouble with the law again.

WFMJ.COM

Women are raping boys and young men

Rape advocacy has been maligned and twisted into a political agenda controlled by radicalized activists. Tim Patten takes a razor keen and well supported look into the manufactured rape culture and...

AVOICEFORMEN.COM | BY TIM PATTEN

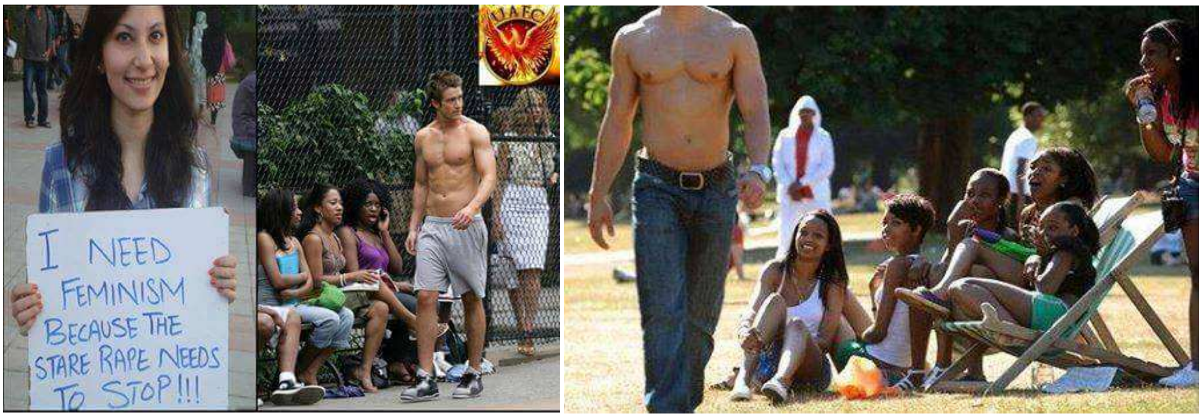
End violence against women . . . .

Bronx Woman Convicted of Poisoning and Drowning Her Children

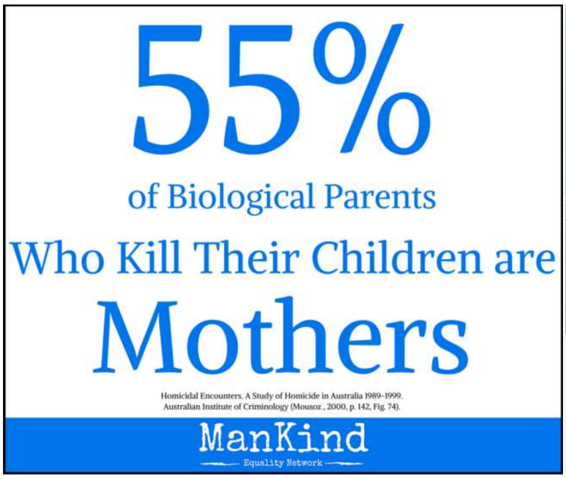
Lisette Bamenga researched methods on the Internet before she killed her son and daughter in 2012.

NYTIMES.COM | BY MARC SANTORA

Monster women have very easy and cozy life. Easy to demand anything and get law in favor !




If the lawmakers submit to these strange demands of say … " Stare Rape ! "; then we can easily see what kind of havoc that will create.


55%
of Biological Parents
Who Kill Their Children are
Mothers
ManKind
Equality Network


FIRST AT 4PM
JOSHUA TREE
EYEWITNESS NEWS
Woman charged with killing baby also had previous infant die
Woman charged with killing baby also had previous infant die
ABC7.COM | BY ROB MCMILLAN


Female Sex Predators: A Crime Epidemic shared a link.
Yesterday at 12:40am
WTVA.com | Woman pleads guilty to having sex with a dog
PITTSBORO, Miss. (WTVA) -- A Calhoun County woman has pleaded guilty to charges she had unnatural intercourse with a dog. Sheriff Greg Poll
WTVA.COM
Like
Comment
Share
Mhra Leander Pallat, Eric Antonio Alvarado and 31 others
Top Comments


Oklahoma Teacher Receives 15-Year Prison Sentence For Sex With 15-Year-Old Boy
A former Oklahoma middle school teacher has pleaded guilty to 6 counts of rape, child enticement...
THREEPERCENTNATION.COM





In several countries or rather in several regions of the world, family system has collapsed, due to bad nature and naughty acts of women. Particularly in Britain, and America, almost 50% people are alone, lonely, separated, divorced or failed marriages. In 2013, 48% children were born out of wedlock. It was projected that by 2016, more than 51% children will be born, to unmarried mothers. In these developed countries " paternity fraud " by women, are close to 20%. You can see several articles in the net, and in wikipedia etc. This means 1 out of 5 children are calling a wrong man as dad. The lonely, alone “ mothers " are frustrated. They see the children as burden. Love in the Society in general is lost, long time ago. The types of “ Mothers “ and “ Women “ we have now …………

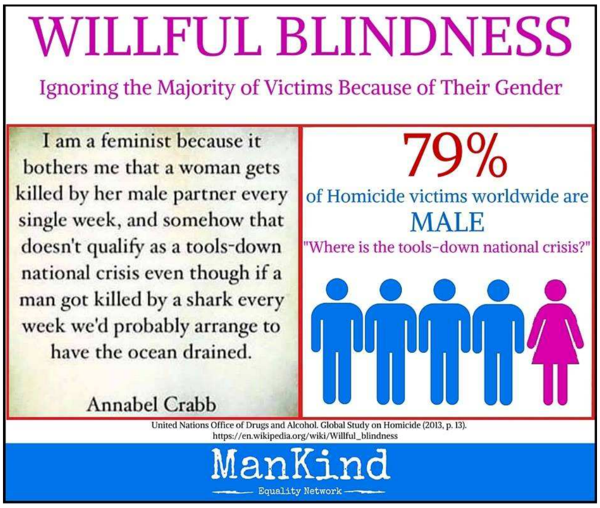

This is the type of women we have in this world. These kind of women were also someones daughter
Mother Stabs Her Baby 90 Times With Scissors After He Bit Her While Breastfeeding Him!
Eight-month-old Xiao Bao was discovered by his uncle in a pool of blood Needed 100 stitches after the incident; he is now recovering in hospital Reports say his...
MOMMABUZZ.COM


MY PUSSY MY RULES
NO GOD IN MY VAGINA
MA CHATTE MES DROITS
c'est sacré!


NATIONAL GO TOPLESS DAY
AUG 23 @ NOON - 3 PM
NYC - CENTRAL PARK
GOING TOPLESS IS LEGAL IN NY !!!


NUDITY ISN'T SEXUAL


TOPLESS : ÉGALITÉ POUR TOUS OU POUR PERSONNE !
SORRY TO BOTHER YOU, BUT WE ARE CHANGING THE WORLD!
LES SEINS DES FEMMES NE SONT PAS INDECENTS




By now if you have assumed that Indian women are not doing any crime then please become friends with MRA Guri https://www.facebook.com/profile.php?id=100004138754180

He has dedicated his life to expose Indian Criminals


MailOnline

U.K. | India | U.S. | News | Sport | TV&Showbiz | Femail | Health | Science | Money | Video |
Latest Headlines | News | World News | Arts | Headlines | France | Pictures | Most read | News Board | Wires

## Mother who had been forced into an arranged marriage is jailed for filming herself having sex with her 14-year-old son and sending the clips to relatives in Pakistan

- Vile mother filmed having sex with her teenage son in sick porn video
- Clips sent to cousin in Pakistan who allegedly asked her to make film
- She also sent her relative indecent images of her three-year-old daughter

By ALEX MATTHEWS FOR MAILONLINE
PUBLISHED: 12:44 GMT, 1 August 2016 | UPDATED: 11:23 GMT, 2 August 2016

Wife Stabs Husband And Runs Away After He Stops Her From Gambling

The husband said his wife had become a habitual gambler who was also addicted to liquor.

INDIATIMES.COM


Teacher learns fate for 6 months of sex with boy

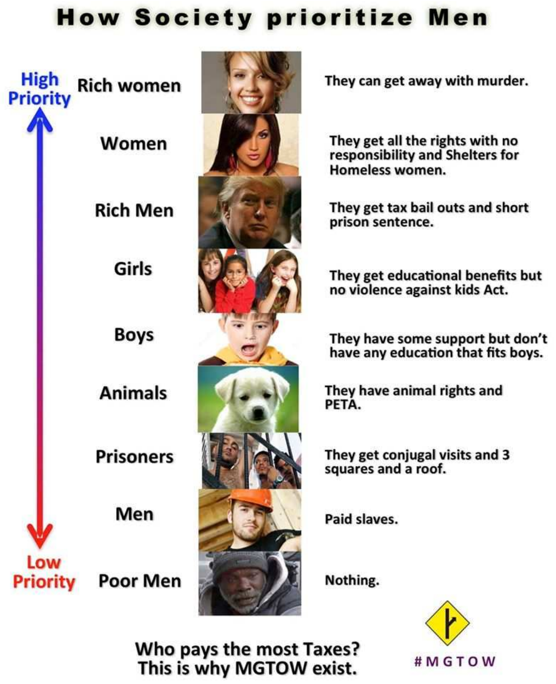
(CBS8) — SAN DIEGO (CNS) – A Crawford High School teacher and coach who carried on a six-month sexual relationship with a 15-year-old male student was sentenced Friday to a two-year prison term. Toni Nicole Sutton, 38, pleaded guilty...

WND.COM

Mom jailed for 40 years after body of daughter, 9, found in fridge

Amber Keyes, 37, was sentenced in the death of Ayahna Comb in Houston on Friday. Ayahna, who had cerebral palsy, had been in the fridge for six months...

DAILYMAIL.CO.UK



Professor Subhashish Chattopadhyay

**Spoon Feeding Series - Trigonometry**

There are hundreds of Trigonometry books and eBooks, which any student can get very easily. In case a student doesn't have money to buy " new " books, he can get " used " or " second hand " books very easily from any "book street ".

The topics, problems, tricks of Trigonometry are very old, often more than 200 or 300 years. So a 10 or 40 year book, just makes no difference at all.

I have met several lazy or stupid students who are not even motivated or interested in remembering the formulae. So forget about applying the formulae to solve a tricky problem. Some trigonometry problems are quite tricky …. Needs lot of practice, even for very intelligent students to solve.

Start like small Children

Find the degree measures corresponding to the following radian measures

$\left(\text{Use } \pi = \frac{22}{7}\right)$.

(i) $\frac{11}{16}$ (ii) $-4$ (iii) $\frac{5\pi}{3}$ (iv) $\frac{7\pi}{6}$

**Ans:**

(i) $\frac{11}{16}$

We know that $\pi$ radian = $180°$

$$\therefore \frac{11}{16} \text{ radain} = \frac{180}{\pi} \times \frac{11}{16} \text{ degree} = \frac{45 \times 11}{\pi \times 4} \text{degree}$$

$$= \frac{45 \times 11 \times 7}{22 \times 4} \text{ degree} = \frac{315}{8} \text{ degree}$$

$$= 39\frac{3}{8} \text{ degree}$$

$$= 39° + \frac{3 \times 60}{8} \text{ minutes} \qquad [1° = 60']$$

$$= 39° + 22' + \frac{1}{2} \text{ minutes}$$

$$= 39°22'30'' \qquad [1' = 60'']$$

(ii) – 4

We know that $\pi$ radian = 180°

$$-4 \text{ radian} = \frac{180}{\pi} \times (-4) \text{ degree} = \frac{180 \times 7(-4)}{22} \text{ degree}$$
$$= \frac{-2520}{11} \text{ degree} = -229\frac{1}{11} \text{ degree}$$
$$= -229° + \frac{1 \times 60}{11} \text{ minutes} \qquad [1° = 60']$$
$$= -229° + 5' + \frac{5}{11} \text{ minutes}$$
$$= -229°5'27" \qquad [1' = 60"]$$

iii) 5π/3

We know that $\pi$ radian = 180°

$$\therefore \frac{5\pi}{3} \text{ radian} = \frac{180}{\pi} \times \frac{5\pi}{3} \text{ degree} = 300°$$

(iv) $\frac{7\pi}{6}$

We know that $\pi$ radian = 180°

$$\therefore \frac{7\pi}{6} \text{ radian} = \frac{180}{\pi} \times \frac{7\pi}{6} = 210°$$

A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

**Ans:**

Number of revolutions made by the wheel in 1 minute = 360

$\therefore$ Number of revolutions made by the wheel in 1 second $= \frac{360}{60} = 6$

In one complete revolution, the wheel turns an angle of $2\pi$ radian.

Hence, in 6 complete revolutions, it will turn an angle of $6 \times 2\pi$ radian, i.e.,

$12\,\pi$ radian

Thus, in one second, the wheel turns an angle of $12\pi$ radian.

-

Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm $\left(\text{Use } \pi = \frac{22}{7}\right)$.

**Ans:**

We know that in a circle of radius $r$ unit, if an arc of length $l$ unit subtends an angle $\theta$ radian at the centre, then

$$\theta = \frac{l}{r}$$

Therefore, for r = 100 cm, l = 22 cm, we have

$$\theta = \frac{22}{100} \text{ radian} = \frac{180}{\pi} \times \frac{22}{100} \text{ degree} = \frac{180 \times 7 \times 22}{22 \times 100} \text{ degree}$$
$$= \frac{126}{10} \text{ degree} = 12\frac{3}{5} \text{ degree} = 12°36' \qquad [1° = 60']$$

Thus, the required angle is $12°36'$.

-

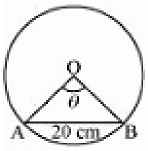
In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

**Ans:**

Diameter of the circle = 40 cm

$\therefore$ Radius ($r$) of the circle $= \frac{40}{2}$ cm = 20 cm

Let AB be a chord (length = 20 cm) of the circle.




In $\Delta OAB$, OA = OB = Radius of circle = 20 cm

Also, AB = 20 cm

Thus, $\Delta OAB$ is an equilateral triangle.

$\therefore \theta = 60° = \frac{\pi}{3}$ radian

We know that in a circle of radius $r$ unit, if an arc of length $l$ unit subtends an angle $\theta$ radian at the centre, then $\theta = \frac{l}{r}$.

$$\frac{\pi}{3} = \frac{\widehat{AB}}{20} \Rightarrow \widehat{AB} = \frac{20\pi}{3} \text{ cm}$$

Thus, the length of the minor arc of the chord is $\frac{20\pi}{3}$ cm.

We know that in a circle of radius $r$ unit, if an arc of length $l$ unit subtends an angle $\theta$ radian at the centre, then $\theta = \frac{l}{r}$.

$$\frac{\pi}{3} = \frac{\widehat{AB}}{20} \Rightarrow \widehat{AB} = \frac{20\pi}{3} \text{ cm}$$

Thus, the length of the minor arc of the chord is $\frac{20\pi}{3}$ cm.

-

If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

**Ans:**

Let the radii of the two circles be $r_1$ and $r_2$. Let an arc of length $l$ subtend an angle of 60° at the centre of the circle of radius $r_1$, while let an arc of length $l$ subtend an angle of 75° at the centre of the circle of radius $r_2$.

Now, $60° = \frac{\pi}{3}$ radian and $75° = \frac{5\pi}{12}$ radian

We know that in a circle of radius $r$ unit, if an arc of length $l$ unit subtends an angle $\theta$ radian at the centre, then $\theta = \frac{l}{r}$ or $l = r\theta$.

$$\therefore l = \frac{r_1\pi}{3} \text{ and } l = \frac{r_2 5\pi}{12}$$

$$\Rightarrow \frac{r_1\pi}{3} = \frac{r_2 5\pi}{12}$$

$$\Rightarrow r_1 = \frac{r_2 5}{4}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{5}{4}$$

Thus, the ratio of the radii is 5:4.

-

Find the angle in radian though which a pendulum swings if its length is 75 cm and the tip describes an arc of length

(i) 10 cm (ii) 15 cm (iii) 21 cm

**Ans:**

We know that in a circle of radius $r$ unit, if an arc of length $l$ unit subtends an angle $\theta$ radian at the centre, then $\theta = \frac{l}{r}$.

It is given that $r = 75$ cm

(i) Here, $l = 10$ cm

$\theta = \frac{10}{75}$ radian $= \frac{2}{15}$ radian

(ii) Here, $l = 15$ cm

$\theta = \frac{15}{75}$ radian $= \frac{1}{5}$ radian

(iii) Here, $l = 21$ cm

$\theta = \frac{21}{75}$ radian $= \frac{7}{25}$ radian

-

Find the values of other five trigonometric functions if $\cos x = -\frac{1}{2}$, $x$ lies in third quadrant.

**Ans:**

$$\cos x = -\frac{1}{2}$$

$$\therefore \sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

$$\sin^2 x + \cos^2 x = 1$$
$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$
$$\Rightarrow \sin^2 x = 1 - \left(-\frac{1}{2}\right)^2$$
$$\Rightarrow \sin^2 x = 1 - \frac{1}{4} = \frac{3}{4}$$
$$\Rightarrow \sin x = \pm\frac{\sqrt{3}}{2}$$

Since $x$ lies in the 3rd quadrant, the value of $\sin x$ will be negative.

$$\therefore \sin x = -\frac{\sqrt{3}}{2}$$

$$\therefore \sin x = -\frac{\sqrt{3}}{2}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$$

-

Find the values of other five trigonometric functions if $\sin x = \frac{3}{5}$, $x$ lies in second quadrant.

**Ans:**

$$\sin x = \frac{3}{5}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3}$$

$$\sin^2 x + \cos^2 x = 1$$
$$\Rightarrow \cos^2 x = 1 - \sin^2 x$$
$$\Rightarrow \cos^2 x = 1 - \left(\frac{3}{5}\right)^2$$
$$\Rightarrow \cos^2 x = 1 - \frac{9}{25}$$
$$\Rightarrow \cos^2 x = \frac{16}{25}$$
$$\Rightarrow \cos x = \pm\frac{4}{5}$$

Since $x$ lies in the $2^{nd}$ quadrant, the value of $\cos x$ will be negative

$$\therefore \cos x = -\frac{4}{5}$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)} = -\frac{3}{4}$$

$$\cot x = \frac{1}{\tan x} = -\frac{4}{3}$$

-

Find the values of other five trigonometric functions if $\cot x = \frac{3}{4}$, $x$ lies in third quadrant.

**Ans:**

$$\cot x = \frac{3}{4}$$

$$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(\frac{4}{3}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{16}{9} = \sec^2 x$$

$$\Rightarrow \frac{25}{9} = \sec^2 x$$

$$\Rightarrow \sec x = \pm\frac{5}{3}$$

Since $x$ lies in the $3^{rd}$ quadrant, the value of $\sec x$ will be negative.

$$\therefore \sec x = -\frac{5}{3}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{5}{3}\right)} = -\frac{3}{5}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow \frac{4}{3} = \frac{\sin x}{\left(\frac{-3}{5}\right)}$$

$$\Rightarrow \sin x = \left(\frac{4}{3}\right) \times \left(\frac{-3}{5}\right) = -\frac{4}{5}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{5}{4}$$

Find the values of other five trigonometric functions if $\sec x = \frac{13}{5}$, $x$ lies in fourth quadrant.

**Ans:**

$$\sec x = \frac{13}{5}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(\frac{13}{5}\right)} = \frac{5}{13}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \left(\frac{5}{13}\right)^2$$

$$\Rightarrow \sin^2 x = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\Rightarrow \sin x = \pm\frac{12}{13}$$

Since $x$ lies in the 4th quadrant, the value of $\sin x$ will be negative.

$$\therefore \sin x = -\frac{12}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{-12}{13}\right)}{\left(\frac{5}{13}\right)} = -\frac{12}{5}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{12}{5}\right)} = -\frac{5}{12}$$

-

Find the values of other five trigonometric functions if $\tan x = -\frac{5}{12}$, $x$ lies in second quadrant.

**Ans:**

$$\tan x = -\frac{5}{12}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(-\frac{5}{12}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{25}{144} = \sec^2 x$$

$$\Rightarrow \frac{169}{144} = \sec^2 x$$

$$\Rightarrow \sec x = \pm\frac{13}{12}$$

Since $x$ lies in the $2^{nd}$ quadrant, the value of $\sec x$ will be negative.

$$\therefore \sec x = -\frac{13}{12}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{13}{12}\right)} = -\frac{12}{13}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow -\frac{5}{12} = \frac{\sin x}{\left(-\frac{12}{13}\right)}$$

$$\Rightarrow \sin x = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right) = \frac{5}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{5}{13}\right)} = \frac{13}{5}$$

Find the value of the trigonometric function sin 765°

**Ans:**

It is known that the values of sin $x$ repeat after an interval of $2\pi$ or $360°$.

$$\therefore \sin 765° = \sin(2\times 360° + 45°) = \sin 45° = \frac{1}{\sqrt{2}}$$

-

Find the value of the trigonometric function cosec (−1410°)

**Ans:**

It is known that the values of cosec $x$ repeat after an interval of $2\pi$ or $360°$

$$\begin{aligned}\therefore \operatorname{cosec}(-1410°) &= \operatorname{cosec}(-1410° + 4\times 360°)\\ &= \operatorname{cosec}(-1410° + 1440°)\\ &= \operatorname{cosec} 30° = 2\end{aligned}$$

-

Find the value of the trigonometric function $\tan\frac{19\pi}{3}$

**Ans:**

It is known that the values of tan $x$ repeat after an interval of $\pi$ or $180°$.

$$\therefore \tan\frac{19\pi}{3} = \tan 6\frac{1}{3}\pi = \tan\left(6\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \tan 60° = \sqrt{3}$$

-

Find the value of the trigonometric function $\sin\left(-\frac{11\pi}{3}\right)$

**Ans:**

It is known that the values of $\sin x$ repeat after an interval of $2\pi$ or $360°$

$$\therefore \sin\left(-\frac{11\pi}{3}\right) = \sin\left(-\frac{11\pi}{3} + 2\times 2\pi\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

-

Find the value of the trigonometric function $\cot\left(-\frac{15\pi}{4}\right)$

**Ans:**

It is known that the values of $\cot x$ repeat after an interval of $\pi$ or $180°$.

$$\therefore \cot\left(-\frac{15\pi}{4}\right) = \cot\left(-\frac{15\pi}{4} + 4\pi\right) = \cot\frac{\pi}{4} = 1$$

-

$$\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4} = -\frac{1}{2}$$

**Ans:**

$$\text{L.H.S.} = \sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4}$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2$$

$$= \frac{1}{4} + \frac{1}{4} - 1 = -\frac{1}{2}$$

$$= \text{R.H.S.}$$

-

Prove that $2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$

**Ans:**

L.H.S. = $2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$=2\left(\frac{1}{2}\right)^2+\operatorname{cosec}^2\left(\pi+\frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$

$=2\times\frac{1}{4}+\left(-\operatorname{cosec}\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$

$=\frac{1}{2}+(-2)^2\left(\frac{1}{4}\right)$

$=\frac{1}{2}+\frac{4}{4}=\frac{1}{2}+1=\frac{3}{2}$

$=$ R.H.S.

-

Prove that $\cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}=6$

**Ans:**

L.H.S. $=\cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}$

$=\left(\sqrt{3}\right)^2+\operatorname{cosec}\left(\pi-\frac{\pi}{6}\right)+3\left(\frac{1}{\sqrt{3}}\right)^2$

$=3+\operatorname{cosec}\frac{\pi}{6}+3\times\frac{1}{3}$

$=3+2+1=6$

$=$ R.H.S

-

Prove that $2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}=10$

**Ans:**

$$\text{L.H.S}=2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}$$

$$=2\left\{\sin\left(\pi-\frac{\pi}{4}\right)\right\}^2+2\left(\frac{1}{\sqrt{2}}\right)^2+2(2)^2$$

$$=2\left\{\sin\frac{\pi}{4}\right\}^2+2\times\frac{1}{2}+8$$

$$=2\left(\frac{1}{\sqrt{2}}\right)^2+1+8$$

$$=1+1+8$$

$$=10$$

$$=\text{R.H.S}$$

-

Find the value of:

(i) sin 75°

(ii) tan 15°

**Ans:**

(i) $\sin 75° = \sin(45° + 30°)$

$= \sin 45° \cos 30° + \cos 45° \sin 30°$

$[\sin(x+y) = \sin x \cos y + \cos x \sin y]$

$$=\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$=\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}=\frac{\sqrt{3}+1}{2\sqrt{2}}$$

-

(ii) $\tan 15° = \tan(45° - 30°)$

$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°} \qquad \left[\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}\right]$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)} = \frac{3+1-2\sqrt{3}}{\left(\sqrt{3}\right)^2 - (1)^2}$$

$$= \frac{4-2\sqrt{3}}{3-1} = 2 - \sqrt{3}$$

-

Prove that: $\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right) = \sin(x+y)$

**Ans:**

$$\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)$$

$$= \frac{1}{2}\left[2\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right)\right] + \frac{1}{2}\left[-2\sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)\right]$$

$$= \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\} + \cos\left\{\left(\frac{\pi}{4} - x\right) - \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$+ \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\} - \cos\left\{\left(\frac{\pi}{4} - x\right) - \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$\left[\begin{array}{l} \because 2\cos A \cos B = \cos(A+B) + \cos(A-B) \\ -2\sin A \sin B = \cos(A+B) - \cos(A-B) \end{array}\right]$$

$$= 2 \times \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$= \cos\left[\frac{\pi}{2} - (x+y)\right]$$

$$= \sin(x+y)$$

$$= \text{R.H.S}$$

Prove that: $\dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left(\dfrac{1+\tan x}{1-\tan x}\right)^2$

**Ans:**

It is known that $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$ and $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

$\therefore$ L.H.S. =

$$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\frac{\left(\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right)}{\left(\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)}=\frac{\left(\dfrac{1+\tan x}{1-\tan x}\right)}{\left(\dfrac{1-\tan x}{1+\tan x}\right)}=\left(\frac{1+\tan x}{1-\tan x}\right)^2=\text{R.H.S.}$$

-

Prove that $\dfrac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\cot^2 x$

**Ans:**

$$\text{L.H.S.}=\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}$$

$$=\frac{[-\cos x][\cos x]}{(\sin x)(-\sin x)}$$

$$=\frac{-\cos^2 x}{-\sin^2 x}$$

$$=\cot^2 x$$

$$=\text{R.H.S.}$$

-

$$\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]=1$$

**Ans:**

$$\text{L.H.S.} = \cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$$

$$= \sin x\cos x[\tan x+\cot x]$$

$$= \sin x\cos x\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)$$

$$= (\sin x\cos x)\left[\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right]$$

$$= 1 = \text{R.H.S.}$$

-

Prove that sin $(n + 1)x$ sin $(n + 2)x$ + cos $(n + 1)x$ cos $(n + 2)x$ = cos $x$

**Ans:**

$$\text{L.H.S.} = \sin\,(n+1)x\,\sin(n+2)x + \cos\,(n+1)x\,\cos(n+2)x$$

$$= \frac{1}{2}\left[2\sin(n+1)x\sin(n+2)x+2\cos(n+1)x\cos(n+2)x\right]$$

$$= \frac{1}{2}\left[\begin{matrix}\cos\{(n+1)x-(n+2)x\}-\cos\{(n+1)x+(n+2)x\}\\ +\cos\{(n+1)x+(n+2)x\}+\cos\{(n+1)x-(n+2)x\}\end{matrix}\right]$$

$$\left[\begin{matrix}\because -2\sin A\sin B=\cos(A+B)-\cos(A-B)\\ 2\cos A\cos B=\cos(A+B)+\cos(A-B)\end{matrix}\right]$$

$$= \frac{1}{2}\times 2\cos\{(n+1)x-(n+2)x\}$$

$$= \cos(-x) = \cos x = \text{R.H.S.}$$

-

Prove that $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$

**Ans:**

It is known that $\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)$.

$\therefore$ L.H.S. = $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$

$$=-2\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right\}.\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right\}$$

$$=-2\sin\left(\frac{3\pi}{4}\right)\sin x$$

$$=-2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$$

$$=-2\sin\frac{\pi}{4}\sin x$$

$$=-2\times\frac{1}{\sqrt{2}}\times\sin x$$

$$=-\sqrt{2}\sin x$$

$$=\text{R.H.S.}$$

-

Prove that $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

**Ans:**

It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\quad \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.} = \sin^2 6x - \sin^2 4x$$

$$= (\sin 6x + \sin 4x)(\sin 6x - \sin 4x)$$

$$= \left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\left[2\cos\left(\frac{6x+4x}{2}\right).\sin\left(\frac{6x-4x}{2}\right)\right]$$

$$= (2\sin 5x \cos x)(2\cos 5x \sin x)$$

$$= (2\sin 5x \cos 5x)(2\sin x \cos x)$$

$$= \sin 10x \sin 2x$$

$$= \text{R.H.S.}$$

-

Prove that $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

**Ans:**

It is known that

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \quad \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$\therefore$ L.H.S. $= \cos^2 2x - \cos^2 6x$

$= (\cos 2x + \cos 6x)\,(\cos 2x - 6x)$

$$= \left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\frac{(2x-6x)}{2}\right]$$

$$= \left[2\cos 4x\cos(-2x)\right]\left[-2\sin 4x\sin(-2x)\right]$$

$= [2\cos 4x \cos 2x]\,[-2\sin 4x\,(-\sin 2x)]$

$= (2\sin 4x \cos 4x)\,(2\sin 2x \cos 2x)$

$= \sin 8x \sin 4x$

$=$ R.H.S.

-

Prove that $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

**Ans:**

$$\text{L.H.S.} = \sin 2x + 2\sin 4x + \sin 6x$$

$$= [\sin 2x + \sin 6x] + 2\sin 4x$$

$$= \left[2\sin\left(\frac{2x+6x}{2}\right)\left(\frac{2x-6x}{2}\right)\right] + 2\sin 4x$$

$$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= 2\sin 4x \cos(-2x) + 2\sin 4x$$

$$= 2\sin 4x \cos 2x + 2\sin 4x$$

$$= 2\sin 4x(\cos 2x + 1)$$

$$= 2\sin 4x(2\cos^2 x - 1 + 1)$$

$$= 2\sin 4x(2\cos^2 x)$$

$$= 4\cos^2 x \sin 4x$$

-

Prove that cot 4*x* (sin 5*x* + sin 3*x*) = cot *x* (sin 5*x* − sin 3*x*)

**Ans:**

$\text{L.H.S} = \cot 4x\,(\sin 5x + \sin 3x)$

$= \frac{\cos 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$

$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$= \left(\frac{\cos 4x}{\sin 4x}\right)[2\sin 4x \cos x]$

$= 2\cos 4x \cos x$

$\text{R.H.S.} = \cot x\,(\sin 5x - \sin 3x)$

$= \frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]$

$\left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$

$= \frac{\cos x}{\sin x}[2\cos 4x \sin x]$

$= 2\cos 4x.\cos x$

$\text{L.H.S.} = \text{R.H.S.}$

-

Prove that $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$

**Ans:**

It is known that

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \quad \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S} = \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$$

$$= \frac{-2\sin\left(\frac{9x+5x}{2}\right).\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right).\sin\left(\frac{17x-3x}{2}\right)}$$

$$= \frac{-2\sin 7x.\sin 2x}{2\cos 10x.\sin 7x}$$

$$= -\frac{\sin 2x}{\cos 10x}$$

$$= \text{R.H.S.}$$

-

Prove that $\dfrac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

**Ans:**

It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \quad \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.} = \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$$

$$= \frac{2\sin\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}$$

$$= \frac{2\sin 4x.\cos x}{2\cos 4x.\cos x}$$

$$= \frac{\sin 4x}{\cos 4x}$$

$$= \tan 4x = \text{R.H.S.}$$

-

Prove that $\dfrac{\sin x - \sin y}{\cos x + \cos y} = \tan \dfrac{x-y}{2}$

**Ans:**

It is known that

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),\quad \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.} = \frac{\sin x - \sin y}{\cos x + \cos y}$$

$$= \frac{2\cos\left(\frac{x+y}{2}\right).\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)}$$

$$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$$

$$= \tan\left(\frac{x-y}{2}\right) = \text{R.H.S.}$$

-

Prove that $\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$

**Ans:**

It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \quad \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.} = \frac{\sin x + \sin 3x}{\cos x + \cos 3x}$$

$$= \frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$$

$$= \frac{\sin 2x}{\cos 2x}$$

$$= \tan 2x$$

$$= \text{R.H.S}$$

-

Prove that $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

**Ans:**

It is known that

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),\ \cos^2 A - \sin^2 A = \cos 2A$$

$$\therefore \text{L.H.S.} = \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$$

$$= \frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x}$$

$$= \frac{2\cos 2x \sin(-x)}{-\cos 2x}$$

$$= -2\times(-\sin x)$$

$$= 2\sin x = \text{R.H.S.}$$

-

Prove that $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

**Ans:**

$$\text{L.H.S.} = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$$

$$= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

$$= \frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$$

$$\left[\because \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= \frac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$$

$$= \frac{\cos 3x(2\cos x + 1)}{\sin 3x(2\cos x + 1)}$$

$$= \cot 3x = \text{R.H.S.}$$

-

Prove that $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

**Ans:**

$$\text{L.H.S.} = \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x$$

$$= \cot x \cot 2x - \cot 3x (\cot 2x + \cot x)$$

$$= \cot x \cot 2x - \cot (2x + x)(\cot 2x + \cot x)$$

$$= \cot x \cot 2x - \left[\frac{\cot 2x \cot x - 1}{\cot x + \cot 2x}\right](\cot 2x + \cot x)$$

$$\left[\because \cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}\right]$$

$$= \cot x \cot 2x - (\cot 2x \cot x - 1)$$

$$= 1 = \text{R.H.S.}$$

Prove that $\tan 4x = \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$

**Ans:**

It is known that $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$.

$\therefore$ L.H.S. $= \tan 4x = \tan 2(2x)$

$$= \frac{2\tan 2x}{1-\tan^2(2x)}$$

$$= \frac{2\left(\frac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2}$$

$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[1-\frac{4\tan^2 x}{(1-\tan^2 x)^2}\right]}$$

$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[\frac{(1-\tan^2 x)^2-4\tan^2 x}{(1-\tan^2 x)^2}\right]}$$

$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[\frac{(1-\tan^2 x)^2-4\tan^2 x}{(1-\tan^2 x)^2}\right]}$$

$$= \frac{4\tan x(1-\tan^2 x)}{(1-\tan^2 x)^2-4\tan^2 x}$$

$$= \frac{4\tan x(1-\tan^2 x)}{1+\tan^4 x-2\tan^2 x-4\tan^2 x}$$

$$= \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x} = \text{R.H.S.}$$

Prove that $\cos 4x = 1 - 8\sin^2 x \cos^2 x$

**Ans:**

$\text{L.H.S.} = \cos 4x$

$= \cos 2(2x)$

$= 1 - 2\sin^2 2x$ [$\cos 2A = 1 - 2\sin^2 A$]

$= 1 - 2(2\sin x \cos x)^2$ [$\sin 2A = 2\sin A \cos A$]

$= 1 - 8\sin^2 x \cos^2 x$

$= \text{R.H.S.}$

-

Prove that: $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

**Ans:**

$\text{L.H.S.} = \cos 6x$

$= \cos 3(2x)$

$= 4\cos^3 2x - 3\cos 2x$ [$\cos 3A = 4\cos^3 A - 3\cos A$]

$= 4[(2\cos^2 x - 1)^3 - 3(2\cos^2 x - 1)$ [$\cos 2x = 2\cos^2 x - 1$]

$= 4[(2\cos^2 x)^3 - (1)^3 - 3(2\cos^2 x)^2 + 3(2\cos^2 x)] - 6\cos^2 x + 3$

$= 4[8\cos^6 x - 1 - 12\cos^4 x + 6\cos^2 x] - 6\cos^2 x + 3$

$= 32\cos^6 x - 4 - 48\cos^4 x + 24\cos^2 x - 6\cos^2 x + 3$

$= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

$= \text{R.H.S.}$

-

Find the principal and general solutions of the equation $\tan x = \sqrt{3}$

**Ans:**

$\tan x = \sqrt{3}$

It is known that $\tan\frac{\pi}{3} = \sqrt{3}$ and $\tan\left(\frac{4\pi}{3}\right) = \tan\left(\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \sqrt{3}$

Therefore, the principal solutions are $x = \frac{\pi}{3}$ and $\frac{4\pi}{3}$.

Now, $\tan x = \tan\frac{\pi}{3}$

$\Rightarrow x = n\pi + \frac{\pi}{3}$, where $n \in Z$

Therefore, the general solution is $x = n\pi + \frac{\pi}{3}$, where $n \in Z$

-

Find the principal and general solutions of the equation $\sec x = 2$

**Ans:**

$\sec x = 2$

It is known that $\sec\frac{\pi}{3} = 2$ and $\sec\frac{5\pi}{3} = \sec\left(2\pi - \frac{\pi}{3}\right) = \sec\frac{\pi}{3} = 2$

Therefore, the principal solutions are $x = \frac{\pi}{3}$ and $\frac{5\pi}{3}$.

Now, $\sec x = \sec\frac{\pi}{3}$

$\Rightarrow \cos x = \cos\frac{\pi}{3}$ $\left[\sec x = \frac{1}{\cos x}\right]$

$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$

Therefore, the general solution is $x = 2n\pi \pm \frac{\pi}{3}$, where $n \in \mathbf{Z}$

Find the principal and general solutions of the equation $\cot x = -\sqrt{3}$

**Ans:**

$\cot x = -\sqrt{3}$

It is known that $\cot\frac{\pi}{6} = \sqrt{3}$

$\therefore \cot\left(\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$ and $\cot\left(2\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$

i.e., $\cot\frac{5\pi}{6} = -\sqrt{3}$ and $\cot\frac{11\pi}{6} = -\sqrt{3}$

Therefore, the principal solutions are $x = \frac{5\pi}{6}$ and $\frac{11\pi}{6}$.

Now, $\cot x = \cot\frac{5\pi}{6}$

$\Rightarrow \tan x = \tan\frac{5\pi}{6}$ $\qquad \left[\cot x = \frac{1}{\tan x}\right]$

$\Rightarrow x = n\pi + \frac{5\pi}{6}$, where $n \in Z$

Therefore, the general solution is $x = n\pi + \frac{5\pi}{6}$, where $n \in Z$

-

Find the general solution of $\operatorname{cosec} x = -2$

**Ans:**

It is known that

$$\operatorname{cosec}\frac{\pi}{6} = 2$$

$$\therefore \operatorname{cosec}\left(\pi + \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2 \text{ and } \operatorname{cosec}\left(2\pi - \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2$$

$$\text{i.e., } \operatorname{cosec}\frac{7\pi}{6} = -2 \text{ and } \operatorname{cosec}\frac{11\pi}{6} = -2$$

Therefore, the principal solutions are $x = \frac{7\pi}{6}$ and $\frac{11\pi}{6}$.

$$\text{Now, } \operatorname{cosec} x = \operatorname{cosec}\frac{7\pi}{6}$$

$$\Rightarrow \sin x = \sin\frac{7\pi}{6} \qquad \left[\operatorname{cosec} x = \frac{1}{\sin x}\right]$$

$$\Rightarrow x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in Z$$

Therefore, the general solution is $x = n\pi + (-1)^n \frac{7\pi}{6}$, where $n \in Z$

-

Find the general solution of the equation $\cos 4x = \cos 2x$

**Ans:**

$\cos 4x = \cos 2x$

$\Rightarrow \cos 4x - \cos 2x = 0$

$\Rightarrow -2\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) = 0$

$\left[\because \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow \sin 3x \sin x = 0$

$\Rightarrow \sin 3x = 0$ or $\sin x = 0$

$\therefore 3x = n\pi$ or $x = n\pi$, where $n \in Z$

$\Rightarrow x = \frac{n\pi}{3}$ or $x = n\pi$, where $n \in Z$

-

Find the general solution of the equation $\cos 3x + \cos x - \cos 2x = 0$

**Ans:**

$\cos 3x + \cos x - \cos 2x = 0$

$\Rightarrow 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - \cos 2x = 0$ $\left[\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow 2\cos 2x \cos x - \cos 2x = 0$

$\Rightarrow \cos 2x(2\cos x - 1) = 0$

$\Rightarrow \cos 2x = 0$ or $2\cos x - 1 = 0$

$\Rightarrow \cos 2x = 0$ or $\cos x = \frac{1}{2}$

$\therefore 2x = (2n+1)\frac{\pi}{2}$ or $\cos x = \cos\frac{\pi}{3}$, where $n \in Z$

$\Rightarrow x = (2n+1)\frac{\pi}{4}$ or $x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$

-

Find the general solution of the equation $\sin 2x + \cos x = 0$

**Ans:**

$\sin 2x + \cos x = 0$

$\Rightarrow 2\sin x \cos x + \cos x = 0$

$\Rightarrow \cos x\left(2\sin x + 1\right) = 0$

$\Rightarrow \cos x = 0 \quad \text{or} \quad 2\sin x + 1 = 0$

Now, $\cos x = 0 \Rightarrow \cos x = (2n+1)\frac{\pi}{2}$, where $n \in Z$

$2\sin x + 1 = 0$

$\Rightarrow \sin x = \frac{-1}{2} = -\sin\frac{\pi}{6} = \sin\left(\pi + \frac{\pi}{6}\right) = \sin\left(\pi + \frac{\pi}{6}\right) = \sin\frac{7\pi}{6}$

$\Rightarrow x = n\pi + (-1)^n \frac{7\pi}{6}$, where $n \in Z$

Therefore, the general solution is $(2n+1)\frac{\pi}{2}$ or $n\pi + (-1)^n \frac{7\pi}{6}$, $n \in Z$

-

Find the general solution of the equation $\sec^2 2x = 1 - \tan 2x$

**Ans:**

$\sec^2 2x = 1 - \tan 2x$

$\Rightarrow 1 + \tan^2 2x = 1 - \tan 2x$

$\Rightarrow \tan^2 2x + \tan 2x = 0$

$\Rightarrow \tan 2x(\tan 2x + 1) = 0$

$\Rightarrow \tan 2x = 0 \qquad \text{or} \qquad \tan 2x + 1 = 0$

Now, $\tan 2x = 0$

$\Rightarrow \tan 2x = \tan 0$

$\Rightarrow 2x = n\pi + 0$, where $n \in Z$

$\Rightarrow x = \frac{n\pi}{2}$, where $n \in Z$

$\tan 2x + 1 = 0$

$\Rightarrow \tan 2x = -1 = -\tan\frac{\pi}{4} = \tan\left(\pi - \frac{\pi}{4}\right) = \tan\frac{3\pi}{4}$

$\Rightarrow 2x = n\pi + \frac{3\pi}{4}$, where $n \in Z$

$\Rightarrow x = \frac{n\pi}{2} + \frac{3\pi}{8}$, where $n \in Z$

Therefore, the general solution is $\frac{n\pi}{2}$ or $\frac{n\pi}{2} + \frac{3\pi}{8}$, $n \in Z$.

-

Find the general solution of the equation $\sin x + \sin 3x + \sin 5x = 0$

**Ans:**

$\sin x + \sin 3x + \sin 5x = 0$

$\Rightarrow \sin 3x\left(2\cos 2x + 1\right) = 0$

$\Rightarrow \sin 3x = 0 \quad \text{or} \quad 2\cos 2x + 1 = 0$

Now, $\sin 3x = 0 \Rightarrow 3x = n\pi$, where $n \in Z$

i.e., $x = \dfrac{n\pi}{3}$, where $n \in Z$

$2\cos 2x + 1 = 0$

$\Rightarrow \cos 2x = \dfrac{-1}{2} = -\cos\dfrac{\pi}{3} = \cos\left(\pi - \dfrac{\pi}{3}\right)$

$\Rightarrow \cos 2x = \cos\dfrac{2\pi}{3}$

$\Rightarrow 2x = 2n\pi \pm \dfrac{2\pi}{3}$, where $n \in Z$

$\Rightarrow x = n\pi \pm \dfrac{\pi}{3}$, where $n \in Z$

$\left(\sin x + \sin 5x\right) + \sin 3x = 0$

$\Rightarrow \left[2\sin\left(\dfrac{x+5x}{2}\right)\cos\left(\dfrac{x-5x}{2}\right)\right] + \sin 3x = 0 \qquad \left[\sin A + \sin B = 2\sin\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)\right]$

$\Rightarrow 2\sin 3x\cos\left(-2x\right) + \sin 3x = 0$

$\Rightarrow 2\sin 3x\cos 2x + \sin 3x = 0$

-

Prove that: $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$

**Ans:**

L.H.S.

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\left(\frac{\frac{3\pi}{13}+\frac{5\pi}{13}}{2}\right)\cos\left(\frac{\frac{3\pi}{13}-\frac{5\pi}{13}}{2}\right) \quad \left[\cos x+\cos y=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right]$$

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\left(\frac{-\pi}{13}\right)$$

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\frac{\pi}{13}$$

$$=2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13}+\cos\frac{4\pi}{13}\right]$$

$$=2\cos\frac{\pi}{13}\left[2\cos\left(\frac{\frac{9\pi}{13}+\frac{4\pi}{13}}{2}\right)\cos\left(\frac{\frac{9\pi}{13}-\frac{4\pi}{13}}{2}\right)\right]$$

$$=2\cos\frac{\pi}{13}\left[2\cos\frac{\pi}{2}\cos\frac{5\pi}{26}\right]$$

$$=2\cos\frac{\pi}{13}\times 2\times 0\times\cos\frac{5\pi}{26}$$

= 0 = RHS

-

Prove that: $(\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x = 0$

**Ans:**

L.H.S.

$= (\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x$

$= \sin 3x \sin x + \sin^2 x + \cos 3x \cos x - \cos^2 x$

$= \cos 3x \cos x + \sin 3x \sin x - \left(\cos^2 x - \sin^2 x\right)$

$= \cos(3x - x) - \cos 2x \qquad \left[\cos(A - B) = \cos A \cos B + \sin A \sin B\right]$

$= \cos 2x - \cos 2x$

$= 0$

$=$ R.H.S.

-

Prove that: $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2 \frac{x+y}{2}$

**Ans:**

L.H.S. $= (\cos x + \cos y)^2 + (\sin x - \sin y)^2$

$= \cos^2 x + \cos^2 y + 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$

$= \left(\cos^2 x + \sin^2 x\right) + \left(\cos^2 y + \sin^2 y\right) + 2(\cos x \cos y - \sin x \sin y)$

$= 1 + 1 + 2\cos(x + y) \qquad \left[\cos(A + B) = (\cos A \cos B - \sin A \sin B)\right]$

$= 2 + 2\cos(x + y)$

$= 2\left[1 + \cos(x + y)\right]$

$= 2\left[1 + 2\cos^2\left(\frac{x+y}{2}\right) - 1\right] \qquad \left[\cos 2A = 2\cos^2 A - 1\right]$

$= 4\cos^2\left(\frac{x+y}{2}\right) =$ R.H.S.

-

Prove that: $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2 \frac{x-y}{2}$

**Ans:**

L.H.S. = $(\cos x - \cos y)^2 + (\sin x - \sin y)^2$

$$= \cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$$
$$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) - 2[\cos x \cos y + \sin x \sin y]$$
$$= 1 + 1 - 2[\cos(x-y)] \qquad [\cos(A-B) = \cos A \cos B + \sin A \sin B]$$
$$= 2[1 - \cos(x-y)]$$
$$= 2\left[1 - \left\{1 - 2\sin^2\left(\frac{x-y}{2}\right)\right\}\right] \qquad [\cos 2A = 1 - 2\sin^2 A]$$
$$= 4\sin^2\left(\frac{x-y}{2}\right) = \text{R.H.S.}$$

-

Prove that: $\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x \cos 2x \sin 4x$

**Ans:**

It is known that $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$.

$\therefore$ L.H.S. = $\sin x + \sin 3x + \sin 5x + \sin 7x$

$$= (\sin x + \sin 5x) + (\sin 3x + \sin 7x)$$
$$= 2\sin\left(\frac{x+5x}{2}\right) \cdot \cos\left(\frac{x-5x}{2}\right) + 2\sin\left(\frac{3x+7x}{2}\right)\cos\left(\frac{3x-7x}{2}\right)$$
$$= 2\sin 3x \cos(-2x) + 2\sin 5x \ \cos(-2x)$$
$$= 2\sin 3x \cos 2x + 2\sin 5x \cos 2x$$
$$= 2\cos 2x[\sin 3x + \sin 5x]$$
$$= 2\cos 2x\left[2\sin\left(\frac{3x+5x}{2}\right) \cdot \cos\left(\frac{3x-5x}{2}\right)\right]$$
$$= 2\cos 2x[2\sin 4x \cdot \cos(-x)]$$
$$= 4\cos 2x \sin 4x \cos x = \text{R.H.S.}$$

-

Prove that: $\dfrac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$

**Ans:**

It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right), \quad \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$$

$$\text{L.H.S.} = \frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)}$$

$$= \frac{\left[2\sin\left(\frac{7x+5x}{2}\right)\cdot\cos\left(\frac{7x-5x}{2}\right)\right] + \left[2\sin\left(\frac{9x+3x}{2}\right)\cdot\cos\left(\frac{9x-3x}{2}\right)\right]}{\left[2\cos\left(\frac{7x+5x}{2}\right)\cdot\cos\left(\frac{7x-5x}{2}\right)\right] + \left[2\cos\left(\frac{9x+3x}{2}\right)\cdot\cos\left(\frac{9x-3x}{2}\right)\right]}$$

$$= \frac{[2\sin 6x \cdot \cos x] + [2\sin 6x \cdot \cos 3x]}{[2\cos 6x \cdot \cos x] + [2\cos 6x \cdot \cos 3x]}$$

$$= \frac{2\sin 6x[\cos x + \cos 3x]}{2\cos 6x[\cos x + \cos 3x]}$$

$= \tan 6x$

$= \text{R.H.S.}$

-

Prove that: $\sin 3x + \sin 2x - \sin x = 4\sin x \cos\frac{x}{2}\cos\frac{3x}{2}$

**Ans:**

L.H.S. = $\sin 3x + \sin 2x - \sin x$

$$= \sin 3x + (\sin 2x - \sin x)$$

$$= \sin 3x + \left[2\cos\left(\frac{2x+x}{2}\right)\sin\left(\frac{2x-x}{2}\right)\right] \qquad \left[\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$= \sin 3x + \left[2\cos\left(\frac{3x}{2}\right)\sin\left(\frac{x}{2}\right)\right]$$

$$= \sin 3x + 2\cos\frac{3x}{2}\sin\frac{x}{2}$$

$$= 2\sin\frac{3x}{2}\cdot\cos\frac{3x}{2} + 2\cos\frac{3x}{2}\sin\frac{x}{2} \qquad [\sin 2A = 2\sin A\cdot\cos B]$$

$$= 2\cos\left(\frac{3x}{2}\right)\left[\sin\left(\frac{3x}{2}\right) + \sin\left(\frac{x}{2}\right)\right]$$

$$= 2\cos\left(\frac{3x}{2}\right)\left[2\sin\left\{\frac{\left(\frac{3x}{2}\right)+\left(\frac{x}{2}\right)}{2}\right\}\cos\left\{\frac{\left(\frac{3x}{2}\right)-\left(\frac{x}{2}\right)}{2}\right\}\right] \qquad \left[\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= 2\cos\left(\frac{3x}{2}\right).2\sin x\cos\left(\frac{x}{2}\right)$$

$$= 4\sin x\cos\left(\frac{x}{2}\right)\cos\left(\frac{3x}{2}\right) = \text{R.H.S.}$$

-

Find $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$, if

$\tan x = -\frac{4}{3}$, $x$ in quadrant II

**Ans:**

Here, $x$ is in quadrant II.

i.e., $\frac{\pi}{2} < x < \pi$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Therefore, $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are all positive.

It is given that $\tan x = -\frac{4}{3}$.

$$\sec^2 x = 1 + \tan^2 x = 1 + \left(\frac{-4}{3}\right)^2 = 1 + \frac{16}{9} = \frac{25}{9}$$

$$\therefore \cos^2 x = \frac{9}{25}$$

$$\Rightarrow \cos x = \pm\frac{3}{5}$$

As $x$ is in quadrant II, $\cos x$ is negative.

$\therefore \cos x = \frac{-3}{5}$

Now, $\cos x = 2\cos^2 \frac{x}{2} - 1$

$\Rightarrow \frac{-3}{5} = 2\cos^2 \frac{x}{2} - 1$

$\Rightarrow 2\cos^2 \frac{x}{2} = 1 - \frac{3}{5}$

$\Rightarrow 2\cos^2 \frac{x}{2} = \frac{2}{5}$

$\Rightarrow \cos^2 \frac{x}{2} = \frac{1}{5}$

$\Rightarrow \cos \frac{x}{2} = \frac{1}{\sqrt{5}}$ $\quad \left[\because \cos \frac{x}{2} \text{ is positive}\right]$

$\therefore \cos \frac{x}{2} = \frac{\sqrt{5}}{5}$

$\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} = 1$

$\Rightarrow \sin^2 \frac{x}{2} + \left(\frac{1}{\sqrt{5}}\right)^2 = 1$

$\Rightarrow \sin^2 \frac{x}{2} = 1 - \frac{1}{5} = \frac{4}{5}$

$\Rightarrow \sin \frac{x}{2} = \frac{2}{\sqrt{5}}$ $\quad \left[\because \sin \frac{x}{2} \text{ is positive}\right]$

$\therefore \sin \frac{x}{2} = \frac{2\sqrt{5}}{5}$

$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\left(\frac{2}{\sqrt{5}}\right)}{\left(\frac{1}{\sqrt{5}}\right)} = 2$

Thus, the respective values of $\sin \frac{x}{2}$, $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$ are $\frac{2\sqrt{5}}{5}$, $\frac{\sqrt{5}}{5}$, and 2

-

Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ for $\cos x = -\frac{1}{3}$, $x$ in quadrant III

**Ans:**

Here, $x$ is in quadrant III.

$$\text{i.e., } \pi < x < \frac{3\pi}{2}$$

$$\Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

Therefore, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are negative, whereas $\sin\frac{x}{2}$ is positive.

It is given that $\cos x = -\frac{1}{3}$.

$$\cos x = 1 - 2\sin^2\frac{x}{2}$$

$$\Rightarrow \sin^2\frac{x}{2} = \frac{1-\cos x}{2}$$

$$\Rightarrow \sin^2\frac{x}{2} = \frac{1-\left(-\frac{1}{3}\right)}{2} = \frac{\left(1+\frac{1}{3}\right)}{2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}$$

$$\Rightarrow \sin\frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \qquad \left[\because \sin\frac{x}{2} \text{ is positive}\right]$$

$$\therefore \sin\frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

Now, $\cos x = 2\cos^2\frac{x}{2} - 1$

$$\Rightarrow \cos^2\frac{x}{2} = \frac{1+\cos x}{2} = \frac{1+\left(-\frac{1}{3}\right)}{2} = \frac{\left(\frac{3-1}{3}\right)}{2} = \frac{\left(\frac{2}{3}\right)}{2} = \frac{1}{3}$$

$$\Rightarrow \cos\frac{x}{2} = -\frac{1}{\sqrt{3}} \qquad \left[\because \cos\frac{x}{2} \text{ is negative}\right]$$

$$\therefore \cos\frac{x}{2} = -\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)}{\left(\frac{-1}{\sqrt{3}}\right)} = -\sqrt{2}$$

Thus, the respective values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\frac{\sqrt{6}}{3}$, $\frac{-\sqrt{3}}{3}$, and $-\sqrt{2}$

-

Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ for $\sin x = \frac{1}{4}$, $x$ in quadrant II

**Ans:**

Here, $x$ is in quadrant II.

i.e., $\frac{\pi}{2} < x < \pi$

$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$

Therefore, $\sin\frac{x}{2}, \cos\frac{x}{2}$, and $\tan\frac{x}{2}$ are all positive.

It is given that $\sin x = \frac{1}{4}$.

$$\cos^2 x = 1 - \sin^2 x = 1 - \left(\frac{1}{4}\right)^2 = 1 - \frac{1}{16} = \frac{15}{16}$$

$\Rightarrow \cos x = -\frac{\sqrt{15}}{4}$ [cos x is negative in quadrant II]

$$\sin^2\frac{x}{2} = \frac{1-\cos x}{2} = \frac{1-\left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4+\sqrt{15}}{8}$$

$$\Rightarrow \sin\frac{x}{2} = \sqrt{\frac{4+\sqrt{15}}{8}} \qquad \left[\because \ \sin\frac{x}{2} \text{ is positive}\right]$$

$$= \sqrt{\frac{4+\sqrt{15}}{8} \times \frac{2}{2}}$$

$$= \sqrt{\frac{8+2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8+2\sqrt{15}}}{4}$$

$$\cos^2\frac{x}{2} = \frac{1+\cos x}{2} = \frac{1+\left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4-\sqrt{15}}{8}$$

$$\Rightarrow \cos\frac{x}{2} = \sqrt{\frac{4-\sqrt{15}}{8}} \qquad \left[\because \ \cos\frac{x}{2} \text{ is positive}\right]$$

$$= \sqrt{\frac{4-\sqrt{15}}{8} \times \frac{2}{2}}$$

$$= \sqrt{\frac{8-2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8-2\sqrt{15}}}{4}$$

$$\tan\frac{x}{2}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=\frac{\left(\frac{\sqrt{8+2\sqrt{15}}}{4}\right)}{\left(\frac{\sqrt{8-2\sqrt{15}}}{4}\right)}=\frac{\sqrt{8+2\sqrt{15}}}{\sqrt{8-2\sqrt{15}}}$$

$$=\sqrt{\frac{8+2\sqrt{15}}{8-2\sqrt{15}}\times\frac{8+2\sqrt{15}}{8+2\sqrt{15}}}$$

$$=\sqrt{\frac{\left(8+2\sqrt{15}\right)^2}{64-60}}=\frac{8+2\sqrt{15}}{2}=4+\sqrt{15}$$

Thus, the respective values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\frac{\sqrt{8+2\sqrt{15}}}{4}$, $\frac{\sqrt{8-2\sqrt{15}}}{4}$,

and $4+\sqrt{15}$

-

Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$

Answer

Let $\sin^{-1}\left(-\frac{1}{2}\right)=y.$ Then $\sin y = -\frac{1}{2}=-\sin\left(\frac{\pi}{6}\right)=\sin\left(-\frac{\pi}{6}\right).$

We know that the range of the principal value branch of $\sin^{-1}$ is

$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and $\sin\left(-\frac{\pi}{6}\right)=-\frac{1}{2}.$

Therefore, the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is $-\frac{\pi}{6}.$

-

Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Answer

Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = y$. Then, $\cos y = \frac{\sqrt{3}}{2} = \cos\left(\frac{\pi}{6}\right)$.

We know that the range of the principal value branch of $\cos^{-1}$ is

$[0,\pi]$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$.

Therefore, the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{6}$.

-

Find the principal value of $\operatorname{cosec}^{-1}(2)$

Answer

Let $\operatorname{cosec}^{-1}(2) = y$. Then, $\operatorname{cosec} y = 2 = \operatorname{cosec}\left(\frac{\pi}{6}\right)$.

We know that the range of the principal value branch of $\operatorname{cosec}^{-1}$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$.

Therefore, the principal value of $\operatorname{cosec}^{-1}(2)$ is $\frac{\pi}{6}$.

-

Find the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$

Answer

Let $\tan^{-1}\left(-\sqrt{3}\right) = y$. Then, $\tan y = -\sqrt{3} = -\tan\frac{\pi}{3} = \tan\left(-\frac{\pi}{3}\right)$.

We know that the range of the principal value branch of $\tan^{-1}$ is

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{3}\right)$ is $-\sqrt{3}$.

Therefore, the principal value of $\tan^{-1}\left(\sqrt{3}\right)$ is $-\frac{\pi}{3}$.

-

Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$

Answer

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y$. Then, $\cos y = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$.

We know that the range of the principal value branch of $\cos^{-1}$ is

$[0, \pi]$ and $\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$.

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

-

Find the principal value of $\tan^{-1}(-1)$

Answer

Let $\tan^{-1}(-1) = y$. Then, $\tan y = -1 = -\tan\left(\frac{\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right).$

We know that the range of the principal value branch of $\tan^{-1}$ is

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{4}\right) = -1.$

Therefore, the principal value of $\tan^{-1}(-1)$ is $-\frac{\pi}{4}.$

-

Find the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Answer

Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$. Then, $\sec y = \frac{2}{\sqrt{3}} = \sec\left(\frac{\pi}{6}\right).$

We know that the range of the principal value branch of $\sec^{-1}$ is

$[0,\pi] - \left\{\frac{\pi}{2}\right\}$ and $\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}.$

Therefore, the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}.$

-

Find the principal value of $\cot^{-1}\left(\sqrt{3}\right)$

Answer

Let $\cot^{-1}\left(\sqrt{3}\right) = y$. Then, $\cot y = \sqrt{3} = \cot\left(\frac{\pi}{6}\right)$.

We know that the range of the principal value branch of $\cot^{-1}$ is $(0,\pi)$ and

$\cot\left(\frac{\pi}{6}\right) = \sqrt{3}$.

Therefore, the principal value of $\cot^{-1}\left(\sqrt{3}\right)$ is $\frac{\pi}{6}$.

-

Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

Answer

Let $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = y$. Then, $\cos y = -\frac{1}{\sqrt{2}} = -\cos\left(\frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right)$

We know that the range of the principal value branch of $\cos^{-1}$ is $[0,\pi]$ and

$\cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$.

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

-

Find the principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$

Answer

Let $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right) = y$. Then, $\operatorname{cosec} y = -\sqrt{2} = -\operatorname{cosec}\left(\frac{\pi}{4}\right) = \operatorname{cosec}\left(-\frac{\pi}{4}\right)$

We know that the range of the principal value branch of $\operatorname{cosec}^{-1}$ is

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ and $\operatorname{cosec}\left(-\frac{\pi}{4}\right) = -\sqrt{2}$.

Therefore, the principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$ is $-\frac{\pi}{4}$.

-

Find the value of $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$

Answer

Let $\tan^{-1}(1)=x$. Then, $\tan x=1=\tan\frac{\pi}{4}$.

$\therefore \tan^{-1}(1)=\frac{\pi}{4}$

Let $\cos^{-1}\left(-\frac{1}{2}\right)=y$. Then, $\cos y=-\frac{1}{2}=-\cos\left(\frac{\pi}{3}\right)=\cos\left(\pi-\frac{\pi}{3}\right)=\cos\left(\frac{2\pi}{3}\right)$

$\therefore \cos^{-1}\left(-\frac{1}{2}\right)=\frac{2\pi}{3}$

Let $\sin^{-1}\left(-\frac{1}{2}\right)=z$. Then, $\sin z=-\frac{1}{2}=-\sin\left(\frac{\pi}{6}\right)=\sin\left(-\frac{\pi}{6}\right)$.

$\therefore \sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$

$\therefore \tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$

$=\frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{6}$

$=\frac{3\pi+8\pi-2\pi}{12}=\frac{9\pi}{12}=\frac{3\pi}{4}$

-

Find the value of $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$

Answer

Let $\cos^{-1}\left(\frac{1}{2}\right)=x$. Then, $\cos x=\frac{1}{2}=\cos\left(\frac{\pi}{3}\right)$.

$\therefore \cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$

Let $\sin^{-1}\left(\frac{1}{2}\right)=y$. Then, $\sin y=\frac{1}{2}=\sin\left(\frac{\pi}{6}\right)$.

$\therefore \sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$

$\therefore \cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}+\frac{2\pi}{6}=\frac{\pi}{3}+\frac{\pi}{3}=\frac{2\pi}{3}$

-

Find the value of if $\sin^{-1} x = y$, then

**(A)** $0\le y\le\pi$ **(B)** $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$

**(C)** $0<y<\pi$ **(D)** $-\frac{\pi}{2}<y<\frac{\pi}{2}$

Answer

It is given that $\sin^{-1} x = y$.

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

Therefore, $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$.

-

Find the value of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$ is equal to

**(A)** π **(B)** $-\frac{\pi}{3}$ **(C)** $\frac{\pi}{3}$ **(D)** $\frac{2\pi}{3}$

Answer

Let $\tan^{-1}\sqrt{3}=x$. Then, $\tan x=\sqrt{3}=\tan\frac{\pi}{3}$.

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$.

$\therefore \tan^{-1}\sqrt{3}=\frac{\pi}{3}$

Let $\sec^{-1}(-2)=y$. Then, $\sec y=-2=-\sec\left(\frac{\pi}{3}\right)=\sec\left(\pi-\frac{\pi}{3}\right)=\sec\frac{2\pi}{3}$.

We know that the range of the principal value branch of $\sec^{-1}$ is $[0,\pi]-\left\{\frac{\pi}{2}\right\}$.

$\therefore \sec^{-1}(-2)=\frac{2\pi}{3}$

Hence, $\tan^{-1}\left(\sqrt{3}\right)-\sec^{-1}(-2)=\frac{\pi}{3}-\frac{2\pi}{3}=-\frac{\pi}{3}$

-

Prove $3\sin^{-1}x = \sin^{-1}(3x-4x^3),\ x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Answer

To prove: $3\sin^{-1}x = \sin^{-1}(3x-4x^3),\ x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Let $x = \sin\theta$. Then, $\sin^{-1}x = \theta$.

We have,

R.H.S. = $\sin^{-1}(3x-4x^3) = \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$= \sin^{-1}(\sin 3\theta)$

$= 3\theta$

$= 3\sin^{-1}x$

= L.H.S.

-

Prove $3\cos^{-1}x = \cos^{-1}(4x^3-3x),\ x \in \left[\frac{1}{2}, 1\right]$

Answer

To prove: $3\cos^{-1}x = \cos^{-1}(4x^3-3x),\ x \in \left[\frac{1}{2}, 1\right]$

Let $x = \cos\theta$. Then, $\cos^{-1}x = \theta$.

We have,

$$\begin{aligned} \text{R.H.S.} &= \cos^{-1}(4x^3-3x) \\ &= \cos^{-1}(4\cos^3\theta - 3\cos\theta) \\ &= \cos^{-1}(\cos 3\theta) \\ &= 3\theta \\ &= 3\cos^{-1}x \\ &= \text{L.H.S.} \end{aligned}$$

Prove $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}=\tan^{-1}\frac{1}{2}$

Answer

To prove: $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}=\tan^{-1}\frac{1}{2}$

$$\text{L.H.S.}=\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}$$

$$=\tan^{-1}\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\cdot\frac{7}{24}} \qquad \left[\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right]$$

$$=\tan^{-1}\frac{\frac{48+77}{11\times24}}{\frac{11\times24-14}{11\times24}}$$

$$=\tan^{-1}\frac{48+77}{264-14}=\tan^{-1}\frac{125}{250}=\tan^{-1}\frac{1}{2}=\text{R.H.S.}$$

-

Prove $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$

Answer

To prove: $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$

$$\text{L.H.S.}=2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}$$

$$=\tan^{-1}\frac{2\cdot\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}+\tan^{-1}\frac{1}{7} \qquad \left[2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2}\right]$$

$$=\tan^{-1}\frac{1}{\left(\frac{3}{4}\right)}+\tan^{-1}\frac{1}{7}$$

$$=\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{1}{7}$$

$$=\tan^{-1}\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\cdot\frac{1}{7}} \qquad \left[\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right]$$

$$=\tan^{-1}\frac{\left(\frac{28+3}{21}\right)}{\left(\frac{21-4}{21}\right)}$$

$$=\tan^{-1}\frac{31}{17}=\text{R.H.S.}$$

-

Write the function in the simplest form:

$$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x},\ x\neq 0$$

Answer

$$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$$

Put $x=\tan\theta \Rightarrow \theta=\tan^{-1}x$

$$\therefore \tan^{-1}\frac{\sqrt{1+x^2}-1}{x}=\tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$=\tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)=\tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$=\tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$=\tan^{-1}\left(\tan\frac{\theta}{2}\right)=\frac{\theta}{2}=\frac{1}{2}\tan^{-1}x$$

-

Write the function in the simplest form:

$$\tan^{-1}\frac{1}{\sqrt{x^2-1}},\ |x|>1$$

Answer

$$\tan^{-1}\frac{1}{\sqrt{x^2-1}},\ |x|>1$$

Put $x = \operatorname{cosec}\theta \Rightarrow \theta = \operatorname{cosec}^{-1} x$

$$\therefore \tan^{-1}\frac{1}{\sqrt{x^2-1}} = \tan^{-1}\frac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$$

$$= \tan^{-1}\left(\frac{1}{\cot\theta}\right) = \tan^{-1}(\tan\theta)$$

$$= \theta = \operatorname{cosec}^{-1}x = \frac{\pi}{2} - \sec^{-1}x \qquad \left[\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}\right]$$

-

Write the function in the simplest form:

$$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right),\ x<\pi$$

Answer

$$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right),\ x<\pi$$

$$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right) = \tan^{-1}\left(\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}\right)$$

$$= \tan^{-1}\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right) = \tan^{-1}\left(\tan\frac{x}{2}\right)$$

$$= \frac{x}{2}$$

Write the function in the simplest form:

$$\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right),\ 0<x<\pi$$

Answer

$$\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$$

$$=\tan^{-1}\left(\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}\right)$$

$$=\tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$$

$$=\tan^{-1}(1)-\tan^{-1}(\tan x) \qquad \left[\tan^{-1}\frac{x-y}{1-xy}=\tan^{-1}x-\tan^{-1}y\right]$$

$$=\frac{\pi}{4}-x$$

-

Write the function in the simplest form:

$$\tan^{-1}\frac{x}{\sqrt{a^2-x^2}},\ |x|<a$$

Answer

$$\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$$

$$\text{Put } x=a\sin\theta \Rightarrow \frac{x}{a}=\sin\theta \Rightarrow \theta=\sin^{-1}\left(\frac{x}{a}\right)$$

$$\therefore \tan^{-1}\frac{x}{\sqrt{a^2-x^2}}=\tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2-a^2\sin^2\theta}}\right)$$

$$=\tan^{-1}\left(\frac{a\sin\theta}{a\sqrt{1-\sin^2\theta}}\right)=\tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right)$$

$$=\tan^{-1}(\tan\theta)=\theta=\sin^{-1}\frac{x}{a}$$

-

Write the function in the simplest form:

$$\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right),\ a>0;\ \frac{-a}{\sqrt{3}}\le x\le\frac{a}{\sqrt{3}}$$

Answer :

$$\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$$

Put $x = a\tan\theta \Rightarrow \frac{x}{a} = \tan\theta \Rightarrow \theta = \tan^{-1}\frac{x}{a}$

$$\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right) = \tan^{-1}\left(\frac{3a^2\cdot a\tan\theta - a^3\tan^3\theta}{a^3-3a\cdot a^2\tan^2\theta}\right)$$

$$= \tan^{-1}\left(\frac{3a^3\tan\theta - a^3\tan^3\theta}{a^3-3a^3\tan^2\theta}\right)$$

$$= \tan^{-1}\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)$$

$$= \tan^{-1}(\tan 3\theta)$$

$$= 3\theta$$

$$= 3\tan^{-1}\frac{x}{a}$$

-

Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

Answer

Let $\sin^{-1}\frac{1}{2} = x$. Then, $\sin x = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$.

$$\therefore \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$$

$$\therefore \tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] = \tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right]$$

$$= \tan^{-1}\left[2\cos\frac{\pi}{3}\right] = \tan^{-1}\left[2\times\frac{1}{2}\right]$$

$$= \tan^{-1}1 = \frac{\pi}{4}$$

-

Find the value of $\cot\left(\tan^{-1} a + \cot^{-1} a\right)$

Answer

$$\cot\left(\tan^{-1} a + \cot^{-1} a\right)$$

$$= \cot\left(\frac{\pi}{2}\right) \qquad \left[\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}\right]$$

$$= 0$$

-

Find the value of $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right],\ |x| < 1,\ y > 0 \text{ and } xy < 1$

Answer

Let $x = \tan\theta$. Then, $\theta = \tan^{-1} x$.

$$\therefore \sin^{-1}\frac{2x}{1+x^2} = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan^{-1} x$$

Let $y = \tan\Phi$. Then, $\Phi = \tan^{-1} y$.

$$\therefore \cos^{-1}\frac{1-y^2}{1+y^2} = \cos^{-1}\left(\frac{1-\tan^2\phi}{1+\tan^2\phi}\right) = \cos^{-1}(\cos 2\phi) = 2\phi = 2\tan^{-1} y$$

$$\therefore \tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right]$$

$$= \tan\frac{1}{2}\left[2\tan^{-1} x + 2\tan^{-1} y\right]$$

$$= \tan\left[\tan^{-1} x + \tan^{-1} y\right]$$

$$= \tan\left[\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$= \frac{x+y}{1-xy}$$

-

If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, then find the value of $x$.

Answer

$$\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$$

$$\Rightarrow \sin\left(\sin^{-1}\frac{1}{5}\right)\cos\left(\cos^{-1}x\right)+\cos\left(\sin^{-1}\frac{1}{5}\right)\sin\left(\cos^{-1}x\right)=1$$

$$\left[\sin(A+B)=\sin A\cos B+\cos A\sin B\right]$$

$$\Rightarrow \frac{1}{5}\times x+\cos\left(\sin^{-1}\frac{1}{5}\right)\sin\left(\cos^{-1}x\right)=1$$

$$\Rightarrow \frac{x}{5}+\cos\left(\sin^{-1}\frac{1}{5}\right)\sin\left(\cos^{-1}x\right)=1 \qquad ...(1)$$

Now, let $\sin^{-1}\frac{1}{5}=y$.

Then, $\sin y=\frac{1}{5}\Rightarrow \cos y=\sqrt{1-\left(\frac{1}{5}\right)^2}=\frac{2\sqrt{6}}{5}\Rightarrow y=\cos^{-1}\left(\frac{2\sqrt{6}}{5}\right)$.

$$\therefore \sin^{-1}\frac{1}{5}=\cos^{-1}\left(\frac{2\sqrt{6}}{5}\right) \qquad ...(2)$$

Let $\cos^{-1}x=z$.

Then, $\cos z = x \Rightarrow \sin z = \sqrt{1-x^2} \Rightarrow z = \sin^{-1}\left(\sqrt{1-x^2}\right)$.

$\therefore \cos^{-1} x = \sin^{-1}\left(\sqrt{1-x^2}\right)$ ...(3)

From (1), (2), and (3) we have:

$$\frac{x}{5} + \cos\left(\cos^{-1}\frac{2\sqrt{6}}{5}\right)\cdot\sin\left(\sin^{-1}\sqrt{1-x^2}\right) = 1$$

$$\Rightarrow \frac{x}{5} + \frac{2\sqrt{6}}{5}\cdot\sqrt{1-x^2} = 1$$

$$\Rightarrow x + 2\sqrt{6}\sqrt{1-x^2} = 5$$

$$\Rightarrow 2\sqrt{6}\sqrt{1-x^2} = 5 - x$$

On squaring both sides, we get:

$$(4)(6)\left(1-x^2\right) = 25 + x^2 - 10x$$

$$\Rightarrow 24 - 24x^2 = 25 + x^2 - 10x$$

$$\Rightarrow 25x^2 - 10x + 1 = 0$$

$$\Rightarrow (5x-1)^2 = 0$$

$$\Rightarrow (5x-1) = 0$$

$$\Rightarrow x = \frac{1}{5}$$

-

If $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$, then find the value of $x$.

Answer

$$\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right]=\frac{\pi}{4} \qquad \left[\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{(x-1)(x+2)+(x+1)(x-2)}{(x+2)(x-2)-(x-1)(x+1)}\right]=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{x^2+x-2+x^2-x-2}{x^2-4-x^2+1}\right]=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{2x^2-4}{-3}\right]=\frac{\pi}{4}$$

$$\Rightarrow \tan\left[\tan^{-1}\frac{4-2x^2}{3}\right]=\tan\frac{\pi}{4}$$

$$\Rightarrow \frac{4-2x^2}{3}=1$$

$$\Rightarrow 4-2x^2=3$$

$$\Rightarrow 2x^2=4-3=1$$

$$\Rightarrow x=\pm\frac{1}{\sqrt{2}}$$

Hence, the value of $x$ is $\pm\frac{1}{\sqrt{2}}$.

-

Find the values of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

Answer

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

We know that $\sin^{-1}(\sin x) = x$ if $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, which is the principal value branch of $\sin^{-1}x$.

Here, $\frac{2\pi}{3} \notin \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Now, $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ can be written as:

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left[\sin\left(\pi - \frac{2\pi}{3}\right)\right] = \sin^{-1}\left(\sin\frac{\pi}{3}\right) \text{ where } \frac{\pi}{3} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

$$\therefore \sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$$

-

Find the values of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

Answer

$\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

We know that $\tan^{-1}(\tan x) = x$ if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, which is the principal value branch of $\tan^{-1}x$.

Here, $\frac{3\pi}{4} \notin \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

Now, $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ can be written as:

$$\tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}\left[-\tan\left(\frac{-3\pi}{4}\right)\right] = \tan^{-1}\left[-\tan\left(\pi - \frac{\pi}{4}\right)\right]$$

$$= \tan^{-1}\left[-\tan\frac{\pi}{4}\right] = \tan^{-1}\left[\tan\left(-\frac{\pi}{4}\right)\right] \text{ where } -\frac{\pi}{4} \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$$

$$\therefore \tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}\left[\tan\left(\frac{-\pi}{4}\right)\right] = \frac{-\pi}{4}$$

-

Find the values of $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$

Answer

Let $\sin^{-1}\frac{3}{5}=x$ . Then, $\sin x=\frac{3}{5}\Rightarrow \cos x=\sqrt{1-\sin^2 x}=\frac{4}{5}\Rightarrow \sec x=\frac{5}{4}.$

$$\therefore \tan x=\sqrt{\sec^2 x-1}=\sqrt{\frac{25}{16}-1}=\frac{3}{4}$$

$$\therefore x=\tan^{-1}\frac{3}{4}$$

$$\therefore \sin^{-1}\frac{3}{5}=\tan^{-1}\frac{3}{4} \qquad ...(i)$$

$$\text{Now, } \cot^{-1}\frac{3}{2}=\tan^{-1}\frac{2}{3} \qquad ...(ii) \qquad \left[\tan^{-1}\frac{1}{x}=\cot^{-1}x\right]$$

$$\text{Hence, } \tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$$

$$=\tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right) \qquad [\text{Using (i) and (ii)}]$$

$$=\tan\left(\tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}}\right) \qquad \left[\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right]$$

$$=\tan\left(\tan^{-1}\frac{9+8}{12-6}\right)$$

$$=\tan\left(\tan^{-1}\frac{17}{6}\right)=\frac{17}{6}$$

-

Find the values of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to

**(A)** $\frac{7\pi}{6}$ **(B)** $\frac{5\pi}{6}$ **(C)** $\frac{\pi}{3}$ **(D)** $\frac{\pi}{6}$

Answer

We know that $\cos^{-1}(\cos x) = x$ if $x \in [0, \pi]$, which is the principal value branch of $\cos^{-1}x$.

Here, $\frac{7\pi}{6} \notin x \in [0, \pi]$.

Now, $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ can be written as:

$$\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\frac{-7\pi}{6}\right) = \cos^{-1}\left[\cos\left(2\pi - \frac{7\pi}{6}\right)\right] \quad [\cos(2\pi + x) = \cos x]$$

$$= \cos^{-1}\left[\cos\frac{5\pi}{6}\right] \text{ where } \frac{5\pi}{6} \in [0, \pi]$$

$$\therefore \cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\frac{5\pi}{6}\right) = \frac{5\pi}{6}$$

The correct answer is B.

-

Find the values of $\sin\left(\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right)$ is equal to

**(A)** $\frac{1}{2}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{4}$ **(D)** 1

Answer

Let $\sin^{-1}\left(\frac{-1}{2}\right)=x$. Then, $\sin x=\frac{-1}{2}=-\sin\frac{\pi}{6}=\sin\left(\frac{-\pi}{6}\right)$.

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

$\therefore \sin^{-1}\left(\frac{-1}{2}\right)=\frac{-\pi}{6}$

$$\therefore \sin\left(\frac{\pi}{3}-\sin^{-1}\left(\frac{-1}{2}\right)\right)=\sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right)=\sin\left(\frac{3\pi}{6}\right)=\sin\left(\frac{\pi}{2}\right)=1$$

The correct answer is D.

-

Find the value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

Answer

We know that $\cos^{-1}(\cos x) = x$ if $x \in [0, \pi]$, which is the principal value branch of $\cos^{-1}x$.

Here, $\frac{13\pi}{6} \notin [0, \pi]$.

Now, $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ can be written as:

$$\cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left[\cos\left(2\pi + \frac{\pi}{6}\right)\right] = \cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right], \text{ where } \frac{\pi}{6} \in [0, \pi].$$

$$\therefore \cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right] = \frac{\pi}{6}$$

-

Find the value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

Answer

We know that $\tan^{-1}(\tan x) = x$ if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, which is the principal value branch of $\tan^{-1}x$.

Here, $\frac{7\pi}{6} \notin \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Now, $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ can be written as:

$$\tan^{-1}\left(\tan\frac{7\pi}{6}\right) = \tan^{-1}\left[\tan\left(2\pi - \frac{5\pi}{6}\right)\right] \qquad \left[\tan(2\pi - x) = -\tan x\right]$$

$$= \tan^{-1}\left[-\tan\left(\frac{5\pi}{6}\right)\right] = \tan^{-1}\left[\tan\left(-\frac{5\pi}{6}\right)\right] = \tan^{-1}\left[\tan\left(\pi - \frac{5\pi}{6}\right)\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{6}\right)\right], \text{ where } \frac{\pi}{6} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\therefore \tan^{-1}\left(\tan\frac{7\pi}{6}\right) = \tan^{-1}\left(\tan\frac{\pi}{6}\right) = \frac{\pi}{6}$$

-

Prove $2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$

Answer

Let $\sin^{-1}\frac{3}{5} = x$. Then, $\sin x = \frac{3}{5}$.

$\Rightarrow \cos x = \sqrt{1-\left(\frac{3}{5}\right)^2} = \frac{4}{5}$

$\therefore \tan x = \frac{3}{4}$

$\therefore x = \tan^{-1}\frac{3}{4} \Rightarrow \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4}$

Now, we have:

$$\text{L.H.S.} = 2\sin^{-1}\frac{3}{5} = 2\tan^{-1}\frac{3}{4}$$

$$= \tan^{-1}\left(\frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}\right) \qquad \left[2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$$

$$= \tan^{-1}\left(\frac{\frac{3}{2}}{\frac{16-9}{16}}\right) = \tan^{-1}\left(\frac{3}{2}\times\frac{16}{7}\right)$$

$$= \tan^{-1}\frac{24}{7} = \text{R.H.S.}$$

-

Prove $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{77}{36}$

Answer

Let $\sin^{-1}\frac{8}{17}=x$. Then, $\sin x=\frac{8}{17}\Rightarrow \cos x=\sqrt{1-\left(\frac{8}{17}\right)^2}=\sqrt{\frac{225}{289}}=\frac{15}{17}$.

$\therefore \tan x=\frac{8}{15}\Rightarrow x=\tan^{-1}\frac{8}{15}$

$\therefore \sin^{-1}\frac{8}{17}=\tan^{-1}\frac{8}{15}$ ...(1)

Now, let $\sin^{-1}\frac{3}{5}=y$. Then, $\sin y=\frac{3}{5}\Rightarrow \cos y=\sqrt{1-\left(\frac{3}{5}\right)^2}=\sqrt{\frac{16}{25}}=\frac{4}{5}$.

$\therefore \tan y=\frac{3}{4}\Rightarrow y=\tan^{-1}\frac{3}{4}$

$\therefore \sin^{-1}\frac{3}{5}=\tan^{-1}\frac{3}{4}$ ...(2)

Now, we have:

$$\text{L.H.S.}=\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}$$

$$=\tan^{-1}\frac{8}{15}+\tan^{-1}\frac{3}{4} \quad \left[\text{Using (1) and (2)}\right]$$

$$=\tan^{-1}\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\times\frac{3}{4}}$$

$$=\tan^{-1}\left(\frac{32+45}{60-24}\right) \quad \left[\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right]$$

$$=\tan^{-1}\frac{77}{36}=\text{R.H.S.}$$

-

Prove $\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$

Answer

Let $\cos^{-1}\frac{4}{5}=x$. Then, $\cos x=\frac{4}{5}\Rightarrow \sin x=\sqrt{1-\left(\frac{4}{5}\right)^2}=\frac{3}{5}$.

$\therefore \tan x=\frac{3}{4}\Rightarrow x=\tan^{-1}\frac{3}{4}$

$\therefore \cos^{-1}\frac{4}{5}=\tan^{-1}\frac{3}{4}$ ...(1)

Now, let $\cos^{-1}\frac{12}{13}=y$. Then, $\cos y=\frac{12}{13}\Rightarrow \sin y=\frac{5}{13}$.

$\therefore \tan y=\frac{5}{12}\Rightarrow y=\tan^{-1}\frac{5}{12}$

$\therefore \cos^{-1}\frac{12}{13}=\tan^{-1}\frac{5}{12}$ ...(2)

Let $\cos^{-1}\frac{33}{65}=z$. Then, $\cos z=\frac{33}{65}\Rightarrow \sin z=\frac{56}{65}$.

$\therefore \tan z=\frac{56}{33}\Rightarrow z=\tan^{-1}\frac{56}{33}$

$\therefore \cos^{-1}\frac{33}{65}=\tan^{-1}\frac{56}{33}$ ...(3)

Now, we will prove that:

$$\text{L.H.S.} = \cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13}$$

$$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{5}{12} \qquad [\text{Using } (1) \text{ and } (2)]$$

$$= \tan^{-1}\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}.\frac{5}{12}} \qquad \left[\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$$

$$= \tan^{-1}\frac{36+20}{48-15}$$

$$= \tan^{-1}\frac{56}{33}$$

$$= \tan^{-1}\frac{56}{33} \qquad [\text{by } (3)]$$

$$= \text{R.H.S.}$$

-

Prove $\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}$

Answer

Let $\sin^{-1}\frac{3}{5}=x$. Then, $\sin x=\frac{3}{5}\Rightarrow \cos x=\sqrt{1-\left(\frac{3}{5}\right)^2}=\sqrt{\frac{16}{25}}=\frac{4}{5}$

$\therefore \tan x=\frac{3}{4}\Rightarrow x=\tan^{-1}\frac{3}{4}$

$\therefore \sin^{-1}\frac{3}{5}=\tan^{-1}\frac{3}{4}$ ...(1)

Now, let $\cos^{-1}\frac{12}{13}=y$. Then, $\cos y=\frac{12}{13}\Rightarrow \sin y=\frac{5}{13}$.

$\therefore \tan y=\frac{5}{12}\Rightarrow y=\tan^{-1}\frac{5}{12}$

$\therefore \cos^{-1}\frac{12}{13}=\tan^{-1}\frac{5}{12}$ ...(2)

Let $\sin^{-1}\frac{56}{65}=z$. Then, $\sin z=\frac{56}{65}\Rightarrow \cos z=\frac{33}{65}$.

$\therefore \tan z=\frac{56}{33}\Rightarrow z=\tan^{-1}\frac{56}{33}$

$\therefore \sin^{-1}\frac{56}{65}=\tan^{-1}\frac{56}{33}$ ...(3)

Now, we have:

$$\text{L.H.S.} = \cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5}$$

$$= \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{3}{4} \qquad \left[\text{Using } (1) \text{ and } (2)\right]$$

$$= \tan^{-1}\frac{\frac{5}{12}+\frac{3}{4}}{1-\frac{5}{12}.\frac{3}{4}} \qquad \left[\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$$

$$= \tan^{-1}\frac{20+36}{48-15}$$

$$= \tan^{-1}\frac{56}{33}$$

$$= \sin^{-1}\frac{56}{65} = \text{R.H.S.} \qquad \left[\text{Using } (3)\right]$$

-

Prove $\tan^{-1}\frac{63}{16}=\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}$

Answer

Let $\sin^{-1}\frac{5}{13}=x$. Then, $\sin x=\frac{5}{13}\Rightarrow\cos x=\frac{12}{13}$

$\therefore\tan x=\frac{5}{12}\Rightarrow x=\tan^{-1}\frac{5}{12}$

$\therefore\sin^{-1}\frac{5}{13}=\tan^{-1}\frac{5}{12}\qquad...(1)$

Let $\cos^{-1}\frac{3}{5}=y$. Then, $\cos y=\frac{3}{5}\Rightarrow\sin y=\frac{4}{5}$.

$\therefore\tan y=\frac{4}{3}\Rightarrow y=\tan^{-1}\frac{4}{3}$

$\therefore\cos^{-1}\frac{3}{5}=\tan^{-1}\frac{4}{3}\qquad...(2)$

Using (1) and (2), we have

$$\text{R.H.S.}=\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}$$

$$=\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{4}{3}$$

$$=\tan^{-1}\left(\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\times\frac{4}{3}}\right)\qquad\left[\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right]$$

$$=\tan^{-1}\left(\frac{15+48}{36-20}\right)$$

$$=\tan^{-1}\frac{63}{16}$$

$$=\text{L.H.S.}$$

-

Prove $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

Answer

$$\text{L.H.S.} = \tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}$$

$$= \tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\times\frac{1}{7}}\right)+\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}\times\frac{1}{8}}\right) \qquad \left[\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right]$$

$$= \tan^{-1}\left(\frac{7+5}{35-1}\right)+\tan^{-1}\left(\frac{8+3}{24-1}\right)$$

$$= \tan^{-1}\frac{12}{34}+\tan^{-1}\frac{11}{23}$$

$$= \tan^{-1}\frac{6}{17}+\tan^{-1}\frac{11}{23}$$

$$= \tan^{-1}\left(\frac{\frac{6}{17}+\frac{11}{23}}{1-\frac{6}{17}\times\frac{11}{23}}\right)$$

$$= \tan^{-1}\left(\frac{138+187}{391-66}\right)$$

$$= \tan^{-1}\left(\frac{325}{325}\right) = \tan^{-1}1$$

$$= \frac{\pi}{4} = \text{R.H.S.}$$

-

Prove $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right),\ x \in [0,\ 1]$

Answer

Let $x = \tan^2\theta$. Then, $\sqrt{x} = \tan\theta \Rightarrow \theta = \tan^{-1}\sqrt{x}$.

$$\therefore \frac{1-x}{1+x} = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$$

Now, we have:

$$\text{R.H.S.} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\cos^{-1}(\cos 2\theta) = \frac{1}{2}\times 2\theta = \theta = \tan^{-1}\sqrt{x} = \text{L.H.S.}$$

-

Prove $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2},\ x \in \left(0,\ \frac{\pi}{4}\right)$

Answer

Consider $\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}$

$$= \frac{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)^2}{\left(\sqrt{1+\sin x}\right)^2-\left(\sqrt{1-\sin x}\right)^2} \qquad \text{(by rationalizing)}$$

$$= \frac{(1+\sin x)+(1-\sin x)+2\sqrt{(1+\sin x)(1-\sin x)}}{1+\sin x-1+\sin x}$$

$$= \frac{2\left(1+\sqrt{1-\sin^2 x}\right)}{2\sin x} = \frac{1+\cos x}{\sin x} = \frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \cot\frac{x}{2}$$

$$\therefore \text{L.H.S.} = \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \cot^{-1}\left(\cot\frac{x}{2}\right) = \frac{x}{2} = \text{R.H.S.}$$

-

Prove $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\ -\frac{1}{\sqrt{2}}\le x\le 1$ [**Hint:** put $x = \cos 2\theta$]

Answer

Put $x=\cos 2\theta$ so that $\theta=\frac{1}{2}\cos^{-1}x$. Then, we have:

$$\text{L.H.S.}=\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$$

$$=\tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$$

$$=\tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$$

$$=\tan^{-1}\left(\frac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}\right)$$

$$=\tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)=\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$=\tan^{-1}1-\tan^{-1}(\tan\theta) \qquad \left[\tan^{-1}\left(\frac{x-y}{1+xy}\right)=\tan^{-1}x-\tan^{-1}y\right]$$

$$=\frac{\pi}{4}-\theta=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x=\text{R.H.S.}$$

-

Prove $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}=\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

Answer

$$\text{L.H.S.}=\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}$$
$$=\frac{9}{4}\left(\frac{\pi}{2}-\sin^{-1}\frac{1}{3}\right)$$
$$=\frac{9}{4}\left(\cos^{-1}\frac{1}{3}\right) \qquad . \qquad ....(1) \quad \left[\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}\right]$$

Now, let $\cos^{-1}\frac{1}{3}=x$. Then, $\cos x=\frac{1}{3}\Rightarrow \sin x=\sqrt{1-\left(\frac{1}{3}\right)^2}=\frac{2\sqrt{2}}{3}$.

$$\therefore x=\sin^{-1}\frac{2\sqrt{2}}{3}\Rightarrow \cos^{-1}\frac{1}{3}=\sin^{-1}\frac{2\sqrt{2}}{3}$$
$$\therefore \text{L.H.S.}=\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}=\text{R.H.S.}$$

-

Solve $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$

Answer

$$2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$$
$$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right)=\tan^{-1}(2\operatorname{cosec} x) \qquad \left[2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2}\right]$$
$$\Rightarrow \frac{2\cos x}{1-\cos^2 x}=2\operatorname{cosec} x$$
$$\Rightarrow \frac{2\cos x}{\sin^2 x}=\frac{2}{\sin x}$$
$$\Rightarrow \cos x=\sin x$$
$$\Rightarrow \tan x=1$$
$$\therefore x=\frac{\pi}{4}$$

Solve $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x,(x>0)$

Answer

$$\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x$$

$$\Rightarrow \tan^{-1}1-\tan^{-1}x=\frac{1}{2}\tan^{-1}x \qquad \left[\tan^{-1}x-\tan^{-1}y=\tan^{-1}\frac{x-y}{1+xy}\right]$$

$$\Rightarrow \frac{\pi}{4}=\frac{3}{2}\tan^{-1}x$$

$$\Rightarrow \tan^{-1}x=\frac{\pi}{6}$$

$$\Rightarrow x=\tan\frac{\pi}{6}$$

$$\therefore x=\frac{1}{\sqrt{3}}$$

-

Solve $\sin\left(\tan^{-1}x\right),\ |x|<1$ is equal to

**(A)** $\frac{x}{\sqrt{1-x^2}}$ **(B)** $\frac{1}{\sqrt{1-x^2}}$ **(C)** $\frac{1}{\sqrt{1+x^2}}$ **(D)** $\frac{x}{\sqrt{1+x^2}}$

Answer

Let $\tan^{-1}x = y$. Then, $\tan y = x \Rightarrow \sin y=\frac{x}{\sqrt{1+x^2}}$.

$$\therefore y=\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\Rightarrow \tan^{-1}x=\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$$\therefore \sin\left(\tan^{-1}x\right)=\sin\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right)=\frac{x}{\sqrt{1+x^2}}$$

The correct answer is D.

-

Solve $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$, then $x$ is equal to

**(A)** $0, \frac{1}{2}$ **(B)** $1, \frac{1}{2}$ **(C)** 0 **(D)** $\frac{1}{2}$

Answer

$$\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$$

$$\Rightarrow -2\sin^{-1}x=\frac{\pi}{2}-\sin^{-1}(1-x)$$

$$\Rightarrow -2\sin^{-1}x=\cos^{-1}(1-x) \qquad ...(1)$$

Let $\sin^{-1}x=\theta \Rightarrow \sin\theta = x \Rightarrow \cos\theta=\sqrt{1-x^2}$.

$$\therefore \theta=\cos^{-1}\left(\sqrt{1-x^2}\right)$$

$$\therefore \sin^{-1}x=\cos^{-1}\left(\sqrt{1-x^2}\right)$$

Therefore, from equation (1), we have

$$-2\cos^{-1}\left(\sqrt{1-x^2}\right)=\cos^{-1}(1-x)$$

Put $x$ = sin $y$. Then, we have:

$$-2\cos^{-1}\left(\sqrt{1-\sin^2 y}\right)=\cos^{-1}(1-\sin y)$$

$$\Rightarrow -2\cos^{-1}(\cos y)=\cos^{-1}(1-\sin y)$$

$$\Rightarrow -2y = \cos^{-1}(1-\sin y)$$
$$\Rightarrow 1-\sin y = \cos(-2y) = \cos 2y$$
$$\Rightarrow 1-\sin y = 1-2\sin^2 y$$
$$\Rightarrow 2\sin^2 y - \sin y = 0$$
$$\Rightarrow \sin y(2\sin y - 1) = 0$$
$$\Rightarrow \sin y = 0 \text{ or } \frac{1}{2}$$
$$\therefore x = 0 \text{ or } x = \frac{1}{2}$$

But, when $x = \frac{1}{2}$, it can be observed that:

$$\text{L.H.S.} = \sin^{-1}\left(1-\frac{1}{2}\right) - 2\sin^{-1}\frac{1}{2}$$
$$= \sin^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\frac{1}{2}$$
$$= -\sin^{-1}\frac{1}{2}$$
$$= -\frac{\pi}{6} \neq \frac{\pi}{2} \neq \text{R.H.S.}$$

$\therefore x = \frac{1}{2}$ is not the solution of the given equation.

Thus, $x = 0$.

Hence, the correct answer is **C**.

-

Solve $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\frac{x-y}{x+y}$ is equal to

**(A)** $\frac{\pi}{2}$ **(B).** $\frac{\pi}{3}$ **(C)** $\frac{\pi}{4}$ **(D)** $\frac{-3\pi}{4}$

Answer

$$\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\frac{x-y}{x+y}$$

$$=\tan^{-1}\left[\frac{\frac{x}{y}-\frac{x-y}{x+y}}{1+\left(\frac{x}{y}\right)\left(\frac{x-y}{x+y}\right)}\right] \qquad \left[\tan^{-1}y-\tan^{-1}y=\tan^{-1}\frac{x-y}{1+xy}\right]$$

$$=\tan^{-1}\left[\frac{\frac{x(x+y)-y(x-y)}{y(x+y)}}{\frac{y(x+y)+x(x-y)}{y(x+y)}}\right]$$

$$=\tan^{-1}\left(\frac{x^2+xy-xy+y^2}{xy+y^2+x^2-xy}\right)$$

$$=\tan^{-1}\left(\frac{x^2+y^2}{x^2+y^2}\right)=\tan^{-1}1=\frac{\pi}{4}$$

Hence, the correct answer is **C.**

-

Learn the Formulae from Videos

https://archive.org/details/11BasicTrigonometryFormulaeProblemsAndPractiseSolutionsAlliedExamples

Another bunch of videos explaining formulae

https://archive.org/details/TrigonometryFormulaePart1

AIEEE 2009 ( Now known as IIT JEE main )

AIEEE-Trigonometry Identities-Cos and Sin of Alpha Beta and Gamma sums=0 https://archive.org/details/AIEEETrigonometryIdentitiesCosAndSinOfAlphaBetaAndGammaSums0

Video Solutions to some problems from Book of Professor R S Agarwal ( page T 101 )

https://archive.org/details/2RSAgarwalPgT101TrigonometricEquationsPr20SinXTanXMinus1IsSinXTanX

Some problems related to Trigonometry Identities

https://archive.org/details/6TrigonometricIdentitiesCosecXMinusSinXIsACubeAndBCubeAlsoThere

AIEEE 2002 Trigonometry Solutions

https://archive.org/details/AIEEE2002TrigonometryInverseCircularFunctionsTrickToBeUsed

Cos square A - 120 plus cos square A plus Cos square A plus 120 is zero

$Cos^2$ (A - 120 ) + $Cos^2$ A + $Cos^2$ (A + 120 ) = 0

https://archive.org/details/CosSquareA120PlusCosSquareAPlusCosSquareAPlus120IsZero

if A + B + C = Pi then PT Sin A + Sin B + Sin C = 4 Cos A by 2 Cos B by 2 Cos C by 2

if A + B + C = π then Prove That Sin A + Sin B + Sin C = 4 Cos A/2 Cos B/2 Cos C/2

https://archive.org/details/IfABCPiThenPTSinASinBSinC4CosABy2CosBBy2CosCBy2

if sin A = k Sin B then ST tan A minus B by 2 is k minus 1 by k plus 1 X tan A plus B by 2

if sin A = k Sin B then Show That tan ( A - B )/2 = ( k - 1 )/( k + 1 ) ( tan A + tan B )/2

https://archive.org/details/IfSinAKSinBThenSTTanAMinusBBy2IsKMinus1ByKPlus1XTanAPlusBBy2

MNR ( Motilal Nehru Regional Engineering College ) Utterpradesh Joint Entrance Exam 1986 and 1990 Trigonometry Trick

https://archive.org/details/MNRUtterpradeshJointEntranceExam1986TrigonometryTrick

An interesting problem in inverse functions

https://archive.org/details/TrigonometryInverseFunctionCBSEIScIITJEEDezrinaSouthBangaloreSKMClasses

WB Joint JEE 1995 Trigonometric Identities Play with Theta ( West Bengal Joint )

https://archive.org/details/WBJointJEE1995TrigonometricIdentitiesPlayWithTheta

Before I list out the formulae ( which are given in all books ) let us see a few tricky problems

$$\text{If } a \left(\theta - \frac{\pi}{6}\right) = b \tan\left(\theta + \frac{2\pi}{3}\right), \text{ prove that } \cos 2\theta = \frac{a+b}{2(a-b)}$$

**Sol.**

$$a \tan\left(\theta - \frac{\pi}{6}\right) = b \tan\left(\theta + \frac{2\pi}{3}\right)$$

$$\Rightarrow \quad \frac{a}{b} = \frac{\tan\left(\theta + \frac{2\pi}{3}\right)}{\tan\left(\theta - \frac{\pi}{6}\right)}$$

Applying componendo and dividendo, we have

$$\frac{a+b}{a-b} = \frac{\tan\left(\theta + \frac{2\pi}{3}\right) + \tan\left(\theta - \frac{\pi}{6}\right)}{\tan\left(\theta + \frac{2\pi}{3}\right) - \tan\left(\theta - \frac{\pi}{6}\right)}$$

$$= \frac{\dfrac{\sin\left(\theta + \frac{2\pi}{3}\right)}{\cos\left(\theta + \frac{2\pi}{3}\right)} + \dfrac{\sin\left(\theta - \frac{\pi}{6}\right)}{\cos\left(\theta - \frac{\pi}{6}\right)}}{\dfrac{\sin\left(\theta + \frac{2\pi}{3}\right)}{\cos\left(\theta + \frac{2\pi}{3}\right)} - \dfrac{\sin\left(\theta - \frac{\pi}{6}\right)}{\cos\left(\theta - \frac{\pi}{6}\right)}}$$

or $$\frac{a+b}{a-b} = \frac{\sin\left(\theta+\frac{2\pi}{3}\right)\cos\left(\theta-\frac{\pi}{6}\right)+\cos\left(\theta+\frac{2\pi}{3}\right)\sin\left(\theta-\frac{\pi}{6}\right)}{\sin\left(\theta+\frac{2\pi}{3}\right)\cos\left(\theta-\frac{\pi}{6}\right)-\cos\left(\theta+\frac{2\pi}{3}\right)\sin\left(\theta-\frac{\pi}{6}\right)},$$

taking L.C.M.

or $$\frac{a+b}{a-b} = \frac{\sin\left[\left(\theta+\frac{2\pi}{3}\right)+\left(\theta-\frac{\pi}{6}\right)\right]}{\sin\left[\left(\theta+\frac{2\pi}{3}\right)-\left(\theta-\frac{\pi}{6}\right)\right]},$$

Using, $\sin\alpha\cos\beta + \cos\alpha\sin\beta = \sin(\alpha+\beta)$ and $\sin\alpha\cos\beta - \cos\alpha\sin\beta = \sin(\alpha-\beta)$

$$= \frac{\sin\left(\frac{\pi}{2}+2\theta\right)}{\sin\frac{5\pi}{6}} = \frac{\cos 2\theta}{\sin 150^\circ} = \frac{\cos 2\theta}{\left(\frac{1}{2}\right)}$$

$\Rightarrow$ $$\frac{a+b}{a-b} = 2\cos 2\theta$$

Hence, $$\cos 2\theta = \frac{a+b}{2(a-b)}.$$

So you see this problem was solved using Componendo Dividendo

There is a separate eBook only on Componendo Dividendo tricks. That is better seen.

Spoonfeeding Arithmatic Mean ≥ Geometric Mean

The minimum value of $27\tan^2\theta + 3\cot^2\theta$ is

(a) 9 (b) 18 (c) 27 (d) 30

*Ans.* (b)

A.M. $\geq$ G.M.

$\Rightarrow \quad \dfrac{27\tan^2\theta + 3\cot^2\theta}{2} \geq \sqrt{27\tan^2\theta \cdot 3\cot^2\theta}$

$\Rightarrow \quad 27\tan^2\theta + 3\cot^2\theta \geq 18.$

Spoonfeeding a Square of an expression equal to zero

If $\sin\theta + \operatorname{cosec}\theta = 2$, then $\sin^n\theta + \operatorname{cosec}^n\theta$ is equal to

(a) 2 (b) $2^n$ (c) $4^n$ (d) none of these

*Ans.* (a)

Solution We can write $\sin^2\theta + 1 = 2\sin\theta$

$\Rightarrow \quad \sin^2\theta - 2\sin\theta + 1 = 0 \quad \Rightarrow \quad (\sin\theta - 1)^2 = 0$

$\Rightarrow \quad \sin\theta = 1 \quad \Rightarrow \quad \operatorname{cosec}\theta = 1$

and thus $\sin^n\theta + \operatorname{cosec}^n\theta = 2$.

Spoonfeeding

$2\sec^2\alpha - \sec^4\alpha - 2\operatorname{cosec}^2\alpha + \operatorname{cosec}^4\alpha =$ 15/4

then tan a is equal to ?

(a) $1/\sqrt{2}$ (b) 1/2

(c) $1/2\sqrt{2}$ (d) 1/4

*Ans.* (a)

L.H.S. $= 2(1 + \tan^2\alpha - 1 - \cot^2\alpha) - [(1 + \tan^2\alpha)^2 - (1 + \cot^2\alpha^2)^2]$

$= \cot^4\alpha - \tan^4\alpha = 15/4$

$\tan^4\alpha = 1/4 \Rightarrow \tan\alpha = \pm 1/\sqrt{2}$.

Spoonfeeding

If $a = \cos\phi\cos\psi + \sin\phi\sin\psi\cos\delta$

$b = \cos\phi\sin\psi - \sin\phi\cos\psi\cos\delta$

and $c = \sin\phi\sin\delta$. Then $a^2 + b^2 + c^2 =$

(a) $-1$ (b) 0 (c) 1 (d) none of these

*Ans.* (c)

Solution $a^2 + b^2 + c^2 = \cos^2\phi\cos^2\psi + \sin^2\phi\sin^2\psi\cos^2\delta$

$+ \cos^2\phi\sin^2\psi + \sin^2\phi\cos^2\psi\cos^2\delta + \sin^2\phi\sin^2\delta$

$= \cos^2\phi + \sin^2\phi\cos^2\delta + \sin^2\phi\sin^2\delta$

$= \cos^2\phi + \sin^2\phi = 1.$

See another problem

*If* $\cos\theta = \dfrac{\cos\alpha - \cos\beta}{1 - \cos\alpha.\cos\beta}$, then prove that one of the values of $\tan\dfrac{\theta}{2}$ is $\tan\dfrac{\alpha}{2}\ \cot\dfrac{\beta}{2}$.

**Solution**

We have $\dfrac{\cos\theta}{1} = \dfrac{\cos\alpha - \cos\beta}{1 - \cos\alpha.\cos\beta}$

Now by componendo Dividendo we get,

$$\frac{1 - \cos\theta}{1 + \cos\theta} = \frac{1 - \cos\alpha.\cos\beta - \cos\alpha + \cos\beta}{1 - \cos\alpha.\cos\beta + \cos\alpha - \cos\beta}$$

$$\text{or, } \tan^2\frac{\theta}{2} = \frac{(1 - \cos\alpha) + \cos\beta(1 - \cos\beta)}{(1 - \cos\beta) + \cos\alpha(1 - \cos\beta)}$$

$$\text{or, } \tan^2\frac{\theta}{2} = \frac{(1 - \cos\alpha)(1 + \cos\beta)}{(1 - \cos\beta)(1 + \cos\alpha)} = \frac{2\sin^2\frac{\alpha}{2}.2\cos^2\frac{\alpha}{2}}{2\sin^2\frac{\beta}{2}.2\cos^2\frac{\alpha}{2}}$$

$$\text{or, } \tan^2\frac{\theta}{2} = \tan^2\frac{\alpha}{2}\ .\ \cot^2\frac{\beta}{2}$$

$$\therefore\ \tan\frac{\theta}{2} = \pm\tan\frac{\alpha}{2}\ .\ \cot\frac{\beta}{2}$$

Hence one of the values of $\tan\dfrac{\theta}{2}$ is $\tan\dfrac{\alpha}{2}\ .\ \cot\dfrac{\beta}{2}$.

Remember the surd values for various angles

| | 0° | 15° | 18° | 30° |
|---|---|---|---|---|
| sin | 0 | $\frac{\sqrt{6}-\sqrt{2}}{4}$ | $\frac{\sqrt{5}-1}{4}$ | $\frac{1}{2}$ |
| cos | 1 | $\frac{\sqrt{6}+\sqrt{2}}{4}$ | $\frac{\sqrt{10+2\sqrt{5}}}{4}$ | $\frac{\sqrt{3}}{2}$ |
| tan | 0 | $2-\sqrt{3}$ | $\frac{\sqrt{25-10\sqrt{5}}}{5}$ | $\frac{1}{\sqrt{3}}$ |
| | **36°** | **45°** | **60°** | **90°** |
| sin | $\frac{\sqrt{10-2\sqrt{5}}}{4}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| cos | $\frac{\sqrt{5}+1}{4}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |
| tan | $\sqrt{5-2\sqrt{5}}$ | 1 | $\sqrt{3}$ | not defined |

$$\sin\theta\sin(60^\circ-\theta)\sin(60^\circ+\theta)=(1/4)\sin 3\theta.$$
$$\cos\theta\cos(60^\circ-\theta)\cos(60^\circ+\theta)=(1/4)\cos 3\theta.$$
$$\tan\theta\tan(60^\circ-\theta)\tan(60^\circ+\theta)=\tan 3\theta$$
$$\sin 9^\circ=(1/4)\left[\sqrt{3+\sqrt{5}}-\sqrt{5-\sqrt{5}}\right]=\cos 81^\circ$$
$$\cos 9^\circ=(1/4)\left[\sqrt{3+\sqrt{5}}+\sqrt{5-\sqrt{5}}\right]=\sin 81^\circ$$
$$\cos 36^\circ-\cos 72^\circ=1/2$$
$$\cos 36^\circ\cos 72^\circ=1/4$$
$$\sin 22\frac{1}{2}^\circ=\left(\frac{1}{2}\right)\left[\sqrt{2-\sqrt{2}}\right]$$
$$\cos 22\frac{1}{2}^\circ=\left(\frac{1}{2}\right)\left[\sqrt{2+\sqrt{2}}\right]$$

Spoonfeeding

$\cos(540° - \theta) - \sin(630° - \theta)$ is equal to

(a) 0 (b) $2\cos\theta$

(c) $2\sin\theta$ (d) $\sin\theta - \cos\theta$

*Ans.* (a)

**Solution** $\cos(540° - \theta) = \cos(6(\pi/2) - \theta) = -\cos\theta$

$\sin(630° - \theta) = \sin(7(\pi/2) - \theta) = -\cos\theta$

All problems are not solved by substituting the value of the angles

The value of Sin 12 Sin 48 Sin 54

( a ) Sin 30 (b) $\sin^2 30°$ (c) $\sin^3 30°$ (d) $\cos^3 30°$

Ans : ( c )

Solution $\sin 12° \sin 48° \sin 54°$

$$= \frac{\sin 12° \sin(60° - 12°)\cos 36° \sin(60° + 12°)}{\sin 72°}$$

$$= \frac{\sin 12°(\sqrt{3}\cos 12° - \sin 12°)(\sqrt{3}\cos 12° + \sin 12°)\cos 36°}{4\sin 72°}$$

$$= \frac{\sin 12°(3\cos^2 12° - \sin^2 12°)\cos 36°}{4\sin 72°}$$

$$= \frac{(3\sin 12° - 4\sin^3 12°)\cos 36°}{4\sin 72°} = \frac{\sin 36° \cos 36°}{4\sin 72°} = \frac{1}{8}$$

$= \sin^3 30°.$

(a) Prove that $\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\ldots\ldots \tan 89^\circ = 1$

(b) $\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \ldots + \sin^2 85^\circ + \sin^2 90^\circ = 9\frac{1}{2}$

(Karnataka C.E.E. 1999)

(c) One of the roots of the equation $8x^3 - 6x + 1 = 0$ is

(a) $\cos 10^\circ$ (b) $\cos 30^\circ$

(c) $\sin 30^\circ$ (d) $\cos 80^\circ$

(a) By complementary rule

$(\tan 1^\circ \cot 1^\circ)(\tan 2^\circ \cot 2^\circ)(\tan 3^\circ \cot 3^\circ)\ldots.$

$(\tan 44^\circ \cot 44^\circ)(\tan 45^\circ) = 1$

(b) $8 + 1 + \frac{1}{2} = 9\frac{1}{2}$. The angles are in A.P. of 18 terms of which $\sin^2 90^\circ = 1$,

$\sin^2 45^\circ = \frac{1}{2}$ and 16 terms form 8 pairs like

$\sin^2 5^\circ + \sin^2 85^\circ = \sin^2 5^\circ + \cos^2 5^\circ = 1$

**by Complementary Rule.**

(c) Ans. (d). From given equation $4x^3 - 3x = -1/2$

If $x = \sin\theta$, then $-\sin 3\theta = -1/2$ or $3\theta = 30^\circ$

or $\theta = 10^\circ$

$\therefore \quad x = \sin 10^\circ = \cos 80^\circ \Rightarrow$ (d)

Formulae to be remembered

$$\frac{1-\cos\theta}{1+\cos\theta} = \tan^2\frac{\theta}{2}.$$

$$\sin 3x = 3\sin x - 4\sin^3 x \qquad \cos 3x = 4\cos^3 x - 3\cos x$$

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}.$$

$$\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A\tan B\tan C}{1 - \tan B\tan C - \tan C\tan A - \tan A\tan B}.$$

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}. \qquad (Note)$$

$$\tan A + \tan B = \frac{\sin(A+B)}{\cos A\cos B}$$

$$\tan A - \tan B = \frac{\sin(A-B)}{\cos A\cos B}.$$

First try yourself

(a) Prove : $\cot\theta - \cot 2\theta = \operatorname{cosec} 2\theta$

(b) $\tan 70° - \tan 20^{0} = 2\tan 50°$

(a) Changing to $\sin\theta$ and $\cos\theta$, we get

$$\cot\theta - \cot 2\theta = \frac{\cos\theta}{\sin\theta} - \frac{\cos 2\theta}{\sin 2\theta}$$

$$= \frac{\sin 2\theta \cos\theta - \cos 2\theta \sin\theta}{\sin\theta \sin 2\theta}$$

$$= \frac{\sin(2\theta - \theta)}{\sin\theta \sin 2\theta} = \operatorname{cosec} 2\theta.$$

(b) $\because \quad 70^\circ = 50^\circ + 20^\circ,$

$$\therefore \quad \tan 70^\circ = \frac{\tan 50^\circ + \tan 20^\circ}{1 - \tan 50^\circ \tan 20^\circ}$$

or $\tan 70^\circ - \tan 70^\circ \tan 50^\circ \tan 20^\circ$

$$= \tan 50^\circ + \tan 20^\circ$$

or $\tan 70^\circ - \tan 20^\circ = \tan 50^\circ + \tan 50^\circ$

$$= 2\tan 50^\circ$$

$[\because \tan 70^\circ \tan 20^\circ = \tan(90^\circ - 20^\circ)\tan 20^\circ$

$= \cot 20^\circ \tan 20^\circ = 1]$

**Alt.** $\tan 70^\circ - \tan 20^\circ = \dfrac{\sin(70^\circ - 20^\circ)}{\cos 70^\circ \cos 20^\circ}$

$$= \frac{2\sin 50^\circ}{2\sin 20^\circ \cos 20^\circ} \qquad \textbf{(Comp. Rule)}$$

$$= \frac{2\sin 50^\circ}{\sin 40^\circ} = \frac{2\sin 50^\circ}{\cos 50^\circ} = 2\tan 50^\circ$$

**if A+B=45 then 2=( 1 + tan A ) ( 1 + tan B )**

**( 1 + tan 1 ) ( 1 + tan 2 ) ( 1 + tan 3 ) …. ( 1 + tan 45 ) = 2 to the power n then how much is n**

If $(1+\tan 1^\circ)(1+\tan 2^\circ) \ldots (1+\tan 45^\circ) = 2^n$ then $n = ?$

We have $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

If $A+B=45$

$\Rightarrow 1 - \tan A \tan B = \tan A + \tan B$

$\Rightarrow 2 = \tan A + \tan B + \tan A \tan B + 1$

$= (1+\tan A)(1+\tan B)$

Solution:

$[(1+\tan 1^\circ)(1+\tan 44^\circ)][(1+\tan 2^\circ)(1+\tan 43^\circ)]$

……… 22 times $](1+\tan 45^\circ) = 2^n$

Now if $A + B = 45^\circ$ then

$(1+\tan A)(1+\tan B) = 2$

so (2. 2……. 22 times) $(1+1) = 2^n$

$2^{22} \cdot 2^1 = 2^n$

$2^{23} = 2^n \quad \Rightarrow n = 23$

**tan (50) = tan (40+ 10) ( i have omitted degree sign)**
**= (tan 40 + tan 10)/ (1- tan 40 tan 10)**

**so tan 50 (1- tan 40 tan 10) = tan 40 + tan 10**
**or tan 50 - tan 50 tan 40 tan 10 = tan 40 + tan 10**
**or tan 50 - tan 10 (as tan 50 tan 40 = 1) = tan 40 + tan 10**
**or tan 50 = tan 40 + 2 tan 10**
**or tan 50 - tan 40 = 2 tan 10**

**Another method**
**tan(A)-tan(B) = sin(A-B)/(cos(A)cos(B))**
**cos(A+B)+cos(A-B) = 2cos(A)cos(B)**

**tan(A)-tan(B) = sin(A-B)/((cos(A+B)+cos(A-B))/2 )**
**tan(A)-tan(B) = 2*sin(A-B)/(cos(A+B)+cos(A-B))**

**A=50°, B=40°**
**tan(50°)-tan(40°) = 2*sin(50°-40°)/(cos(50°+40°)+cos(50°-40°…‡**
**tan(50°)-tan(40°) = 2*sin(10°)/(cos(90°)+cos(10°))**
**tan(50°)-tan(40°) = 2*sin(10°)/(0+cos(10°))**
**tan(50°)-tan(40°) = 2*sin(10°)/cos(10°)**
**tan(50°)-tan(40°) = 2*tan(10°)**

**Another Method**

tan(45+5)-tan(45-5)

=[ (tan45+tan5) / (1-tan45*tan5) ] - [(tan45-tan5) / (1+tan45*tan5)]

=[ tan45+tan5+(tan^2 45 tan5)+(tan^2 5 tan45)-tan45+tan5+
(tan^2 45 tan5)-(tan^2 5 tan45) ] / 1-(tan 45 tan 5)^2

=[2tan5+2tan5] / 1-(tan5)^2

=2 tan 10

Spoonfeeding substitution of values after simplification

If $\cos A = 3/4$, then the value of $16\cos^2(A/2) - 32\sin(A/2)\sin(5A/2)$ is

(a) $-4$ (b) $-3$ (c) $3$ (d) $4$

*Ans.* (c)

**Solution** The given expression is equal to

$$8(1+\cos A) - 16(\cos 2A - \cos 3A)$$
$$= 8(1+\cos A) - 16[2\cos^2 A - 1 - \cos A(4\cos^2 A - 3)]$$
$$= 8\left(1+\frac{3}{4}\right) - 16\left[2\times\frac{9}{16} - 1 - \frac{3}{4}\left(4\times\frac{9}{16} - 3\right)\right]$$
$$= 14 - (18 - 16 - 27 + 36) = 3.$$

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$
$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$
$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$
$$2\sin A\sin B = \cos(A-B) - \cos(A+B) \quad (\textit{Note})$$
$$\sin nA = \cos^n A\,({}^nC_1\tan A - {}^nC_3\tan^3 A + {}^nC_5\tan^5 A - \ldots)$$
$$\cos nA = \cos^n A\,(1 - {}^nC_2\tan^2 A + {}^nC_4\tan^4 A \ldots)$$
$$\tan nA = \frac{{}^nC_1\tan A - {}^nC_3\tan^3 A + {}^nC_5\tan^5 A \ldots}{1 - {}^nC_2\tan^2 A + {}^nC_4\tan^4 A \ldots}$$
$$\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \ldots + \sin(\alpha+(n-1)\beta)$$
$$= \frac{\sin(\alpha+(n-1)\beta/2)}{\sin(\beta/2)}\sin(n\beta/2)$$
$$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \ldots + \cos(\alpha+(n-1)\beta)$$
$$= \frac{\cos(\alpha+(n-1)\beta/2)}{\sin(\beta/2)}\sin(n\beta/2)$$

$$\tan 22\frac{1}{2}^\circ = \sqrt{2} - 1$$

$$\cot 22\frac{1}{2}^\circ = \sqrt{2} + 1$$

$$-\sqrt{a^2+b^2} \le a \sin x + b \cos x \le \sqrt{a^2+b^2} \quad \text{for all } x \in \mathbf{R}.$$

Spoonfeeding

$\tan 203^\circ + \tan 22^\circ + \tan 203^\circ \tan 22^\circ =$

(a) $-1$ (b) $0$ (c) $1$ (d) $2$

*Ans.* (c)

**Solution** $\tan(203^\circ + 22^\circ) = \dfrac{\tan 203^\circ + \tan 22^\circ}{1 - \tan 203^\circ \tan 22^\circ}$

$\Rightarrow \quad 1 = \tan(180^\circ + 45^\circ) = \dfrac{\tan 203^\circ + \tan 22^\circ}{1 - \tan 203^\circ \tan 22^\circ}$

$\Rightarrow \quad \tan 203^\circ + \tan 22^\circ + \tan 203^\circ \tan 22^\circ = 1.$

Conditional Identities

if $A + B + C = \pi$, then

1. $\sin(B + C) = \sin A, \quad \cos B = -\cos(C + A)$
2. $\cos(A + B) = -\cos C, \quad \sin C = \sin(A + B)$
3. $\tan(C + A) = -\tan B, \quad \cot A = -\cot(B + C).$
4. $\cos\dfrac{A+B}{2} = \sin\dfrac{C}{2}, \quad \cos\dfrac{C}{2} = \sin\dfrac{A+B}{2}$
5. $\sin\dfrac{C+A}{2} = \cos\dfrac{B}{2}, \quad \sin\dfrac{A}{2} = \cos\dfrac{B+C}{2}.$
6. $\tan\dfrac{B+C}{2} = \cot\dfrac{A}{2}, \quad \tan\dfrac{B}{2} = \cot\dfrac{C+A}{2}.$

*Some Important Identities* If $A + B + C = \pi$, then

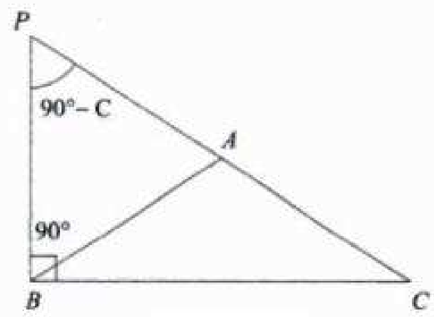
1. $\tan A + \tan B + \tan C = \tan A \tan B \tan C$.
2. $\cot B \cot C + \cot C \cot A + \cot A \cot B = 1$.
3. $\tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} + \tan \frac{A}{2} \tan \frac{B}{2} = 1$.
4. $\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \cot \frac{A}{2} \cot \frac{B}{2} \cot \frac{C}{2}$.
5. $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$.
6. $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$.
7. $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A \cos B \cos C$.
8. $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$.
9. $\cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$

Problems combined with Geometry

In a triangle $ABC$, $BP$ is drawn perpendicular to $BC$ to meet $CA$ in $P$, such that $CA = AP$, then $\frac{BP}{AB} =$

(a) $2 \sin A$ (b) $2 \sin B$
(c) $2 \sin C$ (d) none of these

*Ans.* (c)




Solution We have $\frac{BP}{AB} = \frac{BP}{AC}$

($AB$, $AC$ are the radii of the circle on $CP$ as diametre)

$= 2\frac{BP}{CP} = 2 \sin C$.

Spoonfeeding

If $\cos \theta = \cos \alpha \cos \beta$, then $\tan \frac{\theta + \alpha}{2} \tan \frac{\theta - \alpha}{2}$ is equal to

(a) $\tan^2 (\alpha/2)$ (b) $\tan^2 (\beta/2)$ (c) $\tan^2 (\theta/2)$ (d) $\cot^2 (\beta/2)$

*Ans.* (b)

Solution :

$$\tan\frac{\theta+\alpha}{2}\tan\frac{\theta-\alpha}{2}$$

$$=\frac{\tan^2(\theta/2)-\tan^2(\alpha/2)}{1-\tan^2(\theta/2)\tan^2(\alpha/2)}$$

$$=\frac{\frac{1-\cos\theta}{1+\cos\theta}-\frac{1-\cos\alpha}{1+\cos\alpha}}{1-\frac{1-\cos\theta}{1+\cos\theta}\cdot\frac{1-\cos\alpha}{1+\cos\alpha}}$$

$$=\frac{2(\cos\alpha-\cos\theta)}{2(\cos\alpha+\cos\theta)}=\frac{\cos\alpha(1-\cos\beta)}{\cos\alpha(1+\cos\beta)}=\tan^2\frac{\beta}{2}$$

Spoonfeeding

$$\tan^6\frac{\pi}{9}-33\tan^4\frac{\pi}{9}+27\tan^2\frac{\pi}{9}=$$

(a) $\tan\frac{\pi}{3}$

(b) $\tan^2\frac{\pi}{3}$

(c) $\tan\frac{\pi}{6}$

(d) $\tan^2\frac{\pi}{6}$

*Ans.* (b)

**Solution** $\tan 3\theta=\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}$

$\Rightarrow$ $\sqrt{3}=\tan 3\times\frac{\pi}{9}=\frac{3\tan\frac{\pi}{9}-\tan^3\frac{\pi}{9}}{1-3\tan^2\frac{\pi}{9}}$

$\Rightarrow$ $\left[\sqrt{3}\left(1-3\tan^2\frac{\pi}{9}\right)\right]^2=\left(3\tan\frac{\pi}{9}-\tan^3\frac{\pi}{9}\right)^2$

$$\Rightarrow\tan^6\frac{\pi}{9}-33\tan^4\frac{\pi}{9}+27\tan^2\frac{\pi}{9}=3=\tan^2\frac{\pi}{3}$$

Spoonfeeding modulus trick

The equation $a \sin x + b \cos x = c$, where $|c| > \sqrt{a^2 + b^2}$ has

(a) a unique solution
(b) infinite number of solutions
(c) no solution
(d) none of these

*Ans.* (c)

**Solution** Let $a = r \cos \alpha$, $b = r \sin \alpha$ so that $r = \sqrt{a^2 + b^2}$.

The given equation can be written as

$$r \sin (x + \alpha) = c \Rightarrow \sin (x + \alpha) = \frac{c}{\sqrt{a^2 + b^2}}$$

$$\Rightarrow \quad |\sin (x + \alpha)| = \frac{|c|}{\sqrt{a^2 + b^2}} > 1 \text{ as } |c| > \sqrt{a^2 + b^2}$$

which is not possible for any value of $x$.

Spoonfeeding Inequality trick

If $\cot \alpha$ equals the integral solution of the inequality $4x^2 - 16x + 15 < 0$ and $\sin \beta$ equals to the slope of the bisector of the first quadrant, then $\sin (\alpha + \beta) \sin (\alpha - \beta)$ is equal to

(a) $-3/5$ (b) $-4/5$ (c) $2/\sqrt{5}$ (d) 3

*Ans.* (b)

**Solution** We have $4x^2 - 16x + 15 < 0$

$$\Rightarrow \quad \frac{3}{2} < x < \frac{5}{2}$$

$\Rightarrow \cot \alpha = 2$, the integral solution of the given inequality

and $\sin \beta = \tan 45° = 1$

$$\therefore \quad \sin (\alpha + \beta) \sin (\alpha - \beta) = \sin^2 \alpha - \sin^2 \beta$$

$$= \frac{1}{1 + \cot^2 \alpha} - 1 = \frac{1}{1+4} - 1 = -\frac{4}{5}.$$

Spoonfeeding a π/7 problem

The value of $\cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} + \cos \frac{7\pi}{7}$ is

(a) 1 (b) $-1$ (c) 1/2 (d) $-3/2$

*Ans.* (d)

Solution $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}+\cos\frac{7\pi}{7}$

$$=\frac{1}{2\sin\frac{\pi}{7}}\left[2\sin\frac{\pi}{7}\cos\frac{2\pi}{7}+2\sin\frac{\pi}{7}\cos\frac{4\pi}{7}+2\sin\frac{\pi}{7}\cos\frac{6\pi}{7}\right]-1$$

$$=\frac{1}{2\sin\frac{\pi}{7}}\left[\sin\frac{3\pi}{7}-\sin\frac{\pi}{7}+\sin\frac{5\pi}{7}-\sin\frac{3\pi}{7}+\sin\frac{7\pi}{7}-\sin\frac{5\pi}{7}\right]-1$$

$$=\frac{1}{2\sin\frac{\pi}{7}}\left(-\sin\frac{\pi}{7}\right)-1=-\frac{1}{2}-1=-\frac{3}{2}.$$

Spoonfeeding Maxima of a Trigonometry Function

The greatest value of $f(x) = 2\sin x + \sin 2x$ on $[0, 3\pi/2]$, is given by

(a) 9/2 (b) 5/2 (c) $3\sqrt{3}/2$ (d) 3/2

*Ans.* (c)

Solution $f'(x) = 2\cos x + 2\cos 2x$ and $f''(x) = -2\sin x - 4\sin 2x$

For extreme value $f'(x) = 0$

$\Rightarrow$ $\cos x + 2\cos^2 x - 1 = 0$

$\Rightarrow$ $\cos x = -1$ or $1/2$

$\Rightarrow$ $x = \pi$ or $\pi/3$ as $x \in [0, 3\pi/2]$

Now $f(\pi) = 0$ and $f(\pi/3) = \frac{2\sqrt{3}}{2}+\frac{\sqrt{3}}{2}=\frac{3\sqrt{3}}{2}$

Also $f(0) = 0$ and $f(3\pi/2) = -2$ so the greatest value of $f(x)$ is $3\sqrt{3}/2$

A trick problem independent of θ

If $x = a\cos^3\theta\sin^2\theta$, $y = a\sin^3\theta\cos^2\theta$ and $\frac{(x^2+y^2)^p}{(xy)^q}$ $(p, q, \in \mathbf{N})$ is independent of $\theta$, then

(a) $4p = 5q$ (b) $4q = 5p$ (c) $p + q = 9$ (d) $pq = 20$

*Ans.* (a)

Solution $\frac{(x^2+y^2)^p}{(xy)^q}=\frac{[a\sin^2\theta\cos^2\theta]^{2p}}{[a^2\sin^5\theta\cos^5\theta]^q}=\frac{a^{2p}(\sin\theta\cos\theta)^{4p}}{a^{2q}(\sin\theta\cos\theta)^{5q}}$

which is independent of $\theta$ if $4p = 5q$.

Spoonfeeding

If $\cos\alpha + \cos\beta = a$, $\sin\alpha + \sin\beta = b$ and $\alpha - \beta = 2\theta$, then $\dfrac{\cos 3\theta}{\cos\theta} =$

(a) $a^2 + b^2 - 2$ (b) $a^2 + b^2 - 3$ (c) $3 - a^2 - b^2$ (d) $(a^2 + b^2)/4$

*Ans.* (b)

**Solution** From the given relations we have

$$a^2 + b^2 = \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta + \sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta$$

$$= 2 + 2\cos(\alpha - \beta) = 2 + 2\cos 2\theta = 4\cos^2\theta$$

Now $\dfrac{\cos 3\theta}{\cos\theta} = \dfrac{4\cos^3\theta - 3\cos\theta}{\cos\theta} = 4\cos^2\theta - 3 = a^2 + b^2 - 3.$

Spoonfeeding

If $\dfrac{1}{\cos\alpha\cos\beta} + \tan\alpha\tan\beta = \tan\gamma$, $0 < \alpha, \beta < \pi$ then $1 - \tan^2\gamma < 0$ for

(a) all values of $\alpha$ and $\beta$
(b) no values of $\alpha$ and $\beta$
(c) finite number of values of $\alpha$ and $\beta$
(d) infinite number of values of $\alpha$ and $\beta$

*Ans.* (a)

**Solution** We have $1 - \tan^2\gamma = \dfrac{\cos^2\alpha\cos^2\beta - (1 + \sin\alpha\sin\beta)^2}{\cos^2\alpha\cos^2\beta}$

$$= \frac{(1 - \sin^2\alpha)(1 - \sin^2\beta) - (1 + 2\sin\alpha\sin\beta + \sin^2\alpha\sin^2\beta)}{\cos^2\alpha\cos^2\beta}$$

$$= \frac{-(\sin\alpha + \sin\beta)^2}{\cos^2\alpha\cos^2\beta} < 0 \quad (\sin\alpha + \sin\beta \neq 0, \text{ as } 0 < \alpha, \beta < \pi)$$

Spoonfeeding

If $\sin 32° = k$ and $\cos x = 1 - 2k^2$; $\alpha, \beta$ are the values of $x$ between 0° and 360° with $\alpha < \beta$, then

(a) $\alpha + \beta = 180°$
(b) $\beta - \alpha = 200°$
(c) $\beta = 4\alpha + 40°$
(d) $\beta = 5\alpha - 20°$

*Ans.* (c)

**Solution** $\cos x = 1 - 2k^2 = 1 - 2\sin^2 32° = \cos 64°$

$\Rightarrow$ $x = 64°$ or $296°$

$\therefore$ $\alpha = 64°$ and $\beta = 296°$

which satisfy (c).

Spoonfeeding problem with Determinant

If $D = \begin{vmatrix} 1 & \cos\theta & 1 \\ -\sin\theta & 1 & -\cos\theta \\ -1 & \sin\theta & 1 \end{vmatrix}$ then $D$ lies in the interval

(a) [0, 4]
(b) [2, 4]
(c) $[2-\sqrt{2}, 2+\sqrt{2}]$
(d) [–2, 2]

*Ans.* (c)

**Solution** Expanding $D$, we get

$$D = 1 + \sin\theta\cos\theta - \cos\theta(-\sin\theta - \cos\theta) + (-\sin^2\theta + 1)$$
$$= 2 + \sin 2\theta + \cos 2\theta = 2 + \sqrt{2}\cos(2\theta - \pi/4)$$

As $-1 \le \cos(2\theta - \pi/4) \le 1$, $2 - \sqrt{2} \le D \le 2 + \sqrt{2}$

Spoonfeeding problem with Determinant

The value of $\theta$ lying between $\theta = 0$ and $\theta = \pi/2$ and satisfying the equation

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0 \text{ is}$$

(a) $3\pi/24$ (b) $5\pi/24$ (c) $11\pi/24$ (d) $\pi/24$

*Ans.* (c)

Solution Applying $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$ to the given determinant we get

$$\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

$\Rightarrow \quad 1 + 4\sin 4\theta + \cos^2\theta + \sin^2\theta = 0$

$\Rightarrow \quad \sin 4\theta = -1/2 \Rightarrow 4\theta = \pi + \pi/6$ or $2\pi - \pi/6$ $\quad [\because 0 < 4\theta < 2\pi]$

$\Rightarrow \quad \theta = 7\pi/24$ or $11\pi/24$.

Spoonfeeding

The value of the determinant

$$\begin{vmatrix} 1 & a & a^2 \\ \cos(n-1)x & \cos nx & \cos(n+1)x \\ \sin(n-1)x & \sin nx & \sin(n+1)x \end{vmatrix} \qquad (a \neq 1)$$

is zero if

(a) $\sin x = 0$ (b) $\cos x = 0$

(c) $a = 0$ (d) $\cos x = \dfrac{1+a^2}{2a}$

*Ans.* (a).

*Solution* Applying $C_1 \to C_1 + C_3 - 2\cos x\, C_2$, the given determinant is equal to

$$\begin{vmatrix} 1+a^2-2a\cos x & a & a^2 \\ 0 & \cos nx & \cos(n+1)x \\ 0 & \sin nx & \sin(n+1)x \end{vmatrix}$$

$$= (1 + a^2 - 2a \cos x) [\cos nx \sin(n + 1)x - \sin nx \cos (n + 1) x]$$
$$= (1 + a^2 - 2a \cos x) \sin (n + 1 - n) x$$
$$= (1 + a^2 - 2a \cos x) \sin x$$

which is zero if $\sin x = 0$ or $\cos x = (1 + a^2)/2a$. As $a \neq 1$,

$$(1 + a^2)/2a > 1$$

Therefore, $\cos x = (1 + a^2)/2a$ is not possible.

Spoonfeeding number of solutions

Number of solutions of the equations $\tan x + \sec x = 2 \cos x$ lying in the interval $[0, 2\pi]$ is

(a) 0 (b) 1 (c) 2 (d) 3

Ans. (c)

Solution The given equation can be written as

$\frac{\sin x + 1}{\cos x} = 2 \cos x$ $(\cos x \neq 0)$

$\Rightarrow$ $\sin x + 1 = 2 \cos^2 x \Rightarrow 2 \sin^2 x + \sin x - 1 = 0$

$\Rightarrow$ $(2 \sin x - 1)(\sin x + 1) = 0 \Rightarrow \sin x = 1/2 (\because \sin x + 1 \neq 0$ as $\cos x \neq 0)$

$\Rightarrow$ $x = \pi/6, 5\pi/6$ in$[0, 2\pi]$ so that required number of solutions is 2.

Spoonfeeding number of Solutions

The numbers of solutions of the pair of equations

$$2 \sin^2\theta - \cos 2\theta = 0$$
$$2 \cos^2\theta - 3 \sin \theta = 0$$

in the interval $[0, 2\pi]$ is

(a) zero (b) one (c) two (d) four

*Ans.* (c)

**Solution** $2 \sin^2\theta - \cos 2\theta = 0$

$\Rightarrow$ $1 - \cos 2\theta - \cos 2\theta = 0 \Rightarrow \cos 2\theta = 1/2$

$\Rightarrow$ $2 \cos^2\theta - 1 = 1/2 \Rightarrow 2 \cos^2 \theta = 3/2$

So that from $2 \cos^2\theta - 3 \sin \theta = 0$, we have

$\sin \theta = 1/2 \Rightarrow \theta = \pi/6, 5\pi/6$ as $\theta \in [0, 2\pi]$.

Spoonfeeding Solution of a Trigonometric Equation

The solution set of the equation

$\tan(\pi \tan x) = \cot(\pi \cot x)$ is

(a) $= \{0\}$ (b) $\{\pi/4\}$ (c) $\phi$ (d) none of these

*Ans.* (c)

**Solution** $\tan(\pi \tan x) = \tan\left(\pi/2 - \pi \cot x\right)$

$\Rightarrow$ $\pi \tan x = \pi/2 - \pi \cot x \Rightarrow \tan x + \cot x = 1/2$

$\Rightarrow$ $2 \tan^2 x - \tan x + 2 = 0$

$\Rightarrow$ $\tan x = \dfrac{1 \pm \sqrt{1 - 16}}{4}$

which does not give real values of $\tan x$.

Spoonfeeding a Trigonometric simplification technique

If $15\sin^4 x + 10\cos^4 x = 6$, Then $\tan^2 x =$

(a) 1/5 (b) 2/5 (c) 2/3 (d) 1/3

*Ans.* (c)

**Solution** $15\sin^4 x + 10\cos^4 x = 6(\sin^2 x + \cos^2 x)^2$

$\Rightarrow\ 9\sin^4 x + 4\cos^4 x - 12\sin^2 x\cos^2 x = 0$

$\Rightarrow\ (3\sin^2 x - 2\cos^2 x)^2 = 0$

$\Rightarrow\ \tan^2 x = 2/3.$

Spoonfeeding Trigonometry problem along with Theory of Equations

Sum of the root of the equation $2\sin^2\theta + \sin^2 2\theta = 2$ $0 \le \theta \le \pi/2$ is

(a) $\pi/2$ (b) $3\pi/4$ (c) $7\pi/2$ (d) $5\pi/12$

*Ans.* (b)

**Solution** $4\sin^2\theta\cos^2\theta = 2(1 - \sin^2\theta)$

$\Rightarrow\ (2\sin^2\theta - 1)\cos^2\theta = 0$

$\Rightarrow\ \sin^2\theta = 1/2$ or $\cos^2\theta = 0$

$\Rightarrow\ \theta = \pi/4$ or $\theta = \pi/2$

Spoonfeeding to find a value if a condition is given

If $\tan x/2 = \operatorname{cosec} x - \sin x$, then $\sec^2(x/2) =$

(a) $\sqrt{5}+1$ (b) $\sqrt{5}-1$ (c) $\sqrt{5}-2$ (d) $\sqrt{5}+2$

*Ans.* (b)

**Solution** $\tan(x/2) = \dfrac{1+\tan^2(x/2)}{2\tan(x/2)} - \dfrac{2\tan(x/2)}{1+\tan^2(x/2)}$

$\Rightarrow\ 2\tan^2(x/2)(1 + \tan^2(x/2) = [1 + \tan^2(x/2)]^2 - 4\tan^2(x/2)$

$\Rightarrow\ 2\tan^4(x/2) + 2\tan^2(x/2) = 1 + \tan^4(x/2) - 2\tan^2(x/2)$

$\Rightarrow\ \tan^4(x/2) + 4\tan^2(x/2) - 1 = 0$

$\Rightarrow\ \tan^2(x/2) = \sqrt{5} - 2 \ \Rightarrow\ \sec^2(x/2) = \sqrt{5} - 1$

Spoonfeeding Trigonometry problem statements ( True / False )

Let $A$ and $B$ denote the statements $A$ : $\cos\alpha + \cos\beta + \cos\gamma = 0$

$B$ : $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -3/2$

Then

(a) both $A$ and $B$ are true (b) both $A$ and $B$ are false

(c) $A$ is true $B$ is false (d) $A$ is false $B$ is true.

*Ans.* (a)

**Solution** $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -3/2$

$$\Rightarrow \quad 2[\cos\beta\cos\gamma + \cos\gamma\cos\alpha + \cos\alpha\cos\beta + \sin\beta\sin\gamma + \sin\gamma\sin\alpha + \sin\alpha\sin\beta]$$

$$+ (\sin^2\alpha + \cos^2\alpha) + (\sin^2\beta + \cos^2\beta) + (\sin^2\gamma + \cos^2\gamma) = 0$$

$$\Rightarrow \quad (\sin\alpha + \sin\beta + \sin\gamma)^2 + (\cos\alpha + \cos\beta + \cos\gamma)^2 = 0$$

$$\Rightarrow \quad \cos\alpha + \cos\beta + \cos\gamma = 0$$

and $\sin\alpha + \sin\beta + \sin\gamma = 0$

so both $A$ and $B$ are true.

Spoonfeeding Trigonometry Condition problem

$\cos^2 u + \cos^2(u + x) - 2\cos u \cos x \cos(u + x) = 1/2$ if

(a) $x = \pi/4$ (b) $u = \pi/4$ (c) $x = \pi/2$ (d) $u = \pi/2$

*Ans.* (a)

**Solution** $\text{L.H.S} = \cos^2 u + \cos^2(u + x) - [\cos(u + x) + \cos(u - x)]\cos(u + x)$

$= \cos^2 u - \cos(u - x)\cos(u + x)$

$= \cos^2 u - (\cos^2 u - \sin^2 x) = \sin^2 x.$

so $\sin^2 x = 1/2 \Rightarrow \sin x = \pm 1/\sqrt{2}$

Spoonfeeding Properties of Triangle Problem

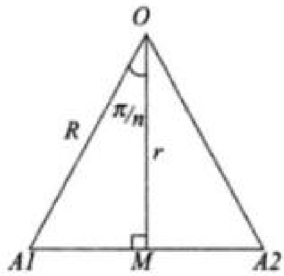
For a regular polygon, let $r$, $R$ be the radii of the inscribed and circumscribed circles. There is no regular polygon with

(a) $\frac{r}{R}=\frac{2}{3}$ (b) $\frac{r}{R}=\frac{\sqrt{3}}{2}$ (c) $\frac{r}{R}=\frac{1}{2}$ (d) $\frac{r}{R}=\frac{1}{\sqrt{2}}$.

*Ans.* (a)

**Solution** We have $\frac{r}{R}=\cos\frac{\pi}{n}$

When $\frac{r}{R}=\frac{\sqrt{3}}{2}$




$$\cos\frac{\pi}{n} = \cos\frac{\pi}{6}.$$

$\Rightarrow$ $n=6$

when $\frac{r}{R}=\frac{1}{2}$, $\cos\frac{\pi}{n} = \cos\frac{\pi}{3} \Rightarrow n=3$

and when $\frac{r}{R}=\frac{1}{\sqrt{2}}$, $\cos\frac{\pi}{n}=\cos\frac{\pi}{4} \Rightarrow n=4$

But when $\frac{r}{R}=\frac{2}{3}$, $\cos\frac{\pi}{n}=\frac{2}{3}$

Which does not give a positive integral value of $n$.

Spoonfeeding Inequality Trick

If $\cos(\alpha+\beta)=4/5$ and $\sin(\alpha-\beta)=5/13$ where $0 \le \alpha, \beta \le \pi/4$, then $\tan 2\alpha =$

(a) $\frac{19}{12}$ (b) $\frac{20}{7}$ (c) $\frac{25}{16}$ (d) $\frac{56}{33}$

*Ans.* (d)

**Solution** $0 \le \alpha, \beta \le \pi/4$

$\Rightarrow$ $0 \le \alpha + \beta \le \pi/2 \Rightarrow -\pi/4 \le \alpha - \beta \le \pi/4$

Now $\cos(\alpha + \beta) = 4/5 \Rightarrow \tan(\alpha + \beta) = 3/4$

and $\sin(\alpha - \beta) = 5/13 \Rightarrow \tan(\alpha - \beta) = 5/12$

we have $\tan 2\alpha = \tan[(\alpha + \beta) + (\alpha - \beta)]$

$$= \frac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$= \frac{(3/4)+(5/12)}{1-(3/4)(5/12)} = \frac{14/12}{33/48}$$

$$= \frac{56}{33}.$$

Spoonfeeding

If cos A = 3/4 then value of 32 sin (A/2) sin (5 A/2) is equal to

(a) $\sqrt{11}$ (b) $-\sqrt{11}$ (c) 11 (d) $-11$

*Ans.* (c)

**Solution** 32 sin (A/2) sin (5A/2)

$= 16 [\cos 2A - \cos 3A]$

$= 16 [2\cos^2 A - 1 - 4\cos^3 A + 3\cos A]$

$$= 16\left[2 \times \frac{9}{16} - 1 - 4 \times \frac{27}{64} + 3 \times \frac{3}{4}\right]$$

$= 18 - 16 - 27 + 36 = 11.$

Spoonfeeding

$$\frac{\cos 10^\circ + \sin 10^\circ}{\cos 10^\circ - \sin 10^\circ} \text{ is equal to}$$

(a) $\tan 55^\circ$ (b) $\cot 55^\circ$ (c) $-\tan 35^\circ$ (d) $-\cot 35^\circ$

*Ans.* (a)

**Solution** $$\frac{\cos 10^\circ + \sin 10^\circ}{\cos 10^\circ - \sin 10^\circ} = \frac{1 + \tan 10^\circ}{1 - \tan 10^\circ}$$

$$= \tan (45^\circ + 10^\circ) = \tan 55^\circ.$$

Spoonfeeding

$$\text{If } \tan x = \frac{b}{a} \text{ then } \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} \text{ is equal to}$$

(a) $\dfrac{2 \sin x}{\sqrt{\sin 2x}}$ (b) $\dfrac{2 \cos x}{\sqrt{\cos 2x}}$ (c) $\dfrac{2 \cos x}{\sqrt{\sin 2x}}$ (d) $\dfrac{2 \sin x}{\sqrt{\cos 2x}}$

*Ans.* (b)

**Solution** $$\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$$

$$= \frac{a+b+a-b}{\sqrt{a^2-b^2}} = \frac{2a}{a\sqrt{1-\frac{b^2}{a^2}}}$$

$$= \frac{2}{\sqrt{1-\tan^2 x}} = \frac{2 \cos x}{\sqrt{\cos 2x}}.$$

Spoonfeeding

If $\dfrac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = x$, then $\dfrac{\cos\alpha}{1+\sin\alpha}$ is equal to

(a) $1/x$ (b) $x$ (c) $1+x$ (d) $1-x$

*Ans.* (d)

**Solution** $\dfrac{\cos\alpha}{1+\sin\alpha} = 1 - \dfrac{1+\sin\alpha-\cos\alpha}{1+\sin\alpha}$

Now $$\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} = \frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}\cdot\frac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha}$$

$$= \frac{(1+\sin\alpha)^2-\cos^2\alpha}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}$$

$$= \frac{(1+\sin\alpha)^2-(1+\sin\alpha)(1-\sin\alpha)}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}$$

$$= \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = x.$$

Spoonfeeding

If $\sin x + \cos y = a$ and $\cos x + \sin y = b$, then $\tan\dfrac{x-y}{2}$ is equal to

(a) $a+b$ (b) $a-b$ (c) $\dfrac{a+b}{a-b}$ (d) $\dfrac{a-b}{a+b}$

*Ans.* (d)

**Solution** From the given relations we have

$$\sin x + \sin((\pi/2)-y) = a \text{ and } \cos x + \cos((\pi/2)-y) = b$$

$\Rightarrow$ $$2\sin\frac{x+(\pi/2)-y}{2}\cos\frac{x-(\pi/2)+y}{2} = a$$

and $$2\cos\frac{x+(\pi/2)-y}{2}\cos\frac{x-(\pi/2)+y}{2} = b$$

Dividing we get,

$$\tan\left(\frac{\pi}{4}+\frac{x-y}{2}\right) = \frac{a}{b} \quad\Rightarrow\quad \frac{1+\tan\dfrac{x-y}{2}}{1-\tan\dfrac{x-y}{2}} = \frac{a}{b}$$

or $$\tan\frac{x-y}{2} = \frac{a-b}{a+b}.$$

Spoonfeeding

$\dfrac{\sin 3\alpha}{\cos 2\alpha} < 0$ if $\alpha$ lies in

(a) $(13\pi/48,\ 14\pi/48)$
(b) $(14\pi/48,\ 18\pi/48)$
(c) $(18\pi/48,\ 23\pi/48)$
(d) any of these intervals

*Ans.* (a)

**Solution** $\dfrac{\sin 3\alpha}{\cos 2\alpha} < 0$ if $\sin 3\alpha > 0$ and $\cos 2\alpha < 0$

or $\sin 3\alpha < 0$ and $\cos 2\alpha > 0$

i.e. if $3\alpha \in (0, \pi)$ and $2\alpha \in (\pi/2, 3\pi/2)$

or $3\alpha \in (\pi, 2\pi)$ and $2\alpha \in (-\pi/2, \pi/2)$

i.e. if $\alpha \in (0, \pi/3)$ and $\alpha \in (\pi/4, 3\pi/4)$

or $\alpha \in (\pi/3, 2\pi/3)$ and $\alpha \in (-\pi/4, \pi/4)$

i.e. if $\alpha \in (\pi/4, \pi/3)$

since $(13\pi/48, 14\pi/48) \subset (\pi/4, \pi/3)$, (a) is correct

Spoonfeeding

If $\cos\alpha + \cos\beta = a$, $\sin\alpha + \sin\beta = b$ and $\theta$ is the arithmetic mean between $\alpha$ and $\beta$ then $\sin 2\theta + \cos 2\theta$ is equal to

(a) $(a + b)^2/(a^2 + b^2)$
(b) $(a - b)^2/(a^2 + b^2)$
(c) $(a^2 - b^2)/(a^2 + b^2)$
(d) none of these

*Ans.* (d)

**Solution** From the given relations we have

$$2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = a \text{ and } 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = b$$

By dividing we get $\tan\dfrac{\alpha+\beta}{2} = \dfrac{b}{a} \Rightarrow \tan\theta = \dfrac{b}{a}$ $\left[\because \theta = \dfrac{\alpha+\beta}{2}\right]$

so that $\cos 2\theta = \dfrac{1-\frac{b^2}{a^2}}{1+\frac{b^2}{a^2}} = \dfrac{a^2-b^2}{a^2+b^2}$ and $\sin 2\theta = \dfrac{2ab}{a^2+b^2}$.

$$\therefore \sin 2\theta + \cos 2\theta = \frac{a^2 - b^2 + 2ab}{a^2 + b^2}$$

Spoonfeeding

**Find the value of $2\tan^{-1}\left(\frac{1}{5}\right)+\sec^{-1}\left(\frac{5\sqrt{2}}{7}\right)+2\tan^{-1}\frac{1}{8}$**

**Sol.** Consider $2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{5\sqrt{2}}{7}+2\tan^{-1}\frac{1}{8}$

Consider $\alpha = \sec^{-1}\left(\frac{5\sqrt{2}}{7}\right) \Rightarrow \sec\alpha = \frac{5\sqrt{2}}{7}$

$\Rightarrow \tan\alpha = \sqrt{\frac{50}{49}-1} = \frac{1}{7} \Rightarrow \alpha = \tan^{-1}\left(\frac{1}{7}\right)$

From (*i*) we get

$$2\left[\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}\right]+\tan^{-1}\frac{1}{7}$$

$$= 2\tan^{-1}\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{5}\times\frac{1}{8}} + \tan^{-1}\frac{1}{7} = 2\tan^{-1}\frac{13}{39}+\tan^{-1}\frac{1}{7}$$

$$= 2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{2\times\frac{1}{3}}{1-\frac{1}{9}} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{3}{4}+\tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}} = \tan^{-1}\frac{25}{25} = \tan^{-1}1 = \frac{\pi}{4}$$

Spoonfeeding

If $x + y = z$, then $\cos^2 x + \cos^2 y + \cos^2 z - 2\cos x \cos y \cos z$ is equal to

(a) $\cos^2 z$ (b) $\sin^2 z$ (c) 0 (d) 1

*Ans.* (d)

**Solution** The given expression can be written as

$\cos^2 x + \cos^2 y + \cos^2 z - \cos z\,[\cos(x+y) + \cos(x-y)]$

$= \cos^2 x + \cos^2 y + \cos^2 z - \cos^2 z \quad - \cos(x+y)\cos(x-y)$

$= \cos^2 x + \cos^2 y - (1/2)[\cos 2x + \cos 2y]$

$= (1/2)\,[2\cos^2 x + 2\cos^2 y - \cos 2x - \cos 2y]$

$= (1/2)\,[2\cos^2 x + 2\cos^2 y - 2\cos^2 x + 1 - 2\cos^2 y + 1] = 1$

Spoonfeeding

If $\sin 2\theta = k$, then the value of $\dfrac{\tan^3\theta}{1+\tan^2\theta} + \dfrac{\cot^3\theta}{1+\cot^2\theta}$ is equal to

(a) $\dfrac{1-k^2}{k}$ (b) $\dfrac{2-k^2}{2}$ (c) $k^2+1$ (d) $2-k^2$

*Ans.* (b)

**Solution** We have $\dfrac{\tan^3\theta}{1+\tan^2\theta} + \dfrac{\cot^3\theta}{1+\cot^2\theta}$

$$= \frac{\sin^3\theta\cdot\cos^2\theta}{\cos^3\theta} + \frac{\cos^3\theta}{\sin^3\theta}\cdot\sin^2\theta = \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta}$$

$$= \frac{\sin^4\theta + \cos^4\theta}{\sin\theta\cos\theta} = \frac{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1 - k^2/2}{k/2} = \frac{2-k^2}{k}. \qquad \left[\because \sin\theta\cos\theta = \frac{k}{2}\right]$$

Spoonfeeding

If $\sin^2 A = x$, then $\sin A \sin 2A \sin 3A \sin 4A$ is a polynomial in $x$, the sum of whose coefficients is

(a) 0 (b) 40 (c) 168 (d) 336

*Ans.* (a)

**Solution** We have $\sin A \sin 2A \sin 3A \sin 4A$

$$= \sin A\,(2 \sin A \cos A)\,(3 \sin A - 4 \sin^3 A) \times 2 \sin 2A \cos 2A$$

$$= 2 \sin^2 A \cos A \times \sin A\,(3 - 4 \sin^2 A) \times 2 \times 2 \sin A \cos A\,(1 - 2 \sin^2 A)$$

$$= 8\sin^4 A \cos^2 A\,(3 - 4 \sin^2 A)\,(1 - 2 \sin^2 A) = 8x^2 (1 - x)(3 - 4x)(1 - 2x)$$

$$= 24x^2 - 104x^3 + 144x^4 - 64x^5.$$

The required sum $= 24 - 104 + 144 - 64 = 0$.

Spoonfeeding

If $\dfrac{\sin A}{\sin B} = \dfrac{\sqrt{3}}{2}$ and $\dfrac{\cos A}{\cos B} = \dfrac{\sqrt{5}}{2}$, $0 < A, B < \pi/2$, then

$\tan A + \tan B$ is equal to

(a) $\sqrt{3}/\sqrt{5}$ (b) $\sqrt{5}/\sqrt{3}$ (c) 1 (d) $(\sqrt{3}+\sqrt{5})/\sqrt{5}$

*Ans.* (d)

**Solution** From the given relation we have

$$\Rightarrow \quad \frac{\tan A}{\sqrt{3}} = \frac{\tan B}{\sqrt{5}} = k \text{ (say), (clearly } k > 0)$$

Also $2 \sin A = \sqrt{3} \sin B$.

$$\Rightarrow \quad \frac{2\tan A}{\sqrt{1+\tan^2 A}} = \frac{\sqrt{3}\tan B}{\sqrt{1+\tan^2 B}} \Rightarrow \frac{2\sqrt{3}k}{\sqrt{1+3k^2}} = \frac{\sqrt{3}\times\sqrt{5}k}{\sqrt{1+5k^2}}$$

$$\Rightarrow \quad 4(1 + 5k^2) = 5(1 + 3k^2)$$

$$\Rightarrow \quad k^2 = 1/5 \Rightarrow k = 1/\sqrt{5}$$

so that $\tan A = \dfrac{\sqrt{3}}{\sqrt{5}}$, $\tan B = 1 \Rightarrow \tan A + \tan B = \dfrac{\sqrt{3}+\sqrt{5}}{\sqrt{5}}$.

Spoonfeeding

To introduce Manipulations by forseeing future steps.

If $m^2 + m'^2 + 2mm'\cos\theta = 1$

$n^2 + n'^2 + 2nn'\cos\theta = 1$

& $mn + m'n' + (mn' + m'n)\cos\theta = 0$

Prove that $m^2 + n^2 = \operatorname{cosec}^2\theta$.

$m^2 + m'^2 + 2mm'\cos\theta = 1$

$\Rightarrow m^2 + m^2\cos^2\theta + m'^2 + 2mm'\cos\theta - m^2\cos^2\theta = 1$

$m^2 - m^2\cos^2\theta + (m' + m\cos\theta)^2 = 1$

~~$m^2(1-0$~~ $m^2\sin^2\theta + (m' + m\cos\theta)^2 = 1$

$\Rightarrow (m' + m\cos\theta)^2 =$ ~~$1 - m^2\cos^2\theta$~~ $1 - m^2\sin^2\theta$.

Similarly $(n' + n\cos\theta)^2 = 1 - n^2\sin^2\theta$.

Now

$(m' + m\cos\theta)(n' + n\cos\theta) = m'n' + m'n\cos\theta + mn'\cos\theta + mn\cos^2\theta$

$\Rightarrow m'n' + \cos\theta(m'n + mn') + mn\cos^2\theta$

$\Rightarrow mn + m'n' + (mn' + m'n)\cos\theta + mn\cos^2\theta - mn$

$\Rightarrow mn(\cos^2\theta - mn \Rightarrow -mn(1 - \cos^2\theta)$

$\Rightarrow -mn\sin^2\theta$.

squaring both sides

$(m' + m\cos\theta)^2 (n' + n\cos\theta)^2 = n^2 m^2 \sin^4\theta$

$(1 - m^2\sin^2\theta)(1 - n^2\sin^2\theta) = m^2 n^2 \sin^4\theta$

$1 - n^2\sin^2\theta - m^2\sin^2\theta + m^2n^2\sin^4\theta = m^2n^2\sin^4\theta$

$1 = \sin^2\theta\,(n^2 + m^2)$

$\frac{1}{\sin^2\theta} = n^2 + m^2 \Rightarrow \operatorname{cosec}^2\theta = n^2 + m^2$. H.P

Spoonfeeding

If $0 < \alpha, \beta < \pi$ and $\cos\alpha + \cos\beta - \cos(\alpha + \beta) = 3/2$ then $\sin\alpha + \cos\beta$ is equal to

(a) 0 (b) 1 (c) $(\sqrt{3}+1)/2$ (d) $\sqrt{3}$

*Ans.* (c)

**Solution** From the given equation we have

$$2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} - 2\cos^2\frac{\alpha+\beta}{2} + 1 = \frac{3}{2}$$

$$\Rightarrow \quad 4\cos^2\frac{\alpha+\beta}{2} - 4\cos\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2} + 1 = 0$$

$$\Rightarrow \quad \left(2\cos\frac{\alpha+\beta}{2} - \cos\frac{\alpha-\beta}{2}\right)^2 = \cos^2\frac{\alpha-\beta}{2} - 1 \qquad \text{(i)}$$

so the only possibility is $\cos^2\frac{\alpha-\beta}{2} - 1 = 0$ since $\cos^2\left(\frac{\alpha-\beta}{2}\right) \le 1$

As $0 < \alpha, \beta < \pi$, we have $\alpha = \beta$

From (i) we get $\cos\alpha = \frac{1}{2} = \cos\beta$

and $\sin\beta = \sin\alpha = \frac{\sqrt{3}}{2}$. so that $\sin\alpha + \cos\beta = \frac{\sqrt{3}+1}{2}$

Spoonfeeding

The general solution of the equation

$$\frac{1-\sin x+\ \dots\ +(-1)^n \sin^n x+\dots}{1+\sin x+\dots+\sin^n x+\dots}=\frac{1-\cos 2x}{1+\cos 2x},\ x \neq (2n+1)\ \pi/2,\ n \in \mathbf{I} \text{ is}$$

(a) $(-1)^n\ (\pi/3)+n\pi$  (b) $(-1)^n\ (\pi/6)+n\pi$

(c) $(-1)^{n+1}\ (\pi/6)+n\pi$  (d) $(-1)^{n-1}\ (\pi/3)+n\pi,\ (n \in \mathbf{I})$

*Ans.* (b)

**Solution** The equation

$$\frac{1-\sin x+\dots+(-1)^n \sin^n x+\dots}{1+\sin x+\dots+\sin^n x+\dots}=\frac{1-\cos 2x}{1+\cos 2x}$$

$$\Rightarrow \quad \frac{1}{1+\sin x}\times\frac{1-\sin x}{1}=\frac{2\sin^2 x}{2\cos^2 x} \qquad \text{as } -1<\sin x<1$$

$$\Rightarrow \quad 1-\sin x=\frac{\sin^2 x\,(1+\sin x)}{1-\sin^2 x}$$

$$\Rightarrow \quad (1-\sin x)^2=\sin^2 x \ \Rightarrow\ 1-2\sin x=0$$

$$\Rightarrow \quad \sin x=1/2=\sin(\pi/6)$$

$$\Rightarrow \quad x=n\pi+(-1)^n\ \pi/6.$$

Spoonfeeding

If $\sin^4 x+\cos^4 y+2=4\sin x\cos y,\ 0\le x,\ y\le \pi/2$ then $\sin x+\cos y=$

(a) $-2$  (b) $0$  (c) $2$  (d) none of these

*Ans.* (c)

**Solution** The given equation can be written as

$$\sin^4 x+\cos^4 y+2-4\sin x\cos y=0$$

$$\Rightarrow \quad (\sin^2 x-1)^2+(\cos^2 y-1)^2+2\sin^2 x+2\cos^2 y-4\sin x\cos y=0$$

$$\Rightarrow \quad (\sin^2 x-1)^2+(\cos^2 y-1)^2+2(\sin x-\cos y)^2=0$$

which is true if $\sin^2 x=1$, $\cos^2 y=1$ and $\sin x=\cos y$, so $\sin x+\cos y=2$ as $0\le x,\ y\le\pi/2$.

Spoonfeeding

The value of $\sin^{-1}(\sin 10)$ is

(a) 10 (b) $3\pi - 10$ (c) $10 - 3\pi$ (d) none of these

*Ans.* (b)

**Solution** $y = \sin^{-1}(\sin 10)$

$\Rightarrow$ $\sin y = \sin 10$

$= \sin(3\pi + (10 - 3\pi))$ $(\because 3\pi < 10 < 3\pi + \pi/2)$

$= -\sin(10 - 3\pi)$

$= \sin(3\pi - 10)$

$\Rightarrow$ $y = 3\pi - 10$

Spoonfeeding

If $\sin^{-1} x + \sin^{-1} y = 2\pi/3$, then $\cos^{-1} x + \cos^{-1} y$ is equal to

(a) $2\pi/3$ (b) $\pi/3$ (c) $\pi/6$ (d) $\pi$

*Ans.* (b)

**Solution** Let $\cos^{-1} x + \cos^{-1} y = \theta$

then $\sin^{-1} x + \cos^{-1} x + \sin^{-1} y + \cos^{-1} y = 2\pi/3 + \theta$

$\Rightarrow$ $\frac{\pi}{2} + \frac{\pi}{2} = \frac{2\pi}{3} + \theta \Rightarrow \theta = \frac{\pi}{3}$.

Spoonfeeding

When the angles of product of cos are in multiples of 2 then Multiply & divide by sin θ to successively simplify the equation :

$$16\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{14\pi}{15} = 1$$

$$\Rightarrow 16\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15} = \frac{1}{16} \quad \left[\because \cos\frac{14\pi}{15} = \cos\frac{16\pi}{15}\right]$$

LHS =

$$\Rightarrow \frac{1}{2\sin\frac{2\pi}{15}}\; 2\sin\frac{2\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}$$

$$\Rightarrow \frac{1}{2^2\sin\frac{2\pi}{15}}\; 2\sin\frac{4\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}$$

$$\Rightarrow \frac{1}{2^3\sin\frac{2\pi}{15}}\; 2\cos\frac{8\pi}{15}\sin\frac{8\pi}{15}\cos\frac{16\pi}{15}$$

$$\Rightarrow \frac{1}{2^4}\; 2\sin\frac{16\pi}{15}\cos\frac{16\pi}{15} \Rightarrow \frac{1}{2^4}\frac{\sin\frac{32\pi}{15}}{\sin\frac{2\pi}{15}} = \frac{1}{2^4}.$$

Spoonfeeding

If $\dfrac{\cos^4\alpha}{\cos^2\beta} + \dfrac{\sin^4\alpha}{\sin^2\beta} = 1.$

Prove that

(i) $\sin^4\alpha + \sin^4\beta = 2\sin^2\alpha\sin^2\beta$

(2) $\dfrac{\cos^4\beta}{\cos^2\alpha} + \dfrac{\sin^4 B}{\sin^2\alpha} = 1.$

$$\frac{\cos^4\alpha}{\cos^2\beta} + \frac{\sin^4\alpha}{\sin^2\beta} = 1$$

$$\Rightarrow \cos^4\alpha\sin^2\beta + \sin^4\alpha\cos^2\beta = \cos^2\beta\sin^2\beta$$

$$\cos^4\alpha(1-\cos^2\beta) + \cos^2\beta(1-\cos^2\alpha)^2 = \cos^2\beta(1-\cos^2\beta)$$

$$\Rightarrow \cos^4\alpha - 2\cos^2\alpha\cos^2\beta + \cos^4\beta = 0$$

$$\Rightarrow (\cos^2\alpha - \cos^2\beta)^2 = 0$$

$$\Rightarrow \cos^2\alpha = \cos^2\beta$$

$$\text{\&} \quad \sin^2\alpha = \sin^2\beta$$

$$\Rightarrow \alpha = \pm\beta.$$

$\sin^4 \alpha + \sin^4 \beta$

$\Rightarrow (\sin^2 \alpha - \cos^2 \alpha) \pm 2\sin^2 \alpha \cos^2 \alpha$ .

$\underbrace{(\sin^2 \alpha - \sin^2 \beta)}_{0} + 2\sin^2 \alpha \sin^2 \beta$

$\Rightarrow 2\sin^2 \alpha \sin^2 \beta$  H.P.

$\dfrac{\cos^4 \beta}{\cos^2 \alpha} + \dfrac{\sin^4 \beta}{\sin^2 \alpha} = 1$

$\dfrac{\cos^2 \beta \cos^2 \alpha}{\cos^2 \alpha} + \dfrac{\sin^2 \beta \sin^2 \alpha}{\sin^2 \alpha} = 1$

$\cos^2 \beta + \sin^2 \beta = 1$  H.P.

Spoonfeeding

$\tan^{-1}(1/4) + \tan^{-1}(2/9)$ is equal to

(a) $(1/2)\cos^{-1}(3/5)$  (b) $\sin^{-1}(4/5)$

(c) $(1/2)\tan^{-1}(3/5)$  (d) $\tan^{-1}(8/9)$

*Ans*. (b)

**Solution** $\tan^{-1}(1/4) + \tan^{-1}(2/9)$

$$= \tan^{-1}\frac{\frac{1}{4}+\frac{2}{9}}{1-\left(\frac{1}{4}\right)\left(\frac{2}{9}\right)} = \tan^{-1}\frac{17}{34} = \tan^{-1}\left(\frac{1}{2}\right)$$

If $\theta = \tan^{-1}(1/2) \Rightarrow \tan\theta = (1/2)$

$$\sin\theta = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{4}{5}$$

$\Rightarrow$ $\theta = \sin^{-1}(4/5) = \tan^{-1}(1/2)$

Spoonfeeding

If $3 \sin \beta = \sin (2\alpha + \beta)$, then $\tan (\alpha + \beta) - 2 \tan \alpha$ is

(a) independent of $\alpha$
(b) independent of $\beta$
(c) independent of both $\alpha$ and $\beta$
(d) independent of none of them

*Ans.* (c)

**Solution** $\sin (2\alpha + \beta) = 3 \sin \beta$

$$\Rightarrow \quad \frac{\sin(2\alpha + \beta) + \sin \beta}{\sin(2\alpha + \beta) - \sin \beta} = \frac{3+1}{3-1}$$

$$\Rightarrow \quad \frac{2\sin(\alpha + \beta)\cos\alpha}{2\cos(\alpha + \beta)\sin\alpha} = 2 \Rightarrow \tan(\alpha + \beta) - 2\tan \alpha = 0$$

Recall

$$\sin 18^\circ = \frac{\sqrt{5}-1}{4} \qquad \cos 18 = \frac{\sqrt{10+2\sqrt{5}}}{4}$$

$$\cos 36^\circ = \frac{\sqrt{5}+1}{4}$$

Spoonfeeding

If there are products of sin and cos then they are not necessarily grouped up as per the sequence given, but they are grouped in such a way so that the known angles are generated.

P.T. $\tan 6^\circ \tan 42^\circ \tan 66^\circ \tan 78^\circ = 1$

$$LHS = \frac{\sin 6^\circ \sin 42^\circ \sin 66^\circ \sin 78^\circ}{\cos 6^\circ \cos 42^\circ \cos 66^\circ \cos 78^\circ}$$

$$\Rightarrow \frac{(2\sin 66^\circ \sin 6^\circ)(2\sin 78^\circ \sin 42^\circ)}{(2\cos 66^\circ \cos 6^\circ)(2\cos 78^\circ \cos 42^\circ)}$$

$$\Rightarrow \frac{(\cos 60 - \cos 72^\circ)(\cos 36^\circ - \cos 120^\circ)}{(\cos 60 + \cos 72^\circ)(\cos 36^\circ + \cos 120^\circ)}$$

$$\Rightarrow \left(\frac{\cos 60 - \sin 18^\circ}{\cos 60 + \sin 18}\right)\left(\frac{\cos 36 + \sin 30}{\cos 36 - \sin 30}\right)$$

$$\Rightarrow \frac{\left(\frac{1}{2} - \frac{(\sqrt{5}-1)}{4}\right)\left(\frac{\sqrt{5}+1}{4} + \frac{1}{2}\right)}{\left(\frac{1}{2} + \frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4} - \frac{1}{2}\right)}$$

$$\Rightarrow \frac{\left(\frac{2-\sqrt{5}+1}{4}\right)\left(\frac{\sqrt{5}+1+2}{4}\right)}{\left(\frac{2+\sqrt{5}-1}{4}\right)\left(\sqrt{5}+1-2\right)} \Rightarrow \frac{(3-\sqrt{5})(3+\sqrt{5})}{(\sqrt{5}+1)(\sqrt{5}-1)} \Rightarrow \frac{9-5}{5-1} =$$

Recall

$$\sin 36^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

$$\sin 27 = \frac{\sqrt{5+\sqrt{5}} - \sqrt{3-\sqrt{5}}}{4}$$

Prove that $4\sin 27^\circ = \sqrt{(5+\sqrt{5})} - \sqrt{(3-\sqrt{5})}$

$16\sin^2 27 = 8 \times 2\sin 27 \sin 27 =$

$= 8(1-\cos 54^\circ) = 8(1-\sin 36^\circ)$

$16\sin^2 27 = 8\left\{1 - \dfrac{\sqrt{10-2\sqrt{5}}}{4}\right\}$

$16\sin^2 27 = 2\left\{4 - \sqrt{10-2\sqrt{5}}\right\}$

$16\sin^2 27 = 8 - 2\sqrt{10-2\sqrt{5}}$

$16\sin^2 27 = (5+\sqrt{5}) + (3-\sqrt{5}) - 2\sqrt{(5+\sqrt{5})(3-\sqrt{5})}$

$16\sin^2 27 = \left(\sqrt{5+\sqrt{5}} - \sqrt{3-\sqrt{5}}\right)^2$

$4\sin 27 = \sqrt{5+\sqrt{5}} - \sqrt{3-\sqrt{5}}$

Spoonfeeding

If $A = \sin^2\theta + \cos^4\theta$, then for all values of $\theta$

(a) $1 \le A \le 2$ (b) $3/4 \le A \le 1$

(c) $13/16 \le A \le 1$ (d) $3/4 \le A \le 13/16$

*Ans.* (b)

**Solution** $A = \sin^2\theta + (1-\sin^2\theta)^2 = 1 + \sin^2\theta(\sin^2\theta - 1)$

$= 1 - \sin^2\theta\cos^2\theta \le 1$

**Also** $A = 1 - (1/4)\sin^2 2\theta \ge 1 - (1/4) = (3/4)$. Hence $3/4 \le A \le 1$

Spoonfeeding

Let $n$ be a fixed positive integer such that

$\sin(\pi/2n) + \cos(\pi/2n) = \sqrt{n}/2$, then

(a) $n < 4$ (b) $n > 8$

(c) $n = 6$ (d) none of these

*Ans.* (c)

**Solution** $\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \sqrt{2}\sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right)$

$\Rightarrow \quad \frac{\sqrt{n}}{2} = \sqrt{2}\sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right)$

so for $n > 1$, $\frac{\sqrt{n}}{2\sqrt{2}} = \sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right) > \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ thus $n > 4$

Since $\sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right) < 1$ for all $n > 2$, we get $\frac{\sqrt{n}}{2\sqrt{2}} < 1$ or $n < 8$

so that $4 < n < 8$. By actual verification we find that only $n = 6$ satisfies the given relation.

Spoonfeeding

If $A$ and $B$ are acute angles such that $A + B$ and $A - B$ satisfy the equation $\tan^2\theta - 4\tan\theta + 1 = 0$, then

(a) $A = \pi/4$ (b) $A = \pi/6$

(c) $B = \pi/4$ (d) $B = \pi/6$

*Ans.* (a) and (d)

***Solution*** From the given equation, we have

$$\tan(A + B) + \tan(A - B) = 4 \quad (1)$$

$$\tan(A + B)\tan(A - B) = 1 \quad (2)$$

From (1) and (2) we get

$\tan[A + B + A - B] = \tan\pi/2$

$\Rightarrow \quad 2A = \pi/2 \quad \Rightarrow \quad A = \pi/4$

and from (1) we get

$$\frac{1+\tan B}{1-\tan B}+\frac{1-\tan B}{1+\tan B}=4 \Rightarrow \frac{(1+\tan B)^2+(1-\tan B)^2}{1-\tan^2 B}=4$$

$$\Rightarrow \quad \frac{2(1+\tan^2 B)}{1-\tan^2 B}=4 \Rightarrow \frac{1-\tan^2 B}{1+\tan^2 B}=\frac{1}{2}$$

$$\Rightarrow \quad \cos 2B = 1/2 \quad \Rightarrow \quad 2B = \pi/3 \quad \Rightarrow \quad B = \pi/6$$

Spoonfeeding

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1} x \text{ for.}$$

(a) $|x| \ge 1$ (b) $x \ge 0$ (c) $|x| \le 1$ (d) all $x \in \mathbb{R}$.

*Ans.* (c)

**Solution** $-\frac{\pi}{2} \le \sin^{-1}\left(\frac{2x}{1+x^2}\right) \le \frac{\pi}{2}$

$\Rightarrow$ $-(\pi/2) \le 2\tan^{-1} x \le (\pi/2)$

$\Rightarrow$ $-(\pi/4) \le \tan^{-1} x \le (\pi/4)$

$\Rightarrow$ $\tan(-\pi/4) \le x \le \tan\ (\pi/4)$

$\Rightarrow$ $-1 \le x \le 1 \Rightarrow |x| \le 1$

Spoonfeeding

$$\cot^{-1}[(\cos\alpha)^{1/2}] - \tan^{-1}[(\cos\alpha)^{1/2}] = x \text{ then } \sin x =$$

(a) $\tan^2(\alpha/2)$ (b) $\cot^2(\alpha/2)$ (c) $\tan\alpha$ (d) $\cot\alpha$

*Ans.* (a)

**Solution** $x = (\pi/2) - 2\tan^{-1}[(\cos\alpha)^{1/2}]$

$\Rightarrow$ $(\pi/2) - x = 2\tan^{-1}[(\cos\alpha)^{1/2}]$

$\Rightarrow$ $\tan\ (\pi/2 - x) = \frac{2(\cos\alpha)^{1/2}}{1-\cos\alpha} \Rightarrow \cot x = \frac{2(\cos\alpha)^{1/2}}{1-\cos\alpha}$

$\Rightarrow$ $\operatorname{cosec}^2 x = 1 + \frac{4\cos\alpha}{(1-\cos\alpha)^2} = \left(\frac{1+\cos\alpha}{1-\cos\alpha}\right)^2$

$\Rightarrow$ $\sin x = \frac{1-\cos\alpha}{1+\cos\alpha} = \frac{2\sin^2(\alpha/2)}{2\cos^2(\alpha/2)} = \tan^2(\alpha/2)$

Spoonfeeding

The trigonometric equation $\sin^{-1} x = 2 \sin^{-1} a$ has a solution for

(a) all real values of a (b) $|a| \le \frac{1}{\sqrt{2}}$

(c) $|a| \ge \frac{1}{\sqrt{2}}$ (d) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$

*Ans.* (b)

**Solution** $\sin^{-1} x = \sin^{-1} 2a\sqrt{1-a^2}$ if $|a| \le 1/\sqrt{2}$

$\Rightarrow$ $x = 2a\sqrt{1-a^2}$ which is possible [$\because -1 \le \sin^{-1} x \le 1$]

if $x^2 = 4a^2(1-a^2) \le 1$

or if $4a^4 - 4a^2 + 1 \ge 0$ if $(2a^2 - 1)^2 \ge 0$

which is true, so $|a| \le 1/\sqrt{2}$

Spoonfeeding

If $0 \le x \le 0\ 1$ and $\theta = \sin^{-1} x + \cos^{-1} x - \tan^{-1} x$, then

(a) $\theta \le \pi/2$ (b) $\theta \ge \pi/4$

(c) $\theta = \pi/4$ (d) $\pi/4 \le \theta \le \pi/2$

*Ans.* (d)

**Solution** $\theta = \sin^{-1} x + \cos^{-1} x - \tan^{-1} x = \pi/2 - \tan^{-1} x$

Since $0 \le x \le 1 \Rightarrow 0 \le \tan^{-1} x \le \pi/4 \Rightarrow \pi/4 \le \theta \le \pi/2$.

Spoonfeeding

If $\tan^{-1}\frac{1}{1+2}+\tan^{-1}\frac{1}{1+(2)(3)}+\tan^{-1}\frac{1}{1+(3)(4)}+\ldots\tan^{-1}\frac{1}{1+n(n+1)}=\tan^{-1}\theta$, then $\theta=$

(a) $\frac{n}{n+1}$ (b) $\frac{n+1}{n+2}$ (c) $\frac{n}{n+2}$ (d) $\frac{n-1}{n+2}$

*Ans.* (c)

**Solution** $\tan^{-1}\frac{1}{1+n(n+1)}=\tan^{-1}\frac{n+1-n}{1+n(n+1)}$

$$=\tan^{-1}(n+1)-\tan^{-1}(n)$$

so that L.H.S. of the given equation is

$$\tan^{-1}2-\tan^{-1}1+\tan^{-1}3-\tan^{-1}2+\cdots+\tan^{-1}(n+1)-\tan^{-1}n.$$

$$=\tan^{-1}(n+1)-\tan^{-1}1=\tan^{-1}\frac{n+1-1}{1+(n+1)}=\tan^{-1}\frac{n}{n+2}$$

so that $\tan^{-1}\frac{n}{n+2}=\tan^{-1}\theta \Rightarrow \theta=\frac{n}{n+2}$.

Spoonfeeding

A value of $x$ satisfying $\tan^{-1}(x+3)-\tan^{-1}(x-3)=\sin^{-1}(3/5)$ is

(a) 0 (b) 2 (c) 4 (d) 8

Ans. (c)

**Solution** $\tan^{-1}(x+3)-\tan^{-1}(x-3)=\sin^{-1}(3/5)$

$$\Rightarrow \tan^{-1}\frac{(x+3)-(x-3)}{1+(x+3)(x-3)}=\tan^{-1}(3/4)$$

$$\Rightarrow \frac{6}{1+x^2-9}=\frac{3}{4} \Rightarrow x^2=16$$

$$\Rightarrow x=\pm 4.$$

Spoonfeeding

If $x = \text{cosec}\,(\tan^{-1}(\cos(\cot^{-1}(\sec(\sin^{-1}a)))))$ $a \in [0, 1]$, then

(a) $x^2 - a^2 = 3$
(b) $x^2 + a^2 = 3$
(c) $x^2 - a^2 = 2$
(d) $x^2 + a^2 = 2$

**Ans. (b)**

**Solution** $x = \text{cosec}\left(\tan^{-1}\left(\cos\left(\cot^{-1}\left(\frac{1}{\sqrt{1-a^2}}\right)\right)\right)\right)$

$$= \text{cosec}(\tan^{-1}(\cos(\sec^{-1}(\sqrt{2-a^2}))))$$

$$= \text{cosec}\left(\tan^{-1}\frac{1}{\sqrt{2-a^2}}\right)$$

$$= \text{cosec}(\cot^{-1}\sqrt{2-a^2}) = \sqrt{3-a^2}$$

$\Rightarrow$ $x^2 + a^2 = 3$

Spoonfeeding

If $\sin^{-1}\left(\sin\frac{33\pi}{7}\right) + \cos^{-1}\left(\cos\frac{46\pi}{7}\right) + \tan^{-1}\left(-\tan\frac{13\pi}{8}\right) + \cot^{-1}\left(\cot\frac{-19\pi}{8}\right) = \frac{a\pi}{b}$

where $a$ and $b$ are in their lowest form, then $(a + b)$ is equal to

(a) 17
(b) 20
(c) 23
(d) none of these

**Ans. (b)**

**Solution** We have L.H.S $= \sin^{-1}(\sin(5\pi - 2\pi/7)) + \cos^{-1}(\cos(7\pi - 3\pi/7)) + \tan^{-1}(-\tan(2\pi - 3\pi/8))$ $+ \cot^{-1}(-\cot(3\pi - 5\pi/8))$

$$= \sin^{-1}(\sin(2\pi/7)) + \cos^{-1}(-\cos(3\pi/7)) + \tan^{-1}(\tan(3\pi/8)) + \cot^{-1}(\cot(5\pi/8))$$

$$= \frac{2\pi}{7} + \pi - \frac{3\pi}{7} + \frac{3\pi}{8} + \frac{5\pi}{8}$$

$$= \frac{13\pi}{7} = \frac{a\pi}{b} \quad \Rightarrow a + b = 20$$

Spoonfeeding

If $0 < x < 1$, the number of solutions of the equation $\tan^{-1}(x-1) + \tan^{-1}(x) + \tan^{-1}(x+1) = \tan^{-1}(3x)$ is

(a) 0 (b) 1 (c) 2 (d) 3

Ans. (b)

**Solution** $\tan^{-1}(x-1) + \tan^{-1}(x+1) = \tan^{-1}3x - \tan^{-1}x$

$$\Rightarrow \quad \frac{(x-1)+(x+1)}{1-(x+1)(x-1)} = \frac{3x-x}{1+3x^2}$$

$\Rightarrow \quad 1 + 3x^2 = 2 - x^2, \quad x \neq 0$

$\Rightarrow \quad x = \pm 1/2$. so $x = 1/2$ as $0 < x < 1$.

Spoonfeeding

$\tan^{-1}(\tan 4) - \tan^{-1}(\tan(-6)) + \cos^{-1}(\cos 10)$ is equal to

(a) 0 (b) $\pi$ (c) $-\pi$ (d) $5\pi$

Ans. (b)

**Solution** $\pi < 4 < 3\pi/2 \quad \Rightarrow 0 < 4 - \pi < \pi/2$

$\Rightarrow \quad \tan^{-1}(\tan 4) = 4 - \pi \quad \because \tan(4 - \pi) = \tan 4$

$2\pi - 6 \in (-\pi/2, \pi/2)$, $\tan(2\pi - 6) = -\tan 6$

$\Rightarrow \quad \tan^{-1}(\tan(-6)) = \tan^{-1}(-\tan 6) = 2\pi - 6$ and $4\pi - 10 \in (0, \pi)$, $\cos(4\pi - 10) = \cos 10$

so the given expression is equal to $4 - \pi - (2\pi - 6) + 4\pi - 10 = \pi$

Spoonfeeding

If $x < -1/\sqrt{3}$, then the value of $3\tan^{-1}x - \tan^{-1}\dfrac{3x - x^3}{1 - 3x^2}$ equals

(a) $-\pi$ (b) $\pi$ (c) $-\pi/2$ (d) 0

Ans. (a)

**Solution** Let $x = \tan\theta$, then $x < -1/\sqrt{3}$

$\Rightarrow \quad \tan\theta < -1/\sqrt{3} \quad \Rightarrow -\pi/2 < \theta < -\pi/6$

$\Rightarrow \quad -3\pi/2 < 3\theta < -\pi/2$

$\Rightarrow \quad -\pi/2 < \pi + 3\theta < \pi/2$

so $\quad 3\tan^{-1}x - \tan^{-1}\dfrac{3x - x^3}{1 - 3x^2}$

$= 3\tan^{-1}(\tan\theta) - \tan^{-1}(\tan 3\theta)$

$= 3\tan^{-1}(\tan\theta) - \tan^{-1}(\tan(\pi + 3\theta))$

$= 3\theta - (\pi + 3\theta) = -\pi$

Spoonfeeding

$\sin^{-1}\dfrac{4}{5} + \sin^{-1}\dfrac{5}{13} + \sin^{-1}\dfrac{16}{65}$ is equal to

(a) 0 (b) $\pi/2$ (c) $\pi$ (d) $\sin^{-1}\dfrac{63}{65}$

Ans. (b)

**Solution** $\sin^{-1}\left[\dfrac{4}{5}\sqrt{1 - \left(\dfrac{5}{13}\right)^2} + \dfrac{5}{13}\sqrt{1 - \left(\dfrac{4}{5}\right)^2}\right] + \sin^{-1}\dfrac{16}{65}$

$= \sin^{-1}\left[\dfrac{4}{5} \times \dfrac{12}{13} + \dfrac{5}{13} \times \dfrac{3}{5}\right] + \sin^{-1}\dfrac{16}{65}$

$= \sin^{-1}\dfrac{63}{65} + \sin^{-1}\dfrac{16}{65}$

$$= \cos^{-1}\sqrt{1-\left(\frac{63}{65}\right)^2} + \sin^{-1}\frac{16}{65}$$

$$= \cos^{-1}\frac{16}{65} + \sin^{-1}\frac{16}{65} = \frac{\pi}{2}$$

Spoonfeeding

If $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\cos^{-1}x$ then $x$ is equal to

(a) 1/2 (b) 2/5 (c) 3/5 (d) none of these

Ans. (c)

Solution $\frac{1}{2}\cos^{-1}x = \tan^{-1}\frac{(1/4)+(2/9)}{1-(1/4)(2/9)} = \tan^{-1}\left(\frac{1}{2}\right)$

$$= \frac{1}{2}\times 2\tan^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}\cos^{-1}\left[\frac{1-\frac{1}{4}}{1+\frac{1}{4}}\right] \quad \left(\text{using } 2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) \text{for } x \geq 0\right)$$

$$= \frac{1}{2}\cos^{-1}\frac{3}{5}.$$

Spoonfeeding

If $u = \cot^{-1}\sqrt{\tan\alpha} - \tan^{-1}\sqrt{\tan\alpha}$, then $\tan\left(\frac{\pi}{4}-\frac{u}{2}\right) =$

(a) $\sqrt{\tan\alpha}$ (b) $\sqrt{\cot\alpha}$ (c) $\tan\alpha$ (d) $\cot\alpha$

*Ans.* (a)

Solution Let $\sqrt{\tan\alpha} = \tan x$, then $u = \cot^{-1}(\tan x) - \tan^{-1}(\tan x)$

$$= (\pi/2) - x - x = (\pi/2) - 2x$$

$\Rightarrow \quad 2x = (\pi/2) - u \quad \Rightarrow \quad x = (\pi/4) - (u/2)$

$\Rightarrow \quad \tan x = \tan\left(\frac{\pi}{4}-\frac{u}{2}\right) \quad \Rightarrow \quad \sqrt{\tan\alpha} = \tan\left(\frac{\pi}{4}-\frac{u}{2}\right)$

Spoonfeeding

If $\tan^{-1} y = 4 \tan^{-1} x$, then $1/y$ is zero for

(a) $x = 1 \pm \sqrt{2}$
(b) $x = \sqrt{2} \pm \sqrt{3}$
(c) $3 \pm 2\sqrt{2}$
(d) all values of $x$

*Ans.* (a)

**Solution** If we put $x = \tan\theta$, the given equality becomes $\tan^{-1} y = 4\theta$.

$$\Rightarrow \quad y = \tan 4\theta = \frac{2\tan 2\theta}{1-\tan^2 2\theta} = \frac{2\left[\dfrac{2\tan\theta}{1-\tan^2\theta}\right]}{1-\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)^2}$$

$$= \frac{2\times 2x\left(1-x^2\right)}{\left(1-x^2\right)^2-4x^2} = \frac{4x\left(1-x^2\right)}{1-6x^2+x^4}$$

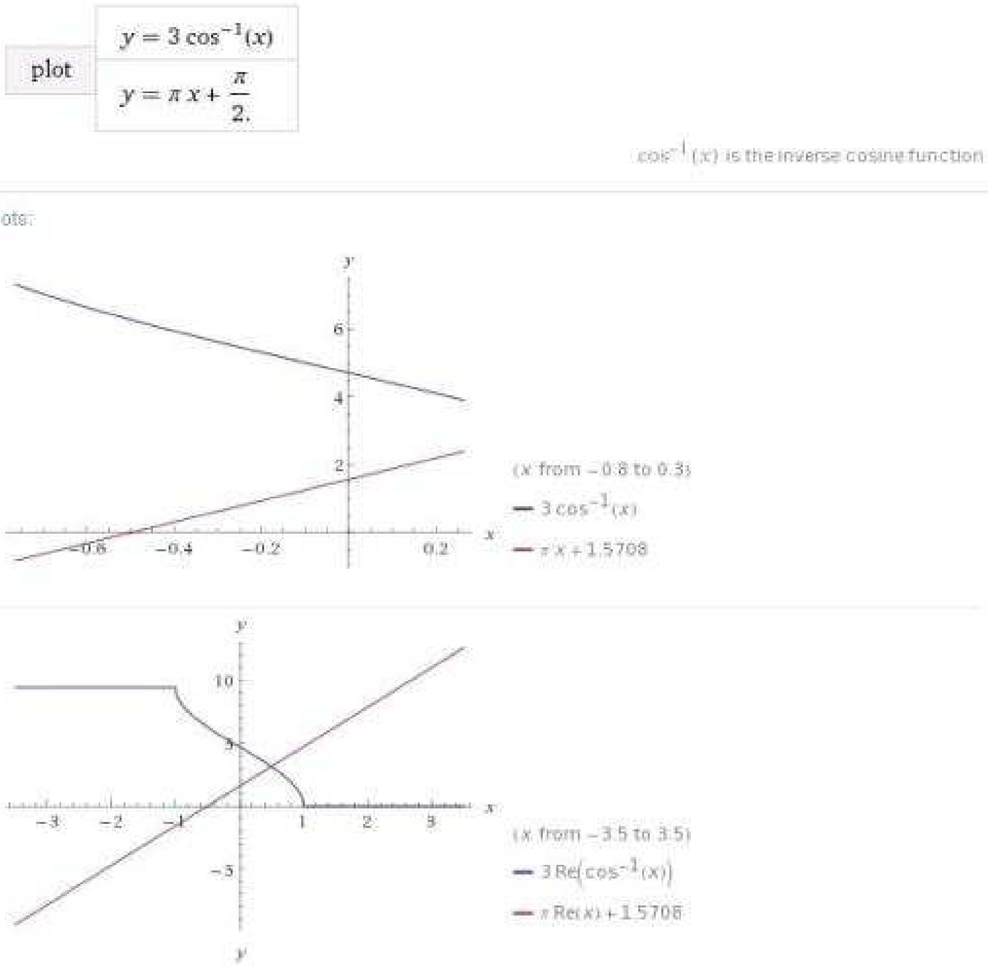
so that $1/y$ is zero if $x^4 - 6x^2 + 1 = 0$

$$\Rightarrow \quad x^2 = \frac{6\pm\sqrt{36-4}}{2} = 3 \pm 2\sqrt{2} = (1\pm\sqrt{2})^2$$

Spoonfeeding a Graphical concept

How many solutions does the equation have 3 Cos inverse x – π x – π/2 = 0

We can solve this graphically by superimposing both the graphs.




Spoonfeeding

$3\cos^{-1}x - \pi x - \pi/2 = 0$ has

(a) one solution (b) one and only one solution

(c) no solution (d) more than one solution

*Ans.* (b)

Solution $x = 1/2$ is clearly a solution of the given equation which can be obtained by trial and error method. The given equation can be written as

$$3\cos^{-1}x = \pi x + \pi/2 \tag{1}$$

since the L.H.S. of (1) is a decreasing function and R.H.S. of (1) is an increasing function of $x$. The equation (1) has either no solution or only one solution. So $x = 1/2$ is one and only one solution of the given equation.

Spoonfeeding

If $\cos^{-1} x = \tan^{-1} x$, then $\sin(\cos^{-1} x) =$

(a) $x$ (b) $x^2$ (c) $1/x$ (d) $1/x^2$

***Ans.*** **(b)**

**Solution** $\cos^{-1} x = \tan^{-1} x = \theta$ (say) $\Rightarrow x = \cos\theta = \tan\theta$

$\Rightarrow \cos^2\theta = \sin\theta \Rightarrow \sin^2\theta + \sin\theta - 1 = 0$

$$\Rightarrow \sin\theta = \frac{-1 \pm \sqrt{1+4}}{2} \Rightarrow \sin\theta = \frac{\sqrt{5}-1}{2}$$

So $$x^2 = \cos^2\theta = \frac{\sqrt{5}-1}{2}$$

and $$\sin(\cos^{-1} x) = \sin\theta = \frac{\sqrt{5}-1}{2} = x^2.$$

Spoonfeeding

$x = n\pi - \tan^{-1} 3$ is a solution of the equation $12\tan 2x + \dfrac{\sqrt{10}}{\cos x} + 1 = 0$ if

(a) $n$ is any integer (b) $n$ is an even integer
(c) $n$ is a positive integer (d) $n$ is an odd integer

*Ans.* (d)

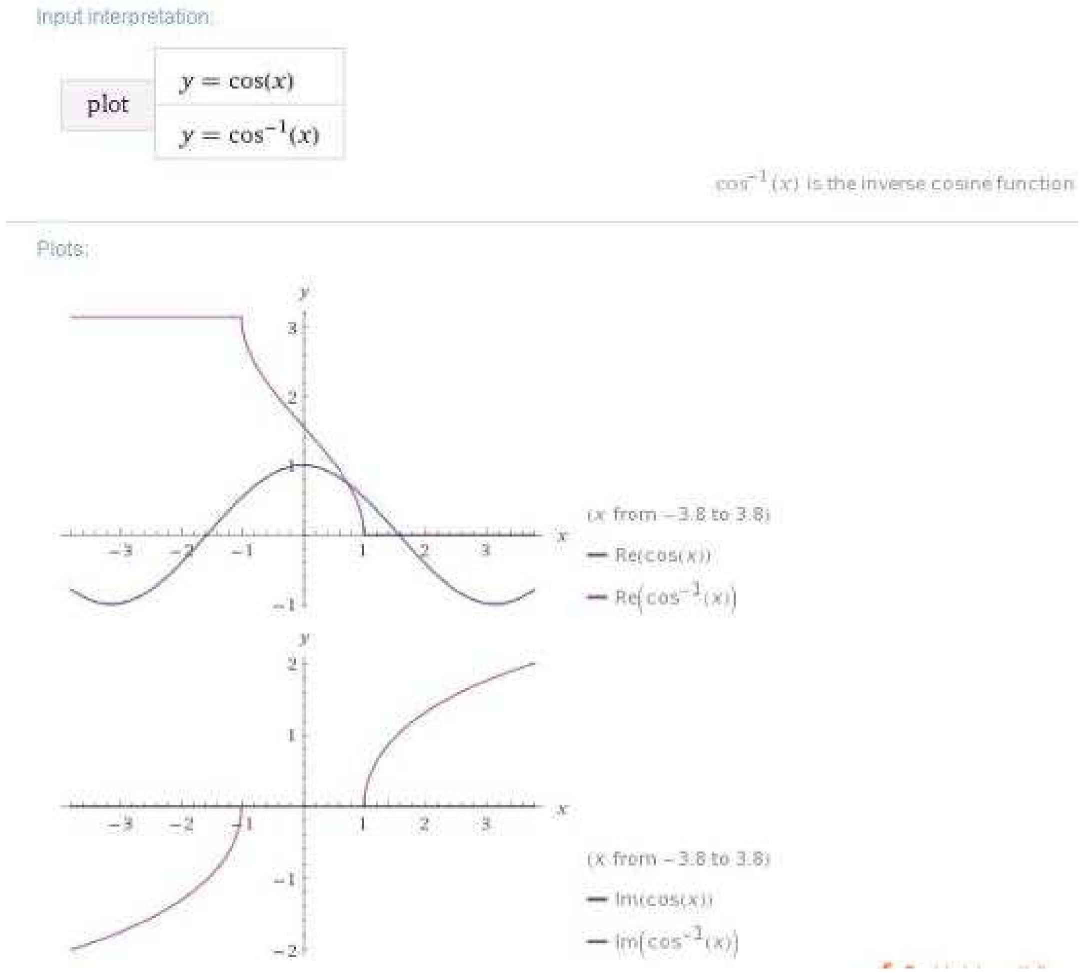
**Solution** $x = n\pi - \tan^{-1} 3 \Rightarrow \tan^{-1} 3 = n\pi - x$

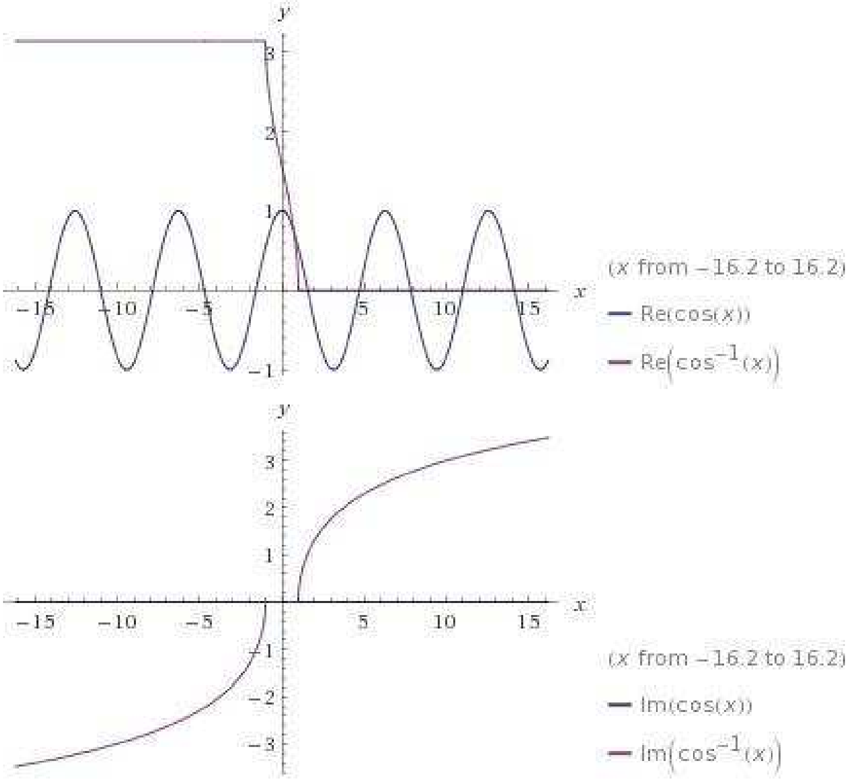
$\Rightarrow \tan(n\pi - x) = 3 \Rightarrow -\tan x = 3$

$$\Rightarrow \tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{3}{4}$$

and $$\cos x = \pm\frac{1}{\sqrt{1+\tan^2 x}} = \pm\frac{1}{\sqrt{10}}$$

on substituting these value in the given equation we find only $\cos x = -1/\sqrt{10}$ satisfies the equation. So that the given equation holds for values of $x$ for which $\tan x = -3$ and $\cos x = -1/\sqrt{10}$. Which is possible if $x$ lies in the second quadrant only and so $n$ must be an odd integer.

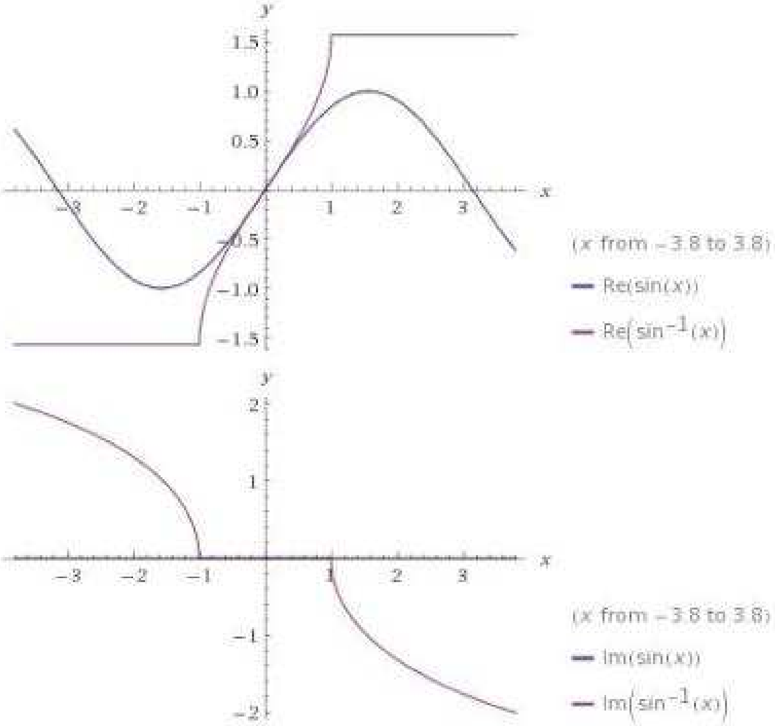

Input interpretation:
plot
y = cos(x)
y = cos^-1(x)
cos^-1(x) is the inverse cosine function
Plots:
(x from -3.8 to 3.8)
Re(cos(x))
Re(cos^-1(x))
(x from -3.8 to 3.8)
Im(cos(x))
Im(cos^-1(x))

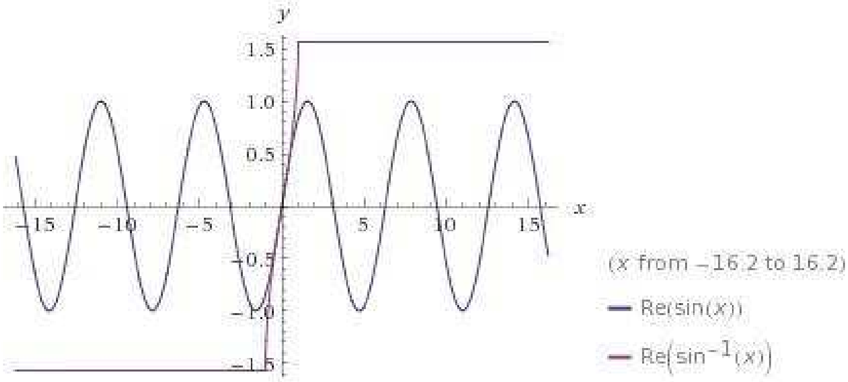

(x from −16.2 to 16.2)
Re(cos(x))
Re(cos^−1(x))
(x from −16.2 to 16.2)
Im(cos(x))
Im(cos^−1(x))

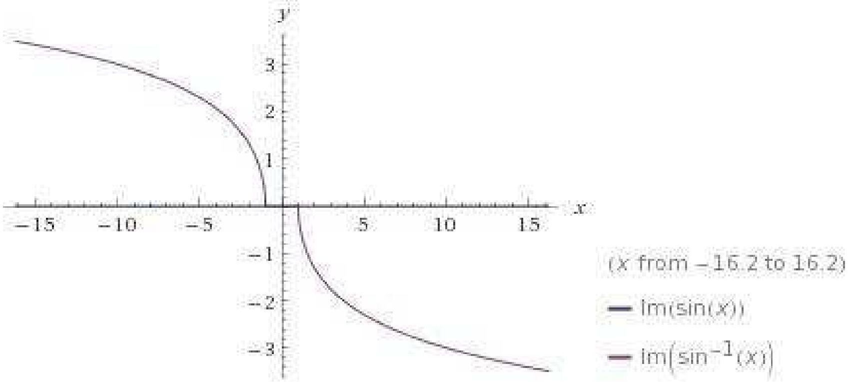
| plot | $y = \sin(x)$ |
|---|---|
| | $y = \sin^{-1}(x)$ |

$\sin^{-1}(x)$ is the inverse sine function

Plots:




y
1.5
1.0
0.5
−0.5
−1.0
−1.5
x
−15
−10
−5
5
10
15
(x from −16.2 to 16.2)
Re(sin(x))
Re(sin⁻¹(x))

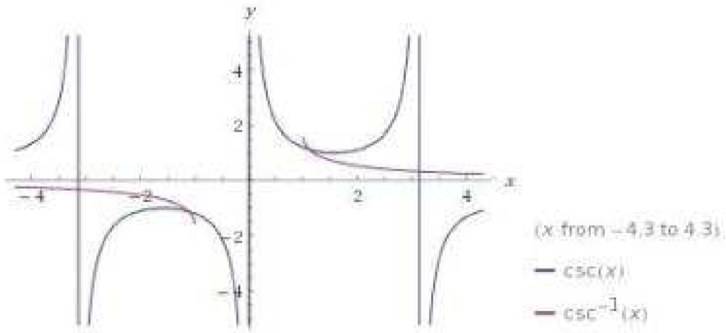

y
3
2
1
−1
−2
−3
x
−15
−10
−5
5
10
15
(x from −16.2 to 16.2)
Im(sin(x))
Im(sin⁻¹(x))

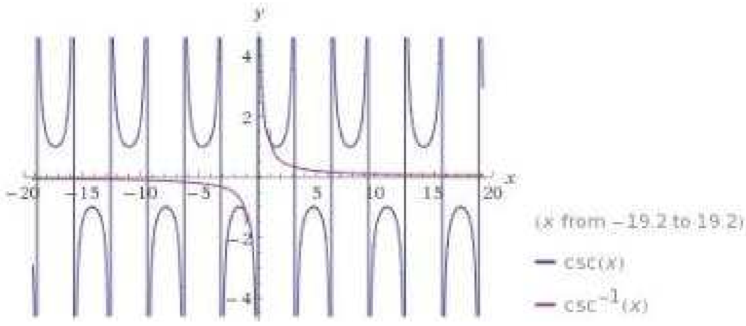
| plot | $y = \csc(x)$ |
|---|---|
| | $y = \csc^{-1}(x)$ |

$\csc(x)$ is the cosecant function

$\csc^{-1}(x)$ is the inverse cosecant function

Plots:



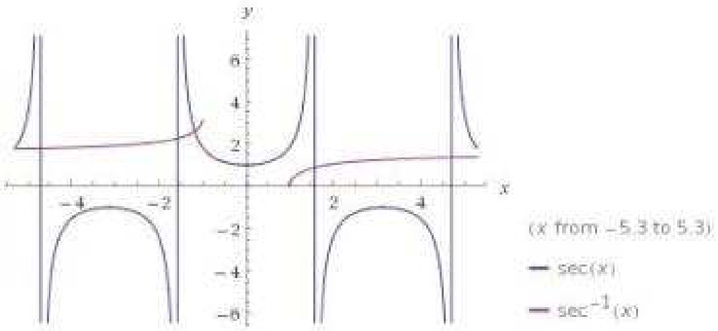



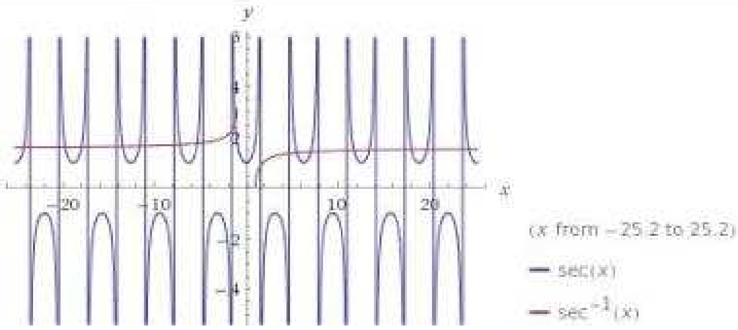
| plot | $y = \sec(x)$ |
| --- | --- |
| | $y = \sec^{-1}(x)$ |

$\sec(x)$ is the secant function

$\sec^{-1}(x)$ is the inverse secant function

Plots:

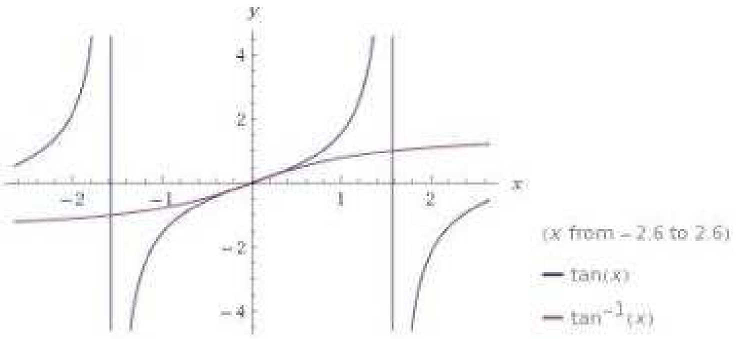


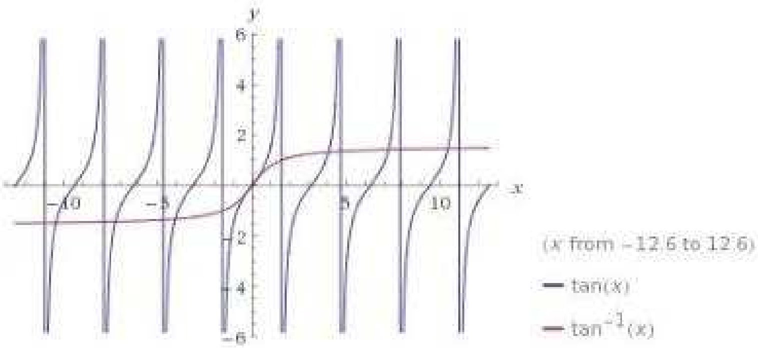



| plot | $y = \tan(x)$ |
|---|---|
| | $y = \tan^{-1}(x)$ |

$\tan^{-1}(x)$ is the inverse tangent function

Plots:



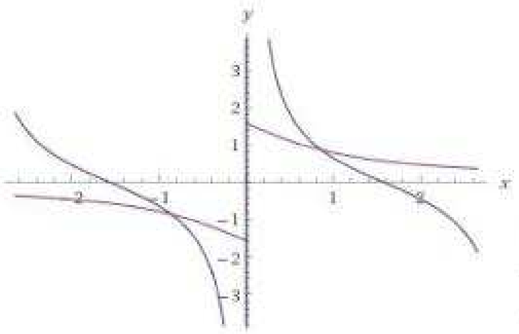



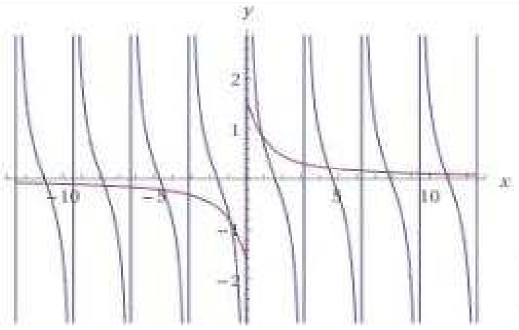
plot $y = \cot(x)$, $y = \cot^{-1}(x)$

$\cot(x)$ is the cotangent function

$\cot^{-1}(x)$ is the inverse cotangent function

Plots:



Spoonfeeding

If $\cot\alpha + \tan\alpha = m$ and $\frac{1}{\cos\alpha} - \cos\alpha = n$, then

(a) $m(mn^2)^{1/3} - n(nm^2)^{1/3} = 1$

(b) $m(m^2n)^{1/3} - n(mn^2)^{1/3} = 1$

(c) $n(mn^2)^{1/3} - m(nm^2)^{1/3} = 1$

(d) $n(m^2n)^{1/3} - m(mn^2)^{1/3} = 1$.

*Ans.* (a).

***Solution*** Clearly $\alpha \neq 0$.

$$\cot\alpha + \tan\alpha = m \Rightarrow 1 + \tan^2\alpha = m\tan\alpha$$

$$\Rightarrow \sec^2\alpha = m\tan\alpha \qquad (1)$$

and $\frac{1}{\cos\alpha} - \cos\alpha = n \Rightarrow \sec^2\alpha - 1 = n\sec\alpha$

$$\Rightarrow \tan^2\alpha = n\sec\alpha \Rightarrow \tan^4\alpha = n^2\sec^2\alpha$$

$$\Rightarrow \tan^4\alpha = n^2m\tan\alpha \qquad \text{[by (1)]}$$

$$\Rightarrow \tan^3\alpha = n^2m \Rightarrow \tan\alpha = (n^2m)^{1/3}$$

and $\sec^2\alpha = m(n^2m)^{1/3}$ (by (1)]

Now $\sec^2\alpha - \tan^2\alpha = 1 \Rightarrow m(n^2m)^{1/3} - (n^2m)^{2/3} = 1$

$$\Rightarrow m(mn^2)^{1/3} - n(nm^2)^{1/3} = 1.$$

Spoonfeeding

If $a\cos^3\alpha + 3a\cos\alpha\sin^2\alpha = m$ and $a\sin^3\alpha + 3a\cos^2\alpha\sin\alpha = n$, then $(m+n)^{2/3} + (m-n)^{2/3}$ is equal to

(a) $2a^2$ (b) $2a^{1/3}$ (c) $2a^{2/3}$ (d) $2a^3$.

*Ans*. (c).

*Solution* From the given relations, we get

$$m + n = a\cos^3\alpha + 3a\cos\alpha\sin^2\alpha + 3a\cos^2\alpha\sin\alpha + a\sin^3\alpha$$

$$= a(\cos\alpha + \sin\alpha)^3$$

Similarly $m - n = a(\cos\alpha - \sin\alpha)^3$

$\therefore$ $(m+n)^{2/3} + (m-n)^{2/3}$

$$= a^{2/3}[(\cos\alpha + \sin\alpha)^2 + (\cos\alpha - \sin\alpha)^2]$$

$$= a^{2/3}[2(\cos^2\alpha + \sin^2\alpha)] = 2a^{2/3}$$

Spoonfeeding

If $\dfrac{2 \sin \alpha}{1 + \cos \alpha + \sin \alpha} = y$, then

$\dfrac{1 - \cos \alpha + \sin \alpha}{1 + \sin \alpha}$ is equal to

(a) $1/y$ (b) $y$ (c) $1 - y$ (d) $1 + y$.

*Ans.* (b)

*Solution* $\dfrac{1 - \cos \alpha + \sin \alpha}{1 + \sin \alpha}$

$$= \frac{1 - \cos \alpha + \sin \alpha}{1 + \sin \alpha} \cdot \frac{1 + \cos \alpha + \sin \alpha}{1 + \cos \alpha + \sin \alpha}$$

$$= \frac{(1 + \sin \alpha)^2 - \cos^2 \alpha}{(1 + \sin \alpha)(1 + \cos \alpha + \sin \alpha)}$$

$$= \frac{1 + 2\sin \alpha + \sin^2 \alpha - 1 + \sin^2 \alpha}{(1 + \sin \alpha)(1 + \cos \alpha + \sin \alpha)}$$

$$= \frac{2\sin \alpha(1 + \sin \alpha)}{(1 + \sin \alpha)(1 + \cos \alpha + \sin \alpha)}$$

$$= \frac{2 \sin \alpha}{1 + \cos \alpha + \sin \alpha} = y$$

Spoonfeeding

If $x_i > 0$ for $1 \le i \le n$ and $x_1 + x_2 + \dots x_n = \pi$ then the greatest value of the sum $\sin x_1 + \sin x_2 + \dots + \sin x_n$ is equal to

(a) $n$ (b) 0

(c) $n \sin (\pi/x)$ (d) $\pi$

*Ans.* (c)

*Solution* The value of the sum is greatest when $x_1 = x_2 = \dots = x_n = \pi/n$ and the required value is $n \sin(\pi/n)$.

Spoonfeeding

Minimum value of $4x^2 - 4x\,|\sin\theta| - \cos^2\theta$ is equal to

(a) –2 (b) –1 (c) –1/2 (d) 0

*Ans.* (b)

*Solution* $4x^2 \pm 4x \sin\theta - (1 - \sin^2\theta)$

$= 4x^2 \pm 4x \sin\theta + \sin^2\theta - 1$

$= (2x \pm \sin\theta)^2 - 1 \ge -1$

Hence the required value is –1.

Spoonfeeding

If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 - bx + c = 0$, then $a$, $b$ and $c$ satisfy the relation

(a) $a^2 + b^2 + 2ac = 0$ (b) $a^2 - b^2 + 2ac = 0$
(c) $a^2 + c^2 + 2ab = 0$ (d) $a^2 - b^2 - 2ac = 0$.

*Ans.* (b).

***Solution*** Since $\sin\theta$ and $\cos\theta$ are roots of the given quadratic equation, we have $\sin\theta + \cos\theta = b/a$ and $\sin\theta\cos\theta = c/a$.

$$\Rightarrow \quad (\sin\theta + \cos\theta)^2 = b^2/a^2$$

$$\Rightarrow \quad \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = b^2/a^2$$

$$\Rightarrow \quad 1 + 2\frac{c}{a} = \frac{b^2}{a^2} \quad \Rightarrow \quad a^2 + 2ac - b^2 = 0.$$

Spoonfeeding

If $\sin x + \sin^2 x = 1$, then the value of $\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x - 1$ is equal to

(a) 0 (b) 1 (c) $-1$ (d) 2.

*Ans.* (a).

***Solution*** From $\sin x + \sin^2 x = 1$, we get $\sin x = \cos^2 x$. Now, the given expression is equal to

$$\cos^6 x(\cos^6 x + 3\cos^4 x + 3\cos^2 x + 1) - 1$$
$$= \cos^6 x(\cos^2 x + 1)^3 - 1$$
$$= \sin^3 x(\sin x + 1)^3 - 1$$
$$= (\sin^2 x + \sin x)^3 - 1 = 1 - 1 = 0.$$

Spoonfeeding

The value of

$\cos y \cos (\pi/2 - x) - \cos (\pi/2 - y) \cos x +$
$\sin y \cos (\pi/2 - x) + \cos x \sin (\pi/2 - y)$ is zero if

(a) $x = 0$ (b) $y = 0$
(c) $x = y$ (d) $x = n\pi - \pi/4 + y \ (n \in \mathbf{I})$.

*Ans.* (d).

*Solution* The given expression is equal to
$\cos y \sin x - \sin y \cos x + \sin y \sin x + \cos x \cos y =$
$\sin (x - y) + \cos (x - y)$

$$= \sqrt{2}\left[\frac{1}{\sqrt{2}}\sin(x-y)+\frac{1}{\sqrt{2}}\cos(x-y)\right]$$

$$= \sqrt{2}\sin\left[\frac{\pi}{4}+(x-y)\right] = 0 \text{ if}$$

$$\left[\frac{\pi}{4}+(x-y)\right] = n\pi \Rightarrow x - y = n\pi - \frac{\pi}{4}.$$

Spoonfeeding

If $\theta$ lies in the first quadrant and $\cos\theta = 8/17$, then the value of $\cos(30° + \theta) + \cos(45° - \theta) + \cos(120° - \theta)$ is

(a) $\left(\dfrac{\sqrt{3}-1}{2}+\dfrac{1}{\sqrt{2}}\right)\dfrac{23}{17}$ (b) $\left(\dfrac{\sqrt{3}+1}{2}+\dfrac{1}{\sqrt{2}}\right)\dfrac{23}{17}$

(c) $\left(\dfrac{\sqrt{3}-1}{2}-\dfrac{1}{\sqrt{2}}\right)\dfrac{23}{17}$ (d) $\left(\dfrac{\sqrt{3}+1}{2}-\dfrac{1}{\sqrt{2}}\right)\dfrac{23}{17}$.

*Ans*. (a).

*Solution* $\cos\theta = \dfrac{8}{17} \Rightarrow \sin\theta = \dfrac{\sqrt{(17)^2 - 8^2}}{17} = \dfrac{15}{17}$

Now the given expression is equal to

$$\cos 30° \cos\theta - \sin 30° \sin\theta + \cos 45° \cos\theta + \sin 45° \sin\theta + \cos 120° \cos\theta + \sin 120° \sin\theta$$

$$= \cos\theta(\cos 30° + \cos 45° + \cos 120°) - \sin\theta(\sin 30° - \sin 45° - \sin 120°)$$

$$= \frac{8}{17}\left(\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}-\frac{1}{2}\right)-\frac{15}{17}\left(\frac{1}{2}-\frac{1}{\sqrt{2}}-\frac{\sqrt{3}}{2}\right)$$

$$= \left(\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right)\frac{23}{17}.$$

Spoonfeeding

If $A$ lies in the second quadrant and $3 \tan A + 4 = 0$, the value of $2 \cot A - 5 \cos A + \sin A$ is equal to

(a) $-53/10$ (b) $23/10$ (c) $37/10$ (d) $7/10$

*Ans.* (b).

*Solution* From $3 \tan A + 4 = 0$, we get $\tan A = -4/3$, so that

$$\sin A = \frac{-\tan A}{\sqrt{1+\tan^2 A}} = \frac{4/3}{\sqrt{1+16/9}} = \frac{4}{5}$$

[$\because$ $\sin A > 0$ and $\tan A < 0$ in quad. II]

and $$\cos A = -\frac{1}{\sqrt{1+\tan^2 A}} = -\frac{3}{5}$$

[$\because$ $\cos A$ is negative in quad. II]

Hence $2 \cot A - 5 \cos A + \sin A$

$$= 2\left(-\frac{3}{4}\right) - 5\left(-\frac{3}{5}\right) + \frac{4}{5} = \frac{23}{10}$$

Spoonfeeding

The value of $\cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{5\pi}{8}\right) + \cos^4\left(\frac{7\pi}{8}\right)$ is equal to

(a) 1/2 (b) 3/4

(c) 3/2 (d) 1

*Ans.* (c)

*Solution* We can write the given expression as

$$\cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\pi - \frac{3\pi}{8}\right) + \cos^4\left(\pi - \frac{\pi}{8}\right)$$

$$= 2\left[\cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right)\right]$$

$$= 2\left[\cos^4\left(\frac{\pi}{8}\right) + \sin^4\left(\frac{\pi}{2} - \frac{3\pi}{8}\right)\right]$$

$$= 2\left[\cos^4\left(\frac{\pi}{8}\right) + \sin^4\frac{\pi}{8}\right]$$

$$= 2\left[\left(\cos^2\left(\frac{\pi}{8}\right) + \sin^2\left(\frac{\pi}{8}\right)\right)^2 - 2\cos^2\left(\frac{\pi}{8}\right)\sin^2\left(\frac{\pi}{8}\right)\right]$$

$$= 2\left[1 - \frac{1}{2}\sin^2\left(\frac{\pi}{4}\right)\right] = 2\left[1 - \frac{1}{2} \times \frac{1}{2}\right] = \frac{3}{2}$$

Spoonfeeding

An angle $\alpha$ is divided into two parts so that the ratio of the tangents of these parts is $\lambda$. If the difference between these parts is $x$ then $\sin x/\sin\alpha$ is equal to

(a) $\lambda/(\lambda + 1)$ (b) $(\lambda - 1)/\lambda$

(c) $\dfrac{\lambda-1}{\lambda+1}$ (d) none of these

*Ans.* (c)

*Solution* Let $\theta_1 + \theta_2 = \alpha$ and $\theta_1 - \theta_2 = x$

$$\frac{\tan\theta_1}{\tan\theta_2} = \lambda \Rightarrow \frac{\tan\theta_1 - \tan\theta_2}{\tan\theta_1 + \tan\theta_2} = \frac{\lambda-1}{\lambda+1}$$

$$\Rightarrow \frac{\sin(\theta_1-\theta_2)}{\sin(\theta_1+\theta_2)} = \frac{\lambda-1}{\lambda+1}$$

$$\Rightarrow \frac{\sin x}{\sin\alpha} = \frac{\lambda-1}{\lambda+1}$$

Spoonfeeding

If $\alpha \in (0, \pi/2)$, then the expression $\sqrt{x^2+x} + \dfrac{\tan^2 x}{\sqrt{x^2+x}}$ is always greater than or equal to

(a) $2 \tan \alpha$ (b) 2

(c) 1 (d) $\sec^2\alpha$

*Ans.* (a).

*Solution* Since A.M $\geq$ G.M, we get

$$\frac{1}{2}\left[\sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}}\right] \geq \sqrt{\sqrt{x^2+x} \times \frac{\tan^2 x}{\sqrt{x^2+x}}} = |\tan\alpha|$$

Spoonfeeding

Given $\theta \in (0, \pi/4)$ and $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$ then

(a) $t_1 > t_2 > t_3 > t_4$
(b) $t_4 > t_3 > t_1 > t_2$
(c) $t_3 > t_1 > t_2 > t_4$
(d) $t_2 > t_3 > t_1 > t_4$

*Ans.* (b)

*Solution* $\theta \in (0, \pi/4)$

$$0 < \tan\theta < 1 \text{ and } \cot\theta > 1 \Rightarrow \log \cot\theta > 0$$

Now $t_1 = (\tan\theta)^{\tan\theta} \Rightarrow \log t_1 = \tan\theta \log(\tan\theta)$

$$\Rightarrow \log t_1 = \tan\theta \log\frac{1}{\cot\theta} = \tan\theta[\log 1 - \log(\cot\theta)]$$

$$= -\tan\theta\log(\cot\theta)$$

Similarly $\log t_2 = -\cot\theta\log(\cot\theta)$

$\log t_3 = \tan\theta\log(\cot\theta)$, $\log t_4 = \cot\theta\log(\cot\theta)$

As $\cot\theta > \tan\theta$, we have

$$\log t_4 > \log t_3 > \log t_1 > \log t_2$$

$$\Rightarrow t_4 > t_3 > t_1 > t_2.$$

Spoonfeeding

If $x = \sin\alpha$, $y = \sin\beta$, $z = \sin(\alpha + \beta)$ then $\cos(\alpha + \beta) =$

(a) $\dfrac{x^2 + y^2 + z^2}{2xy}$ (b) $\dfrac{x^2 + y^2 - z^2}{xy}$

(c) $\dfrac{z^2 - x^2 - y^2}{2xy}$ (d) $\dfrac{z^2 - x^2 - y^2}{xy}$

*Ans.* (d)

*Solution* $z^2 - x^2 - y^2 = \sin^2(\alpha + \beta) - \sin^2\alpha - \sin^2\beta$

$= \sin(\alpha + \beta + \alpha)\sin(\alpha + \beta - \alpha) - \sin^2\beta$

$= \sin\beta[\sin(2\alpha + \beta) - \sin\beta]$

$= \sin\beta\left[2\cos\frac{(2\alpha + \beta + \beta)}{2}\sin\frac{(2\alpha + \beta - \beta)}{2}\right]$

$= 2\sin\alpha\sin\beta\cos(\alpha + \beta)$

$\Rightarrow \cos(\alpha + \beta) = \dfrac{z^2 - x^2 - y^2}{2xy}$.

Spoonfeeding

The radius of the circle
$2x^2 + 2y^2 - 4x\cos\theta + 4y\sin\theta - 1 - 4\cos\theta - \cos 2\theta = 0$ is

(a) $1 - \cos\theta$ (b) $1 + \cos\theta$
(c) $1 - \sin\theta$ (d) none of these

*Ans.* (b).

***Solution*** Equation of the circle can be written as

$$x^2 + y^2 - 2x\cos\theta + 2y\sin\theta = \frac{1 + 4\cos\theta + \cos 2\theta}{2}$$

$$\Rightarrow (x - \cos\theta)^2 + (y + \sin\theta)^2 = \frac{1 + 4\cos\theta + \cos 2\theta}{2} + \sin^2\theta + \cos^2\theta = r^2$$

$$\Rightarrow r^2 = \frac{1 + 4\cos\theta + \cos 2\theta + 2}{2} = \frac{3 + 4\cos\theta + 2\cos^2\theta - 1}{2}$$

$$= (1 + \cos\theta)^2$$

$\Rightarrow$ The radius $= r = 1 + \cos\theta$.

Spoonfeeding

If $\tan x + \tan(x + \pi/3) + \tan(x + 2\pi/3) = 3$, then

(a) $\tan x = 1$ (b) $\tan 2x = 1$
(c) $\tan 3x = 1$ (d) none of these.

*Ans.* (c).

***Solution*** The given equation can be written as

$$\tan x + \frac{\tan x + \tan(\pi/3)}{1 - \tan x \tan(\pi/3)} + \frac{\tan x + \tan(2\pi/3)}{1 - \tan x \tan(2\pi/3)} = 3$$

$$\Rightarrow \quad \tan x + \frac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x} + \frac{\tan x - \sqrt{3}}{1 + \sqrt{3}\tan x} = 3$$

$$\Rightarrow \quad \tan x + \frac{(\tan x+\sqrt{3})(1+\sqrt{3}\tan x)+(1-\sqrt{3}\tan x)(\tan x-\sqrt{3})}{1-3\tan^2 x} = 3$$

$$\Rightarrow \quad \tan x + \frac{8\tan x}{1-3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{\tan x(1-3\tan^2 x)+8\tan x}{1-3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{3(3\tan x-\tan^3 x)}{1-3\tan^2 x} = 3 \quad \Rightarrow \quad 3\tan 3x = 3$$

$$\Rightarrow \quad \tan 3x = 1.$$

Spoonfeeding

The equation $\cos 2x + a\sin x = 2a - 7$ possesses a solution if

(a) $a < 2$ (b) $2 \le a \le 6$

(c) $a > 6$ (d) $a$ is any integer.

*Ans* (b).

*Solution* The given equation can be written as

$$1 - 2\sin^2 x + a\sin x = 2a - 7$$

$$\Rightarrow \quad 2\sin^2 x - a\sin x + 2a - 8 = 0$$

$$\Rightarrow \sin x = \frac{a \pm \sqrt{a^2-8(2a-8)}}{4} = \frac{a \pm \sqrt{a^2-16a+64}}{4}$$

$$= \frac{a \pm (a-8)}{4}$$

Hence, $\sin x = (a-4)/2$ (the other value is not possible as $|\sin x| \le 1$). This value is possible only when

$$-1 \le \frac{a-4}{2} \le 1 \quad \text{or} \quad -2 \le a-4 \le 2$$

$$\Rightarrow \quad 2 \le a \le 6.$$

Spoonfeeding

$\sin 47° + \sin 61° - \sin 11° - \sin 25°$ is equal to

(a) $\sin 36°$ (b) $\cos 36°$ (c) $\sin 7°$ (d) $\cos 7°$.

*Ans.* (d).

*Solution* The given expression is equal to

$$(\sin 47° + \sin 61°) - (\sin 11° + \sin 25°)$$
$$= 2 \sin 54° \cos 7° - 2 \sin 18° \cos 7°$$
$$= 2 \cos 7°(\sin 54° - \sin 18°)$$
$$= 2\cos 7°\left[\frac{\sqrt{5}+1}{4} - \frac{\sqrt{5}-1}{4}\right] = \cos 7°.$$

Spoonfeeding

If $\tan \alpha = 1/7$ and $\sin \beta = 1/\sqrt{10}$ where $0 < \alpha, \beta < \pi/2$, then $2\beta$ is equal to

(a) $\pi/4 - \alpha$ (b) $3\pi/4 - \alpha$
(c) $\pi/8 - \alpha/2$ (d) $3\pi/8 - \alpha/2$.

*Ans.* (a).

*Solution* $\sin \beta = \frac{1}{\sqrt{10}} \Rightarrow \cos \beta = \sqrt{1 - \frac{1}{10}} = \frac{3}{\sqrt{10}}$

$\Rightarrow \quad \tan \beta = 1/3$

$\therefore \quad \tan 2\beta = \frac{2 \tan \beta}{1 - \tan^2 \beta} = \frac{2 \cdot 1/3}{1 - 1/9} = \frac{3}{4}$

and $\tan (\alpha + 2\beta)$

$$= \frac{\tan \alpha + \tan 2\beta}{1 - \tan \alpha \tan 2\beta} = \frac{1/7 + 3/4}{1 - (1/7)(3/4)} = \frac{25}{25} = 1$$

Since $0 < \beta < \pi/2$ and $\tan 2\beta = 3/4 > 0$, we get $0 < 2\beta < \pi/2$. Also, $0 < \alpha < \pi/2$. Hence, $0 < \alpha + 2\beta < \pi$ and $\tan (\alpha + 2\beta) = 1$, so that

$$\alpha + 2\beta = \pi/4 \quad \Rightarrow \quad 2\beta = \pi/4 - \alpha.$$

Spoonfeeding

If $3\pi/4 < \alpha < \pi$, then

$\sqrt{2 \cot \alpha + \frac{1}{\sin^2 \alpha}}$ is equal to

(a) $1 + \cot \alpha$ (b) $-1 - \cot \alpha$
(c) $1 - \cot \alpha$ (d) $-1 + \cot \alpha$.

*Ans.* (b).

*Solution*

$$\sqrt{2 \cot \alpha + \frac{1}{\sin^2 \alpha}} = \sqrt{2 \cot \alpha + \operatorname{cosec}^2 \alpha}$$

$$\sqrt{2 \cot \alpha + 1 + \cot^2 \alpha} = \sqrt{(1 + \cot \alpha)^2} = |1 + \cot \alpha|$$

Since $\cot \alpha < -1$ when $3\pi/4 < \alpha < \pi$, we have $|1 + \cot \alpha| = -1 - \cot \alpha$.

Spoonfeeding

$(2\sqrt{3}+4)\sin x + 4\cos x$ lies in the interval

(a) $(-4, 4)$ (b) $(-2\sqrt{5}, 2\sqrt{5})$

(c) $(-2+\sqrt{5}, 2+\sqrt{5})$ (d) $(-2(2+\sqrt{5}), 2(2+\sqrt{5}))$.

*Ans.* (d).

*Solution* The given expression is equal to $2[(\sqrt{3}+2)\sin x + 2\cos x]$. Put $\sqrt{3}+2 = r\cos\theta$ and $2 = r\sin\theta$, so that $r^2 = (\sqrt{3}+2)^2 + 2^2 = 11 + 4\sqrt{3}$. Then the expression can be written as

$$2(r\cos\theta\sin x + r\sin\theta\cos x)$$

$$= 2r\sin(\theta + x) = y \qquad \text{(say)}$$

Since $11 + 4\sqrt{3} < 9 + 4\sqrt{5}$, we have

$$\sqrt{11+4\sqrt{3}} < \sqrt{9+4\sqrt{5}} \quad \Rightarrow \quad \sqrt{11+4\sqrt{3}} < 2+\sqrt{5} \quad (1)$$

Also, since $-1 \le \sin(\theta + x) \le 1$

$$-2r \le 2r\sin(\theta + x) \le 2r$$

$$\Rightarrow \qquad -2\sqrt{11+4\sqrt{3}} \le y \le 2\sqrt{11+4\sqrt{3}}$$

$$\Rightarrow \qquad -2(2+\sqrt{5}) < y < 2(2+\sqrt{5}) \quad \text{[from (1)]}$$

Spoonfeeding

If $\cos(\theta-\alpha) = a$ and $\sin(\theta-\beta) = b$ $(0 < \theta-\alpha, \theta-\beta < \pi/2)$, then $\cos^2(\alpha-\beta) + 2ab\sin(\alpha-\beta)$ is equal to

(a) $4a^2b^2$ (b) $a^2 - b^2$

(c) $a^2 + b^2$ (d) $-a^2b^2$.

*Ans.* (c).

*Solution* We have

$$\sin(\alpha-\beta) = \sin(\alpha-\theta+\theta-\beta)$$
$$= \sin[(\theta-\beta)-(\theta-\alpha)]$$
$$= \sin(\theta-\beta)\cos(\theta-\alpha) - \cos(\theta-\beta)\sin(\theta-\alpha)$$
$$= ba - \sqrt{1-b^2}\sqrt{1-a^2}$$

and
$$\cos(\alpha-\beta) = \cos[(\theta-\beta)-(\theta-\alpha)]$$
$$= \cos(\theta-\beta)\cos(\theta-\alpha) + \sin(\theta-\beta)\sin(\theta-\alpha)$$
$$= a\sqrt{1-b^2} + b\sqrt{1-a^2}$$

Substituting these values in the given expression, we get

$$\cos^2(\alpha-\beta) + 2ab\sin(\alpha-\beta)$$
$$= \left(a\sqrt{1-b^2} + b\sqrt{1-a^2}\right)^2 + 2ab\left[ab - \sqrt{(1-a^2)}\sqrt{(1-b^2)}\right]$$
$$= a^2(1-b^2) + b^2(1-a^2) + 2ab\sqrt{(1-a^2)}\sqrt{(1-b^2)}$$
$$+ 2a^2b^2 - 2ab\sqrt{(1-a^2)}\sqrt{(1-b^2)} = a^2 + b^2.$$

Spoonfeeding

The expression

$$\frac{\cos 6x + 6\cos 4x + 15\cos 2x + 10}{\cos 5x + 5\cos 3x + 10\cos x}$$ is equal to

(a) $\cos 2x$ (b) $2\cos x$

(c) $\cos^2 x$ (d) $1 + \cos x$.

*Ans*. (b).

*Solution* The given expression can be written as

$$\frac{(\cos 6x + \cos 4x) + 5(\cos 4x + \cos 2x) + 10(\cos 2x + 1)}{\cos 5x + 5\cos 3x + 10\cos x}$$

$$= \frac{2\cos 5x\cos x + 5\cdot 2\cos 3x\cos x + 10\cdot 2\cos^2 x}{\cos 5x + 5\cos 3x + 10\cos x}$$

$$= \frac{2\cos x(\cos 5x + 5\cos 3x + 10\cos x)}{\cos 5x + 5\cos 3x + 10\cos x} = 2\cos x.$$

Spoonfeeding

If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$ then $\cos(\theta - \pi/4)$ is equal to

(a) $\pm\frac{1}{2\sqrt{2}}$ (b) $\pm\frac{1}{\sqrt{2}}$

(c) $\pm\sqrt{2}$ (d) $\pm 2\sqrt{2}$

*Ans*. (a).

*Solution* $\tan(\pi\cos\theta)$

$$= \cot(\pi\sin\theta) = \tan\left(\pm\frac{\pi}{2} - \pi\sin\theta\right)$$

$$\Rightarrow \pi\cos\theta = \pm\frac{\pi}{2} - \pi\sin\theta \Rightarrow \cos\theta + \sin\theta = \pm\frac{1}{2}$$

$$\Rightarrow \sqrt{2}\cos\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2} \Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2\sqrt{2}}.$$

Spoonfeeding

If $\tan\theta_1$, $\tan\theta_2$, $\tan\theta_3$ and $\tan\theta_4$ are the roots of the equation

$$x^4 - x^3\sin 2\beta + x^2\cos 2\beta - x\cos\beta - \sin\beta = 0$$

then $\tan(\theta_1 + \theta_2 + \theta_3 + \theta_4)$ is equal to

(a) $\sin\beta$ (b) $\cos\beta$ (c) $\tan\beta$ (d) $\cot\beta$

*Ans.* (d)

*Solution* From the given equation we get

$S_1 = \tan\theta_1 + \tan\theta_2 + \tan\theta_3 + \tan\theta_4 = \sin 2\beta.$

$S_2 = \Sigma\tan\theta_1\tan\theta_2 = \cos 2\beta$

$S_3 = \Sigma\tan\theta_1\tan\theta_2\tan\theta_3 = \cos\beta$

and $S_4 = \tan\theta_1\tan\theta_2\tan\theta_3\tan\theta_4 = -\sin\beta$

Now $\tan(\theta_1 + \theta_2 + \theta_3 + \theta_4) = \dfrac{S_1 - S_3}{1 - S_2 + S_4}.$

$$= \frac{\sin 2\beta - \cos\beta}{1 - \cos 2\beta - \sin\beta} = \frac{\cos\beta(2\sin\beta - 1)}{\sin\beta(2\sin\beta - 1)} = \cot\beta.$$

Spoonfeeding

If $\sin A$, $\cos A$ and $\tan A$ are in geometric progression, then $\cot^6 A - \cot^2 A$ is equal to

(a) $-1$ (b) $a$

(c) $1$ (d) none of these

*Ans.* (c)

*Solution* Since $\sin A$, $\cos A$ and $\tan A$ are in G.P., we have

$\cos^2 A = \sin A\tan A \Rightarrow \cos^3 A = \sin^2 A$

$\Rightarrow \cot^2 A = \sec A$

$\Rightarrow \cot^4 A = 1 + \tan^2 A \Rightarrow \cot^6 A - \cot^2 A = 1$

Spoonfeeding

The expression $\cos^2 \phi + \cos^2 (a + \phi) - 2 \cos a \cos \phi \cos (a + \phi)$ is independent of

(a) $\phi$ (b) $a$

(c) both $a$ and $\phi$ (d) none of $a$ and $\phi$

*Ans.* (a)

*Solution* The given expression is equal to

$\cos^2 \phi + \cos^2 (a + \phi) - [\cos (a + \phi) + \cos (a - \phi)] \cos (a + \phi)$

$$= \cos^2 \phi - \cos (a + \phi) \cos (a - \phi)$$
$$= \cos^2 \phi - (\cos^2 \phi - \sin^2 a) = \sin^2 a$$

which is independent of $\phi$.

Spoonfeeding

If the value of

$$\sin \frac{\pi}{14} \sin \frac{3\pi}{14} \sin \frac{5\pi}{14} \sin \frac{7\pi}{14} \sin \frac{9\pi}{14} \sin \frac{11\pi}{14} \sin \frac{13\pi}{14}$$

is equal to $k^2$, then $k$ is equal to

(a) $-1/8$ (b) $1/8$ (c) $1/64$ (d) $1$

*Ans.* (a)

*Solution* The given expression can be written as

$$\sin \frac{\pi}{14} \sin \frac{3\pi}{14} \sin \frac{5\pi}{14} \sin\left(\frac{\pi}{2}\right) \sin\left(\pi - \frac{5\pi}{14}\right)$$

$$\sin\left(\pi - \frac{3\pi}{14}\right) \sin\left(\pi - \frac{\pi}{14}\right) = k^2$$

where $k = \sin \frac{\pi}{14} \sin \frac{3\pi}{14} \sin \frac{5\pi}{14}$

$$= \cos\left(\frac{\pi}{2} - \frac{\pi}{14}\right) \cos\left(\frac{\pi}{2} - \frac{3\pi}{14}\right) \cos\left(\frac{\pi}{2} - \frac{5\pi}{14}\right)$$

$$= \cos \frac{3\pi}{7} \cos \frac{2\pi}{7} \cos \frac{\pi}{7}$$

$$= \frac{1}{2\sin\frac{\pi}{7}} \times 2\sin\frac{\pi}{7}\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}$$

$$= \frac{1}{4\sin\frac{\pi}{7}}\sin\frac{4\pi}{7}\cos\frac{4\pi}{7}$$

$$= \frac{1}{8\sin\frac{\pi}{7}} \times \sin\frac{8\pi}{7} = -\frac{1}{8}.$$

Spoonfeeding

If $\sin x + \sin^2 x + \sin^3 x = 1$, then $\cos^6 x - 4\cos^4 x + 8\cos^2 x$ is equal to

(a) 0 (b) 2 (c) 4 (d) 8

*Ans.* (c)

*Solution* The given relation can be written as

$\sin x\,(1 + \sin^2 x) = 1 - \sin^2 x = \cos^2 x$

$\Rightarrow \quad \sin x\,(2 - \cos^2 x) = \cos^2 x$

$\Rightarrow \quad \sin^2 x\,(2 - \cos^2 x)^2 = \cos^4 x$

[squaring both sides]

$\Rightarrow \quad (1 - \cos^2 x)(4 - 4\cos^2 x + \cos^4 x) = \cos^4 x$

$\Rightarrow \quad \cos^6 x - 4\cos^4 x + 8\cos^2 x = 4.$

Spoonfeeding

If $k = \sin \pi/18 \sin 5\pi/18 \sin 7\pi/18$, then the numerical value of $k$ is equal to

(a) 1/2 (b) 1/4 (c) 1/8 (d) 1/18

*Ans.* (c)

*Solution* $k = \sin \pi/18 \sin 5\pi/18 \sin 7\pi/18$

$= \sin 10° \sin 50° \sin 70°$

$$= \frac{1}{2} [\cos 40° - \cos 60°] \sin 70°$$

$$= \frac{1}{2} \cos 40° \sin 70° - \frac{1}{4} \sin 70°$$

$$= \frac{1}{4} [\sin 110° + \sin 30°] - \frac{1}{4} \sin 70°$$

$$= \frac{1}{4} \sin (180° - 70°) + \frac{1}{4}\times\frac{1}{2}-\frac{1}{4} \sin 70° = \frac{1}{8}$$

Spoonfeeding

If $A$ and $B$ are acute positive angles satisfying the equations $3 \sin^2 A + 2 \sin^2 B = 1$ and $3 \sin 2A - 2 \sin 2B = 0$, then $A + 2B$ is equal to

(a) $\pi/4$ (b) $\pi/2$ (c) $3\pi/4$ (d) $2\pi/3$

*Ans.* (b)

*Solution* From the given relations, we have $\sin 2B = (3/2) \sin 2A$ and

$$3 \sin^2 A = 1 - 2 \sin^2 B = \cos 2B$$

so that

$$\begin{aligned}\cos (A + 2B) &= \cos A \cos 2B - \sin A \sin 2B \\ &= \cos A \cdot 3 \sin^2 A - (3/2) \sin A \sin 2A \\ &= 3 \cos A \sin^2 A - 3 \sin^2 A \cos A = 0\end{aligned}$$

$\Rightarrow \quad A + 2B = \pi/2.$

Spoonfeeding

If $\alpha$, $\beta$, $\gamma$ are acute angles and $\cos\theta = \sin\beta/\sin\alpha$, $\cos\phi = \sin\gamma/\sin\alpha$ and $\cos(\theta - \phi) = \sin\beta \sin\gamma$, then $\tan^2\alpha - \tan^2\beta - \tan^2\gamma$ is equal to

(a) $-1$ (b) $0$

(c) $1$ (d) none of these

*Ans.* (b)

*Solution* From the third relation we get

$$\cos\theta\cos\phi + \sin\theta\sin\phi = \sin\beta\sin\gamma$$

$$\Rightarrow \sin^2\theta\sin^2\phi = (\cos\theta\cos\phi - \sin\beta\sin\gamma)^2$$

$$\Rightarrow \left(1-\frac{\sin^2\beta}{\sin^2\alpha}\right)\left(1-\frac{\sin^2\gamma}{\sin^2\alpha}\right)=\left(\frac{\sin\beta\sin\gamma}{\sin^2\alpha}-\sin\beta\sin\gamma\right)^2$$

[from the first and second relations]

$$\Rightarrow (\sin^2\alpha - \sin^2\beta)(\sin^2\alpha - \sin^2\gamma)$$

$$= \sin^2\beta\sin^2\gamma(1-\sin^2\alpha)^2$$

$$\Rightarrow \sin^4\alpha(1-\sin^2\beta\sin^2\gamma) - \sin^2\alpha(\sin^2\beta + \sin^2\gamma - 2\sin^2\beta\sin^2\gamma) = 0$$

$$\therefore \sin^2\alpha = \frac{\sin^2\beta + \sin^2\gamma - 2\sin^2\beta\sin^2\gamma}{1-\sin^2\beta\sin^2\gamma}$$

[$\because \sin\alpha \neq 0$]

$$\text{and } \cos^2\alpha = \frac{1-\sin^2\beta-\sin^2\gamma+\sin^2\beta\sin^2\gamma}{1-\sin^2\beta\sin^2\gamma}$$

$$\Rightarrow \tan^2\alpha = \frac{\sin^2\beta-\sin^2\beta\sin^2\gamma+\sin^2\gamma-\sin^2\beta\sin^2\gamma}{\cos^2\beta-\sin^2\gamma(1-\sin^2\beta)}$$

$$= \frac{\sin^2\beta\cos^2\gamma+\cos^2\beta\sin^2\gamma}{\cos^2\beta\cos^2\gamma}$$

$$= \tan^2\beta+\tan^2\gamma$$

$$\Rightarrow \tan^2\alpha - \tan^2\beta - \tan^2\gamma = 0.$$

Spoonfeeding Trigonometry problem with Matrices

If $A = \begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix}$

and $B = \begin{bmatrix} \cos^2\beta & \cos\beta\sin\beta \\ \cos\beta\sin\beta & \sin^2\beta \end{bmatrix}$

are two matrices such that $AB$ is the null matrix, then

(a) $\alpha = \beta$ (b) $\cos(\alpha - \beta) = 0$

(c) $\sin(\alpha - \beta) = 0$ (d) none of these

*Ans.* (b)

*Solution* $AB = 0$

$$\Rightarrow \begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix}\begin{bmatrix} \cos^2\beta & \cos\beta\sin\beta \\ \cos\beta\sin\beta & \sin^2\beta \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos\alpha\cos\beta\cos(\alpha-\beta) & \cos\alpha\sin\beta\cos(\alpha-\beta) \\ \cos\beta\sin\alpha\cos(\alpha-\beta) & \sin\alpha\sin\beta\cos(\alpha-\beta) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \cos(\alpha - \beta) = 0$$

Spoonfeeding

If $\tan \beta = \dfrac{n \sin \alpha \cos \alpha}{1 - n \sin^2 \alpha}$, then $\tan(\alpha - \beta)$ is equal to

(a) $n \tan \alpha$ (b) $(1 - n) \tan \alpha$

(c) $(1 + n) \tan \alpha$ (d) none of these

*Ans.* (b)

***Solution***

$$\tan \beta = \frac{n \tan \alpha}{1 + \tan^2 \alpha - n \tan^2 \alpha}$$

$$= \frac{n \tan \alpha}{1 + (1 - n) \tan^2 \alpha}$$

$$\Rightarrow \tan(\alpha - \beta) = \frac{\tan \alpha - \dfrac{n \tan \alpha}{1 + (1 - n) \tan^2 \alpha}}{1 + \dfrac{\tan \alpha \cdot n \tan \alpha}{1 + (1 - n) \tan^2 \alpha}}$$

$$= \frac{\tan \alpha + (1 - n) \tan^3 \alpha - n \tan \alpha}{1 + (1 - n) \tan^2 \alpha + n \tan^2 \alpha}$$

$$= \frac{(1 - n) \tan \alpha (1 + \tan^2 \alpha)}{1 + \tan^2 \alpha}$$

$= (1 - n) \tan \alpha.$

Spoonfeeding

If $\dfrac{\cos\theta}{a} = \dfrac{\sin\theta}{b}$, then $\dfrac{a}{\sec 2\theta} + \dfrac{b}{\operatorname{cosec} 2\theta}$ is equal to

(a) $a$ (b) $b$ (c) $a/b$ (d) $a + b$

*Ans.* (a)

*Solution* Let $(\cos\theta)/a = (\sin\theta)/b = k$, so that $\cos\theta = ak$ and $\sin\theta = bk$. Then

$a\cos 2\theta + b\sin 2\theta = a(1 - 2\sin^2\theta) + 2b\sin\theta\cos\theta$

$= a - 2ab^2k^2 + 2b \cdot bk \cdot ak = a - 2ab^2k^2 + 2ab^2k^2 = a.$

Spoonfeeding

If $\sin\alpha + \sin\beta + \sin\gamma = 0$ and $\cos\alpha + \cos\beta + \cos\gamma = 0$, then value of $\cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha)$ is

(a) $-3/2$ (b) $-1$

(c) $-1/2$ (d) $0$

*Ans.* (a)

*Solution* We have $(\sin\alpha + \sin\beta)^2 + (\cos\alpha + \cos\beta)^2$

$= \sin^2\gamma + \cos^2\gamma \Rightarrow 1 + 1 + 2\cos(\alpha - \beta) = 1$

$\Rightarrow \cos(\alpha - \beta) = -1/2.$

Similarly, $\cos(\beta - \gamma) = -1/2$

and $\cos(\gamma - \alpha) = -1/2$.

Thus value of given expression is $-3/2$.

Spoonfeeding

If $\sin x \cos y = 1/4$ and $3 \tan x = 4 \tan y$, then $\sin(x + y)$ is equal to

(a) 1/4 (b) 3/4

(c) 1 (d) none of these

*Ans.* (d)

*Solution* $3 \tan x = 4 \tan y \Rightarrow 3 \sin x \cos y = 4 \cos x \sin y$

$\Rightarrow \quad 3/4 = 4 \cos x \sin y \Rightarrow \cos x \sin y = 3/16$

$\therefore \quad \sin(x + y) = \sin x \cos y + \cos x \sin y = 1/4 + 3/16 = 7/16.$

Spoonfeeding

If $k_1 = \tan 27\theta - \tan \theta$

and $$k_2 = \frac{\sin \theta}{\cos 3\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin 9\theta}{\cos 27\theta}$$

then,

(a) $k_1 = k_2$ (b) $k_1 = 2k_2$

(c) $k_1 + k_2 = 2$ (d) $k_2 = 2k_1$

*Ans.* (b)

*Solution* We can write

$k_1 = \tan 27\theta - \tan 9\theta + \tan 9\theta - \tan 3\theta + \tan 3\theta - \tan \theta$

But

$$\tan 3\theta - \tan \theta = \frac{\sin 3\theta \cos \theta - \cos 3\theta \sin \theta}{\cos 3\theta \cos \theta} = \frac{\sin 2\theta}{\cos 3\theta \cos \theta} = \frac{2 \sin \theta}{\cos 3\theta}$$

$$\therefore \quad k_1 = 2\left[\frac{\sin 9\theta}{\cos 27\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin \theta}{\cos 3\theta}\right] = 2\,k_2.$$

Spoonfeeding

$$\begin{vmatrix} \cos(\theta+\alpha) & \sin(\theta+\alpha) & 1 \\ \cos(\theta+\beta) & \sin(\theta+\beta) & 1 \\ \cos(\theta+\gamma) & \sin(\theta+\gamma) & 1 \end{vmatrix}$$

is independent of

(a) $\alpha$ (b) $\beta$ (c) $\gamma$ (d) $\theta$

*Ans.* (d)

*Solution* Differentiating the given determinant w.r.t. $\theta$ we get

$$f'(\theta) = \begin{vmatrix} -\sin(\theta+\alpha) & \sin(\theta+\alpha) & 1 \\ -\sin(\theta+\beta) & \sin(\theta+\beta) & 1 \\ -\sin(\theta+\gamma) & \sin(\theta+\gamma) & 1 \end{vmatrix} + \begin{vmatrix} \cos(\theta+\alpha) & \cos(\theta+\alpha) & 1 \\ \cos(\theta+\beta) & \cos(\theta+\beta) & 1 \\ \cos(\theta+\gamma) & \cos(\theta+\gamma) & 1 \end{vmatrix}$$

$$= 0 + 0 = 0$$

where $f(\theta)$ denotes the given determinant

$\Rightarrow$ $f(\theta)$ is independent of $\theta$.

*Alternately* expanding the determinant along last column we get

$\sin(\theta+\gamma-\theta-\beta) - \sin(\theta+\gamma-\theta-\alpha) + \sin(\theta+\beta-\theta-\alpha)$

$= \sin(\gamma-\beta) + \sin(\beta-\alpha) + \sin(\alpha-\gamma)$

which is independent of $\theta$.

**Spoonfeeding**

If $\theta$ and $\phi$ are acute angles such that $\sin\theta = 1/2$ and $\cos\phi = 1/3$, then $\theta + \phi$ lies in

(a) $]\ \pi/3,\ \pi/2\ [$ (b) $]\ \pi/2,\ 2\pi/3\ [$

(c) $]\ 2\pi/3,\ 5\pi/3\ [$ (d) $]\ 5\pi/6,\ \pi\ [$

*Ans.* (b)

*Solution* $\sin\theta = 1/2 \Rightarrow \theta = \pi/6$

and $\cos\phi = 1/3 \Rightarrow \pi/3 < \phi < \pi/2$

so that $\dfrac{\pi}{2} < (\theta + \phi) < \dfrac{2\pi}{3}$.

**Spoonfeeding**

The value of $\tan 3\alpha \cot\alpha$ cannot lie in

(a) $]\ 0,\ 2/3\ [$ (b) $]\ 1/3,\ 3\ [$

(c) $]\ 4/3,\ 4\ [$ (d) $]\ 2,\ 10/3\ [$

*Ans.* (b)

*Solution* $$\tan 3\alpha \cot\alpha = \frac{3\tan\alpha - \tan^3\alpha}{\tan\alpha(1 - 3\tan^2\alpha)}$$

$$= \frac{3 - \tan^2\alpha}{1 - 3\tan^2\alpha} = x \text{ (say)}$$

$$\Rightarrow \qquad \tan^2\alpha = \frac{x-3}{3x-1} = \frac{(3x-1)(x-3)}{(3x-1)^2}$$

Since $\tan^2\alpha$ is non-negative, either $x < 1/3$ or $x \geq 3$, so $x$ cannot lie between 1/3 and 3.

**Spoonfeeding**

For a given pair of values $x$ and $y$ satisfying $x = \sin \alpha$, $y = \sin \beta$, there are four different values of $z = \sin(\alpha + \beta)$ whose product is equal to

(a) $x^2 - y^2$ (b) $x^2 + y^2$

(c) $(x^2 + y^2)^2$ (d) $(x^2 - y^2)^2$

*Ans.* (d)

*Solution* $z = \sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

$$= \pm\left(x\sqrt{1-y^2}\right) \pm \left(y\sqrt{1-x^2}\right)$$

There are four values of $z$, given by

$$\pm\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right] \text{and} \pm\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]$$

and their product is equal to

$$[x^2(1 - y^2) - y^2(1 - x^2)]^2 = (x^2 - y^2)^2.$$

**Spoonfeeding**

If $0 < A < \pi/2$ the value of the expression

$$\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A} - \sec A \operatorname{cosec} A$$ is equal to

(a) $-1$ (b) $0$

(c) $1$ (d) $\sin A + \cos A$

*Ans.* (c)

*Solution* The given expression is equal to

$$\frac{\sin^2 A}{\cos A(\sin A-\cos A)}+\frac{\cos^2 A}{\sin A(\cos A-\sin A)} - \sec A \operatorname{cosec} A$$

$$= \frac{\sin^3 A-\cos^3 A}{\sin A\cos A(\sin A-\cos A)} - \sec A \operatorname{cosec} A$$

$$= \frac{\sin^2 A+\cos^2 A+\sin A\cos A}{\sin A\cos A} - \sec A \operatorname{cosec} A$$

$$= \sec A \operatorname{cosec} A + 1 - \sec A \operatorname{cosec} A = 1.$$

**Spoonfeeding**

If $a^2 - 2a \cos x + 1 = 674$ and $\tan (x/2) = 7$ then the integral value of $a$ is

(a) 25 (b) 49 (c) 67 (d) 74

*Ans.* (a)

*Solution* $$674 = a^2 - 2a\,\frac{1-\tan^2(x/2)}{1+\tan^2(x/2)} + 1$$

$$= a^2 - 2a \times \frac{1-49}{1+49} + 1$$

$$= a^2 + 2a \times \frac{48}{50} + 1$$

$\Rightarrow$ $$25a^2 + 48a - 673 \times 25 = 0$$

$\Rightarrow \qquad (a - 25)(25a + 673) = 0$

$\Rightarrow \qquad a = 25$ (Taking the integral value of $a$).

**Spoonfeeding**

If $\sin 2x = \alpha - 1$ and $\cos 2x = \beta - 1$, then the value of $\dfrac{\sec^2 x[(\cos^2 x - \sin^2 x) - 2\sin x\cos x]}{1 + \sin 2x}$ is equal to

(a) $2(\alpha - \beta)$ (b) $\dfrac{2}{\alpha} - \dfrac{2}{\beta}$

(c) $\dfrac{2}{\beta} - \dfrac{2}{\alpha}$ (d) $2(\alpha + \beta)$

*Ans*. (b)

*Solution* The given expression can be written as

$$\frac{2(\cos 2x - \sin 2x)}{2\cos^2 x(1 + \sin 2x)}$$

$$= \frac{2(\cos 2x - \sin 2x)}{(1 + \cos 2x)(1 + \sin 2x)}$$

$$= 2\left[\frac{1}{1 + \sin 2x} - \frac{1}{1 + \cos 2x}\right]$$

$$= 2\left[\frac{1}{\alpha} - \frac{1}{\beta}\right]$$

**Spoonfeeding**

$\dfrac{1}{\cos 290^\circ} + \dfrac{1}{\sqrt{3}\sin 250^\circ}$ is equal to

(a) $\sqrt{3}/4$ (b) $4/\sqrt{3}$ (c) $2/\sqrt{3}$ (d) $\sqrt{3}/2$

*Ans.* (b)

*Solution* The given expression is equal to

$$= \frac{1}{\cos(270^\circ + 20^\circ)} + \frac{1}{\sqrt{3}\sin(270^\circ - 20^\circ)}$$

$$= \frac{1}{\sin 20^\circ} + \frac{1}{\sqrt{3}(-\cos 20^\circ)}$$

$$= \frac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sqrt{3}\sin 20^\circ \cos 20^\circ} = \frac{\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ}{\frac{\sqrt{3}}{4}\sin 40^\circ}$$

$$= \frac{\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ}{\frac{\sqrt{3}}{4}\sin 40^\circ}$$

$$= \frac{\sin 40^\circ}{\frac{\sqrt{3}}{4}\sin 40^\circ} = \frac{4}{\sqrt{3}}.$$

**Spoonfeeding**

Let $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$, then $f(\theta)$

(a) $\geq 0$ only when $\theta \geq 0$ (b) $\leq 0$ for all real $\theta$

(c) $\geq 0$ for all real $\theta$ (d) $\leq 0$ only when $\theta \leq 0$

*Ans.* (c)

*Solution* $f(\theta) = (1/2)(1 - \cos 2\theta) + (1/2)(\cos 2\theta - \cos 4\theta)$

$= (1/2)(1 - \cos 4\theta) = \sin^2 2\theta \geq 0$ for all real $\theta$.

**Spoonfeeding**

The maximum value of $(\cos \alpha_1)(\cos \alpha_2) \ldots (\cos \alpha_n)$ under the restriction $0 \le \alpha_1, \alpha_2, \ldots, \alpha_n \le \pi/2$ and $(\cot \alpha_1)(\cot \alpha_2) \ldots (\cot \alpha_n) = 1$ is

(a) $\frac{1}{2^{n/2}}$ (b) $\frac{1}{2^n}$ (c) $\frac{1}{2n}$ (d) 1

*Ans.* (a)

*Solution* From the given relations we have

$$\prod_{i=1}^{n} (\cos \alpha_i) = \prod_{i=1}^{n} (\sin \alpha_i)$$

$$\Rightarrow \prod_{i=1}^{n} (\cos^2 \alpha_i) = \prod_{i=1}^{n} (\cos \alpha_i \sin \alpha_i) = \prod_{i=1}^{n} \left(\frac{\sin 2\alpha_i}{2}\right)$$

Since $0 \le \alpha_i \le \pi/2 \Rightarrow 0 \le 2\alpha_i \le \pi$

$\therefore \prod_{i=1}^{n} (\cos^2 \alpha_i) \le \frac{1}{2^n}$ as max. value of $\sin 2\alpha_i$ is 1 for all $i$.

$$\Rightarrow \prod_{i=1}^{n} (\cos \alpha_i) \le \frac{1}{2^{n/2}}.$$

So the maximum value of the given expression is $\frac{1}{2^{n/2}}$.

**Spoonfeeding**

If $\alpha + \beta = \pi/2$ and $\beta + \gamma = \alpha$, then $\tan \alpha$ equals

(a) $2(\tan \beta + \tan \gamma)$ (b) $\tan \beta + \tan \gamma$
(c) $\tan \beta + 2 \tan \gamma$ (d) $2 \tan \beta + \tan \gamma$

*Ans.* (c)

*Solution* $\alpha + \beta = \pi/2 \Rightarrow \alpha = \pi/2 - \beta \Rightarrow \tan \alpha = \cot \beta$

$\Rightarrow \tan \alpha \tan \beta = 1$

Next, $\beta + \gamma = \alpha$

$$\Rightarrow \alpha - \beta = \gamma \Rightarrow \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} = \tan \gamma$$

$\Rightarrow \tan \alpha - \tan \beta = 2 \tan \gamma$

$\Rightarrow \tan \alpha = \tan \beta + 2 \tan \gamma$.

**Spoonfeeding**

The number of integral values of $k$ for which the equation $7 \cos x + 5 \sin x = 2k + 1$ has a solution is

(a) 4 (b) 8 (c) 10 (d) 12

*Ans.* (b)

*Solution* The given equation can be written as $r \cos(x - \alpha) = 2k + 1$ where $r \cos \alpha = 7$, $r \sin \alpha = 5$

$$\Rightarrow \cos(x - \alpha) = \frac{2k+1}{\sqrt{74}} \text{ as } r^2 = 7^2 + 5^2 = 74$$

$$\Rightarrow -1 \le \frac{2k+1}{\sqrt{74}} \le 1$$

$\Rightarrow -\sqrt{74} \le 2k + 1 \le \sqrt{74}$

$\Rightarrow -8 \le 2k + 1 \le 8$ (For integral values of $k$)

$\Rightarrow -4 \le k \le 3$

$\Rightarrow k = -4, -3, -2, -1, 0, 1, 2, 3$

which gives 8 integral values of $k$.

**Spoonfeeding**

If $x_{n+1} = \sqrt{\frac{1}{2}(1+x_n)}$, then

$\cos\left[\frac{\sqrt{1-x_0^2}}{x_1 x_2 x_3 \ldots \text{to infinite}}\right]$ $(-1 < x_0 < 1)$ is equal to

(a) $-1$ (b) $1$ (c) $x_0$ (d) $1/x_0$

*Ans.* (c)

*Solution* Let $x_0 = \cos\theta$, then $x_1 = \sqrt{\frac{1}{2}(1+\cos\theta)}$

$= \cos\theta/2$, $x_2 = \cos(\theta/2^2)$, $x_3 = \cos(\theta/2^3)$, … and so on.

so that $\left[\frac{\sqrt{1-x_0^2}}{x_1 x_2 x_3 \ldots \text{to infinite}}\right]$

$$= \frac{\sin\theta}{\cos\frac{\theta}{2}\cos\frac{\theta}{2^2}\ldots\cos\frac{\theta}{2^n}\ldots\text{infinite}}$$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}\cos\frac{\theta}{2^2}\ldots\cos\frac{\theta}{2^n}\ldots\text{infinite}}$$

$$= \frac{2^2\sin\frac{\theta}{2^2}\cos\frac{\theta}{2^2}}{\cos\frac{\theta}{2^2}\cos\frac{\theta}{2^3}\ldots\cos\frac{\theta}{2^n}\ldots\text{infinite}}$$

$$= \lim_{n\to\infty}\frac{2^n\sin\frac{\theta}{2^n}}{\cos\frac{\theta}{2^{n+1}}}$$

$$= \lim_{n\to\infty}\theta\left(\frac{\sin\frac{\theta}{2^n}}{\frac{\theta}{2^n}}\right)\frac{1}{\cos\frac{\theta}{2^{n+1}}} = \theta$$

so that $\cos\left[\frac{\sqrt{1-x_0^2}}{x_1 x_2\ldots\text{inf.}}\right] = \cos\ \theta = x_0.$

**Spoonfeeding**

The simplest value of

$$\frac{2}{\sqrt{2+\sqrt{2+\sqrt{2+2\cos 4x}}}} \text{ is}$$

(a) $\sec(x/2)$ (b) $\sec x$ (c) $\operatorname{cosec} x$ (d) 1

Ans. (a)

*Solution* Given expression is equal to

$$\frac{2}{\sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2 2x}}}} = \frac{2}{\sqrt{2+\sqrt{2+2\cos 2x}}}$$

$$= \frac{2}{\sqrt{2+2\cos x}} = \frac{2}{\sqrt{4\cos^2 x/2}} = \sec\,(x/2)$$

**Spoonfeeding**

If $\alpha, \beta$ are positive acute angles and $\cos 2\alpha$

$$= \frac{3\cos 2\beta - 1}{3 - \cos 2\beta}$$, then $\tan\alpha = k\tan\beta$ such that

(a) $k = -\sqrt{2}$ (b) $k = \sqrt{2}$ (c) $k = 1$ (d) $k = \sqrt{3}$

*Ans.* (b)

*Solution* $$\frac{1-\cos 2\alpha}{1+\cos 2\alpha} = \frac{(3-\cos 2\beta)-(3\cos 2\beta - 1)}{(3-\cos 2\beta)+(3\cos 2\beta - 1)}$$

$$\Rightarrow \tan^2\alpha = \frac{3(1-\cos 2\beta)+(1-\cos 2\beta)}{3(1+\cos 2\beta)-(1+\cos 2\beta)}$$

$$= \frac{4(1-\cos 2\beta)}{2(1+\cos 2\beta)} = 2\tan^2\beta$$

$\Rightarrow \tan^2\alpha = \sqrt{2}\tan\beta$ (Rejecting the minus sign as $\alpha$, $\beta$ are positive acute angle)

**Spoonfeeding**

$(m+2)\sin\theta + (2m-1)\cos\theta = 2m+1$, if

(a) $\tan\theta = 3/4$

(b) $\tan\theta = 4/3$

(c) $\tan\theta = 2m/(m^2-1)$

(d) $\tan\theta = 2m/(m^2+1)$

*Ans.* (b) and (c)

***Solution*** The given relation can be written as

$$(m+2)\tan\theta + (2m-1) = (2m+1)\sec\theta$$

$$\Rightarrow \quad (m+2)^2\tan^2\theta + 2(m+2)(2m-1)\tan\theta + (2m-1)^2$$

$$= (2m+1)^2(1+\tan^2\theta)$$

$$\Rightarrow \quad [(m+2)^2 - (2m+1)^2]\tan^2\theta + 2(m+2)(2m-1)\tan\theta + (2m-1)^2 - (2m+1)^2 = 0$$

$$\Rightarrow \quad 3(1-m^2)\tan^2\theta + (4m^2+6m-4)\tan\theta - 8m = 0$$

$$\Rightarrow \quad (3\tan\theta - 4)[(1-m^2)\tan\theta + 2m] = 0$$

which is true if $\tan\theta = 4/3$ or $\tan\theta = 2m/(m^2-1)$.

**Spoonfeeding**

If $x = \sec\phi - \tan\phi$ and $y = \operatorname{cosec}\phi + \cot\phi$, then

(a) $x = \dfrac{y+1}{y-1}$ (b) $x = \dfrac{y-1}{y+1}$

(c) $y = \dfrac{1+x}{1-x}$ (d) $xy + x - y + 1 = 0.$

*Ans.* (b), (c) and (d)

*Solution* We have $x = \dfrac{1-\sin\phi}{\cos\phi}, y = \dfrac{1+\cos\phi}{\sin\phi}$

Multiplying, we get $xy = \dfrac{(1-\sin\phi)(1+\cos\phi)}{\cos\phi\sin\phi}$

$$\Rightarrow xy + 1 = \frac{1-\sin\phi+\cos\phi-\sin\phi\cos\phi+\sin\phi\cos\phi}{\cos\phi\sin\phi}$$

$$= \frac{1-\sin\phi+\cos\phi}{\cos\phi\sin\phi}$$

and $x - y = \dfrac{(1-\sin\phi)\sin\phi-\cos\phi(1+\cos\phi)}{\cos\phi\sin\phi}$

$$= \frac{\sin\phi-\sin^2\phi-\cos\phi-\cos^2\phi}{\cos\phi\sin\phi}$$

$$= \frac{\sin\phi-\cos\phi-1}{\cos\phi\sin\phi} = -(xy+1)$$

Thus, $xy + x - y + 1 = 0.$

$$\Rightarrow \quad x = \frac{y-1}{y+1} \quad \text{and} \quad y = \frac{1+x}{1-x}.$$

**Spoonfeeding**

If $x \cos \alpha + y \sin \alpha = x \cos \beta + y \sin \beta = 2a$ $(0 < \alpha, \beta < \pi/2)$, then

(a) $\cos \alpha + \cos \beta = \dfrac{4ax}{x^2 + y^2}$

(b) $\cos \alpha \cos \beta = \dfrac{4a^2 - y^2}{x^2 + y^2}$

(c) $\sin \alpha + \sin \beta = \dfrac{4ay}{x^2 + y^2}$

(d) $\sin \alpha \sin \beta = \dfrac{4a^2 - x^2}{x^2 + y^2}$.

*Ans.* (a), (b), (c) and (d).

*Solution* We find from the given relations that $\alpha$ and $\beta$ are the roots of the equation

$$x \cos \theta + y \sin \theta = 2a \qquad (1)$$

$$\Rightarrow \quad (x \cos \theta - 2a)^2 = (-y \sin \theta)^2$$

$$\Rightarrow \quad x^2 \cos^2 \theta - 4ax \cos \theta + 4a^2 = y^2 \sin^2 \theta = y^2(1 - \cos^2 \theta)$$

$$\Rightarrow \quad (x^2 + y^2) \cos^2 \theta - 4ax \cos \theta + 4a^2 - y^2 = 0$$

which, being quadratic in $\cos \theta$, has two roots $\cos \alpha$ and $\cos \beta$, such that

$$\cos \alpha + \cos \beta = \frac{4ax}{x^2 + y^2} \quad \text{and} \quad \cos \alpha \cos \beta = \frac{4a^2 - y^2}{x^2 + y^2}$$

Similarly, we can write (1) as a quadratic in $\sin \theta$, giving two values $\sin \alpha$ and $\sin \beta$, such that

$$\sin \alpha + \sin \beta = \frac{4ay}{x^2 + y^2} \quad \text{and} \quad \sin \alpha \sin \beta = \frac{4a^2 - x^2}{x^2 + y^2}.$$

**Spoonfeeding**

If $\tan x = 2b/(a - c)$ $(a \neq c)$,
$y = a\cos^2 x + 2b\sin x\cos x + c\sin^2 x$ and
$z = a\sin^2 x - 2b\sin x\cos x + c\cos^2 x$, then

(a) $y = z$ (b) $y + z = a + c$

(c) $y - z = a - c$ (d) $y - z = (a - c)^2 + 4b^2$.

*Ans.* (b) and (c).

*Solution* Adding the expression for $y$ and $z$, we get

$$y + z = a(\cos^2 x + \sin^2 x) + c(\sin^2 x + \cos^2 x)$$
$$= a + c$$

and subtracting them

$$y - z = a(\cos^2 x - \sin^2 x) + 4b\sin x\cos x - c(\cos^2 x - \sin^2 x)$$
$$= a\cos 2x + 2b\sin 2x - c\cos 2x$$
$$= (a - c)\cos 2x + 2b\sin 2x$$
$$= (a - c)[\cos 2x + \tan x\sin 2x]$$
$$= (a - c)[\cos^2 x - \sin^2 x + 2\sin^2 x]$$
$$= (a - c)$$

As $a \neq c$, we get $y \neq z$.

**Spoonfeeding**

The values of $\theta$ lying between 0 and $\pi/2$ and satisfying the equation

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

are

(a) $7\pi/24$ (b) $5\pi/24$ (c) $11\pi/24$ (d) $\pi/24$.

*Ans.* (a) and (c).

*Solution* Applying $R_1 \to R_1 - R_3$ and $R_2 \to R_2 - R_3$ on the LHS, the given equation can be written as

$$\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

Expanding the LHS along $R_1$, we get
$1 + 4\sin 4\theta + \cos^2\theta + \sin^2\theta = 0$

$\therefore\quad 4\sin 4\theta = -2 \quad\Rightarrow\quad \sin 4\theta = -1/2$

$\therefore\quad 4\theta = 7\pi/6 \quad\text{or}\quad 11\pi/6$

$[\because 0 < \theta < \frac{\pi}{2} \Rightarrow 0 < 4\theta < 2\pi]$

$\therefore\quad \theta = 7\pi/24 \quad\text{or}\quad 11\pi/24.$

**Spoonfeeding**

If $0 \leq x, y \leq 180°$ and $\sin(x - y) = \cos(x + y) = 1/2$, then the values of $x$ and $y$ are given by

(a) $x = 45°, y = 15°$ (b) $x = 45°, y = 135°$
(c) $x = 165°, y = 15°$ (d) $x = 165°, y = 135°$.

*Ans*. (a) and (d).

*Solution*

$$\sin(x - y) = 1/2 \quad \Rightarrow \quad x - y = 30° \text{ or } 150° \quad (1)$$

and $$\cos(x + y) = 1/2 \quad \Rightarrow \quad x + y = 60° \text{ or } 300° \quad (2)$$

Since $x$ and $y$ lie between 0° and 180°, (1) and (2) are simultaneously true when $x = 45°, y = 15°$, or $x = 165°, y = 135°$. But, for the values given by (b) or (c), (1) and (2) do not hold simultaneously.

**Spoonfeeding**

If $\tan(x/2) = \operatorname{cosec} x - \sin x$, then $\tan^2(x/2)$ is equal to

(a) $2 - \sqrt{5}$ (b) $\sqrt{5} - 2$

(c) $(9 - 4\sqrt{5})(2 + \sqrt{5})$ (d) $(9 + 4\sqrt{5})(2 - \sqrt{5})$

*Ans.* (b) and (c).

*Solution* The given relation can be written as

$$\tan(x/2) = \frac{1 - \sin^2 x}{\sin x} = \frac{\cos^2 x}{\sin x}$$

$\Rightarrow$ $2\sin^2(x/2) = [\cos^2(x/2) - \sin^2(x/2)]^2$

$\Rightarrow$ $2\tan^2(x/2) = [1 - \tan^2(x/2)]^2/[(1 + \tan^2 x/2]$

$\Rightarrow$ $2y(1 + y) = (1 - y)^2$

[where $y = \tan^2 x/2$]

$\Rightarrow$ $y^2 + 4y - 1 = 0$

$\Rightarrow$ $y = \dfrac{-4 \pm \sqrt{16 + 4}}{2} = -2 \pm \sqrt{5}$

Since $y > 0$, we get

$$y = \sqrt{5} - 2 = \frac{(\sqrt{5} - 2)^2}{\sqrt{5} - 2} \cdot \frac{2 + \sqrt{5}}{2 + \sqrt{5}}$$

$= (9 - 4\sqrt{5})(2 + \sqrt{5})$

**Spoonfeeding**

If $y = \dfrac{\sqrt{1 - \sin 4A} + 1}{\sqrt{1 + \sin 4A} - 1}$, then one of the values of $y$ is

(a) $-\tan A$ (b) $\cot A$

(c) $\tan\left(\dfrac{\pi}{4} + A\right)$ (d) $-\cot\left(\dfrac{\pi}{4} + A\right)$.

*Ans.* (a), (b), (c) and (d)

*Solution* $$y = \frac{\sqrt{(\cos 2A - \sin 2A)^2} + 1}{\sqrt{(\cos 2A + \sin 2A)^2} - 1}$$

$$\Rightarrow \qquad y = \frac{\pm(\cos 2A - \sin 2A) + 1}{\pm(\cos 2A + \sin 2A) - 1}$$

which gives us four values of $y$, say $y_1$, $y_2$, $y_3$ and $y_4$. We have

$$y_1 = \frac{\cos 2A - \sin 2A + 1}{\cos 2A + \sin 2A - 1} = \frac{(1 + \cos 2A) - \sin 2A}{(\cos 2A - 1) + \sin 2A}$$

$$= \frac{2\cos^2 A - 2\sin A\cos A}{-2\sin^2 A + 2\sin A\cos A}$$

$$= \frac{\cos A(\cos A - \sin A)}{\sin A(\cos A - \sin A)} = \cot A$$

$$y_2 = \frac{-(\cos 2A - \sin 2A) + 1}{-(\cos 2A + \sin 2A) - 1} = \frac{(1 - \cos 2A) + \sin 2A}{-(1 + \cos 2A) - \sin 2A}$$

$$= \frac{2\sin^2 A + 2\sin A\cos A}{-2\cos^2 A - 2\sin A\cos A} = -\tan A$$

$$y_3 = \frac{(\cos 2A - \sin 2A) + 1}{-(\cos 2A + \sin 2A) - 1} = \frac{(1 + \cos 2A) - \sin 2A}{-(1 + \cos 2A) - \sin 2A}$$

$$= \frac{2\cos^2 A - 2\sin A\cos A}{-2\cos^2 A - 2\sin A\cos A} = -\frac{\cos A - \sin A}{\cos A + \sin A}$$

$$= -\frac{1 - \tan A}{1 + \tan A} = -\tan\left(\frac{\pi}{4} - A\right) = -\cot\left(\frac{\pi}{4} + A\right)$$

$$y_4 = \frac{-(\cos 2A - \sin 2A) + 1}{(\cos 2A + \sin 2A) - 1} = \frac{(1 - \cos 2A) + \sin 2A}{-(1 - \cos 2A) + \sin 2A}$$

$$= \frac{2\sin^2 A + 2\sin A\cos A}{-2\sin^2 A + 2\sin A\cos A} = \frac{\cos A + \sin A}{\cos A - \sin A}$$

$$= \frac{1 + \tan A}{1 - \tan A} = \tan\left(\frac{\pi}{4} + A\right).$$

**Spoonfeeding**

If $\cos 5\theta = a \cos \theta + b \cos^3 \theta + c \cos^5 \theta + d$, then

(a) $a = 20$ (b) $b = -20$ (c) $c = 16$ (d) $d = 5$

*Ans.* (b) and (c)

***Solution*** $\cos 5\theta = \cos (4\theta + \theta) = \cos 4\theta \cos \theta - \sin 4\theta \sin \theta$

$= (2 \cos^2 2\theta - 1) \cos \theta - 2 \sin 2\theta \cos 2\theta \sin \theta$

$= [2(2 \cos^2 \theta - 1)^2 - 1] \cos \theta - 2 \cdot 2 \cos \theta \sin^2 \theta (2 \cos^2 \theta - 1)$

$= [2 (4 \cos^4 \theta - 4\cos^2 \theta + 1) - 1] \cos \theta - 4 \cos \theta (2 \cos^2 \theta - 1) (1 - \cos^2 \theta)$

$= \cos \theta (8 \cos^4 \theta - 8 \cos^2 \theta + 1) - 4 \cos \theta (3 \cos^2 \theta - 2 \cos^4 \theta - 1)$

$= \cos \theta (16 \cos^4 \theta - 20 \cos^2 \theta + 5)$

$= 16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos \theta$

clearly, $a = 5, b = -20, c = 16$ and $d = 0$. Satisfy the given relation.

**Spoonfeeding**

If $(x - a) \cos \theta + y \sin \theta = (x - a) \cos \phi + y \sin \phi = a$ and $\tan (\theta/2) - \tan (\phi/2) = 2b$, then

(a) $y^2 = 2ax - (1 - b^2)x^2$

(b) $\tan \dfrac{\theta}{2} = \dfrac{1}{x}(y + bx)$

(c) $y^2 = 2bx - (1 - a^2)x^2$

(d) $\tan \dfrac{\phi}{2} = \dfrac{1}{x}(y - bx)$.

*Ans.* (a), (b) and (d)

*Solution* Let $\tan (\theta/2) = \alpha$ and $\tan (\phi/2) = \beta$, so that $\alpha - \beta = 2b$.

Also $$\cos \theta = \frac{1 - \tan^2 (\theta/2)}{1 + \tan^2 (\theta/2)} = \frac{1 - \alpha^2}{1 + \alpha^2}$$

and $$\sin \theta = \frac{2 \tan (\theta/2)}{1 + \tan^2 (\theta/2)} = \frac{2\alpha}{1 + \alpha^2}$$

Similarly $\cos \phi = \dfrac{1 - \beta^2}{1 + \beta^2}$ and $\sin \phi = \dfrac{2\beta}{1 + \beta^2}$

Therefore, we have from the given relations

$$(x - a)\ \frac{1-\alpha^2}{1+\alpha^2} + y\left(\frac{2\alpha}{1+\alpha^2}\right) = a$$

$\Rightarrow \quad x\alpha^2 - 2y\alpha + 2a - x = 0$

Similarly $\quad x\beta^2 - 2y\beta + 2a - x = 0.$

We see that $\alpha$ and $\beta$ are the roots of the equation $xz^2 - 2yz + 2a - x = 0$, so that $\alpha + \beta = 2y/x$ and $\alpha\beta = (2a - x)/x$. Now, from $(\alpha+\beta)^2 = (\alpha-\beta)^2 + 4\alpha\beta$, we get

$$\left(\frac{2y}{x}\right)^2 = (2b)^2 + \frac{4(2a-x)}{x}$$

$\Rightarrow \quad y^2 = 2ax - (1 - b^2)x^2$

Also, from $\alpha + \beta = 2y/x$ and $\alpha - \beta = 2b$, we get

$$\alpha = y/x + b \text{ and } \beta = y/x - b$$

$\Rightarrow \quad \tan\frac{\theta}{2} = \frac{1}{x}(y + bx)$ and $\tan\frac{\phi}{2} = \frac{1}{x}(y - bx)$

**Spoonfeeding**

For $0 < \phi < \pi/2$, if

$$x = \sum_{n=0}^{\infty} \cos^{2n}\phi, \quad y = \sum_{n=0}^{\infty} \sin^{2n}\phi,$$

$$z = \sum_{n=0}^{\infty} \cos^{2n}\phi \sin^{2n}\phi,$$

then

(a) $xyz = xz + y$ (b) $xyz = xy + z$

(c) $xyz = x + y + z$ (d) $xy^2 = y^2 + x.$

*Ans.* (b) and (c)

*Solution* We have

$$x = \sum_{n=0}^{\infty} \cos^{2n} \phi = \frac{1}{1 - \cos^2 \phi} = \operatorname{cosec}^2 \phi$$

and $$y = \sum_{n=0}^{\infty} \sin^{2n} \phi = \frac{1}{1 - \sin^2 \phi} = \sec^2 \phi$$

and $$z = \sum_{n=0}^{\infty} \cos^{2n} \phi \sin^{2n} \phi$$

$$= \frac{1}{1 - \cos^2 \phi \sin^2 \phi} = \frac{1}{1 - 1/xy} = \frac{xy}{xy - 1}$$

$\Rightarrow$ $xyz - z = xy \Rightarrow xyz = xy + z$

Also $$x\,y = \frac{1}{\sin^2 \phi \cos^2 \phi} = \frac{\sin^2 \phi + \cos^2 \phi}{\sin^2 \phi \cos^2 \phi}$$

$$= \frac{1}{\cos^2 \phi} + \frac{1}{\sin^2 \phi} = x + y.$$

So that we can write $xyz = x + y + z$.

**Spoonfeeding**

If

$$x = \sqrt{a^2 \cos^2 \alpha + b^2 \sin^2 \alpha} + \sqrt{a^2 \sin^2 \alpha + b^2 \cos^2 \alpha}$$

then $x^2 = a^2 + b^2 + 2\sqrt{P(a^2 + b^2) - P^2}$ where $P$ is equal to

(a) $a^2 \cos^2 \alpha + b^2 \sin^2 \alpha$

(b) $a^2 \sin^2 \alpha + b^2 \cos^2 \alpha$.

(c) $(1/2)\ [a^2 + b^2 + (a^2 - b^2) \cos 2\alpha]$

(d) $(1/2)\ [a^2 + b^2 - (a^2 - b^2) \cos 2\alpha]$

*Ans.* (a), (b), (c) and (d)

*Solution*

$$x = \sqrt{a^2 \cos^2 \alpha + b^2 \sin^2 \alpha} + \sqrt{a^2 \sin^2 \alpha + b^2 \cos^2 \alpha}$$

$$\Rightarrow x^2 = a^2 + b^2 + 2\sqrt{(a^2 \cos^2 \alpha + b^2 \sin^2 \alpha)(a^2 \sin^2 \alpha + b^2 \cos^2 \alpha)}$$

$$= a^2 + b^2 + k, \text{ where}$$

$$k = 2\sqrt{[(a^2 + b^2) - (a^2 \sin^2 \alpha + b^2 \cos^2 \alpha)] \times (a^2 \sin^2 \alpha + b^2 \cos^2 \alpha)}$$

$$\therefore \quad x = a^2 + b^2 + 2\sqrt{(a^2 + b^2)P - P^2}$$

where $P = a^2 \sin^2 \alpha + b^2 \cos^2 \alpha$.

or $P = (a^2/2)\ (1 - \cos 2\alpha) + (b^2/2)\ (1 + \cos 2\alpha)$

$= (1/2)\ [a^2 + b^2 - (a^2 - b^2) \cos 2\alpha]$

Also

$$x^2 = a^2 + b^2 + 2\sqrt{(a^2\cos^2\alpha + b^2\sin^2\alpha)[(a^2+b^2) - (a^2\cos^2\alpha + b^2\sin^2\alpha)]}$$

$$= a^2 + b^2 + 2\sqrt{(a^2+b^2)P - P^2}$$

where

$$P = a^2\cos^2\alpha + b^2\sin^2\alpha$$

or $P = (1/2)\,[a^2 + b^2 + (a^2 - b^2)\cos 2\alpha]$

**Spoonfeeding**

For $0 < \theta < \pi/2$,
$\tan\theta + \tan 2\theta + \tan 3\theta = 0$ if

(a) $\tan\theta = 0$ (b) $\tan 2\theta = 0$

(c) $\tan 3\theta = 0$ (d) $\tan\theta\tan 2\theta = 2$.

*Ans.* (c) and (d).

*Solution* Clearly $\tan\theta \neq 0$ and $\tan 2\theta \neq 0$ for $0 < \theta < \pi/2$. We have $\tan 3\theta = \tan(2\theta + \theta)$

$$\Rightarrow \quad \tan 3\theta = \frac{\tan 2\theta + \tan\theta}{1 - \tan 2\theta\tan\theta}$$

$$\therefore \quad 0 = \tan\theta + \tan 2\theta + \tan 3\theta$$

$$= \tan 3\theta\,(1 - \tan 2\theta\tan 3\theta) + \tan 3\theta$$

$$= \tan 3\theta\,[2 - \tan 2\theta\tan\theta]$$

$\Rightarrow \quad \tan 3\theta = 0$ or $\tan 2\theta\tan\theta = 2$.

If $\sin(\alpha + \beta) = 1$ and $\sin(\alpha - \beta) = 1/2$ where $\alpha, \beta \in [0, \pi/2]$ then

(a) $\tan(\alpha + 2\beta) = -\sqrt{3}$
(b) $\tan(2\alpha + \beta) = -1/\sqrt{3}$
(c) $\tan(\alpha + 2\beta) = \sqrt{3}$
(d) $\tan(2\alpha + \beta) = 1/\sqrt{3}$

*Ans.* (a) and (b)

*Solution* From $\sin(\alpha + \beta) = 1$, we get $\alpha + \beta = \pi/2$ (because $\alpha, \beta \in [0, \pi/2]$), and from $\sin(\alpha - \beta) = 1/2$, we get $\alpha - \beta = \pi/6$. Therefore, $\alpha = \pi/3$ and $\beta = \pi/6$, so that $\alpha + 2\beta = 2\pi/3$ and $2\alpha + \beta = 5\pi/6$.

$$\Rightarrow \quad \tan(\alpha + 2\beta) = \tan 2\pi/3 = \tan(\pi - \pi/3)$$
$$= -\tan \pi/3 = -\sqrt{3}$$
and
$$\tan(2\alpha + \beta) = \tan\frac{5\pi}{6} = \tan(\pi - \pi/6)$$
$$= -\tan \pi/6 = -1/\sqrt{3}.$$

**Spoonfeeding**

If $0 < \alpha, \beta < \pi$ and $\cos\alpha + \cos\beta - \cos(\alpha + \beta) = 3/2$ then

(a) $\alpha = \pi/3$ (b) $\beta = \pi/3$
(c) $\alpha = \beta$ (d) $\alpha + \beta = \pi/3$

*Ans.* (a), (b) and (c)

*Solution* The given equation can be written as

$$2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} - \left(2\cos^2\frac{\alpha+\beta}{2} - 1\right) = \frac{3}{2}$$

$$\Rightarrow \quad 4\cos^2\frac{\alpha+\beta}{2} - 4\cos\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2} + 1 = 0$$

$$\Rightarrow \quad \left[2\cos\frac{\alpha+\beta}{2} - \cos\frac{\alpha-\beta}{2}\right]^2 + \sin^2\frac{\alpha-\beta}{2} = 0$$

$$\Rightarrow \quad \sin\frac{\alpha-\beta}{2} = 0 \text{ and } 2\cos\frac{\alpha+\beta}{2} = \cos\frac{\alpha-\beta}{2} \quad (1)$$

Also, since $0 < \alpha, \beta < \pi$, we have $\alpha = \beta$. Therefore, from (1) we get $\cos\alpha = 1/2$, so that $\alpha = \beta = \pi/3$.

**Spoonfeeding**

If $A$ and $B$ are acute angles such that $\sin A = \sin^2 B$, $2\cos^2 A = 3\cos^2 B$; then

(a) $A = \pi/6$ (b) $A = \pi/2$

(c) $B = \pi/4$ (d) $B = \pi/3$

*Ans.* (a) and (c)

*Solution* From the given conditions

$$2(1 - \sin^2 A) = 3(1 - \sin^2 B) = 3(1 - \sin A)$$

$$\Rightarrow \quad 2\sin^2 A - 3\sin A + 1 = 0$$

$$\Rightarrow \quad (2\sin A - 1)(\sin A - 1) = 0$$

$$\Rightarrow \quad \sin A = 1 \quad \text{or} \quad \sin A = 1/2$$

$$\Rightarrow \quad A = \pi/2 \text{ or } \pi/6$$

But since $A$ is acute, we have $A = \pi/6$.

$$\Rightarrow \quad \sin^2 B = \sin(\pi/6) = 1/2$$

$$\Rightarrow \quad \sin B = 1/\sqrt{2} \quad \Rightarrow \quad B = \pi/4$$

**Spoonfeeding**

If $u_n = \sin n\theta \sec^n \theta$, $v_n = \cos n\theta \sec^n \theta$, $n \neq 1$, $\theta \neq p\pi$

$n, p \in \mathbf{I}$, then $\frac{v_n - v_{n-1}}{u_{n-1}} + \frac{1}{n}\frac{u_n}{v_n} = 0$ for

(a) all values of $n$
(b) finite numbers of values of $n$
(c) infinite number of values of $n$
(d) no values of $n$

*Ans.* (d)

**Solution** We have $\frac{u_n}{v_n} = \tan n\theta$

and $\frac{v_n - v_{n-1}}{u_{n-1}} = \frac{\cos n\theta \sec^n \theta - \cos(n-1)\theta \sec^{n-1}\theta}{\sin(n-1)\theta \sec^{n-1}\theta}$

$$= \frac{\cos n\theta \sec\theta - \cos(n-1)\theta}{\sin(n-1)\theta} = \frac{\cos n\theta - \cos(n-1)\theta\cos\theta}{\cos\theta \sin(n-1)\theta}$$

$$= \frac{\cos(n-1)\theta\cos\theta - \sin(n-1)\theta\sin\theta - \cos(n-1)\theta\cos\theta}{\cos\theta\sin(n-1)\theta} = -\tan\theta$$

so that $\frac{v_n - v_{n-1}}{u_{n-1}} + \frac{1}{n}\frac{u_n}{v_n} = -\tan\theta + \frac{\tan n\theta}{n} \neq 0$, for any value of $n$ unless $\theta$ is an integral multiple of $\pi$.

**Spoonfeeding**

If $\frac{\sin x}{a} = \frac{\cos x}{b} = \frac{\tan x}{c} = k$, then $bc + \frac{1}{ck} + \frac{ak}{1+bk}$ is equal to

(a) $k\left(a + \frac{1}{a}\right)$
(b) $\frac{1}{k}\left(a + \frac{1}{a}\right)$
(c) $\frac{1}{k^2}$
(d) $\frac{a}{k}$

*Ans.* (b)

**Solution** The given expression is equal to

$$\frac{\cos x \cdot \tan x}{k^2} + \frac{1}{\tan x} + \frac{\sin x}{1 + \cos x}$$

$$= \frac{\sin x}{k^2} + \frac{\cos x(1+\cos x)+\sin^2 x}{\sin x(1+\cos x)}$$

$$= \frac{a}{k} + \frac{1}{\sin x} = \frac{a}{k} + \frac{1}{ak} = \frac{1}{k}\left(a + \frac{1}{a}\right)$$

**Spoonfeeding**

$\sin^2 \alpha + \cos^2 (\alpha + \beta) + 2 \sin \alpha \sin \beta \cos (\alpha + \beta)$ is independent of

(a) $\alpha$

(b) $\beta$

(c) both $\alpha$ and $\beta$

(d) none

*Ans.* (a)

**Solution** The given expression is equal to

$$\sin^2 \alpha + \cos (\alpha + \beta) [\cos (\alpha + \beta) + 2 \sin \alpha \sin \beta]$$
$$= \sin^2 \alpha + \cos (\alpha + \beta) [\cos \alpha \cos \beta + \sin \alpha \sin \beta]$$
$$= \sin^2 \alpha + \cos (\alpha + \beta) \cos (\alpha - \beta)$$
$$= \sin^2 \alpha + \cos^2 \alpha - \sin^2 \beta = 1 - \sin^2 \beta = \cos^2 \beta$$

which is independent of $\alpha$ only.

**Spoonfeeding**

If $\cos\alpha + \cos\beta = a$, $\sin\alpha + \sin\beta = b$ and $\theta$ is the arithmetic mean between $\alpha$ and $\beta$ then $\sin 2\theta + \cos 2\theta$ is equal to

(a) $(a+b)^2/(a^2+b^2)$ (b) $(a-b)^2/(a^2+b^2)$

(c) $(a^2-b^2)/(a^2+b^2)$ (d) none of these

*Ans.* (d)

**Solution** From the given relations we have

$$2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = a \text{ and } 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = b$$

By dividing we get $\tan\frac{\alpha+\beta}{2} = \frac{b}{a} \Rightarrow \tan\theta = \frac{b}{a}$ $\left[\because \theta = \frac{\alpha+\beta}{2}\right]$

so that $\cos 2\theta = \dfrac{1-\frac{b^2}{a^2}}{1+\frac{b^2}{a^2}} = \dfrac{a^2-b^2}{a^2+b^2}$ and $\sin 2\theta = \dfrac{2ab}{a^2+b^2}$.

$$\therefore \sin 2\theta + \cos 2\theta = \frac{a^2-b^2+2ab}{a^2+b^2}$$

**Spoonfeeding**

$\dfrac{\sin 3\alpha}{\cos 2\alpha} < 0$ if $\alpha$ lies in

(a) $(13\pi/48, 14\pi/48)$ (b) $(14\pi/48, 18\pi/48)$

(c) $(18\pi/48, 23\pi/48)$ (d) any of these intervals

*Ans.* (a)

**Solution** $\dfrac{\sin 3\alpha}{\cos 2\alpha} < 0$ if $\sin 3\alpha > 0$ and $\cos 2\alpha < 0$

or $\sin 3\alpha < 0$ and $\cos 2\alpha > 0$

i.e. if $3\alpha \in (0, \pi)$ and $2\alpha \in (\pi/2, 3\pi/2)$

or $3\alpha \in (\pi, 2\pi)$ and $2\alpha \in (-\pi/2, \pi/2)$

i.e. if $\alpha \in (0, \pi/3)$ and $\alpha \in (\pi/4, 3\pi/4)$

or $\alpha \in (\pi/3, 2\pi/3)$ and $\alpha \in (-\pi/4, \pi/4)$

i.e. if $\alpha \in (\pi/4, \pi/3)$

since $(13\pi/48, 14\pi/48) \subset (\pi/4, \pi/3)$, (a) is correct

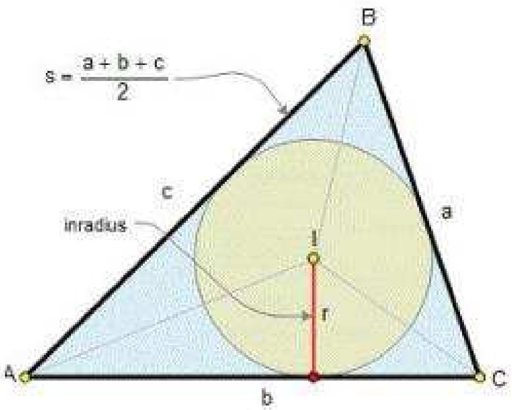
Proof for in radius r = Area / s

Proof for –

## *r = A/s*

Where r = inradius, A = area of the triangle, s = semi-perimeter of a triangle.

1. Consider an acute triangle ABC, A lower left, B lower right and C at the top.




2. Bisect each angle
3. The intersection of the bisectors is the center of the inscribed circle of the triangle with radius r
4. Let the center of this incircle be called I and the three sides a, b and c.
5. Consider the three triangles AIB, BIC and CIA
6. The areas of these triangles are (cr)/2, (ar)/2 and (br)/2 [
7. Therefore the total area of the triangle ABC is **A = (cr)/2 + (ar)/2 + (br)/2**
8. This simplifies to A = r (a + b + c)/2
9. (a + b + c) is the semi-perimeter of the triangle, s.

10. Therefore, A = rs

## Hence, r= A/s

**Spoonfeeding**

If $\tan^2 36° + k(\sin 18° + \cos 36°) = 5$, then the value of $k$ is

(a) 2 (b) $2\sqrt{5}$ (c) 4 (d) $4\sqrt{5}$

*Ans.* (c)

**Solution** From the tables, we have

$$\text{L.H.S.} = 5 - 2\sqrt{5} + K\left(\frac{\sqrt{5}-1}{4} + \frac{\sqrt{5}+1}{4}\right)$$

$$= 5 - 2\sqrt{5} + \frac{\sqrt{5}}{2}K = 5 \text{ (given)}$$

**Spoonfeeding**

If $A = 130°$ and $x = \sin A + \cos A$, then

(a) $x > 0$ (b) $x < 0$ (c) $x = 0$ (d) $x \geq 0$

*Ans.* (a)

**Solution** $x = \sin 130° + \cos 130° = \sin 50° - \sin 40° > 0$

($\because$ $\sin x$ is increasing for $0 < x < \pi/2$)

**Spoonfeeding**

If $\cos x - \sin \alpha \cot \beta \sin x = \cos \alpha$, then the value of $\tan (x/2)$ is.

(a) $-\tan (\alpha/2) \cot (\beta/2)$
(b) $\tan (\alpha/2) \tan (\beta/2)$
(c) $-\cot (\alpha/2) \tan (\beta/2)$
(d) $\cot (\alpha/2) \cot (\beta/2)$

*Ans.* (a) and (b)

*Solution* The given equation can be written as

$$\frac{1-\tan^2 (x/2)}{1+\tan^2 (x/2)} - \sin \alpha \cot \beta \frac{2\tan (x/2)}{1+\tan^2 (x/2)} = \cos \alpha$$

$$\Rightarrow \quad \tan^2 \frac{x}{2}(1 + \cos \alpha) + \sin \alpha \cot \beta \cdot 2 \tan \frac{x}{2} - (1 - \cos \alpha) = 0$$

$$\Rightarrow \quad \tan^2 \frac{x}{2} + \frac{2 \sin \alpha \cot \beta}{1+\cos \alpha} \tan \frac{x}{2} - \frac{1-\cos \alpha}{1+\cos \alpha} = 0$$

$$\Rightarrow \quad \tan^2 \frac{x}{2} + 2\tan \frac{\alpha}{2} \cot \beta \tan \frac{x}{2} - \tan^2 \frac{\alpha}{2} = 0$$

$$\Rightarrow \quad \tan^2 \frac{x}{2} + 2\tan\frac{\alpha}{2}\cdot\frac{1}{2}\left(\cot\frac{\beta}{2} - \tan\frac{\beta}{2}\right)\tan\frac{x}{2} - \tan^2\frac{\alpha}{2} = 0$$

$$\Rightarrow \quad \left(\tan\frac{x}{2} + \cot\frac{\beta}{2}\tan\frac{\alpha}{2}\right)\left(\tan\frac{x}{2} - \tan\frac{\beta}{2}\tan\frac{\alpha}{2}\right) = 0$$

$$\Rightarrow \quad \tan (x/2) = -\tan (\alpha/2) \cot (\beta/2)$$

or $\quad \tan (x/2) = \tan (\alpha/2) \tan (\beta/2)$.

**Spoonfeeding**

The value of the determinant

$$\begin{vmatrix} \sin^2 13^\circ & \sin^2 77^\circ & \tan 135^\circ \\ \sin^2 77^\circ & \tan 135^\circ & \sin^2 13^\circ \\ \tan 135^\circ & \sin^2 13^\circ & \sin^2 77^\circ \end{vmatrix} \text{ is equal to}$$

(a) $-1$ (b) $0$ (c) $1$ (d) $2$

*Ans.* (b)

**Solution** The given determinant is equal to

$$\begin{vmatrix} \sin^2 13^\circ & \cos^2 13^\circ & -1 \\ \cos^2 13^\circ & -1 & \sin^2 13^\circ \\ -1 & \sin^2 13^\circ & \cos^2 13^\circ \end{vmatrix} = \begin{vmatrix} 0 & \cos^2 13^\circ & -1 \\ 0 & -1 & \sin^2 13^\circ \\ 0 & \sin^2 13^\circ & \cos^2 13^\circ \end{vmatrix} = 0$$

(Applying $C_1 \to C_1 + C_2 + C_3$)

**Spoonfeeding**

If $\tan 25^\circ = x$, then $\dfrac{\tan 155^\circ - \tan 115^\circ}{1 + \tan 155^\circ \tan 115^\circ}$ is equal to

(a) $\dfrac{1-x^2}{2x}$ (b) $\dfrac{1+x^2}{2x}$ (c) $\dfrac{1+x^2}{1-x^2}$ (d) $\dfrac{1-x^2}{1+x^2}$

*Ans.* (a)

**Solution** $\dfrac{\tan 155^\circ - \tan 115^\circ}{1 + \tan 155^\circ \tan 115^\circ}$

$$= \frac{\tan(180^\circ - 25^\circ) - \tan(90^\circ + 25^\circ)}{1 + \tan(180^\circ - 25^\circ)\tan(90^\circ + 25^\circ)}$$

$$= \frac{-\tan 25^\circ + \cot 25^\circ}{1 + \tan 25^\circ \cot 25^\circ} = \frac{1}{2}\left(-x + \frac{1}{x}\right) = \frac{1-x^2}{2x}.$$

**Spoonfeeding**

If $\sin x + \cos y = a$ and $\cos x + \sin y = b$, then $\tan \frac{x-y}{2}$ is equal to

(a) $a + b$ (b) $a - b$ (c) $\frac{a+b}{a-b}$ (d) $\frac{a-b}{a+b}$

*Ans.* (d)

**Solution** From the given relations we have

$$\sin x + \sin((\pi/2) - y) = a \text{ and } \cos x + \cos((\pi/2) - y) = b$$

$\Rightarrow$
$$2 \sin \frac{x + (\pi/2) - y}{2} \cos \frac{x - (\pi/2) + y}{2} = a$$

and
$$2 \cos \frac{x + (\pi/2) - y}{2} \cos \frac{x - (\pi/2) + y}{2} = b$$

Dividing we get,

$$\tan\left(\frac{\pi}{4} + \frac{x-y}{2}\right) = \frac{a}{b} \Rightarrow \frac{1 + \tan \frac{x-y}{2}}{1 - \tan \frac{x-y}{2}} = \frac{a}{b}$$

or
$$\tan \frac{x-y}{2} = \frac{a-b}{a+b}.$$

**Spoonfeeding**

If $\cos\alpha = \dfrac{2\cos\beta - 1}{2 - \cos\beta}$ $(0 < \alpha, \beta < \pi)$, $\alpha + \beta = \pi$ then $\tan(\alpha/2)$ is equal to

(a) $3^{1/4}$ (b) $3^{1/2}$

(c) $3$ (d) $3^2$

*Ans.* (a)

**Solution** From the given relation we have

$$1 + \cos\alpha = 1 + \frac{2\cos\beta - 1}{2 - \cos\beta} = \frac{2 - \cos\beta + 2\cos\beta - 1}{2 - \cos\beta}$$

or $$2\cos^2\frac{\alpha}{2} = \frac{1 + \cos\beta}{2 - \cos\beta} = \frac{2\cos^2(\beta/2)}{1 + 2\sin^2(\beta/2)}$$

or $$\cos^2\frac{\alpha}{2} = \frac{\cos^2(\beta/2)}{1 + 2\sin^2(\beta/2)} \qquad (1)$$

$$\Rightarrow 1 - \cos^2\frac{\alpha}{2} = 1 - \frac{\cos^2(\beta/2)}{1 + 2\sin^2(\beta/2)} = \frac{1 + 2\sin^2(\beta/2) - \cos^2(\beta/2)}{1 + 2\sin^2(\beta/2)}$$

or $$\sin^2\frac{\alpha}{2} = \frac{3\sin^2(\beta/2)}{1 + 2\sin^2(\beta/2)} \qquad (2)$$

From (1) and (2) we get

$$\tan^2\frac{\alpha}{2} = 3\tan^2\frac{\beta}{2} \Rightarrow \frac{\tan(\alpha/2)}{\tan(\beta/2)} = \sqrt{3}$$

$$\Rightarrow \tan(\alpha/2) = \sqrt{3}\tan(\beta/2) = \sqrt{3}\cot(\alpha/2)$$

$$\Rightarrow \tan^2(\alpha/2) = \sqrt{3} \Rightarrow \tan(\alpha/2) = 3^{1/4}.$$

**Spoonfeeding**

If $\dfrac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = x$, then $\dfrac{\cos\alpha}{1+\sin\alpha}$ is equal to

(a) $1/x$ (b) $x$
(c) $1 + x$ (d) $1 - x$

*Ans.* (d)

**Solution** $\dfrac{\cos\alpha}{1+\sin\alpha} = 1 - \dfrac{1+\sin\alpha-\cos\alpha}{1+\sin\alpha}$

Now $\dfrac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} = \dfrac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}\cdot\dfrac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha}$

$$= \frac{(1+\sin\alpha)^2-\cos^2\alpha}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}$$

$$= \frac{(1+\sin\alpha)^2-(1+\sin\alpha)(1-\sin\alpha)}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}$$

$$= \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = x.$$

**Spoonfeeding**

If cos $(\theta-\alpha)$, cos $\theta$ and cos $(\theta+\alpha)$ are in harmonic progression, then cos $\theta$ sec $(\alpha/2)$ is equal to

(a) $-\sqrt{2}$ (b) $\sqrt{2}$ (c) 1/2 (d) $-1/2$

*Ans.* (a) and (b)

*Solution* Since cos $(\theta-\alpha)$, cos $\theta$ and cos $(\theta+\alpha)$ are in H.P., we have

$$\cos\theta = \frac{2\cos(\theta-\alpha)\cos(\theta+\alpha)}{\cos(\theta-\alpha)+\cos(\theta+\alpha)} = \frac{2(\cos^2\theta-\sin^2\alpha)}{2\cos\theta\cos\alpha}$$

$\Rightarrow \cos^2\theta\cos\alpha = \cos^2\theta - \sin^2\alpha$

$\Rightarrow (1-\cos\alpha)\cos^2\theta = \sin^2\alpha$

$$\Rightarrow \cos^2\theta = \frac{\sin^2\alpha}{1-\cos\alpha} = \frac{4\sin^2(\alpha/2)\cos^2(\alpha/2)}{2\sin^2(\alpha/2)}$$

$\Rightarrow \cos^2\theta = 2\cos^2(\alpha/2) \Rightarrow \cos\theta\sec(\alpha/2) = \pm\sqrt{2}$.

**Spoonfeeding**

If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -3/2$ then

(a) $\Sigma \cos \alpha = 0$ (b) $\Sigma \sin \alpha = 0$

(c) $\Sigma \cos \alpha \sin \alpha = 0$ (d) $\Sigma (\cos \alpha + \sin \alpha) = 0$

*Ans.* (a), (b) and (d)

*Solution* The given expression can be written as

$$2[\cos\beta\cos\gamma + \cos\gamma\cos\alpha + \cos\alpha\cos\beta] + 2[\sin\beta\sin\gamma + \sin\gamma\sin\alpha + \sin\alpha\sin\beta] + (\sin^2\alpha + \cos^2\alpha) + (\sin^2\beta + \cos^2\beta) + (\sin^2\gamma + \cos^2\gamma) = 0$$

$\Rightarrow$ $(\cos\alpha + \cos\beta + \cos\gamma)^2 + (\sin\alpha + \sin\beta + \sin\gamma)^2 = 0$

$\Rightarrow$ $\Sigma\cos\alpha = 0$ and $\Sigma\sin\alpha = 0$

$\Rightarrow$ $\Sigma(\cos\alpha + \sin\alpha) = 0.$

**Spoonfeeding**

If $\tan 1° \tan 2° \ldots \tan 89° = x^2 - 8$, then the value of $x$ can be

(a) $-1$ (b) $1$

(c) $-3$ (d) $3$

*Ans.* (c), (d)

*Solution* $x^2 - 8 = (\tan 1° \tan 89°)(\tan 2° \tan 88°) \ldots (\tan 44° \tan 46°) \tan 45°$

$$= (\tan 1° \cot 1°)(\tan 2° \cot 2°) \ldots (\tan 44° \cot 44°) \tan 45°$$

$$= 1 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3.$$

**Spoonfeeding**

Which of the following statements are correct

(a) $\sin 1 > \sin 1°$ (b) $\tan 2 < 0$

(c) $\tan 1 > \tan 2$ (d) $\tan 2 < \tan 1 < 0$

*Ans.* (a), (b), (c)

*Solution* Since 1 radian lies between 57° and 58° and $\sin 57° > \sin 1°$, so $\sin 1 > \sin 1°$. Again 1 radian is an acute angle and 2 radian is an obtuse angle, $\tan 1 > 0$, $\tan 2 < 0$, so that $\tan 1 > \tan 2$.

**Spoonfeeding**

If $\sin \alpha + \sin \beta = l$, $\cos \alpha + \cos \beta = m$ and $\tan (\alpha/2) \tan (\beta/2) = n (\neq 1)$, then

(a) $\cos (\alpha - \beta) = \dfrac{l^2 + m^2 - 2}{2}$

(b) $\cos (\alpha + \beta) = \dfrac{m^2 - l^2}{m^2 + l^2}$

(c) $\dfrac{1+n}{1-n} = \dfrac{l^2 + m^2}{2n}$

(d) $\alpha + \beta = \pi/2$ if $l = m$

*Ans.* (a), (b), (c), (d)

*Solution* $l^2 = \sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta$ and

$m^2 = \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta$

$\Rightarrow \quad 2 \cos (\alpha - \beta) = l^2 + m^2 - 2$ (by adding)

and $\cos 2\alpha + \cos 2\beta + 2 \cos (\alpha + \beta) = m^2 - l^2$

(by subtracting)

$\Rightarrow \quad 2 \cos (\alpha + \beta) \cos (\alpha - \beta) + 2 \cos (\alpha + \beta) = m^2 - l^2$

$$\Rightarrow \quad \cos (\alpha + \beta) = \frac{m^2 - l^2}{m^2 + l^2}$$

Next, $$\frac{1+n}{1-n} = \frac{\cos[(\alpha-\beta)/2]}{\cos[(\alpha+\beta)/2]} = \sqrt{\frac{1+\cos(\alpha-\beta)}{1+\cos(\alpha+\beta)}}$$
$$= \frac{l^2+m^2}{2m}.$$

**Spoonfeeding**

If $x = a\cos^3\theta\sin^2\theta$, $y = a\sin^3\theta\cos^2\theta$ and

$\frac{(x^2+y^2)^p}{(xy)^q}$ $(p, q \in \mathbf{N})$ is independent of $\theta$, then

(a) $p = 4$ (b) $p = 5$
(c) $q = 4$ (d) $q = 5$

*Ans.* (b), (c)

*Solution* $x^2 + y^2 = a^2\sin^4\theta\cos^4\theta$
$xy = a^2\sin^5\theta\cos^5\theta$

$$\therefore \quad \frac{(x^2+y^2)^p}{(xy)^q} = \frac{a^{2p}(\sin\theta\cos\theta)^{4p}}{a^{2q}(\sin\theta\cos\theta)^{5q}}$$

which is independent of $\theta$ if $4p = 5q$
i.e. $p = 5, q = 4$.

**Spoonfeeding**

If $\tan\theta + \tan\phi = a$, $\cot\theta + \cot\phi = b$, $\theta - \phi = \alpha \neq 0$ then

(a) $ab > 4$ (b) $ab = 4$

(c) $\tan^2\alpha = \dfrac{ab(ab-4)}{(a+b)^2}$ (d) $\cot^2\alpha = \dfrac{ab(ab+4)}{(a-b)^2}$

*Ans.* (a), (c)

*Solution* $\dfrac{a}{b} = \tan\theta\tan\phi$

$$\tan^2\alpha = \tan^2(\theta - \phi) = \left[\frac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi}\right]^2$$

$$= \frac{a^2 - 4(a/b)}{[1 + (a/b)]^2}$$

$$= \frac{ab(ab-4)}{(a+b)^2}$$

as $\tan^2\alpha > 0$, $ab > 4$.

**Spoonfeeding**

If $\dfrac{\sin^4\alpha}{a} + \dfrac{\cos^4\alpha}{b} = \dfrac{1}{a+b}$, then

(a) $\dfrac{\sin^4\alpha}{a^2} = \dfrac{\cos^4\alpha}{b^2}$ (b) $\dfrac{\sin^4\alpha}{b^2} = \dfrac{\cos^4\alpha}{a^2}$

(c) $\dfrac{\sin^8\alpha}{a^3} + \dfrac{\cos^8\alpha}{b^3} = \dfrac{1}{(a+b)^3}$

(d) $\sin^4\alpha = \dfrac{a^2}{(a+b)^2}$

***Ans.*** (a), (c), (d)

***Solution*** We are given that

$$(a+b)\left(\frac{\sin^4\alpha}{a}+\frac{\cos^4\alpha}{b}\right)=1$$

$$\Rightarrow \quad \sin^4\alpha+\cos^4\alpha+\frac{b}{a}\sin^4\alpha+\frac{a}{b}\cos^4\alpha=1$$
$$=(\sin^2\alpha+\cos^2\alpha)^2$$

$$\Rightarrow \quad \frac{b}{a}\sin^4\alpha-2\sin^2\alpha\cos^2\alpha+\frac{a}{b}\cos^4\alpha=0$$

$$\Rightarrow \quad \left(\sqrt{\frac{b}{a}}\sin^2\alpha-\sqrt{\frac{a}{b}}\cos^2\alpha\right)^2=0$$

$$\Rightarrow \quad \frac{b}{a}\sin^4\alpha=\frac{a}{b}\cos^4\alpha$$

$$\Rightarrow \quad \frac{\sin^4\alpha}{a^2}=\frac{\cos^4\alpha}{b^2}=k \text{ say}$$

Since $\dfrac{\sin^4\alpha}{a}+\dfrac{\cos^4\alpha}{b}=\dfrac{1}{a+b}$, we get

$$ak+bk=\frac{1}{a+b} \quad \Rightarrow \quad k=\frac{1}{(a+b)^2}$$

$$\therefore \quad \frac{\sin^8\alpha}{a^3}+\frac{\cos^8\alpha}{b^3}=\frac{(a^2k)^2}{a^3}+\frac{(b^2k)^2}{b^3}$$
$$=ak^2+bk^2=(a+b)k^2$$
$$=(a+b)\cdot\frac{1}{(a+b)^4}=\frac{1}{(a+b)^3}$$

**Spoonfeeding**

If $P_n = \cos^n \theta + \sin^n \theta$, then

(a) $2P_6 - 3P_4 = -1$

(b) $2P_6 - 3P_4 = 1$

(c) $6P_{10} - 15P_8 + 10P_6 = 0$

(d) $6P_{10} - 15P_8 + 10P_6 = 1$

*Ans.* (a), (d)

*Solution* $2P_6 - 3P_4 + 1$

$$= 2(\cos^6 \theta + \sin^6 \theta) - 3(\cos^4 \theta + \sin^4 \theta) + 1$$

$$= 2[(\cos^2 \theta + \sin^2 \theta)^3 - 3 \sin^2 \theta \cos^2 \theta (\cos^2 \theta + \sin^2 \theta)]$$

$$- 3[(\cos^2 \theta + \sin^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta] + 1$$

$$= 2(1 - 3 \sin^2 \theta \cos^2 \theta) - 3(1 - 2 \sin^2 \theta \cos^2 \theta) + 1 = 0$$

Again for $n \geq 4$, we have

$$P_n - P_{n-2}$$

$= \cos^n \theta + \sin^n \theta - (\cos^{n-2} \theta + \sin^{n-2} \theta)$

$= \cos^{n-2} \theta (\cos^2 \theta - 1) + \sin^{n-2} \theta (\sin^2 \theta - 1)$

$= - \sin^2 \theta \cos^{n-2} \theta - \cos^2 \theta \sin^{n-2} \theta$

$= - \sin^2 \theta \cos^2 \theta (\cos^{n-4} \theta + \sin^{n-4} \theta)$

$= - \sin^2 \theta \cos^2 \theta P_{n-4}$

$6P_{10} - 15P_8 + 10P_6 - 1$

$= 6(P_{10} - P_8) - 9(P_8 - P_6) + (P_6 - P_4) + P_4 - P_2$

$= - \sin^2 \theta \cos^2 \theta (6P_6 - 9P_4 + P_2 + P_0)$

$= - 3 \sin^2 \theta \cos^2 \theta (2P_6 - 3P_4) - \sin^2 \theta \cos^2 \theta (1 + 2)$

$[\because \quad P_2 = 1, P_0 = 2]$

$= - 3 \sin^2 \theta \cos^2 \theta (- 1) - 3 \sin^2 \theta \cos^2 \theta = 0.$

$[\because 2P_6 - 3P_4 + 1 = 0$ (as proved)$]$

**Spoonfeeding**

The equation $\sin^4 x + \cos^4 x = a$ has a real solution for

(a) all values of $a$ (b) $a = 1/2$

(c) $a = 7/10$ (d) $a = 1$

*Ans.* (b), (c), (d)

*Solution* We have $\sin^4 x + \cos^4 x \le \sin^2 x + \cos^2 x$, as $|\sin x| \le 1$ and $|\cos x| \le 1$

$\Rightarrow \quad a \le 1 \qquad (1)$

Next, $\sin^4 x + \cos^4 x = a$

$\Rightarrow \quad (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x = a$

$\Rightarrow \quad \frac{1}{2}\sin^2 2x = 1 - a$

$\Rightarrow \quad 1 - a \le 1/2 \qquad [\because \quad \sin^2 2x \le 1]$

$\Rightarrow \quad 1/2 \le a \qquad (2)$

From (1) and (2) we get $1/2 \le a \le 1$. Note that $a = 1/2$ for $x = \pi/4$ and $a = 1$ for $x = \pi/2$.

**Spoonfeeding**

If $\sin\theta(1 + \sin\theta) + \cos\theta(1 + \cos\theta) = x$ and $\sin\theta(1 - \sin\theta) + \cos\theta(1 - \cos\theta) = y$ then

(a) $x^2 - 2x = \sin 2\theta$ (b) $y^2 + 2y = \sin 2\theta$

(c) $xy = \sin 2\theta$ (d) $x - y = 2$

*Ans.* (a), (b), (c), (d)

*Solution* $\sin\theta + \cos\theta = x - 1 = y + 1$

$$\sin 2\theta = (\sin\theta + \cos\theta)^2 - 1 = x^2 - 2x = y^2 + 2y$$

$$xy = (\sin\theta + \cos\theta)^2 - 1 = \sin 2\theta$$

$$x - y = 2$$

**Spoonfeeding**

For a positive integer $n$, let
$f_n(\theta) = \tan(\theta/2)(1 + \sec\theta)(1 + \sec 2\theta) \ldots (1 + \sec 2^n\theta)$ then

(a) $f_2(\pi/16) = 1$ (b) $f_3(\pi/32) = 1$
(c) $f_4(\pi/64) = 1$ (d) $f_5(\pi/128) = 1$

*Ans.* (a), (b), (c), (d)

*Solution* $f_n(\theta) = \dfrac{\sin(\theta/2)}{\cos(\theta/2)} \times \dfrac{1 + \cos\theta}{\cos\theta}(1 + \sec 2\theta) \ldots (1 + \sec 2^n\theta)$

$$= \frac{\sin(\theta/2) \times 2\cos^2(\theta/2)}{\cos(\theta/2)\cos\theta}(1 + \sec 2\theta) \ldots (1 + \sec 2^n\theta)$$

$$= \tan\theta(1 + \sec 2\theta)(1 + \sec 4\theta) \ldots (1 + \sec 2^n\theta)$$

$$= \tan 2^n\theta$$

$\Rightarrow$ $f_2(\pi/16) = f_3(\pi/32) = f_4(\pi/64) = f_5(\pi/128) = \tan\pi/4.$

**Spoonfeeding**

If $\sec A \tan B + \tan A \sec B = 91$, then the value of $(\sec A \sec B + \tan A \tan B)^2$ is equal to

*Ans*. 8282

*Solution* $(\sec A \sec B + \tan A \tan B)^2 - (\sec A \tan B + \tan A \sec B)^2$

$$= \left[\frac{1+\sin A \sin B}{\cos A \cos B}\right]^2 - \left[\frac{\sin B + \sin A}{\cos A \cos B}\right]^2$$

$$= \frac{1+\sin^2 A \sin^2 B - \sin^2 B - \sin^2 A}{\cos^2 A \cos^2 B}$$

$$= \frac{1-\sin^2 B \cos^2 A - \sin^2 A}{\cos^2 A \cos^2 B}$$

$$= \frac{\cos^2 A \cos^2 B}{\cos^2 A \cos^2 B} = 1$$

$\Rightarrow \quad (\sec A \sec B + \tan A \tan B)^2 = (91)^2 + 1 = 8282.$

**Spoonfeeding**

If $\dfrac{9x}{\cos\theta} + \dfrac{5y}{\sin\theta} = 56$ and $\dfrac{9x\sin\theta}{\cos^2\theta} - \dfrac{5y\cos\theta}{\sin^2\theta} = 0$ then the value of $[(9x)^{2/3} + (5y)^{2/3}]^3$ is

*Ans.* 3136

*Solution* From the second relation $9x\sin^3\theta = 5y\cos^3\theta$.

$$\Rightarrow \quad \frac{\cos^3\theta}{9x} = \frac{\sin^3\theta}{5y} = k^3 \text{ (say)}$$

$\Rightarrow \quad \cos\theta = k(9x)^{1/3}$ and $\sin\theta = k(5y)^{1/3}$

Squaring and adding, we get

$$1 = \cos^2\theta + \sin^2\theta = k^2[(9x)^{2/3} + (5y)^{2/3}]$$

and $\dfrac{9x}{k(9x)^{1/3}} + \dfrac{5y}{k(5y)^{1/3}} = 56$ (From Ist relation)

$$\Rightarrow \quad (9x)^{2/3} + (5y)^{2/3} = 56k$$

$$\Rightarrow \quad [(9x)^{2/3} + (5y)^{2/3}]^2 = (56)^2k^2 = \frac{(56)^2}{(9x)^{2/3} + (5y)^{2/3}}$$

$$\Rightarrow \quad [(9x)^{2/3} + (5y)^{2/3}]^3 = (56)^2 = 3136.$$

**Spoonfeeding**

If $(25)^2 + a^2 + 50\, a \cos\theta = (31)^2 + b^2 + 62\, b \cos\theta = 1$ and $775 + ab + (31a + 25b)\cos\theta = 0$, then the value of $\operatorname{cosec}^2\theta$ is

*Ans.* 1586

*Solution* We can write $(a + 25\cos\theta)^2 + (25)^2 - (25\cos\theta)^2 = 1$ and

$\Rightarrow\ (a + 25\cos\theta)^2 = 1 - (25\sin\theta)^2$

similarly $(b + 31\cos\theta)^2 = 1 - (31\sin\theta)^2$

Multiplying we get

$[(a + 25\cos\theta)(b + 31\cos\theta)]^2 = [1 - (25\sin\theta)^2][1 - (31\sin\theta)^2]$

$$\Rightarrow\ [ab + (31a + 25b)\cos\theta + 775\cos^2\theta]^2 = 1 - (625 + 961)\sin^2\theta + (775\sin^2\theta)^2$$

$\Rightarrow\ (-775 + 775\cos^2\theta)^2 = 1 - 1586\sin^2\theta + (775\sin^2\theta)^2$

$\Rightarrow\ \operatorname{cosec}^2\theta = 1586.$

**Spoonfeeding**

The angle $A$ of the $\Delta ABC$ is obtuse. $x = 2635 - \tan B \tan C$, if $[x]$ denotes the greatest integer function, the value of $[x]$ is

*Ans.* 2634

*Solution* $A > \pi/2 \ \Rightarrow\ B + C < \pi/2$

$$\Rightarrow\ \tan(B + C) > 0 \ \Rightarrow\ \frac{\tan B + \tan C}{1 - \tan B \tan C} > 0$$

$\Rightarrow\ \tan B \tan C < 1$ as $\tan B > 0$, $\tan C > 0$

$\Rightarrow\ [x] = 2635 - 1 = 2634.$

**Spoonfeeding**

If $x + 270\left[\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}\right] =$ 3746 then the value of $x$ is

*Ans.* 3881

*Solution* $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$

$$= \frac{1}{2\sin\frac{2\pi}{7}} \times \left[\sin\frac{4\pi}{7} + \sin\frac{6\pi}{7} - \sin\frac{2\pi}{7} + \sin\frac{8\pi}{7} - \sin\frac{4\pi}{7}\right]$$

$$= \frac{1}{2\sin\frac{2\pi}{7}}\left[-\sin\frac{2\pi}{7}\right] \qquad \left[\therefore\ \sin\frac{6\pi}{7} = -\sin\frac{8\pi}{7}\right]$$

$$= -1/2 \Rightarrow x = 3746 + 135 = 3881.$$

**Spoonfeeding**

If $\alpha + \beta = \gamma$ and $\tan\gamma = 22$, $a$ is the arithmetic and $b$ is the geometric mean respectively between $\tan\alpha$ and $\tan\beta$, then the value of $\frac{a^3}{(1-b^2)^3}$ is equal to

*Ans.* 1331

*Solution* $\tan\gamma = \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$$\Rightarrow \qquad 22 = \frac{2a}{1-b^2}$$

$$\Rightarrow \qquad \frac{a^3}{(1-b^2)^3} = 11^3 = 1331.$$

**Spoonfeeding**

If $\dfrac{1}{\sin 20^\circ} + \dfrac{1}{\sqrt{3}\cos 20^\circ} = 2k\cos 40^\circ$,

then $18k^4 + 162k^2 + 369$ is equal to

*Ans.* 1745

*Solution* $2k\cos 40^\circ = \dfrac{1}{\sin 20^\circ} + \dfrac{1}{\sqrt{3}\cos 20^\circ}$

$$= \frac{\sqrt{3}\cos 20^\circ + \sin 20^\circ}{\sqrt{3}\sin 20^\circ \cos 20^\circ}$$

$$= \frac{\frac{\sqrt{3}}{2}\cos 20^\circ + \frac{1}{2}\sin 20^\circ}{\frac{\sqrt{3}}{4}\sin 40^\circ}$$

$$= \frac{\sin 60^\circ \cos 20^\circ + \cos 60^\circ \sin 20^\circ}{\left(\sqrt{3}/4\right)\sin 40^\circ}$$

$$= \left(4/\sqrt{3}\right)2\cos 40^\circ$$

$\Rightarrow$ $3k^2 = 16$

so $18k^4 + 162k^2 + 369 = 1745$.

**Spoonfeeding**

If $498\,[16\cos x + 12\sin x] = 2k + 60$,

then the maximum value of $k$ is

*Ans.* 4950

*Solution* $16\cos x + 12\sin x = \sqrt{16^2 + 12^2}\ \cos(x - \alpha)$, $\alpha = \tan^{-1}\left(\dfrac{3}{4}\right)$.

$\Rightarrow$ $|2k + 60| \le 498 \times 20$ as $|\cos(x - \alpha)| \le 1$

$\Rightarrow$ $k \le 4950$.

**Spoonfeeding**

Value of $6736 \cos^2 18° + 421 \tan^2 36°$ is

*Ans.* 6315

*Solution* $421 (16 \cos^2 18° + \tan^2 36°)$

$= 421 \left[10 + 2\sqrt{5} + 5 - 2\sqrt{5}\right] = 421 \times 15 = 6315.$

**Spoonfeeding**

If $A + B + C = 180°$,

$$\frac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C} = k \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

then the value of $3k^3 + 2k^2 + k + 1$ is equal to

*Ans.* 1673

*Solution* From conditional identities we have

$$\frac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C}$$

$$= \frac{4 \sin A \sin B \sin C}{4 \cos(A/2) \cos(B/2) \cos(C/2)}$$

$$= 8 \sin (A/2) \sin (B/2) \sin (C/2)$$

$\Rightarrow$ $k = 8$

and $3k^3 + 2k^2 + k + 1 = 1536 + 128 + 8 + 1 = 1673.$

**Spoonfeeding**

$\tan 70° = 2\tan 50° + \tan 20°$;

***Solution:***

$$\tan 70° = \tan(50° + 20°) \quad \therefore \tan 70° = \frac{\tan 50° + \tan 20°}{1 - \tan 50° \tan 20°}$$

$$\Rightarrow \quad \tan 70° - \tan 70° \tan 50° \tan 20° = \tan 50° + \tan 20°$$

$$\Rightarrow \quad \tan 70° - \frac{1}{\tan 20°} \tan 50° \tan 20° = \tan 50° + \tan 20°$$

$$\left[\because \tan 70° = \tan(90° - 20°) = \cot 20° = \frac{1}{\tan 20°}\right]$$

$$\Rightarrow \quad \tan 70° - \tan 50° = \tan 50° + \tan 20° \Rightarrow \tan 70° = 2\tan 50° + \tan 20°$$

**Spoonfeeding**

*Prove that* $\tan 3A - \tan 2A - \tan A = \tan 3A \tan 2A \tan A$

$$\tan 3A = \tan(2A + A) = \frac{\tan 2A + \tan A}{1 - \tan 2A \tan A}$$

$$\Rightarrow \quad \tan 3A[1 - \tan 2A \tan A] = \tan 2A + \tan A$$

$$\Rightarrow \quad \tan 3A - \tan 3A \tan 2A \tan A = \tan 2A + \tan A$$

$$\Rightarrow \quad \tan 3A - \tan 2A - \tan A = \tan 3A \tan 2A \tan A$$

**Spoonfeeding**

*Prove that* $\dfrac{\tan(45^\circ + x)}{\tan(45^\circ - x)} = \left(\dfrac{1+\tan x}{1-\tan x}\right)^2$

$$\tan(45^\circ + x) = \frac{\tan 45^\circ + \tan x}{1 - \tan 45^\circ \tan x} = \frac{1+\tan x}{1-\tan x}$$

$$\tan(45^\circ - x) = \frac{\tan 45^\circ + \tan x}{1 - \tan 45^\circ \tan x} = \frac{1-\tan x}{1+\tan x}$$

$$\text{L.H.S.} = \frac{\tan(45^\circ + x)}{\tan(45^\circ - x)} = \frac{\left(\dfrac{1+\tan x}{1-\tan x}\right)}{\left(\dfrac{1-\tan x}{1+\tan x}\right)} = \left(\frac{1+\tan x}{1-\tan x}\right)^2$$

**Spoonfeeding**

*Prove that* $\dfrac{\tan\left(\dfrac{\pi}{4}+A\right)+\tan\left(\dfrac{\pi}{4}-A\right)}{\tan\left(\dfrac{\pi}{4}+A\right)-\tan\left(\dfrac{\pi}{4}-A\right)} = \operatorname{cosec} 2A$

Let $\dfrac{\pi}{4} + A = \alpha$ and $\dfrac{\pi}{4} - A = \beta$

$$\text{L.H.S.} = \frac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta} = \frac{\dfrac{\sin\alpha}{\cos\alpha} + \dfrac{\sin\beta}{\cos\beta}}{\dfrac{\sin\alpha}{\cos\alpha} - \dfrac{\sin\beta}{\cos\beta}}$$

(Multiplying num. and denom. by $\cos\alpha\cos\beta$)

$$= \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\cos\beta - \cos\alpha\sin\beta} = \frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{\sin\left(\dfrac{\pi}{4} + A + \dfrac{\pi}{4} - A\right)}{\sin\left(\dfrac{\pi}{4} + A - \dfrac{\pi}{4} + A\right)}$$

$$= \frac{\sin \frac{\pi}{2}}{\sin 2A} = \frac{1}{\sin 2A} = \operatorname{cosec} 2A = \text{R.H.S}$$

**Spoonfeeding**

*Prove that* $\dfrac{\tan(A+B)}{\cot(A-B)} = \dfrac{\sin^2 A - \sin^2 B}{\cos^2 A - \sin^2 B}$

$$\text{R.H.S.} = \frac{\sin^2 A - \sin^2 B}{\cos^2 A - \sin^2 B} = \frac{\sin(A+B)\sin(A-B)}{\cos(A+B)\cos(A-B)}$$

$$= \frac{\sin(A+B)}{\cos(A+B)} \cdot \frac{\sin(A-B)}{\cos(A-B)} = \tan(A+B) \cdot \tan(A-B)$$

$$= \tan(A+B) \cdot \frac{1}{\cot(A-B)} = \frac{\tan(A+B)}{\cot(A-B)} = \text{L.H.S.}$$

**Spoonfeeding**

*If* $\tan\beta = \dfrac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}$; *prove that* $\tan(\alpha-\beta) = (1-n)\tan\alpha$.

***Solution:*** L.H.S. $= \tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$

Putting $\tan\beta = \dfrac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}$, we have

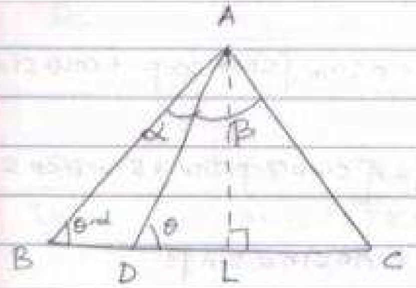
$$\text{L.H.S.} = \frac{\dfrac{\sin\alpha}{\cos\alpha} - \dfrac{m\sin\alpha\cos\alpha}{1-n\sin^2\alpha}}{1+\dfrac{\sin\alpha}{\cos\alpha}\cdot\dfrac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}} \qquad \left[\because \tan\alpha = \frac{\sin\alpha}{\cos\alpha}\right]$$

$$= \frac{\sin\alpha(1-n\sin^2\alpha) - n\sin\alpha\cos^2\alpha}{\cos\alpha(1-n\sin^2\alpha)+n\sin^2\alpha\cos\alpha} = \frac{\sin\alpha - n\sin^3\alpha - n\sin\alpha\cos^2\alpha}{\cos\alpha - n\sin^2\alpha + n\sin^2\alpha\cos\alpha}$$

$$= \frac{\sin\alpha - n\sin\alpha(\sin^2 a + \cos^2\alpha)}{\cos\alpha} = \frac{\sin\alpha - n\sin\alpha}{\cos\alpha} \qquad [\because \sin^2\alpha + \cos^2\alpha = 1]$$

$$= \frac{\sin\alpha(1-n)}{\cos\alpha} = (1-n)\tan\alpha$$

**Spoonfeeding Apollonius and Mollweide's Formula for Solving a few Trigonometry Problems**




In a $\Delta ABC$ if D is any point on the base BC such that

$BD:DC = m:n$ $\angle BAD = \alpha$, $\angle CAD = \beta$

$\angle CDA = \theta$ & $AD = x$.

Then

$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$

& $(m+n)\cot\theta = n\cot B - m\cot C$

$\frac{BD}{DC} = \frac{m}{n} \Rightarrow n(BD) = m(DC)$

In $\Delta$ ABD

$\frac{BD}{\sin\alpha} = \frac{AD}{\sin(\theta-\alpha)} \Rightarrow \frac{BD}{AD} = \frac{\sin\alpha}{\sin(\theta-\alpha)}$

In $\Delta$ ACD

$\frac{CD}{\sin\beta} = \frac{AD}{\sin(\pi-(\theta+\beta))} \Rightarrow \frac{CD}{\sin\beta} = \frac{AD}{\sin(\theta+\beta)}$

$\Rightarrow \frac{CD}{AD} = \frac{\sin\beta}{\sin(\theta+\beta)} \Rightarrow \frac{AD}{CD} = \frac{\sin(\theta+\beta)}{\sin\beta}$

$$\frac{m}{n} = \frac{BD}{DC} \Rightarrow \frac{BD}{AD} \times \frac{AD}{CD}$$

$$\frac{m}{n} = \frac{\sin\alpha}{\sin(\theta-\alpha)} \times \frac{\sin(\theta+\beta)}{\sin\beta}$$

$$m \sin\beta \sin(\theta-\alpha) = n \sin\alpha \sin(\theta+\beta)$$

$$m \sin\beta [\sin\theta\cos\alpha - \cos\theta\sin\alpha] = n\sin\alpha [\sin\theta\cos\beta + \cos\theta\sin\beta]$$

$$m[\sin\beta\sin\theta\cos\alpha - \sin\beta\cos\theta\sin\alpha] = n[\sin\theta\cos\beta\sin\alpha + \sin\alpha\cos\theta\sin\beta]$$

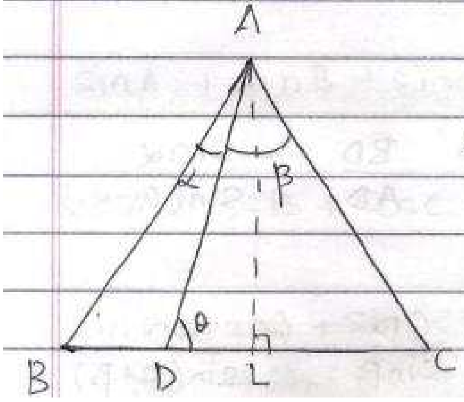
Dividing both sides by $\sin\theta\sin\alpha\sin\beta$

$$m\left[\frac{\sin\beta\sin\theta\cos\alpha}{\sin\theta\sin\alpha\sin\beta} - \frac{\sin\beta\cos\theta\sin\alpha}{\sin\theta\sin\alpha\sin\beta}\right] = n\left[\frac{\sin\theta\sin\alpha\cos\beta}{\sin\theta\sin\alpha\sin\beta} + \frac{\sin\alpha\cos\theta\sin\beta}{\sin\theta\sin\alpha\sin\beta}\right]$$

$$\Rightarrow m[\cot\alpha - \cot\theta] = n[\cot\beta + \cot\theta]$$

$$m\cot\alpha - m\cot\theta = n\cot\beta + n\cot\theta.$$

$$(m+n)\cot\theta = m\cot\alpha - n\cot\beta.$$




let AL be ⊥ on BC

then

$$\cot B = \frac{BL}{AL} = \frac{BD + DL}{AL} \Rightarrow \frac{BD}{AL} + \cot\theta$$

$$\frac{BD}{AL} = \cot B - \cot\theta$$

$$\cot C = \frac{CL}{AL} = \frac{CD - DL}{AL} = \frac{CD}{AL} - \cot\theta$$

$$\frac{CD}{AL} = \cot C + \cot\theta$$

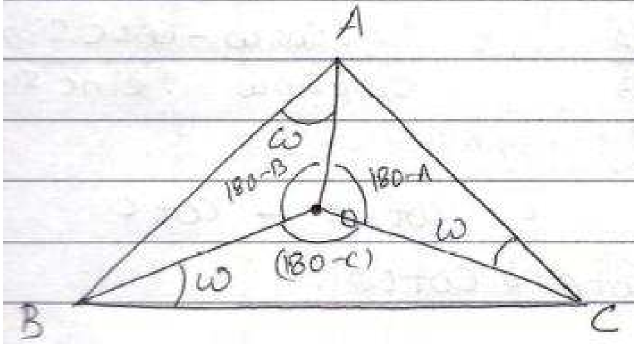
$$\frac{BD}{DC} = \frac{\frac{BD}{AL}}{\frac{CD}{AL}}$$

$$\frac{BD}{DC} = \frac{\cot B - \cot\theta}{\cot C + \cot\theta} = \frac{m}{n}$$

$m(\cot C + \cot\theta) = n(\cot B - \cot\theta)$

$\Rightarrow m\cot\theta + m\cot C = n\cot B - n\cot\theta$

$\Rightarrow (m+n)\cot\theta = n\cot B - m\cot C.$




O is a point inside $\Delta ABC$

$\angle OAB = \angle OBC = \angle OCA = \omega$

Then $\cot\omega = \cot A + \cot B + \cot C.$

$\angle OCB = \angle C - \omega$

$\angle BOC = 180 - [\omega - (C-\omega)] = 180 - C$

In $\Delta OAB$.

$\dfrac{OB}{\sin\omega} = \dfrac{AB}{\sin(180-B)} \Rightarrow \dfrac{OB}{\sin\omega} = \dfrac{c}{\sin B}$

$OB = \dfrac{c\sin\omega}{\sin B}.$ —(1)

In $\Delta OBC$.

$$\frac{OB}{\sin(C-\omega)} = \frac{BC}{\sin(180-C)} \Rightarrow OB = \frac{a\sin(C-\omega)}{\sin C} \quad \text{—(2}$$

From (i) & (2)

$$\frac{c\sin\omega}{\sin B} = \frac{a\sin(C-\omega)}{\sin C}$$

$c = k\sin C$ & $a = k\sin A$

$$\frac{k\sin C\sin\omega}{\sin B} = \frac{k\sin A(C-\omega)}{\sin C}$$

$$\sin^2 C\sin\omega = \sin A\sin B\sin(C-\omega)$$

$$\sin C\sin(A+B)\sin\omega = \sin A\sin B\sin(C-\omega)$$

$$\frac{\sin(A+B)}{\sin A\sin B} \Rightarrow \frac{\sin(C-\omega)}{\sin C\sin\omega}$$

$$\frac{\sin A\cos B}{\sin A\sin B} + \frac{\cos A\sin B}{\sin A\sin B} = \frac{\sin C\cos\omega}{\sin C\sin\omega} - \frac{\cos C\sin\omega}{\sin C\sin\omega}$$

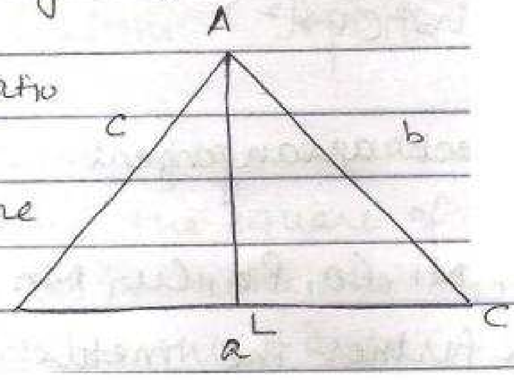
$$\cot B + \cot A = \cot\omega - \cot C$$

$$\Rightarrow \cot A + \cot B + \cot C = \cot\omega .$$

Trick of manipulating to get a desired expression

In a Δ of base a, the ratio of other 2 sides is $r<1$, show that the altitude of the Δ is less than or equal to:

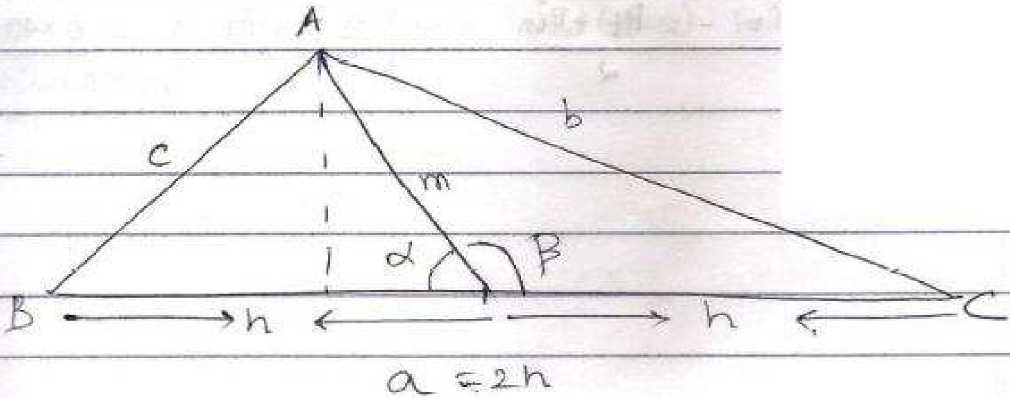
$$\frac{ar}{1-r^2}$$




Let AL be the altitude, In Δ ABL

Theorem of the Medians Apollonius Theorem.

In every Δ, the sum of the squares of any two sides is equal to twice the square of half the third side together with twice the square of the median that bisects the third side.



In $\Delta ABC$ $b^2+c^2 = 2(h^2+m^2)$.

Mollweide's Formula.

$$\frac{a+b}{c} = \frac{\cos\frac{1}{2}(A-B)}{\sin\frac{1}{2}C}, \quad \frac{a-b}{c} = \frac{\sin\frac{(A-B)}{2}}{\cos\frac{C}{2}}.$$

A beam whose centre of gravity divides it into two portions $a$ and $b$, is placed inside a smooth horizontal sphere. If $\theta$ be its inclination to the horizontal in the position of equilibrium and $2\alpha$ be the angle subtended the by beam at the centre of the sphere, then

(a) $\tan\theta = (b-a)(b+a)\tan\alpha$

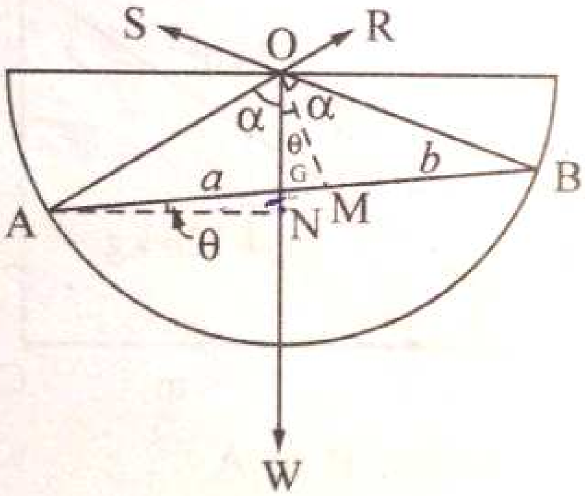
(b) $\tan\theta = \dfrac{(b-a)}{(b+a)}\tan\alpha$

(c) $\tan\theta = \dfrac{(b+a)}{(b-a)}\tan\alpha$

(d) $\tan\theta = \dfrac{1}{(b-a)(b+a)}$.

**(b)** Applying m-n theorem in $\Delta ABO$, we get

$(AG + GB) \cot \angle OGB = GB \cot \angle OAB - AG \cot \angle OBG$




$$\Rightarrow (a+b)\cot\left(\frac{\pi}{2}-\theta\right) = b\cot\left(\frac{\pi}{2}-\alpha\right) - a\cot\left(\frac{\pi}{2}-\alpha\right)$$

$$\Rightarrow (a+b)\tan\theta = b\tan\alpha - a\tan\alpha$$

$$\Rightarrow \tan\theta = \frac{(b-a)}{(b+a)}\tan\alpha.$$

:-{D

## To recall standard integrals

| $f(x)$ | $\int f(x)dx$ | $f(x)$ | $\int f(x)dx$ |
|---|---|---|---|
| $x^n$ | $\frac{x^{n+1}}{n+1} \quad (n \neq -1)$ | $[g(x)]^n g'(x)$ | $\frac{[g(x)]^{n+1}}{n+1} \quad (n \neq -1)$ |
| $\frac{1}{x}$ | $\ln\|x\|$ | $\frac{g'(x)}{g(x)}$ | $\ln\|g(x)\|$ |
| $e^x$ | $e^x$ | $a^x$ | $\frac{a^x}{\ln a} \quad (a > 0)$ |
| $\sin x$ | $-\cos x$ | $\sinh x$ | $\cosh x$ |
| $\cos x$ | $\sin x$ | $\cosh x$ | $\sinh x$ |
| $\tan x$ | $-\ln\|\cos x\|$ | $\tanh x$ | $\ln\cosh x$ |
| $\operatorname{cosec} x$ | $\ln\left\|\tan\frac{x}{2}\right\|$ | $\operatorname{cosech} x$ | $\ln\left\|\tanh\frac{x}{2}\right\|$ |
| $\sec x$ | $\ln\|\sec x + \tan x\|$ | $\operatorname{sech} x$ | $2\tan^{-1} e^x$ |
| $\sec^2 x$ | $\tan x$ | $\operatorname{sech}^2 x$ | $\tanh x$ |
| $\cot x$ | $\ln\|\sin x\|$ | $\coth x$ | $\ln\|\sinh x\|$ |
| $\sin^2 x$ | $\frac{x}{2} - \frac{\sin 2x}{4}$ | $\sinh^2 x$ | $\frac{\sinh 2x}{4} - \frac{x}{2}$ |
| $\cos^2 x$ | $\frac{x}{2} + \frac{\sin 2x}{4}$ | $\cosh^2 x$ | $\frac{\sinh 2x}{4} + \frac{x}{2}$ |

| $f(x)$ | $\int f(x)\,dx$ | $f(x)$ | $\int f(x)\,dx$ |
|---|---|---|---|
| $\frac{1}{a^2+x^2}$ | $\frac{1}{a}\tan^{-1}\frac{x}{a}$ | $\frac{1}{a^2-x^2}$ | $\frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| \quad (0<\|x\|<a)$ |
| | $(a > 0)$ | $\frac{1}{x^2-a^2}$ | $\frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| \quad (\|x\| > a>0)$ |
| $\frac{1}{\sqrt{a^2-x^2}}$ | $\sin^{-1}\frac{x}{a}$ | $\frac{1}{\sqrt{a^2+x^2}}$ | $\ln\left\|\frac{x+\sqrt{a^2+x^2}}{a}\right\| \quad (a > 0)$ |
| | $(-a < x < a)$ | $\frac{1}{\sqrt{x^2-a^2}}$ | $\ln\left\|\frac{x+\sqrt{x^2-a^2}}{a}\right\| \quad (x>a>0)$ |
| $\sqrt{a^2-x^2}$ | $\frac{a^2}{2}\left[\sin^{-1}\left(\frac{x}{a}\right) + \frac{x\sqrt{a^2-x^2}}{a^2}\right]$ | $\sqrt{a^2+x^2}$ | $\frac{a^2}{2}\left[\sinh^{-1}\left(\frac{x}{a}\right) + \frac{x\sqrt{a^2+x^2}}{a^2}\right]$ |
| | | $\sqrt{x^2-a^2}$ | $\frac{a^2}{2}\left[-\cosh^{-1}\left(\frac{x}{a}\right) + \frac{x\sqrt{x^2-a^2}}{a^2}\right]$ |

Some series Expansions -

$$\frac{\pi}{2} = \left(\frac{2}{1}\frac{2}{3}\right)\left(\frac{4}{3}\frac{4}{5}\right)\left(\frac{6}{5}\frac{6}{7}\right)\left(\frac{8}{7}\frac{8}{9}\right)\ldots$$

$$\pi = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \frac{4}{13} - \cdots$$

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$$

$$\pi = \sqrt{12}\left(1 - \frac{1}{3\cdot 3} + \frac{1}{5\cdot 3^2} - \frac{1}{7\cdot 3^3} + \cdots\right)$$

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots = \sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\int_0^{\pi/2} \log \sin x \, dx = -\frac{\pi}{2}\log 2 = \frac{\pi}{2}\log\frac{1}{2}$$

**Solve a series problem**

If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$ upto $\infty = \frac{\pi^2}{6}$, then value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ up to $\infty$ is

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{6}$ (c) $\frac{\pi^2}{8}$ (d) $\frac{\pi^2}{12}$

*Ans.* (c)

**Solution** We have $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ upto $\infty$

$$= \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} \cdots \text{ upto } \infty$$

$$- \frac{1}{2^2}\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right]$$

$$= \frac{\pi^2}{6} - \frac{1}{4}\left(\frac{\pi^2}{6}\right) = \frac{\pi^2}{8}$$

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \ldots \infty = \frac{\pi^2}{12}$$

$$\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \ldots \infty = \frac{\pi^2}{24}$$

$$\frac{\sin\sqrt{x}}{\sqrt{x}} = 1 - \frac{x}{3!} + \frac{x^2}{5!} - \frac{x^3}{7!} + \frac{x^4}{9!} - \frac{x^5}{11!} + \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = \sum_{k=0}^{n} \frac{(-1)^k x^{2k}}{(2k)!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots = \sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots = \sum_{k=0}^{n} \frac{x^{2k}}{(2k)!}$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots = \sum_{k=0}^{n} \frac{x^{2k+1}}{(2k+1)!}$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad (-1 \le x < 1)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} \cdots + \frac{2^{2n}\left(2^{2n}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \qquad |x| < \frac{\pi}{2}$$

$$\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \cdots + \frac{E_n x^{2n}}{(2n)!} + \cdots \quad |x| < \frac{\pi}{2}$$

$$\csc x = \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^5}{15120} + \cdots + \frac{2\left(2^{2n-1}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \qquad 0 < |x| < \pi$$

$$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - - \cdots - \frac{2^{2n} B_n x^{2n-1}}{(2n)!} - \cdots \quad 0 < |x| < \pi$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$$

$$\sec x = 1 + \frac{x^2}{\underline{|2}} + \frac{5x^4}{\underline{|4}} + \cdots$$

$$\log(\cos x) = -\frac{x^2}{\underline{|2}} - \frac{2x^4}{\underline{|4}} - \cdots$$

$$\log(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \cdots$$

$$\sin^{-1} x = x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \cdots \quad |x| < 1$$

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$= \frac{\pi}{2} - \left(x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \cdots\right) \quad |x| < 1$$

$$\tan^{-1} x = \begin{cases} x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots & |x| < 1 \\ \pm\frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \cdots & \begin{cases} + \text{ if } x \geq 1 \\ - \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$$

$$= \frac{\pi}{2} - \left(\frac{1}{x} + \frac{1}{2\cdot 3x^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5x^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \cdots\right) \quad |x| > 1$$

$$\csc^{-1} x = \sin^{-1}(1/x)$$

$$= \frac{1}{x} + \frac{1}{2\cdot 3x^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5x^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \cdots \quad |x| > 1$$

$$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$$

$$= \begin{cases} \frac{\pi}{2} - \left(x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots\right) & |x| < 1 \\ p\pi + \frac{1}{x} - \frac{1}{3x^3} + \frac{1}{5x^5} + \cdots & \begin{cases} p = 0 \text{ if } x \geq 1 \\ p = 1 \text{ if } x \leq -1 \end{cases} \end{cases}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\ln x = 2\left[\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \ldots\right]$$

$$= 2\sum_{n=1}^{\infty} \frac{1}{2n-1}\left(\frac{x-1}{x+1}\right)^{2n-1} \quad (x > 0)$$

$$\ln x = \frac{x-1}{x} + \frac{1}{2}\left(\frac{x-1}{x}\right)^2 + \frac{1}{3}\left(\frac{x-1}{x}\right)^3 + \ldots$$

$$= \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{x-1}{x}\right)^n \quad (x > \frac{1}{2})$$

$$\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}(x-1)^n \quad (0 < x \le 2)$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} x^n \quad (|x| < 1)$$

$$\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots \infty \, (-1 \le x < 1)$$

$$\log_e(1+x) - \log_e(1-x) =$$

$$\log_e \frac{1+x}{1-x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots \infty\right) (-1 < x < 1)$$

$$\log_e\left(1+\frac{1}{n}\right) = \log_e \frac{n+1}{n} = 2 \left[\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \ldots \infty\right]$$

$$\log_e (1 + x) + \log_e (1 - x) = \log_e(1-x^2) = -2\left(\frac{x^2}{2} + \frac{x^4}{4} + \ldots \infty\right)(-1 < x < 1)$$

$$\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots \quad = \frac{1}{1 \cdot 2} + \frac{1}{3.4} + \frac{1}{5.6} + \ldots$$

# Important Results

**(i)** (a) $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cos^n x}{\sin^n x + \cos^n x} dx$

(b) $\int_0^{\pi/2} \frac{\tan^n x}{1 + \tan^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{dx}{1 + \tan^n x}$

(c) $\int_0^{\pi/2} \frac{dx}{1 + \cot^n x} = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{1 + \cot^n x} dx$

(d) $\int_0^{\pi/2} \frac{\tan^n x}{\tan^n x + \cot^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{\tan^n x + \cot^n x} dx$

(e) $\int_0^{\pi/2} \frac{\sec^n x}{\sec^n x + \operatorname{cosec}^n x} dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\operatorname{cosec}^n x}{\sec^n x + \operatorname{cosec}^n x} dx$ where, $n \in R$

**(ii)** $\int_0^{\pi/2} \frac{a^{\sin^n x}}{a^{\sin^n x} + a^{\cos^n x}} dx = \int_0^{\pi/2} \frac{a^{\cos^n x}}{a^{\sin^n x} + a^{\cos^n x}} dx = \frac{\pi}{4}$

**(iii)** (a) $\int_0^{\pi/2} \log \sin x \, dx = \int_0^{\pi/2} \log \cos x \, dx = -\frac{\pi}{2} \log 2$

(b) $\int_0^{\pi/2} \log \tan x \, dx = \int_0^{\pi/2} \log \cot x \, dx = 0$

(c) $\int_0^{\pi/2} \log \sec x \, dx = \int_0^{\pi/2} \log \operatorname{cosec} x \, dx = \frac{\pi}{2} \log 2$

**(iv)** (a) $\int_0^{\infty} e^{-ax} \sin bx \, dx = \frac{b}{a^2 + b^2}$

(b) $\int_0^{\infty} e^{-ax} \cos bx \, dx = \frac{a}{a^2 + b^2}$

(c) $\int_0^{\infty} e^{-ax} x^n \, dx = \frac{n!}{a^n + 1}$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left(x + \sqrt{x^2 - a^2}\right) + C$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left(\frac{x-a}{x+a}\right) + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left(\frac{a+x}{a-x}\right) + C$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{a^2 + x^2}\, dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right) + C$$

( In 2016 Celebrating 27 years of Excellence in Teaching )

Good Luck to you for your Preparations, References, and Exams

All Other Books written by me can be downloaded from

https://zookeepersblog.wordpress.com/free-pdf-e-book-download-for-iit-jee-cbse-isc-cet-physics-chemistry-maths-from-professor-subhashish-skmclasses/

Professor Subhashish Chattopadhyay

Learn more at http://skmclasses.weebly.com/iit-jee-home-tuitions-bangalore.html

Twitter - https://twitter.com/ZookeeperPhy

Facebook - https://www.facebook.com/IIT.JEE.by.Prof.Subhashish/

Blog - http://skmclasses.kinja.com